QCD bound-state effect on dark matter relic abundance

Ellis, FL, Olive, 2015
Ellis, Evans, FL, Olive, 2016
Liew, FL, 2017
Ellis, Evans, FL, Olive, Zheng, 2018

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Why are we interested in calculating dark matter relic abundance?

Because

this is the ***only precise quantity*** we know about dark matter.

Therefore, need to work out its implications to the underlying particle theory models as much as possible by carefully calculating it.

$$\Omega_{CDM} h^2 = 0.1193 \pm 0.0014 \quad (\textit{1-}\sigma\textit{, Planck 2015})$$

The framework we use for the calculations is supersymmetry.

Why use it?

Because

supersymmetry is one of the ***best*** candidates for physics beyond the Standard Model.

We study neutralino dark matter,
and
we use thermal freeze-out mechanism.

Why focus on these?

Because

neutralino is one of the ***best*** candidates for dark matter

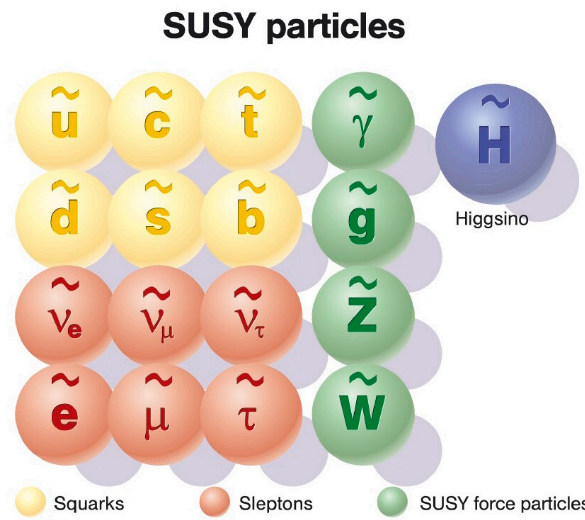
--- a typical WIMP

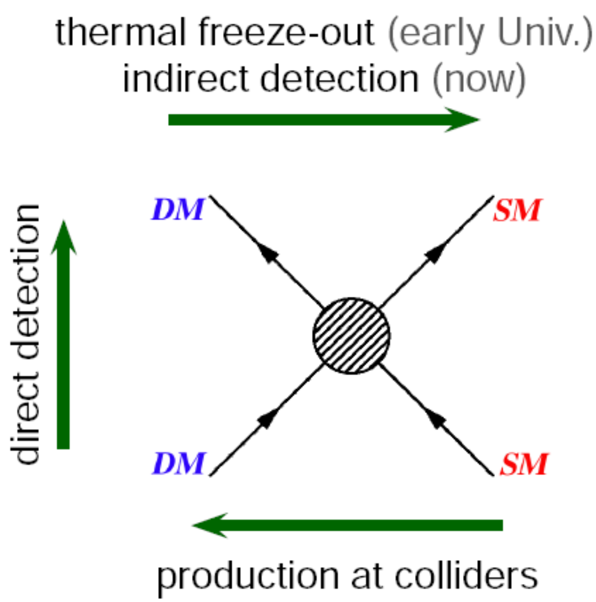
Thermal freeze-out mechanism is a

standard mechanism to get the dark matter relic abundance.

Supersymmetric Dark Matter (SUSY DM)

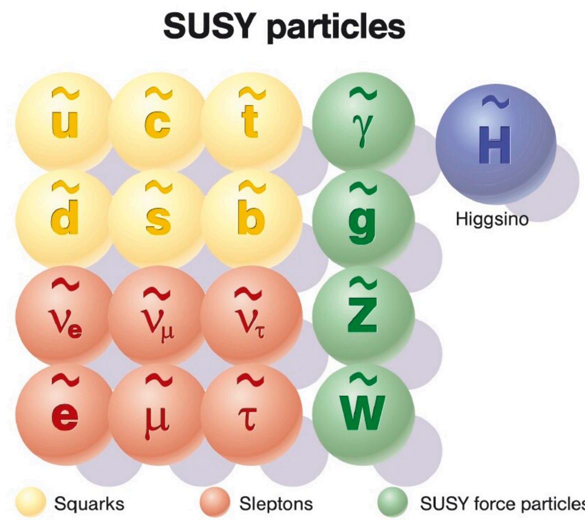
- ✓ theoretically *well motivated*
- ✓ *testable* in the current and forthcoming experiments

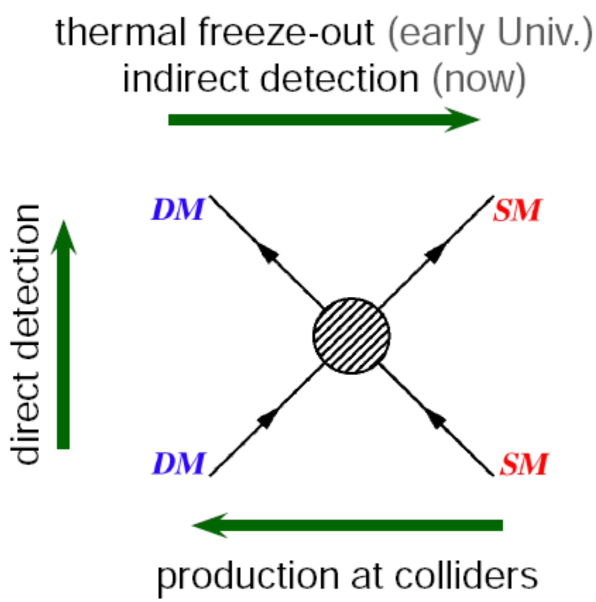




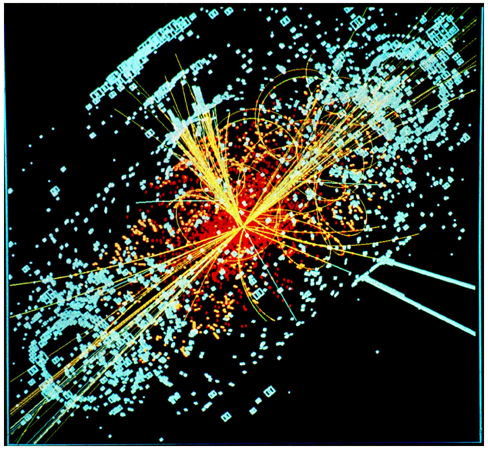
Supersymmetric Dark Matter (SUSY DM)

HOWEVER,
no signal yet.



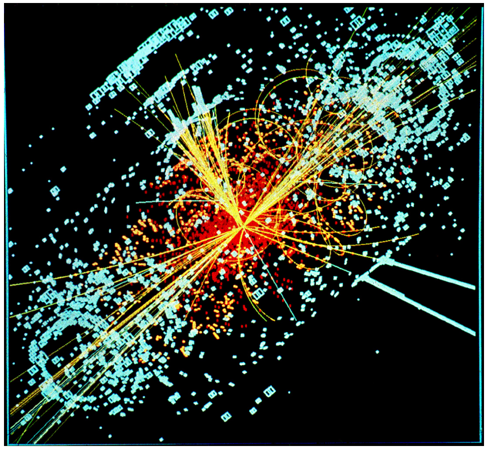


Maybe just too heavy to be produced?



How heavy can dark matter be in supersymmetry?

The answer is useful in assessing the energy needed for a (future) collider to be "*guaranteed*" to discover or exclude supersymmetric dark matter.



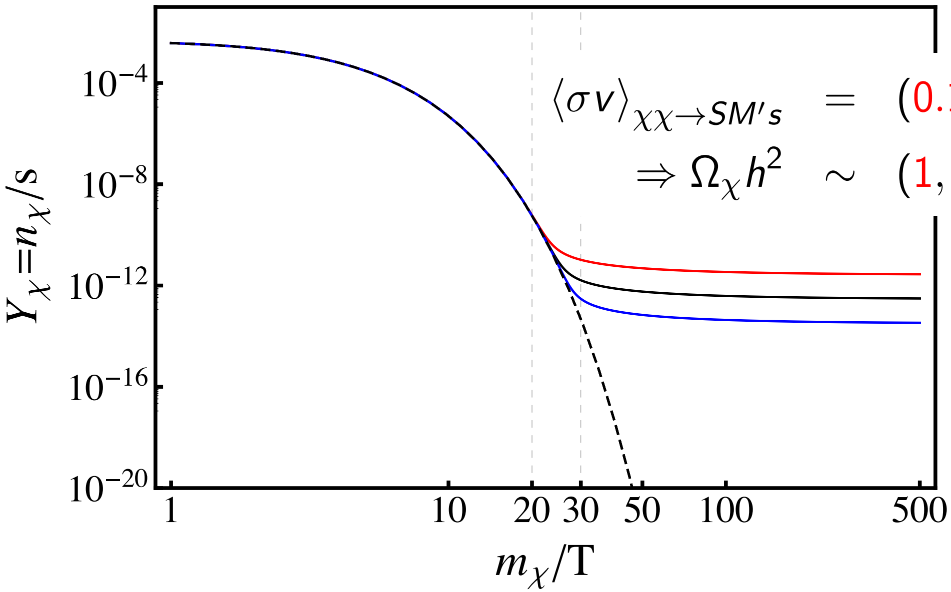
specify the question

We consider

- The *simplest* version of SUSY --- R-parity conserving MSSM
- The *most studied* DM candidate --- neutralino
- The *standard* mechanism to calculate relic abundance --- freeze-out
- *Coannihilation* between neutralino and some colored particle

thermal freeze-out mechanism

$$\frac{dn_\chi}{dt} + 3H(T)n_\chi = -\langle \sigma v \rangle_{\chi\chi \to SM's} \left[n_\chi^2 - \left(n_\chi^{eq}\right)^2 \right]$$



$\langle \sigma v \rangle_{\chi\chi \to SM's} \sim \alpha^2/m_\chi^2$ (perturbative regime $\alpha < 1$),
larger $m_\chi \Rightarrow$ smaller $\langle \sigma v \rangle_{\chi\chi \to SM's} \Rightarrow$ larger $\Omega_\chi h^2$,
$\Rightarrow$ an upper limit for m_χ

I'm a Bino.

I'm the expanding Universe.

$$\frac{dn_\chi}{dt} + 3H(T)n_\chi = -\langle \sigma v \rangle_{\chi\chi \to SM's} \left[n_\chi^2 - \left(n_\chi^{eq} \right)^2 \right]$$

Conditions for coannihilation to reduce DM relic density

If there is another R-odd species χ_2 almost degenerate in mass with the LSP χ_1,

and if χ_2 has a big annihilation cross section with itself and/or with χ_1,

and if χ_1 can efficiently convert to χ_2,

then χ_1 and χ_2 can freeze out together, resulting in a smaller dark matter abundance than if without the existence of χ_2.

Griest and Seckel, 1991

I’m a coannihilator.

I’m a Bino.

I’m the expanding Universe.

Conditions for coannihilation to reduce DM relic density

Define $n \equiv n_1 + n_2$ and $n_{eq} \equiv n_1^{eq} + n_2^{eq}$,

$$\frac{dn}{dt} + 3Hn = -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2} \left[n^2 - n_{eq}^2 \right]$$

(Recall w/o coannihilation: $\boxed{\frac{dn_\chi}{dt} + 3H(T)n_\chi = -\langle \sigma v \rangle_{\chi\chi \to SM's} \left[n_\chi^2 - (n_\chi^{eq})^2 \right]}$)

Note that $n_i^{eq} = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T}$ for $T \ll m_i$

- if $m_2 \gg m_1$, then $n_{eq} \approx n_1^{eq}$, ●● $\approx \langle \sigma v \rangle_{11 \to SM}$

  i.e., no coannihilation

- if $m_2 = m_1$, then ●● $= \frac{g_1^2 \langle \sigma v \rangle_{11 \to SM} + g_2^2 \langle \sigma v \rangle_{22 \to SM} + 2 g_1 g_2 \langle \sigma v \rangle_{12 \to SM}}{(g_1 + g_2)^2}$

  if the middle term dominates, then ●● $\approx \left(\frac{g_2}{g_1 + g_2} \right)^2 \langle \sigma v \rangle_{22 \to SM}$

I’m a coannihilator.

I’m a Bino.

I’m the expanding Universe.

To get the largest Bino dark matter mass, we just need to find his fastest running and most muscular friend.

Bino-gluino coannihilation

$$\chi\chi \leftrightarrow SM,\ \chi\tilde{g} \leftrightarrow q\bar{q},\ \tilde{g}\tilde{g} \leftrightarrow q\bar{q} \text{ or } gg,$$

$$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g, \tilde{R} \leftrightarrow gg,$$

$$\chi q \leftrightarrow \tilde{g}q,\ \tilde{g} \leftrightarrow \chi q\bar{q}$$

Bino-gluino coannihilation

$$\chi\chi \leftrightarrow SM,\ \chi\tilde{g} \leftrightarrow q\bar{q},\ \tilde{g}\tilde{g} \leftrightarrow q\bar{q} \text{ or } gg$$

(1) Sommerfeld effects for $\tilde{g}\tilde{g} \to q\bar{q}$ or gg

Explanation:

Depending on the colour configuration of the initial $\tilde{g}\tilde{g}$, the long range Coulomb-like potential between $\tilde{g}\tilde{g}$ can be attractive or repulsive.

$\Rightarrow$ modify the otherwise free initial particle wave function

Baer, Cheung and Gunion, 1999
Profumo and Yaguna, 2004
De Simone, Giudice and Strumia, 2014
Harigaya, Kaneta and Matsumoto, 2014

Bino-gluino coannihilation

(2) Gluino bound-state effect

$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g$, $\tilde{R} \leftrightarrow gg$

Explanation:

- $\tilde{g}\tilde{g}$ can form a positronium-like bound state $\tilde{R}$
- $\tilde{R} \to gg$ removes two R-odd particles $\Longrightarrow$ decreases the final R-odd particle number density (i.e., DM number density)

Bino-gluino coannihilation

(2) Gluino bound-state effect

$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g$, $\tilde{R} \leftrightarrow gg$

Coulomb potential	$\sim$	$-\alpha_s/r$
Bohr radius	$\sim$	$(\alpha_s m_{\tilde{g}})^{-1}$
binding energy	$\sim$	$\alpha_s^2 m_{\tilde{g}}$
$\tilde{R}$ annihilation decay rate	$\sim$	$\alpha_s^5 m_{\tilde{g}}$
individual $\tilde{g}$ decay rate	$\sim$	$(m_{\tilde{g}} - m_\chi)^5 m_{\tilde{q}}^{-4}$

Explanation:

- $\tilde{g}\tilde{g}$ can form a positronium-like bound state $\tilde{R}$
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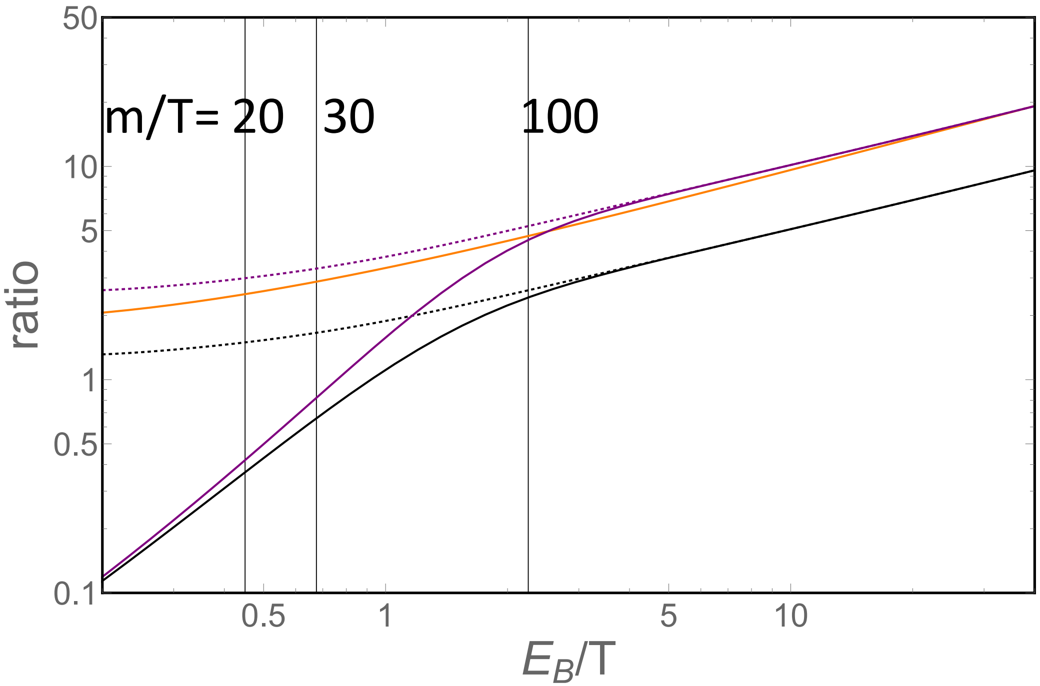
Bino-gluino coannihilation



Due to dissociation, bound-state effect catches up Sommerfeld effect after $T \lesssim E_B$

Solid lines: compare Sommerfeld enhancement with bound-state effect

The “ratios” are normalized to the tree-level annihilation cross section.
Purple lines enlarge the bound-state effect by a factor of 2 comparing to black lines.

Dashed lines: if there were no dissociation process

Bino-gluino coannihilation

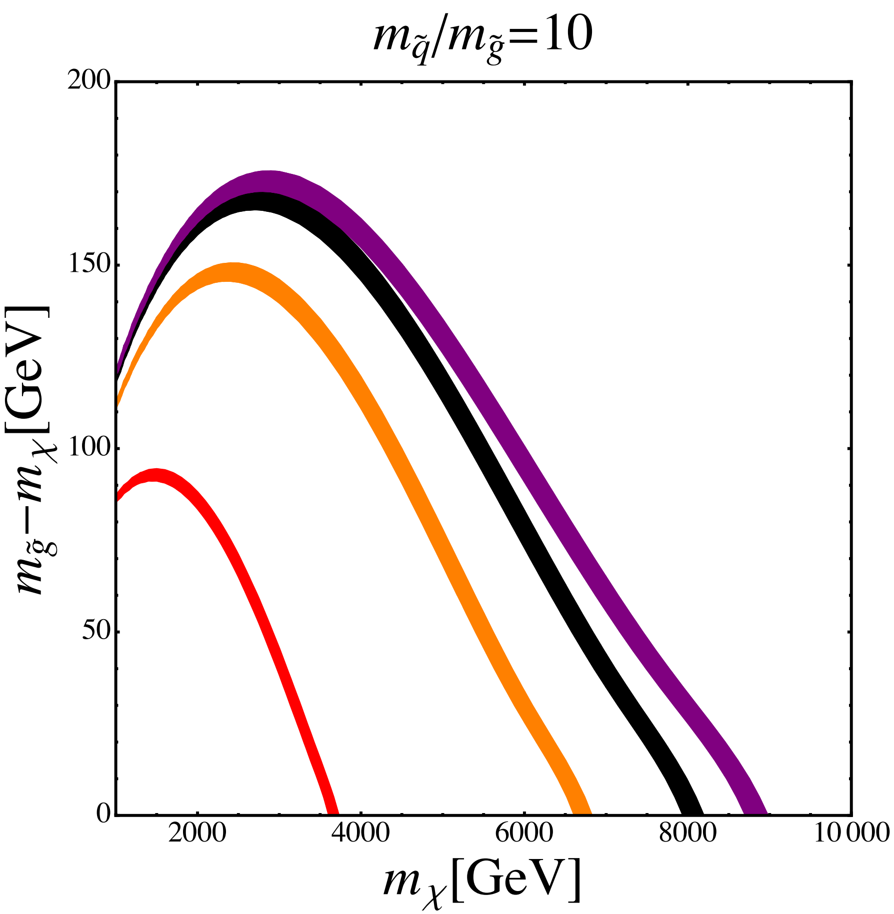
(2) Gluino bound-state effect

$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g$, $\tilde{R} \leftrightarrow gg$

$$\Rightarrow \frac{dn}{dt} + 3Hn \approx \quad - \sum_{i,j=\chi,\tilde{g}} \langle \sigma v \rangle_{ij \to SM} \left[n_i n_j - n_i^{eq} n_j^{eq} \right]$$

$$- \langle \sigma v \rangle_{\tilde{g}\tilde{g} \to \tilde{R}g} \frac{\langle \Gamma \rangle_{\tilde{R} \to gg}}{\langle \Gamma \rangle_{\tilde{R} \to gg} + \langle \Gamma \rangle_{\tilde{R}g \to \tilde{g}\tilde{g}}} \left[n_{\tilde{g}} n_{\tilde{g}} - n_{\tilde{g}}^{eq} n_{\tilde{g}}^{eq} \right]$$

Bino-gluino coannihilation



The bands give correct DM relic abundance: $\Omega_{\chi}h^2 = 0.1193 \pm 0.0042$ (i.e., 3-σ)

red:	w/o Sommerfeld and w/o bound-state
orange:	w/ Sommerfeld but w/o bound-state
black:	w/ Sommerfeld and w/ bound-state
purple:	w/ Sommerfeld and w/ 2 times bound-state

coannihilation

I'm a coannihilator.

I'm a Bino.

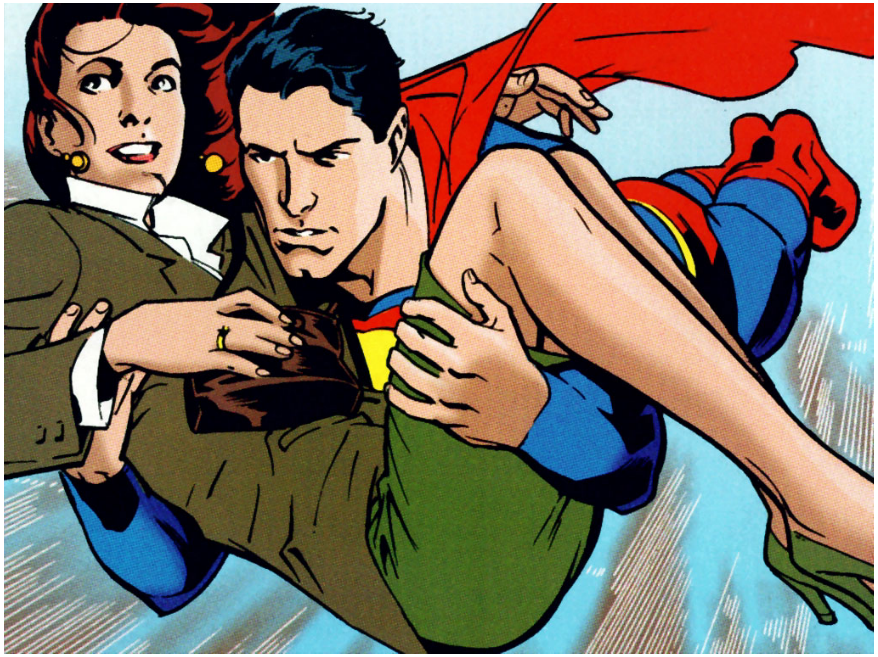
I'm the expanding Universe.

"Dear Gluino, are you the fastest running and most muscular guy?"

"Yes!"

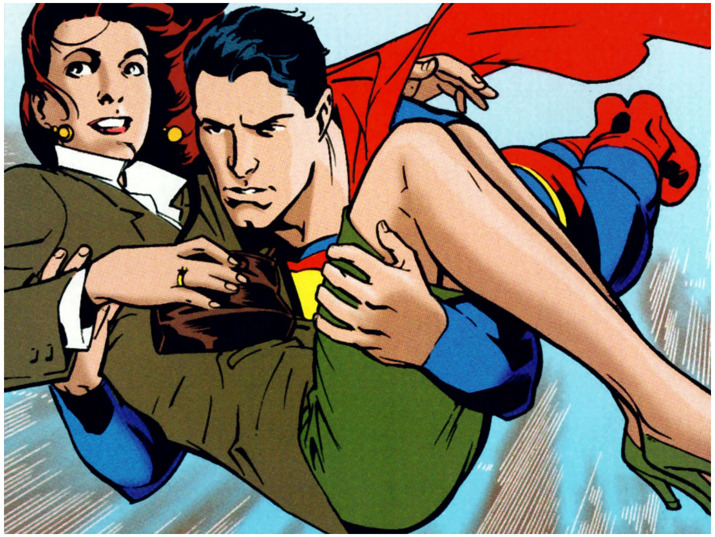
coannihilation with Sommerfeld and bound-state effects

I’m a Bino. I’m a gluino.



I’m the expanding Universe.

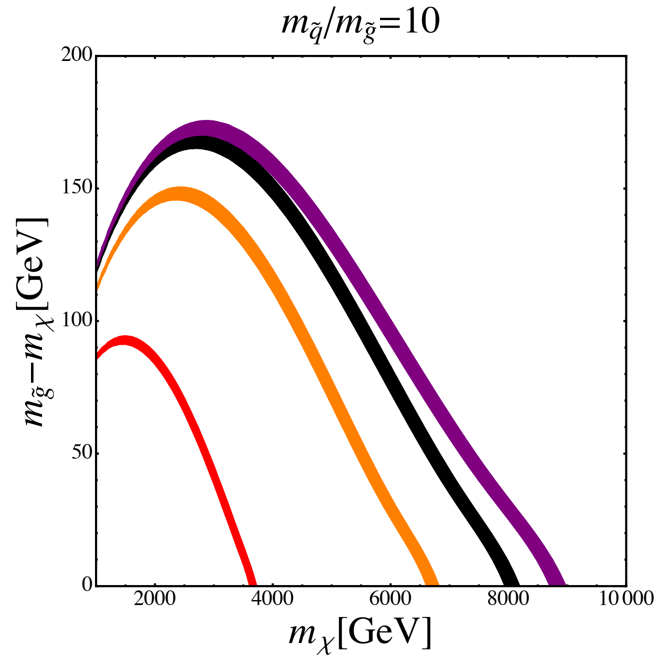
coannihilation with Sommerfeld and bound-state effects

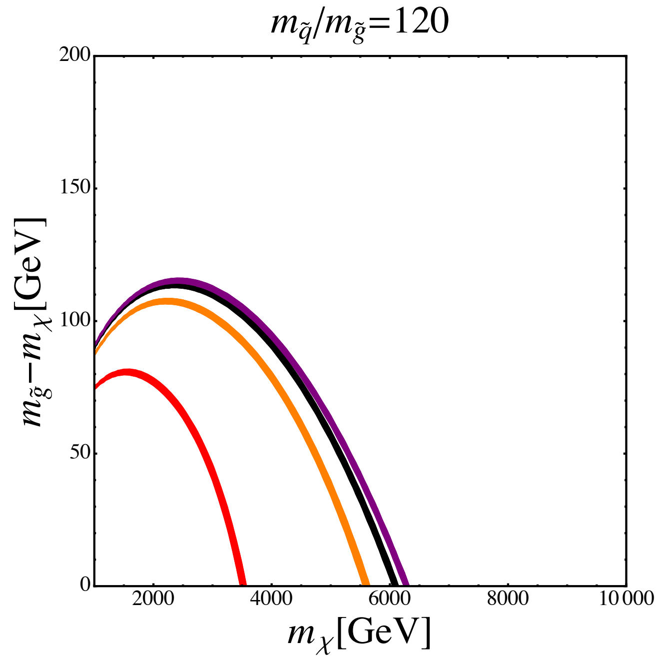


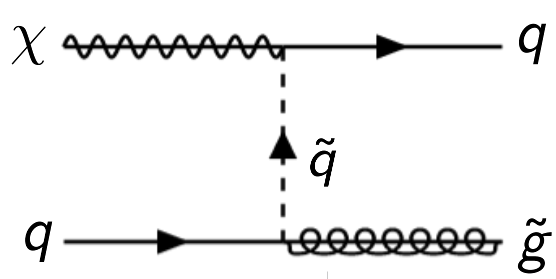
The gluino, $\tilde{g}$, with the largest colour charge, is the strongest coannihilation particle in the MSSM.

The gluino-neutralino coannihilation scenario may give the largest possible neutralino DM mass within the coannihilation thermal freeze-out mechanism.

Bino-gluino coannihilation







Bino-gluino coannihilation

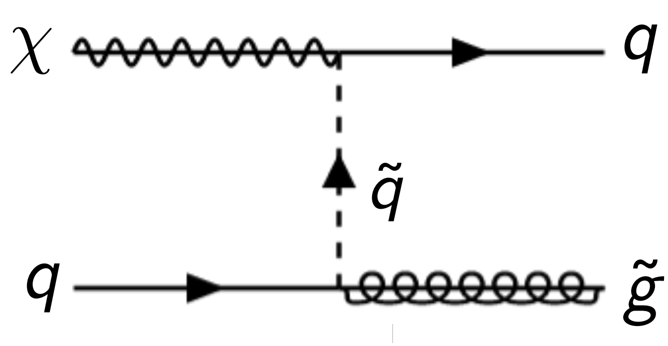
$\chi\chi \leftrightarrow SM$, $\chi\tilde{g} \leftrightarrow q\bar{q}$, $\tilde{g}\tilde{g} \leftrightarrow q\bar{q}$ or gg,
$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g, \tilde{R} \leftrightarrow gg$,
$\chi q \leftrightarrow \tilde{g}q$, $\tilde{g} \leftrightarrow \chi q\bar{q}$

(3) Breakdown of coannihilation by large squark masses

$\chi q \leftrightarrow \tilde{g}q$, $\tilde{g} \leftrightarrow \chi q\bar{q}$

Chung, Farrar and Kolb, 1997

Explanation:



$\Rightarrow$ coannihilation mechanism breaks down, and therefore Sommerfeld enhancement and bound-state effect cannot reduce the χ number density even if they are large and even if $\tilde{g}$ and χ are degenerate in mass

coannihilation breaks down

Sorry, squarks are too heavy.
I cannot give you a hand...

Bino-stop coannihilation

$$\tilde{t}\tilde{t}^* \leftrightarrow q\bar{q}, gg, W^+W^-, ZZ, \ldots$$

$$\tilde{t}\tilde{t}^* \leftrightarrow \tilde{R}g, \tilde{t}\tilde{t}^* \leftrightarrow \tilde{R}\gamma$$

$$\tilde{R} \leftrightarrow gg, W^+W^-, ZZ, \ldots$$

Bino-stop coannihilation

$$\tilde{t}\tilde{t}^* \leftrightarrow q\bar{q}, gg, W^+W^-, ZZ, \ldots$$
$$\tilde{t}\tilde{t}^* \leftrightarrow \tilde{R}g, \tilde{t}\tilde{t}^* \leftrightarrow \tilde{R}\gamma$$
$$\tilde{R} \leftrightarrow gg, W^+W^-, ZZ, \ldots$$

New ingredients compared to the gluino case:

✓ stop anti-stop color **potential prior** to forming a bound state is **repulsive**, while the one for gluino pair is attractive

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

vs. $$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_S \oplus \mathbf{8}_A \oplus \mathbf{8}_S \oplus \mathbf{10}_A \oplus \overline{\mathbf{10}}_A \oplus \mathbf{27}_S$$

stop is a scalar triplet

gluino is a fermion octet

✓ stop has electric charge, while gluino does not

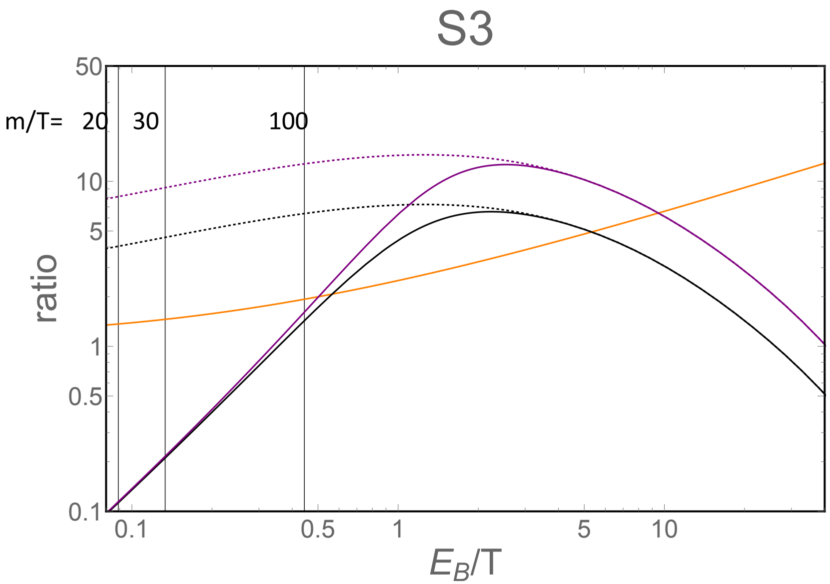
(1) affect the potential

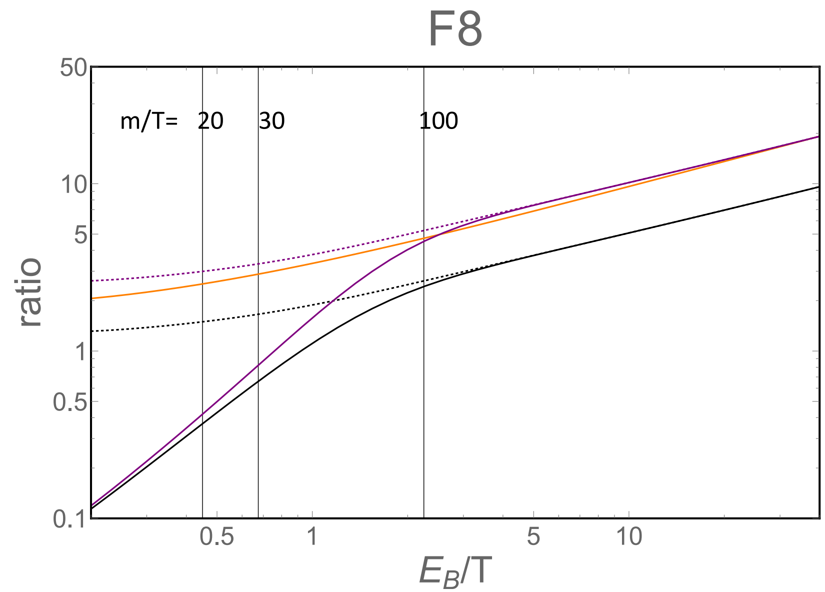
(2) photon emission/absorption processes

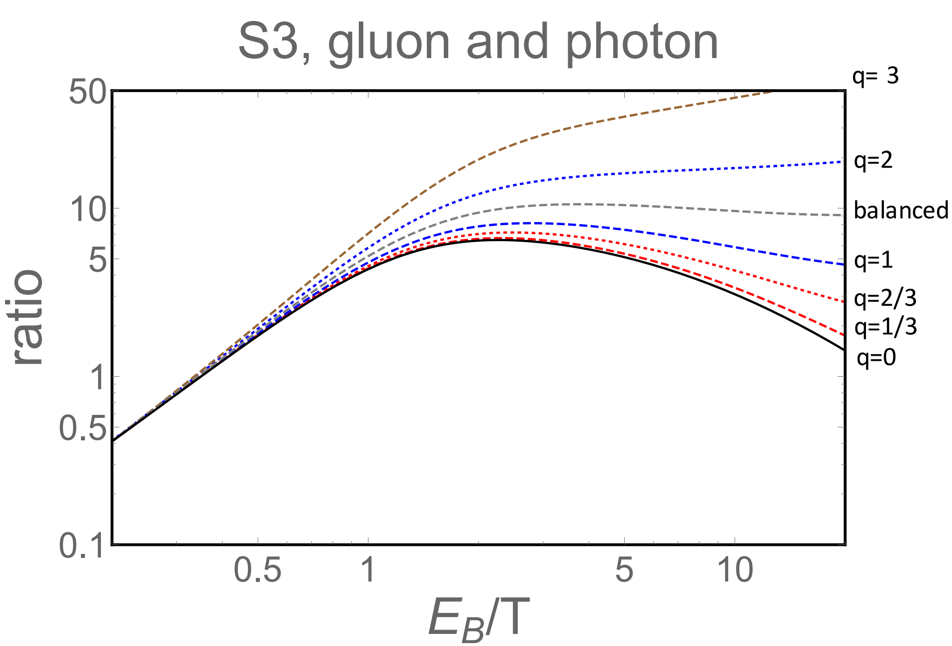
✓ stop anti-stop has more annihilation channels and more annihilation decay channels

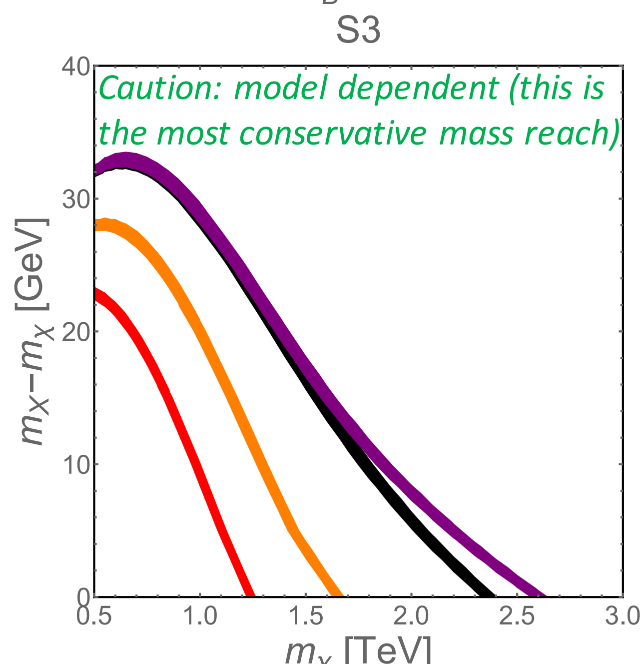
stop = S3
gluino = F8

Bino-stop coannihilation









probe strongly interacting particle coannihilation scenarios in colliders

✓ monojet searches (Low & Wang, 1404.0682)

coannihilator	bkgd. syst.	14 TeV		100 TeV	
		95% limit	5σ discovery	95% limit	5σ discovery
gluino	1%	1.1 TeV	950 GeV	6.2 TeV	5.2 TeV
	2%	1.0 TeV	850 GeV	5.8 TeV	4.8 TeV
stop	1%	530 GeV	420 GeV	2.8 TeV	2.1 TeV
	2%	470 GeV	330 GeV	2.4 TeV	1.7 TeV
squark	1%	740 GeV	600 GeV	4.0 TeV	3.0 TeV
	2%	630 GeV	495 GeV	3.5 TeV	2.6 TeV

✓ long-lived colored particles with displaced vertices
(Nagata, Otono & Shirai, 1504.00504)

$$c\tau_{\tilde{g}} = \mathcal{O}(1) \times \left(\frac{\Delta M}{100\,\mathrm{GeV}}\right)^{-5} \left(\frac{m_{\tilde{q}}}{100\,\mathrm{TeV}}\right)^{4} \mathrm{cm}$$

✓ squark-gluino associated production (S. Ellis & B. Zheng, 1506.02644)

Summary

(1) In the coannihilation scenario, bound-state effect can significantly **enhance** the DM effective **annihilation cross section**. The size of the bound-state effect is comparable to the Sommerfeld effect. ***Note that these two effects are independent.***

(2) Too large squark masses can **break down** the neutralino-gluino **coannihilation mechanism**, due to not fast enough conversion rate between neutralino and gluino.

(3) The potential between the massive colored particles after forming a bound state is attractive, but the potential between them **prior** to forming a bound state can be **either attractive or repulsive**.

How heavy can dark matter be in supersymmetry?

Answer: neutralino dark matter can be as heavy as ~ 8 TeV in neutralino-gluino coannihilation scenario, and $\gtrsim 2.5$ TeV in neutralino-stop coannihilation scenario.

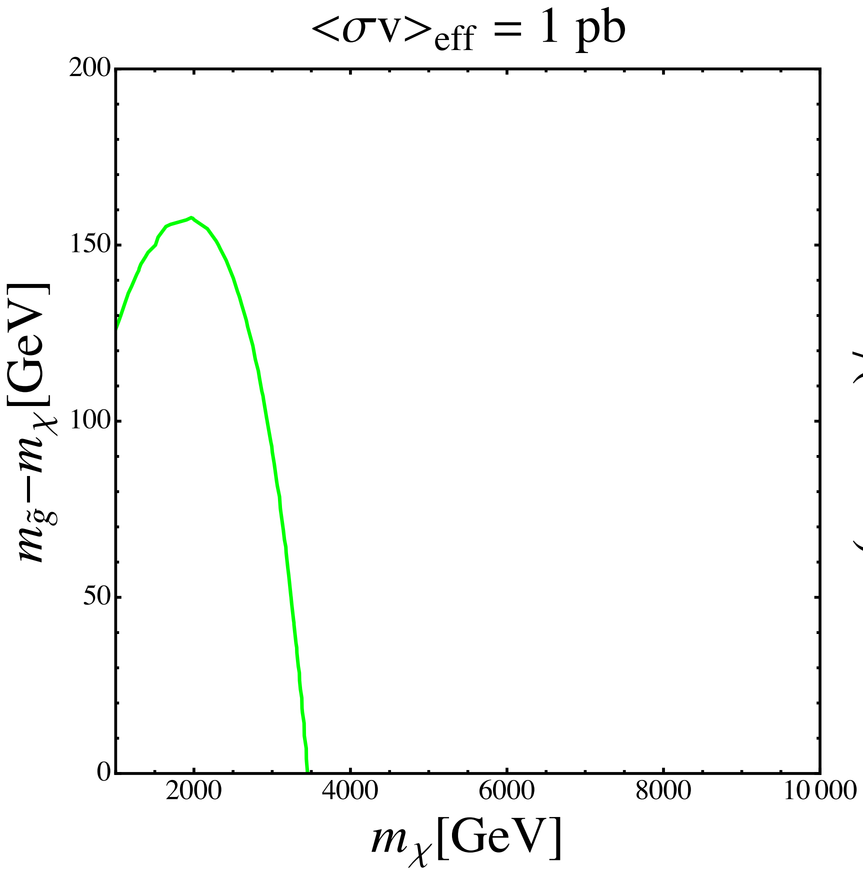
How heavy can dark matter be in supersymmetry?

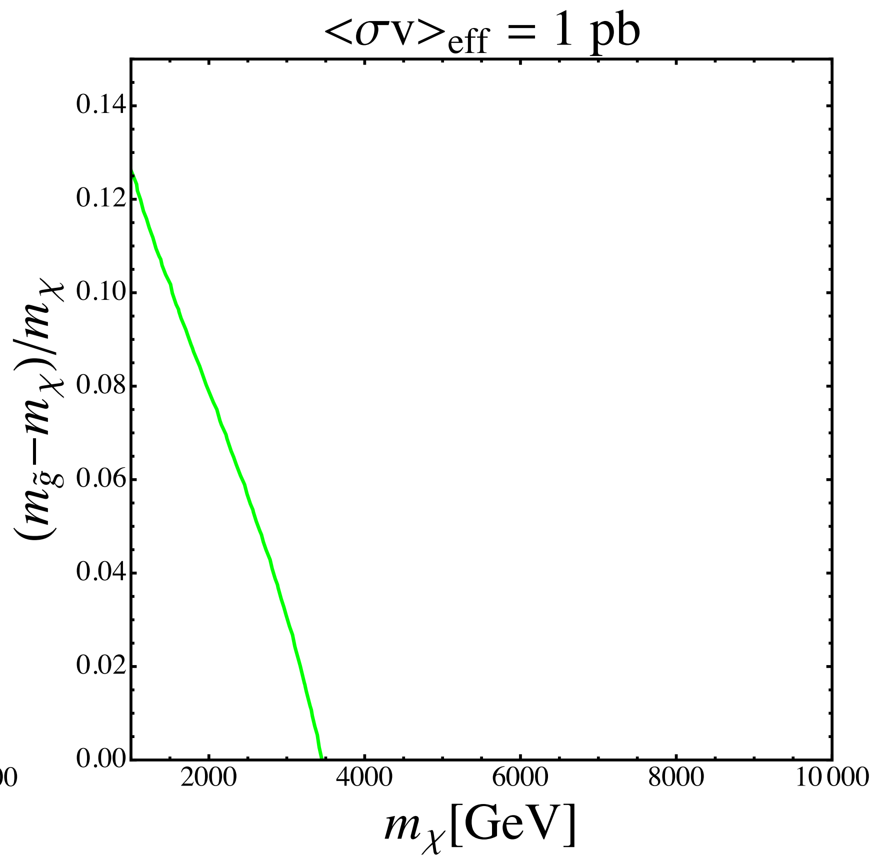
Answer: neutralino dark matter can be as heavy as ~ 8 TeV in neutralino-gluino coannihilation scenario, and $\gtrsim 2.5$ TeV in neutralino-stop coannihilation scenario.

Hajime Fukuda will give a different answer tonight ☺

Thank you!

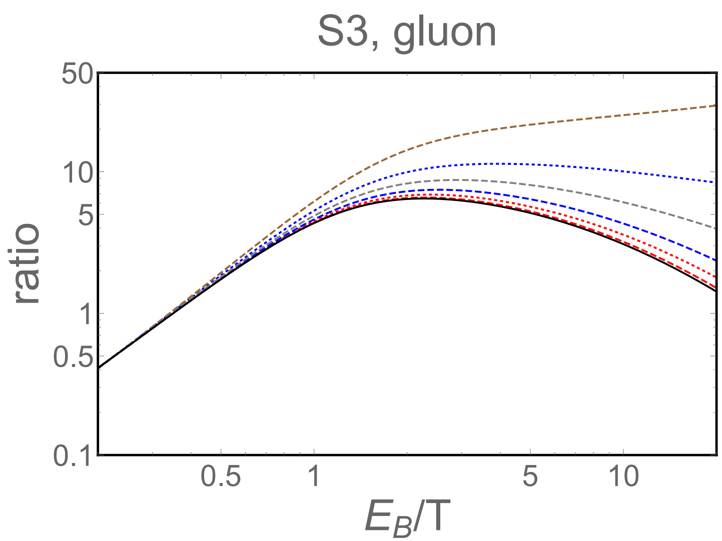
backup: the reason why the Δm vs. m_χ plot has the shape

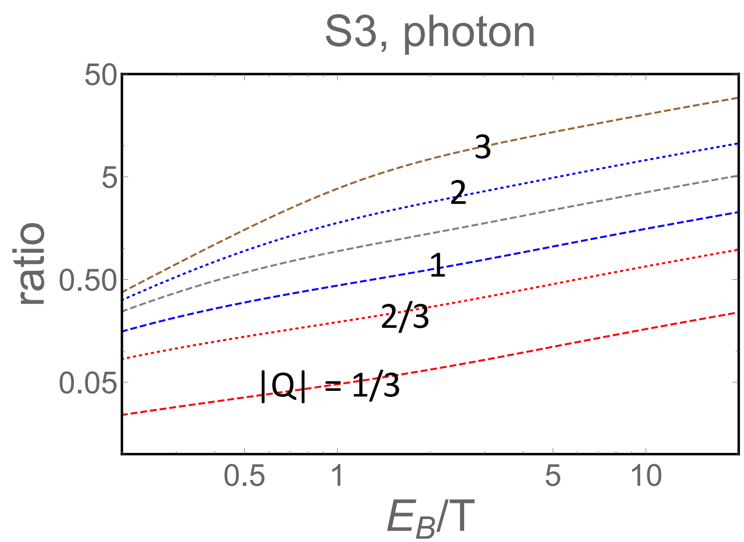


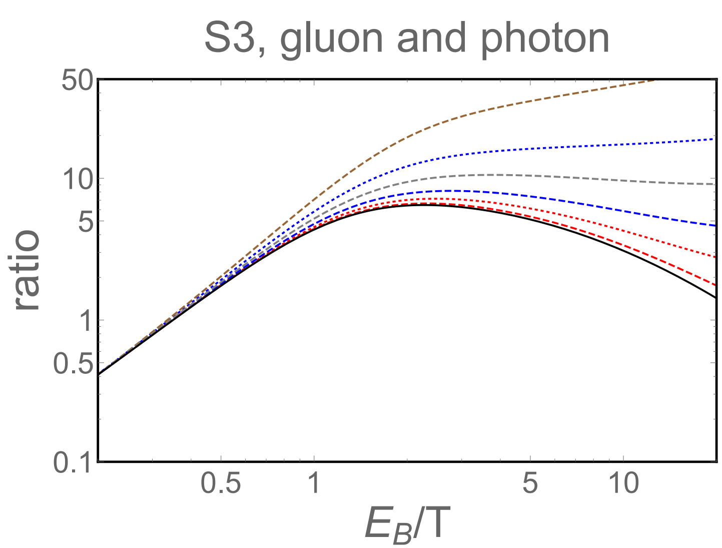


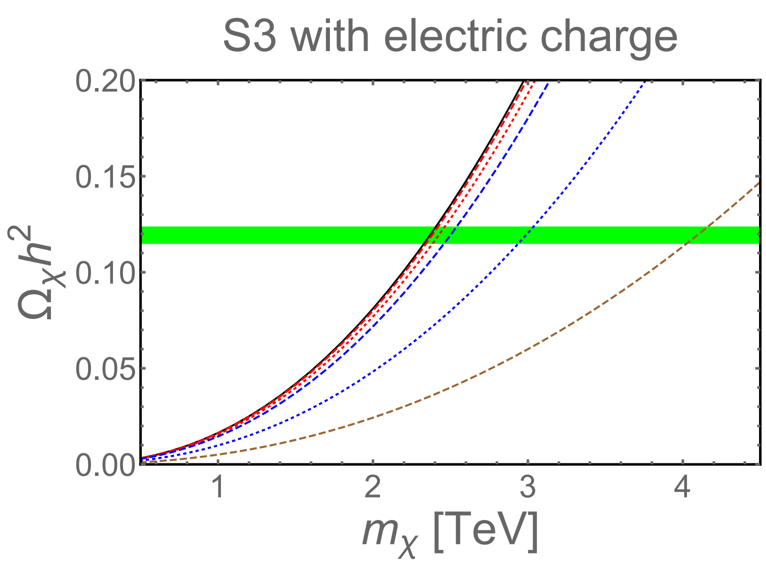
Bino-stop coannihilation

$$\tilde{t}\tilde{t}^* \leftrightarrow \tilde{R}g,\, \tilde{t}\tilde{t}^* \leftrightarrow \tilde{R}\gamma,\, \tilde{R} \leftrightarrow gg$$

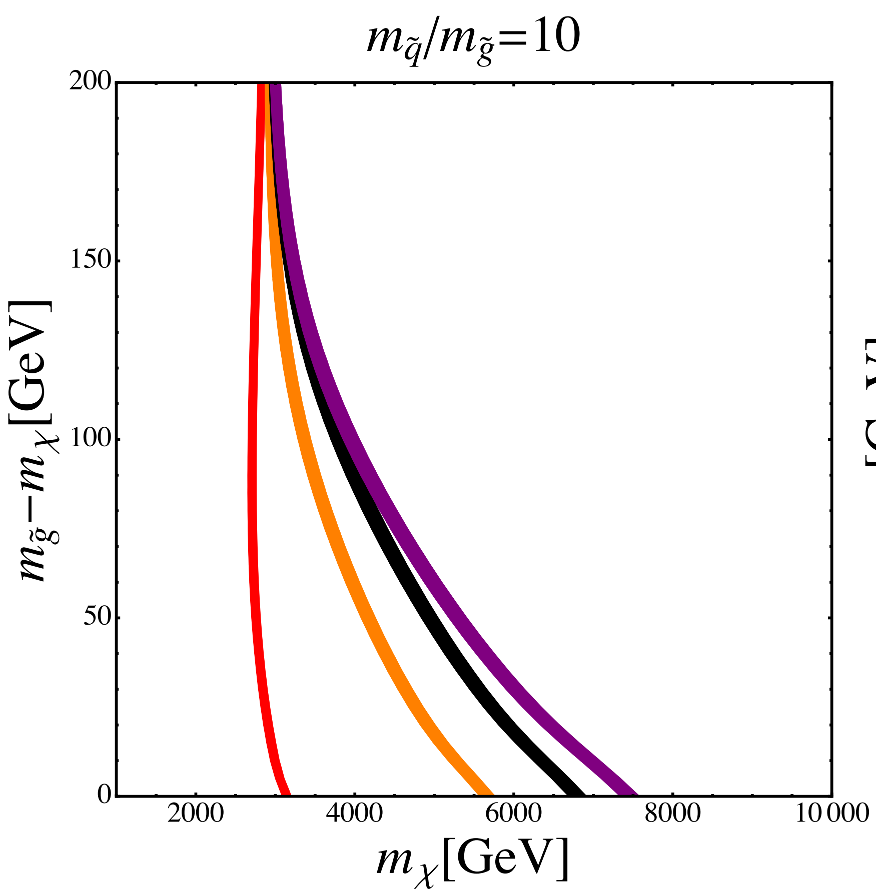


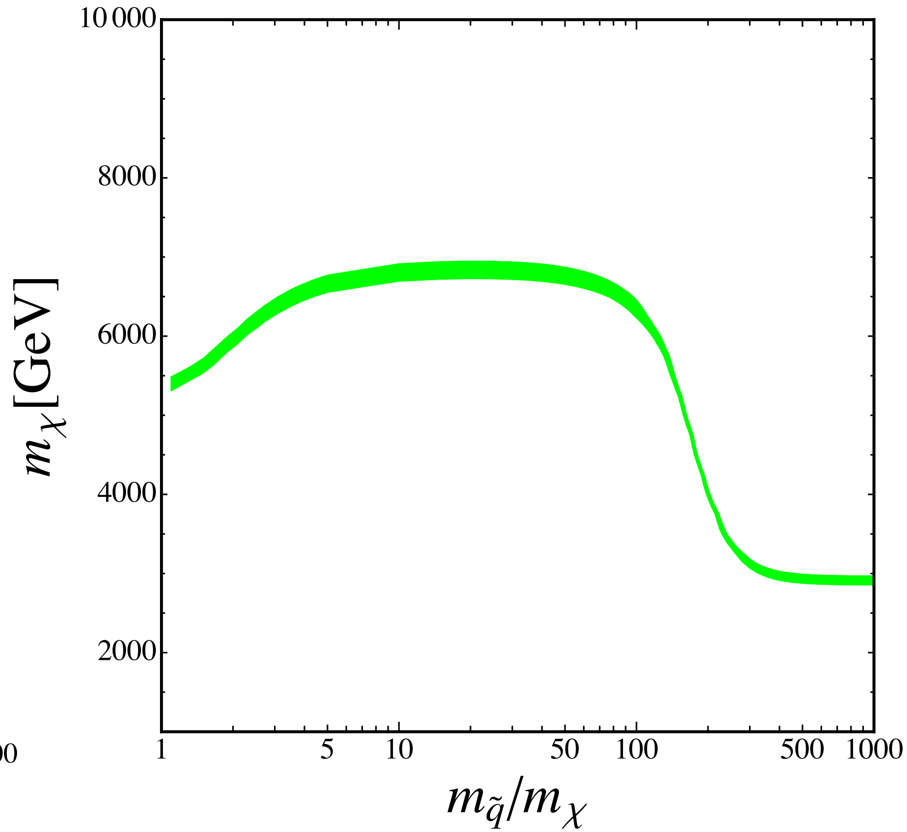




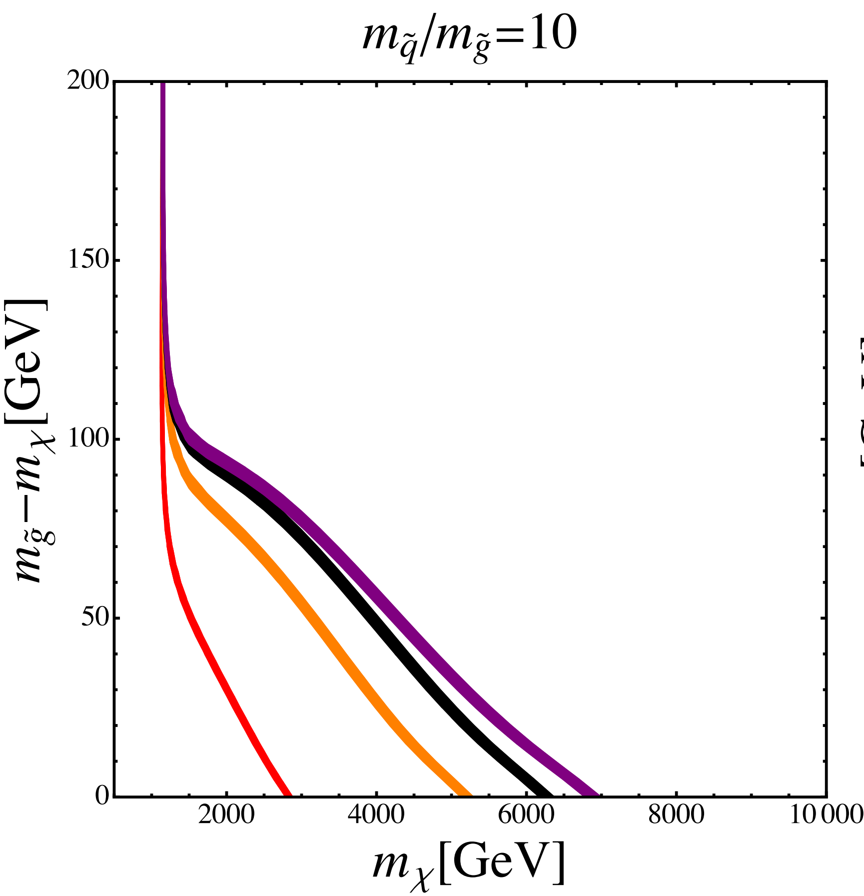


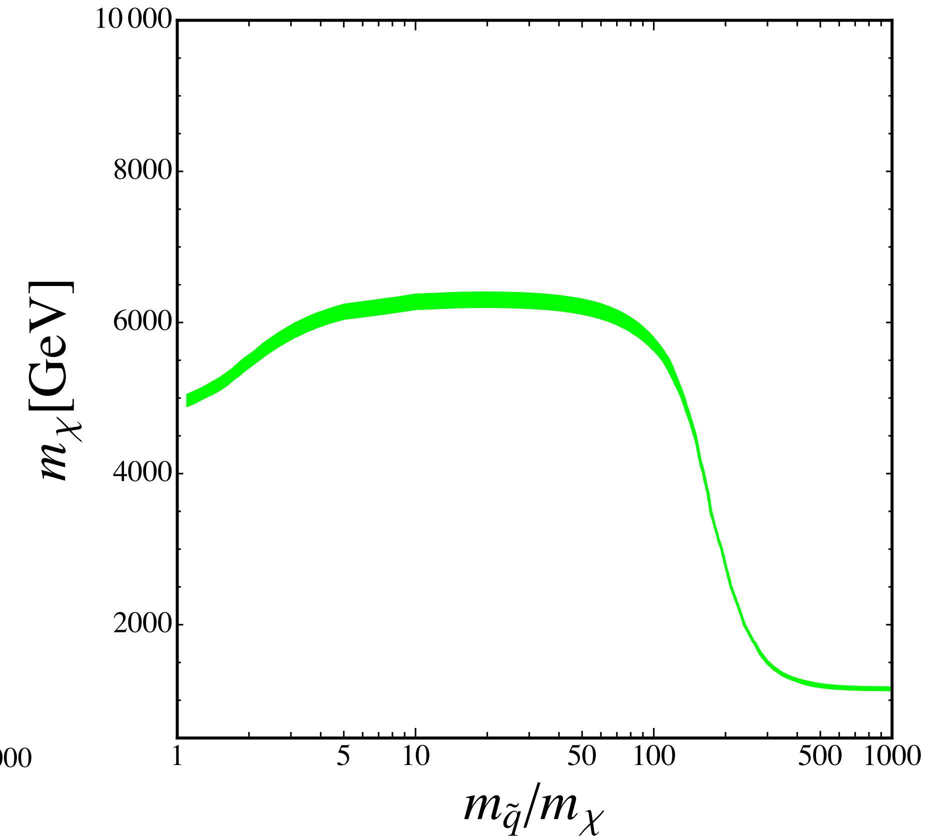
Wino-gluino coannihilation





Higgsino-gluino coannihilation





A remark

Why the maximum LSP mass is smaller for a Wino ($\sim$ 7 TeV) or a Higgsino ($\sim$ 6 TeV) compared to a Bino ($\sim$ 8 TeV)?

Because there are more *inert* degrees of freedom for Wino (=6) or Higgsino (=8) compared to Bino (=2) at large mass when $\chi\chi$ and $\chi\tilde{g}$ (co)annihilation cross sections are much smaller than $\tilde{g}\tilde{g}$ annihilation cross section.

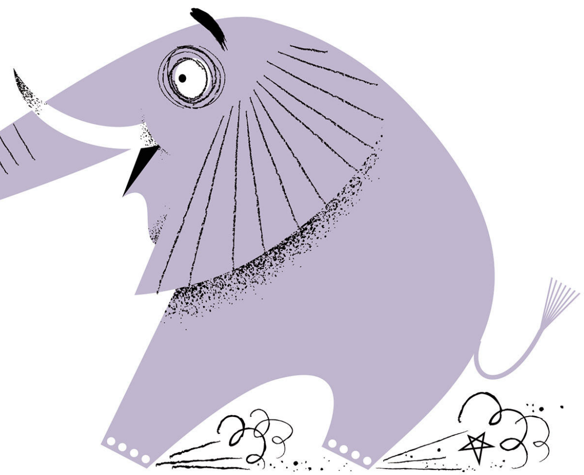
$$\frac{dn}{dt} + 3Hn = -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2} \left[n^2 - n_{eq}^2 \right]$$

- if $m_2 = m_1$, then ●● $= \frac{g_1^2 \langle \sigma v \rangle_{11 \to SM} + g_2^2 \langle \sigma v \rangle_{22 \to SM} + 2 g_1 g_2 \langle \sigma v \rangle_{12 \to SM}}{(g_1 + g_2)^2}$

  if the middle term dominates, then ●● $\approx \left(\frac{g_2}{g_1 + g_2} \right)^2 \langle \sigma v \rangle_{22 \to SM}$

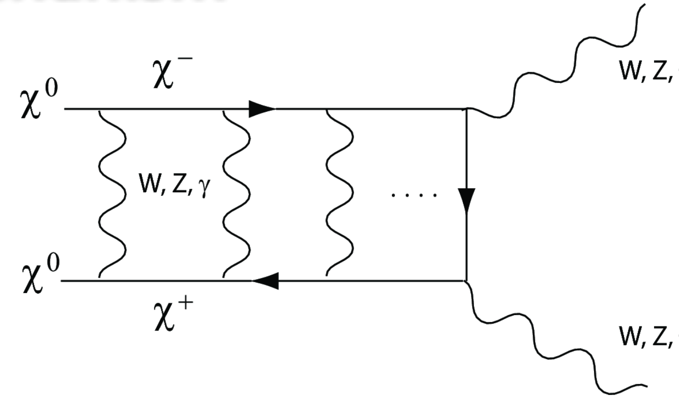
I’m a gluino…

I’m a Wino (Higgsino).



I’m the expanding Universe.

coannihilation mechanism



3 TeV Wino ✓

1 TeV Higgsino ✓

Here coannihilation is an *unavoidable* add-on.

also note the Sommerfeld effect

Hisano, Matsumoto, Nagai, Saito, Senami, 2007

Bino?

- Bino couples to slepton, squark and Higgsino, but not to another Bino.
- Therefore, it usually requires some coannihilation (e.g., with a stau or a stop) to reduce the relic abundance for a Bino of TeV scale.
- Bino-gluino coannihilation is possible by the help of a squark.