Dark Matter early kinetic decoupling and finite temperature corrections to long-range force systems

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based on
[1706.07433] in collaboration with T.Bringmann, M.Gustafsson, A.Hryczuk,

[1808.06472] in collaboration with L. Covi, K. Mukaida.

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elusives
neutrinos, dark matter & dark energy physics

GK1493
DFG

$[q,p]=i\hbar$

Motivation

Standard description of DM chemical decoupling process:

$$\dot{n} + 3Hn = -\langle \sigma v \rangle_{T_{\rm SM}} \left[n^2 - n_{\rm eq}^2(T_{\rm SM}) \right]$$

Lee&Weinberg 77', Gondolo&Gelmini 91'

- implemented in DarkSUSY and MicrOMEGAs, efficiently and accurately solving for DM relic density.

  Captures special cases like co-annihilation, threshold and resonances.

- relies on certain assumptions and has limitations.
- developed refined description accounting for i) **early kinetic decoupling** ii) self-consistent treatment of **thermal corrections**.

  Planck precision era: $\Omega_{\rm DM} h^2 = 0.1198 \pm 0.0015$ (!) **, refinement of DM overlosure bound.**

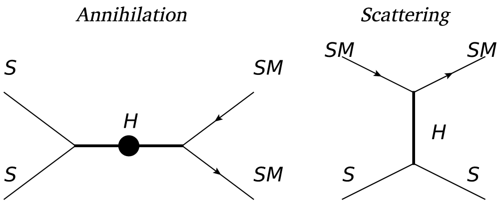
Early kinetic decoupling of dark matter: when the standard way of calculating the thermal relic density fails Binder+ 17'

- Early kd if $\Gamma_{\rm sc}|_{\rm cd} \sim \Gamma_{\rm an}|_{\rm cd}$. Not only a theoretical possibility!
- DM annihilation $\longleftrightarrow$ temperature
- developed **two methods** to accurately compute the DM relic abundance for such case

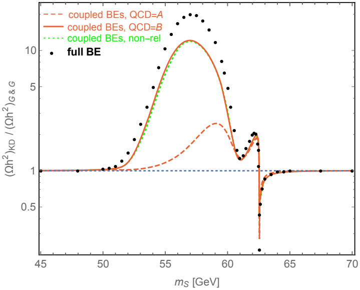
Two methods to deal with early kd

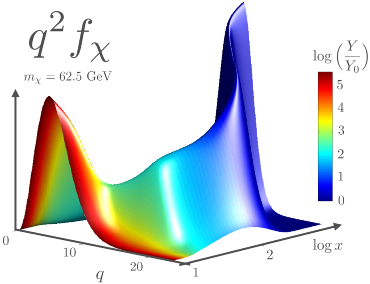
1. Full **phase-space density** solution (full BE)
   - Numerical solution of the non-linear integro PDE: $E(\partial_t - 2Hp\partial_p)f(t,p) = C_{\rm an}[f, f_{\rm eq}] + C_{\rm sc}[f, f_{\rm SM}]$
   - accounting for phase-space density distortion caused by velocity-dependent annihilation.
2. Solving coupled system of **momentum moments** (cBE)
   - Coupled system of ordinary differential equations: $\dot{n} + 3Hn = g\int \frac{d^3p}{(2\pi)^3 E} C_{\rm an}[f, f_{\rm eq}]$ $\dot{T} + \frac{\dot{n}}{n}T + 2HTw(T) = \frac{g}{3n}\int \frac{d^3p}{(2\pi)^3 E}\frac{\mathbf{p}^2}{E}\left(C_{\rm an}[f, f_{\rm eq}] + C_{\rm sc}[f, f_{\rm SM}]\right)$
   - Closer of Boltzmann hierarchy by form Ansatz $f = \frac{n}{n_{\rm eq}(T)} e^{-\frac{E}{T}}$.

Simplest example: Scalar singlet dark matter



- Two parameter model: m_S, λ_S.
- Breit-Wigner **enhanced annihilation** for $m_S \sim m_H/2$.
- Higgs couples preferably to heavy SM particles
  $\rightarrow$ Boltzmann **suppressed scattering**



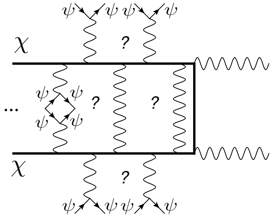


- Already in simplest scalar singlet DM model, the relic abundance can be affected by more than **one order of magnitude**!
- Public phase space density solver soon available

T.Binder, T.Bringmann, M.Gustafsson, A.Hryczuk in prep. 2018, provide many more examples: Higgs- and Z-portals, forbidden DM, Sommerfeld enhanced annihilation,...

Dark matter Sommerfeld-enhanced annihilation and Bound-state decay at finite temperature Binder+ 18'

$$\mathscr{L} \supset g_\chi \bar{\chi}\gamma^\mu \chi A_\mu + g_\psi \bar{\psi}\gamma^\mu \psi A_\mu$$



- DM (χ) longe-range force enhanced annihilation in the presence of hot and dense plasma background.
- Vacuum computation well understood.
- Finite temp. effects: at first place a conceptional question.

  Charge screening? Dissociation ? No (correct) formal description available, claims about strong finite temperature corrections based on linear response theory
- comprehensive ab initio derivation from non-eq. QFT ✓

  mostly new research in quantum statistical mechanics
- main result: new number density equation ✓

Master equation for s-wave annihilation:

$$\dot{n}_\eta + 3Hn_\eta = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)|_{\text{eq}}\left[e^{2\beta\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}|_{\text{eq}} = e^{-2\beta M}\int_{-\infty}^{\infty}\frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3}e^{-\beta\mathbf{P}^2/4M}\int_{-\infty}^{\infty}\frac{\mathrm{d}E}{(2\pi)}e^{-\beta E}G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E)|_{l=0}.$$

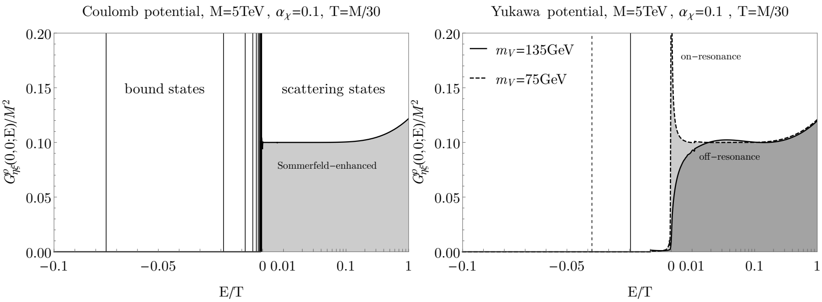
- General equation, accounting for non-ideal and finite temperature corrections.
- Chemical potential $\mu[n_\eta]$ sets functional dependence of r.h.s on n_η
- Four point correlator $G^{++--}_{\eta\xi}(x,x,x,x)|_{\text{eq}}$ in chemical equilibrium includes Sommerfeld enhanced annihilation and bound state decay simultaneously via two-particle spectral function $G^{\rho}_{\eta\xi}$. Form invariant under interpretation of bound or scattering state at finite temperature.
- Self-consistent solution of $\mu[n_\eta]$ and $G^{++--}_{\eta\xi}(x,x,x,x)|_{\text{eq}}$ is important!
- Checked various limits and compared to existing literature: vacuum limit ✓ (Lee-Weinberg eq.), linear regime close to chemical equilibrium ✓ (linear response theory)

Vacuum spectral function

Vacuum relations between s-wave two-particle spectral function $G^{\rho}_{\eta\xi}$, Sommerfeld enhancement factor S, and bound-state decay width Γ:

$$(\sigma v_{\rm rel})_0 G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E)|_{E>0,l=0} = \frac{1}{2\pi}\mathrm{Tr}[\mathbf{1}_{2\times 2}]M(ME)^{1/2}(\sigma v_{\rm rel})_0 S(E),$$

$$(\sigma v_{\rm rel})_0 G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E)|_{E<0,l=0} = \frac{\pi}{2}\mathrm{Tr}[\mathbf{1}_{2\times 2}]\sum_n \delta(E-E_{B_n})\Gamma_n.$$

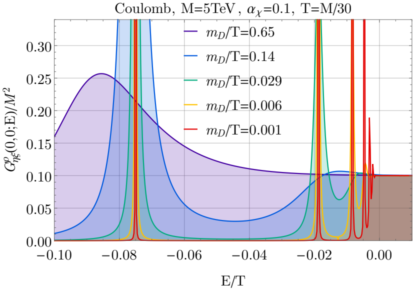


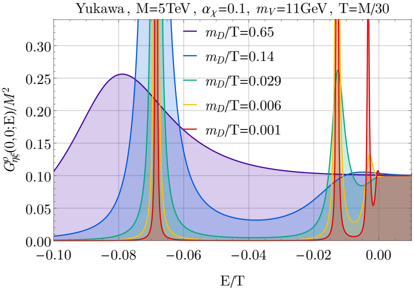
Spectral function at finite temperature

Effective in-medium potential (in HTL resummed effective theory):

$$V_{\text{eff}}(\mathbf{r}) = -\alpha_\chi m_D - \frac{\alpha_\chi}{r} e^{-m_D r} - i\alpha_\chi T \phi(m_D r),$$

where $\phi(0) = 0$ and $\phi(\infty) = 1$, and Debye $m_D^2 = g_\psi^2 T^2/3$.





Notation of spectral function more general than using S or Γ.
Shift towards lower energies affects total rate exponentially.

Summary, Conclusion, Outlook

- Questioning the assumptions entering the standard description of the DM freeze-out can lead to surprising new phenomena.
- Early kinetic decoupling can have huge impact on DM overclosure bound.
- Public phase-space density code with many more examples soon available.
- Developed rigorous theoretical understanding of long-range force systems at finite temperature.
- If bound state solution exist ($m_V \lesssim \alpha_\chi M$) they are important to include in the relic density computation.
- New number density equation allows to include finite temperature effects self-consistently.
- Impact of finite temperature effects on relic density needs careful investigation, work in progress.