

DM shifts away from direct detection

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Ikaho, Hotel Tenbo
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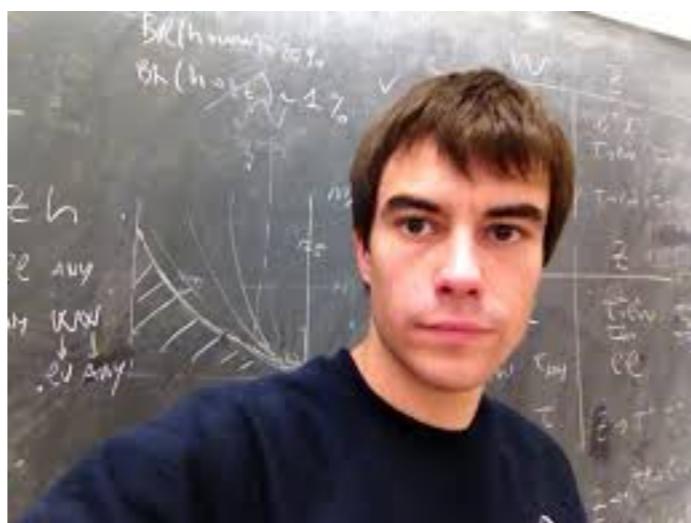
Excellence Cluster Universe



In collaboration with

[arXiv:1707.07685](https://arxiv.org/abs/1707.07685) , [arXiv:1809.09106](https://arxiv.org/abs/1809.09106)

Reuven Balkin (PhD student)



Ennio Salvioni (Post-doc)



Max Ruhdorfer (PhD student)

Goldstone DM

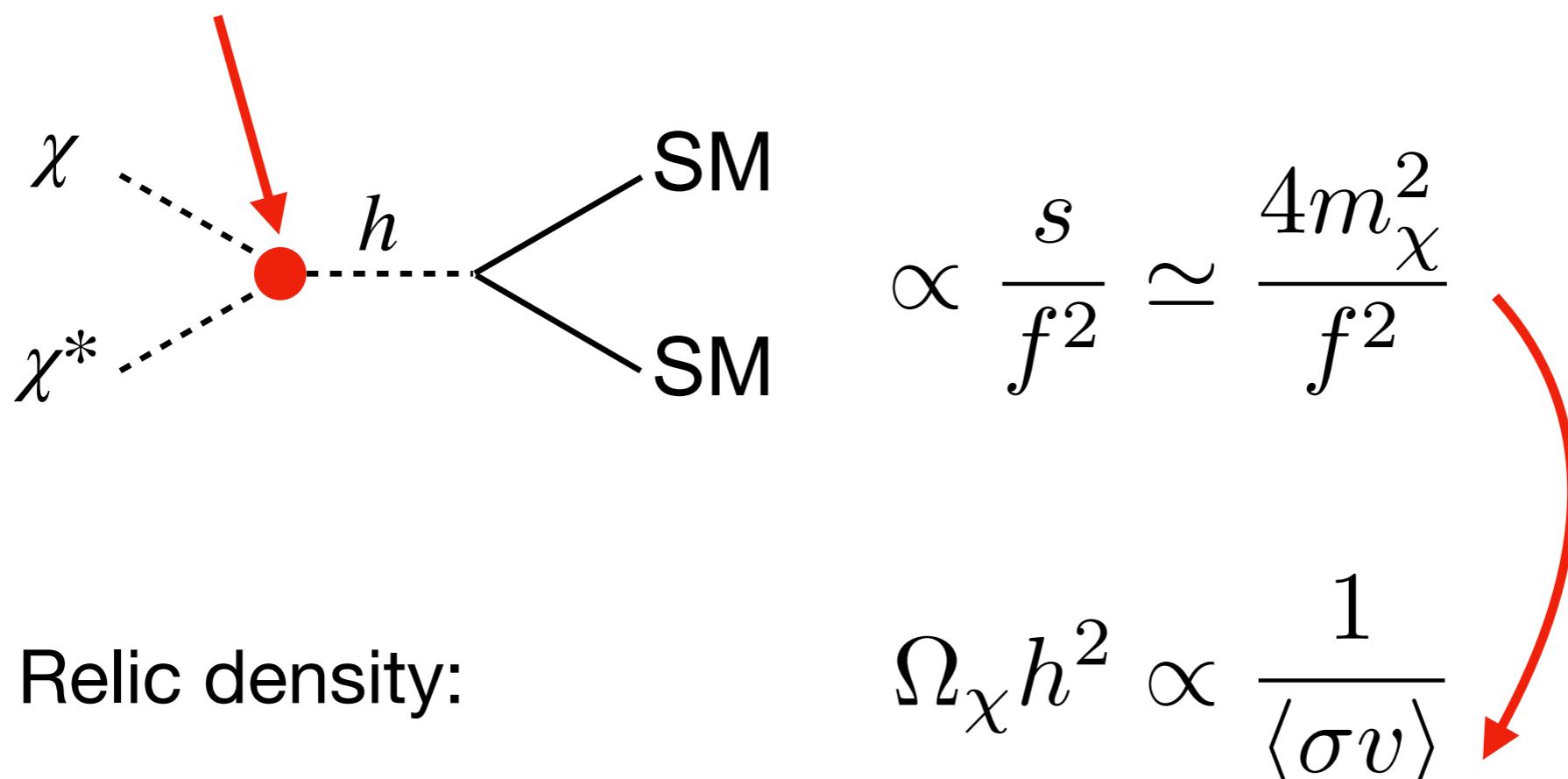
$$\chi \rightarrow \chi + f \quad \text{GB shift symmetry}$$

Leading coupling between DM and SM involves derivative:

$$\frac{1}{f^2} \partial_\mu (h^2) \partial^\mu (\chi^* \chi) + \dots$$

Annihilation

$$\frac{1}{f^2} \partial_\mu (h^2) \partial^\mu (\chi^* \chi) + \dots$$



Relic density:

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle}$$

This fixes a one-to-one relation between m_χ and f

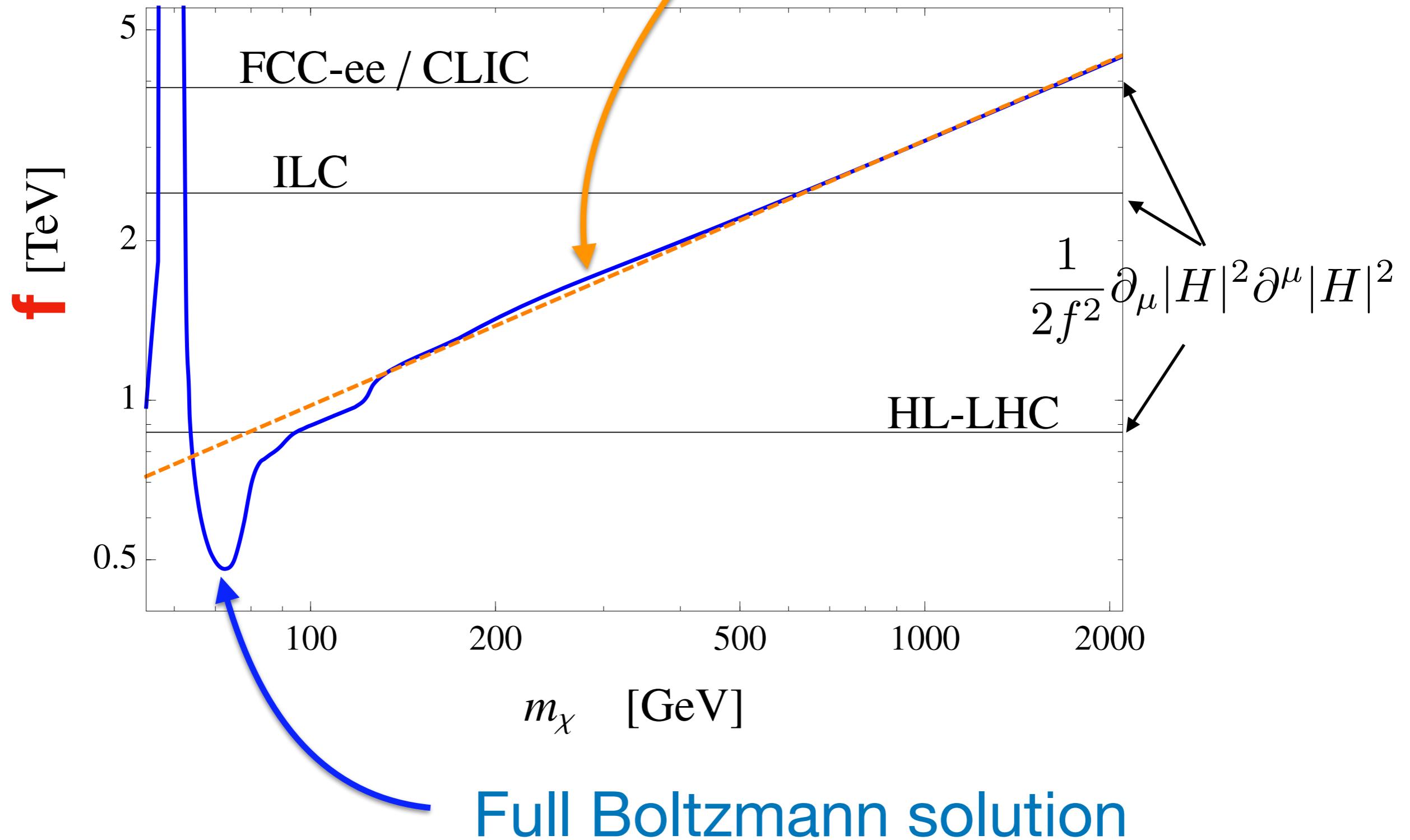
Correct relic density requires:

$$f \approx 1.1 \text{ TeV} \left(\frac{m_\chi}{130 \text{ GeV}} \right)^{1/2}$$

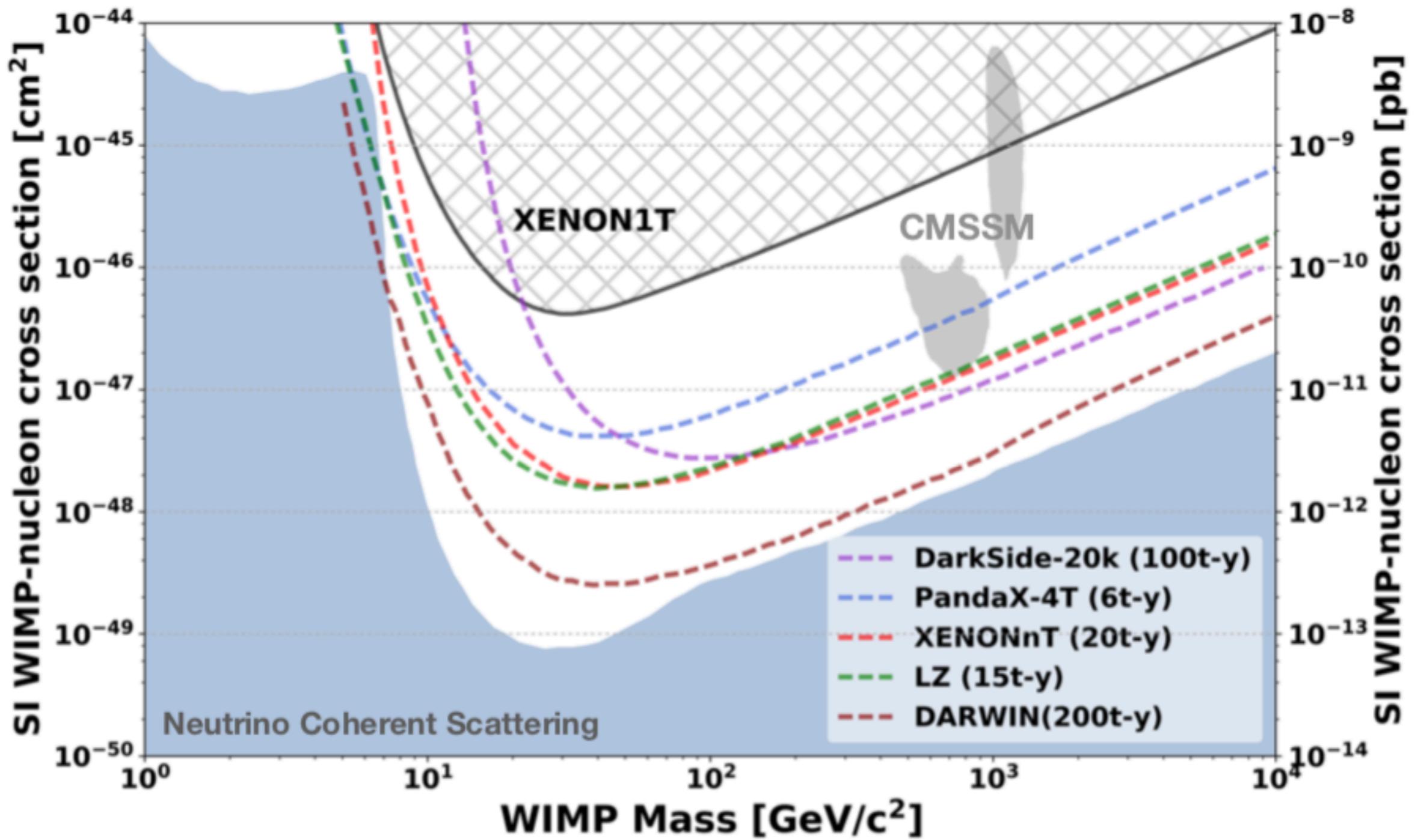
Interesting !

This corresponds to expectation for a natural pGB (compare to minimal composite Higgs models).

Estimation: $f \approx 1.1 \text{ TeV} \left(\frac{m_\chi}{130 \text{ GeV}} \right)^{1/2}$

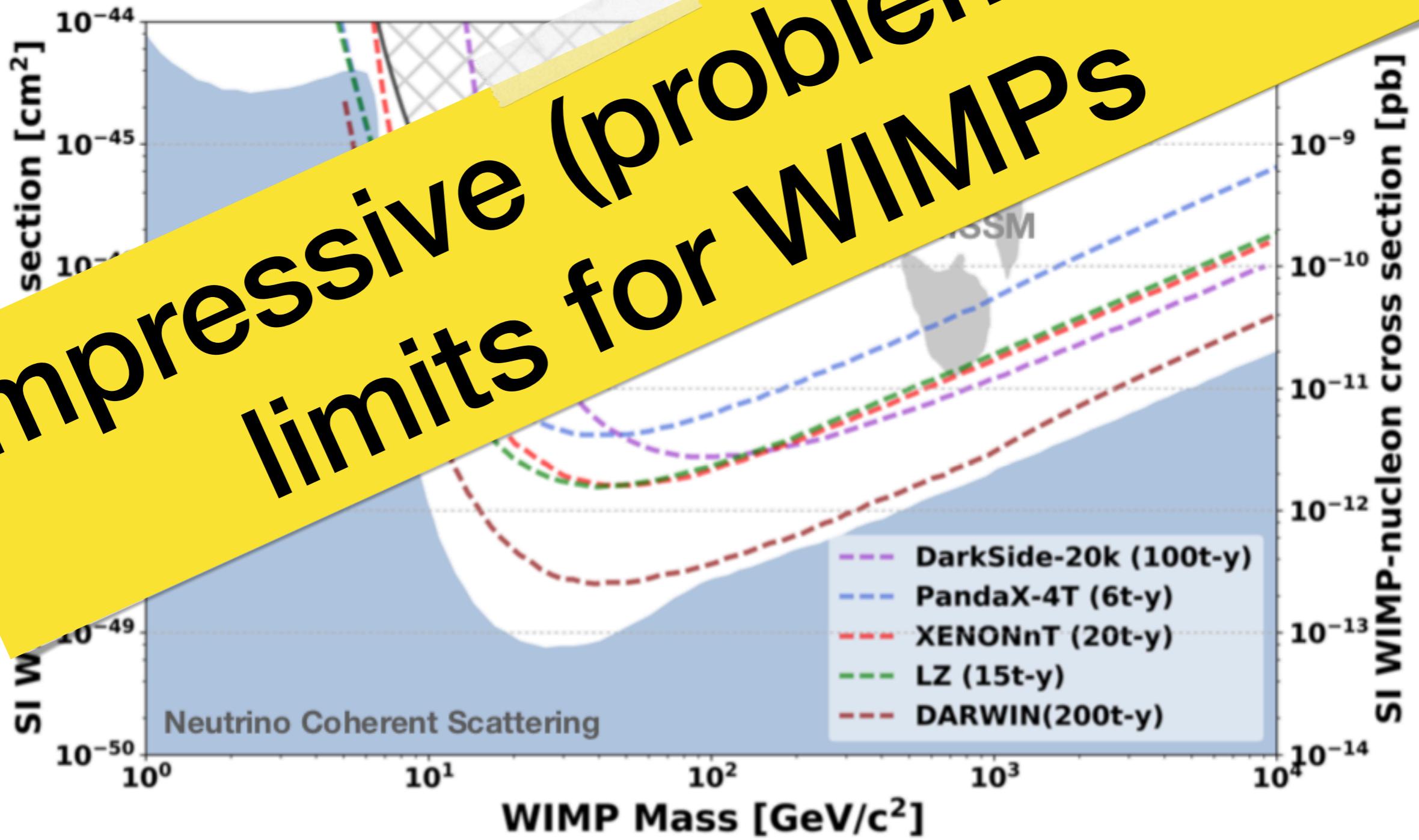


Direct detection



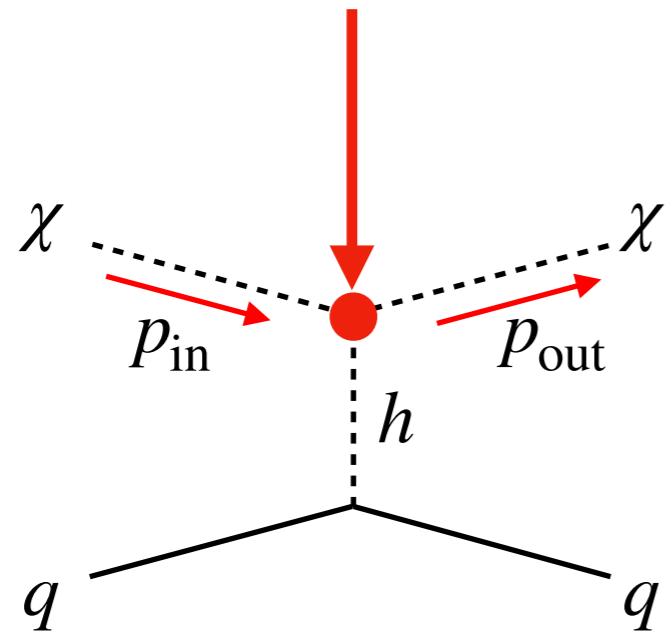
Direct detection problematic?

impressive (problematic)
limits for WIMPs



Direct detection of pure Goldstone DM

$$\frac{1}{f^2} \partial_\mu (h^2) \partial^\mu (\chi^* \chi) + \dots$$



$$\propto \frac{(p_{\text{in}} - p_{\text{out}})^2}{f^2} \lesssim \frac{(1 \text{ MeV})^2}{(1 \text{ TeV})^2}$$

Explains absence of current (and future) direct detection signals.

Wait ...



- DM cannot be an exact Goldstone b/c of DM mass:

$$m_\chi^2 |\chi|^2 + \dots \quad \text{means} \quad \chi \rightarrow \chi + f$$

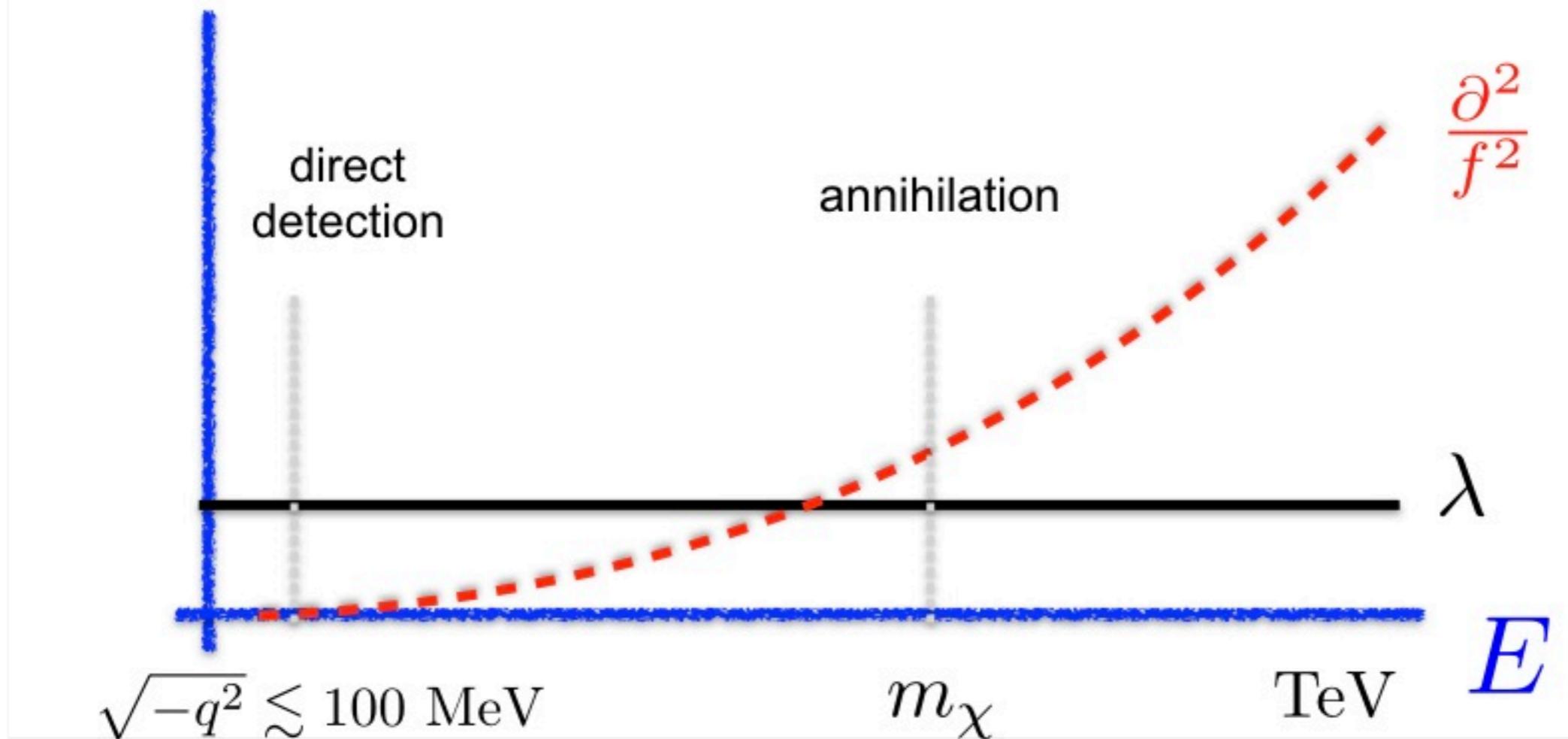
Controlled breaking of DM shift symmetry

$$m_\chi^2 \chi^* \chi + \lambda h^2 \chi^* \chi + \frac{1}{f^2} \partial_\mu(h^2) \partial^\mu(\chi^* \chi)$$

induces mass but also **marginal** portal.

Direct detection is very sensitive to λ .

$g_{\text{DM-SM}}^2(E)$

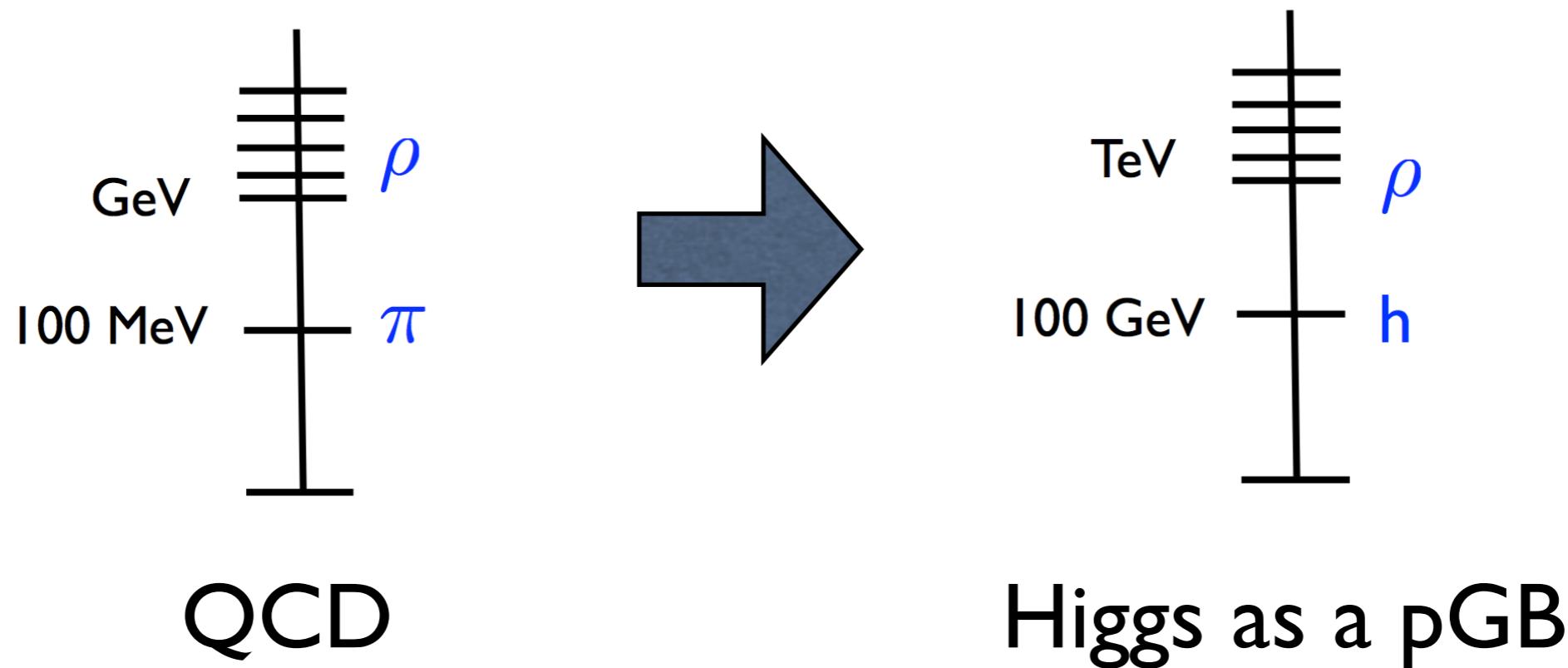


Want to be close to Goldstone limit: $\lambda \ll 1$

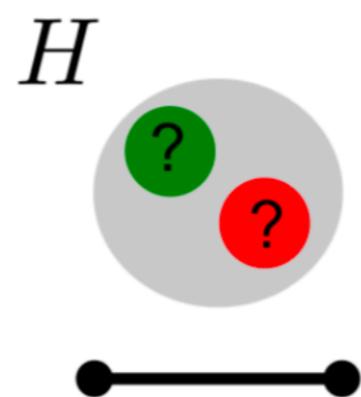
Framework

A solution to the hierarchy problem:

the Higgs as a pGB bound state of a new strong force.



Description changes above confinement scale (\sim TeV)



Higgs mass is naturally
“screened”.

$$r_H \sim (\text{TeV})^{-1}$$

Spontaneous breaking of a global symmetry \mathcal{G}

$$\mathcal{G} \xrightarrow[H, \dots]{f} \mathcal{H}$$

Minimal Model: $SO(5) \xrightarrow[H]{f} SO(4)$

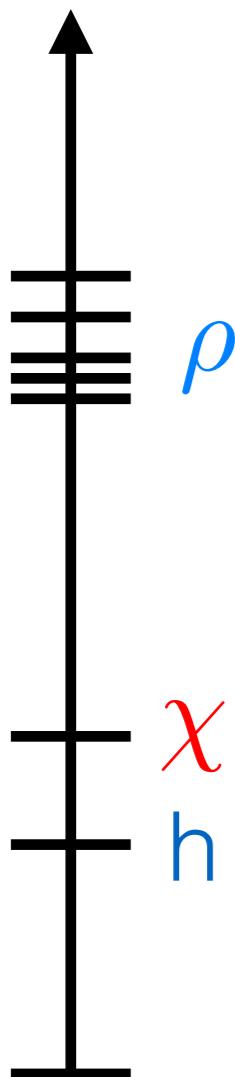
Agashe, Contino, Pomarol 2004

Enlarge global symmetry group: non-minimal pNGB

$$\mathcal{G} \xrightarrow[H, \chi]{f} \mathcal{H} \quad \text{additional pNGB } \chi \text{ as WIMP}$$

E.g. SO(6)/SO(5) or SO(7)/SO(6), etc.

Goldstone dark matter



Symmetry that protects the (Higgs mass)² also

- keeps WIMP χ light: $\chi(x) \rightarrow \chi(x) + \alpha$

- renders χ stable: $\chi(x) \rightarrow -\chi(x)$ parity

or

$$\chi(x) \rightarrow e^{i\beta} \chi(x) \quad U(1)_{\text{DM}}$$

will use here

A pNGB Model

Balkin, Ruhdorfer, Salvioni, AW 1707.07685

$SO(7)/SO(6)$

(H, χ)

Higgs + complex scalar

$SO(7)$ has only real rep's: automatically UV safe
→ no WZW anomaly term

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Unbroken symmetry

$$SO(6) \sim SO(4) \times SO(2)$$

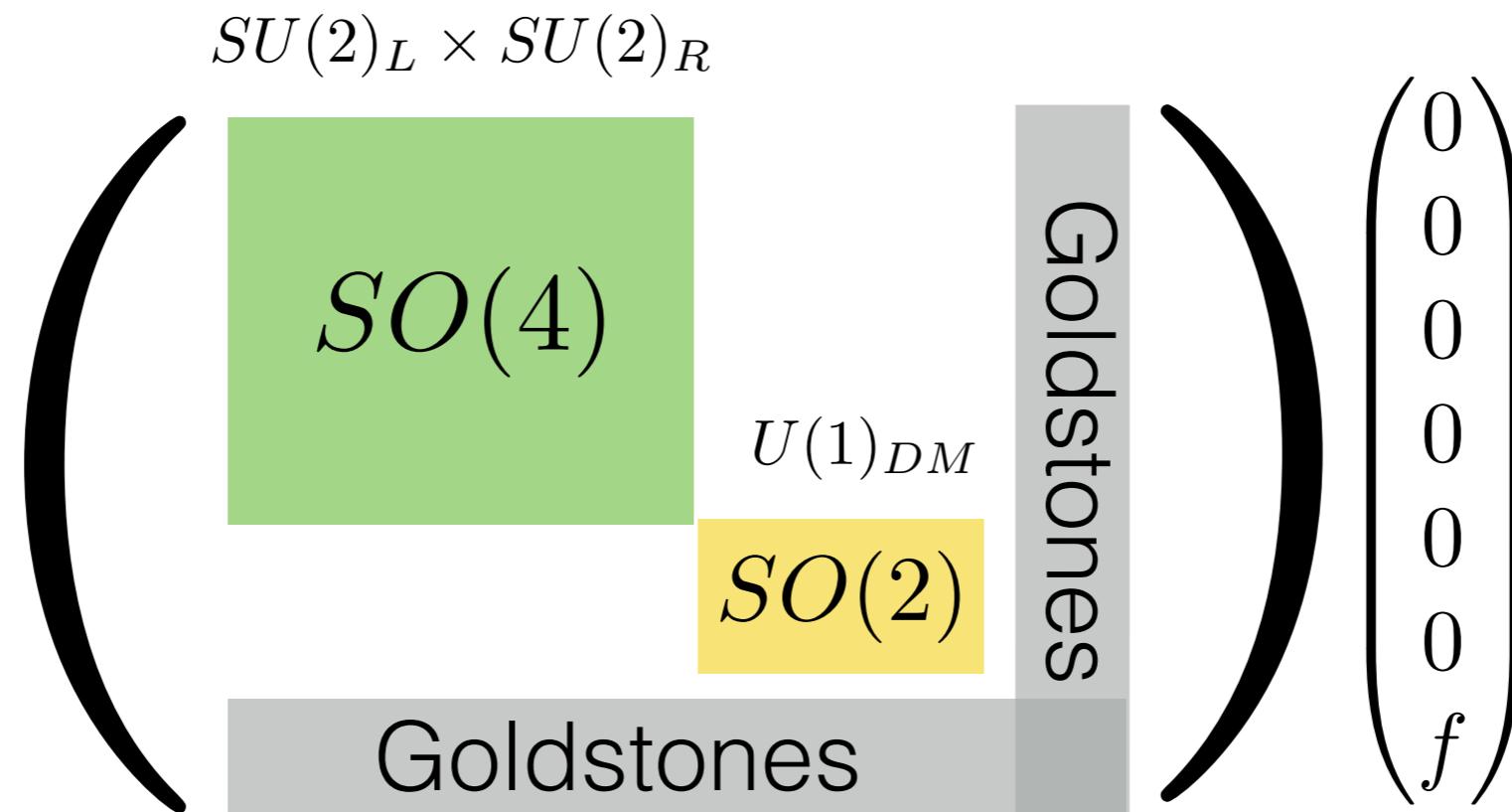
$$\sim SU(2)_L \times SU(2)_R \times U(1)_{DM}$$

EW

custod'

DM stabilisation

$SO(7)/SO(6)$



Breaking the DM shift symmetry

[arXiv:1707.07685](#)

- Top quark mass: $\lambda \sim \frac{\lambda_h}{2}$
- Bottom quark mass: $\lambda \propto y_b^2 \ll 1$ [arXiv:1809.09106](#)
- Partial gauging of DM Goldstone symmetry $U(1)_{\text{DM}}$: $\lambda \propto \text{higher-loop} \ll 1$

$$m_\chi^2 \chi^* \chi + \lambda h^2 \chi^* \chi + \frac{1}{f^2} \partial_\mu(h^2) \partial^\mu(\chi^* \chi)$$

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DM shift symmetry broken
by b-quark embedding

NGB \rightarrow pNGB

Shift symmetry $\chi(x) \rightarrow \chi(x) + \alpha$ broken by

- Gauging of $U(1)_{DM}$
- SM fermions in incomplete **G=SO(7)** multiplets, couple linearly to strong sector

$$\mathcal{L}_{\text{mix}} \sim \epsilon_q \bar{q}_L O_q + \epsilon_t \bar{t}_R O_t$$

partial compositeness

Partial compositeness

D. B. Kaplan '91

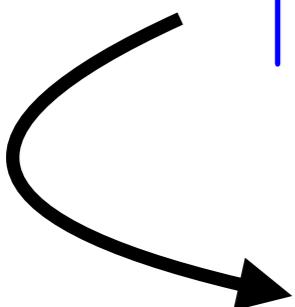
$$\mathcal{L} = \lambda_L \bar{q}_L O_R + \lambda_R \bar{u}_R O_L + h.c.$$

(linear couplings)

elementary

composite

$$|SM\rangle = \cos \alpha |t\rangle + \sin \alpha |T\rangle$$



$$\frac{m_t}{v} \approx \sin \alpha_L \cdot Y \cdot \sin \alpha_R$$

SM matter embedding

- Useful to classify according to

$$SO(7) \supset SO(4) \times SO(3) = SU(2)_L \times SU(2)_R \times SU(2)_{DM}$$

where the $SU(2)_{DM}$ is generated by

$$\{T^\pm, T_{DM}\}$$

broken generators ->
charged DM pNGB

$U(1)_{DM}$: stability of DM

Example: \mathbf{t}_R embedding

$$SO(7) \supset SU(2)_L \times SU(2)_R \times SU(2)_{DM}$$

- $7 = (2, 2, 1) \oplus (1, 1, 3), \quad t_R$
- $8 = (2, 1, 2) \oplus (1, 2, 2),$
- $21 = (2, 2, 3) \oplus (3, 1, 1) \oplus (1, 3, 1) \oplus (1, 1, 3),$
- $27 = (3, 3, 1) \oplus (2, 2, 3) \oplus (1, 1, 5) \oplus (1, 1, 1).$
- ...

Example: \mathbf{t}_R embedding

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Higgs & DM mass

Integrate out heavy composite resonances

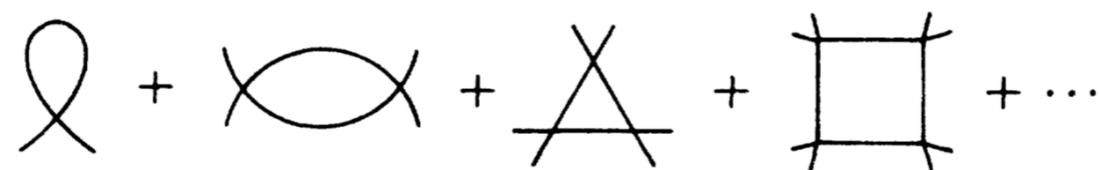
$$\mathcal{L}_{eff} = \Pi_L \bar{t}_L \not{p} t_L + \Pi_R \bar{t}_R \not{p} t_R - (\Pi_{LR} \bar{t}_L t_R + h.c.)$$

with $\left\{ \begin{array}{l} \Pi_L = \Pi_{L_0} + \frac{h^2}{f^2} \Pi_{L_1} \\ \Pi_R = \Pi_{R_0} + \left(\frac{h^2}{f^2} + \frac{2\chi^\dagger \chi}{f^2} \right) \Pi_{R_1} \quad \text{w/ eg.} \quad \Pi_{R_0} = 1 - \sum_{j=1}^{N_S} \frac{|\epsilon_{tS}^j|^2}{p^2 - m_{S_j}^2} \\ \Pi_{LR} = i \frac{h}{f} \sqrt{1 - \frac{h^2}{f^2} - \frac{2\chi^\dagger \chi}{f^2}} \Pi_1. \end{array} \right.$ etc

Higgs & DM mass 2

Coleman-Weinberg

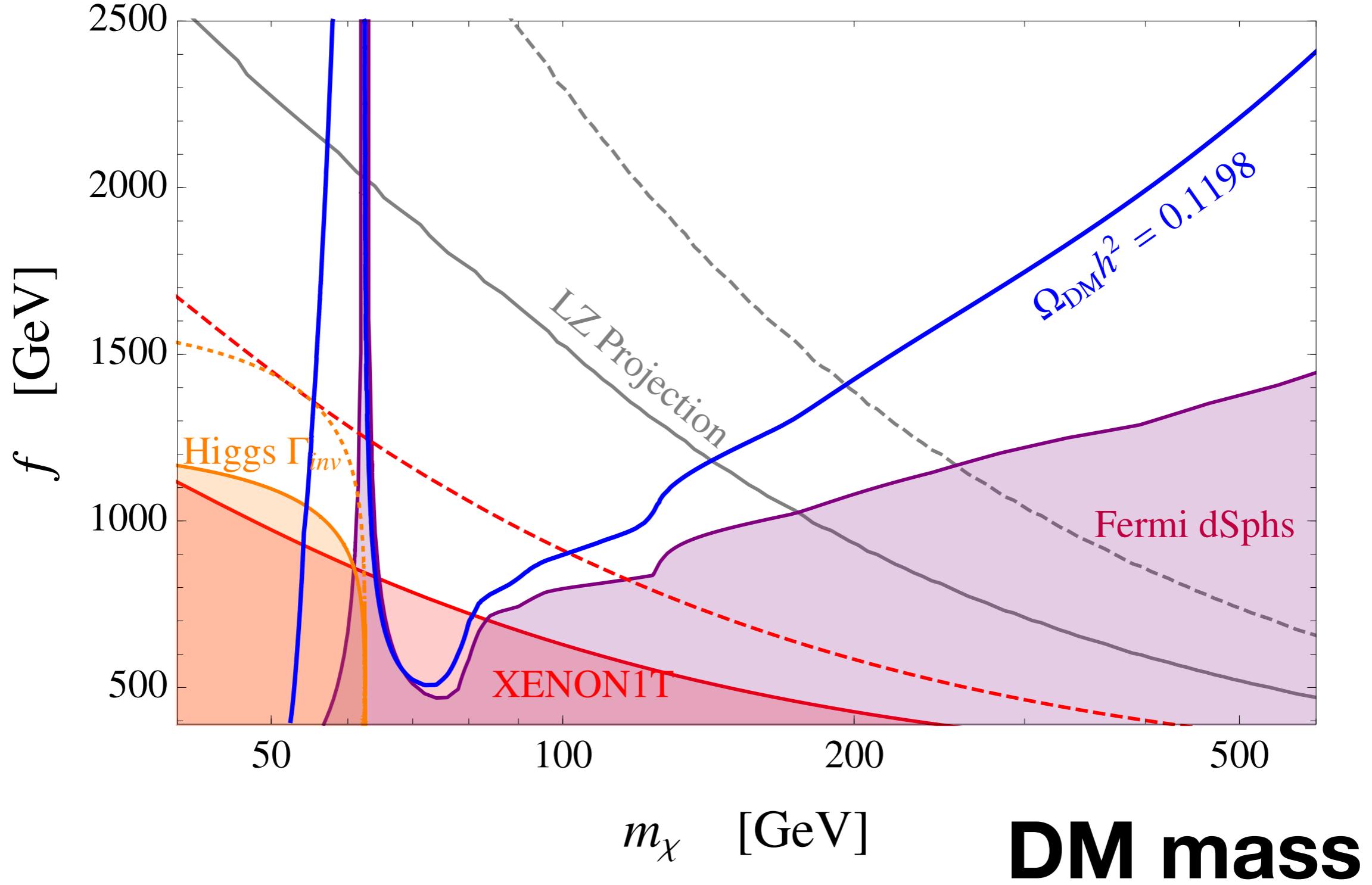
$$V_{eff}(h, \chi) = \frac{-2N_c}{(4\pi)^2} \int_0^\infty dk_E^2 k_E^2 \log \left[1 + \frac{h^2}{f^2} f_1(k_E^2) \right. \\ \left. + \frac{h^2}{f^2} \left(\frac{h^2}{f^2} + \frac{2\chi^\dagger \chi}{f^2} \right) f_2(k_E^2) + \frac{2\chi^\dagger \chi}{f^2} f_3(k_E^2) \right]$$



DM shift symmetry broken by
b-quark embedding:

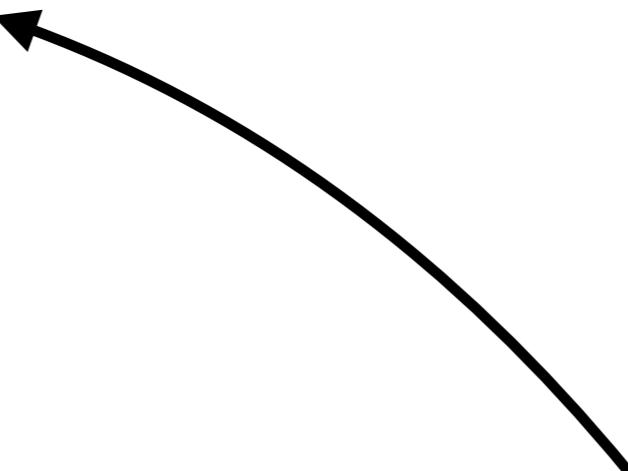
DM phenomenology

strong scale



DM shift symmetry broken by gauging of $U(1)_{\text{DM}}$

$$|\partial^\mu \chi|^2 \rightarrow |(\partial^\mu - ig_D A_D^\mu)\chi|^2 - \frac{1}{4} F_D^{\mu\nu} F_{D\mu\nu} + \frac{1}{2} m_{\gamma_D}^2 A_{D\mu} A_D^\mu$$



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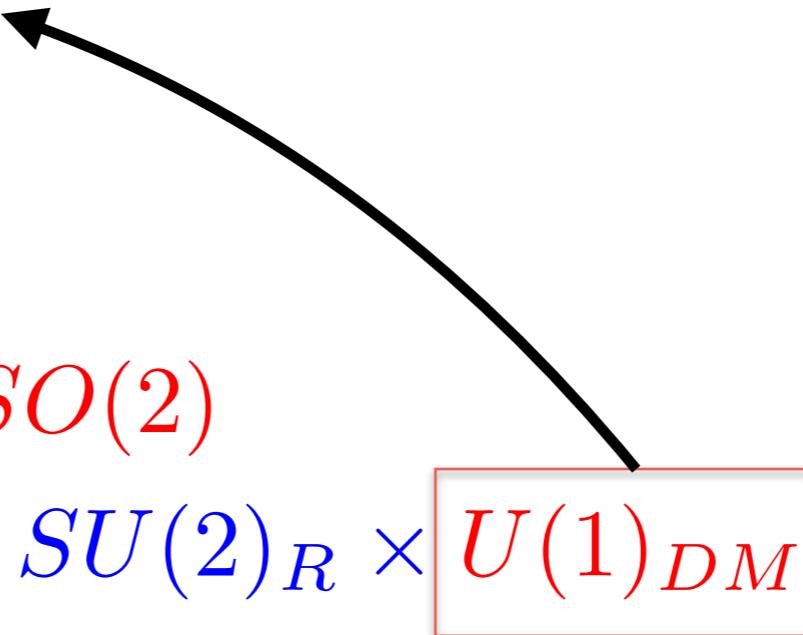
Unbroken symmetry

$$\begin{aligned} SO(6) &\sim SO(4) \times SO(2) \\ &\sim SU(2)_L \times SU(2)_R \times U(1)_{\text{DM}} \end{aligned}$$

EW

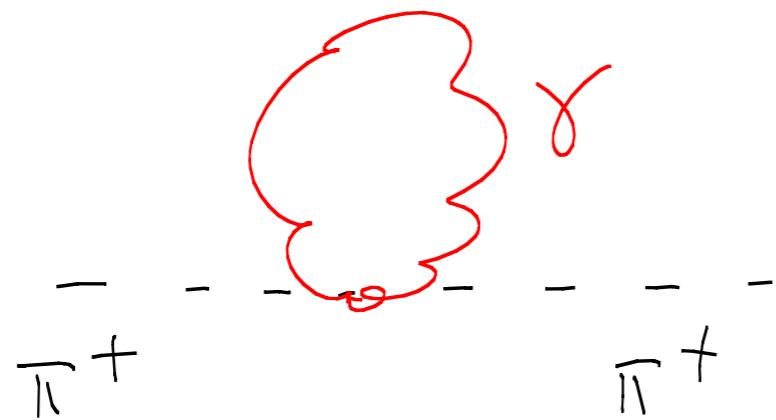
custod'

DM stabilisation



Gauging $U(1)_{\text{DM}}$ generates
DM mass

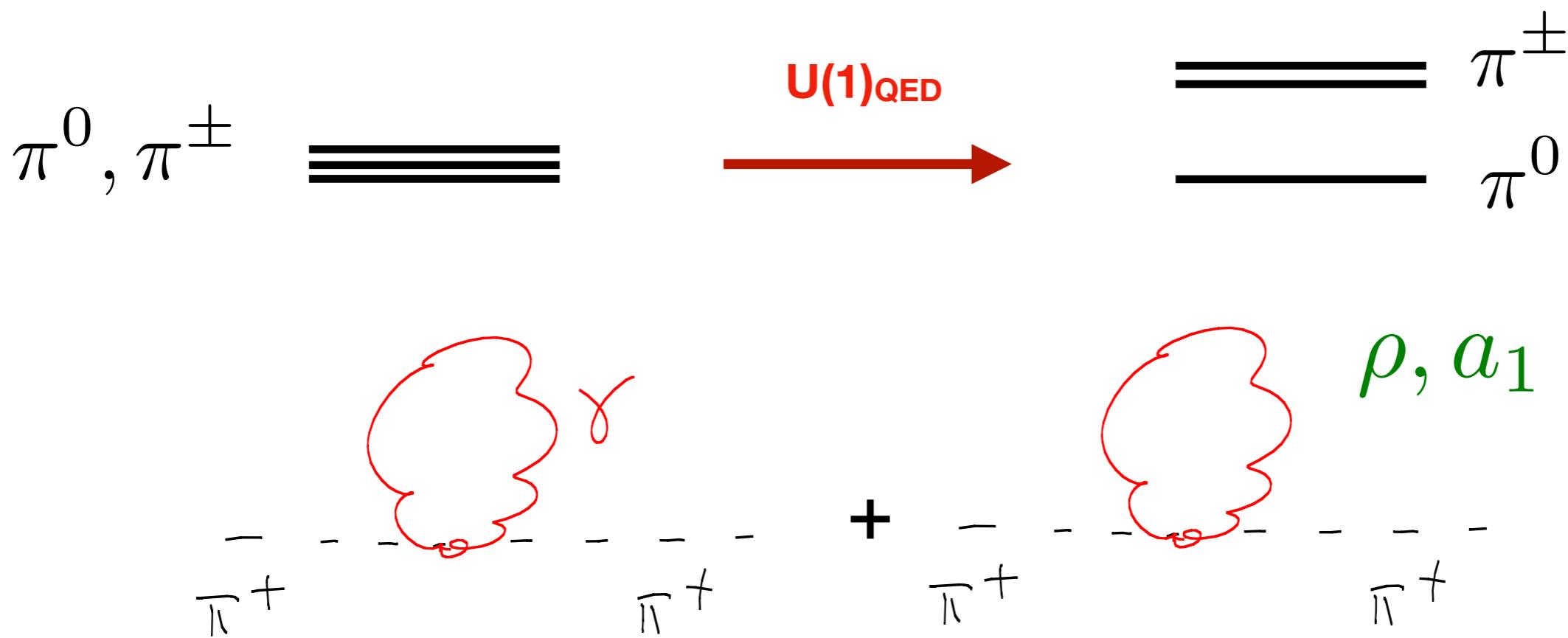
QCD Analogy



Das et al '67

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 \simeq \frac{3 \alpha_{em}}{4\pi} \frac{m_\rho^2 m_{a_1}^2}{m_{a_1}^2 - m_\rho^2} \log \left(\frac{m_{a_1}^2}{m_\rho^2} \right)$$

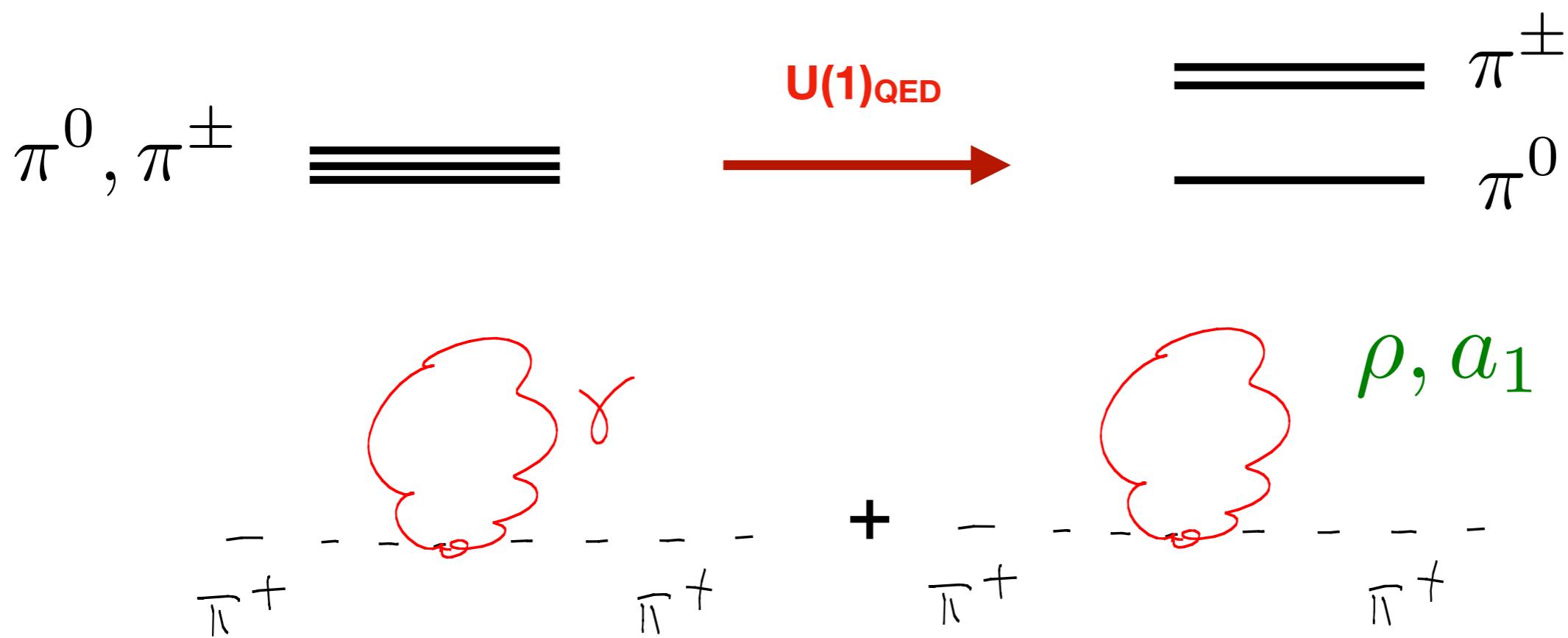
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Dark gauge interaction $U(1)_{\text{DM}}$ breaks shift symmetry.

Radiatively generates non-derivative terms:

$$m_\chi^2 \chi^* \chi + \lambda h^2 \chi^* \chi + \frac{1}{f^2} \partial_\mu (h^2) \partial^\mu (\chi^* \chi)$$

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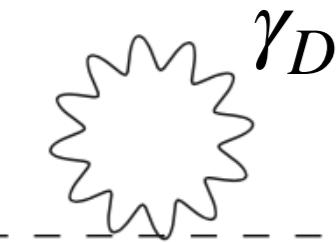
Not generated at
1-loop!

$$\lambda \ll 1$$

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Radiative DM mass:

$$m_\chi \simeq \sqrt{\frac{3\alpha_D}{2\pi}} m_\rho \approx 100 \text{ GeV} \left(\frac{\alpha_D}{10^{-3}}\right)^{1/2} \left(\frac{m_\rho}{5 \text{ TeV}}\right)$$

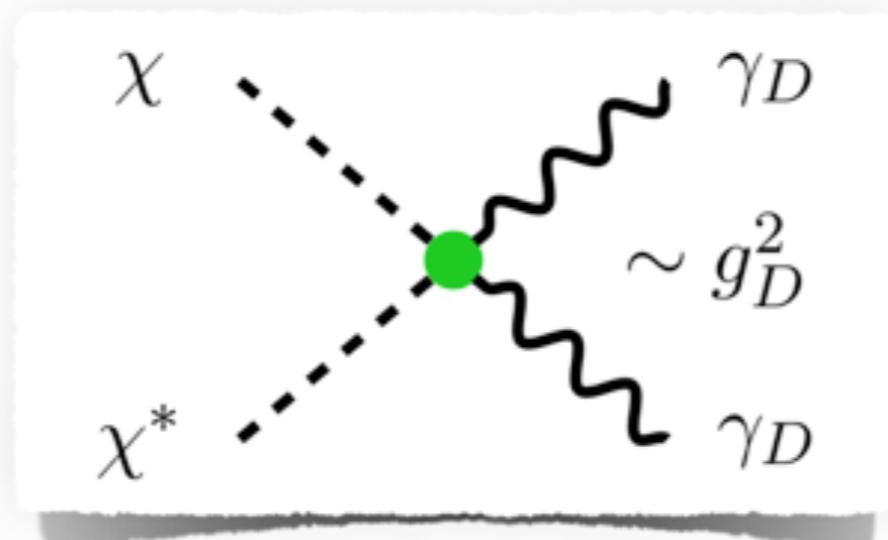
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$$\lambda \ll 1$$

New annihilation channel to dark photons

$$\chi^* \chi \rightarrow A_D A_D$$

$$\langle \sigma v \rangle \simeq \frac{2\pi\alpha_D^2}{m_\chi^2}$$



$U(1)_{\text{DM}}$ and $U(1)_Y$ do not mix kinetically (in $SO(7)/SO(6)$)

Accidental parity: $P_6 = \text{diag}(1, 1, 1, 1, 1, -1, 1) \in O(7)$

$\mathbf{C}_{\text{D}}: A_D^\mu \rightarrow -A_D^\mu$ and $\chi \rightarrow -\chi^*$

Dark charge conjugation \mathbf{C}_{D} forbids kinetic photon mixing

$$\cancel{\epsilon B_{\mu\nu} F_D^{\mu\nu}}$$

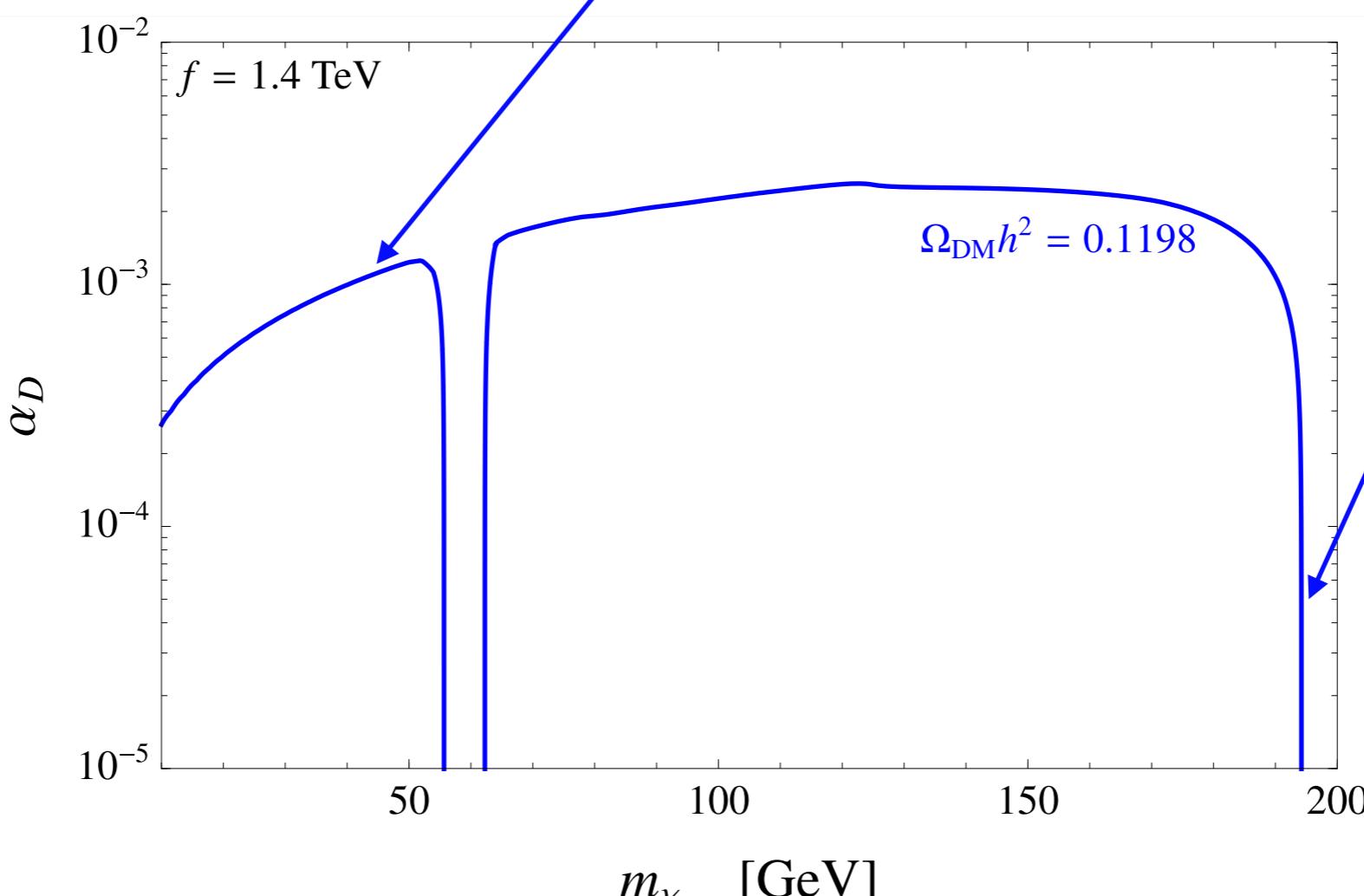
Respected by fermion embedding (since $Q_{\text{DM}} = 0$ for SM).

- 1) Massless dark photon
- 2) Massive dark photon

DM gauge coupling

Massless γ_D

Annihilation mainly to dark photon

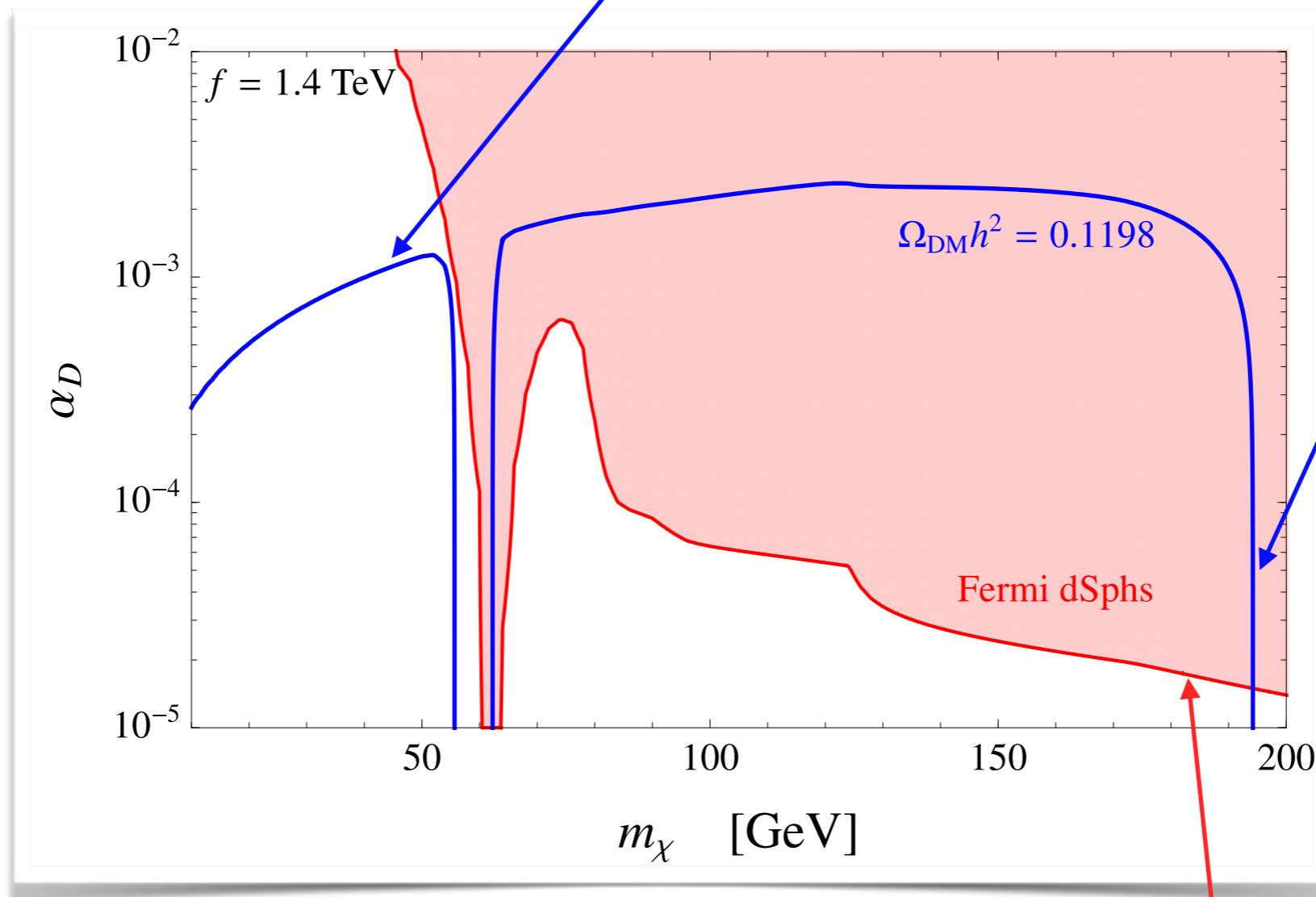


DM mass

Massless γ_D

Annihilation mainly to dark photon

$$\chi^*\chi \rightarrow A_D A_D$$



Annihilation purely
to SM through
 $\frac{1}{f^2} \partial_\mu(h^2) \partial^\mu(\chi^*\chi)$

Annihilation is Sommerfeld Enhanced
in dwarf galaxies today:

$$\frac{\alpha_D}{v_{\text{rel}}} \gtrsim \frac{10^{-3}}{10^{-4}} \gg 1$$

Massless γ_D

Non-evaporation of dwarf galaxies until today while traveling through DM halo of MW

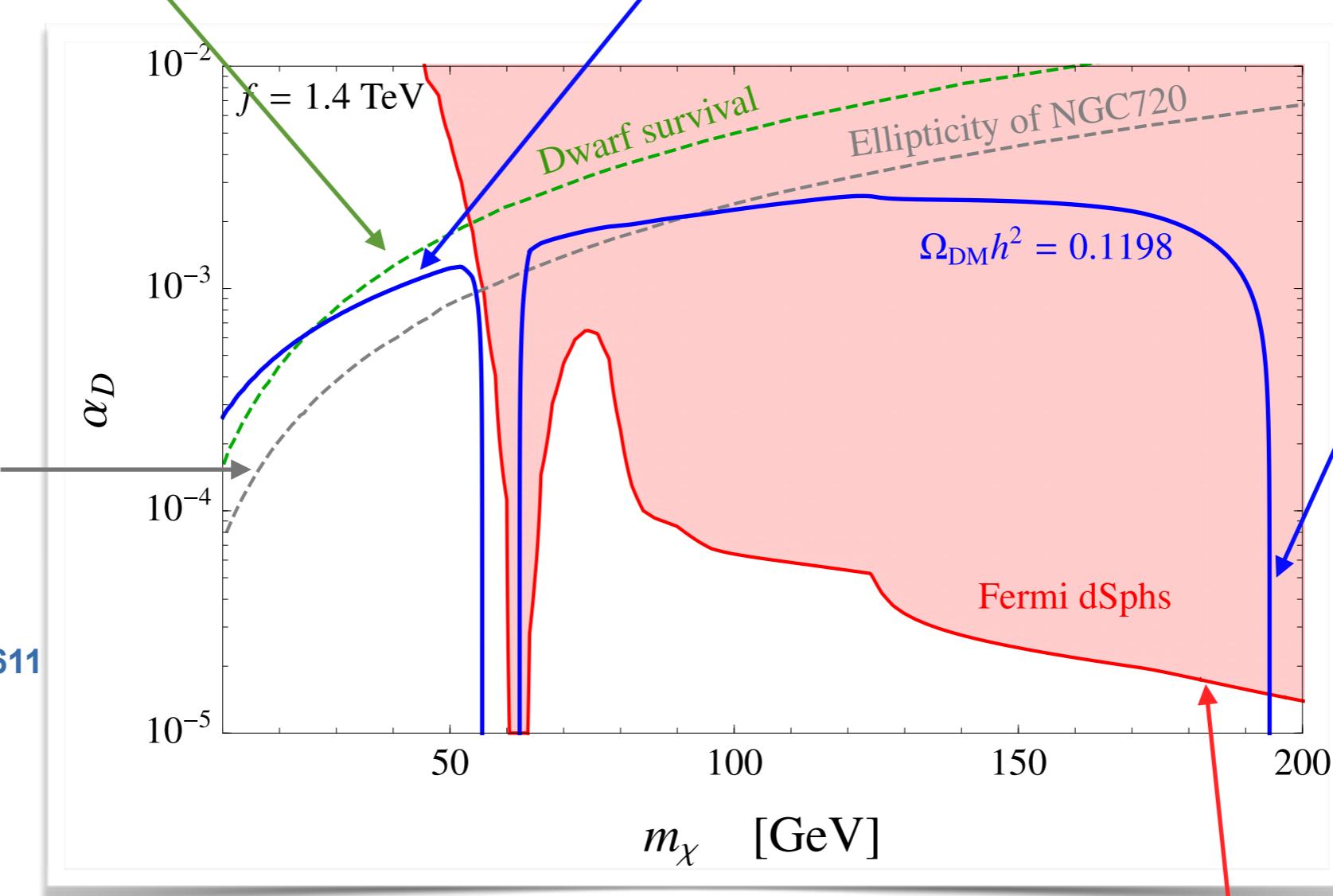
[1308.3419](#)

Ellipticity of the halo of NGC720:
Self-interactions erase anisotropy of DM velocity distribution

[Agrawal et al. 1610.04611](#)

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Massive dark photon

Can add a **Stückelberg mass**

$$\frac{m_A^2}{2} A_D^\mu A_D^\mu$$

$$m_A < 2m_\chi$$



Dark photon is **stable**

$$m_A > 2m_\chi$$



Dark photon **unstable**

$$A_D \rightarrow \chi^* \chi$$

Negligible impact on pheno

$$m_A < 2m_\chi$$

Constraints

$m_{\gamma_D} < 6 \times 10^{-4}$ eV	✓ / X	γ_D is dark radiation today, strong constraints from SE of $\chi\chi^* \rightarrow \text{SM}$
6×10^{-4} eV $< m_{\gamma_D} \lesssim 3m_\chi/25$	X	γ_D is relativistic at freeze-out, ruled out by warm DM bounds/overabundant
$3m_\chi/25 < m_{\gamma_D} < m_\chi$	X	γ_D is non-relativistic at freeze-out, overabundant
$m_\chi \lesssim m_{\gamma_D} < 2m_\chi$	✓	both γ_D and χ are cold DM

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✓ / X

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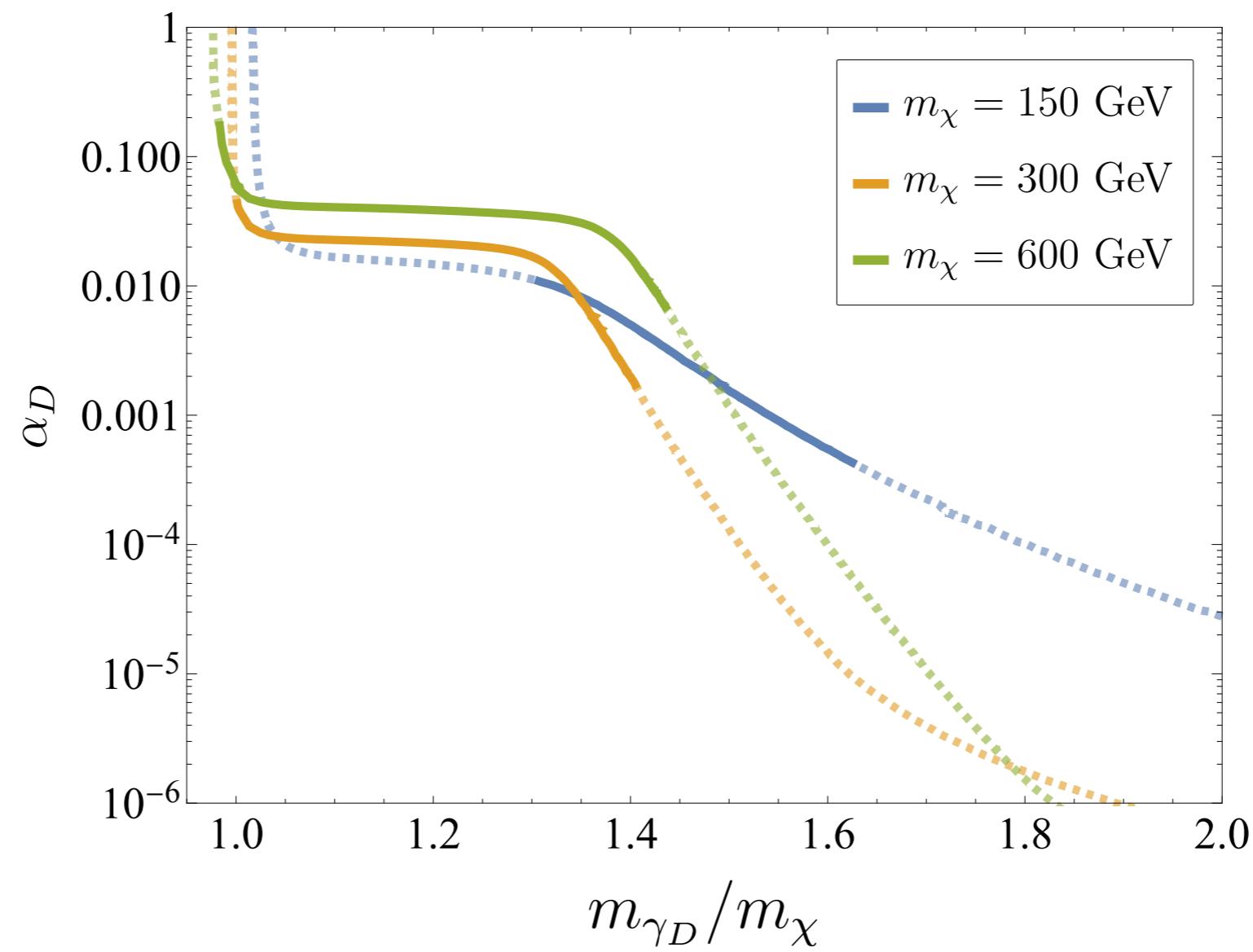
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two component DM

Two-component DM

$$\begin{aligned}\hat{\lambda}^{-1}x^2 \frac{dY_\chi}{dx} &= -\langle \sigma v_{\text{rel}} \rangle_{\text{SM}} Y_\chi^2 + \tfrac{1}{2} \langle \sigma v_{\text{rel}} \rangle_{\gamma_D \gamma_D} Y_{\gamma_D}^2 \\ \hat{\lambda}^{-1}x^2 \frac{dY_{\gamma_D}}{dx} &= -\langle \sigma v_{\text{rel}} \rangle_{\gamma_D \gamma_D} Y_{\gamma_D}^2\end{aligned}$$

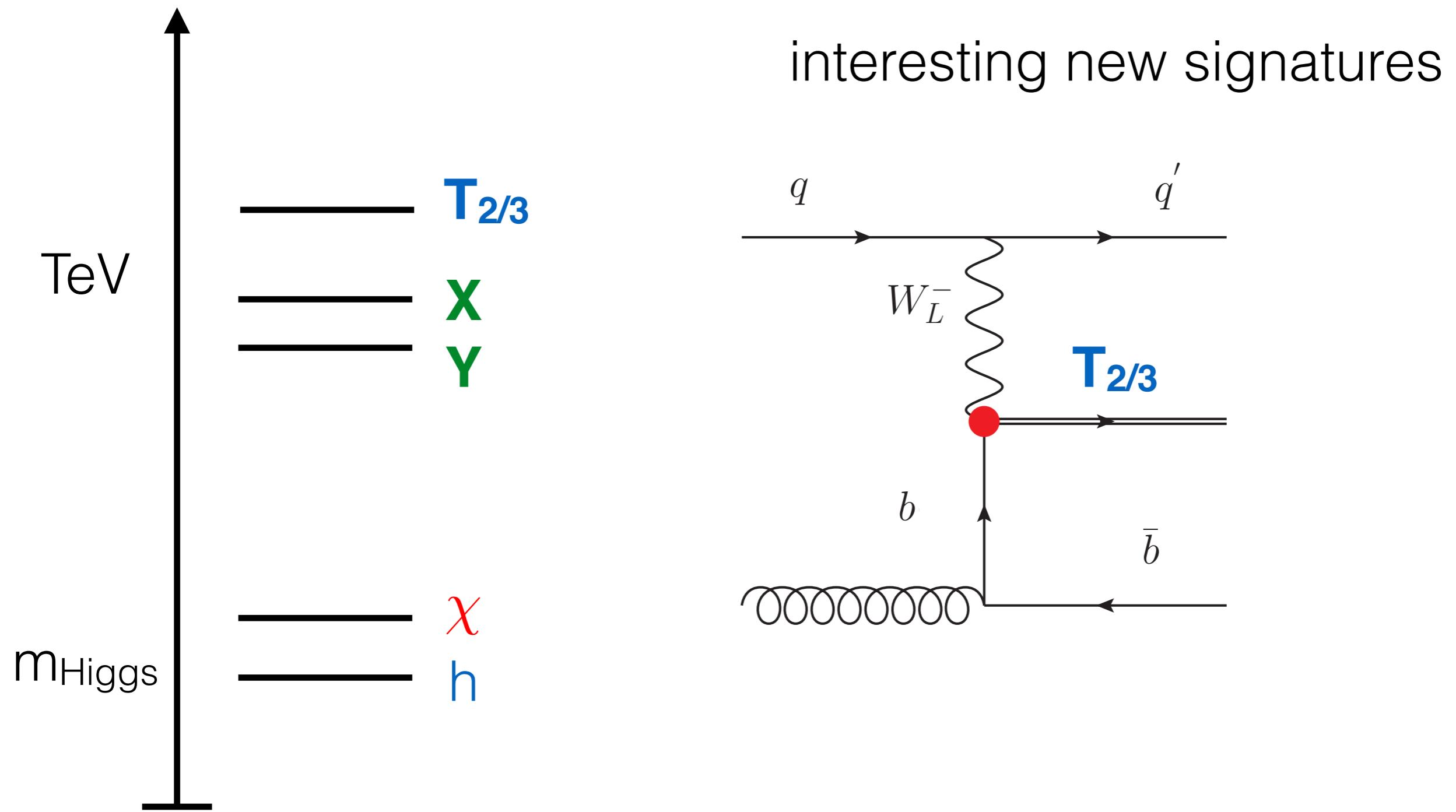


LHC signatures

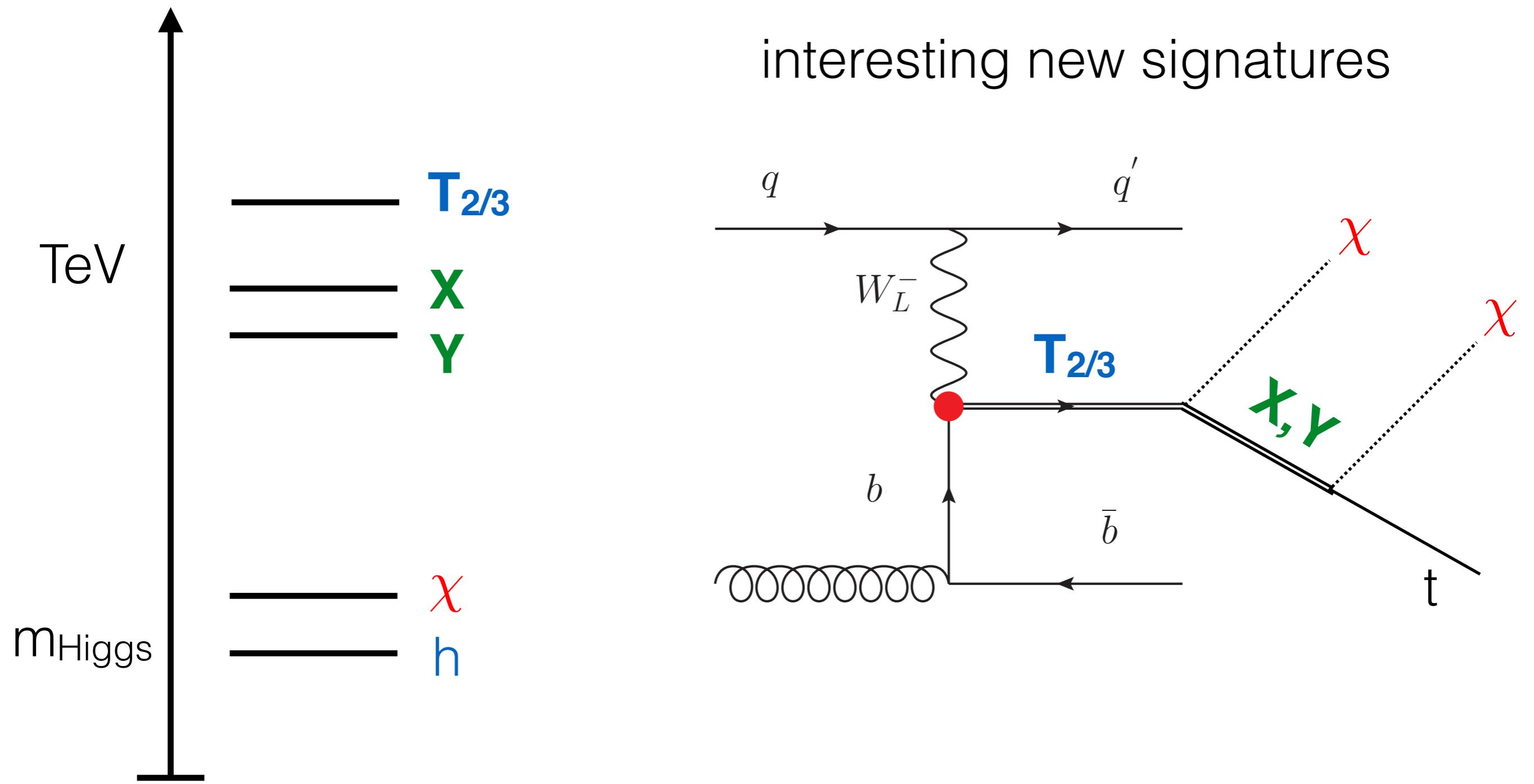
$$Q = \frac{1}{\sqrt{2}} \begin{pmatrix} iB - iX_{5/3} \\ B + X_{5/3} \\ iT + iX_{2/3} \\ -T + X_{2/3} \\ -iY + iZ \\ Y + Z \end{pmatrix}$$

- Composite Top partners, Higgs/DM mass light
-> at least one top partner ~ 1 TeV
- Additional $U(1)_{DM}$ charged particles -> MET
- Interesting new signatures

Single Production + MET



Single Production + MET

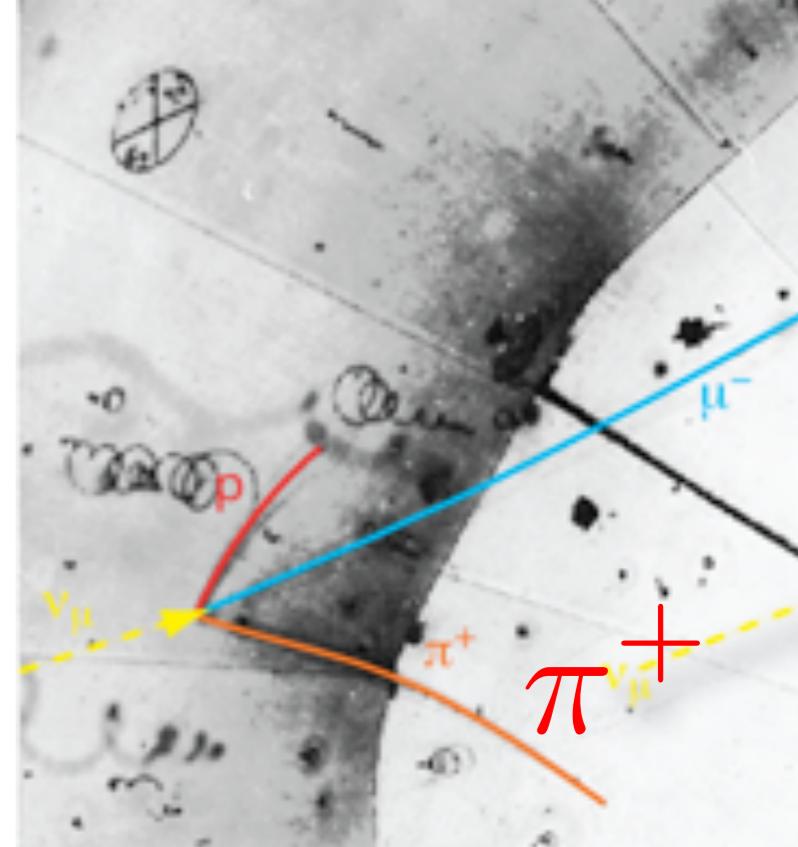


Conclusions

- Additional pNGB scalars in Composite Higgs models can be attractive WIMP candidates
- Robust **UV-safe** stabilisation of DM by exact $U(1)_{\text{DM}}$
- Stabilizing $U(1)_{\text{DM}}$ can be **weakly gauged**:
 - A. Gauging of $U(1)_{\text{DM}}$ generates DM mass
 - B. Negligible direct detection cross-section
 - C. Massless dark photon model is in tension with Fermi-LAT gamma-ray data for $m_\chi > m_h/2$
 - D. Massive dark photon with $m_A > m_\chi$ satisfies constraints from astroph and features rich DM phenomenology

QCD-Pions as pNGBs

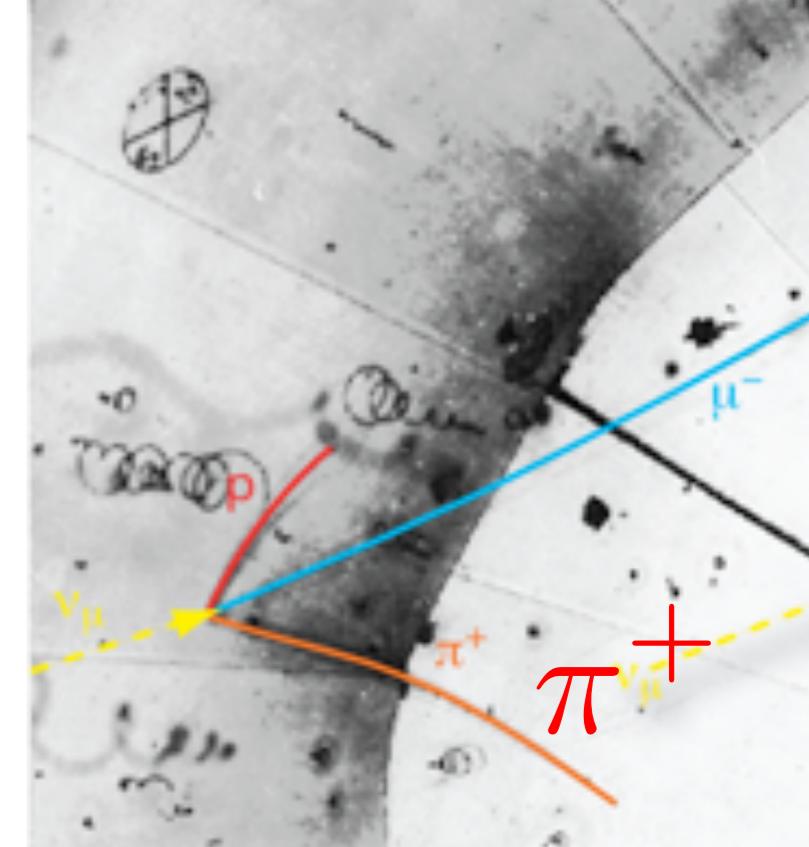
$$SU(2)_L \times SU(2)_R / SU(2)_{L+R}$$



$$U = e^{i\pi/f} \langle \phi \rangle \quad \mathcal{L}_{\text{eff}} = \frac{f^2}{4} \text{tr}(\partial_\mu U^\dagger \partial^\mu U) + \dots$$

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$$\boxed{\pi \rightarrow -\pi}$$

$$(U \rightarrow U^\dagger)$$

Good symmetry? Depends on the UV...

IR probe of UV physics

π^0 DECAY MODES

For decay limits to particles which are not established, see the appropriate Search sections (A^0 (axion) and Other Light Boson (X^0) Searches, etc.).

[pdg](#)

Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1 2γ	$(98.823 \pm 0.034) \%$	S=1.5
Γ_2 $e^+ e^- \gamma$	$(1.174 \pm 0.035) \%$	S=1.5
Γ_3 γ positronium	$(1.82 \pm 0.29) \times 10^{-9}$	
Γ $\gamma + \gamma + \gamma - \gamma - \gamma -$	$(0.001 \pm 0.001) \times 10^{-5}$	

Wess, Zumino '71, Witten '83

IR probe of UV physics

π^0 DECAY MODES		
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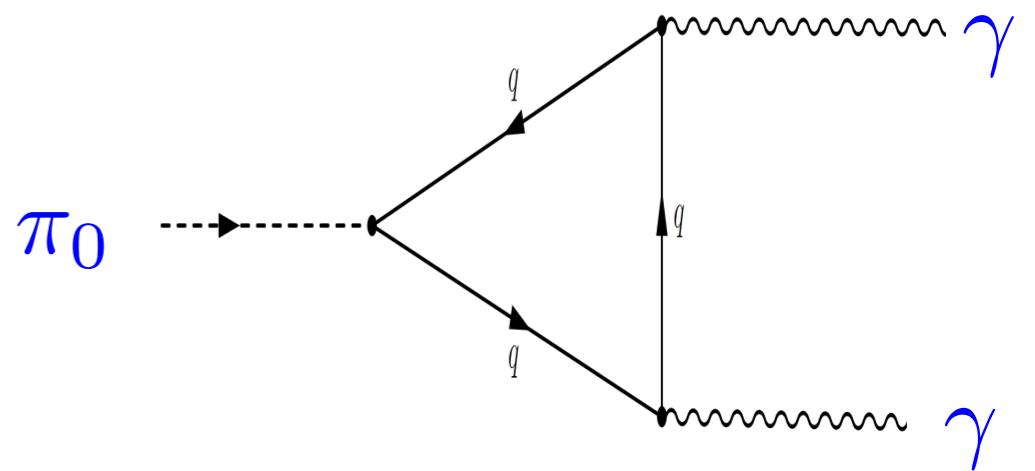
[pdg](#)

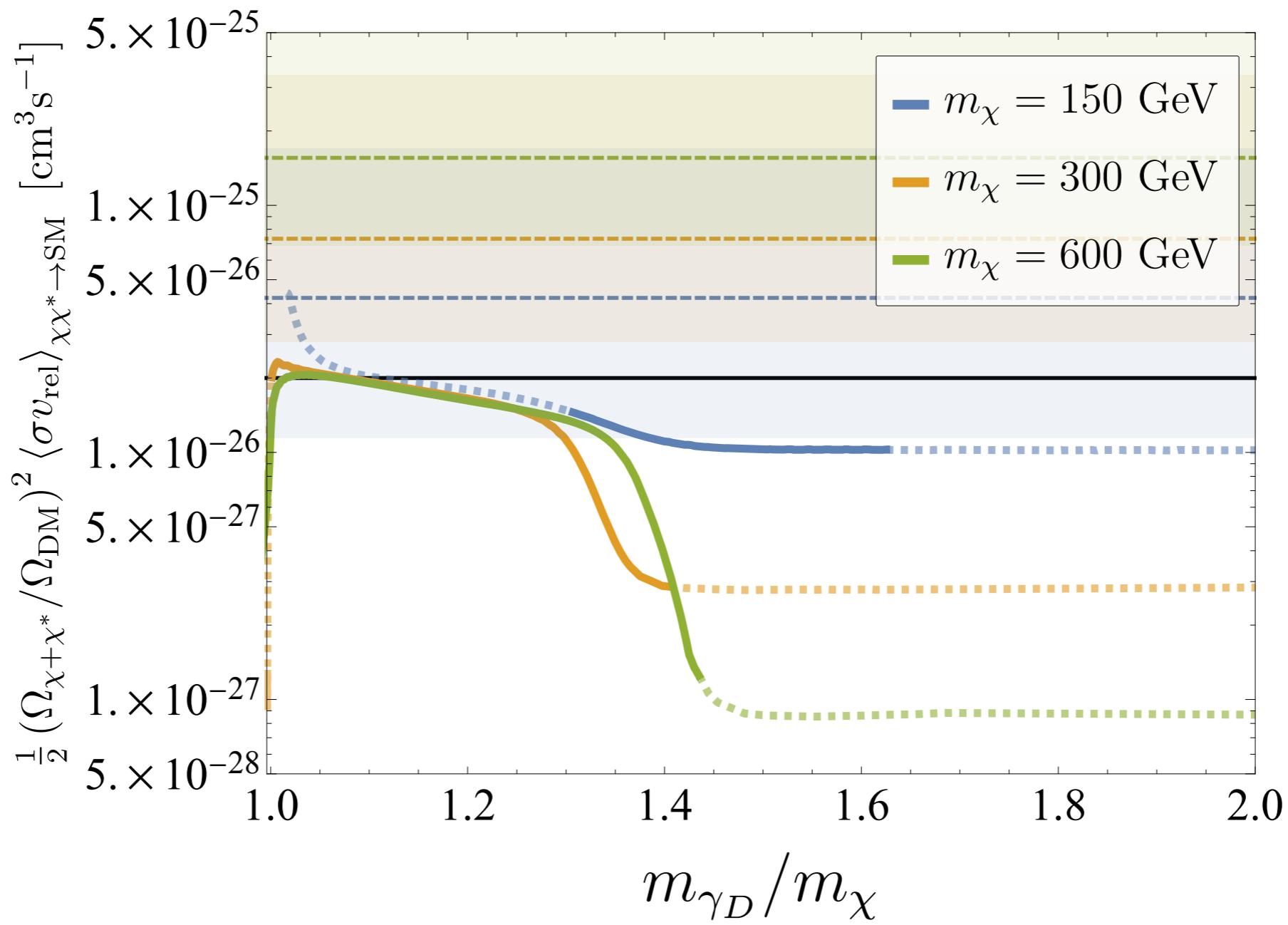
Anomaly: WZW term

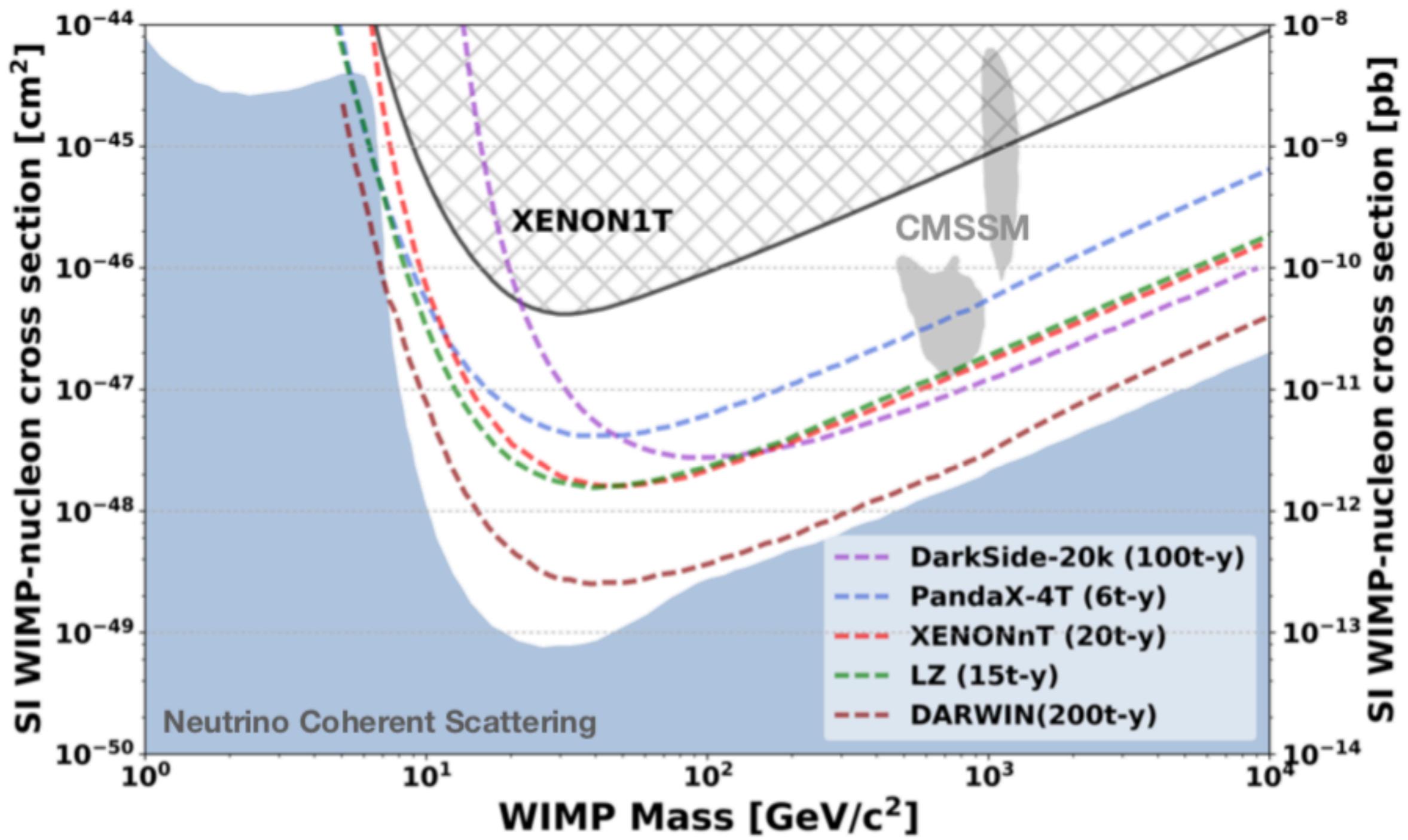
[Wess, Zumino '71, Witten '83](#)

$$\mathcal{L}_A = \frac{e^2 N_c}{48\pi^2 F_\pi} 3 \text{Tr} (Q^2 \tau_3) \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} A_\alpha \partial_\beta \pi^0$$

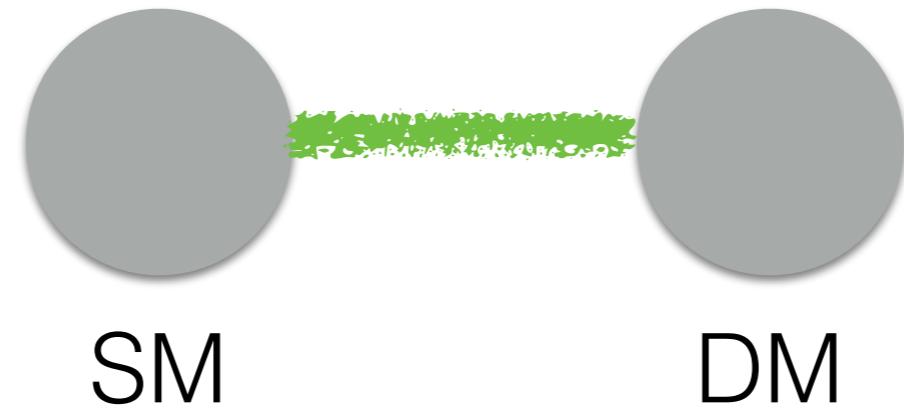
$$\pi \cancel{\rightarrow} -\pi$$







$$\mathcal{L}_{SM}(h) + \mathcal{L}_{DM}(\chi) + \lambda h^2 \chi^* \chi + \frac{1}{f^2} \partial_\mu(h^2) \partial^\mu(\chi^* \chi) + \dots$$



see e.g. 1607.02474

$$\mathcal{L}_{SM}(h) + \mathcal{L}_{DM}(\chi) + \lambda h^2 \chi^* \chi + \frac{1}{f^2} \partial_\mu(h^2) \partial^\mu(\chi^* \chi) + \dots$$

