1-4 October, 2018

4th IPMU-IPPP-KEK-KIAS

`Ikaho Onsen Workshop’

Semiclassical Higgsplosion

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IPPP Durham

- VVK & Spannowsky 1704.03447, 1707.01531, 1809.11141
- VVK 1806.05648 & 1705.04365
- VVK, Reiness, Spannowsky, Waite 1709.08655

- In this talk: I'll imagine n~150 of Higgs bosons produced in a final state at n lambda >> 1. Kinematically possible for scattering at E ~100 TeV

- HIGGSPLOSION: n-particle rates computed in a weakly-coupled theory can become unsuppressed above certain critical values of n and E.

- will consider an intrinsically Non-perturbative — semiclassical set-up
  $n \propto \sqrt{s}/m \propto 1/\lambda \gg 1$
- it incorporates correctly the tree-level results and
- the leading-order quantum effects = leading loops

already known

In this talk:

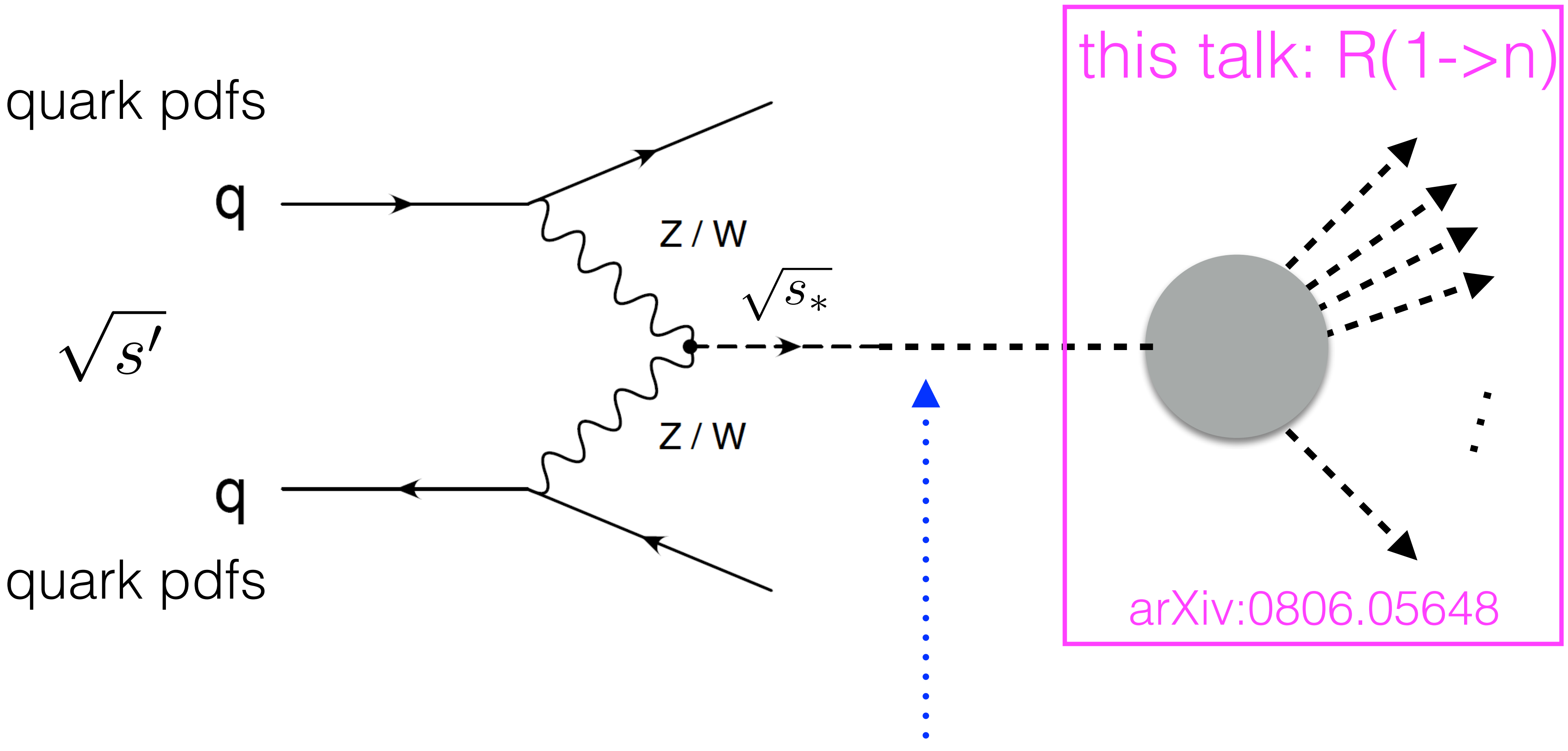
- compute quantum effects in the large lambda n limit

new

1->n processes of interest

for Higgsplosion

e.g.:Vector boson fusion in high-energy
pp collisions at ~100 TeV



n non-relativistic Higgses
Higgsplosion at $\sqrt{s_*}$

$$\frac{i}{s_* - m_h^2 - Re\tilde{\Sigma}(s_*) + i m_h \Gamma(s_*)}$$

Propagator with Higgspersion at $\sqrt{s_*}$

- VVK & Spannowsky 1704.03447,1707.01531

Factorial growth of tree-level amplitudes at thresholds:

$$\boxed{\mathcal{L} = \frac{1}{2}\,\partial^\mu h\,\partial_\mu h - \frac{\lambda}{4}\left(h^2 - v^2\right)^2}$$

prototype of the Higgs in the unitary gauge

The classical equation for the spatially uniform field $h(t)$,

$$d_t^2 h = -\lambda\, h^3 + \lambda v^2\, h\,,$$

has a closed-form solution with correct initial conditions $h_{\rm cl} = v + z + \ldots$

$$\boxed{h_0(z_0;t) = v\left(\frac{1 + z_0\, e^{imt}/(2v)}{1 - z_0\, e^{imt}/(2v)}\right), \quad m = \sqrt{2\lambda}v}$$

$$h_0(z) = v + 2v \sum_{n=1}^{\infty}\left(\frac{z}{2v}\right)^n, \quad z = z(t) = z_0\, e^{imt}$$

$$\mathcal{A}_{1\to n} = \left(\frac{\partial}{\partial z}\right)^n h_{\rm cl}\Big|_{z=0} = \boxed{n!\,(2v)^{1-n}}$$

Factorial growth

L. Brown 9209203

Analytic continuation & singularities in complex time:

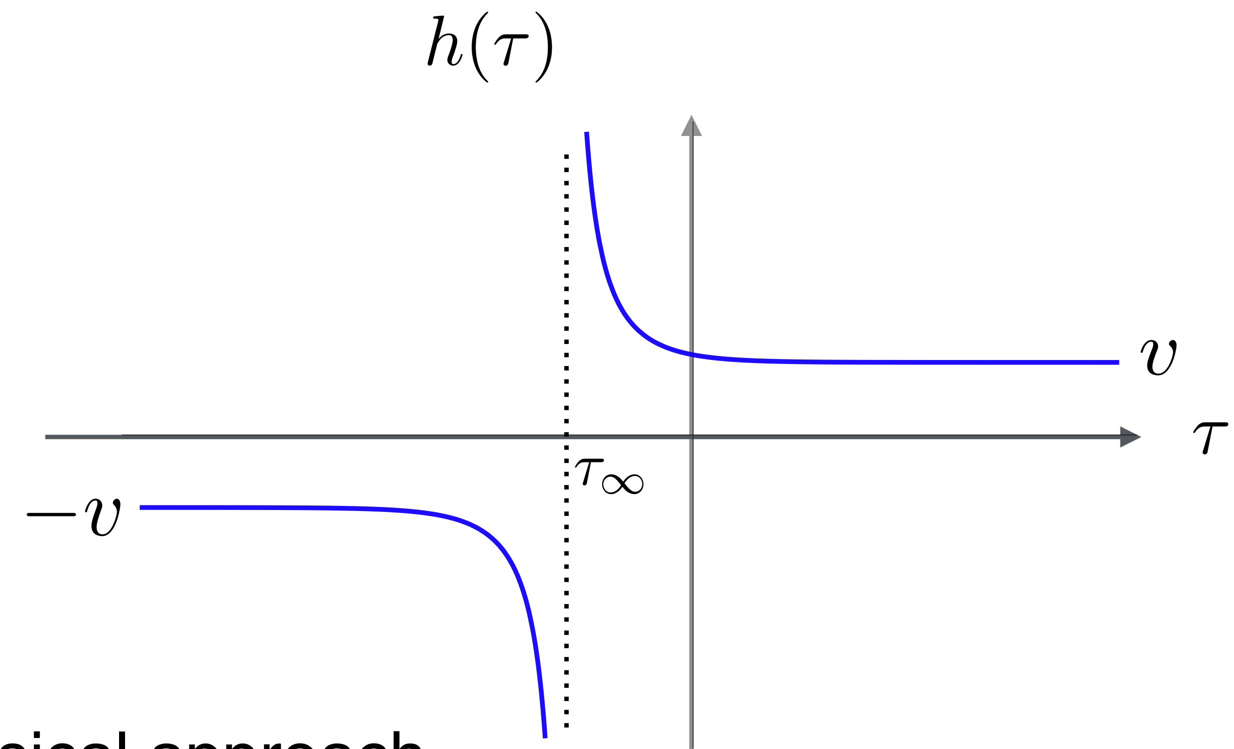
$$t \longrightarrow t_{\mathbb{C}} = t + i\tau$$

$$h_0(t_{\mathbb{C}}) \;=\; v \left(\frac{1 \;+\; e^{im(t_{\mathbb{C}} - i\tau_\infty)}}{1 \;-\; e^{im(t_{\mathbb{C}} - i\tau_\infty)}} \right) ,$$

$$\tau_\infty := \frac{1}{m} \log\left(\frac{z_0}{2v}\right)$$

Our simple example of a classical solution

$$h_0(\tau) \;=\; v \left(\frac{1 \;+\; e^{-m(\tau - \tau_\infty)}}{1 \;-\; e^{-m(\tau - \tau_\infty)}} \right)$$



Such solutions will emerge in the semiclassical approach

Main idea of the semiclassical approach

$\mathcal{R}_n(E)$ is the probability rate for a local operator $\mathcal{O}(0)$ to create n particles of total energy E from the vacuum,

$$\mathcal{R}_n(E) \;=\; \int \frac{1}{n!} d\Phi_n \,\langle 0|\, \mathcal{O}^\dagger\, S^\dagger\, P_E|n\rangle\langle n|\, P_E\, S\, \mathcal{O}\,|0\rangle$$

P_E is the projection operator on states with fixed energy E.

$$\mathcal{O} \;=\; e^{j\,h(0)}\,,$$

and the limit $j \to 0$ is taken in the computation of the probability rates,

$$\mathcal{R}_n(E) \;=\; \lim_{j\to 0} \int \frac{1}{n!} d\Phi_n \,\langle 0|\, e^{j\,h(0)}{}^\dagger\, S^\dagger\, P_E|n\rangle\langle n|\, P_E\, S\, e^{j\,h(0)}\,|0\rangle\,.$$

Note: non-dynamical (non-propagating) initial state $\mathcal{O}|0\rangle$.
The semi-classical (steepest descent) limit:

$$\varepsilon = \frac{E-nm}{nm}$$

$$\boxed{\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed}\,, \quad \varepsilon = \text{fixed}\,.}$$

Evaluate the path integral in this double-scaling limit.
n enters via the coherent state formalism.

Rubakov & Tinyakov; DT Son '95

Main idea of the semiclassical approach

Note:

The initial state is not a semiclassical, it contains few (1 or 2) rather than many particles.

Son argued that it can be approximated in the semiclassical method by a certain local operator acting on the vacuum:

$$|X\rangle = \mathcal{O}(0)\,|0\rangle$$

$$\mathcal{O}(x) = j^{-1}\, e^{j\phi(x)}\,,$$

j is a constant $j = c/\lambda$. Finally one takes the limit $c \to 0$ (or equivalently $j \to 0$)

A refinement:

operator localized in the vicinity of a point x

$$\mathcal{O}_g(x) = \int d^4x'\, g(x'-x)\,\mathcal{O}(x')\,, \qquad |X\rangle = \mathcal{O}_g(0)\,|0\rangle = \int d^4x'\, g(x')\,\mathcal{O}(x')\,|0\rangle$$

The Semiclassical formalism of Son: results in four steps

1. Solve the classical equation without the source-term:

$$\frac{\delta S}{\delta h(x)} = 0$$

   a complex-valued solution $h(x)$ with a point-like singularity at $x^\mu = 0$. The singularity is due to $\mathcal{O}(x=0)$.

2. Impose the initial and final-time boundary conditions:

$$\lim_{t\to-\infty} h(x) = v + \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\, a^\dagger_{\mathbf{k}}\, e^{ik_\mu x^\mu}$$

$$\lim_{t\to+\infty} h(x) = v + \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left(b_{\mathbf{k}}\, e^{\omega_{\mathbf{k}} T - \theta}\, e^{-ik_\mu x^\mu} + b^\dagger_{\mathbf{k}}\, e^{ik_\mu x^\mu} \right)$$

- Son hep-ph/055338

The Semiclassical formalism of Son: results in four steps

3. Compute E and n of the final state using the $t \to +\infty$ asymptotics

$$E = \int d^3k \; \omega_{\mathbf{k}} \, b^{\dagger}_{\mathbf{k}} \, b_{\mathbf{k}} \, e^{\omega_{\mathbf{k}} T - \theta} \,, \qquad n = \int d^3k \; b^{\dagger}_{\mathbf{k}} \, b_{\mathbf{k}} \, e^{\omega_{\mathbf{k}} T - \theta}$$

At $t \to -\infty$ the energy and the particle number are vanishing.
The energy changes discontinuously from 0 to E at the singularity at $t = 0$.

4. Eliminate the T and θ parameters in favour of E and n.
Finally, compute the function $W(E, n)$

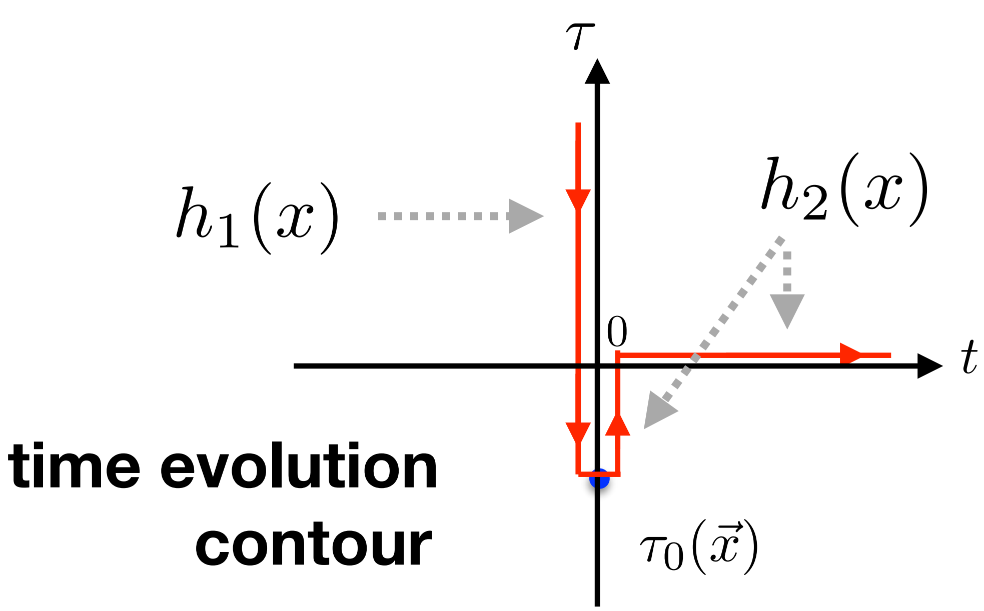
$$W(E, n) = ET - n\theta - 2\mathrm{Im}S[h]$$

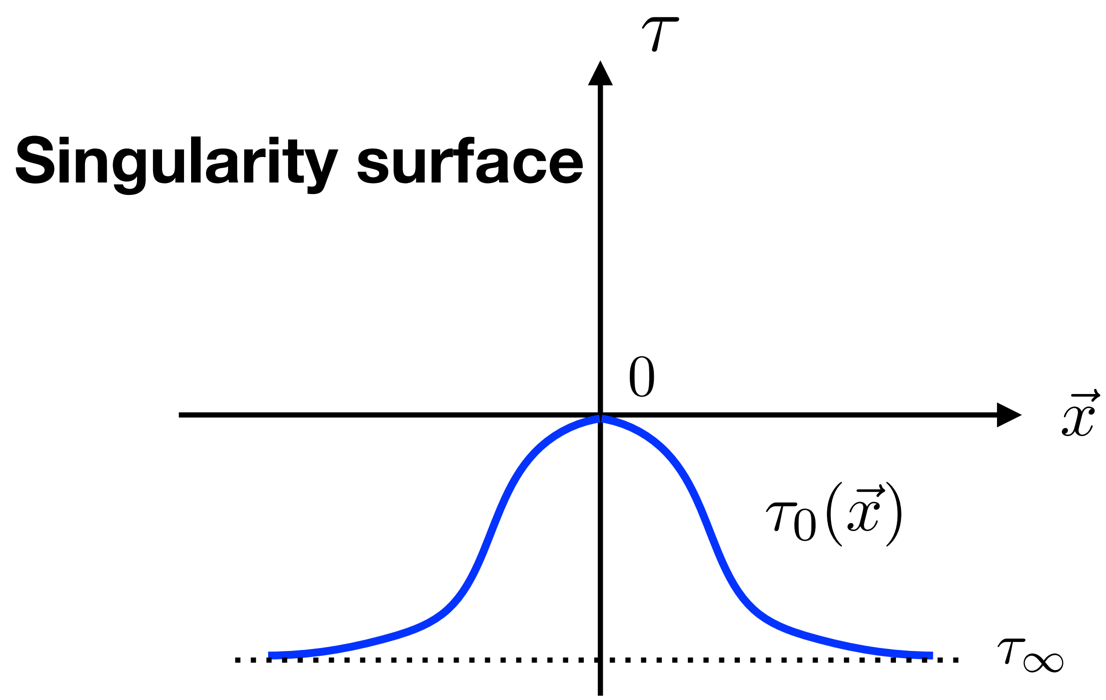
on the set $\{h(x), T, \theta\}$ and fine the semiclassical rate $\mathcal{R}_n(E) = \exp\left[W(E, n)\right]$

- Son hep-ph/055338

Refining the method in complex time

- In the Euclidean space-time, $(\tau, \vec{x})$ the solution will be singular a 3-dimensional hypersurface $\tau = \tau_0(\vec{x})$ located at $t = 0$.





- Find a classical trajectory $h_1(\tau, \vec{x})$ on the first segment $+\infty > \tau > \tau_0(\vec{x})$
- Find another classical solution $h_2(\tau, \vec{x})$ on the remaining part of the contour that at $\tau \to \tau_0(\vec{x})$ is singular and $h_2(\tau_0, \vec{x}) = h_1(\tau_0, \vec{x})$.

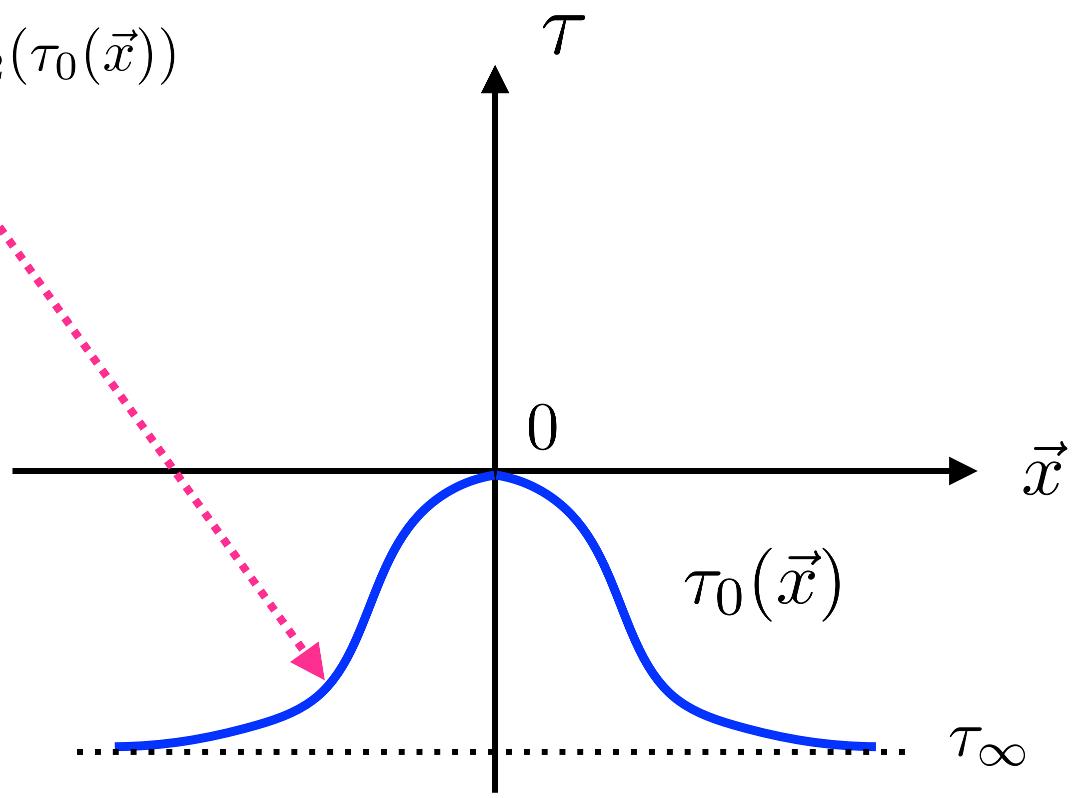
- For the combined configuration $h(x)$ to solve classical equations everywhere, including the τ_0 surface:

need to extremize the action integral over all singularity surfaces $\tau = \tau_0(\vec{x})$ containing the point $t = 0 = \vec{x}$.

$$iS[h] = \int d^3x \left(\left| \int_{+\infty}^{\tau_0(\vec{x})} d\tau\, \mathcal{L}_{\rm Eucl}(h_1) \right| - \left| \int_{\tau_0(\vec{x})}^{0} d\tau\, \mathcal{L}_{\rm Eucl}(h_2) \right| + i \int_0^{\infty} dt\, \mathcal{L}(h_2) \right)$$

$h_1(\tau_0(\vec{x})) = h_2(\tau_0(\vec{x}))$

Extremize the action S over all such singularity surfaces:



Computing the semiclassical rate

Classical solution singular on a generic tau_0 surfaces:

$$h(t_{\mathbb{C}}, \vec{x}) = v \left(\frac{1 + e^{im(t_{\mathbb{C}} - i\tau_\infty)}}{1 - e^{im(t_{\mathbb{C}} - i\tau_\infty)}} \right) + \tilde{\phi}(t_{\mathbb{C}}, \vec{x})$$

Find that:

$$W(E,n) = ET - n\theta - 2\mathrm{Re}S_{\mathrm{Eucl}}[h]$$

$$= n \log \frac{\lambda n}{4} + \frac{3n}{2} \left(\log \frac{3\pi}{\varepsilon} + 1 \right) - 2nm\,\tau_\infty - 2\mathrm{Re}S_{\mathrm{Eucl}}[h]$$

$W(E,n)^{\mathrm{tree}}$

agrees with the known result of tree-level contributions

$\Delta W^{\mathrm{quant}}$

need to compute by extremizing w.r.t tau_0(x)

Computing the semiclassical rate

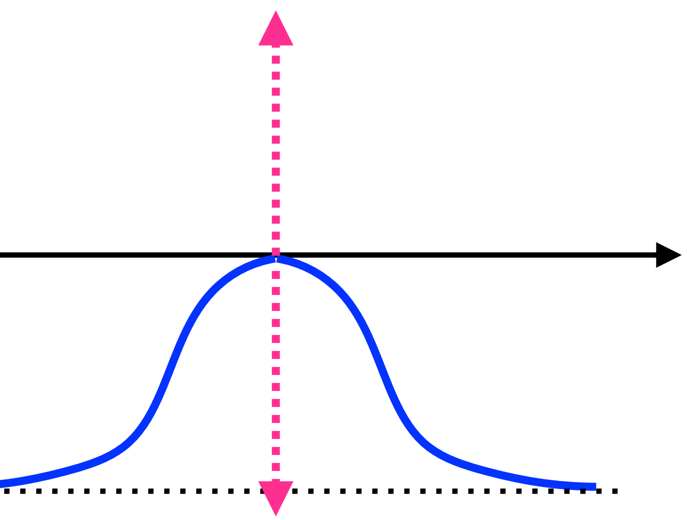
$$\Delta W^{\text{quant}} = -2nm\,\tau_\infty - 2\text{Re}\,S^{(1,2)}_{\text{Eucl}}$$

$$= 2nm\,|\tau_\infty| + 2\int d^3x \left[\int_{\tau_0(\vec{x})}^{+\infty} d\tau\,\mathcal{L}_{\text{Eucl}}(h_1) - \int_{\tau_0(\vec{x})}^{0} d\tau\,\mathcal{L}_{\text{Eucl}}(h_2) \right]$$

E=0 configuration

E=mn configuration

Force x height



Surface-energy

$$\frac{1}{2}\Delta W^{\text{quant}} = nm\,|\tau_\infty| - \underbrace{\int_{A+i\epsilon}^{0+i\epsilon} d\tau\, L_{\text{Eucl}}(h_2;\tau_0(\vec{x}))}_{\equiv S_{\text{Eucl}}[\tau_0(\vec{x})]} + \frac{4\pi}{3}\,\mu R^3$$

Force x height

Surface-energy

Mechanical analogy: surface at equilibrium/balance of forces

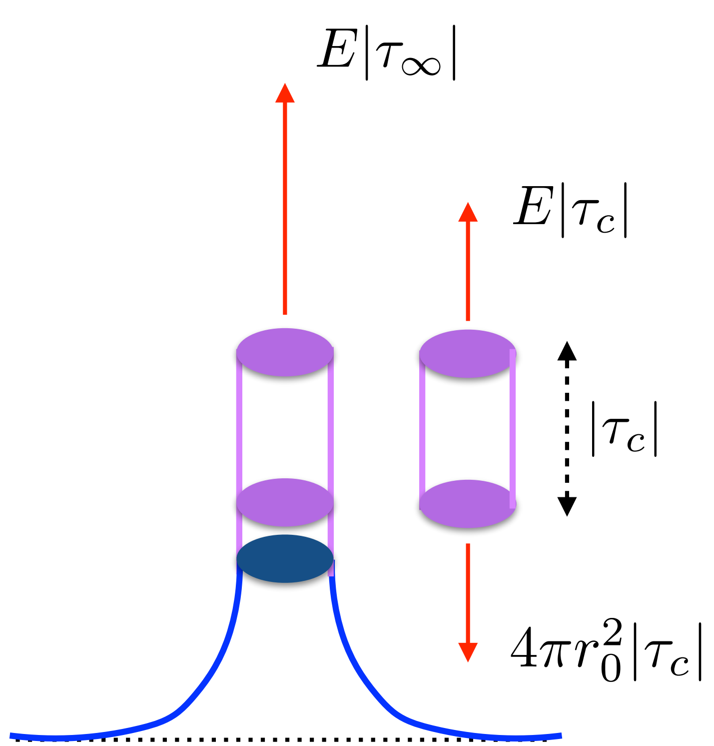
Computing the semiclassical rate

Use *thin wall* approximation:

$$\frac{1}{2}\Delta W^{\text{quant}}_{\text{stationary}} = -\int_R^0 p(E)\,dr + \frac{4\pi}{3}\,\mu R^3\,, \qquad E = nm$$

final result

$$\boxed{\Delta W^{\text{quant}} = \frac{E^{3/2}}{\sqrt{\mu}}\,\frac{2}{3}\,\frac{\Gamma(5/4)}{\Gamma(3/4)} = \frac{1}{\lambda}\,(\lambda n)^{3/2}\,\frac{2}{\sqrt{3}}\,\frac{\Gamma(5/4)}{\Gamma(3/4)} \simeq 0.854\,n\sqrt{\lambda n}}$$



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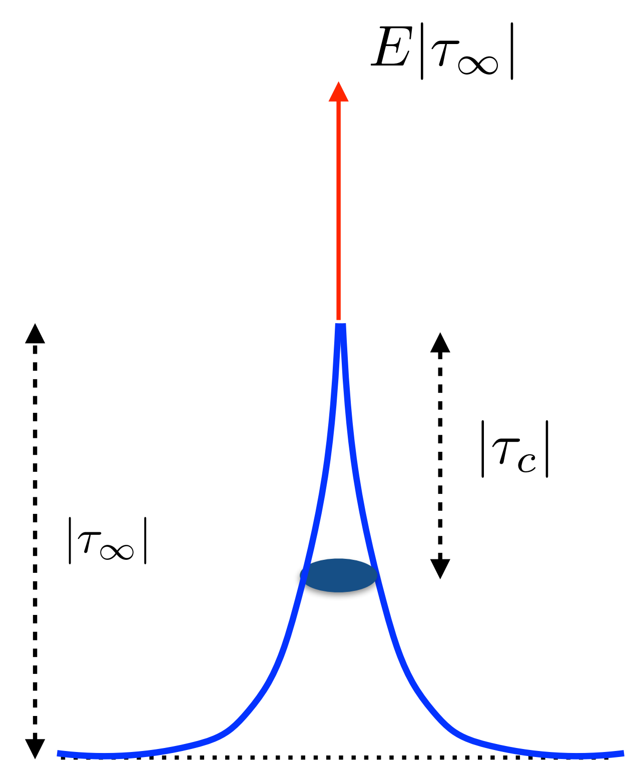
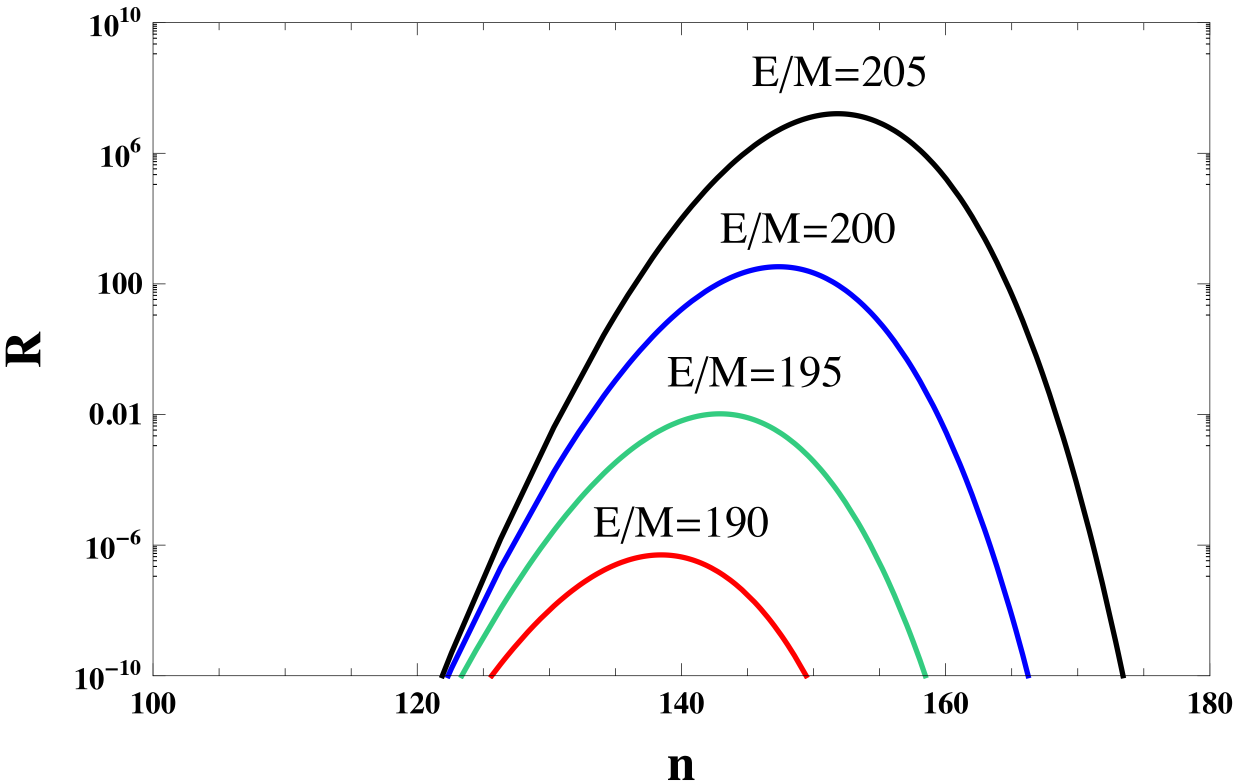


Summary of the main result

- VVK 1806.05648

$$\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed} \gg 1\,, \quad \varepsilon = \text{fixed} \ll 1$$

$$\mathcal{R}_n(E) \,=\, e^{W(E,n)} \,=\, \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + 0.85\sqrt{\lambda n} - 1 + \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\,\varepsilon\right)\right]$$



$$E/m \,=\, (1+\varepsilon)\, n$$

positive
(quantum effects)

negative
(phase space)

Can always make this term win =>
unsuppressed R at high Energies

Higher order corrections are suppressed by extra powers of $\lambda \to 0$ and $1/n \to 0$ and by $\mathcal{O}(1/\sqrt{\lambda n})$ as well as by $\mathcal{O}(\varepsilon)$.

Conclusions:

- The semiclassical calculation reviewed in the talk was aimed towards developing a theoretical foundation for the mechanism of Higgsplosion

- $$\Delta_R(p) \;=\; \frac{i}{p^2 - m^2 \,-\, \mathrm{Re}\,\Sigma_R(p^2) \,+\, \underset{\substack{\uparrow \\ \mathrm{R}\;\longleftrightarrow\;\text{Higgsplosion}}}{im\Gamma(p^2)} \,+ i\epsilon}$$

Loop integrals are effectively cut off at E_* by the exploding width $\Gamma(p^2)$ of the propagating state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta $k_i^2 \sim m^2 \lll E_*^2$.

The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the n soft particle quanta of the same field ϕ.

- VVK & Spannowsky 1704.03447, 1707.01531