Polarization Effects in Electroweak PDFs at Very High Energies

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Work in collaboration with Christian Bauer and Nicolas Ferland, LBNL

1703.08562 = JHEP 08(2017)036, 1808.08831

Motivation

- Electroweak corrections becoming essential
  - Fixed order adequate at present energies
  - Enhanced higher orders important for FCC
- SM may be valid up to much higher energies
  - Implications for cosmology and astrophysics
- Need full simulations of VHE interactions: parton shower event generators for full SM
  - First step: event generators need PDFs

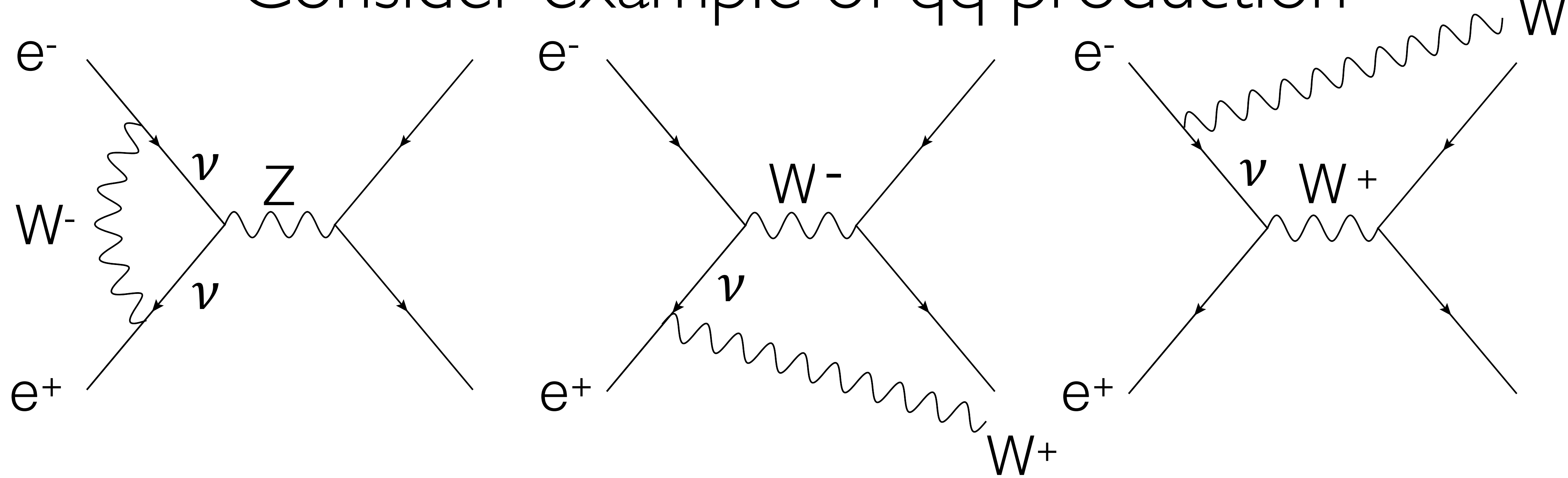
Outline

- Electroweak effects at high energies
  - Non-cancelling large logarithms
  - Sudakov factors
- SM parton distributions
  - DGLAP evolution: double log, LL and NLL
  - L-R and isospin asymmetries
- Polarization of gauge bosons
  - Matching to below EW scale
- Conclusions and prospects

Electroweak Effects at High Energies

Electroweak effects: e^+e^-

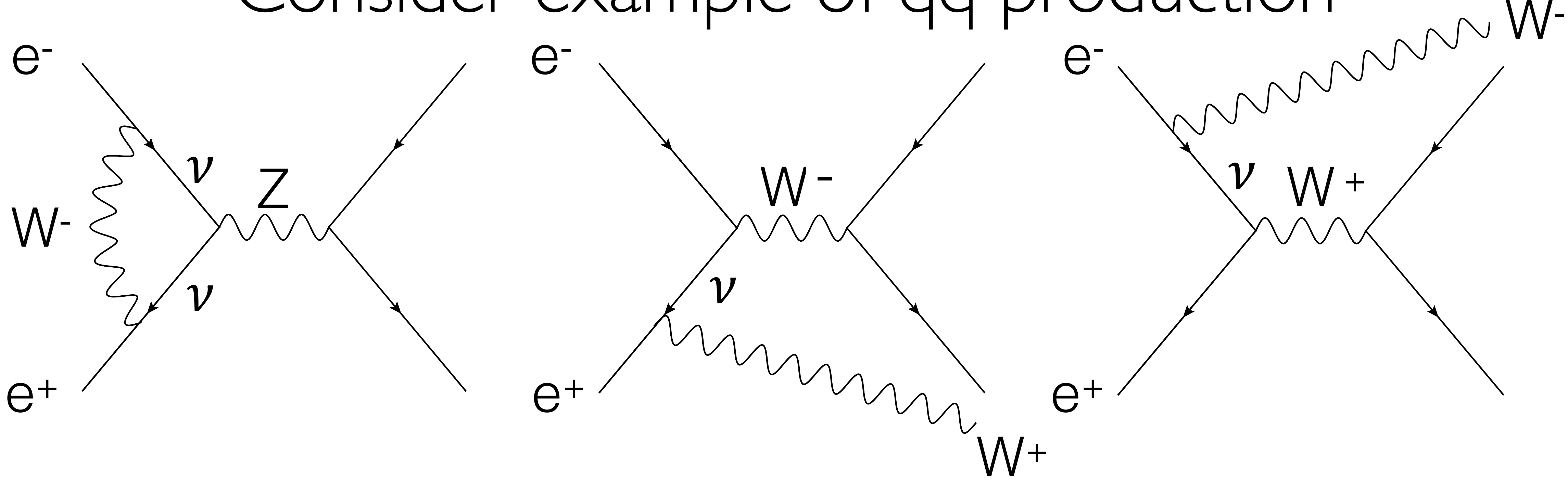
Consider example of $q\bar{q}$ production



- For massless bosons, IR divergences in each graph, cancel in inclusive sum over SU(2) multiplets
- For massive bosons, divergences become $\log(m_W^2/s)$, generally two per power of α_W

Electroweak effects: e^+e^-

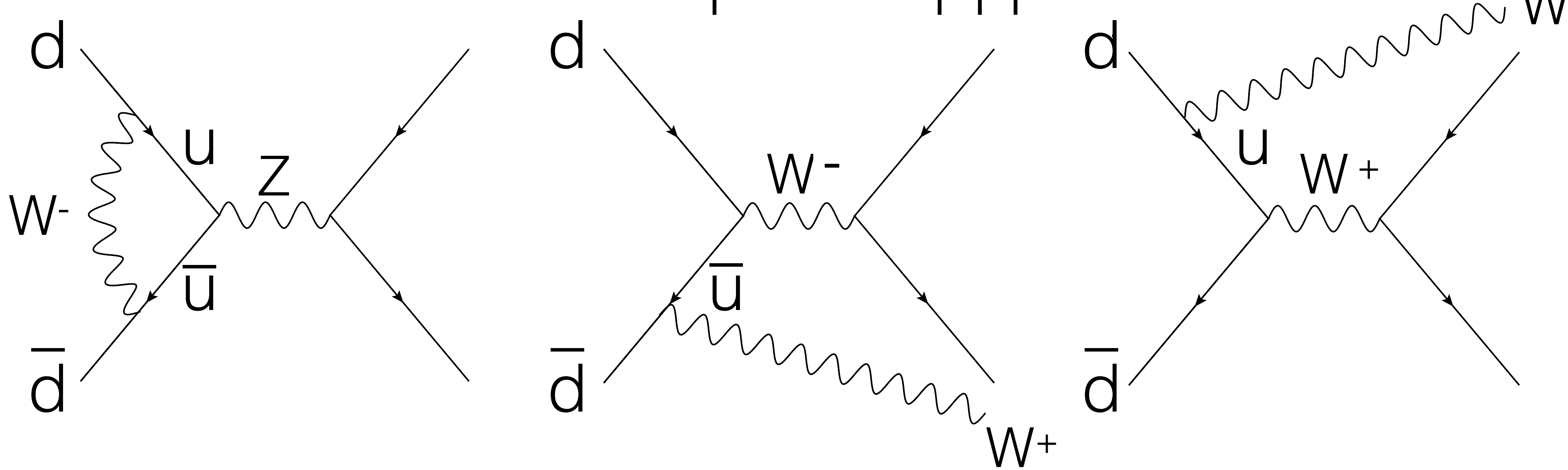
Consider example of $q\bar{q}$ production



- $\alpha_W \log^2(m_W^2/s)$ from each graph, cancel in inclusive sum over SU(2) multiplets
- But we don't have νν or eν colliders, so cancellation is incomplete

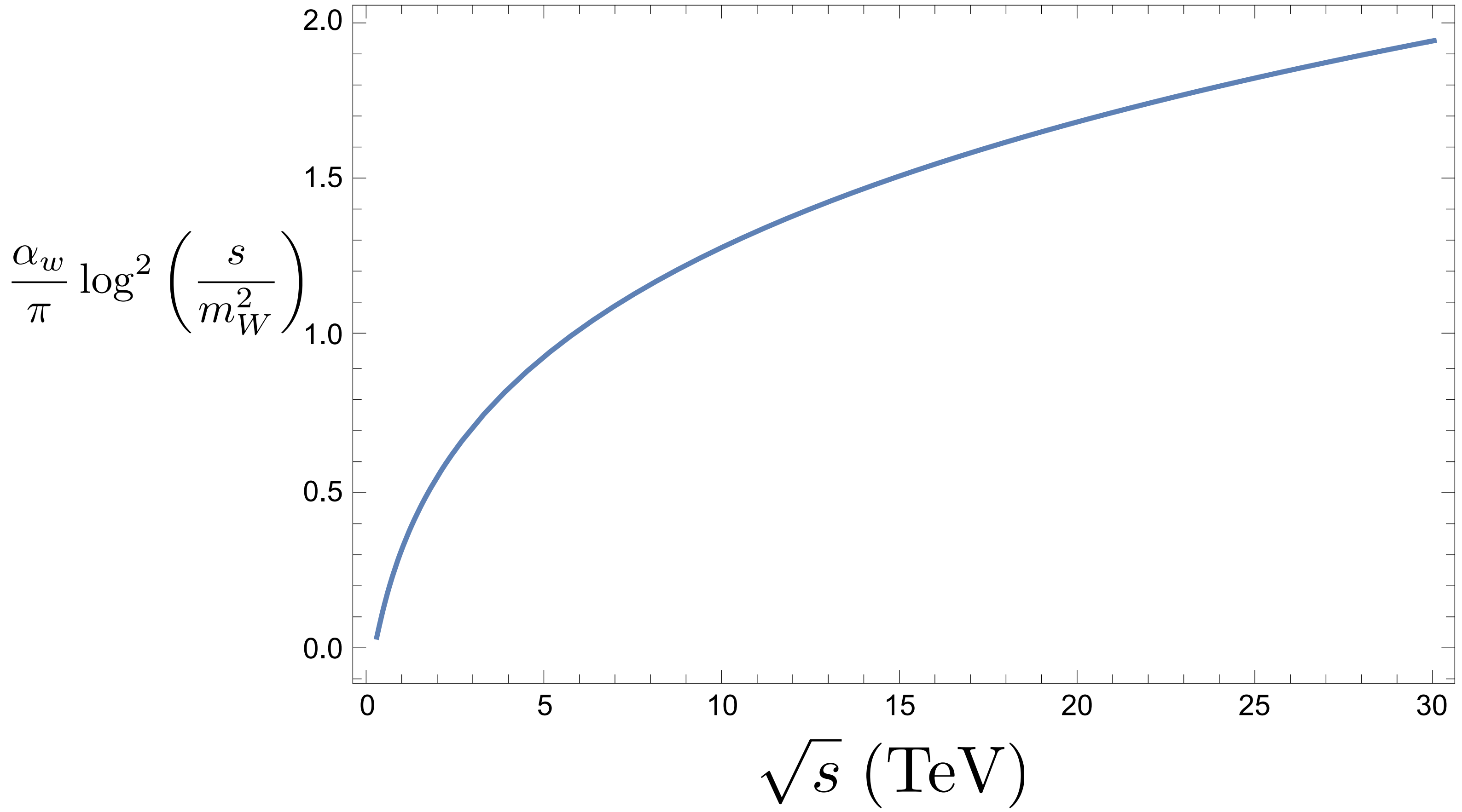
Electroweak effects: $q\bar{q}$

Consider example of $q\bar{q}$ production



- $\alpha_W \log^2(m_W^2/s)$ from each graph, cancel in inclusive sum over SU(2) multiplets
- In pp, u-quark PDF ≠ d-quark PDF, so cancellation is incomplete

Electroweak logarithms

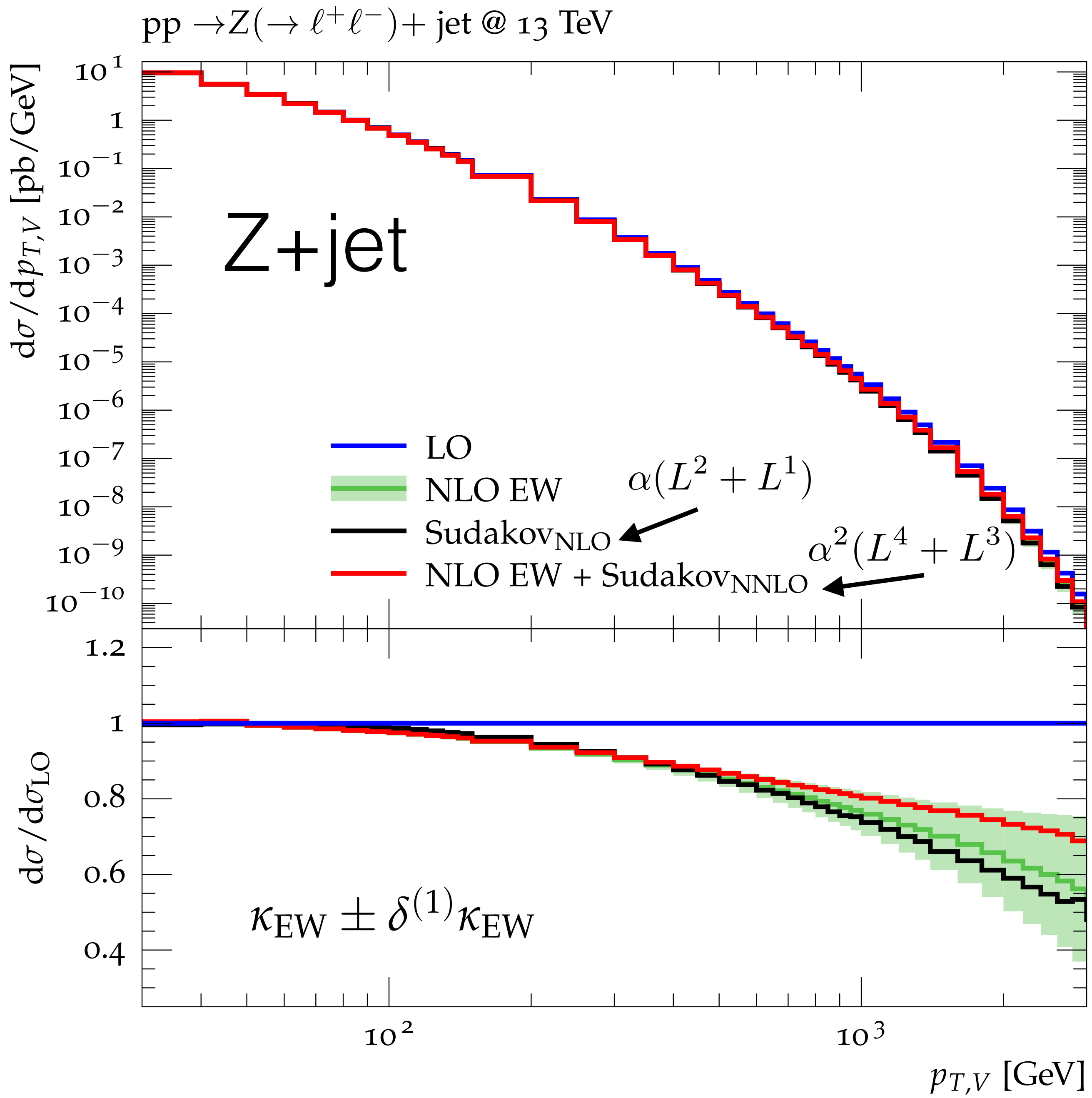


- Electroweak logs get large at high energy
- Virtual corrections exponentiate as Sudakov factor

$$\Delta_i(s) \sim \exp\left[-C_i \frac{\alpha_w}{\pi} \log^2\left(\frac{s}{m_W^2}\right)\right]$$

Electroweak effects in Z+jet

J Lindert, PSR2017



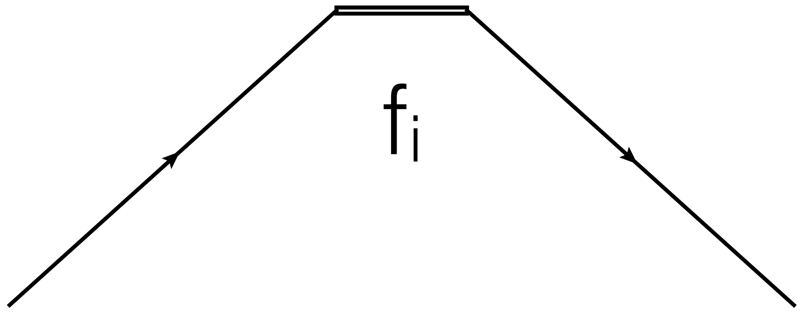
- ~25% at p_T~1 TeV

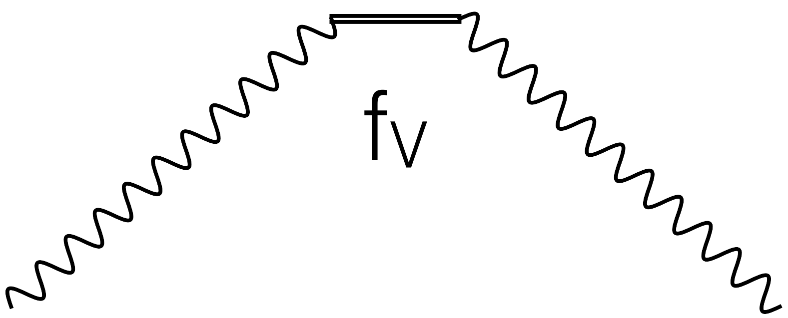
Parton Distribution Functions

PDFs as bilocal operator MEs

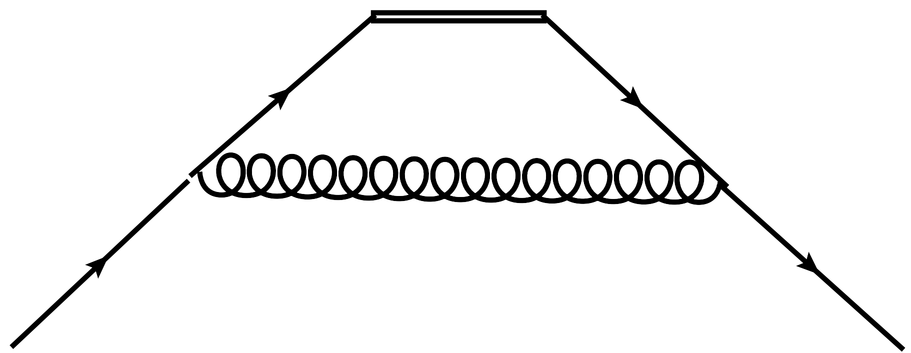
$$f_i(x) = x \int \frac{dy}{2\pi} \, e^{-i\,2x\bar{n}\cdot p\,y} \big\langle p \big| \, \bar{\psi}^{(i)}(y) \, \bar{\not{n}} \, \psi^{(i)}(-y) \big| p \big\rangle$$

$$f_V(x) = \frac{2}{\bar{n}\cdot p} \int \frac{dy}{2\pi} \, e^{-i\,2x\bar{n}\cdot p\,y} \, \bar{n}_\mu \bar{n}^\nu \big\langle p \big| \, V^{\mu\lambda}(y) V_{\lambda\nu}(-y) \big| p \big\rangle$$

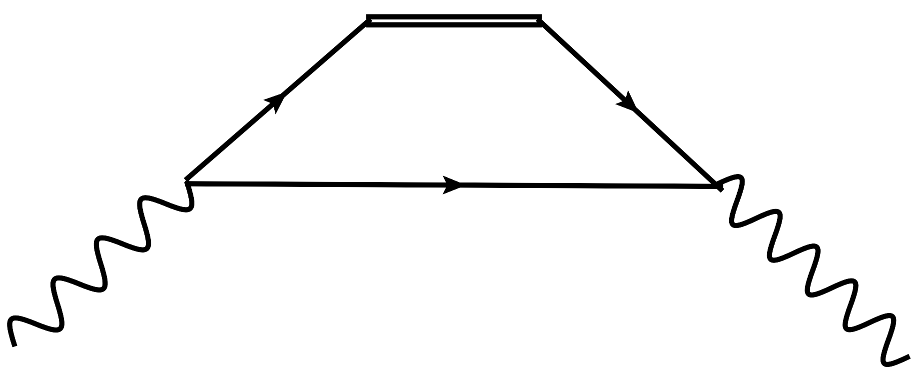




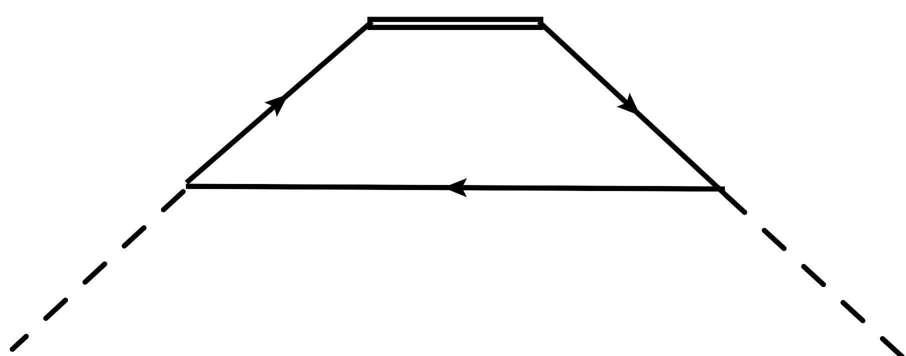
PDF Evolution



$$q\, d/dq\, f = P_{ff} \otimes f$$



$$q\, d/dq\, f = P_{fV} \otimes V$$



$$q\, d/dq\, f = P_{fH} \otimes H$$

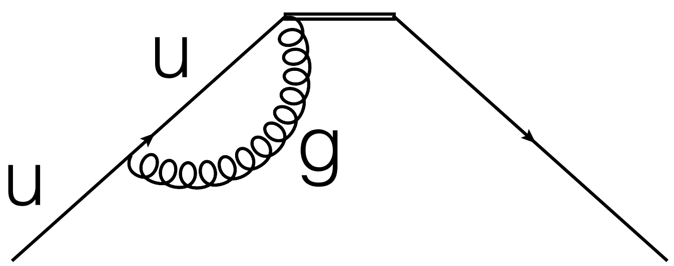
Real and Virtual Contributions

- Reals have loops from one side to the other

- Virtuals have loops on same side

SU(3) Evolution (DGLAP)

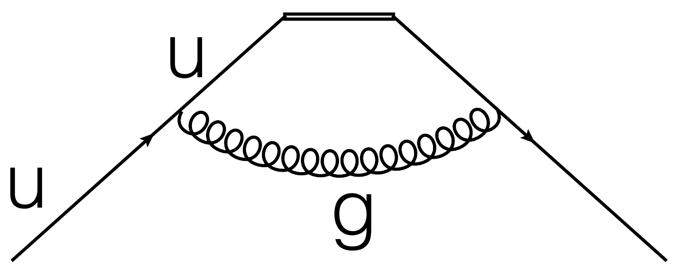
- Consider evolution of u quark PDF



Virtual

$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_3 C_F}{\pi} P_f^V(q) f_u(x,q)$$

$$P_f^V(q) = -\int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z)$$



Real

$\mu \to 0$

$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_3 C_F}{\pi} \int_x^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) f_u(x/z,q)$$

$$P_{ff}(z) = \frac{1+z^2}{1-z}$$

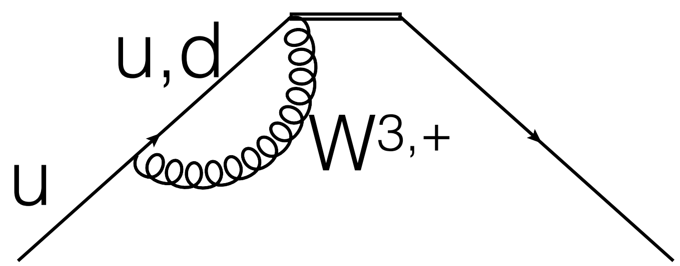
Combination

$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_3 C_F}{\pi} \int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) \left[f_u(x/z,q) - f_u(x)\right]$$

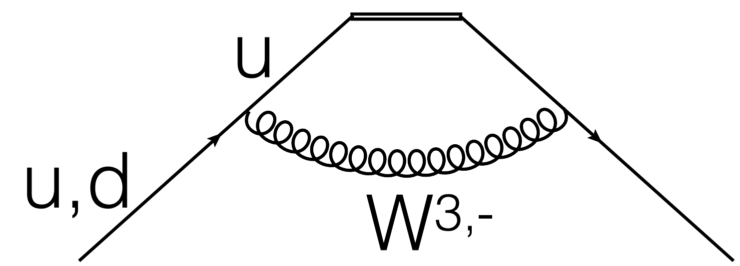
- z=1 singularity cancels ➡ single-log evolution

SU(2) Evolution

- Consider evolution of u_L quark PDF



Virtual



Real

$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_2 C_F}{\pi} P_f^V(q) f_u(x,q)$$

$$P_f^V(q) = -\int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z)$$

$\mu \sim m_W$

$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_2 C_F}{\pi}\int_x^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) \times \left[\frac{1}{3}f_u(x/z,q) + \frac{2}{3}f_d(x/z,q)\right]$$

Combination

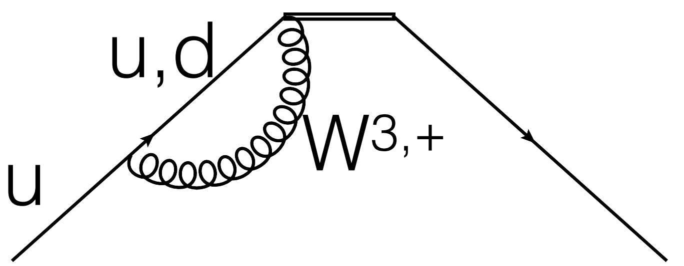
$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_2 C_F}{\pi}\int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z)\left[\frac{1}{3}f_u(x/z,q) + \frac{2}{3}f_d(x/z,q) - f_u(x,q)\right]$$

- z=1 doesn't cancel ➞ double-log evolution

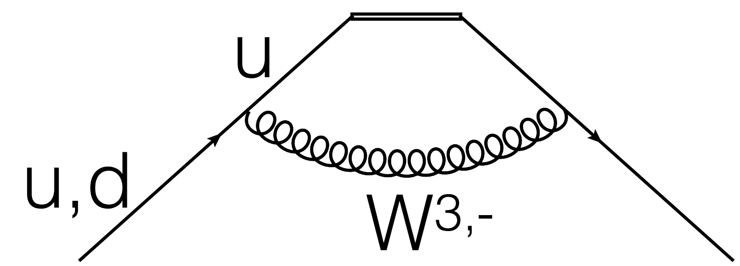
M Ciafaloni, P Ciafaloni, D Comelli, hep-ph/9809321, 0001142, 0111109, 0505047

SU(2) Evolution

- Consider evolution of u_L quark PDF



Virtual



Real

$C_F = 3/4$

$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} P_f^V(q) f_u(x,q)$$

$$P_f^V(q) = -\int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z)$$

$\mu \sim m_W$

$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} \int_x^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) \times \left[\frac{1}{3} f_u(x/z,q) + \frac{2}{3} f_d(x/z,q)\right]$$

Combination

$$\mathcal{L}_{q\bar{q}W} = -\frac{g_2}{2}(\bar{u},\bar{d}) \begin{pmatrix} W^3 & \sqrt{2}\,W^+ \\ \sqrt{2}\,W^- & -W^3 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} \int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) \left[\frac{1}{3} f_u(x/z,q) + \frac{2}{3} f_d(x/z,q) - f_u(x,q)\right]$$

- z=1 doesn't cancel ➡ double-log evolution

M Ciafaloni, P Ciafaloni, D Comelli, hep-ph/9809321, 0001142, 0111109, 0505047

General Evolution

$$q\frac{\partial}{\partial q}f_i(x,q) = \sum_I \frac{\alpha_I(q)}{\pi}\left[P^V_{i,I}(q)\, f_i(x,q) + \sum_j C_{ij,I}\int_x^{z^{ij,I}_{\rm max}(q)} {\rm d}z\, P^R_{ij,I}(z) f_j(x/z,q)\right]$$

$$(I = 1,2,3,\ldots)$$

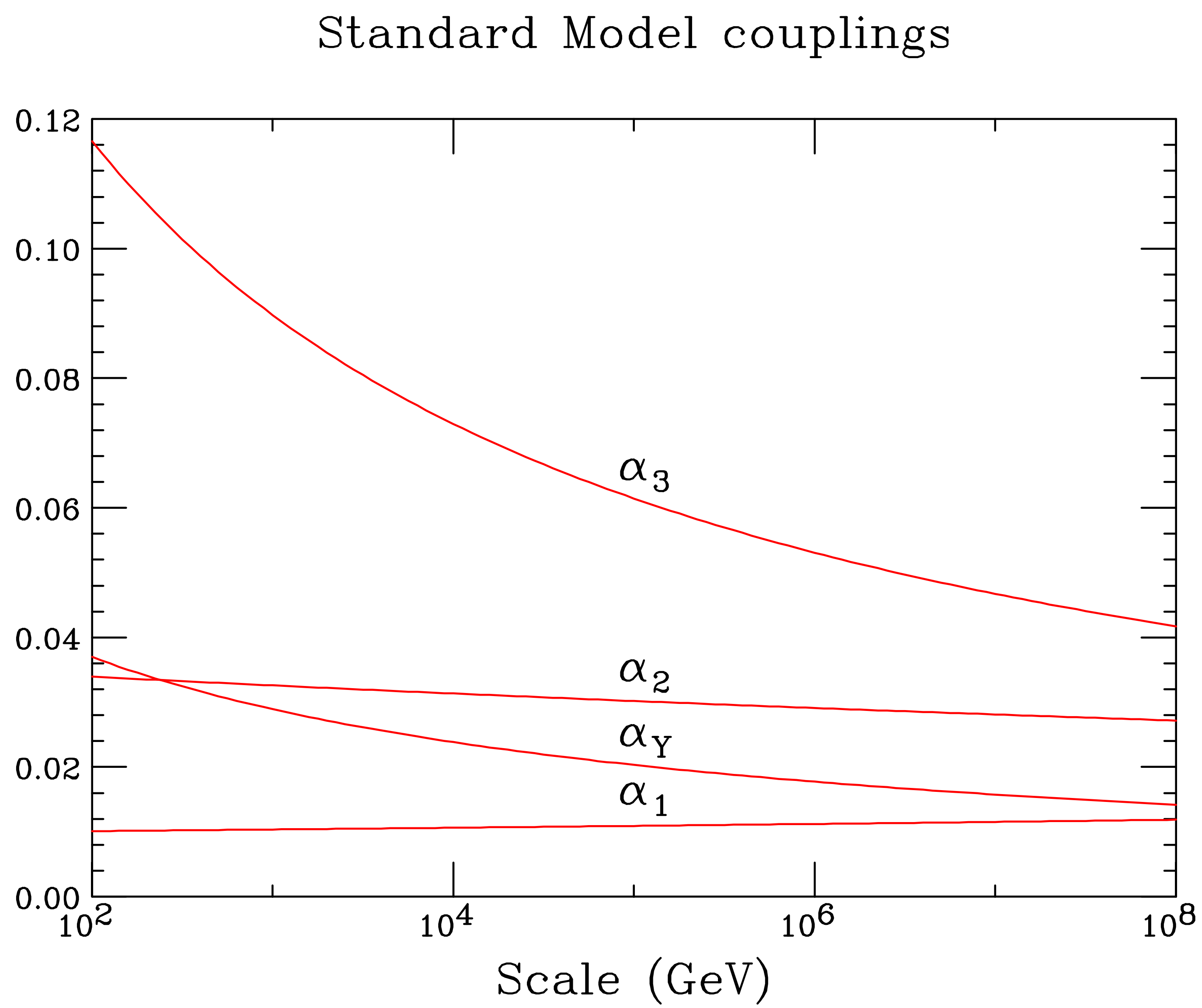
- $z^{ij,I}_{\rm max} = 1-\mu/q$ **with** $\mu = \mathcal{O}(m_V)$ **for** $j\to iV$
- Introduce Sudakov factor

$$\Delta_{i,I}(q) = \exp\left[\int_{q_0}^{q}\frac{{\rm d}q'}{q'}\frac{\alpha_I(q')}{\pi}P^V_{i,I}(q')\right]$$

- Then

$$\Delta_i(q)\, q\frac{\partial}{\partial q}\left[\frac{f_i(x,q)}{\Delta_i(q)}\right] = \sum_I\frac{\alpha_I(q)}{\pi}\sum_j C_{ij,I}P^R_{ij,I}\otimes f_j$$

Couplings



- Far above EW scale ~ unbroken SU(3)xSU(2)xU(1)

Polarized Splitting Functions

$$
\begin{aligned}
P^R_{f_L f_L,G}(z) &= P^R_{f_R f_R,G}(z) = \frac{2}{1-z} - (1+z)\,,\\
P^R_{V_+ f_L,G}(z) &= P^R_{V_- f_R,G}(z) = \frac{(1-z)^2}{z}\,,\\
P^R_{V_- f_L,G}(z) &= P^R_{V_+ f_R,G}(z) = \frac{1}{z}\,,\\
P^R_{f_L V_+,G}(z) &= P^R_{f_R V_-,G}(z) = \frac{1}{2}(1-z)^2\,,\\
P^R_{f_L V_-,G}(z) &= P^R_{f_R V_+,G}(z) = \frac{1}{2}z^2\,,\\
P^R_{V_+ V_+,G}(z) &= P^R_{V_- V_-,G}(z) = \frac{2}{1-z} + \frac{1}{z} - 1 - z(1+z)\,,\\
P^R_{V_+ V_-,G}(z) &= P^R_{V_- V_+,G}(z) = \frac{(1-z)^3}{z}\,,
\end{aligned}
$$

- L and R fermions evolve differently
  - Gauge bosons get polarized

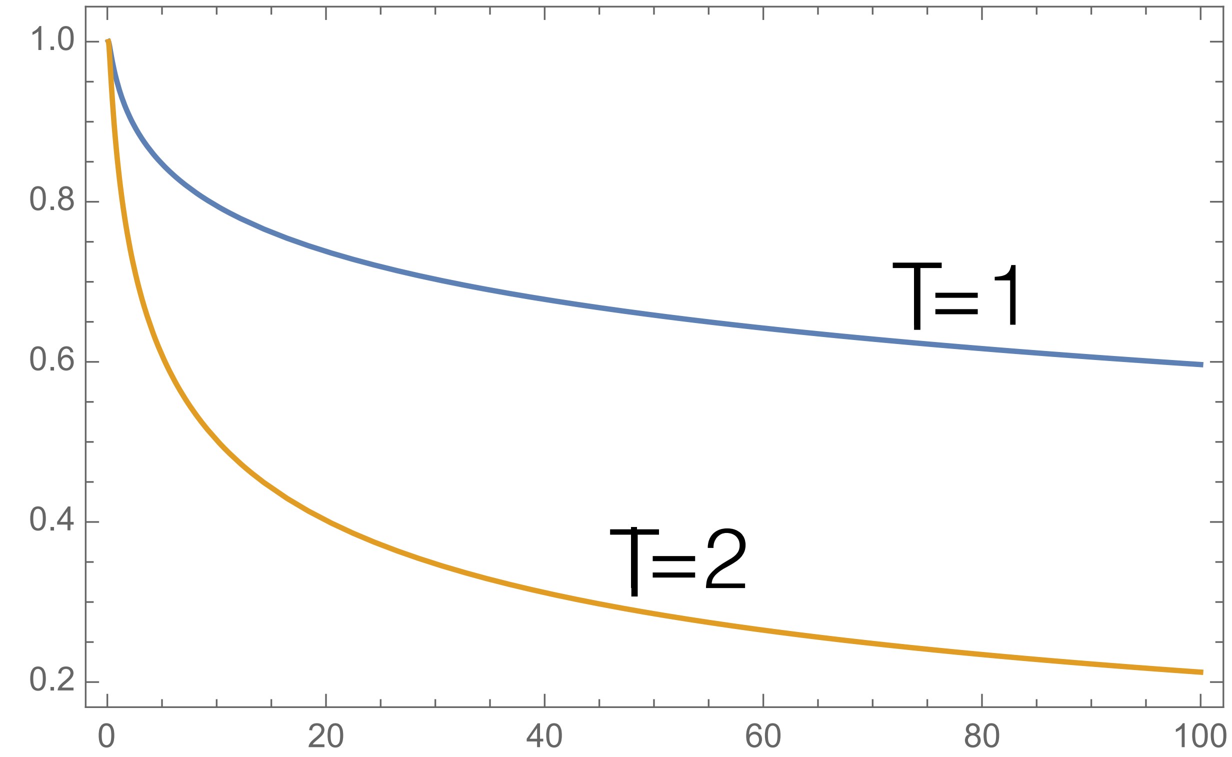
Altarelli & Parisi, NP B126 (1977) 298
Manohar & Waalewijn, arXiv:1802.08687

Isospin (T) + CP PDFs

$$f_{f_L}^{0\pm} = \frac{1}{4}\left[(f_{u_L} + f_{d_L}) \pm (f_{\bar{u}_L} + f_{\bar{d}_L})\right],$$
$$f_{f_L}^{1\pm} = \frac{1}{4}\left[(f_{u_L} - f_{d_L}) \pm (f_{\bar{u}_L} - f_{\bar{d}_L})\right],$$
$$f_W^{0\pm} = \frac{1}{3}\left[\left(f_{W_+^+} + f_{W_+^-} + f_{W_+^3}\right) \pm \left(f_{W_-^+} + f_{W_-^-} + f_{W_-^3}\right)\right],$$
$$f_W^{1\pm} = \frac{1}{2}\left[\left(f_{W_+^+} - f_{W_+^-}\right) \mp \left(f_{W_-^+} - f_{W_-^-}\right)\right],$$
$$f_W^{2\pm} = \frac{1}{6}\left[\left(f_{W_+^+} + f_{W_+^-} - 2f_{W_+^3}\right) \pm \left(f_{W_-^+} + f_{W_-^-} - 2f_{W_-^3}\right)\right].$$

- Double logs only appear in T≠0 PDFs

$$f_i^{T\pm}(x,q) \sim \exp\left[-\frac{T(T+1)}{2}\frac{\alpha_2}{\pi}\log^2\left(\frac{q}{m_W}\right)\right]$$
$$\sim \left[\Delta_{i,2}(q)\right]^{T(T+1)/2C_i}$$



Next-to-leading log accuracy

$$\Delta_2(q) = \exp\left[Lg_1(\alpha_2 L) + g_2(\alpha_2 L) + \ldots\right] \sim \exp\left[-C\alpha_2 L^2\right]$$

where $\alpha_2 = \alpha_2(q)\,, \quad L = \log(q/m_W)$

- Can get the whole of g_1 and g_2 by the following change in evolution equations:
  - Replace $\alpha_2(q) \to \alpha_2\left(k_{\mathrm{CKW}}(1-z)q\right)$

    where $k_{\mathrm{CMW}} = \exp\left(-\frac{1}{\beta_0^{(2)}}\frac{\Gamma^{(2)}_{\mathrm{cusp,f}}}{\Gamma^{(1)}_{\mathrm{cusp,f}}}\right) = \exp\left(\frac{6\pi^2-70}{57}\right) = 0.828\,.$

- Effect is small as long as $\alpha_2 L \ll 1$

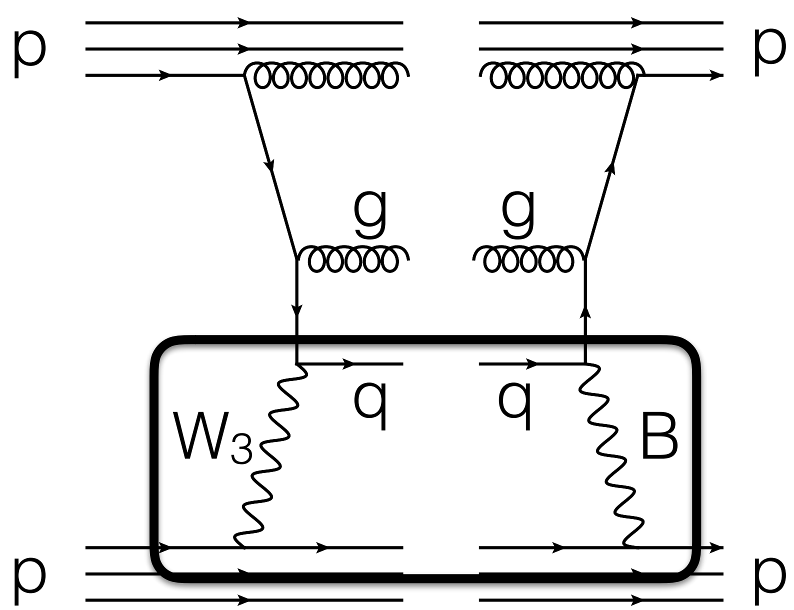
Counting PDFs

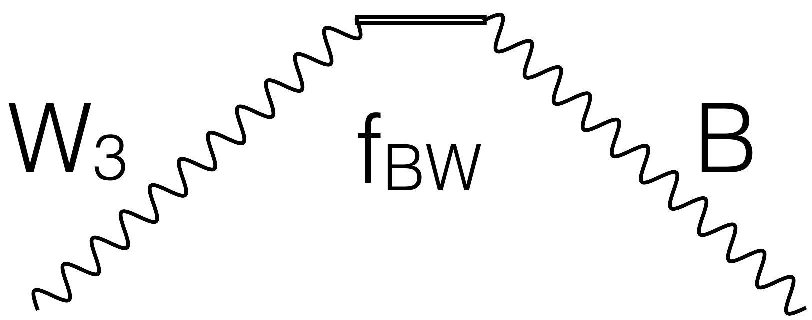
$\{\mathbf{T}, \mathrm{CP}\}$	fields	
$\{0, \pm\}$	$2n_g \times q_R\,, n_g \times \ell_R\,, n_g \times q_L\,, n_g \times \ell_L\,, g\,, W\,, B\,, H$	38
$\{1, \pm\}$	$n_g \times q_L\,, n_g \times \ell_L\,, W\,, BW, H\,, HH$	20
$\{2, \pm\}$	W	2
		60

- 60 SM PDFs for unpolarised proton (48 distinct)
- Only those with same {T,CP} can mix
- Only {0,+} contribute to momentum
- Momentum conserved for each interaction

Mixed U(1)xSU(2) PDF

$$f_{BW}(x) = \frac{2}{\bar{n}\cdot p}\int\frac{dy}{2\pi}\, e^{-i\,2x\bar{n}\cdot p\,y}\,\bar{n}^{\mu}\bar{n}_{\nu}\langle p|\,B_{\mu\lambda}(y)W_3^{\lambda\nu}(-y)\big|p\rangle\ \text{+h.c.}$$

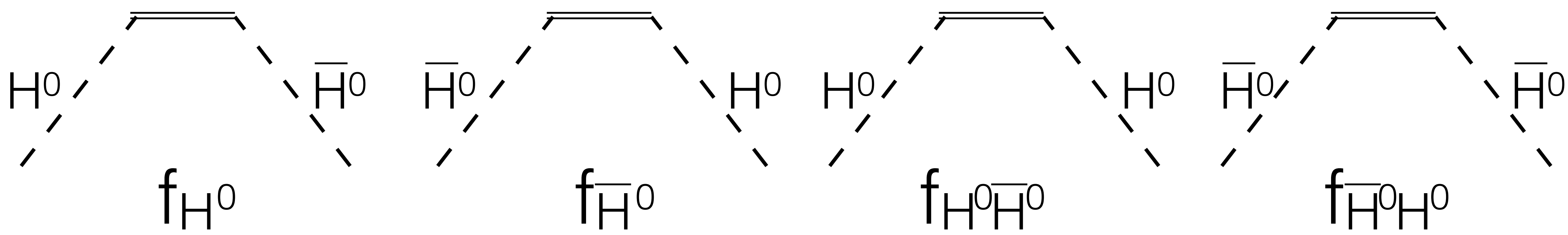




$$\alpha_M = \sqrt{\alpha_1\alpha_2}$$

- Left-handed quarks have isospin and hypercharge, so they can generate f_{BW}
- This means in broken basis we have f_γ, f_Z and $f_{\gamma Z}$

Mixed Higgs PDF



$$H^0 = \frac{1}{\sqrt{2}}(h - iZ_L)\,, \quad \bar{H}^0 = \frac{1}{\sqrt{2}}(h + iZ_L)$$

$$h = \frac{1}{\sqrt{2}}\left(H^0 + \bar{H}^0\right)\,, \quad Z_L = \frac{i}{\sqrt{2}}\left(H^0 - \bar{H}^0\right)$$

$$f^{1\pm}_{HH} = \frac{1}{2}\left(f_{H^0\bar{H}^0} \pm f_{\bar{H}^0H^0}\right)$$

- f^{1+}_{HH} distinguishes between Higgs and Z_L

$$f_{Z_L} = f^{0+}_{H} - f^{1+}_{H} - f^{1+}_{HH}\,,$$
$$f_h = f^{0+}_{H} - f^{1+}_{H} + f^{1+}_{HH}\,.$$

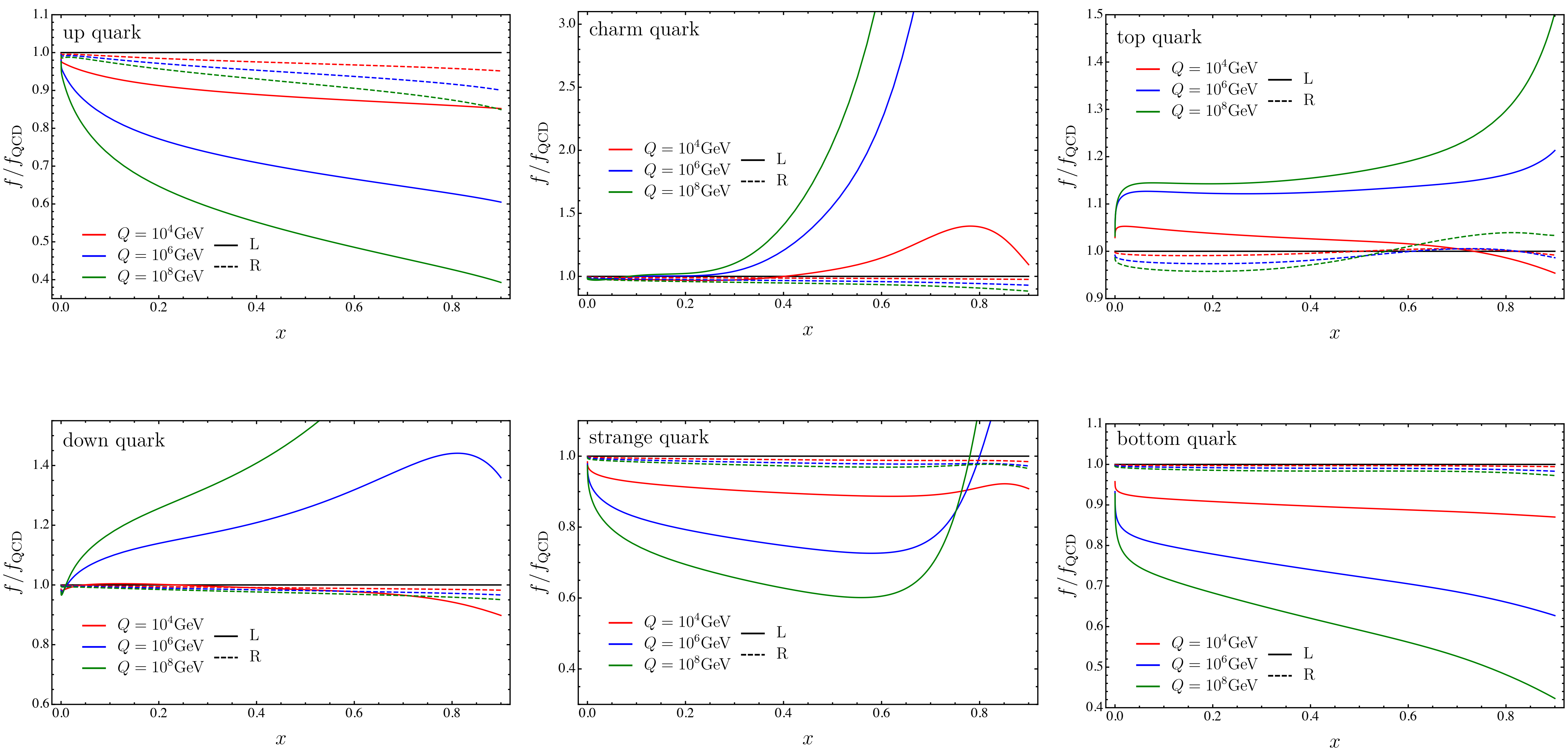
Matching at 100 GeV

$$\begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & c_W s_W \\ s_W^2 & c_W^2 & -c_W s_W \\ -2c_W s_W & 2c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_B \\ f_{W_3} \\ f_{BW} \end{pmatrix}$$

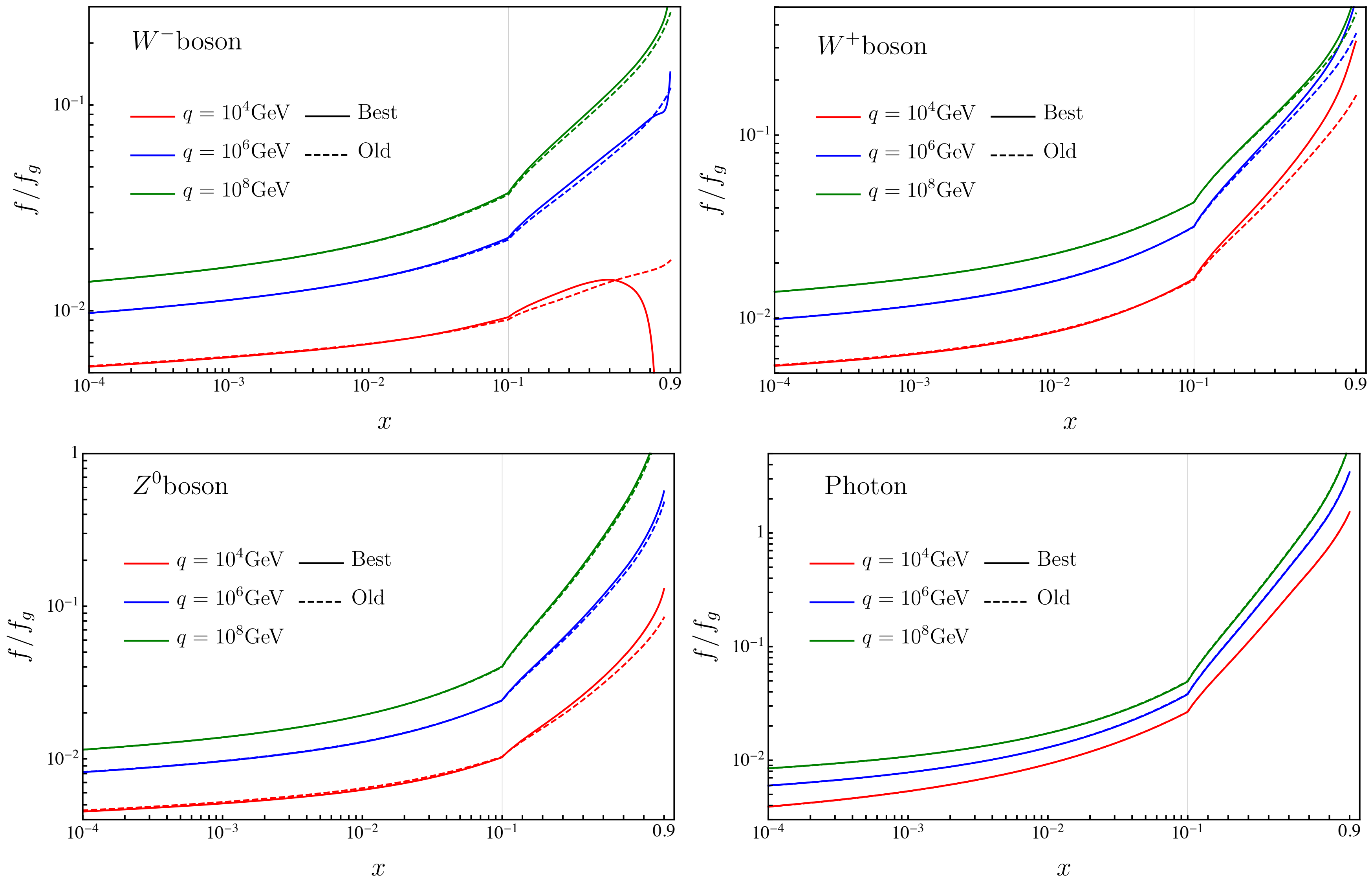
- At q=100 GeV, match to:
  - CT14 QCD partons Schmidt, Pumplin, Stump, Yuan, 1509.02905
  - LUX photon Manohar, Nason, Salam, Zanderighi, 1708.01256
  - FMW Z^0 & $W^\pm$ Fornal, Manohar, Waalewijn, 1803.06347
- Project back on f_γ, f_Z and $f_{\gamma Z}$ at higher scales
- f_h=0 at q≤100 GeV, f_t=0 at q≤$m_t(m_t)$=163 GeV

Results

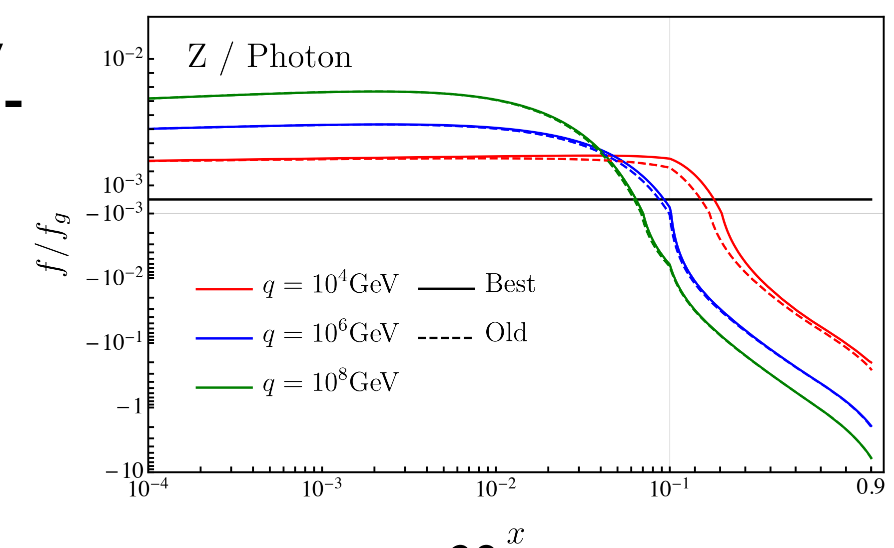
Quarks relative to QCD



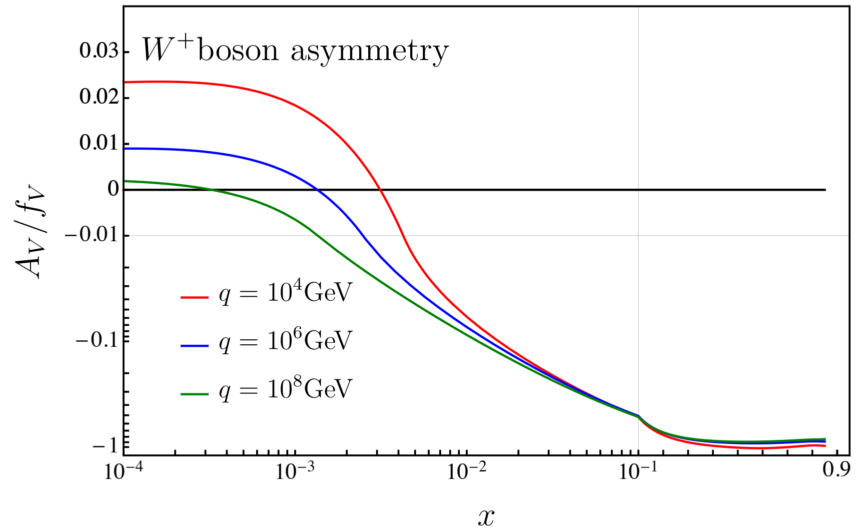
EW bosons relative to gluon

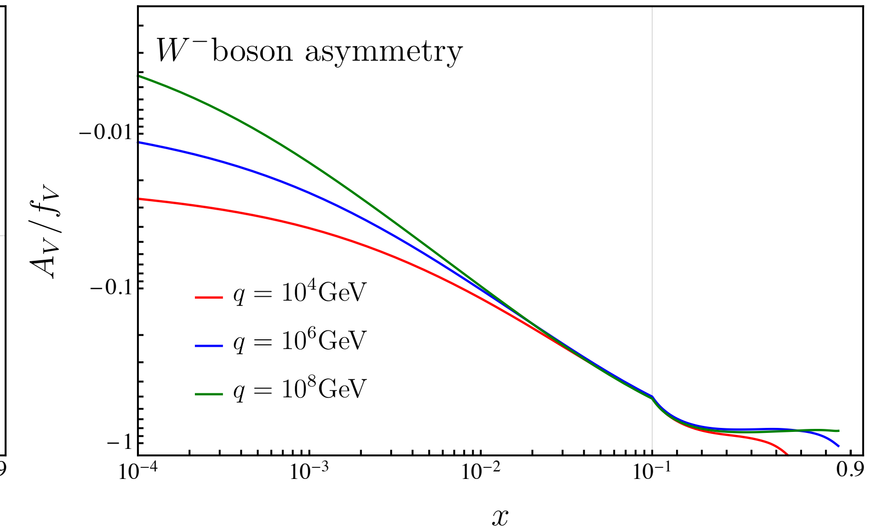


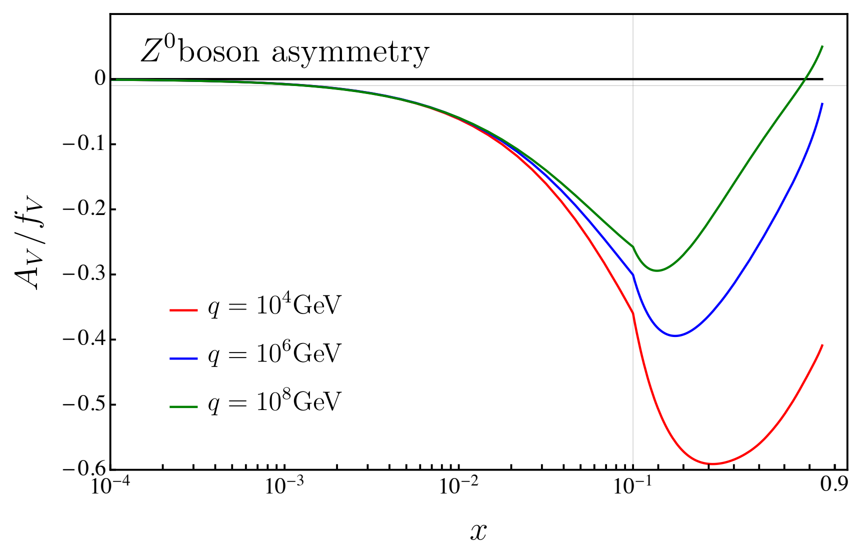
- Transverse, $V_+ + V_-$

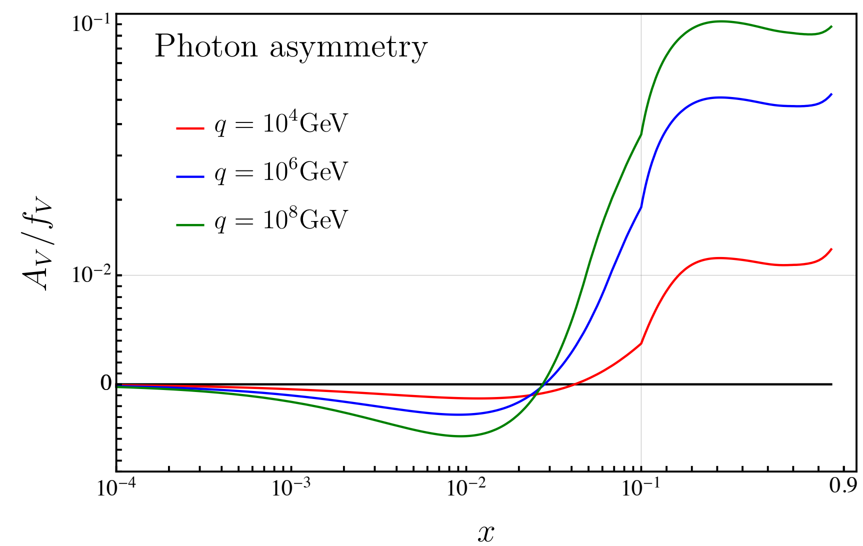


EW boson polarization

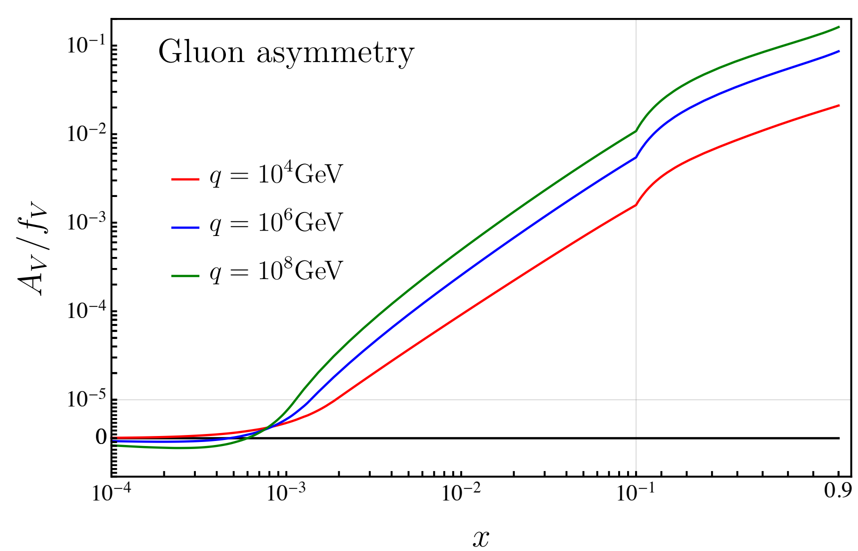




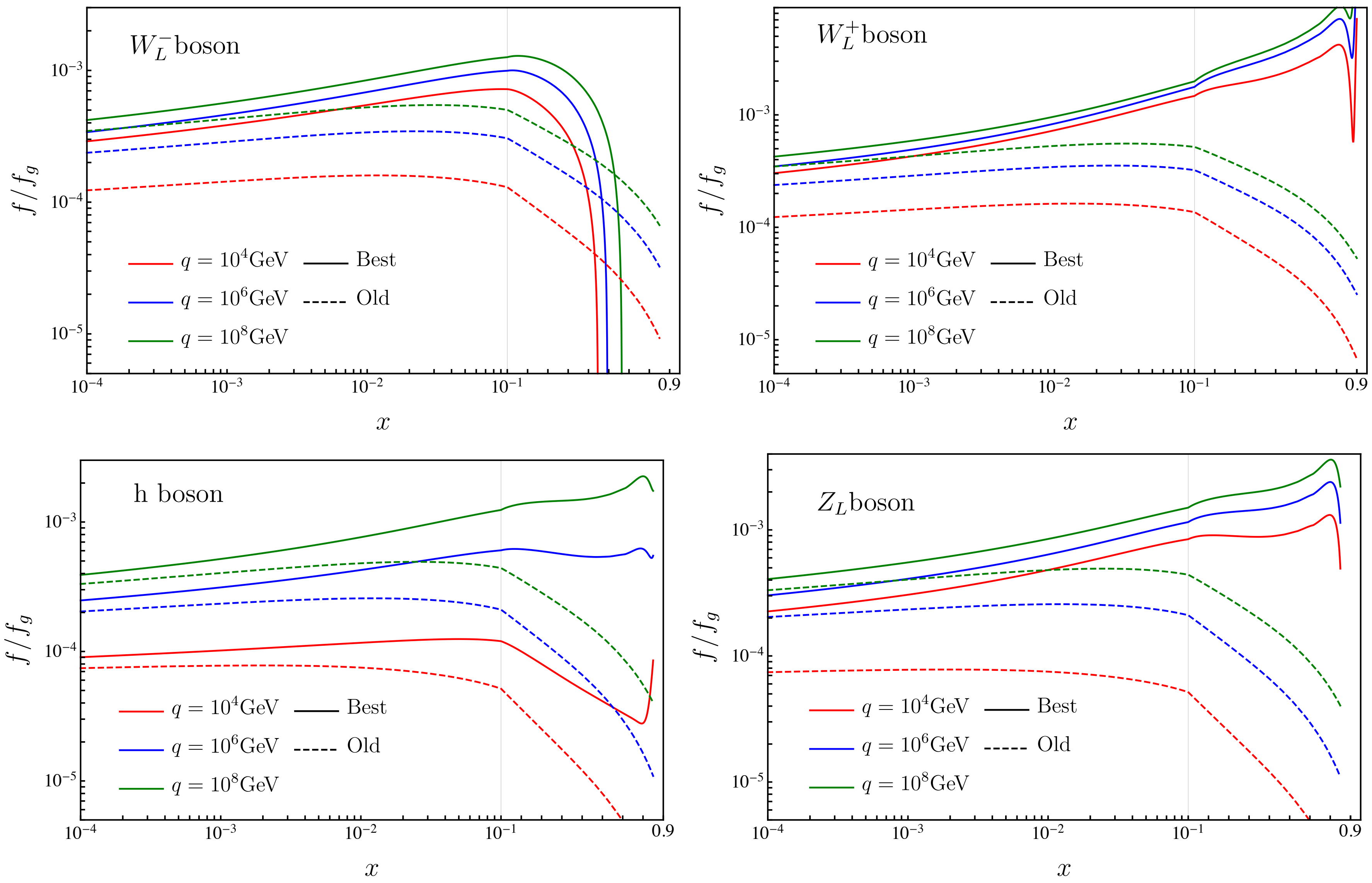




- $(V_+ - V_-)/(V_+ + V_-)$



EW bosons relative to gluon



- Longitudinal + h

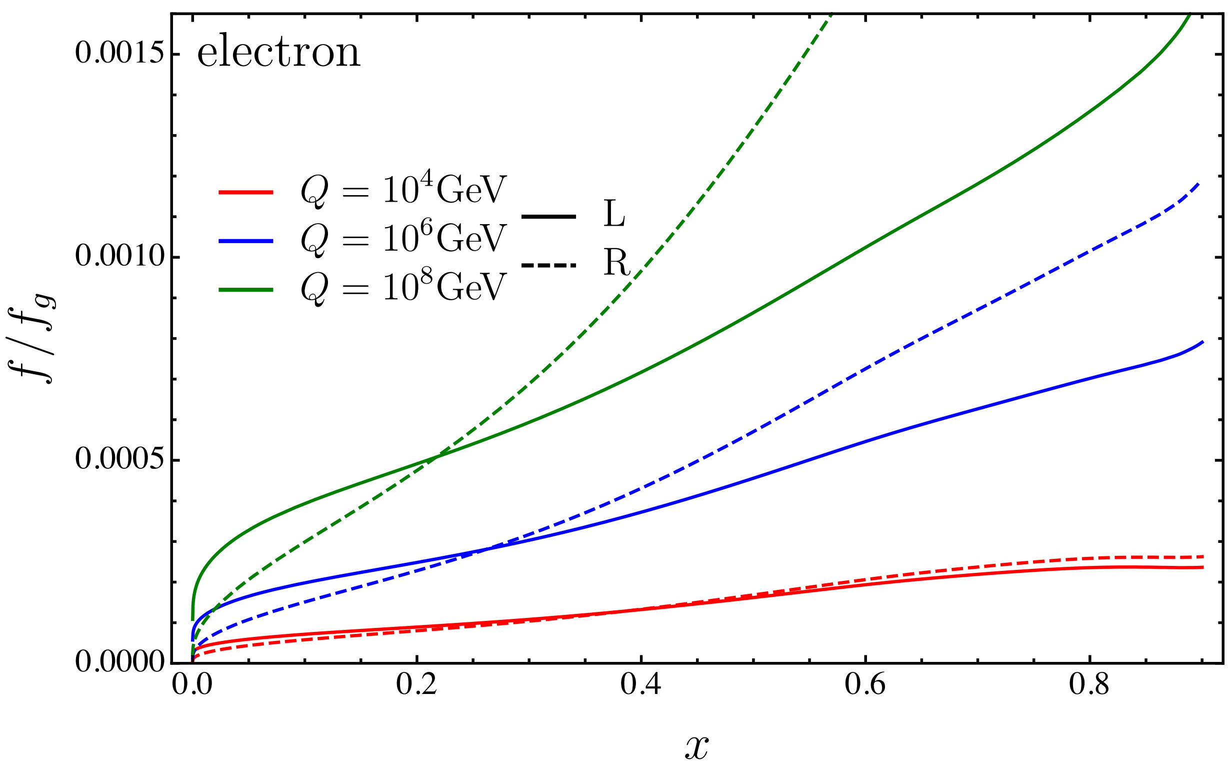
Conclusions and Prospects

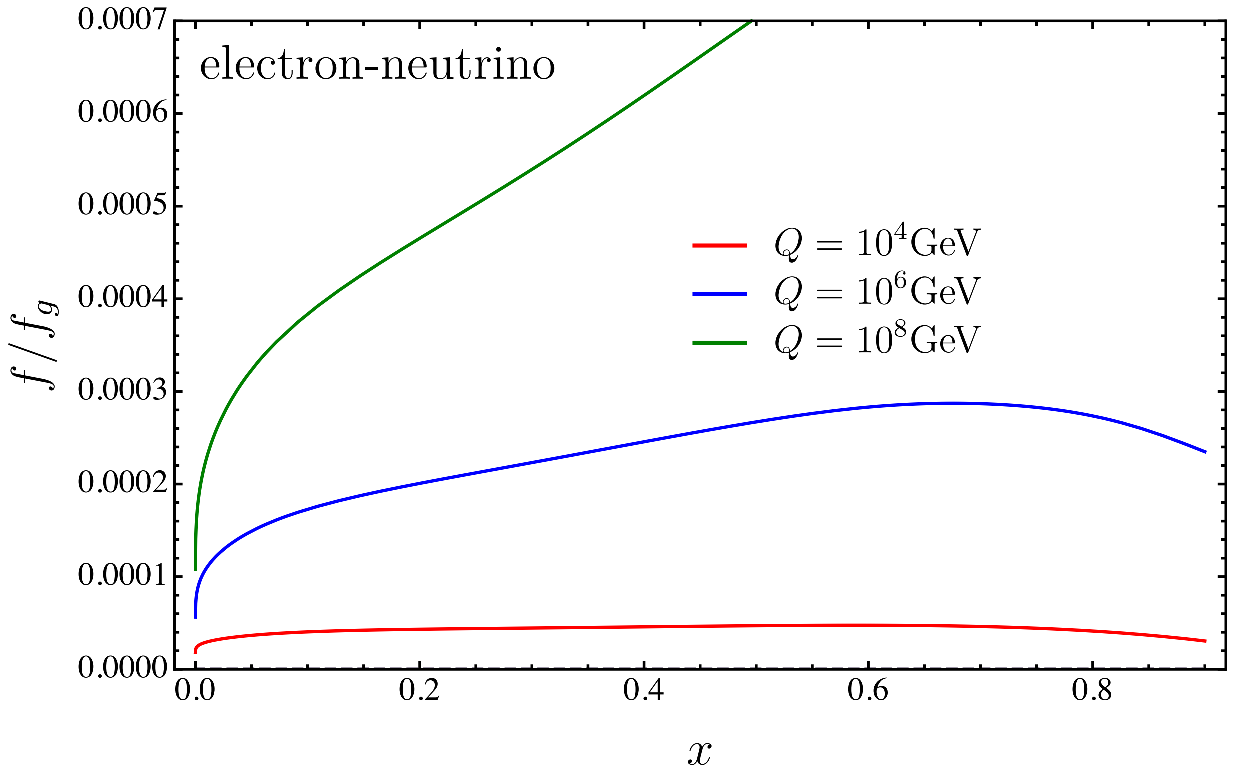
- Rich SM structure inside the proton
  - 60 parton distributions (48 distinct)
- Symmetries restored double-logarithmically, distinct left and right-handed PDFs
  - Onset of large effects around 10 TeV
  - Significant for ~100 TeV collider
  - Large EW boson polarizations
- Next step: complete SM event generator
  - Electroweak jets, ISR, MET, …

Thanks for your attention!

Backup

Leptons relative to gluon

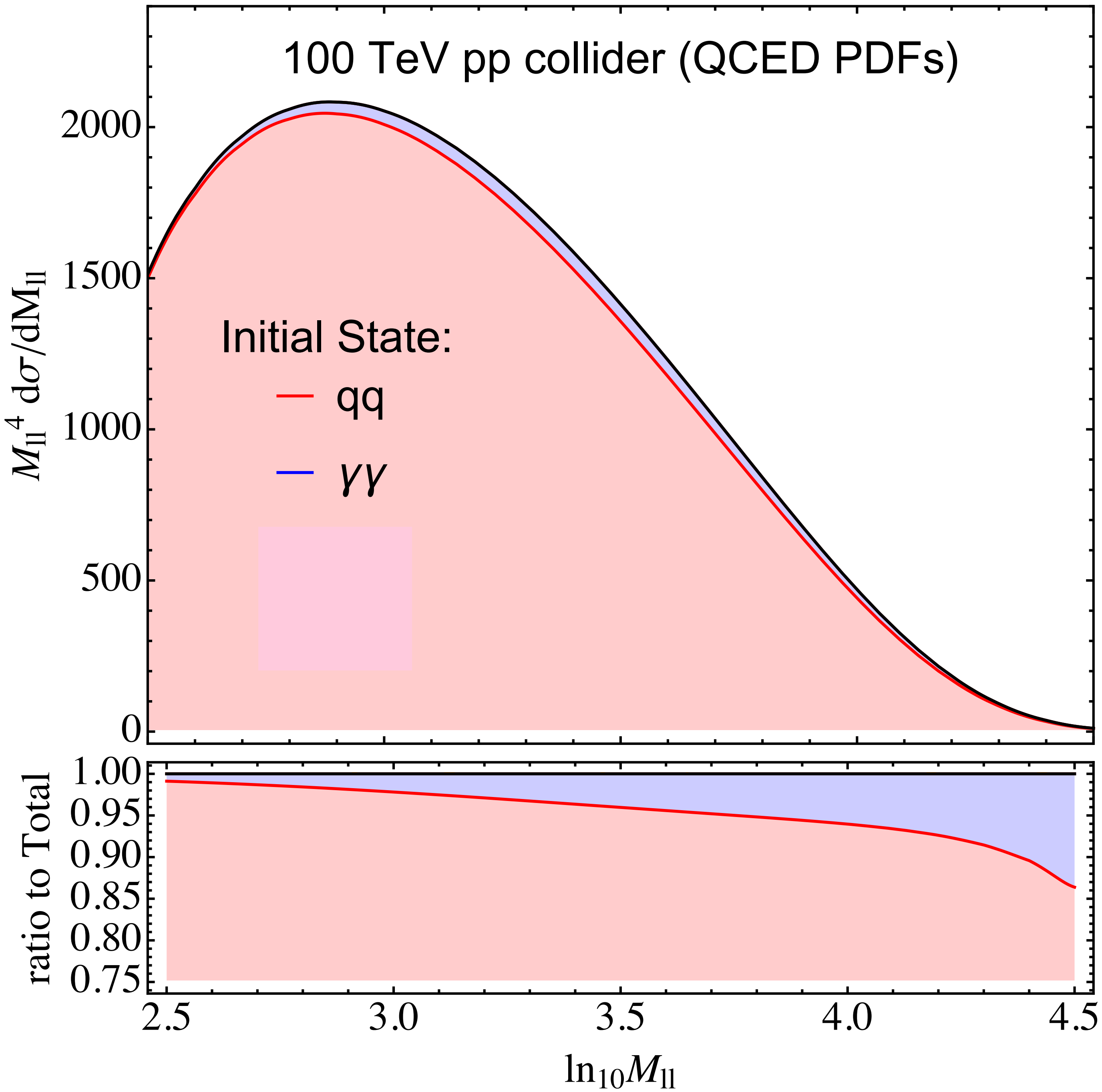




- Masses neglected ➡ all generations equal

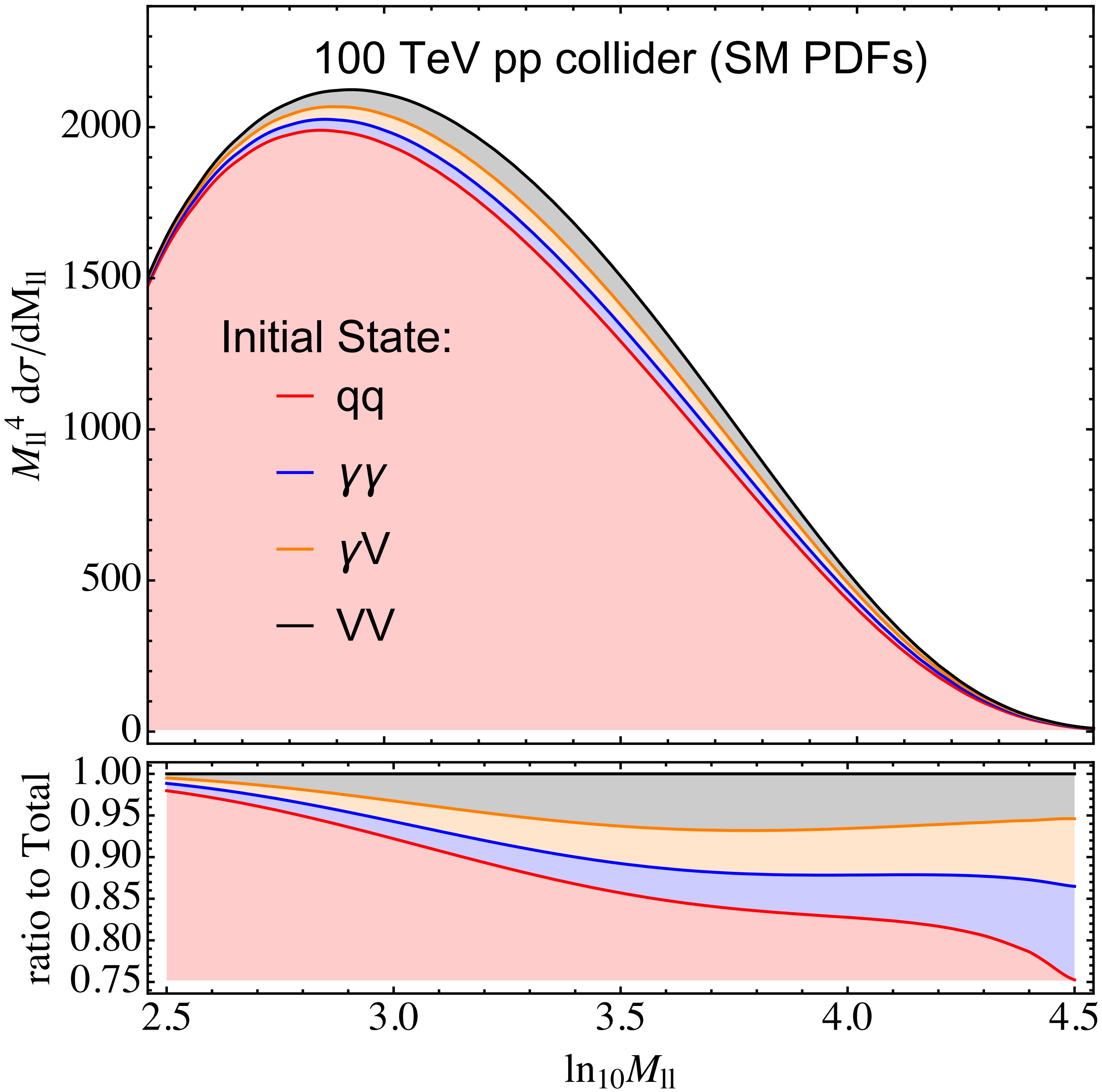
Lepton Pair Production at 100 TeV Collider

Lepton Pair Production



- QCED = SU(3)xU(1)$_{em}$

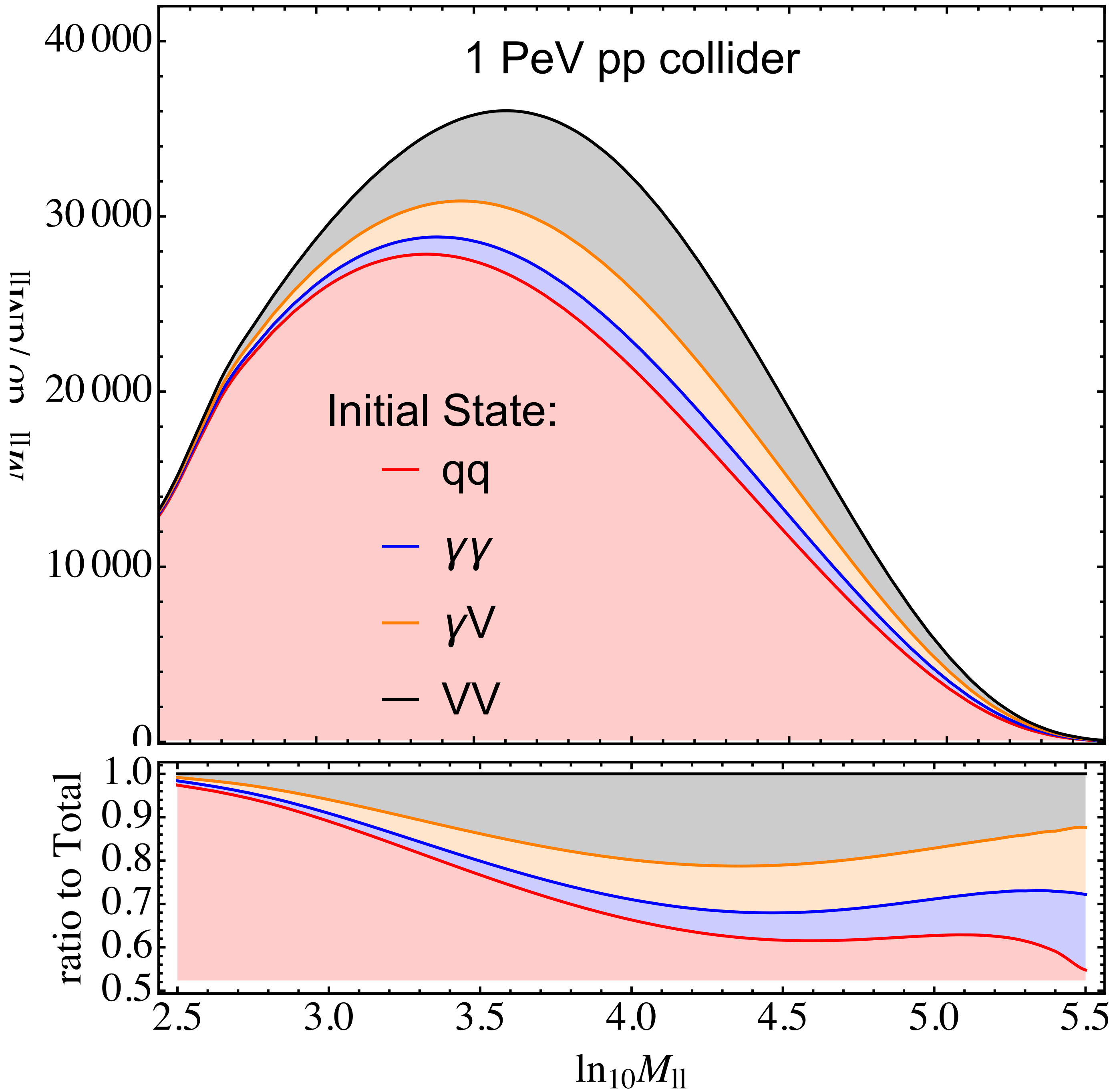
Lepton Pair Production



- SM = SU(3)xSU(2)$_L$xU(1)$_Y$

PeV Collider!

Lepton Pair Production



- SM = SU(3)xSU(2)$_L$xU(1)$_Y$