

Limits on Electroweak Instanton-induced Processes

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In collaboration with

Andreas Ringwald, Bryan Webber

[arXiv:1809.10833](https://arxiv.org/abs/1809.10833) [hep-ph]

Beyond BSM conference 2018/10/02 @ Ikaho

- In pure SU(2) YM theory, there are infinitely many gauge equivalent vacua.

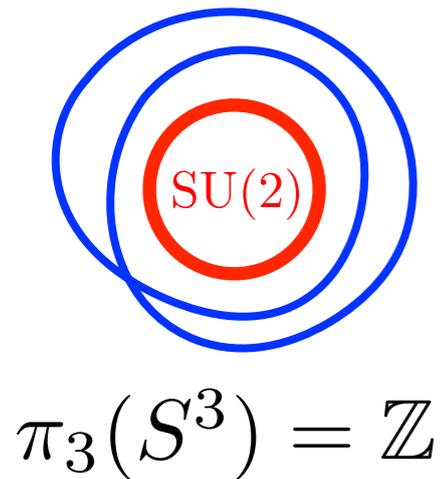
$$A_{\mu}^a T^a = 0 \xrightarrow{\text{gauge trans.}} \frac{i}{g} U \partial_{\mu} U^{\dagger}$$

They are as many as $U_{ij}(\mathbf{x})$

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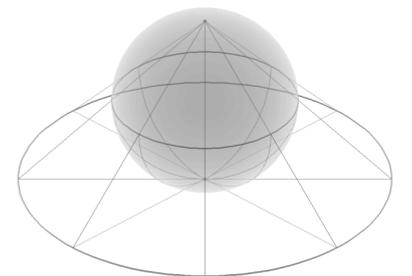
- Some of those vacuum configurations are not continuously connected.



$$SU(2) \simeq S^3 \xleftarrow{\text{map}} S^3 \simeq R^3$$

$U_{ij}(\mathbf{x})$

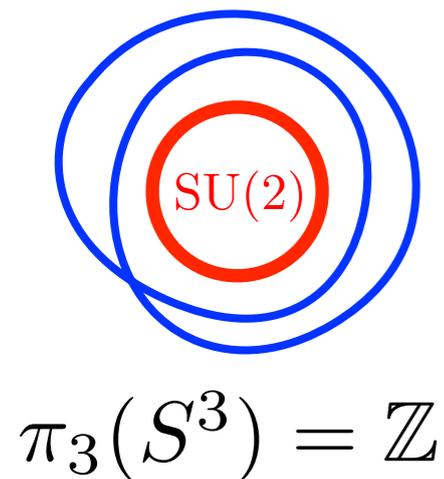
topological gauge
 $U(\infty) \rightarrow \mathbf{1}$



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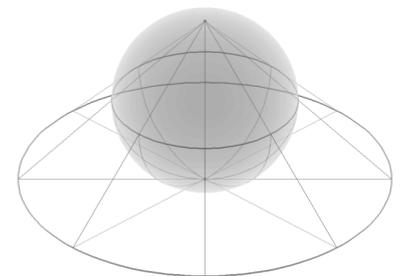
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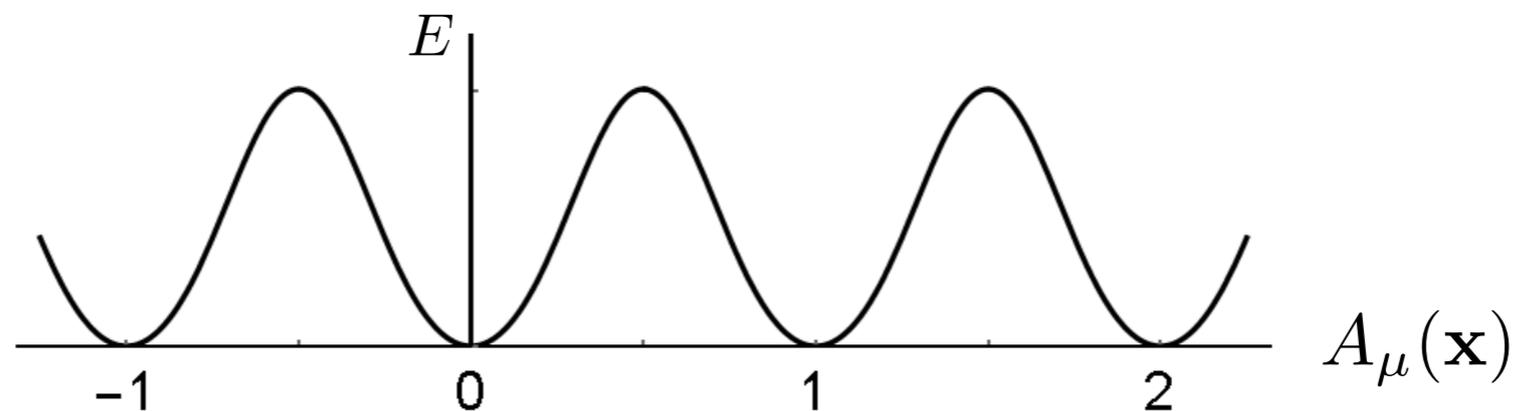
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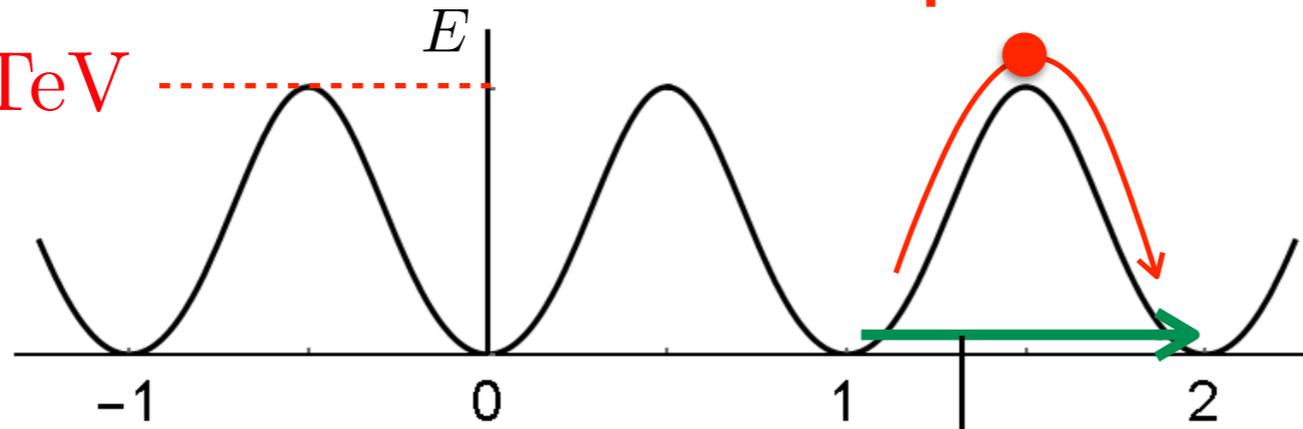
- These disconnected vacua are characterised by the integer N_{cs} .



- One can think of transitions between disconnected vacua.

[Klinkhamer, Manton '84]

$$E_{\text{sph}} \simeq 9 \text{ TeV}$$



sphaleron

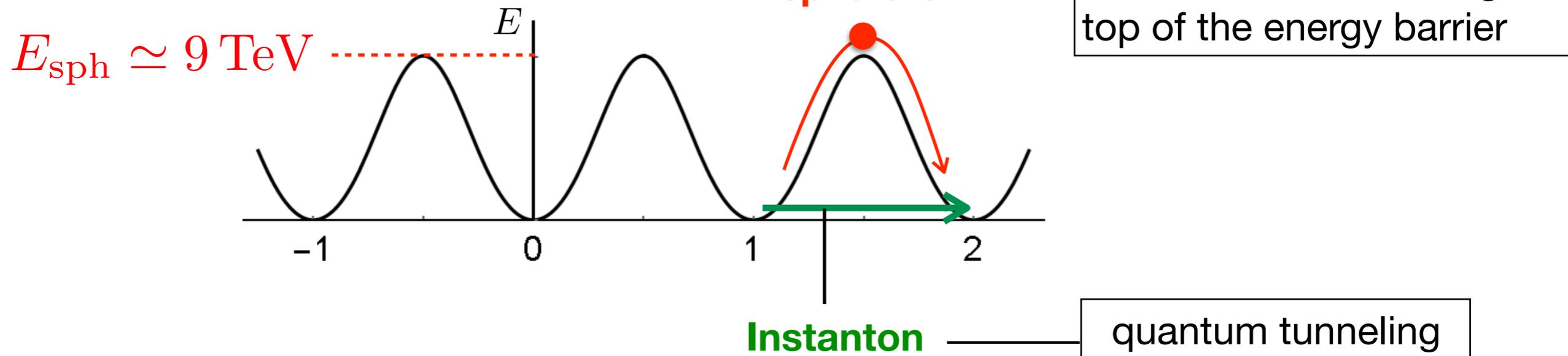
unstable solution sitting on top of the energy barrier

Instanton

quantum tunneling

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[Klinkhamer, Manton '84]

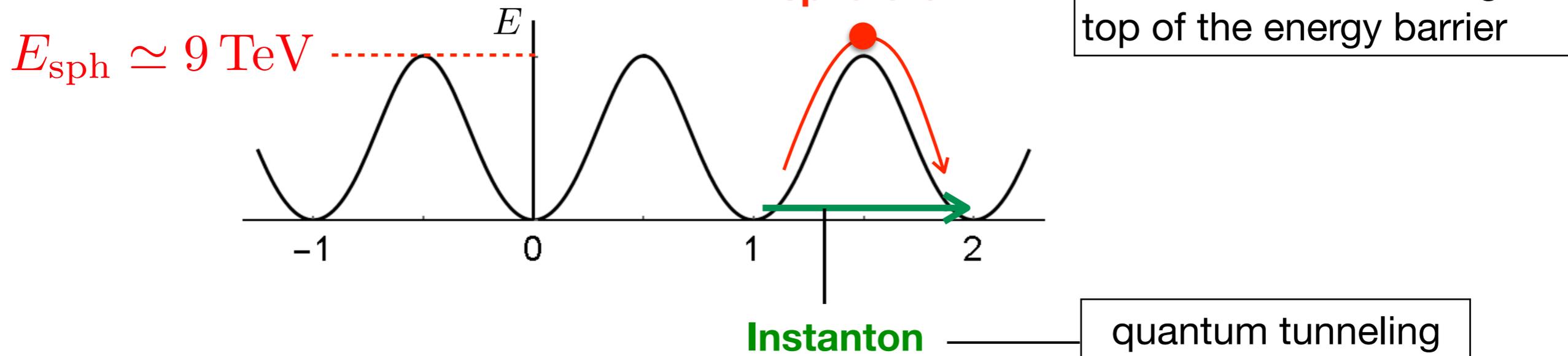


- At zero temperature/energy, the instanton rate is exponentially small:

$$\langle N_{CS} | N_{CS} + 1 \rangle_{\text{instanton}} \sim e^{-S[A_{cl}]} = e^{-\frac{2\pi}{\alpha_W}} \sim e^{-180}$$

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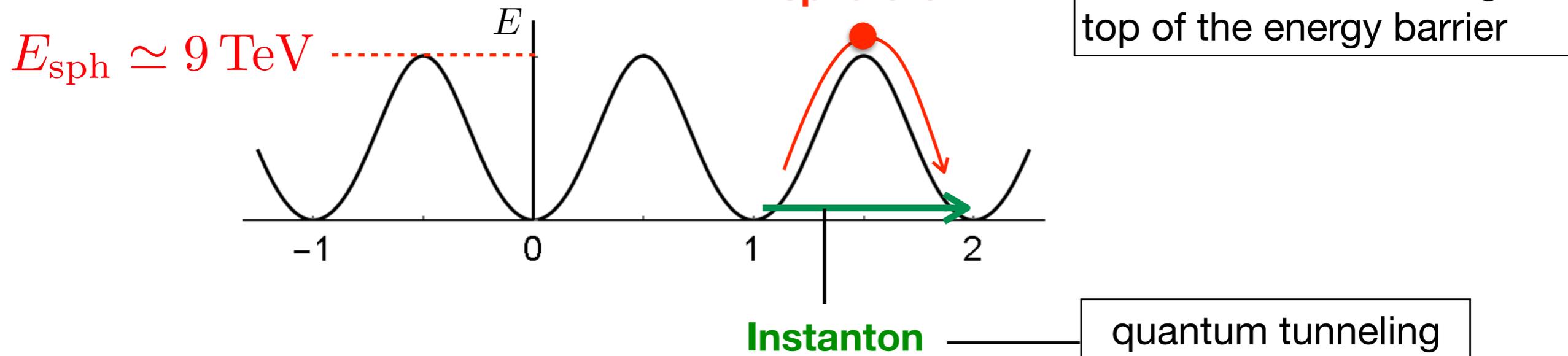
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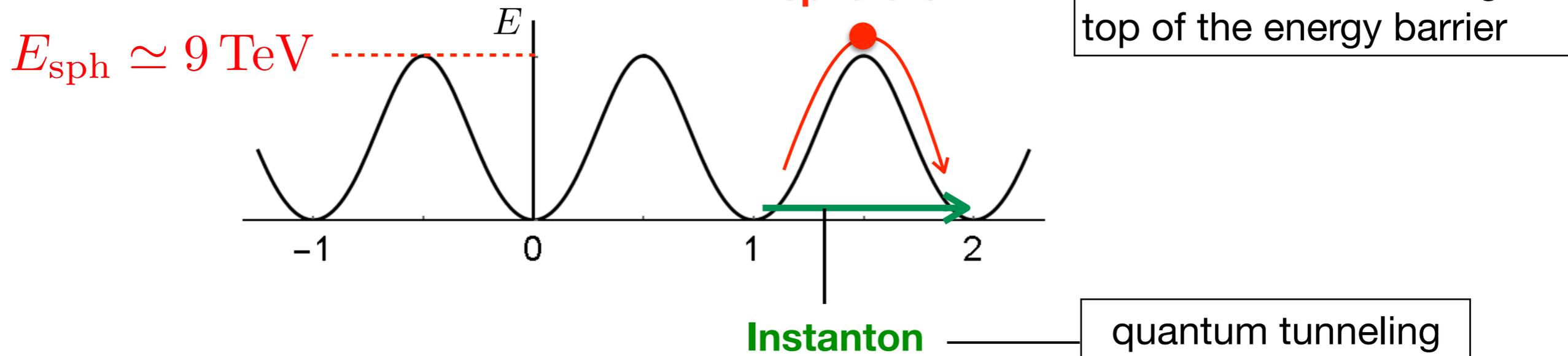
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Setting limits on vacuum transitions from the LHC ← **This talk**

- The change of N_{CS} is related to the following quantity:

$$\Delta N_{CS} = \frac{g^2}{16\pi^2} \int \text{Tr} \left[F_{\mu\nu} \tilde{F}^{\mu\nu} \right] d^4x$$

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anomaly

SU(2) charged fermion

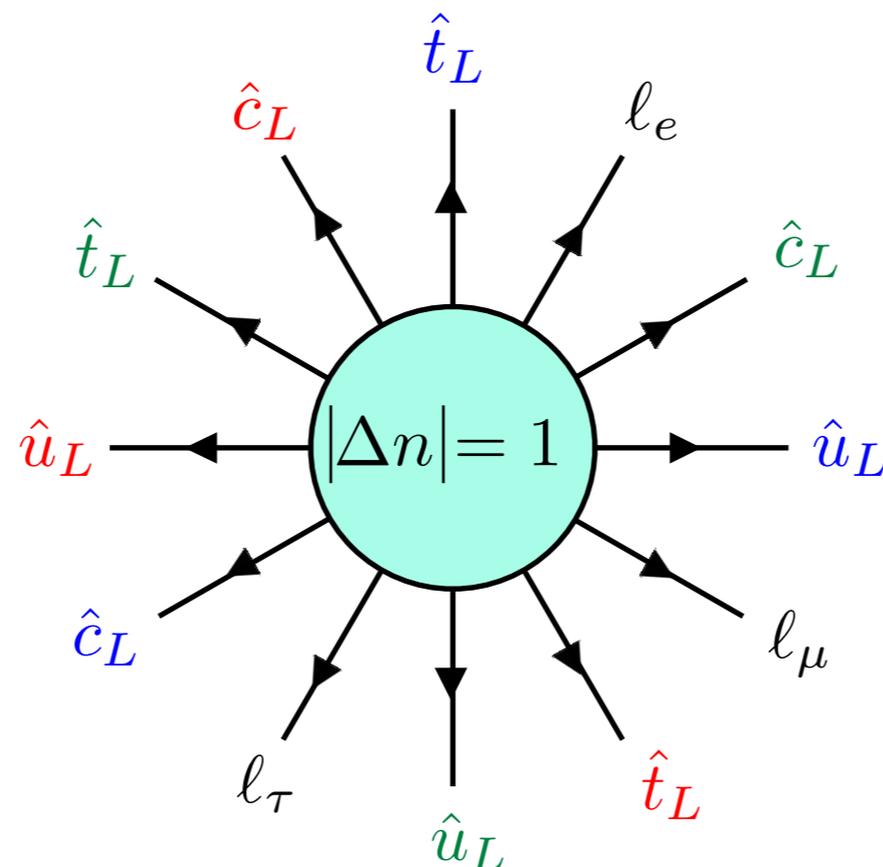
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- ΔN_{CS} is related to the change of SU(2) charged fermion numbers.



$|\Delta N_{CS}| = 1$ transition
creates 12 fermions
altogether!

Party at collider!

- The LO Matrix Element in the *instanton background*

$$i\mathcal{M} \sim \int \mathcal{D}q \mathcal{D}W \mathcal{D}\phi q(x_1) \cdots q(x_{12}) W(y_1) \cdots W(y_{n_W}) \phi(z_1) \cdots \phi(z_{n_h}) \exp(-S_E) \Big|_{\text{LSZ}}$$

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- Evaluate it at the instanton configuration:

$$W_{\text{inst}}^{\mu a} \simeq \frac{2\rho^2}{g} U_{ab} \frac{\bar{\eta}_{b\mu\nu} (x - x_0)_\nu}{(x - x_0)^2 [(x - x_0)^2 + \rho^2]} \quad \phi_{\text{inst}}(x) \simeq v \left[\frac{(x - x_0)^2}{(x - x_0)^2 + \rho^2} \right]^{1/2}$$

orientation
position
size

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- Result

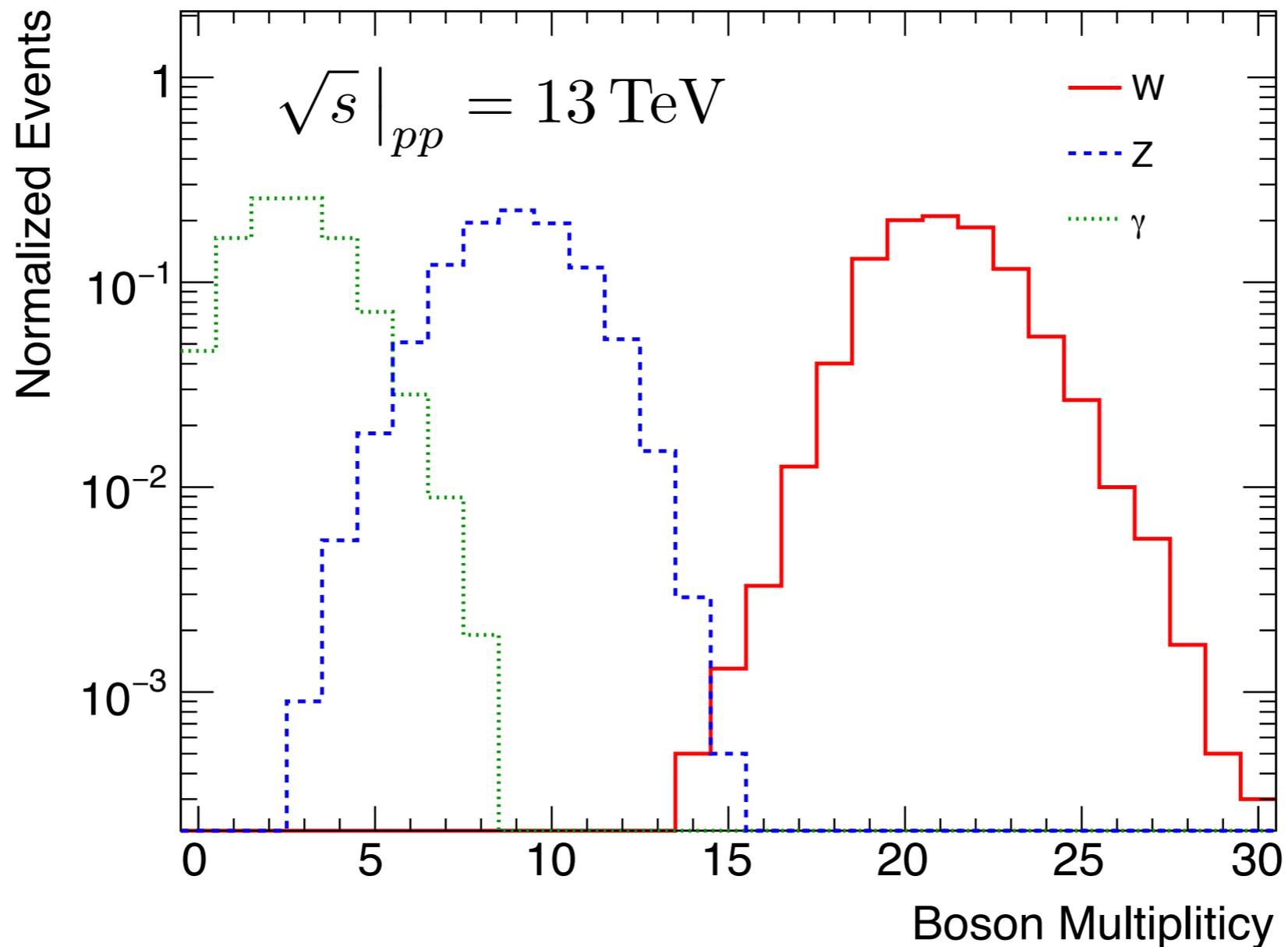
[Ringwald '90, Espinosa '90]

$$\sigma_{\text{LO}}(n_W, n_h) \sim \mathcal{G}^2 2^{n_W} v^{-2n} \left[\frac{\Gamma(n + 103/12)}{\Gamma(103/12)} \right]^2 \frac{1}{n_B! n_H!} \quad \mathcal{G} = 1.6 \times 10^{-101} \text{ GeV}^{-14}$$

$$\times \int \prod_{i=1}^{10} \frac{d^3 p_i}{(2\pi)^3 2E_i} E_i \prod_{j=1}^{n_B} \frac{d^3 p_j}{(2\pi)^3 2E_j} \frac{2(4E_j^2 - m_W^2)}{m_W^2} \prod_{k=1}^{n_H} \frac{d^3 p_k}{(2\pi)^3 2E_k} (2\pi)^4 \delta^{(4)} \left(P_{\text{in}} - \sum_{i=1}^{10} p_i - \sum_{j=1}^{n_B} p_j - \sum_{k=1}^{n_H} p_k \right)$$

The cross-section becomes very large for large n_W and n_h !

- The MC Event Generator (**HERBVI**) was developed by Gibbs and Webber based on the LO ME formula:

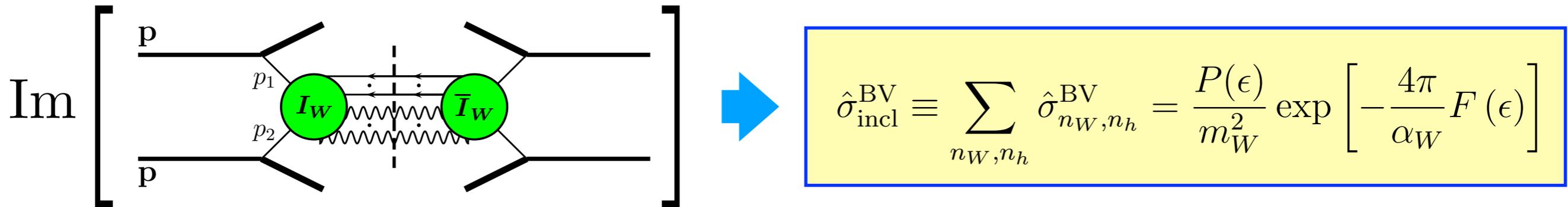


O(30) EW gauge bosons are produced!

Festival at collider!

- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald '91]



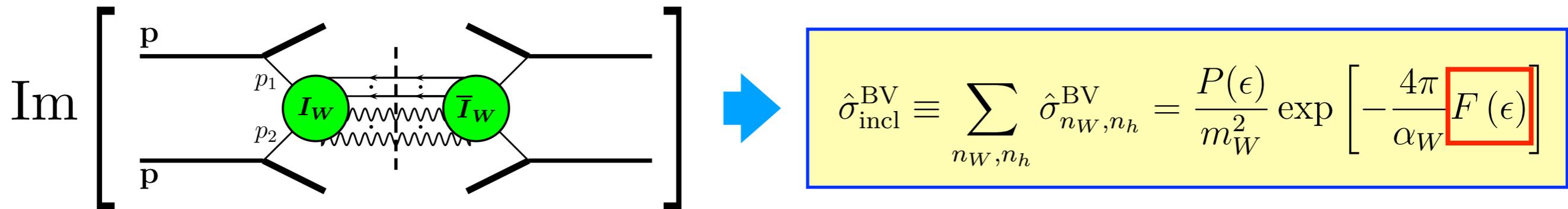
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$$P(\epsilon) = \frac{\pi^{15/2}}{1024} \left(\frac{3}{2} \right)^{2/3} d^2 \left(\frac{4\pi}{\alpha_W} \right)^{7/2} \epsilon^{74/9} [1 + \mathcal{O}(\epsilon^{2/3})] \quad d \simeq 0.15$$

$$\epsilon \equiv \frac{\sqrt{\hat{s}}}{M_0} \quad M_0 \equiv \sqrt{6}\pi \frac{m_W}{\alpha_W} \simeq 18.3 \text{ TeV}$$

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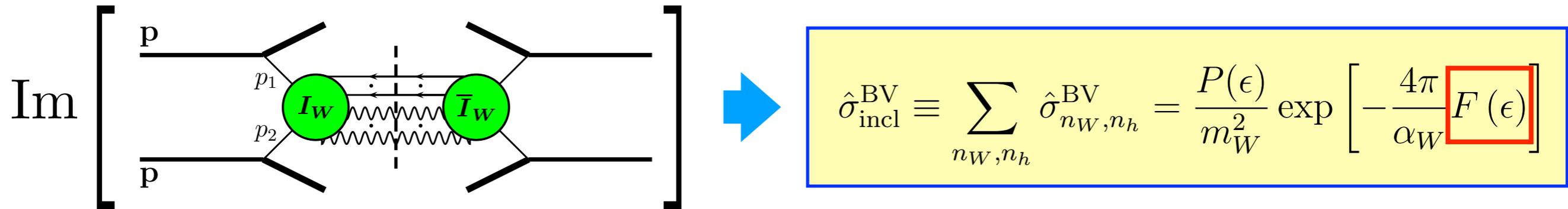
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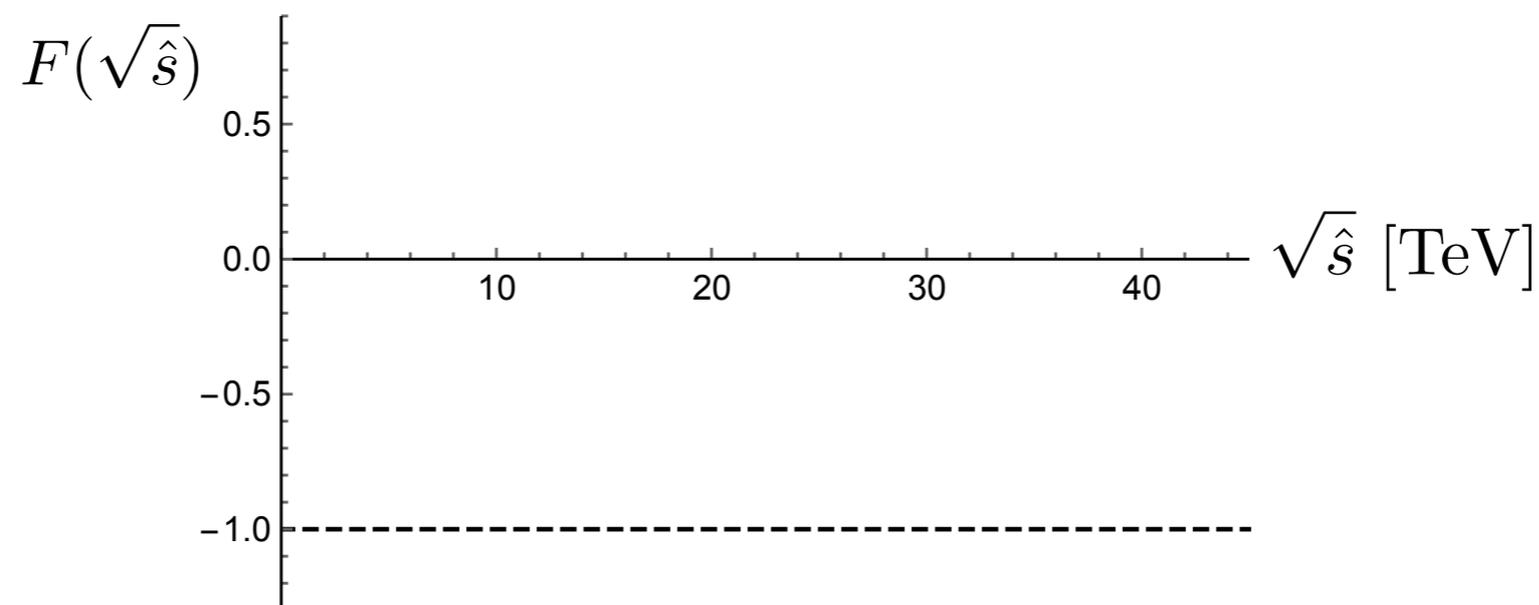


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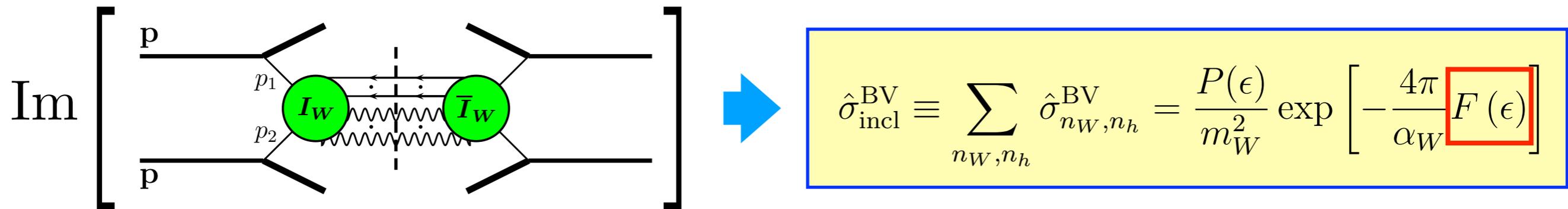
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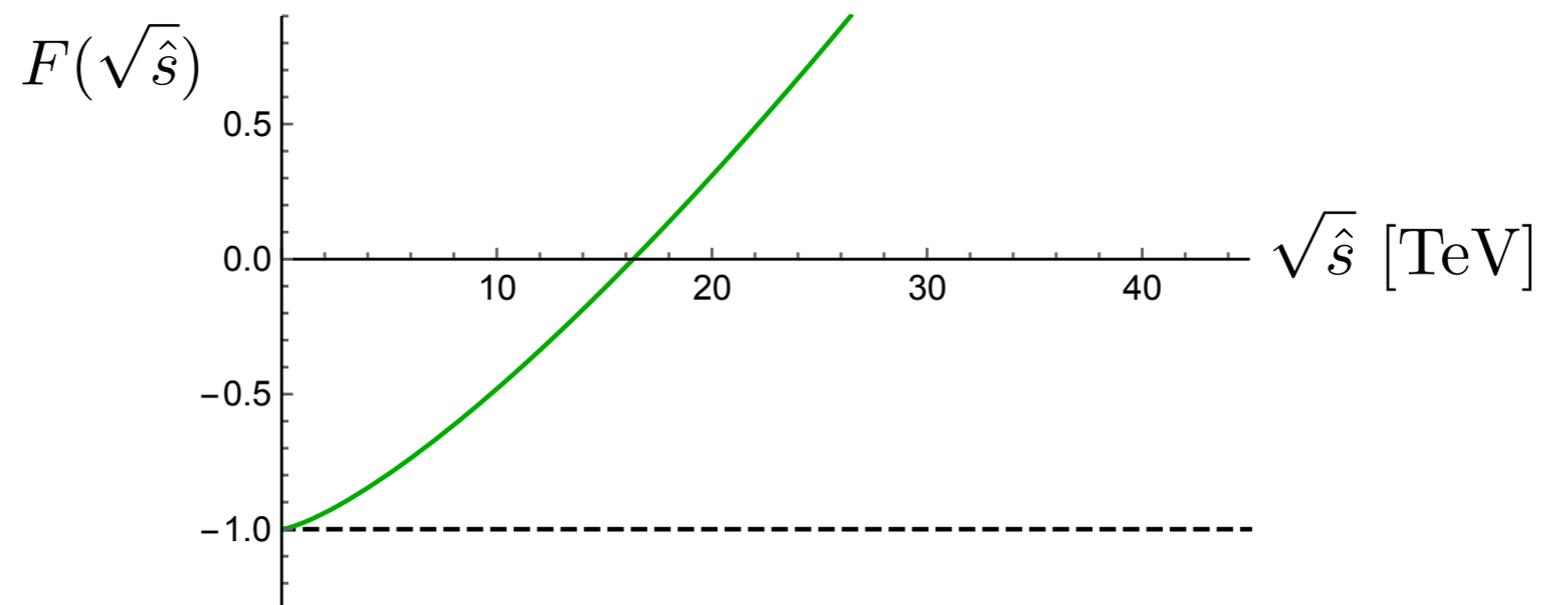


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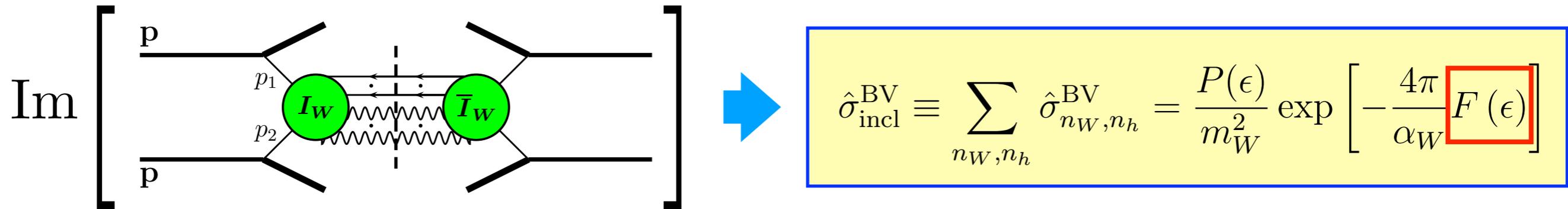
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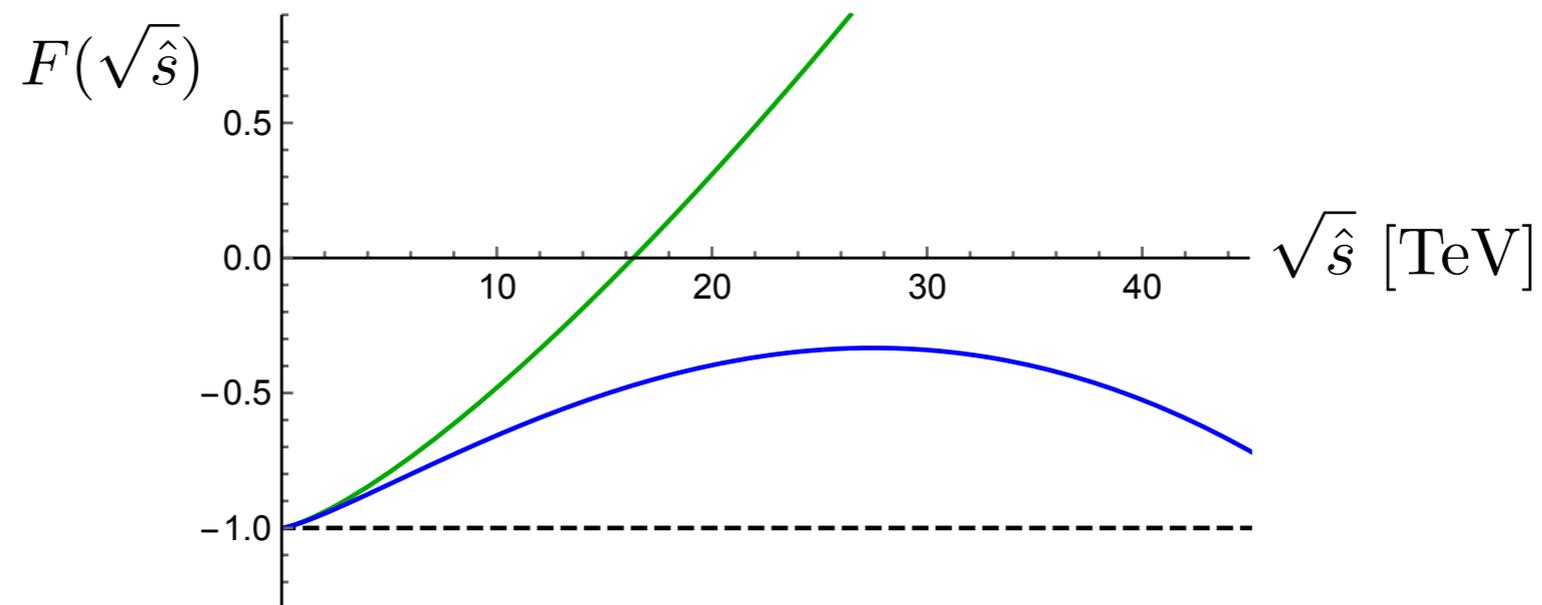


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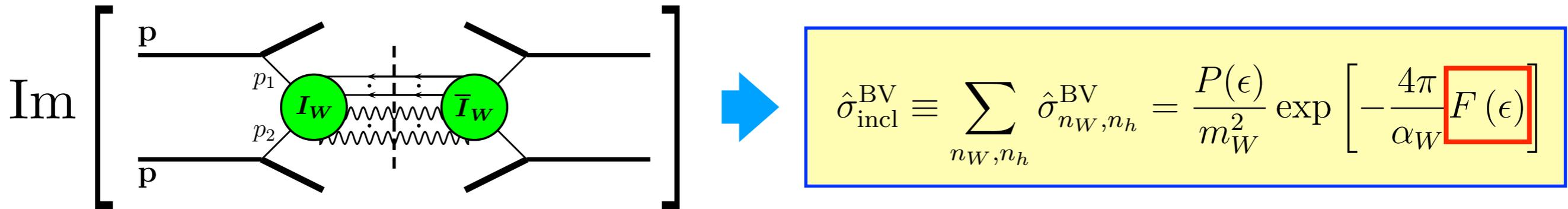
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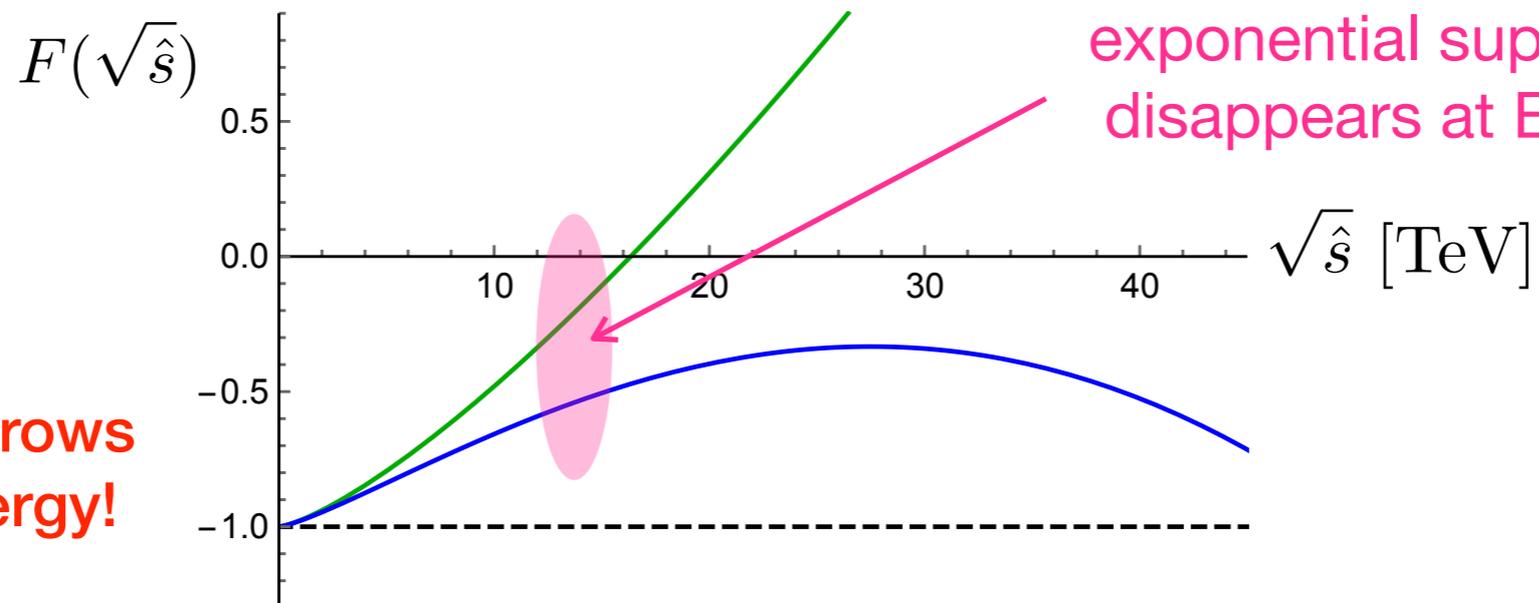


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Cross-section grows rapidly with energy!

- The time evolution of the sphaleron field configuration has been simulated numerically, and it found $O(30)$ gauge bosons in the final state.

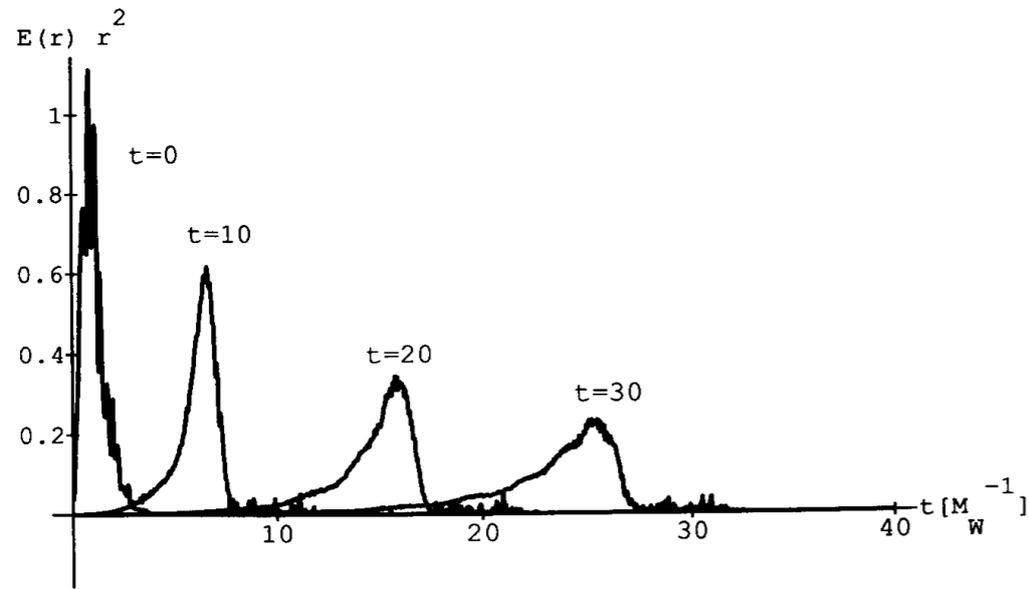


Fig. 1. Radial energy distribution (in units of M_W) of the decaying sphaleron at different times (in units of M_W^{-1}).

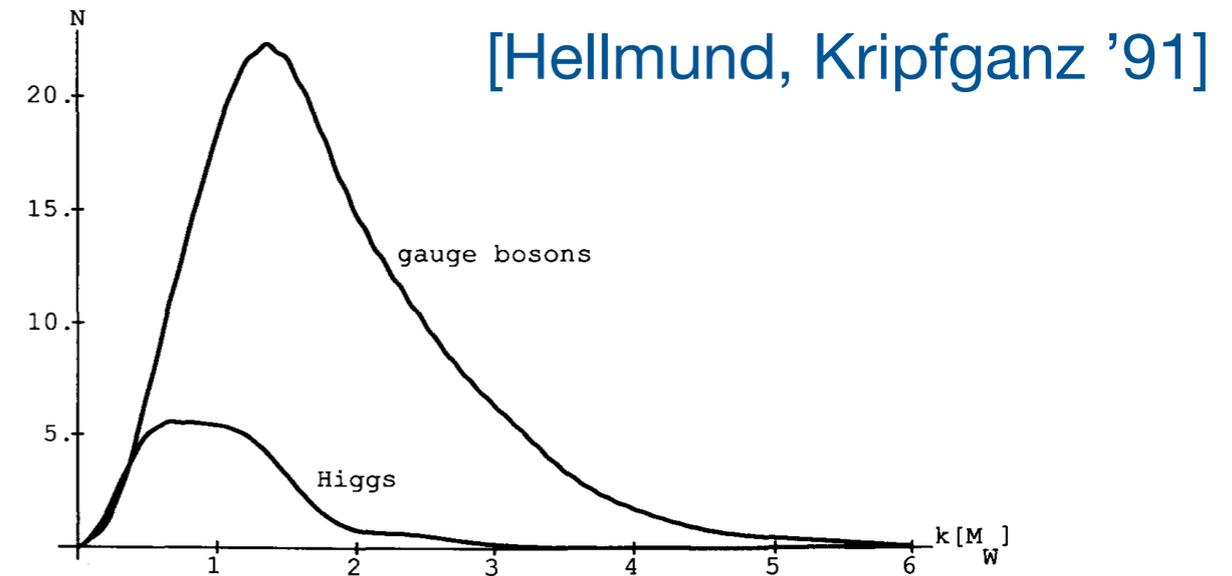


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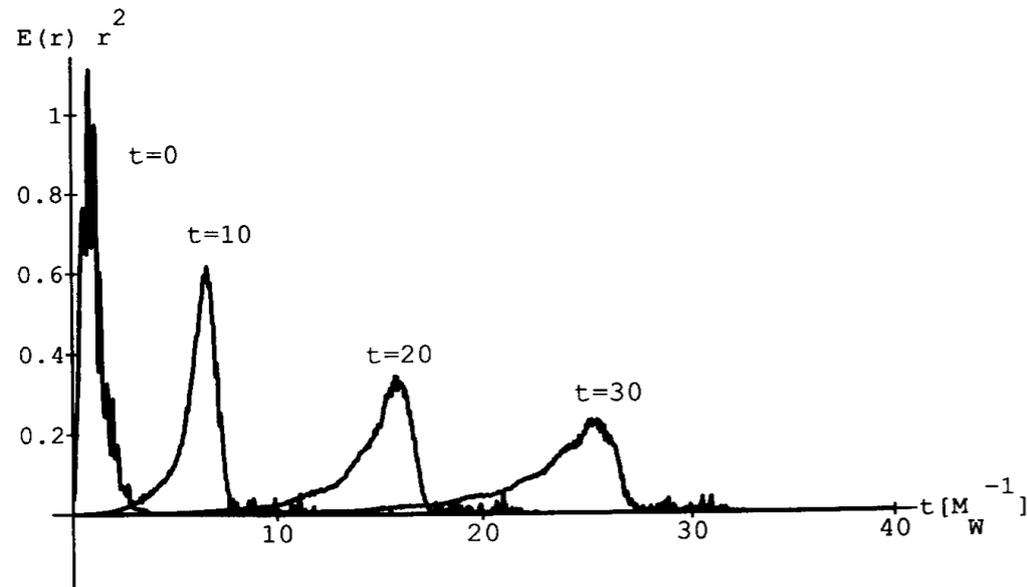


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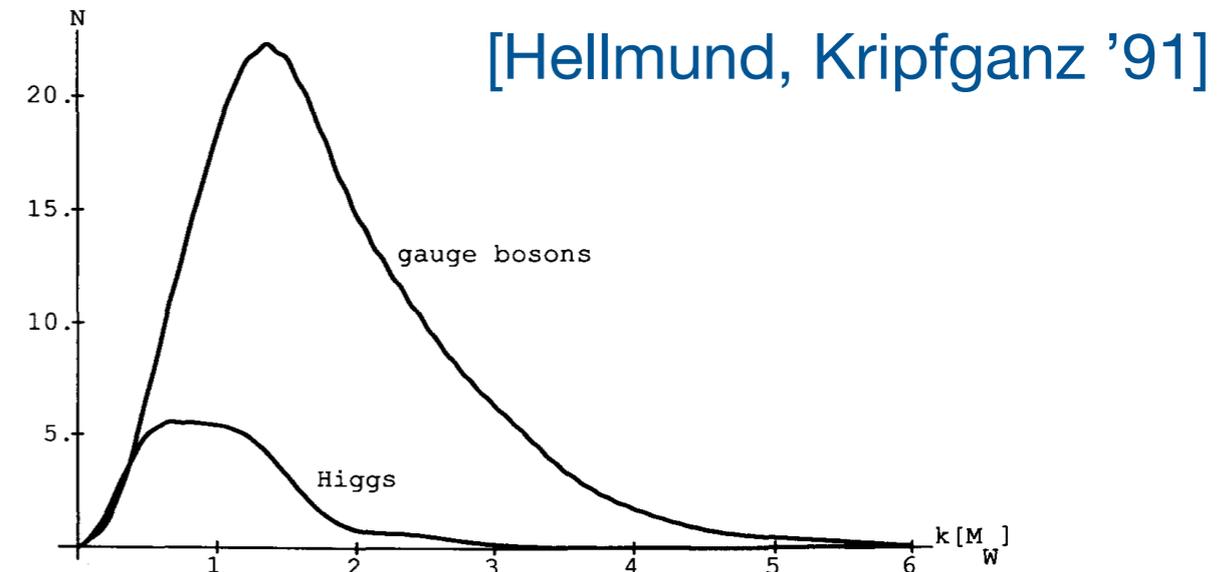
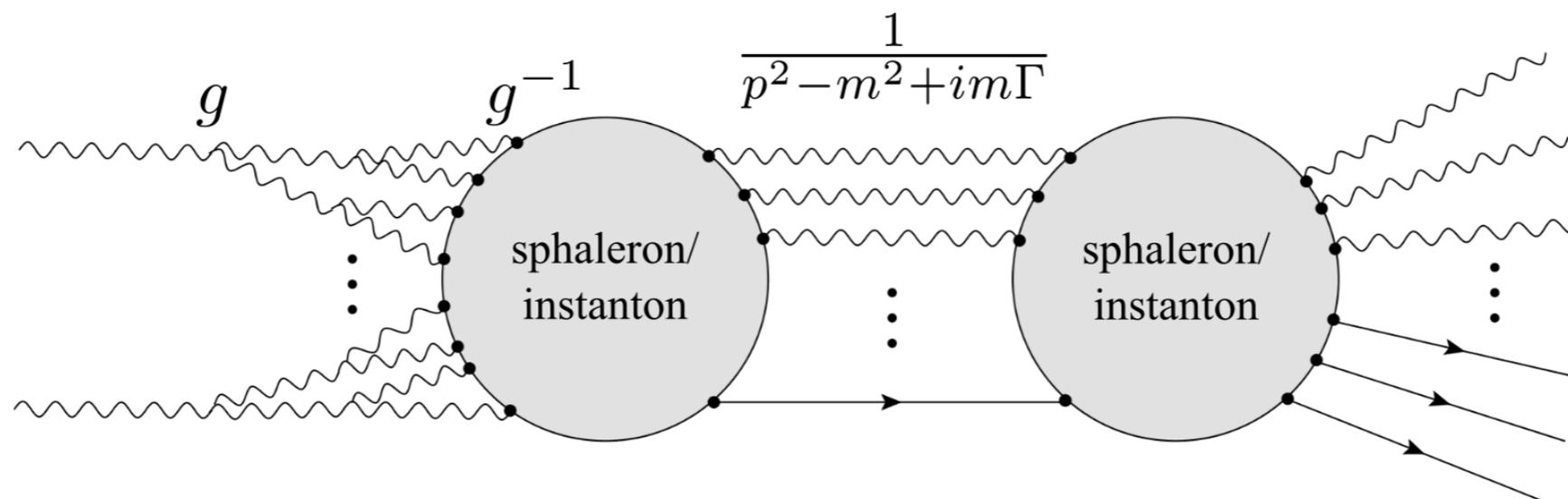


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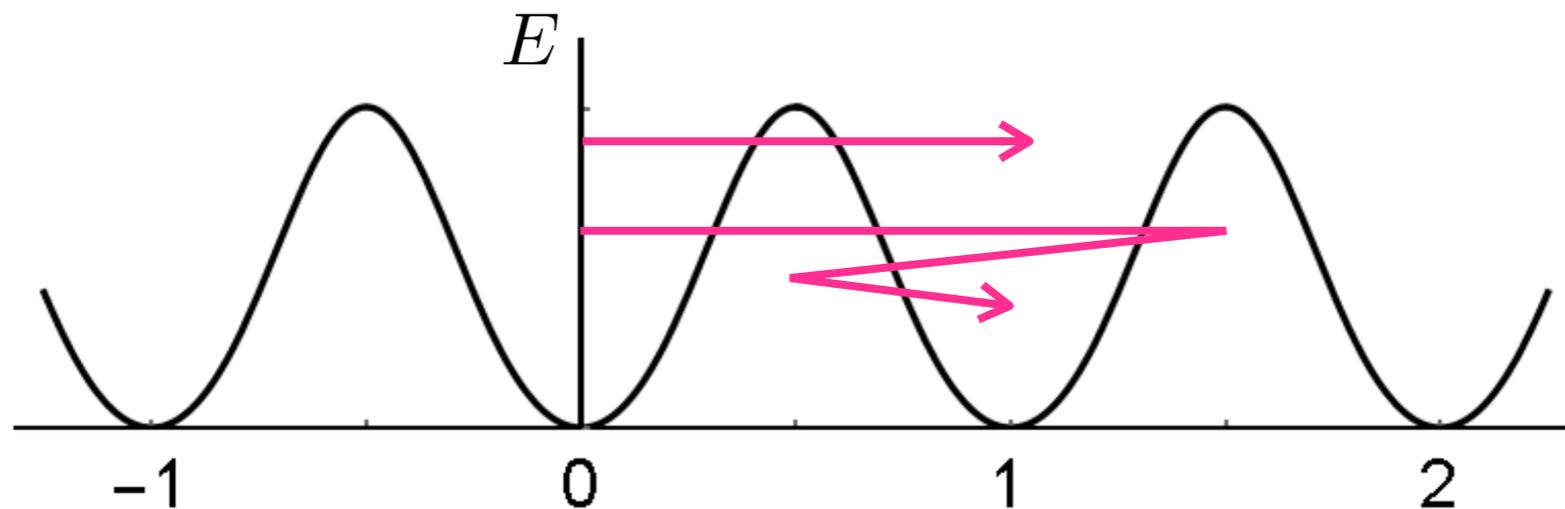
- These studies seem to suggest that many W-bosons are produced in the instanton BG, which produces a sphaleron-like field configuration at $E \sim E_{\text{sph}}$, and vacuum transitions occur semi-classically in some sense.



- More recently (2015), Tye and Wong claims that at zero temperature **instanton rate overcomes the exponential suppression for $E > E_{\text{sph}} \sim 9$ TeV**, if the periodicity of the EW potential is taken into account, due to *resonant tunnelling*.

Resonant tunneling:

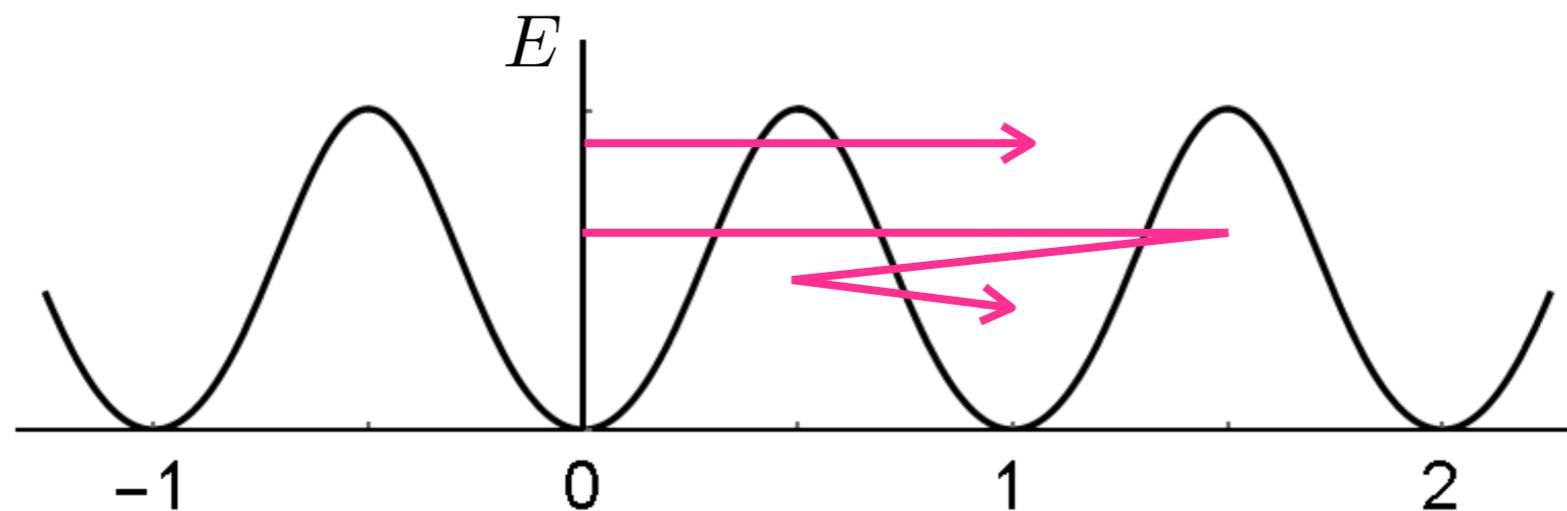
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- This triggers many theoretical and phenomenological studies:

[Ellis, KS '16], [Brooijmans, Schichtel, Spannowsky '16],
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[Cerdeno, Reimitz, KS, Tamarit '18], [Y.Jho, S.C.Park '18], ...

- CMS has published their search for sphaleron-like events. [CMS-EXO-17-023]
- However, they consider zero-boson final state.

$$qq \rightarrow n_q q + 3\ell \quad n_q = 7, 9 \text{ or } 11$$

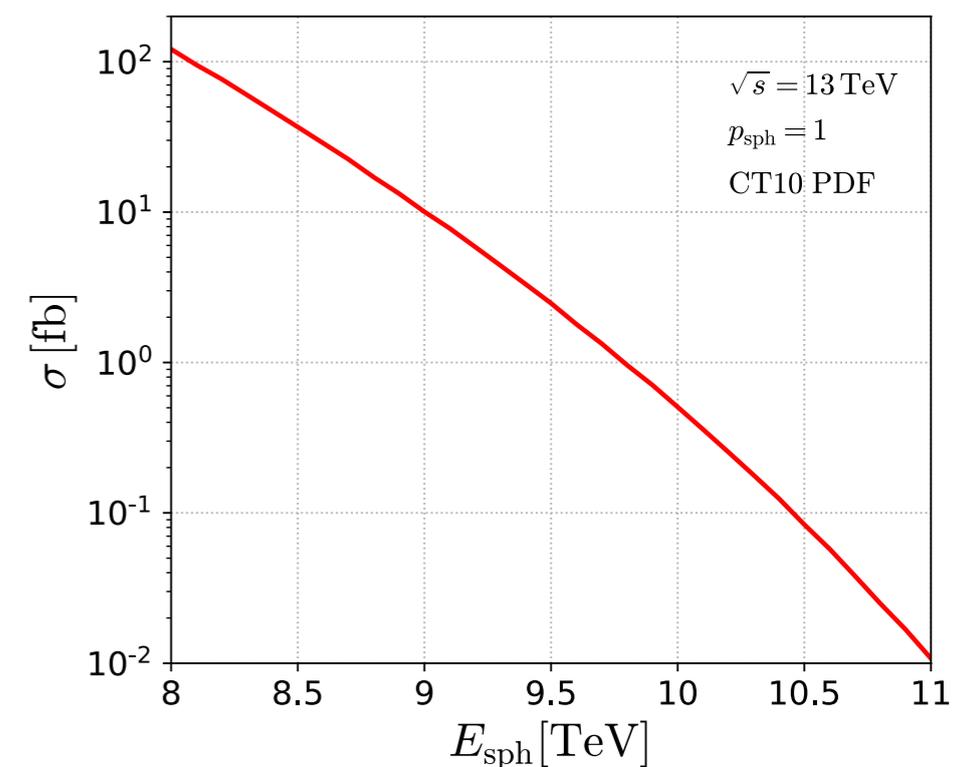
- How does their simulation compare with (more realistic) multi-boson events?

$$qq \rightarrow \begin{cases} n_q q + 3\ell & (\text{BaryoGEN}) \\ 7q + 3\ell + \sum n_B B & (\text{HERBVI}) \end{cases}$$

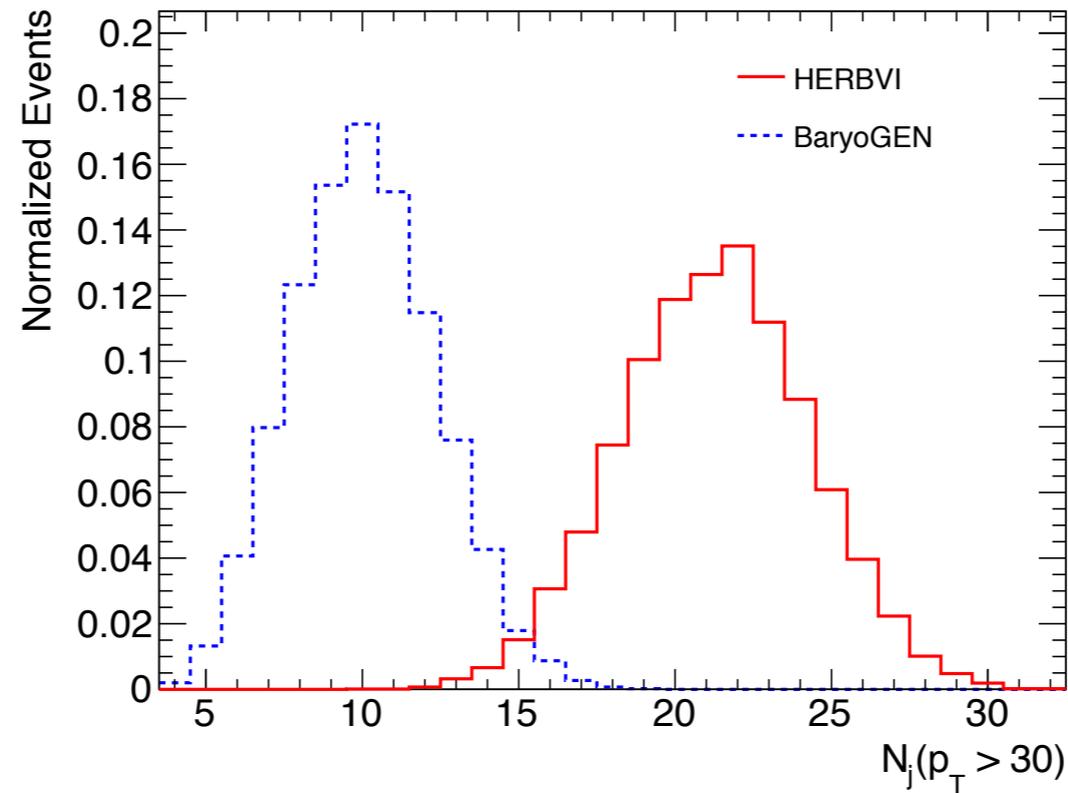
We use *Delphes* to simulate CMS detector.

- We model the partonic cross-section as:

$$\hat{\sigma}(\sqrt{\hat{s}}) = \frac{p_{\text{sph}}}{m_W^2} \Theta(\sqrt{\hat{s}} - E_{\text{sph}})$$

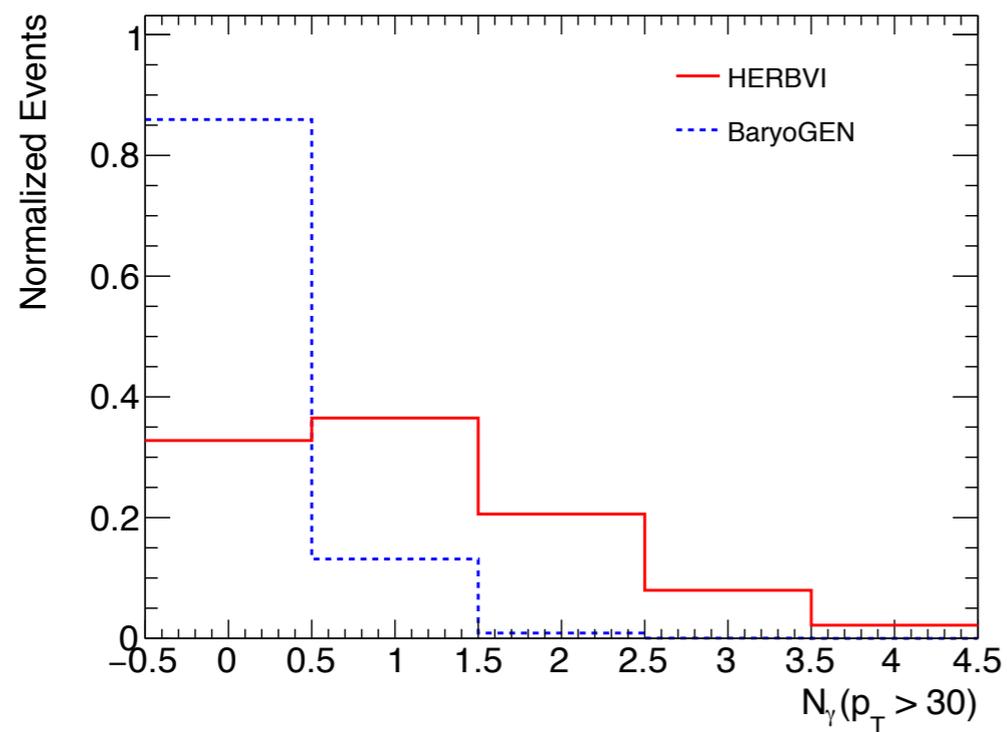


jet multiplicity

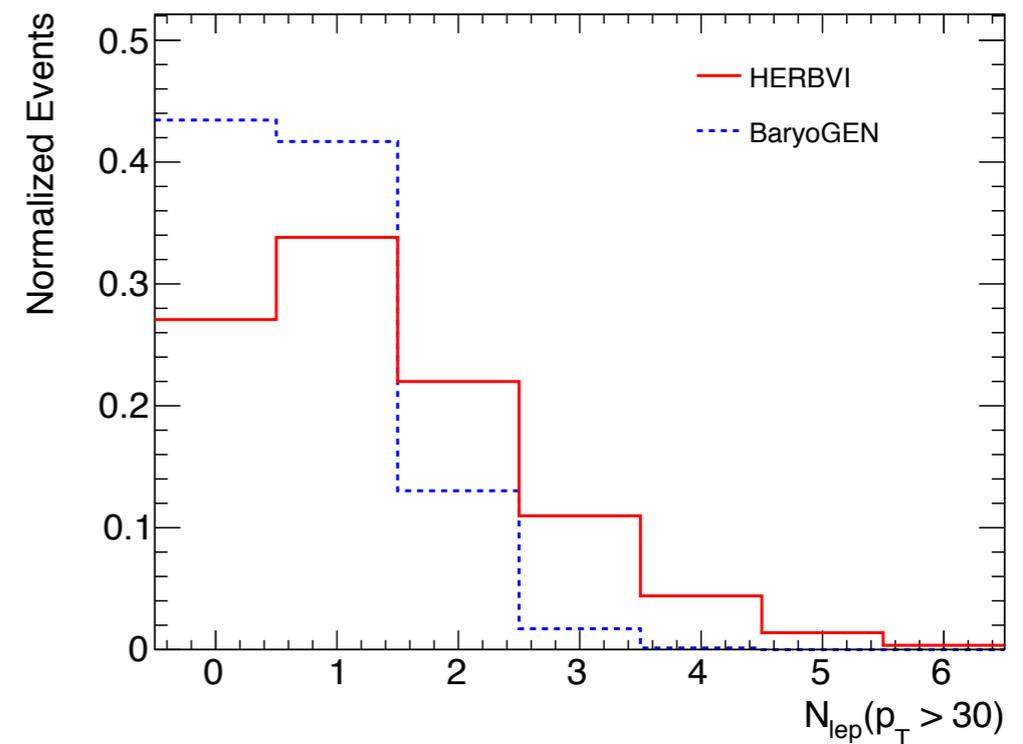


— multi-boson
..... zero-boson

lepton multiplicity



photon multiplicity



- In the CMS analysis the cuts impose on S_T and multiplicity.

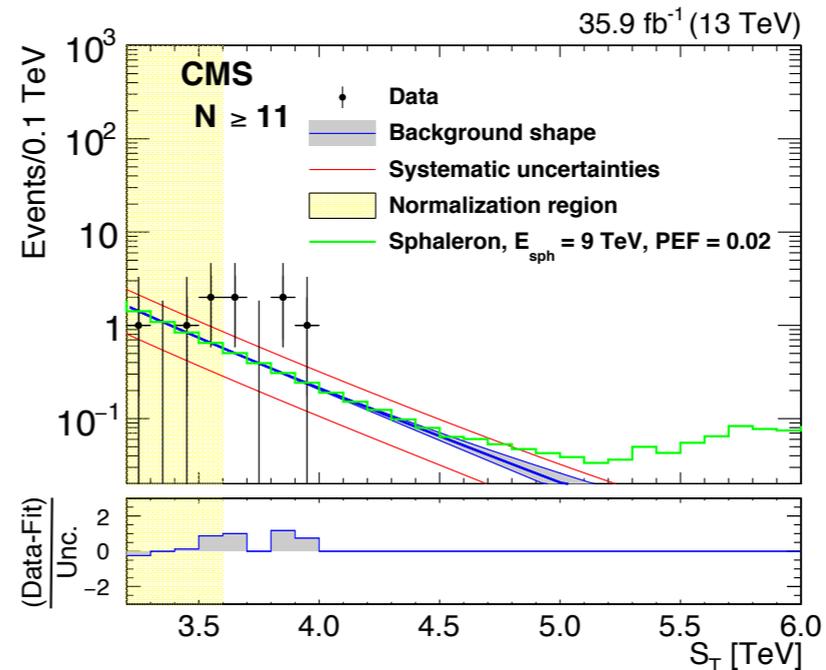
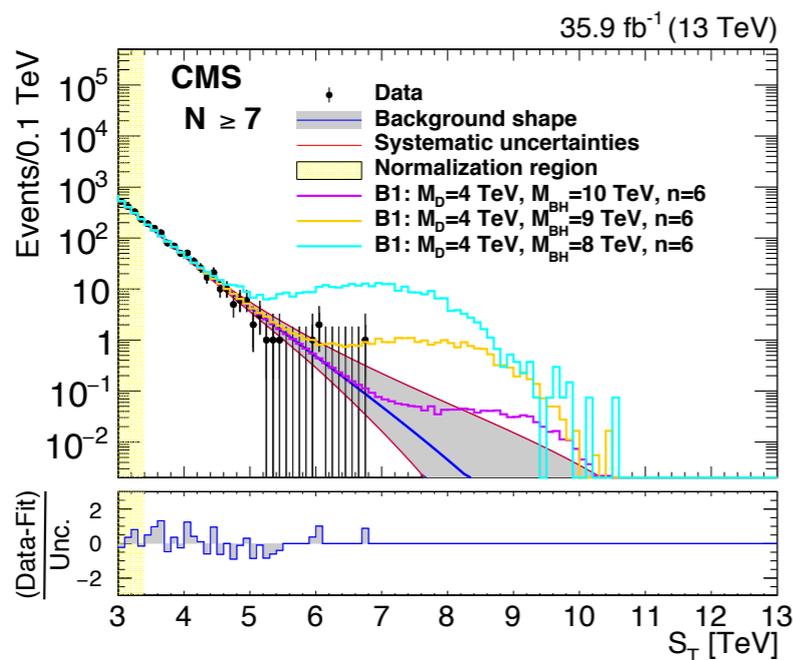
$$S_T \equiv E_T^{\text{miss}} + \sum_i^{p_T > 70 \text{ GeV}} p_T^{(i)} > S_T^{\text{min}} \quad 3.8 < S_T^{\text{min}} / \text{TeV} < 8$$

$$N(p_T > 70 \text{ GeV}) \geq N_{\text{min}} \quad N_{\text{min}} = 3, \dots, 11$$

[Ringwald, KS, Webber 1809.10833]

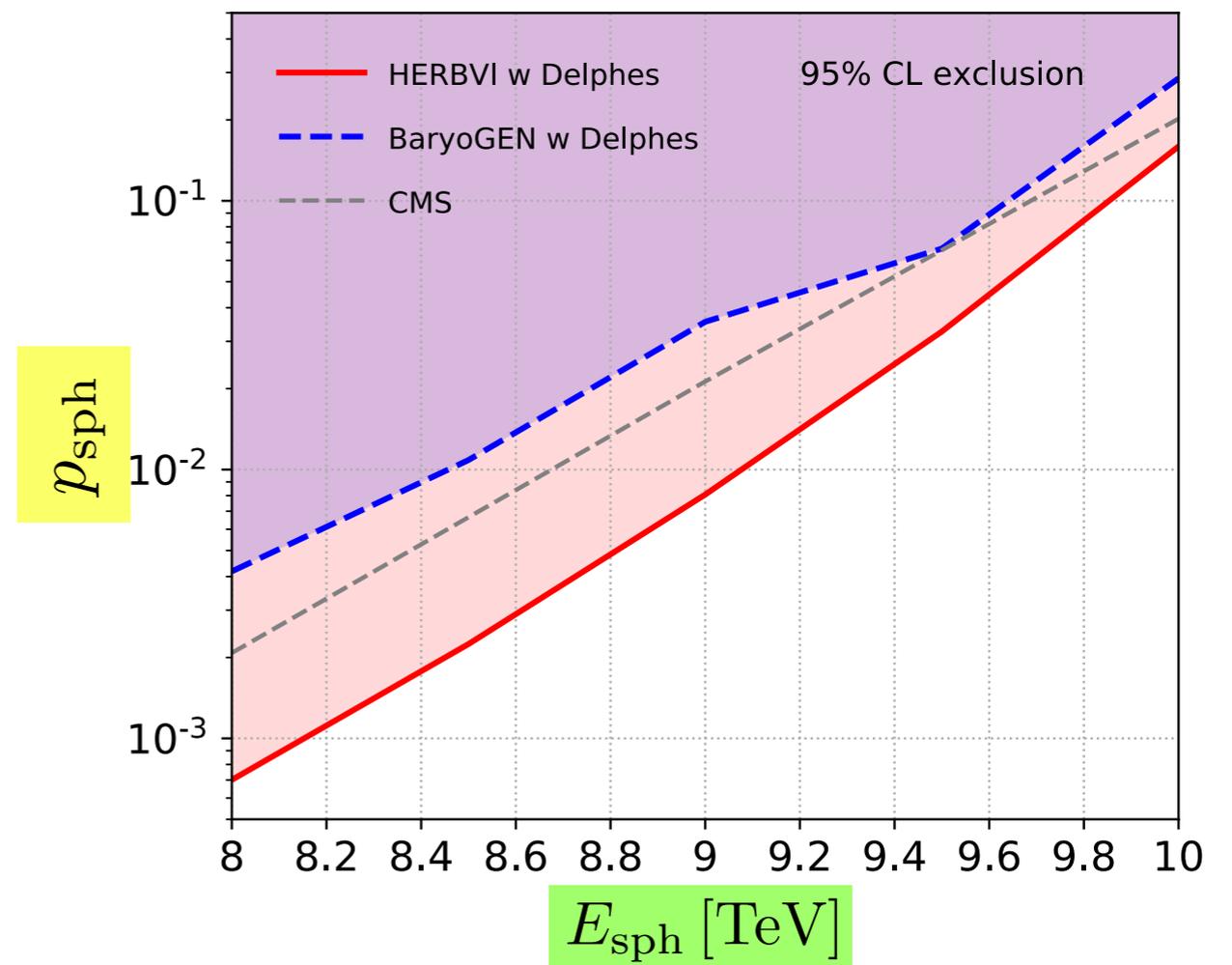
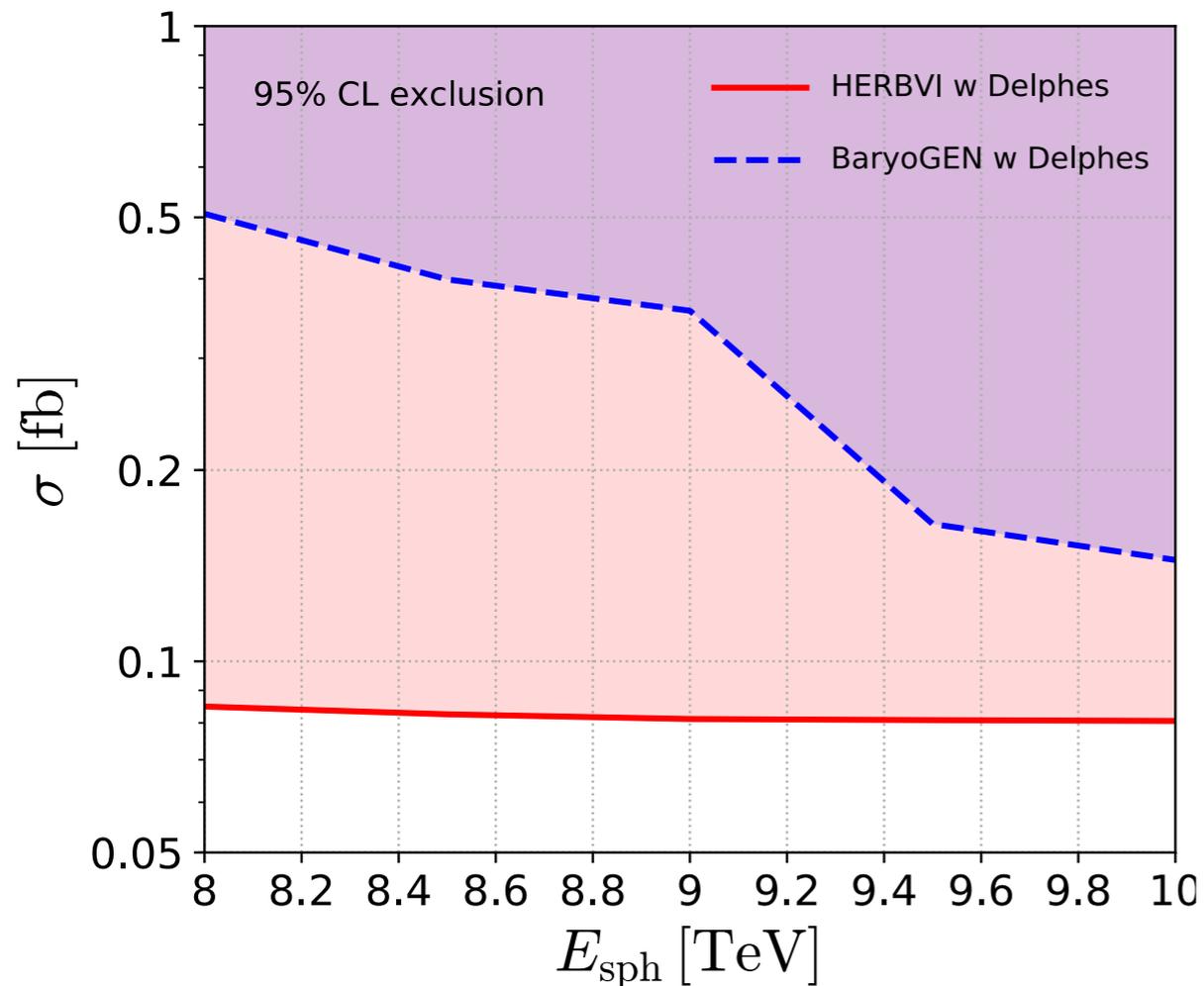
E_{sph} [TeV]		8	8.5	9	9.5	10
multi-boson	$(N_{\text{min}}, S_T^{\text{min}} [\text{TeV}])^*$	(11, 4.2)	(11, 4.2)	(11, 4.2)	(11, 4.2)	(11, 4.2)
	$\epsilon^{(a^*)} [\%]$	94.8	97.5	99.2	99.6	99.9
	$N_{\text{obs}}^{\text{max}(a^*)}$	3.0	3.0	3.0	3.0	3.0
Zero Boson	$(N_{\text{min}}, S_T^{\text{min}} [\text{TeV}])^*$	(9, 5.4)	(9, 5.6)	(9, 5.6)	(8, 6.2)	(8, 6.2)
	$\epsilon^{(a^*)} [\%]$	37.7	40.5	45.3	50.5	57.5
	$N_{\text{obs}}^{\text{max}(a^*)}$	6.9	5.8	5.8	3.0	3.0

most sensitive SR
signal efficiency
limit on signal events



Limits on Sphaleron-like process

$$\hat{\sigma}(\sqrt{\hat{s}}) = \frac{p_{\text{sph}}}{m_W^2} \Theta(\sqrt{\hat{s}} - E_{\text{sph}})$$



- The limit on the multi-boson cross-section: $\sigma_{\text{sph}} < 0.8 \text{ fb}^{-1}$
- We found that their analysis can be more tuned for multi-boson analysis by extending their range of N_{min} cut for higher energy or luminosity options.

Conclusions

- The rate of zero temperature, high energy instanton-induced process is still an open question.
- Many studies suggest that such process may have observably large rate at collider if multiple gauge bosons are produced together with fermions.
- We recast CMS analysis and found the limit, 0.8 fb^{-1}
- Their analysis can be more tuned for multi-boson final state by extending the range of N_{min} cut for higher energy or lumi options.

