Neutral naturalness and (SIMP) dark matter

Hyun Min Lee

Chung-Ang University, Korea

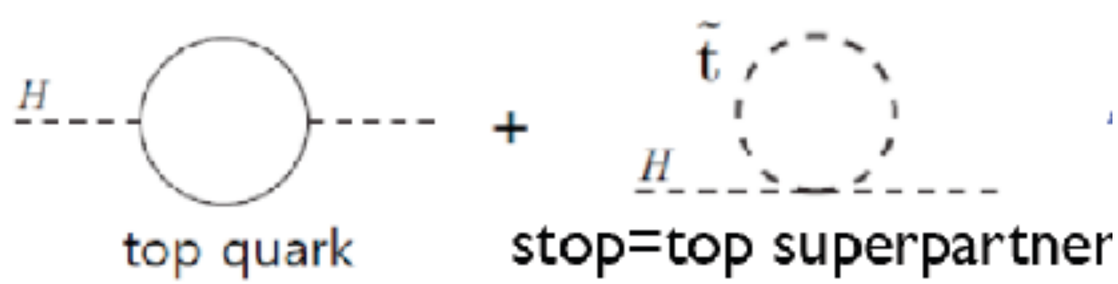
Beyond the Standard Model
4th Kvali IPMU-IPPP-KEK-KIAS Workshop
Ikaho Onsen, Japan, Oct 1-4, 2018

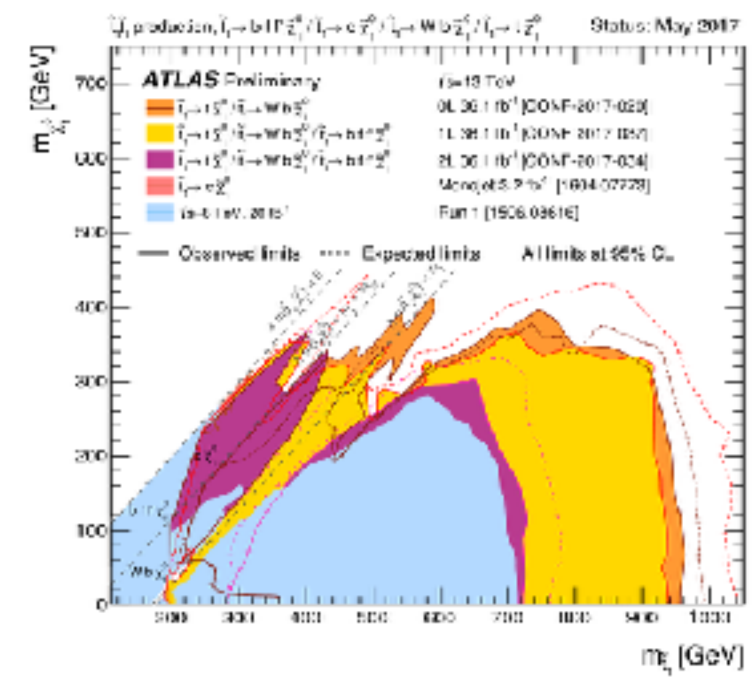
Outline

- Motivation
- SIMP paradigm
- Some stories on dark mesons
- Conclusions

New physics beyond SM

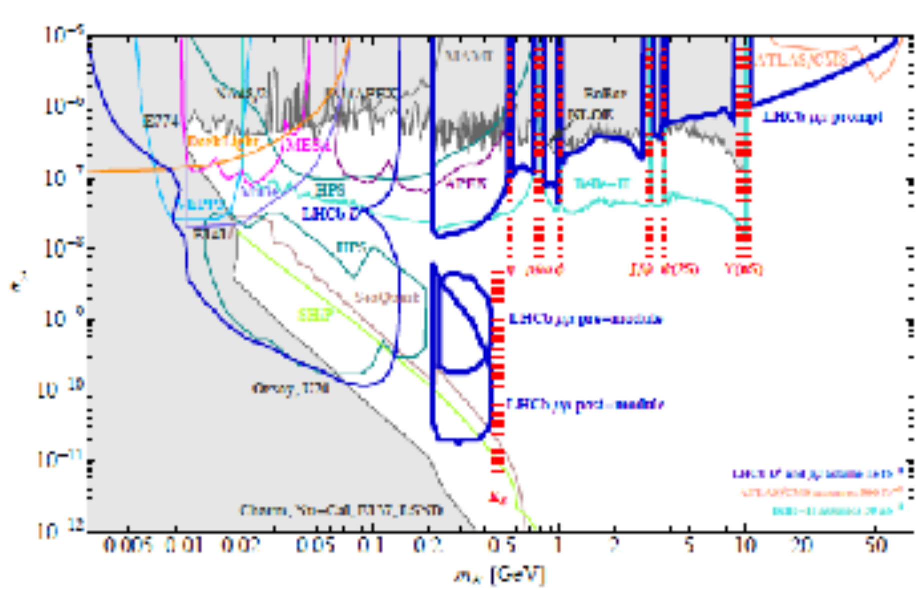
- New symmetry for solving the hierarchy problem predicts new particles at weak scale.





No direct hint yet for new physics.

- Other good motivations for new physics: dark matter, QCD axion, new forces, origin of baryon asymmetry, neutrino interactions, etc.

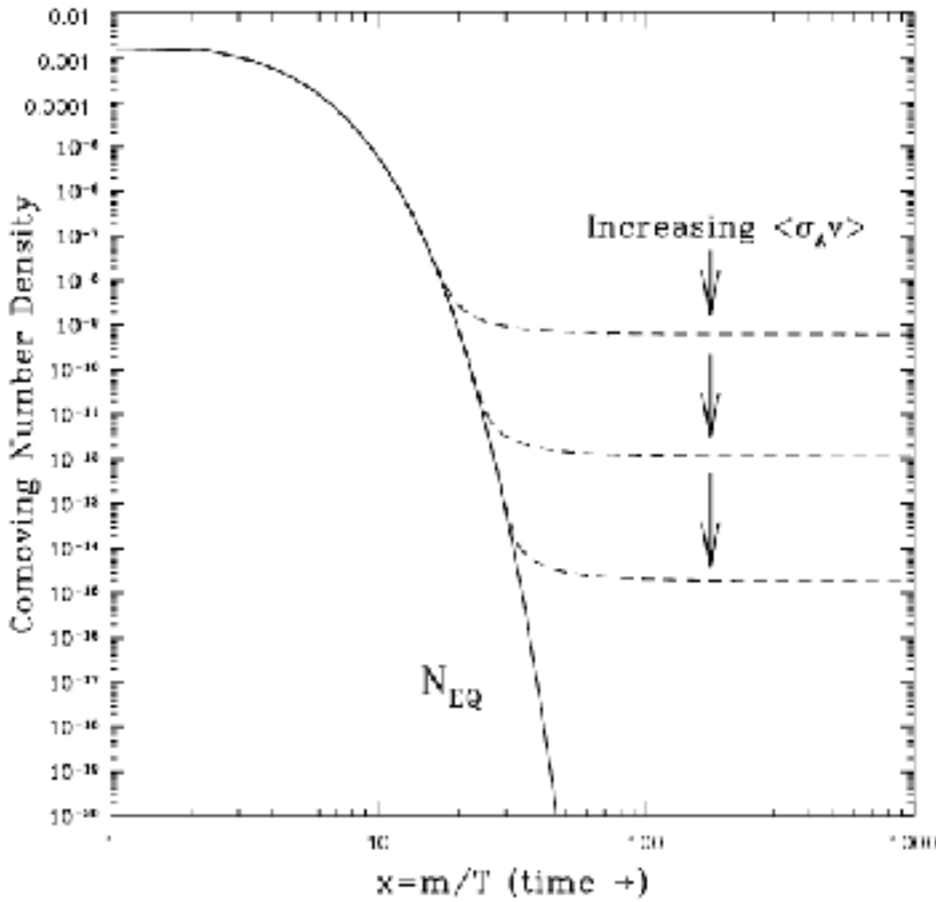


But, we don't have a good understanding on how new physics is realized.

Extensive searches are needed.

WIMP dark matter

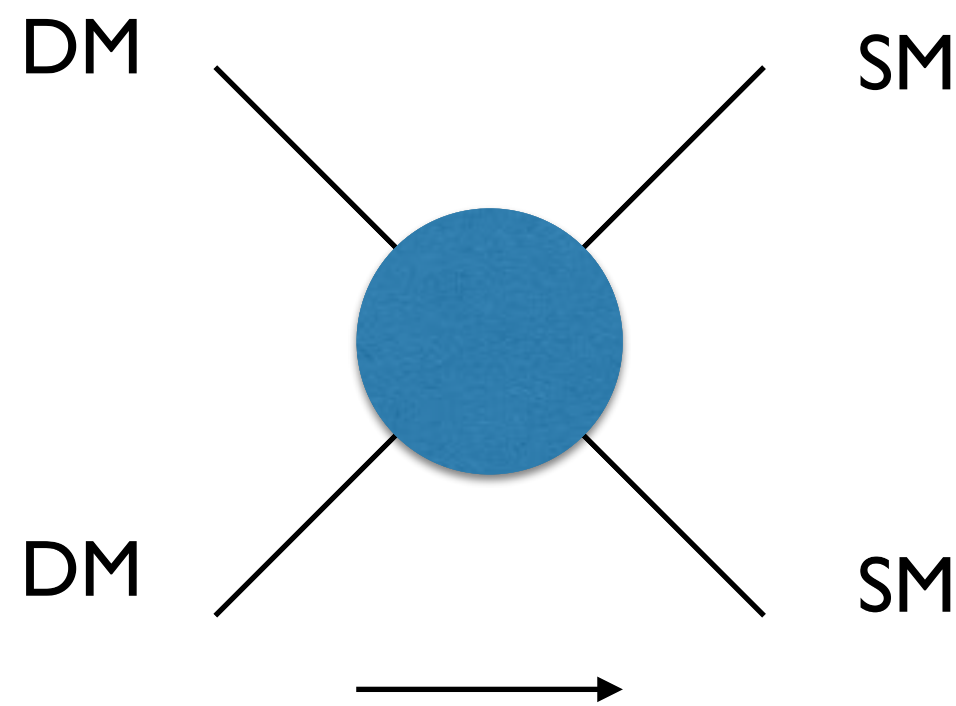
DM number



time

[Lee,Weinberg(1977)]

Dark matter abundance is determined by freeze-out.

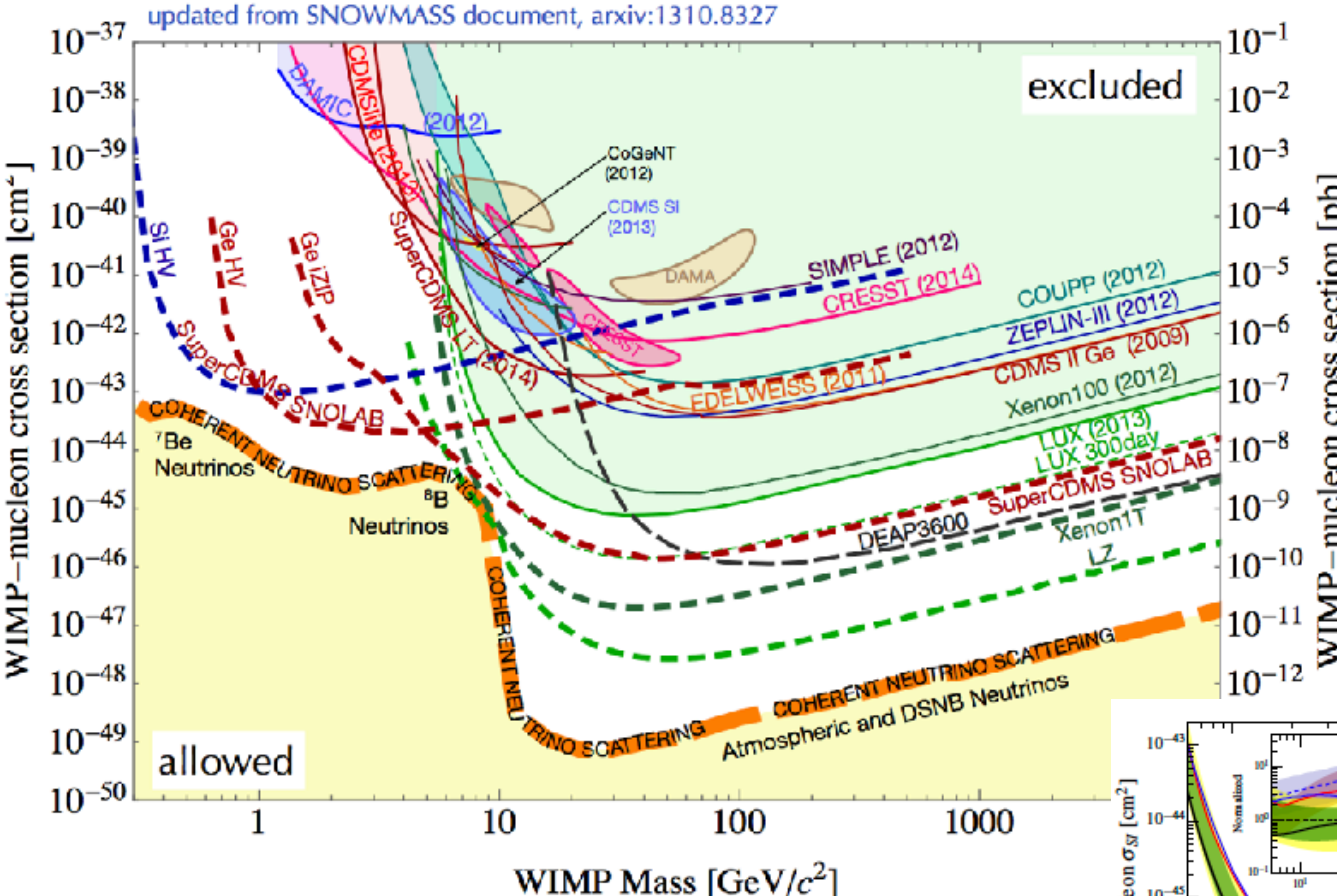


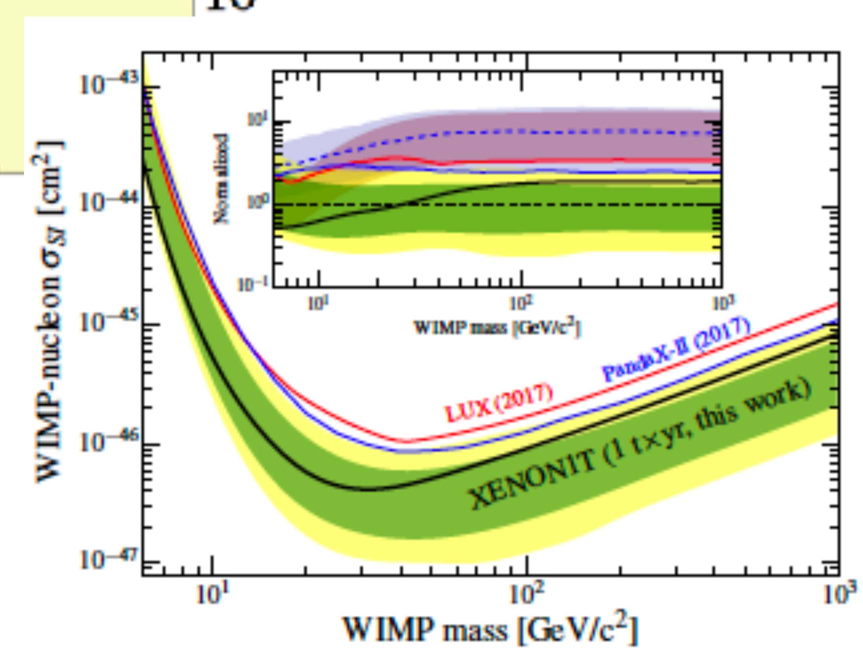
Weak interaction with 100GeV~1TeV mass:

$$\Omega_{\rm DM}h^2 = 0.1\left(\frac{1\,{\rm pb}\cdot c}{\langle\sigma_A v\rangle}\right)$$

Predictive for direct, indirect and collider detections!

WIMP in challenge





No direct evidence for WIMP yet.

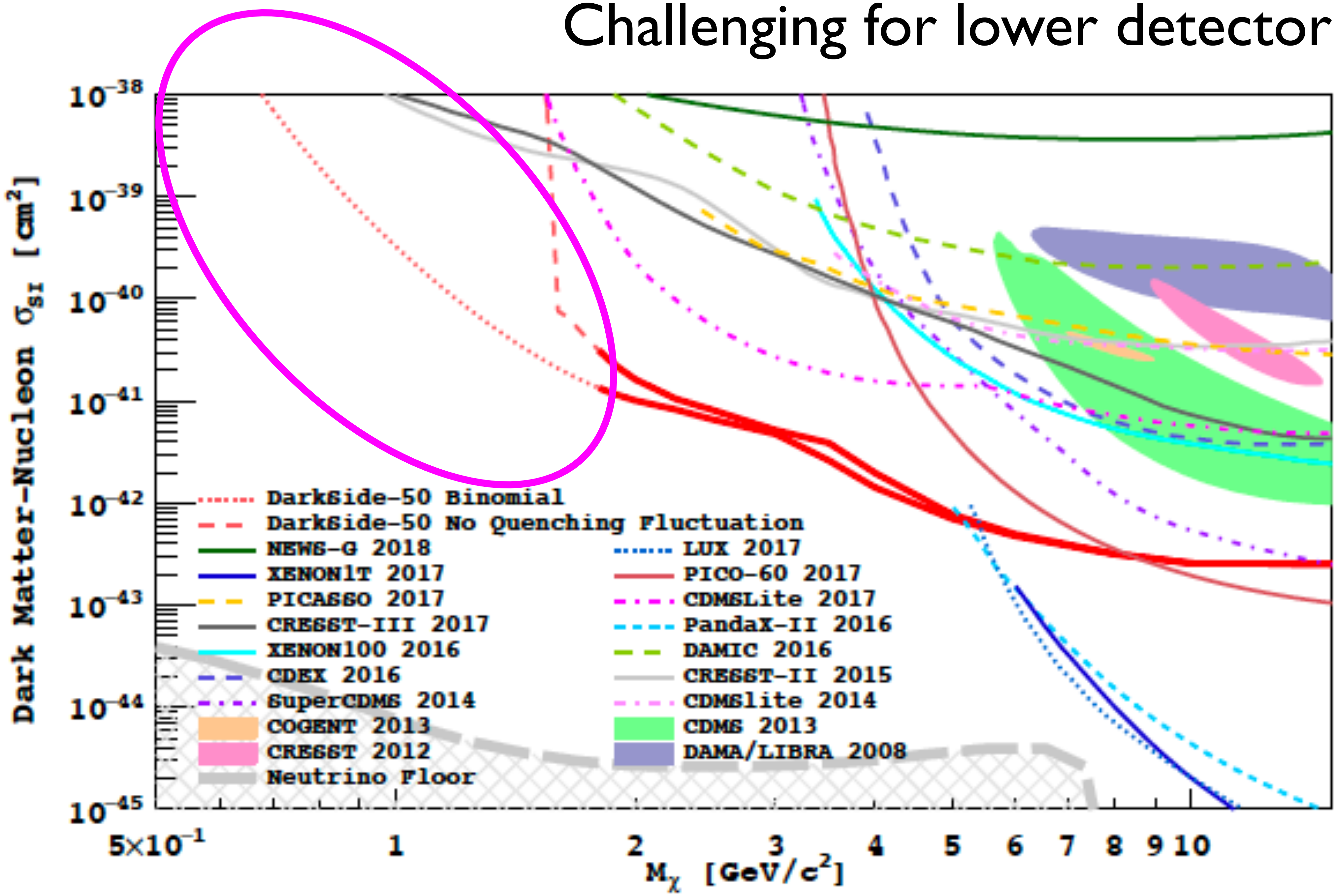
Light dark matter

Below typical WIMP masses: $1\,\mathrm{keV} \lesssim m_{\mathrm{DM}} \lesssim 10\,\mathrm{GeV}$

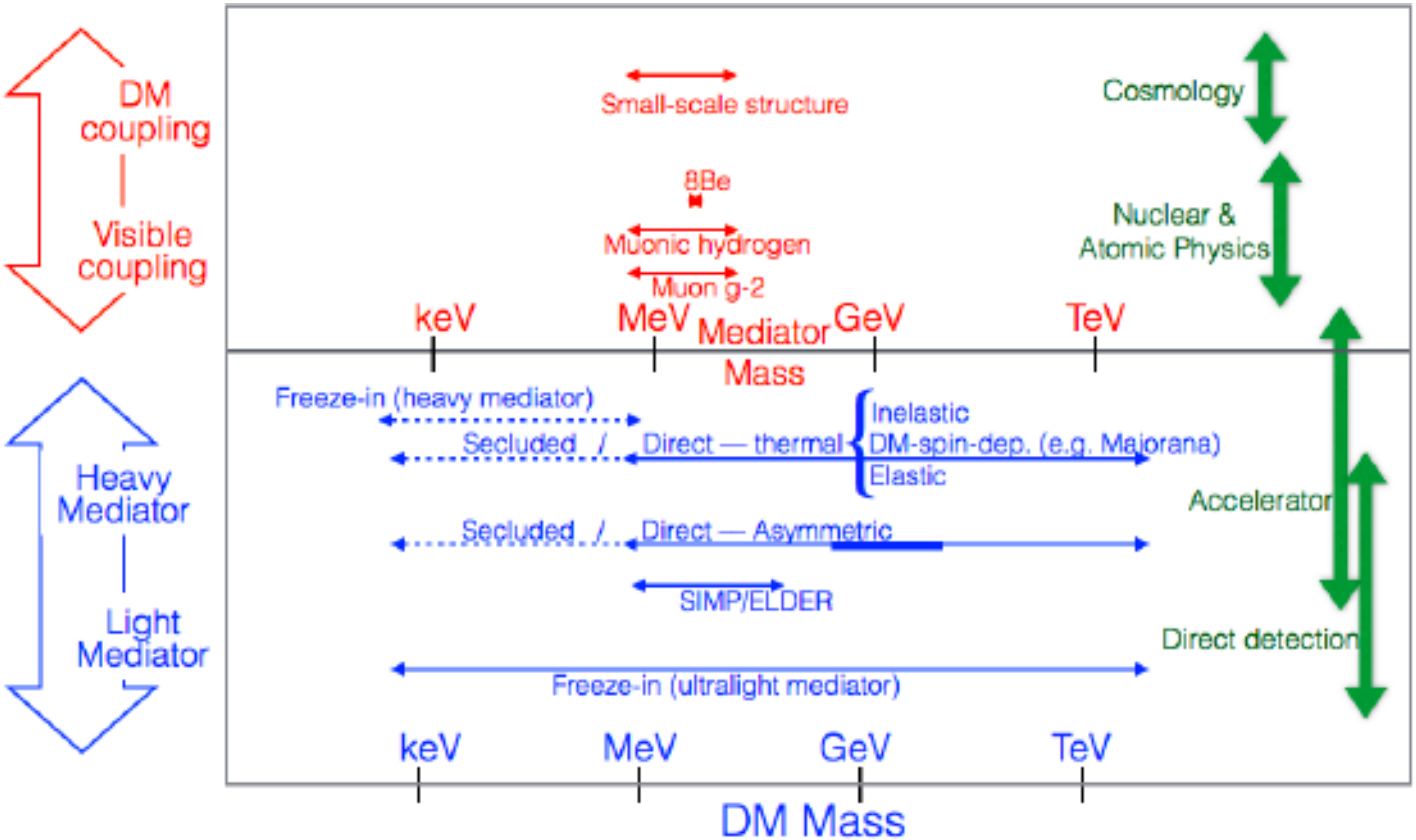
(Fermion DM)

$$E_R = \frac{\vec{q}^2}{2m_N} = \frac{(m_{\mathrm{DM}}v)^2}{2m_N} \lesssim \mathrm{keV}$$: Small recoil energy

Challenging for lower detector threshold.



Status of DM models

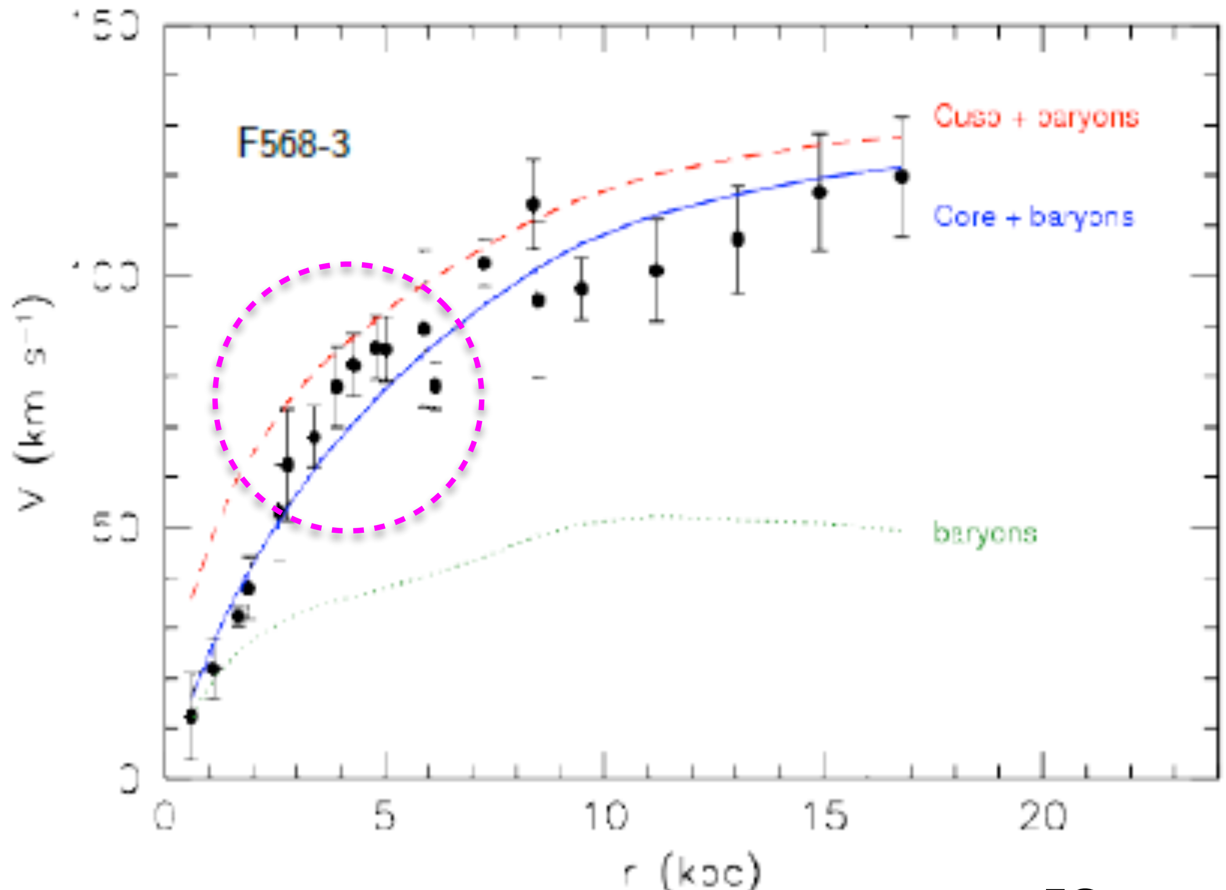


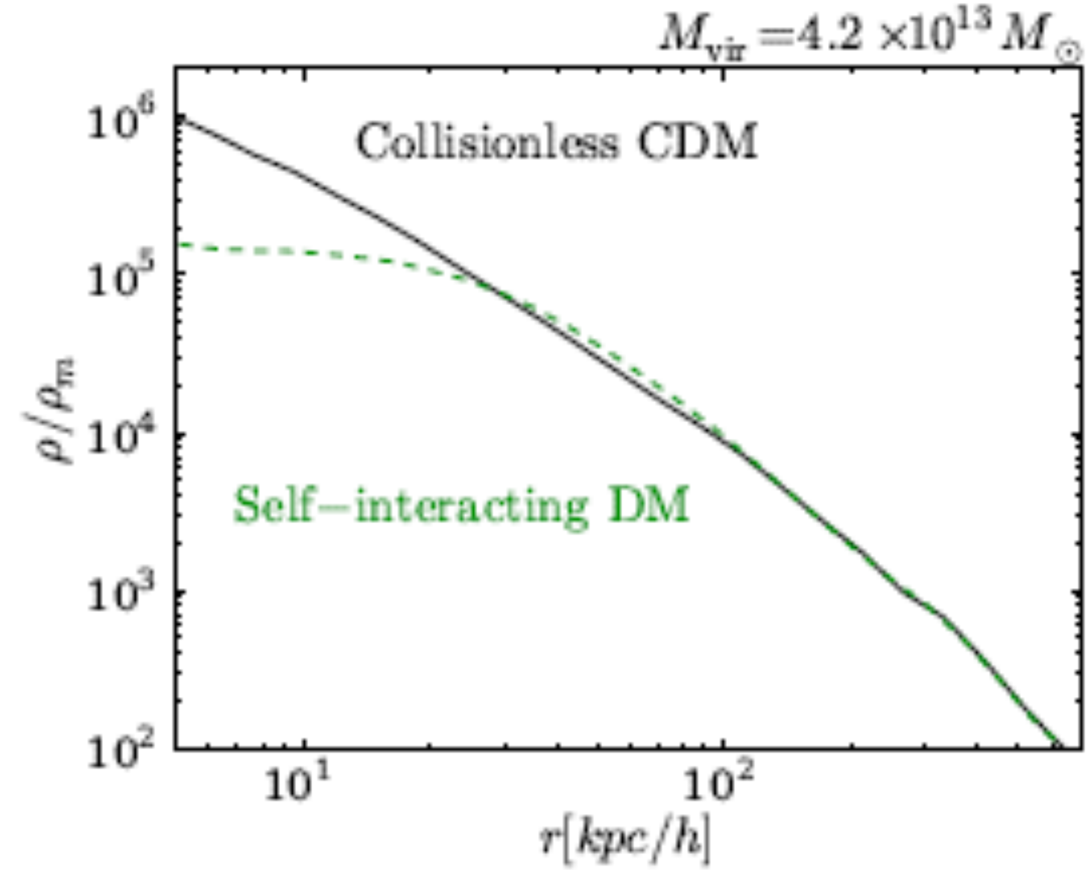
US Cosmic Visions, 1707.04591

Light dark matter models have started being developed.

New production mechanisms call for detection strategies.

Beyond WIMP





[Spergel, Steinhardt, 2000; Weinberg et al, 2013]

- Core-cusp problem: galaxy rotation curves favor cored DM profile, in conflict with N-body simulations.

Baryons (SN feedback) erase small structures, but many dwarf galaxies are lack of baryons.

May need Self-Interacting Dark Matter!

$$\frac{\sigma_{\rm self}}{m_{\rm DM}} \sim 1{\rm cm}^2/{\rm s}$$

cf. missing satellites, too-big-to-fail problems.

How to approach new physics

- New physics not far from weak scale. [Chacko et al, 2005]

  "Neutral naturalness" neutral top partners

- New physics couples too weakly. [W.Bardeen, 1995; Shaposhnikov, Zenhausern, 2008; Ghilencea, HML, 2018]

  "Scale invariance"

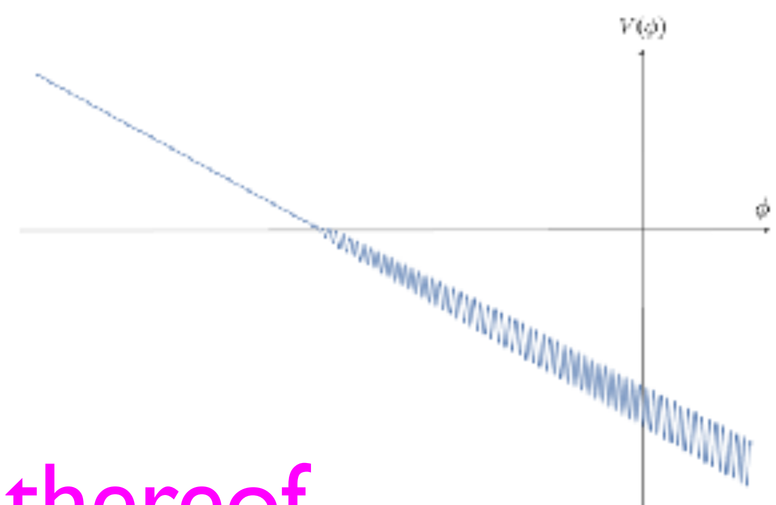
$$M_P^2 \sim \xi \langle \phi^2 \rangle, \;\; m_H^2 \sim \lambda_{H\phi} \langle \phi^2 \rangle : \;\; \lambda_{H\phi} \ll \xi.$$

- New dynamics of early cosmology. [D.E. Kaplan et al, 2015; A. Hook, 2016]

  "Cosmological relaxion"

$$m^2_{H,\mathrm{eff}} = \pm m_H^2 + g\phi :$$

scanning stops at $\phi \sim \mp \frac{m_H^2}{g}$.



True solutions might be combinations thereof.

Naturalness and dark QCD

[Chacko,Goh,Harnik, 2005]

- Double the SM gauge groups with twin Higgs and matter.

$$[SU(3)_A \times SU(2)_A] \times [SU(3)_B \times SU(2)_B] \times Z_2$$

$$Z_2: \quad g_A = g_B, \quad H_A \leftrightarrow H_B$$

$$\begin{pmatrix} H_A \\ H_B \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} e^{i\pi/f}$$: SU(4) ⟶ SU(3) “15 - 8 = 7 pNGBs”

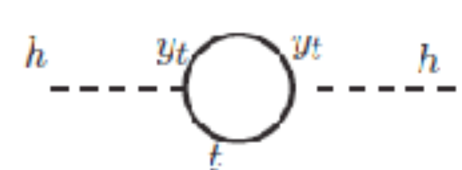
⟹ Higgs = pBGBs on $SU(4)/SU(3)$

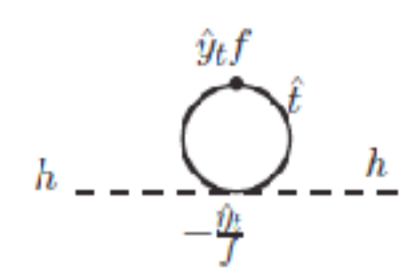
- Neutral top partners cancel quadratic divergences of Higgs mass at one-loop.

“Neutral naturalness”

$$\mathcal{L}_Y = -y_A H_A q_A u_A^c - y_B H_B q_B u_B^c, \; y_A = y_B.$$

⟹ $$\delta m_h^2 = \frac{3}{4\pi^2}(y_A^2 - y_B^2)\Lambda^2 = 0.$$





WIMP from neutral partners

[Craig et al; March-Russell et al, 2015]

- $U(1)_Y$ loops to Higgs mass is about 100GeV for Λ ~ 5 TeV.

→ No need of twin $U(1)_Y$. (Also to avoid too large $N_{eff.}$)

Similar arguments apply to twin light fermions.

"Fraternal" twin Higgs:

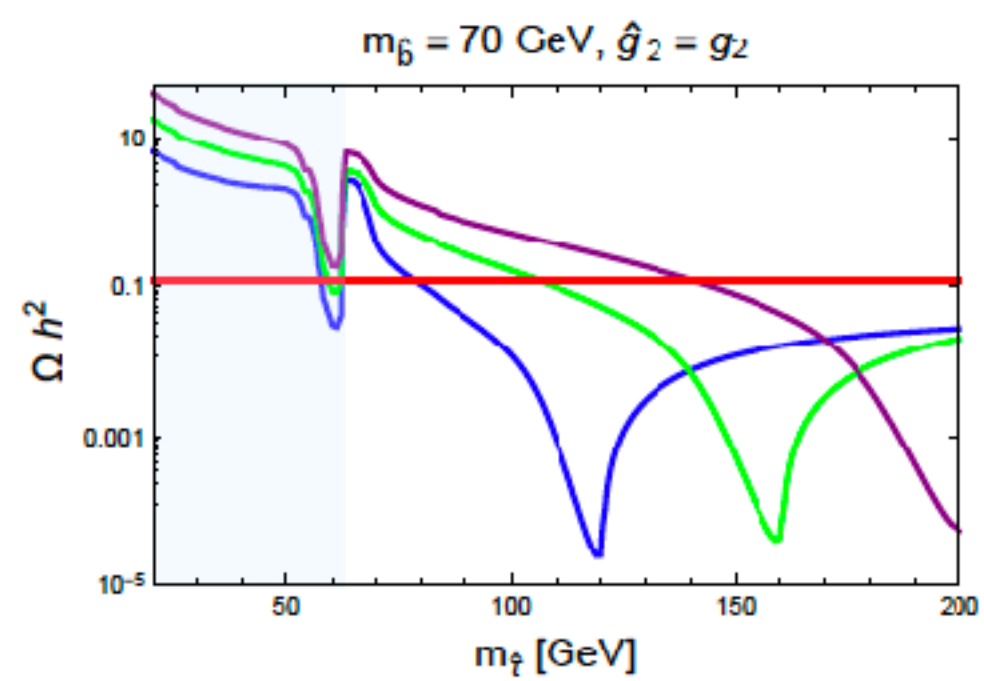
Twin tops, Twin bottoms and twin taus, twin neutrino

- Twin taus can be WIMP dark matter. [Craig et al, 2015]

$$m_{\hat{\tau}} \sim y_{\hat{\tau}} f \sim 100\,\mathrm{GeV}$$

twin W/Z + $\mathcal{L} \supset \frac{m_{\hat{\tau}}}{f}\frac{v}{f} h \hat{\tau}\hat{\bar{\tau}} + \mathrm{h.c.}$

→ $\hat{\tau}\hat{\tau} \to \hat{\nu}\hat{\nu},\ \hat{b}\hat{b},\ Z'Z'$



SIDM from Dark QCD

Higgs mass from gluon loops → "Dark QCD"

$$\delta m_h^2 \approx \frac{3 y_t^2 \Lambda^2}{4\pi^4}(g_3^2 - \hat{g}_3^2)$$

Dark light fermions
$G = SU(N_f)_L \times SU(N_f)_R$

→ dark condensate →

Dark mesons
$SU(N_f)_V$ flavor sym.

[Hochberg et al, 2014, 2018; Seo, HML, 2015]

$$\mathcal{L}_\pi \supset \mathrm{Tr}[(\partial_\mu \pi)(\partial^\mu \pi)] \; - m_\pi^2 \, \mathrm{Tr}[\pi^2]$$
$$+ \frac{2}{3 f_\pi^2} \mathrm{Tr}[(\partial_\mu \pi)\pi(\partial^\mu \pi)\pi - \pi^2 (\partial_\mu \pi)(\partial^\mu \pi))].$$

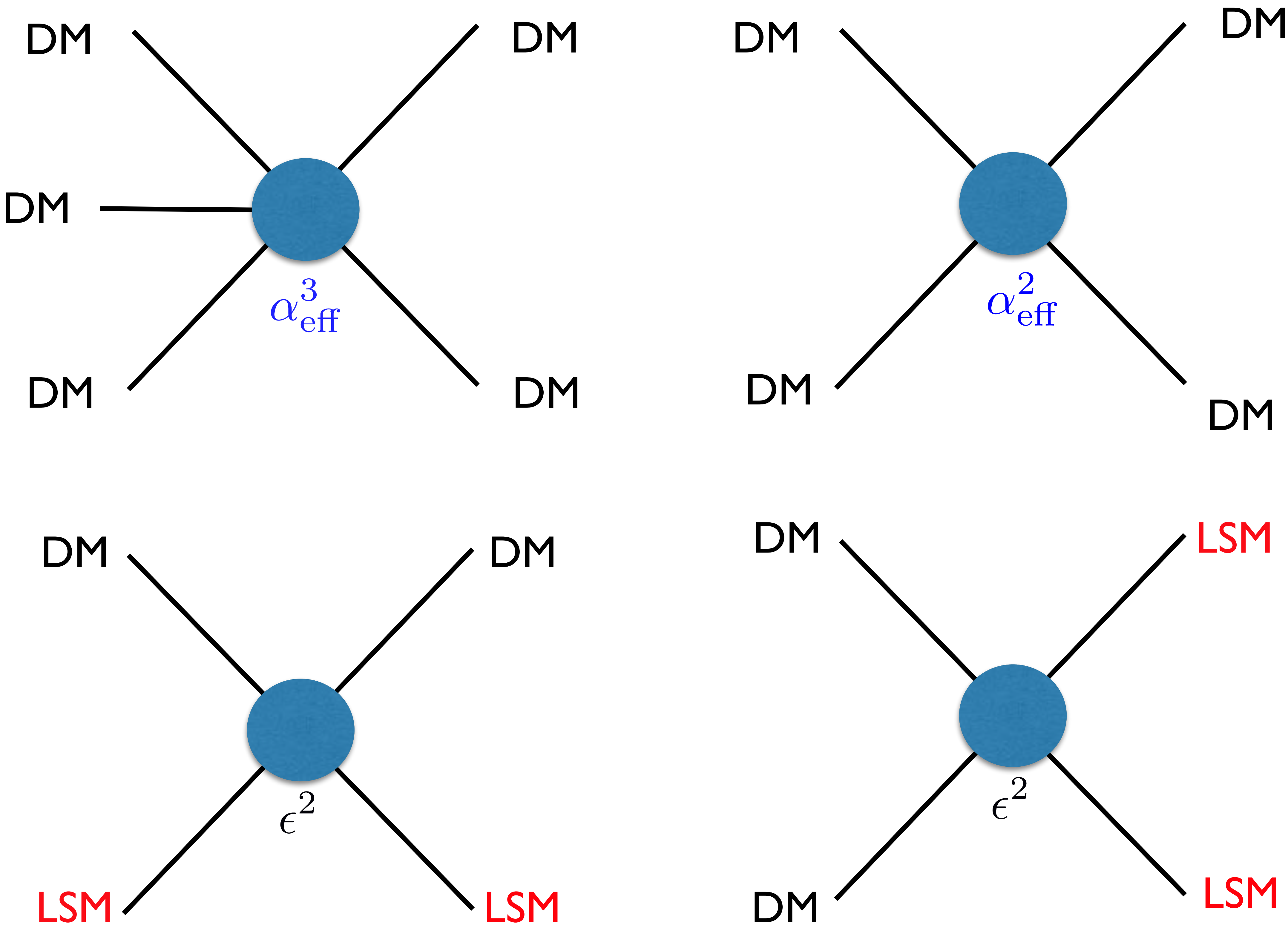
Dark mesons: self-interacting in dark ChPT

Massive "dark U(1)" decouples before dark condensate.

→ Mediator for kinetic equilibrium of dark mesons.

SIMP paradigm

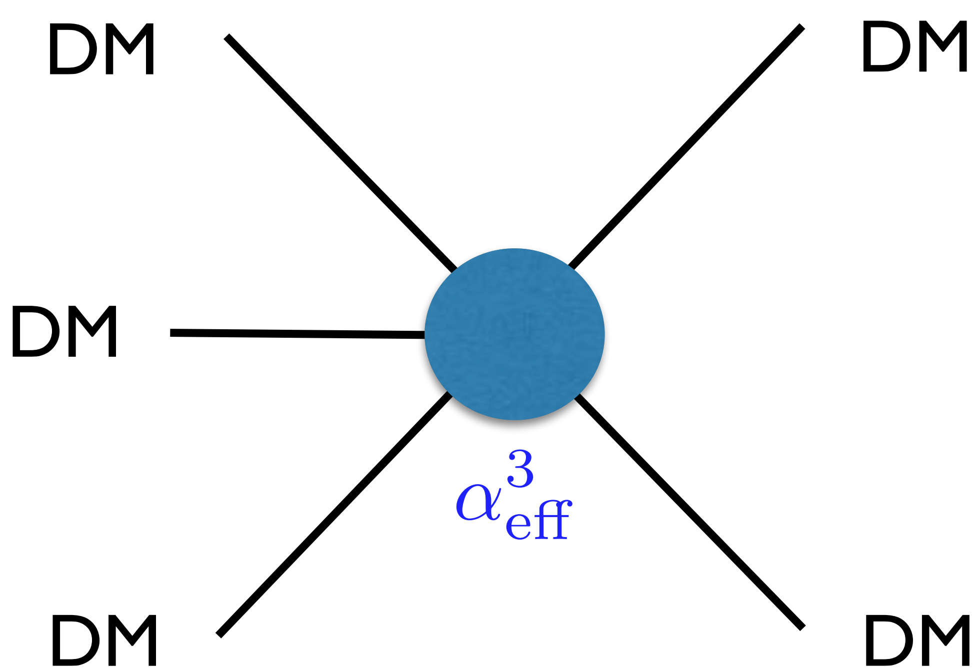
SIMP interplay

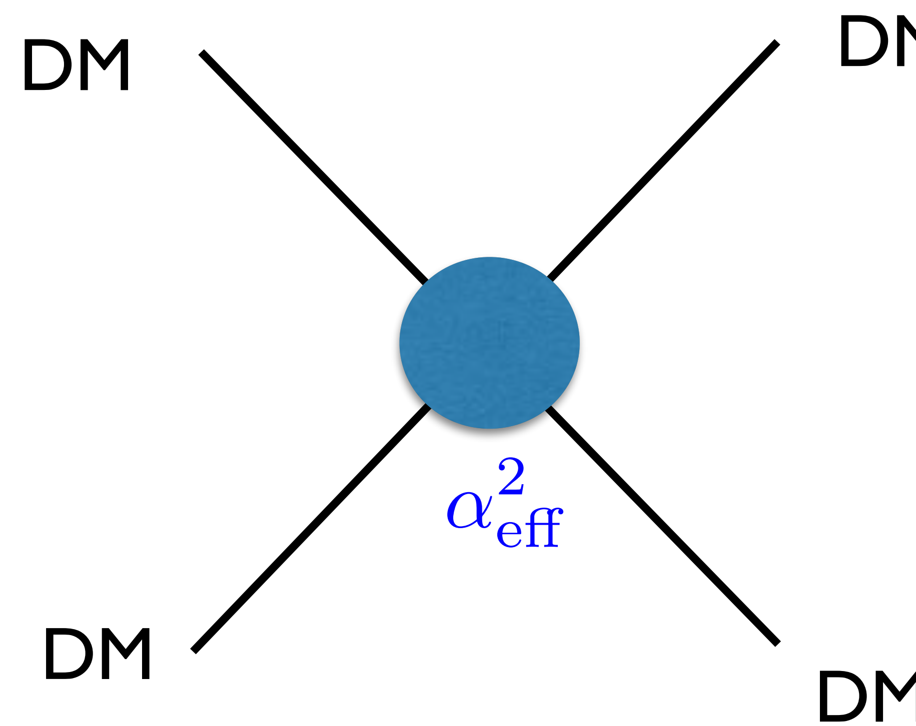


SIMP production

- Strongly Interacting Massive Particles (SIMP) in chemical equilibrium by self-interactions (3→2).

[Carlson et al (1992); Hochberg et al (2014)]





$$\frac{dn_{\rm DM}}{dt} + 3Hn_{\rm DM} \approx -\langle\sigma v^2\rangle_{3\to 2}(n_{\rm DM}^3 - n_{\rm DM}^2 n_{\rm DM}^{\rm eq})$$

Large self-scattering vs small annihilation: $\langle\sigma_{\rm eff} v\rangle = n_{\rm DM}\langle\sigma v^2\rangle \ll \sigma_{\rm self}$

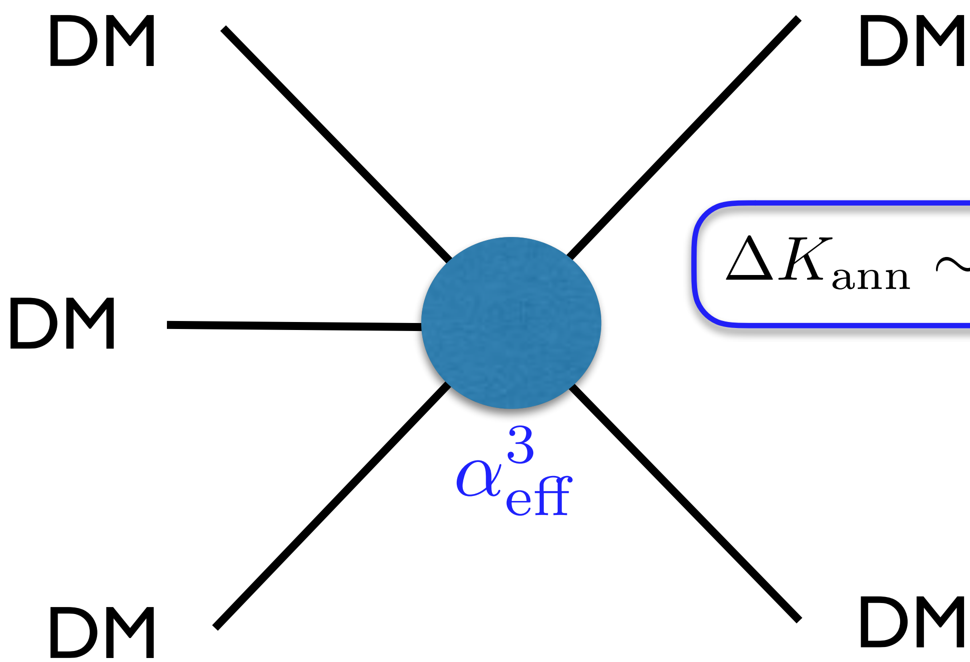
Relic density: $\Omega_{\rm DM}h^2 \simeq 0.1\left(\frac{m_{\rm DM}/35\,{\rm MeV}}{\alpha_{\rm eff}}\right)^{3/2}, \quad \frac{\sigma_{\rm self}}{m_{\rm DM}} \sim \left(\frac{5}{\alpha_{\rm eff}}\right) 1{\rm cm}^2/{\rm g}$

cf. forbidden channels, coscattering, etc.

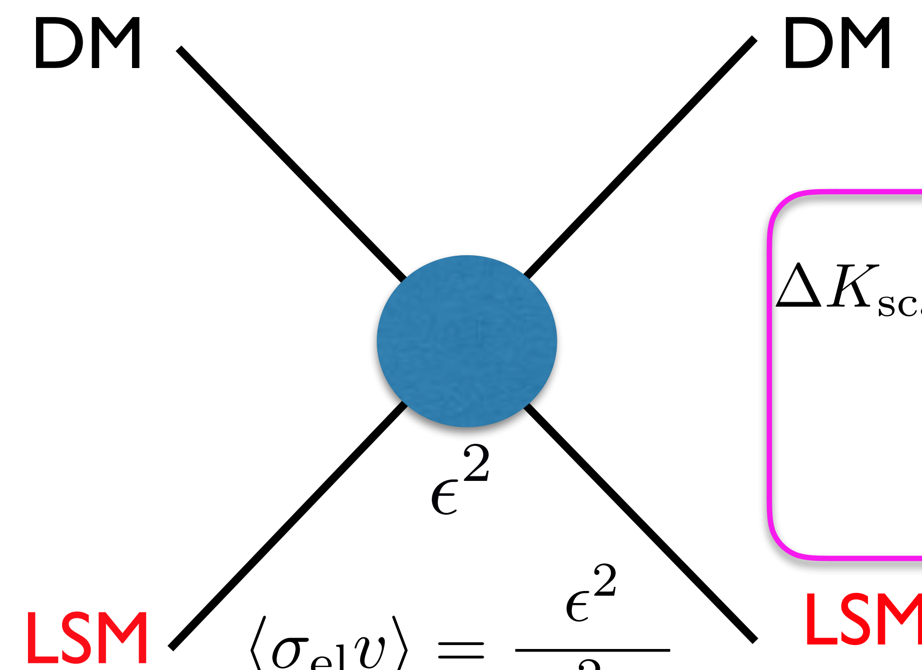
SIMP in equilibrium

- Equilibriate SIMP kinetic energy by kinetic equilibrium for the structure formation.

[de Laix et al, 1995; Murayama, HML et al, 2017]



$$\Delta K_{\rm ann} \sim m_{\rm DM}$$



$$\Delta K_{\rm scatt} = \frac{q^2}{2m_{\rm DM}} \sim \frac{T^2}{m_{\rm DM}}$$

$$\langle \sigma_{\rm el} v \rangle = \frac{\epsilon^2}{m_{\rm DM}^2}$$

$$\dot{K} + 2HK = -m_{\rm DM}^2 H T^{-1} + T\gamma(T), \qquad \gamma(T) \sim \left\langle n_Y \sigma_{\rm el} v_Y \frac{T}{m_{\rm DM}} \right\rangle.$$

3→2 elastic scatt.

Kinetic equilibrium: $\gamma(T) > H\left(\frac{m_{\rm DM}}{T}\right)^2$ ⇒ $\epsilon > 10^{-8}\alpha_{\rm eff}^{1/2}$

cf. WIMP: $\Delta K_{\rm ann} \sim \frac{T^2}{m_{\rm DM}} \longrightarrow \gamma(T) > H$

SIMP direct detection

- DM-nucleon scattering: cryogenic semi-conductors

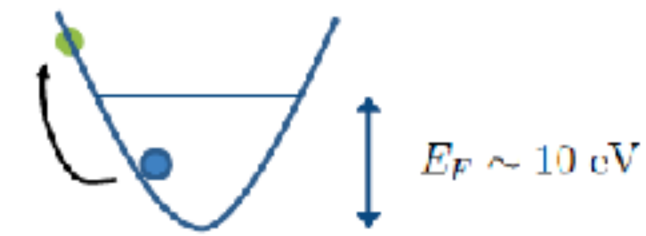
  e.g. CDMSlite: Ge, E_{th}=56eV → $m_{\rm DM} \gtrsim 1.6\,{\rm GeV}$

- DM-electron scattering: break Cooper-pairs and scatter

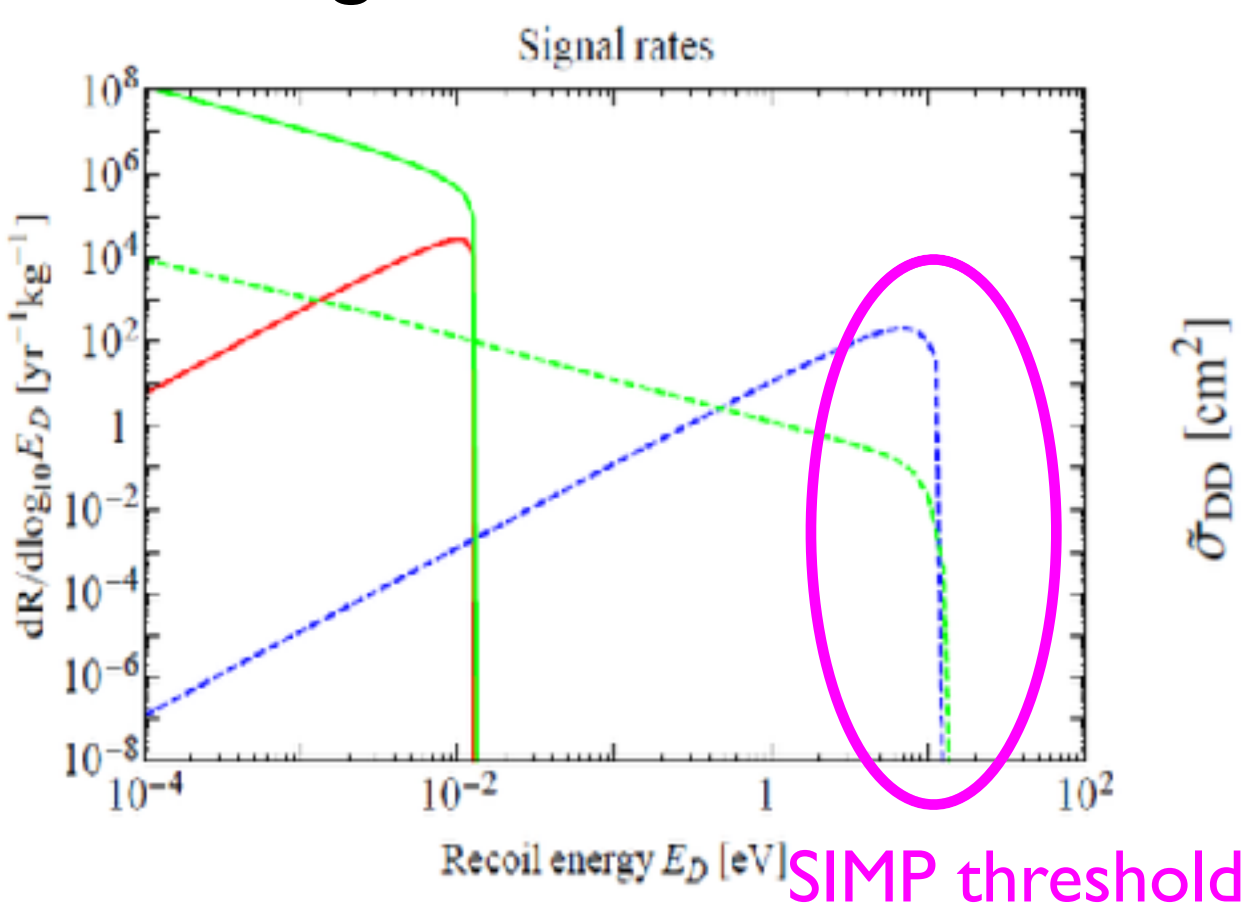
  $m_{\rm DM} v^2 \gtrsim \Delta \sim {\rm meV}$

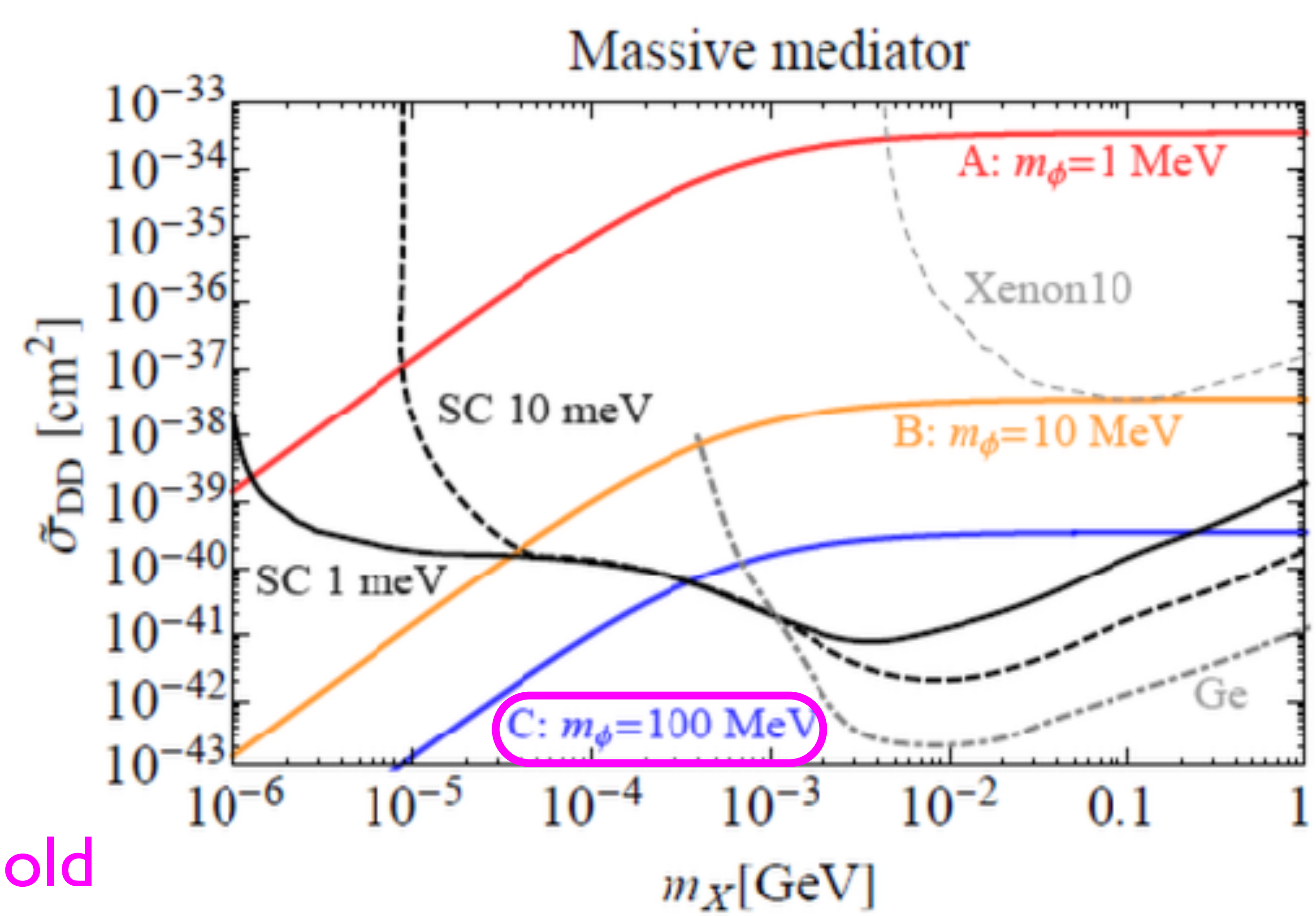
  [Hochberg et al (2015)]

  $m_{\rm DM} \gtrsim m_e: \quad E_R \lesssim 20\,{\rm eV}$

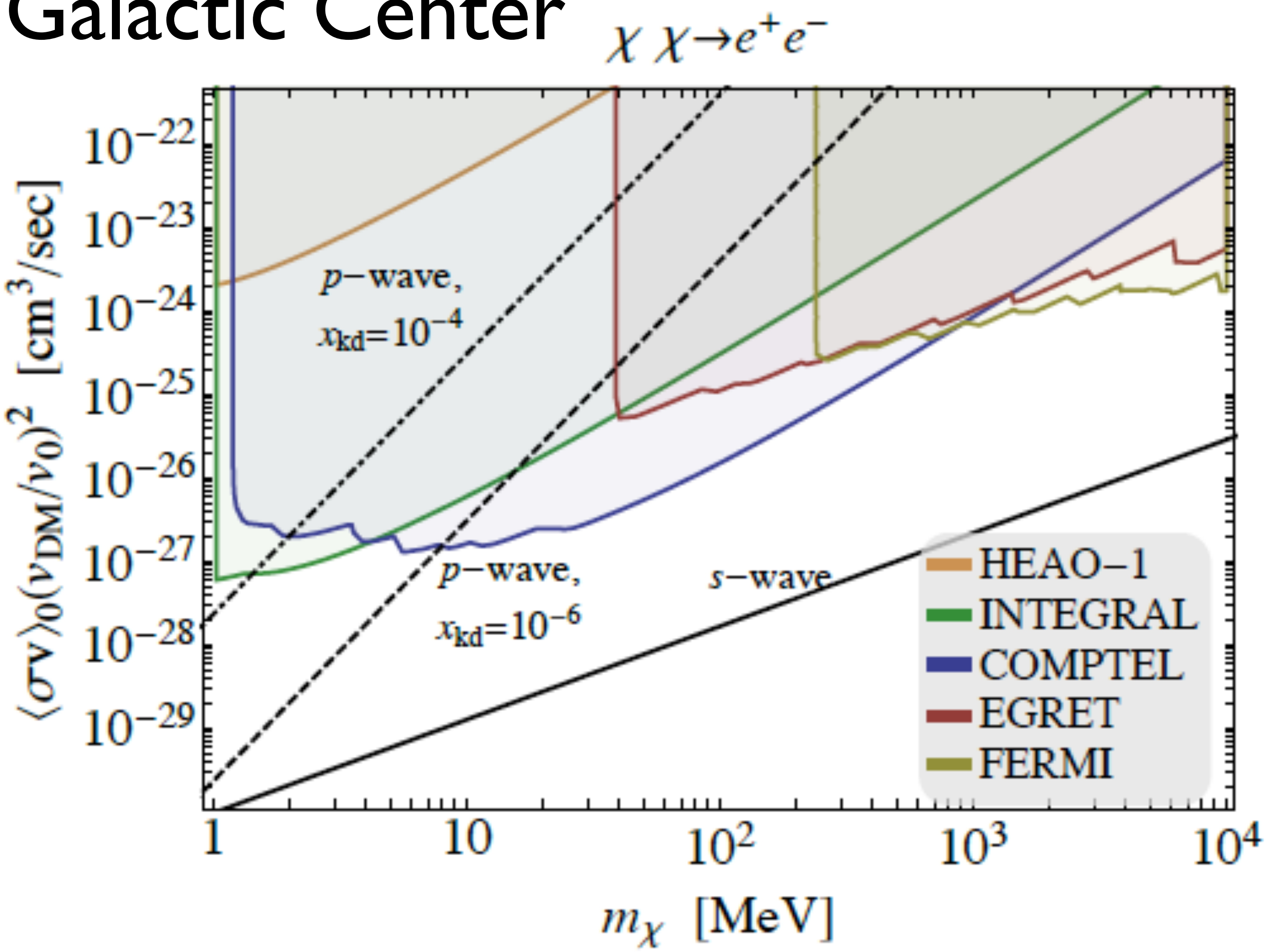
  e.g. Al: E_F=11.7eV, v_{rel}~v_F=10^{-2}





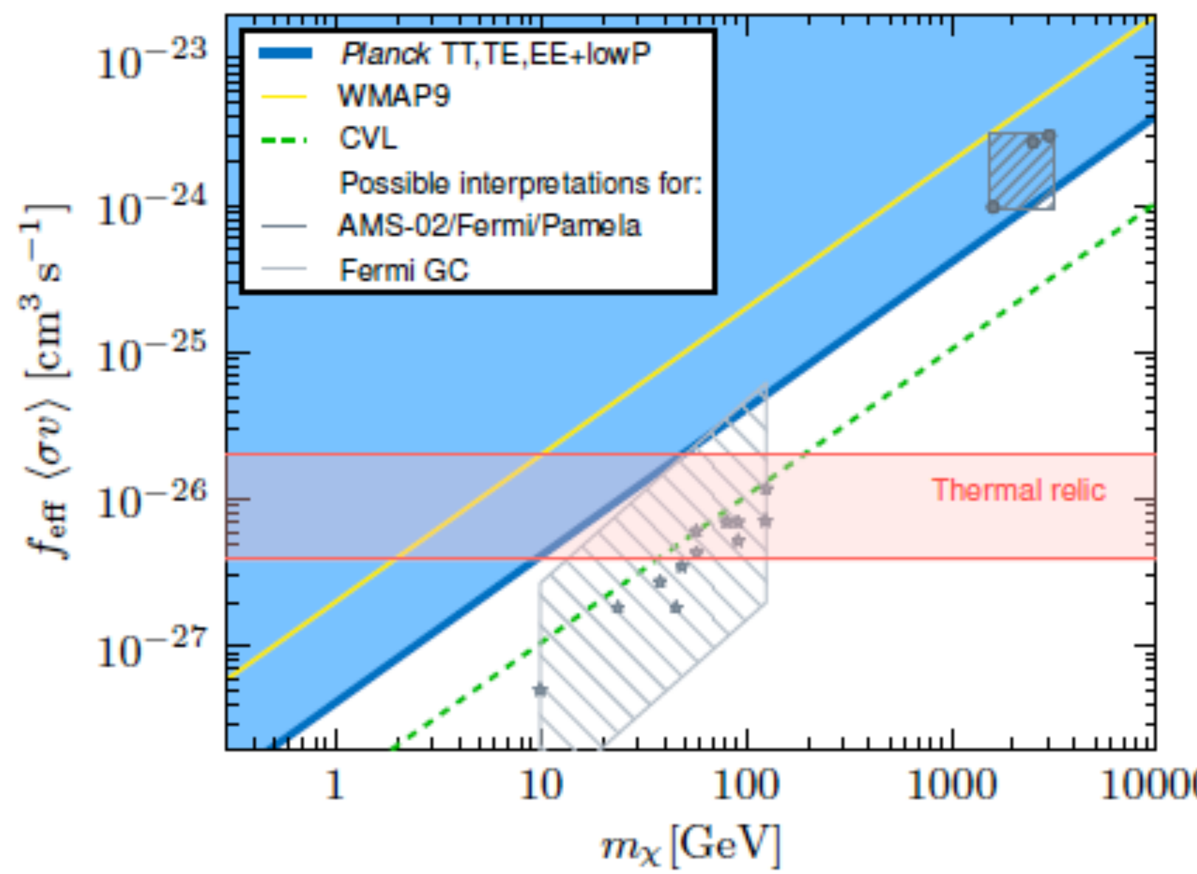
SIMP indirect detection

Galactic Center



[Essig et al, 2013]

CMB

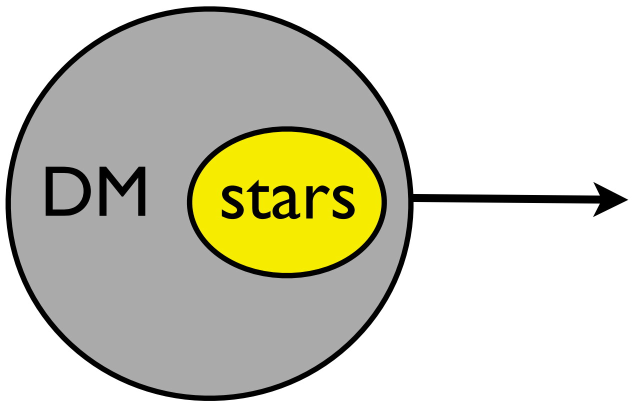


- Gamma-ray & CMB can constrain SIMP DM.
- Need of velocity suppression: scalar SIMP ~ p-wave

$$\langle\sigma v_{\rm rel}\rangle_{l^+l^-} = \left(1.2\times 10^{-27}{\rm cm}^3/{\rm s}\right)\left(\frac{\varepsilon_2}{10^{-6}}\right)^2\left(\frac{100\,{\rm MeV}}{m_\pi}\right)^2, \quad \varepsilon_2(v)=\varepsilon_2(v_F)(v/v_F)$$

SIMP in merging galaxies

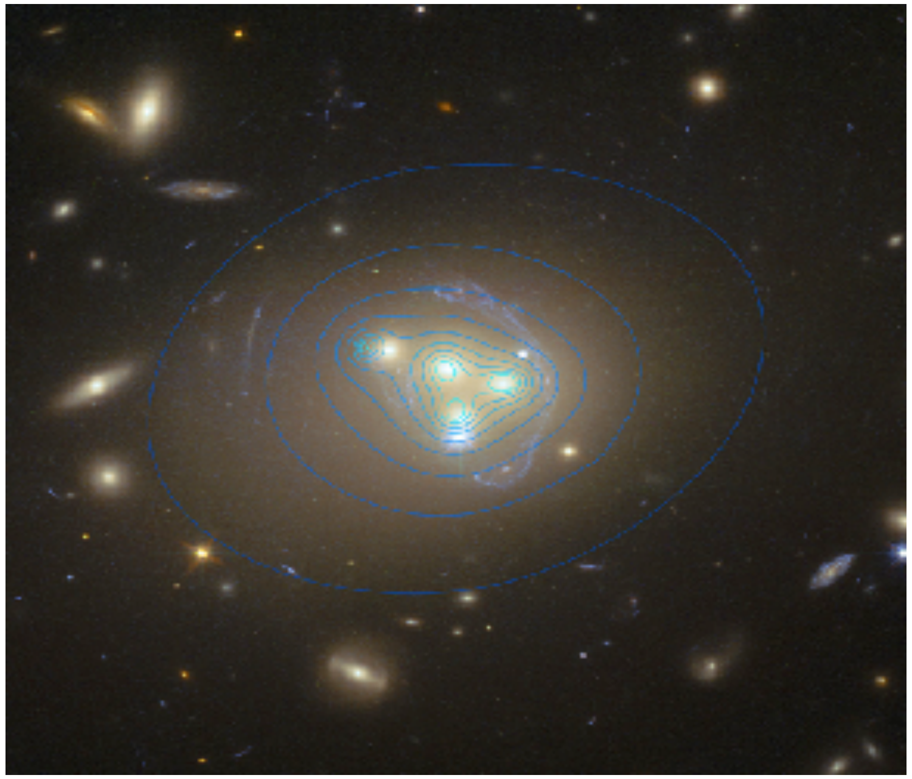
- DM subhalo can be separated from its bounded stars due to DM self-interactions.

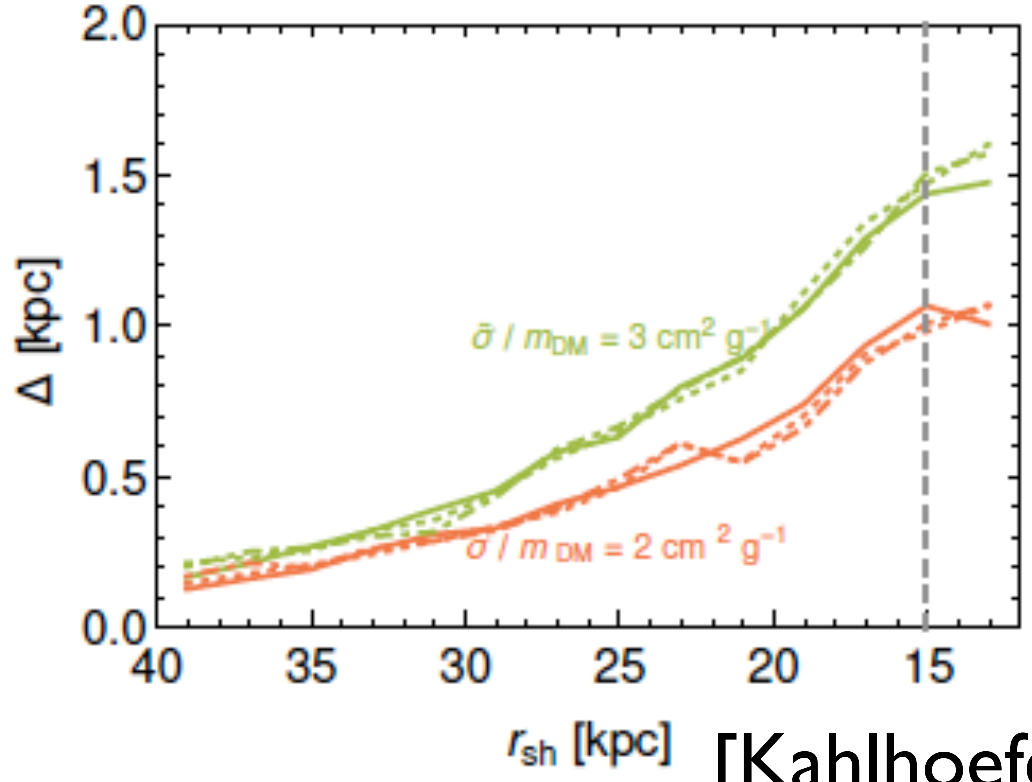


"Drag force" between DM subhalo and larger DM halo is greater than "Gravity" between DM subhalo and star.

e.g. Abell 3827 cluster

$\Delta = 1.62^{+0.47}_{-0.49}\,\mathrm{kpc}$ [Massey et al(2015)]





[Kahlhoefer et al(2015)]

Models of SIMP

- Fundamental scalars with Z_3

  [Bernal et al (2015); Choi, HML (2015)]

- Composite scalars (dark mesons)

  [Hochberg et al (2014); Seo, HML (2015)]

- Non-abelian vectors (minimal: SU(2))

  [Bernal et al (2015); Murayama, HML et al (2017)]

- Fermions (assisted SIMP)

  [J. Cline et al (2017)]

- Generalizations

  [Choi, Seo, HML (2017)]

- $4 \rightarrow 2$ processes and more

Scalar SIMP

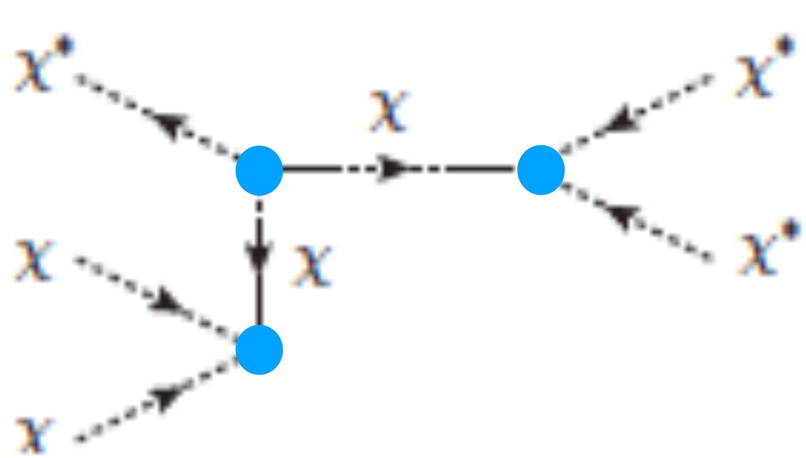
[S.-M.Choi, HML, 2015]

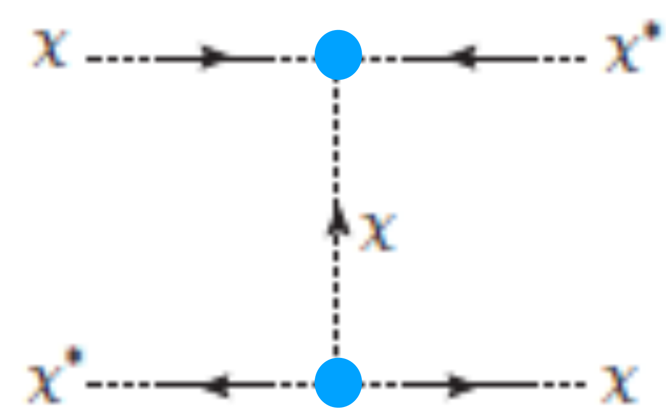
- Z_3 is the minimal discrete symmetry from a dark local U(1) and Z' portal is built-in.

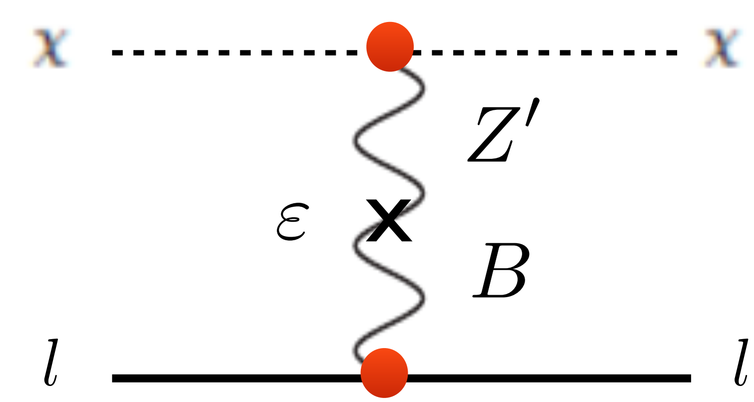
	ϕ	χ
$U(1)_V$	$+3$	$+1$

Table 1: $U(1)_V$ charges.

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}v' \quad \Rightarrow \quad U(1)_V \to Z_3.$$





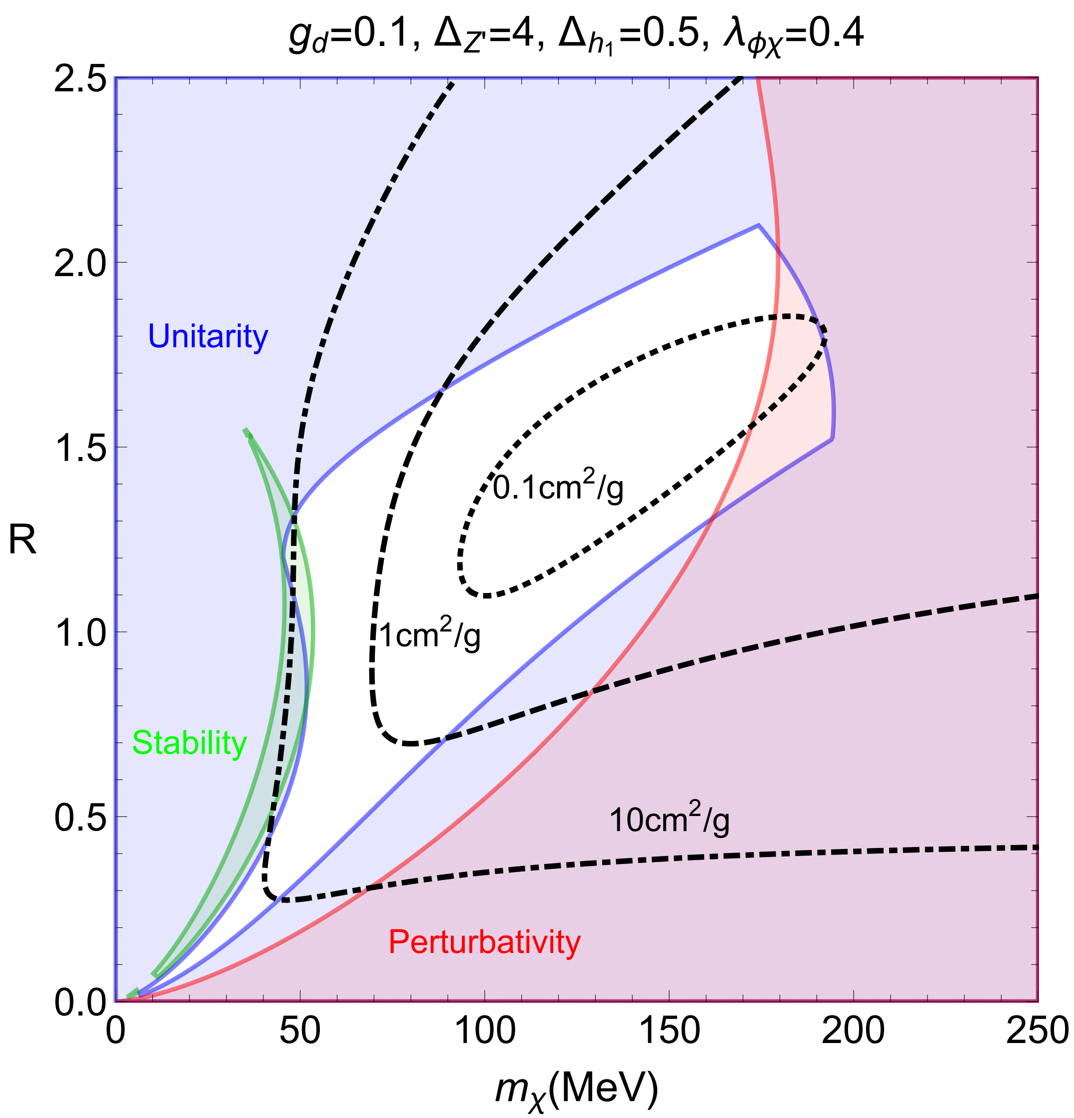


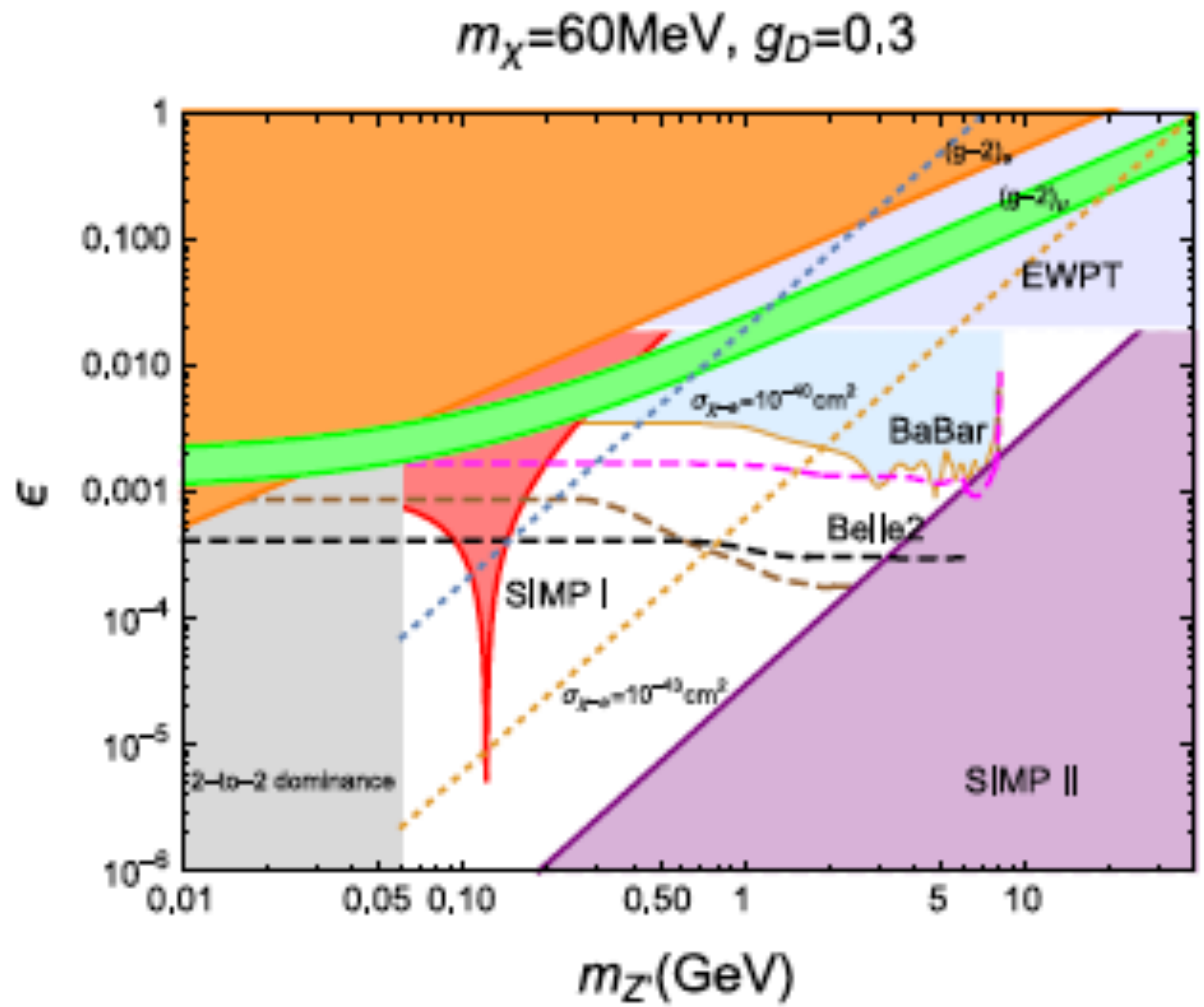
- Correlation btw self-scattering and 3→2 annihilation.

$$\sigma_{\rm self} \sim \frac{\kappa^4}{m_\chi}, \qquad \langle\sigma v^2\rangle \sim \frac{\kappa^6}{m_\chi^5} \quad \Rightarrow \quad \alpha_{\rm eff} \sim \kappa^2$$

Bounds on SIMP

[S.-M. Choi, HML, 2015]

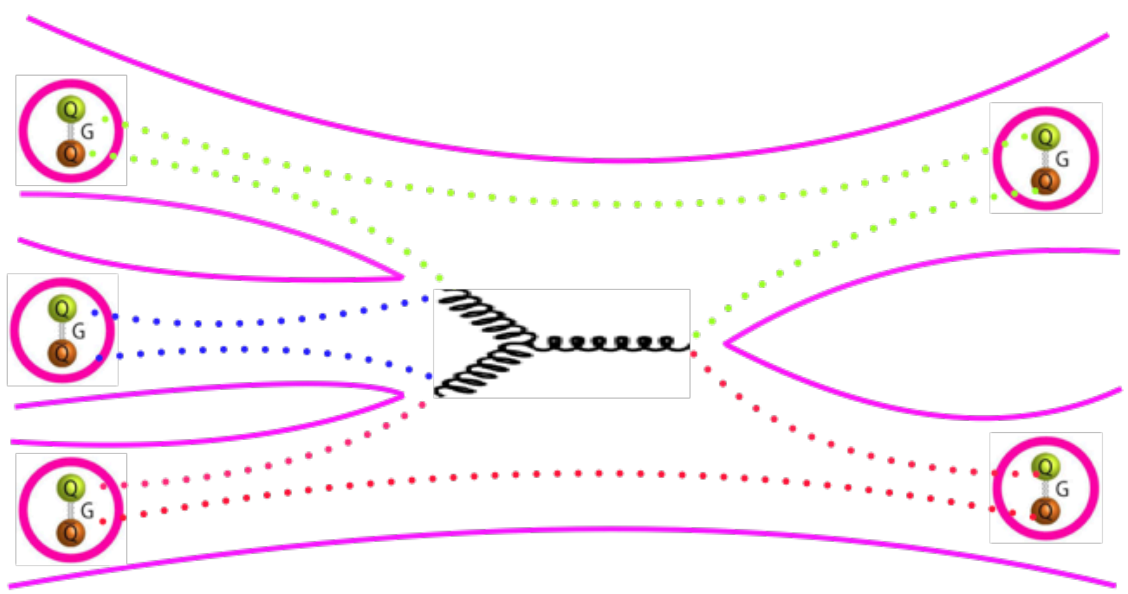


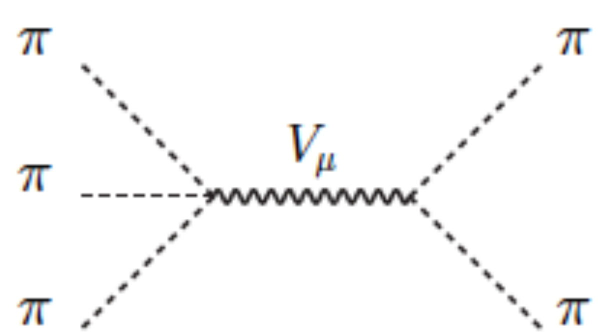


Consistent self-interactions for relic density & small-scale problems

Belle2, LHCb (Z' production), Beam dump exp, Meson decays, DM-electron scattering, etc.

Some stories on dark mesons





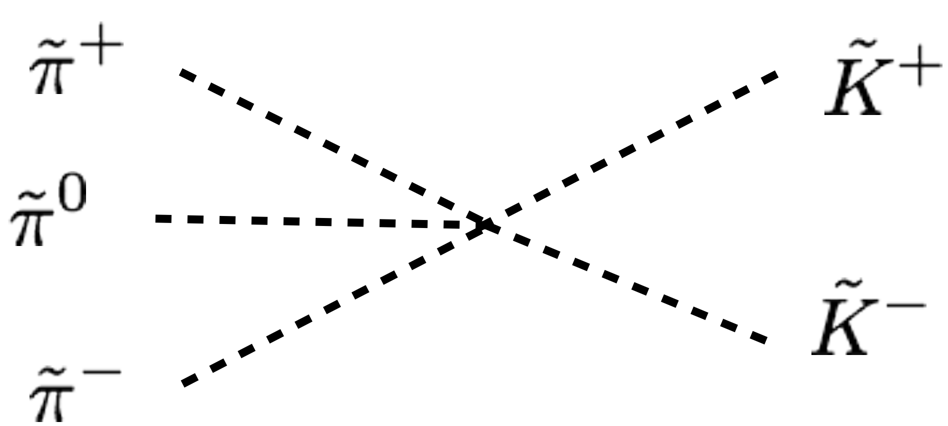
Dark meson dark matter

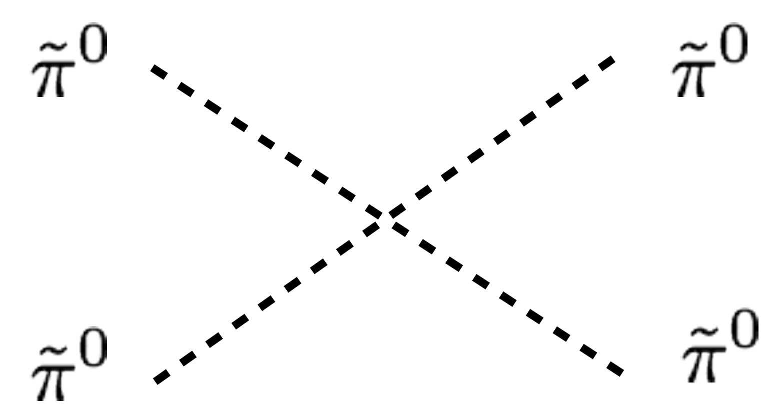
- 5-point self-interactions from Wess-Zumino-Witten term for $\pi_5(G/H)=Z$ (i.e. $N_f \geq 3$). [Wess, Zumino, 1971; Witten, 1983]

$$\mathcal{L}_{WZW} = \frac{2N_c}{15\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi]$$

N_C : topological invariant of 5-sphere in SU(3)

SIMP process: [Hochberg et al, 2014]



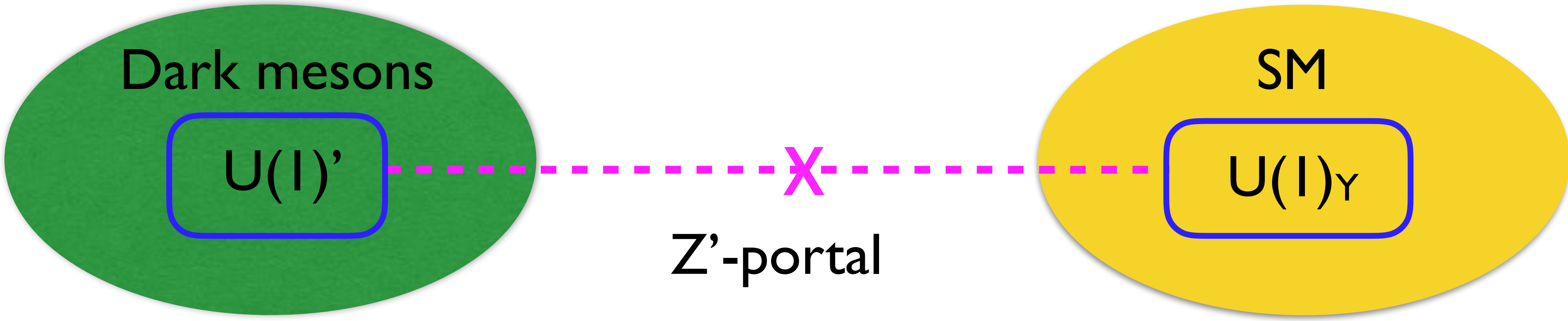


$\Lambda'_{\mathrm{QCD}} \sim \Lambda_{\mathrm{QCD}}$ → $m^2_{\tilde{\pi}} \sim m_{\tilde{q}}\dfrac{(\Lambda'_{\mathrm{QCD}})^3}{F^2} \sim (100\,\mathrm{MeV})^2$

Dark QCD

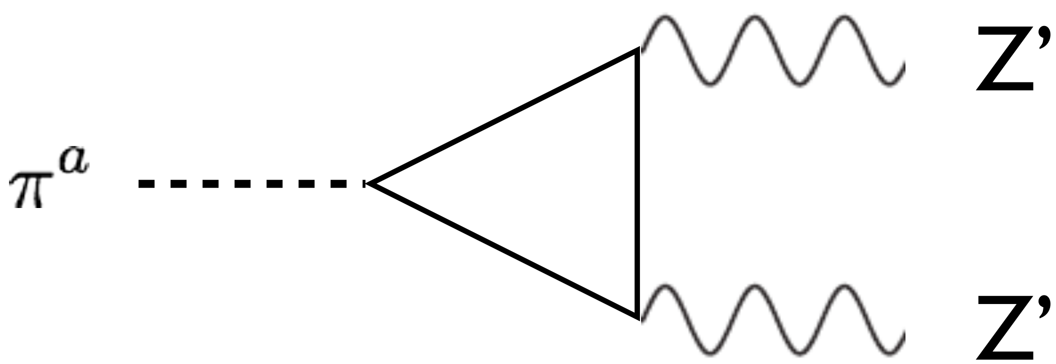
degenerate dark quarks → degenerate meson masses

Z'-portal for dark mesons



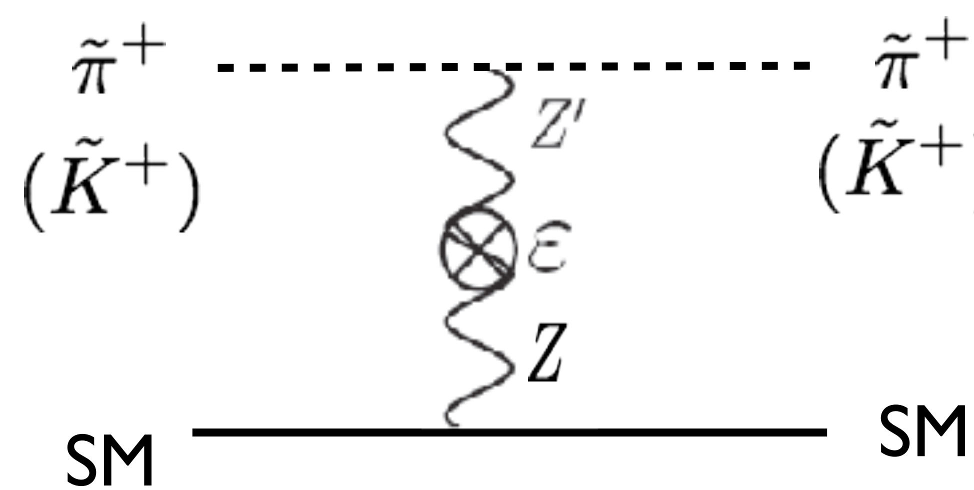
- Vectorial Z' with no chiral anomalies in dark QCD.

[HML, M. Seo, 2015]



$$\propto \mathrm{Tr}(Q_D^2 T^a) = 0$$

$$\Rightarrow \quad Q_D = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

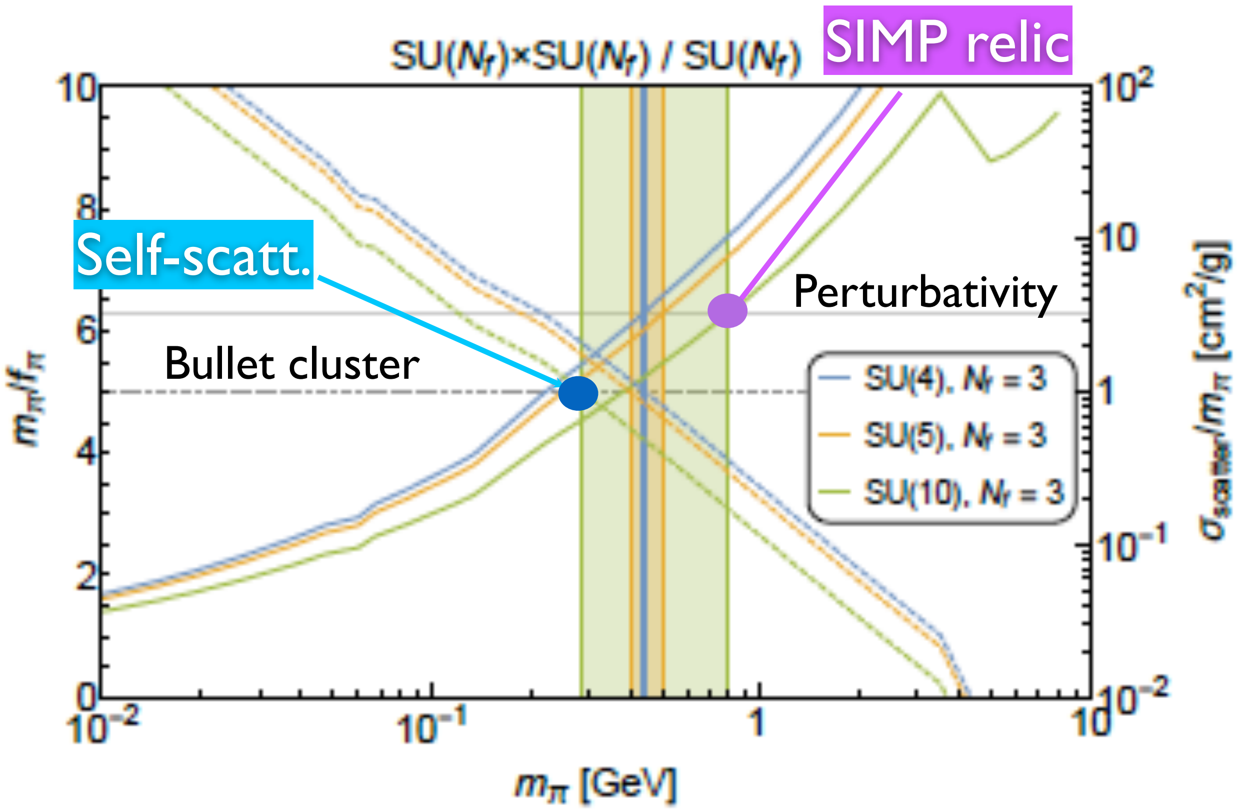


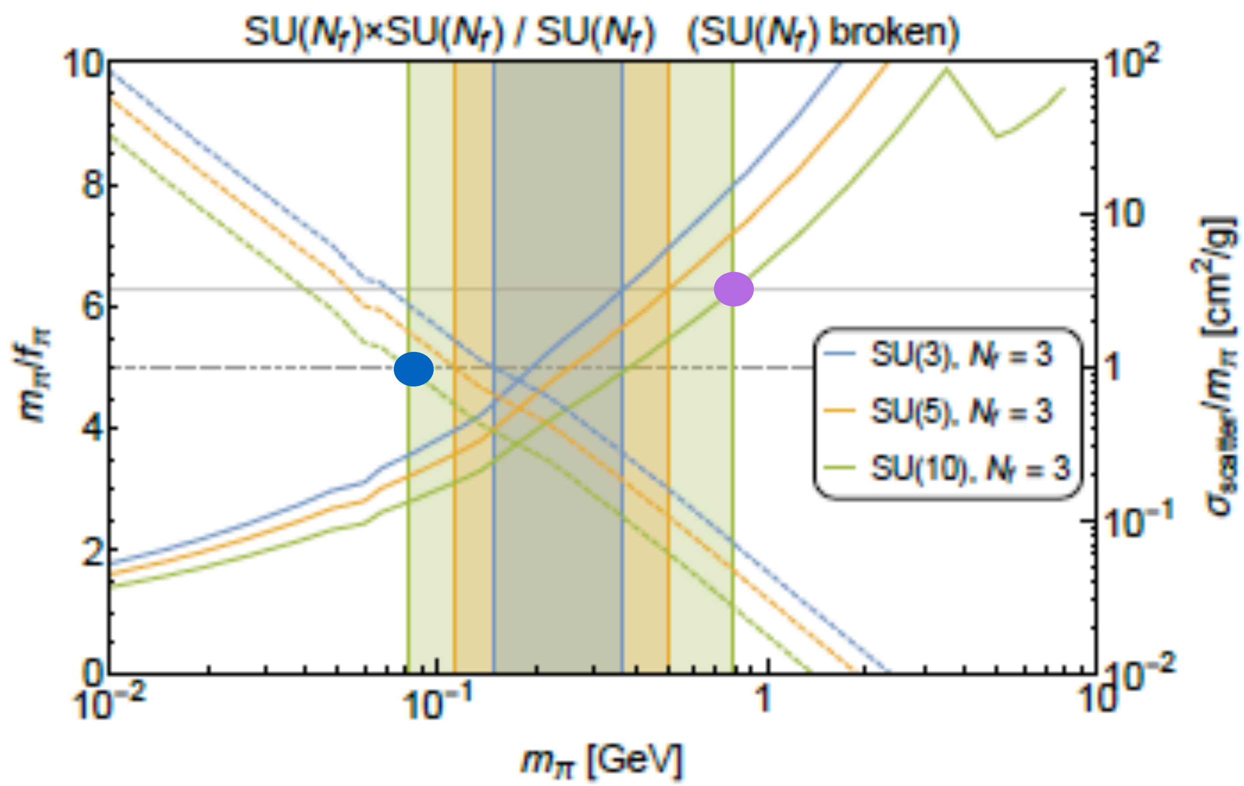
$$\mathcal{L}_{\rm mix} = -\frac{\varepsilon}{2\cos\theta_W} F'_{\mu\nu} F^{\mu\nu}$$

- Small meson mass splitting for SIMP process:

$$\delta\mathcal{L}_{m_\pi} = \alpha_D \Lambda^4 \mathrm{Tr}[Q_D U Q_D U^{-1}] \qquad \delta m_\pi^2 \sim \alpha_D \Lambda^4 / F^2 \lesssim 0.01 m_\pi^2,$$

Dark meson relics





[Hochberg, Kuflik, Murayama, Volansky, Wacker, 2014]

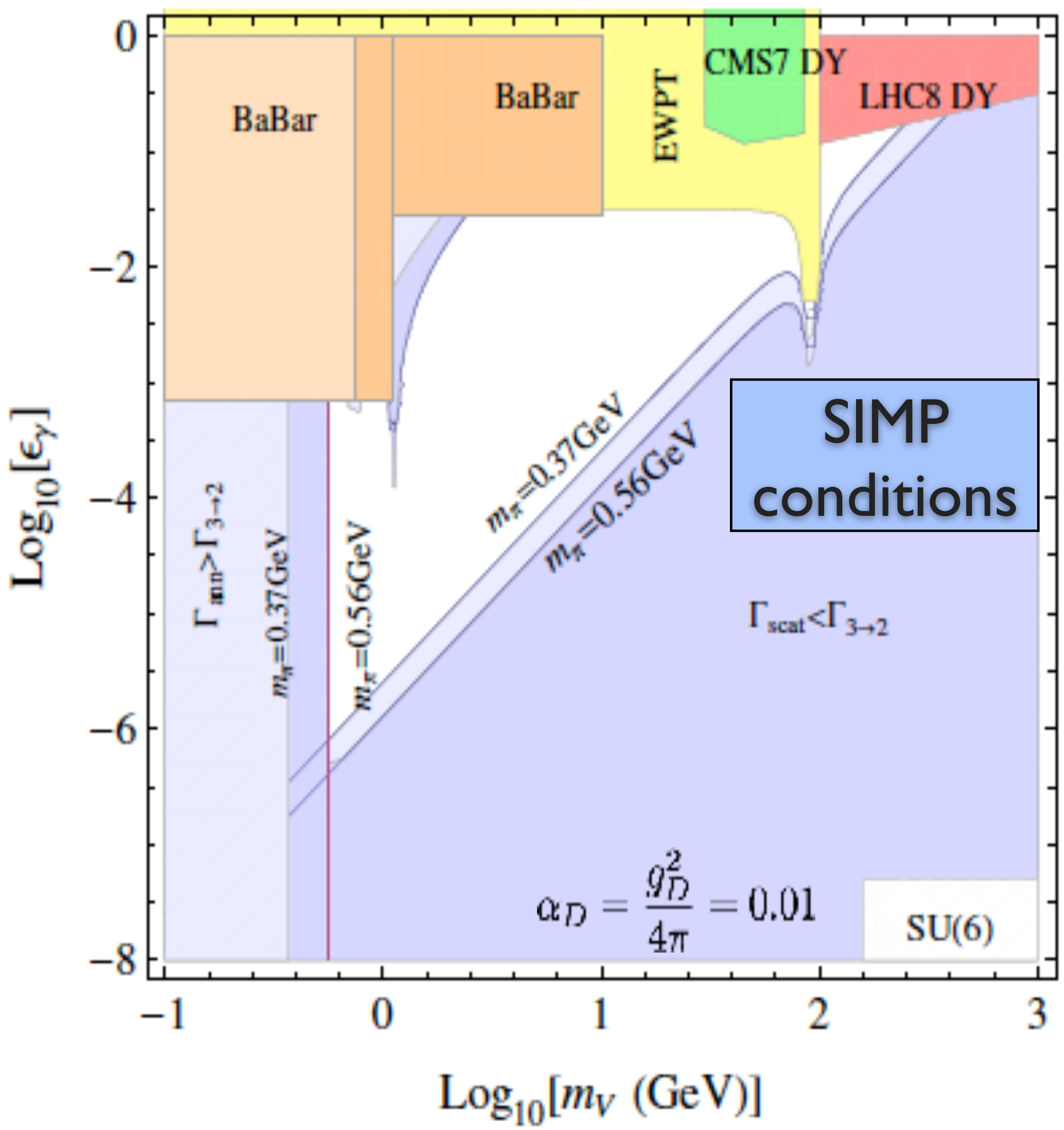
Bullet cluster, Halo shape $\sigma_{\rm self}/m_{\rm DM} < 1\,{\rm cm}^2/{\rm g}$

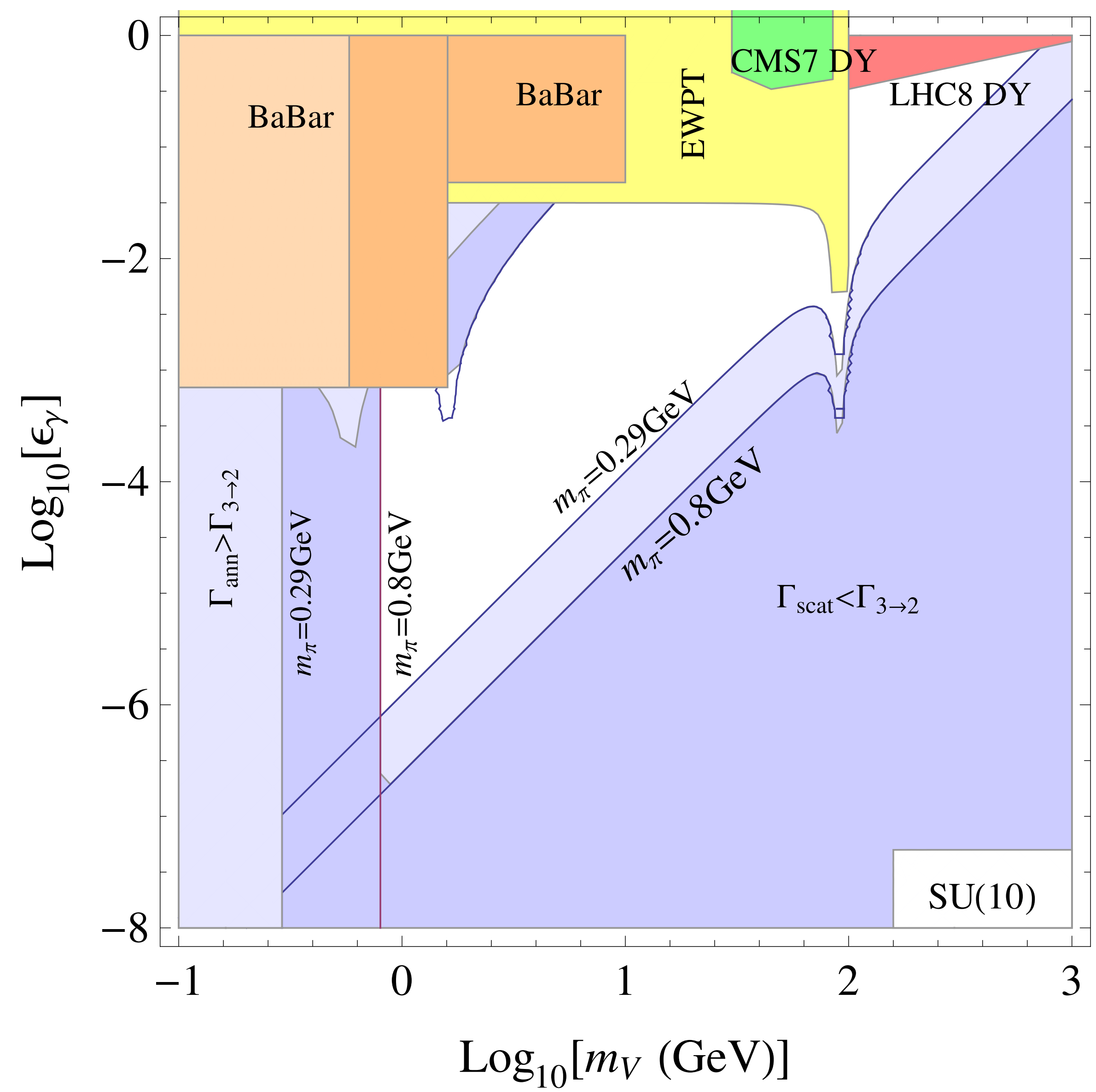
Perturbativity $m_\pi/f_\pi < 2\pi$

Bullet cluster & perturbativity bounds favor large N_c.

Bounds on dark mesons

[M.-S. Seo, HML, 2015]





- SIMP parameter space can be probed by Z' searches.

$e^+e^- \to \gamma Z' \to \gamma(l^+l^-)$, $e^+e^- \to \gamma + \mathrm{MET}$

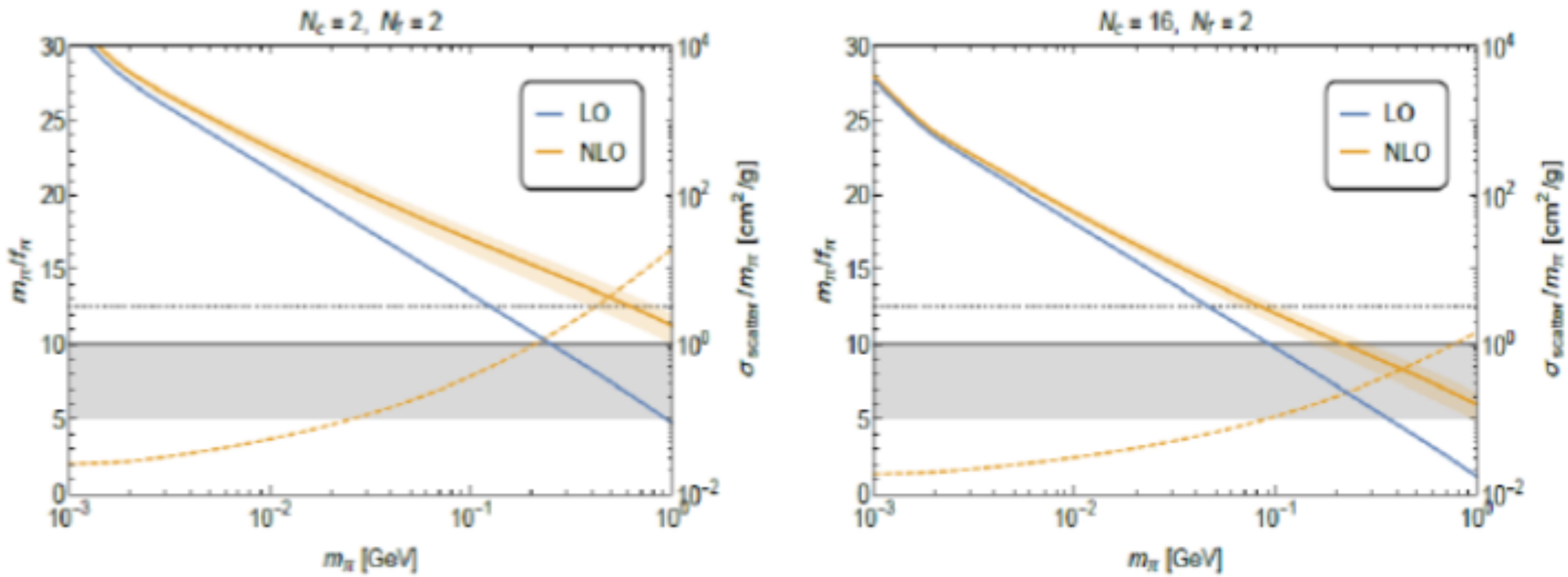
$h \to ZZ'$ (CMS 8TeV), Drell-Yan, dileptons.

cf. Hochberg et al, 2015

Perturbative mesons?

[Hansen et al, 2015]

- 2→2: LO, 3→2: NLO, in SIMP meson scenarios.



NLO corrections enlarge 2→2 cross sections, making it harder to satisfy the Bullet cluster bound within perturbativity.

→ Additional 3→2 annihilation channels?

Resonant annihilations

[S.-M.Choi, HML, 2016]

- Z_5 is the minimal (gauged) discrete symmetry for 5-point interactions for SIMP.

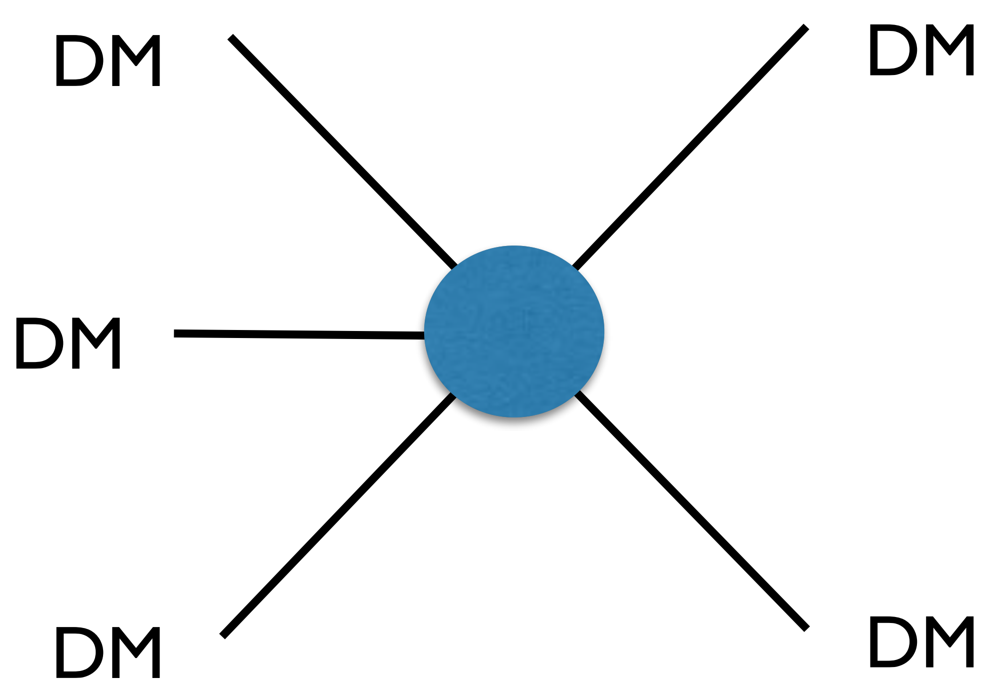
	ϕ	χ	S
$U(1)_V$	$+5$	$+1$	$+3$

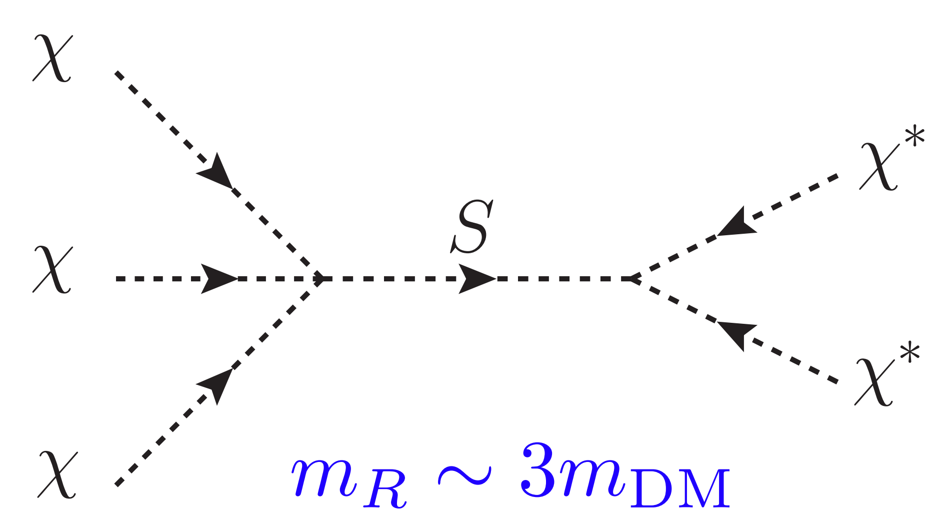
Table 1: $U(1)_V$ charges.

$$\langle\phi\rangle = \frac{1}{2}v' \quad \Rightarrow \quad U(1)_V \to Z_5$$

$$\mathcal{L}_{\rm int} \supset \frac{1}{\sqrt{2}}\lambda_1\phi^\dagger S^2\chi^\dagger + \frac{1}{\sqrt{2}}\lambda_2\phi^\dagger S\chi^2 + \frac{1}{6}\lambda_3 S^\dagger\chi^3 + \text{h.c.}$$

$$\begin{pmatrix}\phi\\ \chi\\ S\end{pmatrix} \to \begin{pmatrix}1 & 0 & 0\\ 0 & \omega & 0\\ 0 & 0 & \omega^3\end{pmatrix}\begin{pmatrix}\phi\\ \chi\\ S\end{pmatrix} \quad \omega = e^{i\frac{2}{5}\pi}$$





"3DM-Resonance"

Averaging on resonances

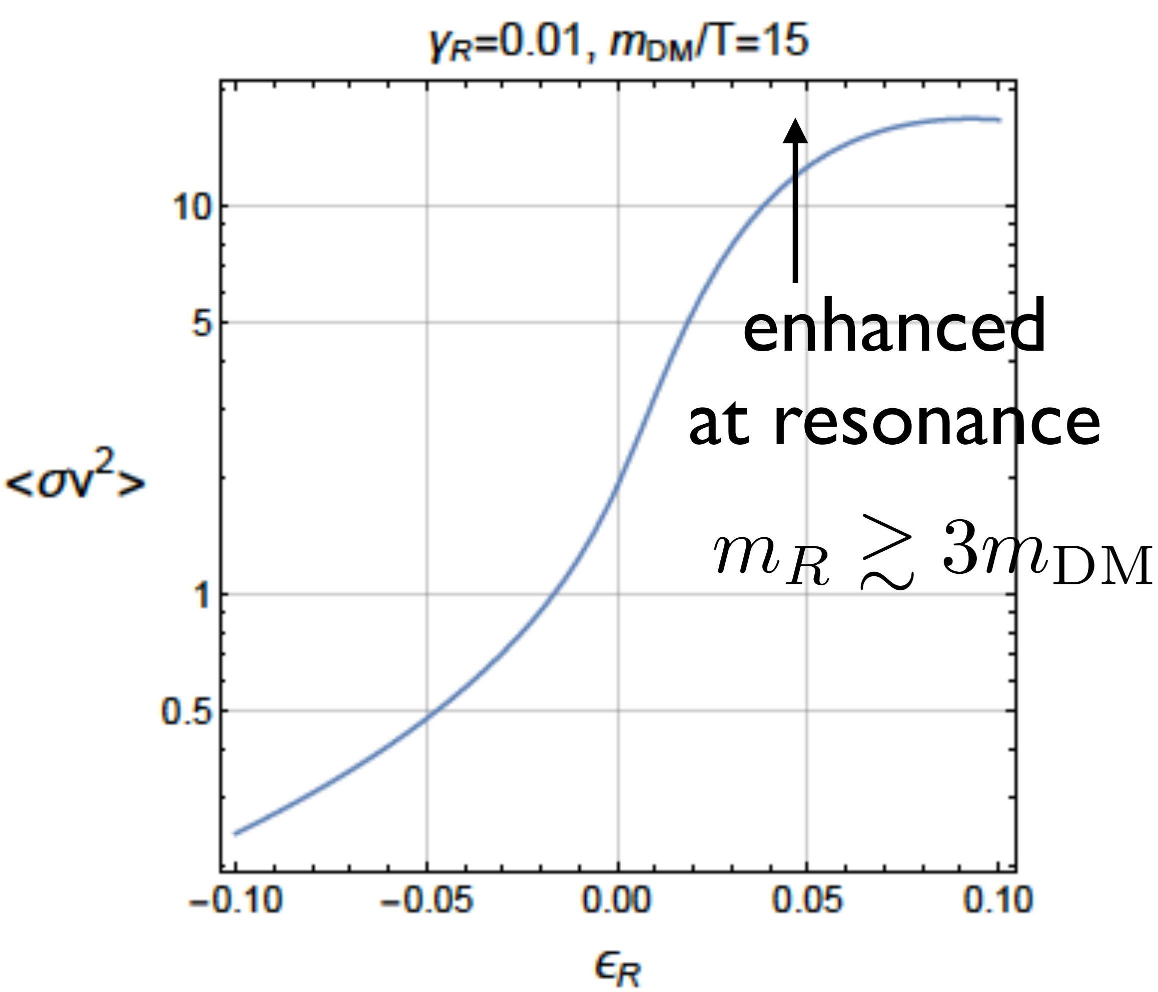
[S.-M. Choi, HML, M.-S. Seo, 2017]

- Thermal average of 3→2 cross section near resonances.

Breit-Wigner form for SIMP:

$$(\sigma v^2)_R = \frac{9\sqrt{5}}{2\beta_\chi \Phi_3 m_R^3} \frac{\gamma_R^2}{(\epsilon_R - \frac{2}{3}\eta)^2 + \gamma_R^2} \mathrm{Br}(R\to\chi\chi\chi)\,\mathrm{Br}(R\to\chi\chi) \equiv b_R \frac{\gamma_R}{(\epsilon_R - \frac{2}{3}\eta)^2 + \gamma_R^2}$$

$$b_R = \sum_{l=0}^\infty \frac{b_R^{(l)}}{l!}\eta^l \qquad \gamma_R \equiv \frac{m_R \Gamma_R}{9 m_{\rm DM}^2} \qquad \epsilon_R \equiv \frac{m_R^2 - 9m_{\rm DM}^2}{9m_{\rm DM}^2}$$



<u>Thermal average with narrow width:</u>

$$\gamma_R \ll 1, \ \langle \sigma v^2\rangle_R \approx \frac{27}{16}\pi \epsilon_R^2 x^3 e^{-\frac{3}{2}x\epsilon_R}\theta(\epsilon_R) \sum_{l=0}^\infty \frac{b_R^{(l)}}{l!}\left(\frac{3}{2}\right)^l \epsilon_R^l.$$

3 DM's: higher dimensional phase space

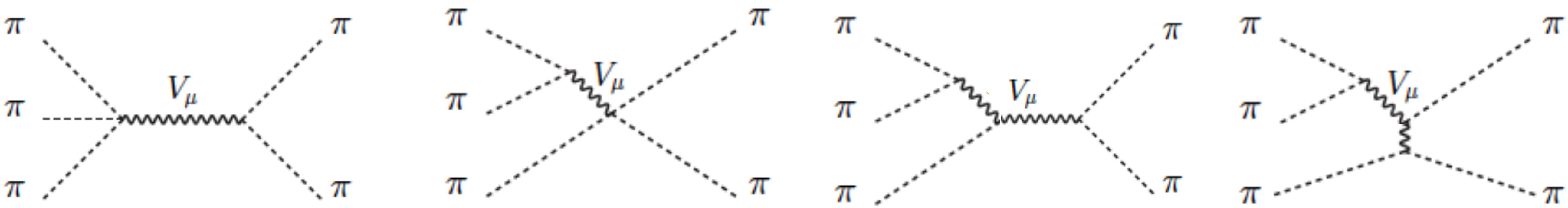
cf. WIMP: $\langle \sigma v\rangle_R \approx 2\sqrt{\pi}\gamma_R \epsilon_R^{1/2} x^{3/2} e^{-x\epsilon_R}\theta(\epsilon_R)\sum_{l=0}^\infty \frac{b_l}{l!}\epsilon_R^l.$

Vector Meson resonances

[S.-Choi, A. Natale, HML, P. Ko, 2018]

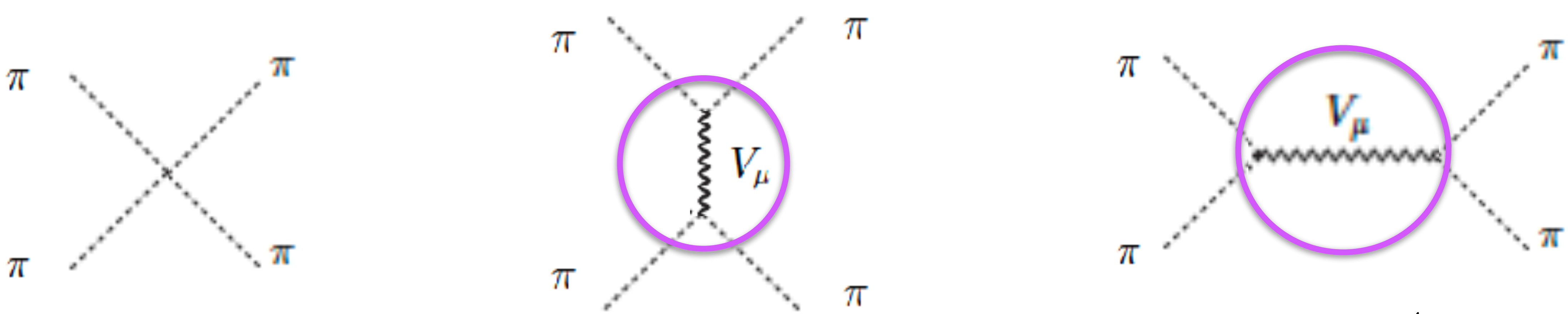
See also P. Ko's talk!

3→2 annihilation:



→ Enhanced near resonances with 2π or 3π.

2→2 self-scattering:

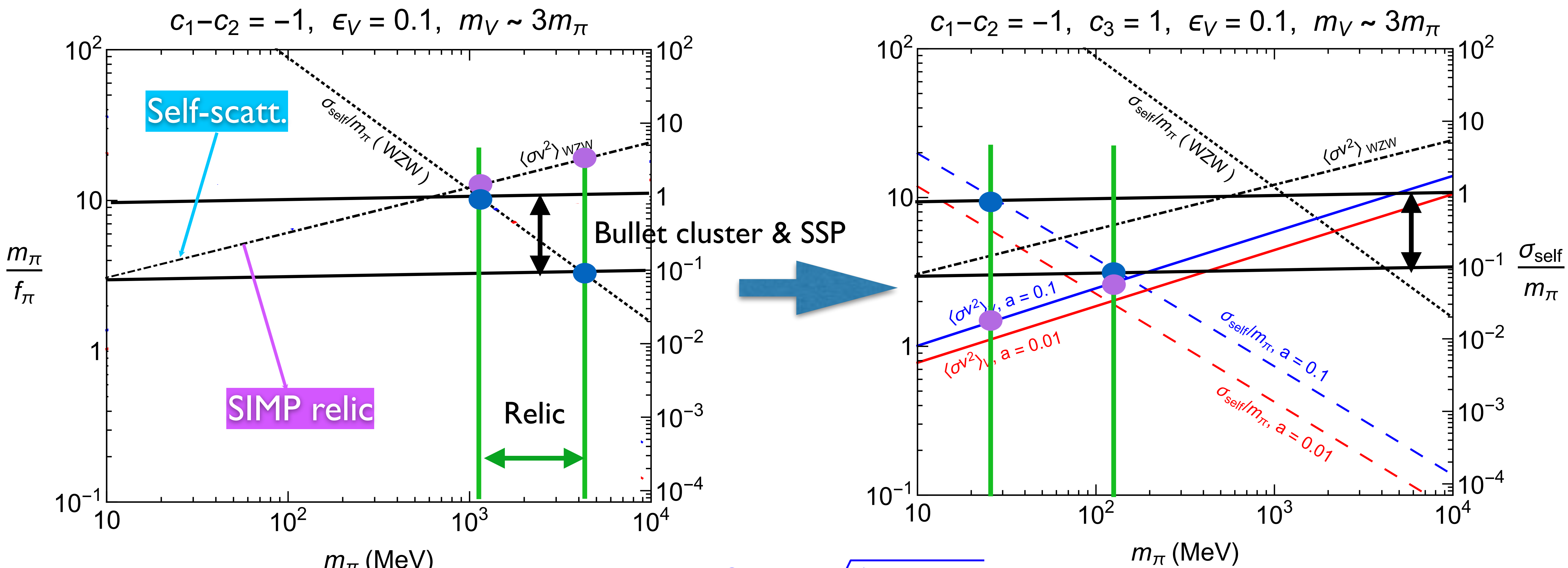


$\sim v^2$ $\sim v^4$

→ VM-induced self-scattering is velocity-suppressed at galaxies and galaxy clusters.

Perturbative mesons!

[S.-Choi, A. Natale, HML, P. Ko, 2018]



$$m_V = 3m_\pi\sqrt{1+\epsilon_V}$$

Vector mesons on-resonance improve perturbativity.

$m_\pi/f_\pi \lesssim 6$ for $g_{V\pi\pi} \lesssim 6$ and $m_\pi \lesssim 1\,\mathrm{GeV}$.

cf. WZW terms only: $m_\pi/f_\pi \lesssim 15.$

Similar improvement at 2-meson resonance.

Conclusions

- Naturalness of Higgs mass and dark matter need new strategies beyond SM and will drive us in next generations.
- Dark mesons are natural candidates for dark matter in models of neutral naturalness where dark QCD and Z' interactions exist by construction.
- SIMP dark matter is a sub-GeV SIDM produced via $3 \rightarrow 2$ freeze-out process, with testable signatures from astrophysical and Earth-based experiments.
- SIMP meson scenarios may be valid solutions to small-scale problems.

Backup

Vector Mesons in dark ChPT

SIMP mesons + Vector mesons in dark QCD:

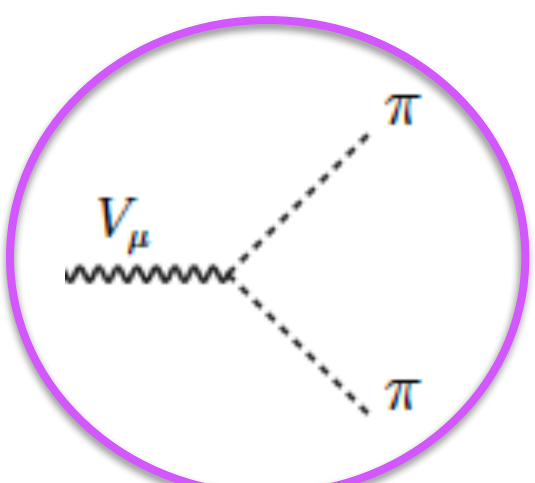
G=SU(N_f)$_V$xSU(N_f)/SU(N_f)$_V$
+ H_{local}=SU(N_f)$_V$ hidden local symmetry

$$V_\mu \equiv V_\mu^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{6}}\omega_{8\mu} + \frac{1}{\sqrt{3}}\omega_{0\mu} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{6}}\omega_{8\mu} + \frac{1}{\sqrt{3}}\omega_{0\mu} & K_\mu^{*0} \\ K_\mu^{*-} & \overline{K_\mu^{*0}} & -\frac{2}{\sqrt{6}}\omega_{8\mu} + \frac{1}{\sqrt{3}}\omega_{0\mu} \end{pmatrix}$$

Extra chiral-invariant term (in unitary gauge):

$$\mathcal{L}_B = -a\frac{f_\pi^2}{4}\mathrm{Tr}\left[(D_\mu\xi_L)\xi_L^\dagger + D_\mu\xi_R)\xi_R^\dagger\right]^2 \qquad \xi_L^\dagger(x) = \xi_R(x) = \exp[i\pi(x)/f_\pi]$$

$$\mathcal{L}_B = m_V^2 \mathrm{Tr} V_\mu V^\mu - 2ig_{V\pi\pi}\mathrm{Tr}\left(V_\mu[\partial^\mu\pi,\pi]\right) - \frac{a}{4f_\pi^2}\mathrm{Tr}\left([\pi,\partial_\mu\pi]^2\right)$$



VM masses: $m_V^2 = a g^2 f_\pi^2$ degenerate

VM-pion couplings: $g_{V\pi\pi} = \frac{1}{2} a g.$ cf. QCD: $a \simeq 2$

WZW with vector mesons

WZW terms gauged by $H_{local}=SU(N_f)_V$

$$\Gamma^{anom} = \int d^4x[\mathcal{L}_{WZW} - 15(c_1\mathcal{L}_1 + c_2\mathcal{L}_2 + c_3\mathcal{L}_3)]$$

$$\mathcal{L}_1 = \mathrm{Tr}\left[\hat{\alpha}_L^3\hat{\alpha}_R - \hat{\alpha}_R^3\hat{\alpha}_L\right] \qquad \hat{\alpha}_L = D\xi_L\cdot\xi_L^\dagger = \alpha_L - igV$$

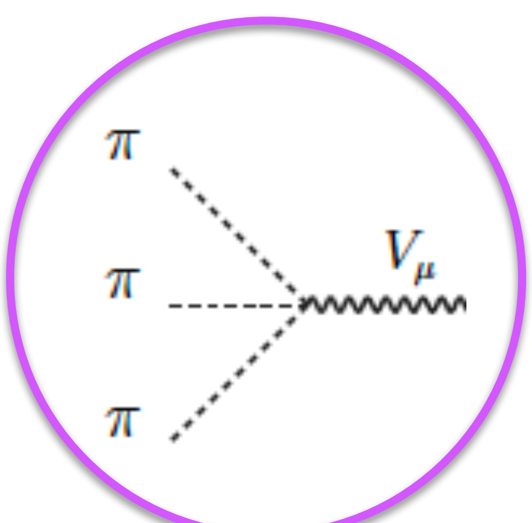
$$\mathcal{L}_2 = \mathrm{Tr}\left[\hat{\alpha}_L\hat{\alpha}_R\hat{\alpha}_L\hat{\alpha}_R\right] \qquad \hat{\alpha}_R = D\xi_R\cdot\xi_R^\dagger = \alpha_R - igV$$

$$\mathcal{L}_3 = i\mathrm{Tr}\left[F_V\left(\hat{\alpha}_L\hat{\alpha}_R - \hat{\alpha}_R\hat{\alpha}_L\right)\right] \qquad F_V = dV - igV^2$$

$$\mathcal{L}_1 = -\mathcal{L}_2 = -\frac{4gC}{f_\pi^3}\mathrm{Tr}[V_\mu\partial_\mu\pi\partial_\rho\pi\partial_\sigma\pi],$$

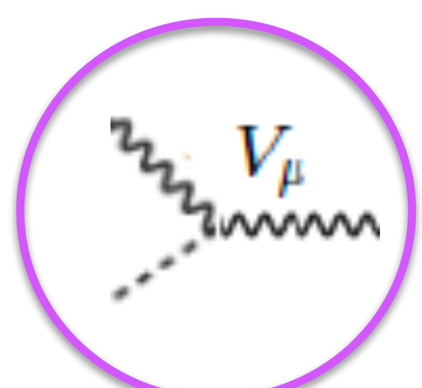
$$\mathcal{L}_3 = -\frac{2igC}{f_\pi}\mathrm{Tr}[\partial_\mu V_\nu(V_\rho\partial_\sigma\pi - \partial_\rho\pi V_\sigma)].$$

$$C = -\frac{iN_c}{240\pi^2}$$



cf. QCD:

$\omega \to 3\pi$



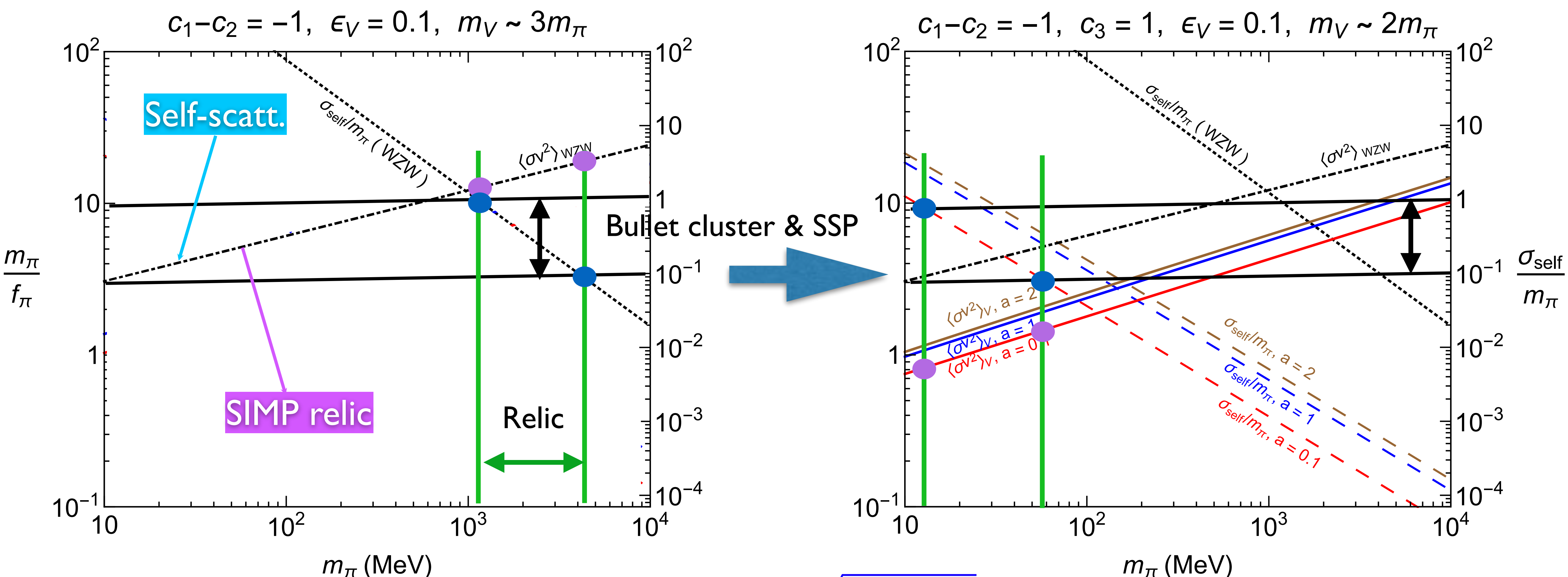
cf. QCD: VM-photon mixing

$\pi^0 \to 2\gamma$

Dark photon case: Berlin et al, 2018

2-meson resonance

[S.-Choi, A. Natale, HML, P. Ko, 2018]



$$m_V = 2m_\pi\sqrt{1+\epsilon_V}$$

Vector mesons on-resonance improve perturbativity.

$m_\pi/f_\pi \lesssim 4$ for $g_{V\pi\pi} \lesssim 1.3$ and $m_\pi \lesssim 1\,\mathrm{GeV}$.

cf. WZW terms only: $m_\pi/f_\pi \lesssim 15.$