Particle Physics, GUT, and Neutrinos

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Plan of the talk

- The Standard Model and the GUT.

  $\longrightarrow$ Lessons from the gauge coupling unification.

  $\longrightarrow$ Baryon number non-conservation ?

- The Standard Model and the Neutrinos.

  $\longrightarrow$ Origin of tiny neutrino masses.

  $\longrightarrow$ Lepton number non-conservation ?

- From recent neutrino oscillation studies.

  $\longrightarrow$ Reactor neutrino oscillation.

  $\longrightarrow$ Accelerator based neutrino oscillation.

  $\longrightarrow$ Atmospheric neutrino oscillation.

- Closing

The Standard Model (SM)

The Standard Model (SM) of elementary particles have 3 gauge interactions, SU(3) for QCD, SU(2)×U(1) for the electroweak theory, which breaks spontaneously down to $U(1)_{EM}$.

We have 8+3+1=12 gauge bosons, 3 generations of 6+3+3+2+1=15 fermions, and at least one doublet of the scalar boson (Minimum SM).

For each generation, the 15 fermions have the following quntum numbers

$$(3,2,1/6),\ (3,1,2/3),\ (3,1,-1/3),\ (1,2,-1/2),\ (1,1,-1)$$

Quantization of hyper-charge in units of 1/6 is probably the most important hint for the origin of the 15 fermions (quarks and leptons).

If we re-write the fermion quantum numbers for their left-hand chirality components, so that they transform the same way under Lorentz transformation, we have

$$(3,2,1/6),\ (3^*,1,-2/3),\ (3^*,1,1/3),\ (1,2,-1/2),\ (1,1,1)$$

and notice that they form 10 and 5^* of SU(5):

$$\begin{aligned} 10 &= (3,2,1/6) + (3^*,1,-2/3) + (1,1,1) \\ 5^* &= (3^*,1,1/3) + (1,2,-1/2) \end{aligned}$$

The charge quantization is a consequence of the tracelessness of the SU(5) generators.

In the SU(5) theory, 16 fermions of 1 generation are represented by 3 multiplets, $\underline{5}^*$, $\underline{10}$, $\underline{1}$:

$$\underline{5}^* = \begin{pmatrix} d_R^c \\ d_R^c \\ d_R^c \\ -e_L \\ \nu_L \end{pmatrix} \qquad \underline{10} = \begin{pmatrix} 0 & u_R^c & -u_R^c & u_L & d_L \\ & 0 & u_R^c & u_L & d_L \\ & & 0 & u_L & d_L \\ & & & 0 & e_R^c \\ & & & & 0 \end{pmatrix} \qquad \underline{1} = \nu_R^c$$

Among the $5 \times 5 - 1 = 24$ gauge bosons, one can be identified as the hypercharge $U(1)_Y$ gauge boson (B), and the quantization of the hypercharges follows from the tracelessness of the SU(5) generators:

$$\underline{24} = \frac{1}{\sqrt{2}} \begin{pmatrix} & & & X^- & Y^- \\ 8 & \text{gluons} & & X^- & Y^- \\ & & & X^- & Y^- \\ X^+ & X^+ & X^+ & W^3/\sqrt{2} & W^+ \\ Y^+ & Y^+ & Y^+ & W^- & -W^3/\sqrt{2} \end{pmatrix} + \frac{B}{2\sqrt{15}} \begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix}$$

In order to break the SU(5) symmetry down to the standard model, we introduce 3 types of Higgs bosons, $\underline{24}$, $\underline{5}$ and $\underline{5}^*$:

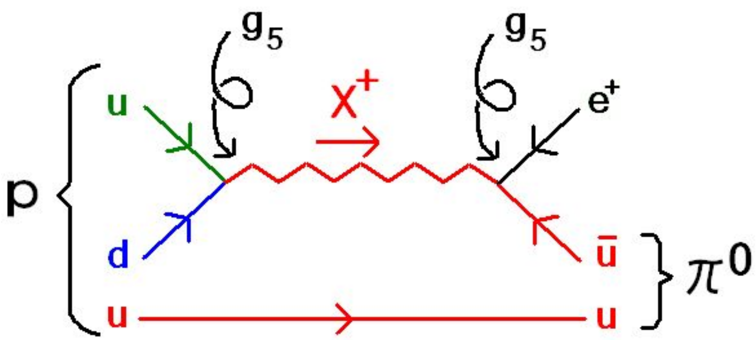
$$\underline{24}_H = \Sigma \qquad \underline{5}_H^* = \begin{pmatrix} D^+ \\ D^+ \\ D^+ \\ h_d^- \\ h_d^0 \end{pmatrix} \qquad \underline{5}_H = \begin{pmatrix} D^- \\ D^- \\ D^- \\ h_u^+ \\ h_u^0 \end{pmatrix}$$

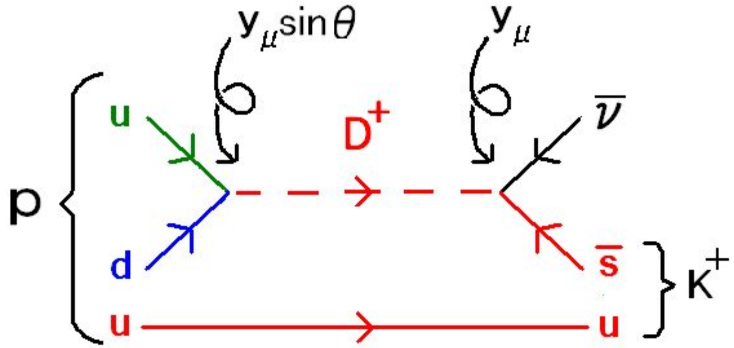
The SU(5) GUT thus gives us a satisfactory answer to the fundamental question of the charge quantization, explaining the origin of the 1/3 units of the quark charge in particular. Once this is recognized, it is no more possible to regard quarks and leptons as independent unrelated particles.

However, this model had two serious phenomenological problems:

- $\alpha_s = \alpha_W = \frac{5}{3}\alpha_Y$
- proton decays

First, the proton decays in this theory as follows:





In order for protons and nuclei to be stable enough, we should require

$$m_X = m_Y \gtrsim 10^{15}\text{GeV} \qquad m_D \gtrsim 10^{13}\text{GeV}$$

After the symmetry breakdown, the 24 gauge bosons split into 12 heavy bosons (X,Y) and the 12 massless gauge bosons which can be identified as 8 gluons, 3 SU(2) gauge bosons and the hyperchage gauge boson B. Half of the 24 Higgs boson are absorbed by X and Y, and the remaining half become heavy. The 5 plets of Higgs bosons split into heavy triplets D and massless doublets, which become the SM Higgs doublet. It may be worth noting that a pair of Higgs quintets are required at this stage, in order to make D massive.

Heavy particles do not contribute to the quantum corrections at energy scales below their masses (decoupling). Only light particles contribute to radiative corrections. Accordingly, the unique gauge coupling of SU(5) receive different radiative corrections below the X, Y, D masses for different vertices, such as:

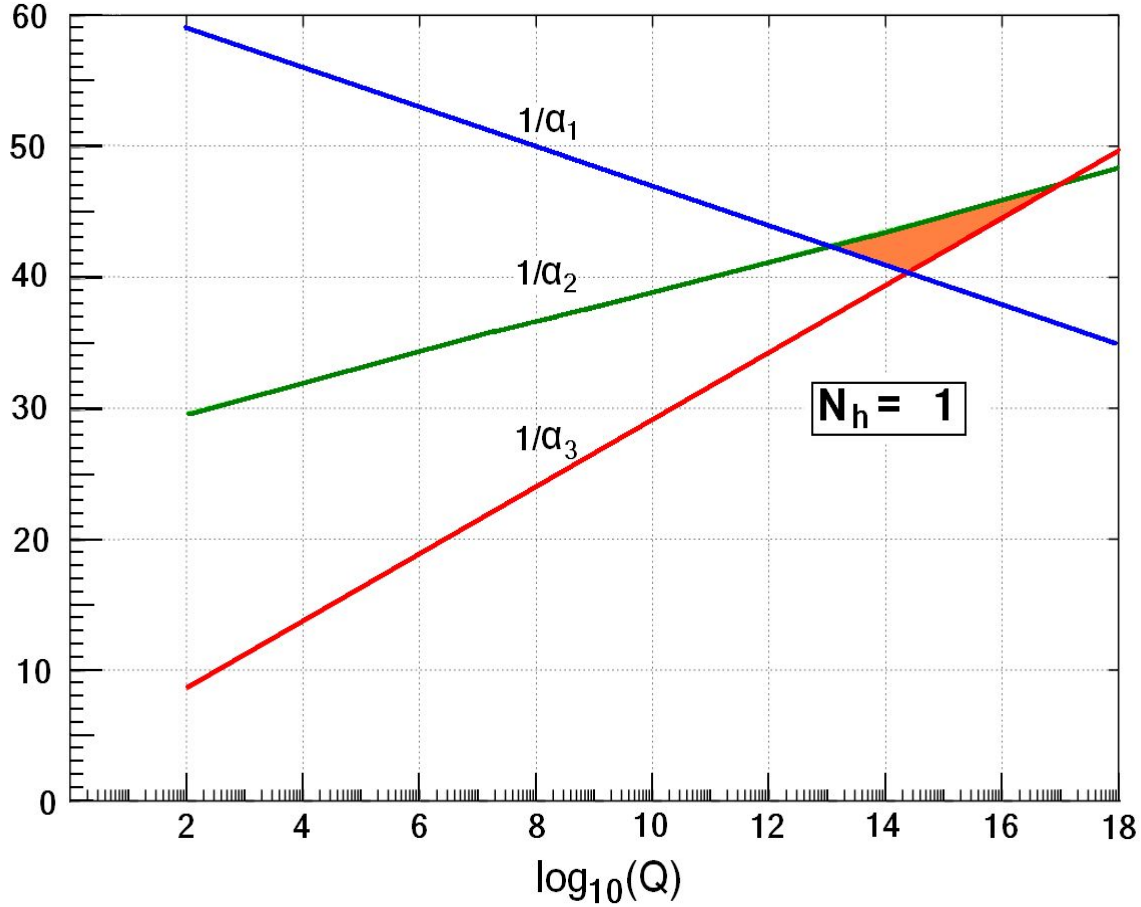
q-q-g, **l-ν-W**, **l-l-B**

vertices. This was noticed by Georgi、Quinn、and Weinberg in 1974. Each effective coupling receives radiative corrections as:

$$
\begin{array}{lll}
\frac{\pi}{\alpha_3(m_Z)} = \frac{\pi}{\alpha_5(m_X)} - b_3 \ln\left(\frac{m_X}{m_Z}\right) & b_3 = \frac{11}{2} \quad 0 & -\frac{2}{3}N_{gen} \\
\frac{\pi}{\alpha_2(m_Z)} = \frac{\pi}{\alpha_5(m_X)} - b_2 \ln\left(\frac{m_X}{m_Z}\right) & b_2 = \frac{11}{3} \quad -\frac{1}{12}N_h & -\frac{2}{3}N_{gen} \\
\frac{\pi}{\alpha_1(m_Z)} = \frac{\pi}{\alpha_5(m_X)} - b_1 \ln\left(\frac{m_X}{m_Z}\right) & b_1 = 0 \quad -\frac{1}{20}N_h & -\frac{2}{3}N_{gen}
\end{array}
$$

where the coefficients b_3, b_2, b_1 receive contributions from all the light particles that couple to the SU(3), SU(2), U(1) gauge bosons, respectively. The first terms are from the self-coupled gauge bosons, which are positive (asymptotic free) and proportional to n for SU(n), the second terms count the number of the Higgs doublets, and the last term receive contributions from quarks and leptons. If the quarks and leptons of each generation are all light, then their contributions are common for all the three couplings, since one generation of quarks and leptons form complete SU(5) multiplets.

If the grand unification of the three couplings takes place, the above equations should give a unique GUT coupling $\alpha_5(m_X)$ at the GUT scale, m_X. This prediction can hence be tested by inserting the measured values of the three couplings at the Z boson mass scale.



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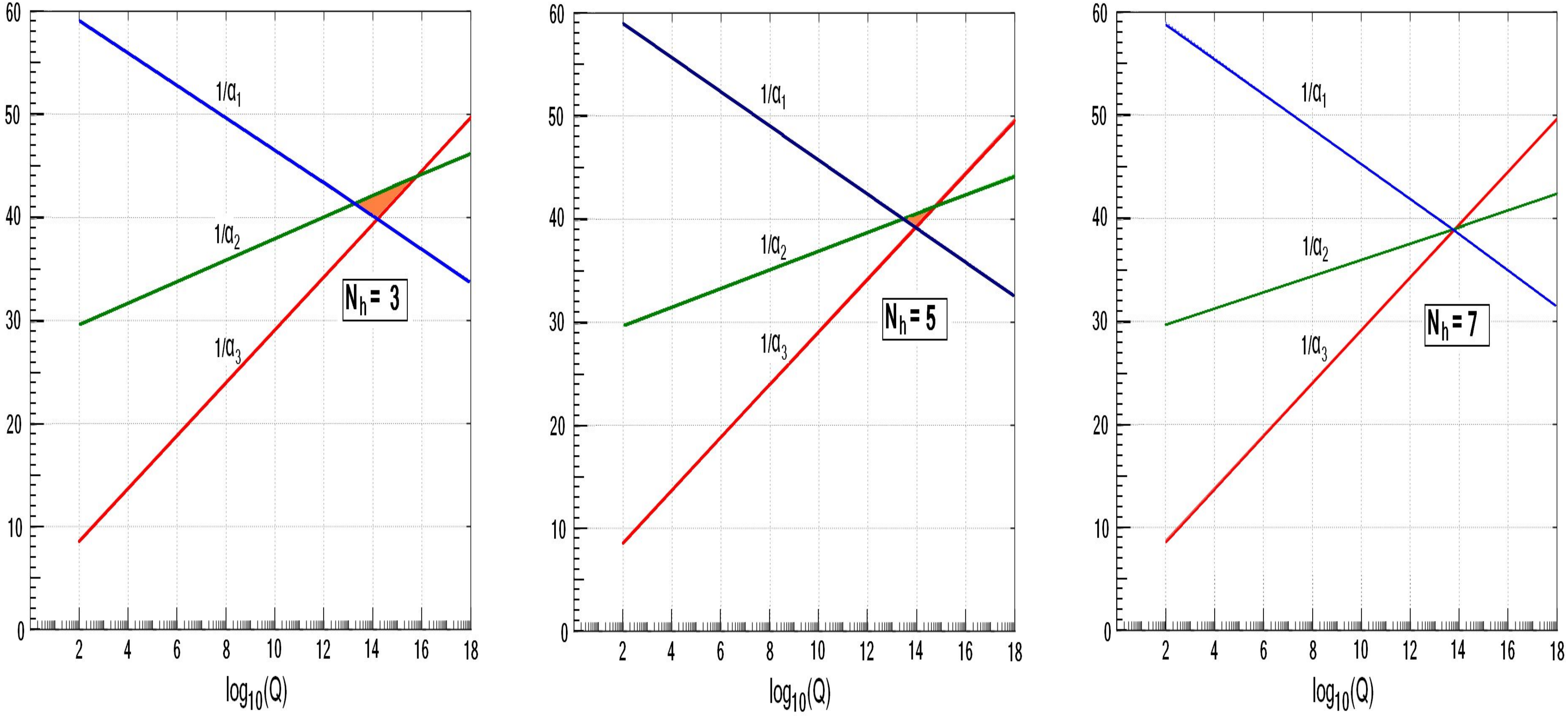
The above result follows from the 3 gauge coupling strengths measured at the m_Z scale, if we assume 3 generation of quarks and leptons and 1 Higgs doublet in the SM (minimum SM). The idea of the Grand Unification of the 3 gauge couplings is clearly a great success qualitatively, since the ordering of the three gauge coupling strengths, $\alpha_3(m_Z) > \alpha_2(m_Z) > \alpha_1(m_Z)$ agree with the ordering $b_3 > b_2 > b_1$, which in tern refrects the ordering $3 > 2 > 1$ of the gauge group SU(**3**), SU(**2**), U(**1**).

The quantitative disagreement of the unification may suggest new particles in the TeV region. For instance, if we introduce N_h Higgs doublets we find



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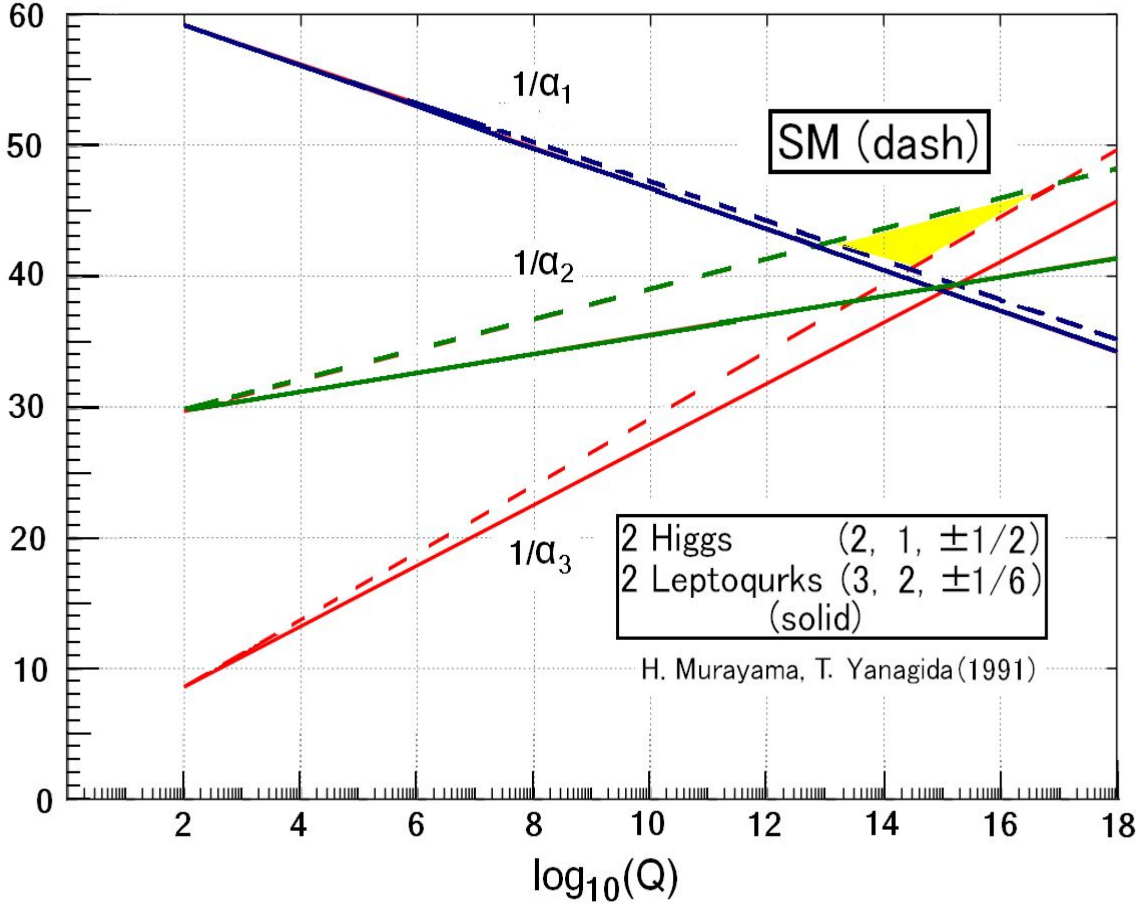
The unification is achieved for $N_h = 7$, but the GUT scale becomes rather small, which contradicts with the observed proton longevity. In order to avoid this, we should make the slope of the SU(3) coupling b_3 small, suggesting new colored particles at TeV scale.

As a simplest example, instead of introducing 6 additional Higgs doublet, we may introduce a pair of color-triplet and SU(2) doublet scalar bosons. Now both the SU(3) and SU(2) couplings run slower, and they meet at large enough mass scale. By arranging their hypercharge to make them ‘lepto-quarks’, the U(1) coupling meets at the same point.



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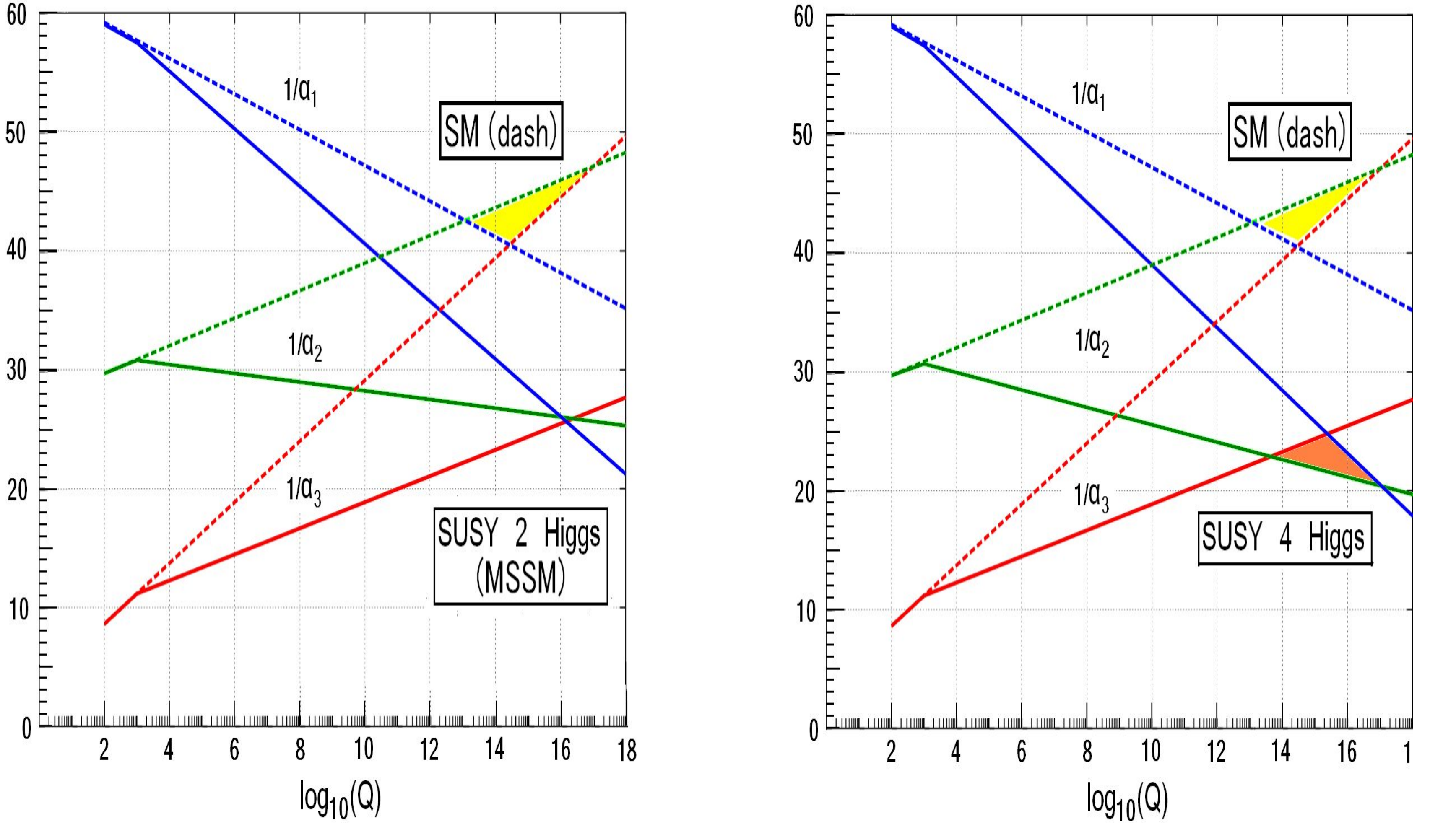
This example shows that it is relatively easy to find a common solution for the unification of the 3 gauge couplings and the proton longevity. How about the Supersymmetric SM ?



The unification occurs only for the MSSM (Minimum Supersymmetric SM), where there is only one pair of Higgs supermultiplets. The effective number of the Higgs doublet is 6, since the Higgsinos contribute twice the Higgs bosons to the running of SU(2) and U(1) couplings. The two couplings meats at higher scale because the winos make the SU(2) couplings run slower. Miraculously, the gluino contribution to the SU(3) couplings make the 3 couplings meet at one point, $m_{MSSMGUT} = 2 \times 10^{16}$ GeV.

Since there are 3 equations for the two unknowns, the GUT scale m_X and the GUT coupling $\alpha_5(m_X)$, there is one constraint among the three effective couplings, if the coefficients b_1, b_2, b_3 are known.

One often expresses this constraint as a prediction for the ratio of the SU(1)$_Y$ and SU(2)$_L$ couplings

$$\sin^2\theta_W(m_Z) = \frac{3}{8} \times \frac{1 + \frac{5}{3}\frac{\alpha}{\alpha_3}X}{1 + \frac{5}{8}X} = \frac{3}{8} \times \frac{1 + 0.118X}{1 + 0.625X},$$

where $\alpha/\alpha_3 = \widehat{\alpha}(m_Z)^{(6)}/\widehat{\alpha}_s(m_Z)^{(6)} = (1/128.12)/(0.110) = 0.0710$, and

$$X = \frac{b_2 - b_1}{b_3 - b_2}$$

For pure gauge, this ratio is $X = (2-0)/(3-2) = 2$ for SM or MSSM, and $\sin^2\theta_W = 0.206$ follows. In order to make this ratio the observed value of $\sin^2\theta_W = 0.233$, we need a theory which gives $X \approx 1.4$. This is made possible only with in-complete SU(5) multiplets (split representations). Since matter (non-gauge) particles can only decrease b_i's, we need to have split representation with the SU(2) charge. The SM Higgs boson and the MSSM Higgs super-multiplets are among the most efficient particles to achieve this goal.

$$\begin{aligned}
X_{SM}(n_H = H) &= \frac{22 - H/5}{11 + H/2} = 1.42 \quad \text{for } H = 7, \\
X_{MSSM}(n_H = H) &= \frac{18 - 3H/5}{9 + 3H/2} = 1.40 \quad \text{for } H = 2, \\
X_{SM}(n_H = H, n_{LQ} = N) &= \frac{22 - H/5 - 7N/5}{11 + H/2 + N/2} = 1.44 \quad \text{for } H = N = 2.
\end{aligned}$$

What is so super about **SUPER-SYMMETRY** ?

It is the only known extention of the Einstein's space-time symmetry, called Lorentz or Poincare symmetry, where the space-time is considered as a part of more general space, the super-space, which contains dimensions of non-commutative (fermionic) coordinates.

Supersymmetry transforms **fermions (matter)** into **bosons (force)**, and vice versa. Since fermions anti-commutes while bosons commutes, their contributions to quantum fluctuations tend to cancell.

We learned that the idea of Grand Unification works only when there is a hierarchy between the unification scale $\sim 10^{16}$ GeV and the electroweak scale $\sim 10^{2}$ GeV.

Supersymmetry is the only known symmetry which can suppress quantum fluctuations of spinless boson mass, the Higgs boson mass, which should be 10^{14} times smaller than the unification scale.

Because we do not observe spinless partners of the photon, electron, quarks and gluons, the supersymmetry should be broken. The beautiful idea is that what we think is the electroweak gauge symmetry breaking scale is indeed the supersymmetry breaking scale, and that we will discover superpartners of all the SM particles in the mass scale of W, Z and the top quarks.

What we learned from the **LHC** at 7 and 8 TeV:

- The Standard Model like Higgs boson at $m_H = 125$ GeV.

- No hint of Supersymmetric particles that are pair produced and decay into a Dark Matter candidate.

The Higgs boson mass of 125 GeV is near the upper border line of the weakly coupled supersymmetric standard models that are compatible with the quantitative gauge coupling unification. Therefore, we have every reason not to abandone the beautiful idea of supersymmetric unification, at about 2×10^{16} GeV. The proton decay via the GUT gauge boson mediation should then give the life time of about 10^{37} years, which may need 10 mega-ton level low background experiment (10 times HK). Smaller life time is possible in SUSY GUT by mediating squarks and sleptons in the loop, but this lifetime is highly dependent on the GUT breaking mechanism (especially on the origin of the doublet-triplet splitting in the Higgs quintet, between the weak-doublet and the color-triplet).

However, the mass 125 GeV is also compatible with the idea that the Standard Model is valid up to the Planck scale. The GUT may or may not be realized with the 12 missing massive gauge bosons in this scenario. If they are, the proton decay should be mediated by their exchange. The likely scale, however, may well be the scale at which the SU(3) and SU(2) couplings meet in the minimum SM, which is about 10^{17} GeV, or the gauge-boson mediated proton life time of 10^{41} years.

Particle masses and physics scales in Logarithms of [GeV] units.

17	Quantum Gravity		←	super-string, extra space dimension
16	Grand Unification (**GUT**)		←	**SUSY-GUT** Unification
15				
14				
13				
12				
11				
10				
9				
8				
7				
6				
5				
4				
3	**Gauge Symmetry Breakdown**		←	**Supersymmetry Breakdown**
2	W, Z, top, Higgs			
1				
0	charm, bottom, tau,	proton		
-1	strange, mu,	pi		
-2				
-3	up, down, electron			
-4				
-5				
-6				
-7				
-8				
-9				
-10	neutrinos			

We seem to have a half-size (in logarithmic scale) desert in the mass spectrum of the fundamental particles. A bautiful idea explaining this hierarchy is to assume the absence of the right-handed neutrino's and that the neutrino masses are generated by the dimension-5 operators

$$\frac{f_{ij}}{\Lambda}(L_i \cdot \phi) \cdot (L_j \cdot \phi)$$

where L_i with (i =1,2,3) are the 3 generations of lepton doublets and ϕ is the SM Higgs doublet with Hyper-charge $+\frac{1}{2}$ that gives masses to the weak bosons and all the quarks and charged leptons. Once the Higgs boson aquires the v.e.v., the lepton number is broken, and the neutrinos obtain the Majorana mass matrix

$$M_{ij} = f_{ij}\frac{v^2}{2\Lambda}$$

If the coefficients f_{ij} are of order unity, then the ratio

$$\frac{v^2}{2\Lambda} = \frac{(246\text{GeV})^2}{2\Lambda} \sim 0.1\text{eV}\,(\frac{10^{14}\text{GeV}}{\Lambda})$$

sets the neutrino mass scale. Λ can be lowered if the coefficients f_{ij}'s are loop suppressed.

Models with $f_{ij} = O(1)$ and $\Lambda = 10^{14}$ GeV (the original See-Saw models) attract me most, since it suggests the intermediate scale in the SM singlet sector, opening the road to higher rank GUT's including SO(10).

Therefore it is my opinion that the experiments in pursuit of the Majorana nature of the neutrino masses (the neutrinoless double-beta decays) are as important as those for proton decays in our quest for unification.

The neutrinoless double beta decay measures the following combination of the Majorana 'electron-neutrino' mass

$$m_{\beta\beta} = m_1 (V_{e1})^2 + m_2 (V_{e2})^2 + m_3 (V_{e3})^2$$

where m_i's (i =1,2,3) are the Majorana neutrino masses and V_{ei} are the neutrino-flavor mixing matrix which is parametrized as V = U diag(1, $e^{i\alpha_2/2}$, $e^{i\alpha_3/2}$) with the observable Majorana phases α_2 and α_3. Explicitly, it reads

$$m_{\beta\beta} = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2\, e^{i\alpha_2} + m_3 s_{13}^2\, e^{i(\alpha_3 - 2\delta)}$$

where δ is the unique CP phase of the 3 neutrino model when the lepton number is conserved (the Dirac mass limit). Let me estimate its magnitude in the limit when the lightest neutrino mass is zero. For the inverted hierarchy, I find

$$|m_{\beta\beta}| \to c_{12} c_{13} \sqrt{\Delta m_{atm}^2} \times \,|\, 1 + \sqrt{1 + \frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} \frac{s_{12}^2}{c_{12}^2}}\; e^{i\alpha_2} \,|$$

and for the normal hierarchy,

$$|m_{\beta\beta}| \to s_{13} \sqrt{\Delta m_{atm}^2} \times \,|\, 1 + \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} \frac{s_{12}^2 c_{13}^2}{s_{13}^2}}\; e^{i(\alpha_2 - \alpha_3 + 2\delta)} \,|$$

I note that in both limits, the coefficients of the phase factor is O(1), and there is a realistic possibility that the Majorana phase of the neutrino mass matrix (CP violation in the lepton-number violating sector) can be measured. This would be a truely fundamental discovery which has profound implications for our quest for unification.

I would even tell that precision measurements of the all the three neutrino model parameters (including δ) are needed to extract a combination of α_2 and α_3 from the neutrinoless double beta decay experiments. The determination of the smallest neutrino mass may be the job of cosmology.

Three neutrino model has 9 parameters:

3	masses	$m_1,\ m_2,\ m_3$
3	angles	$\theta_{23},\ \theta_{12},\ \theta_{13}$
3	phases	$\delta_{\text{MNS}},\ \alpha_1,\ \alpha_2$

Neutrino oscillation experiments can measure 6 out of the 9 parameters:

2	mass-squared differences	$m_2^2 - m_1^2,\ m_3^2 - m_1^2$
3	angles	$\theta_{23},\ \theta_{12},\ \theta_{13}$
1	phase	δ_{MNS}

Both mass-squared differences and **ALL** 3 angles have been measured. The tasks of the future neutrino oscillation experiments are to determine:

the mass hierarchy	$m_3^2 - m_1^2 > 0$ or $m_3^2 - m_1^2 < 0$
the CP phase	δ_{MNS}
the octant degeneracy	$\cos^2\theta_{23} - \sin^2\theta_{23} > 0$ or < 0

besides sharpening of the existing measurements and search for new physics.

Please let me introduce recent works of my colleagues on possible neutrino oscillation experiments in the near future.

- Reactor anti-neutrino oscillation experiments at medium baseline length (DayaBayII, RENO50),
  10 km$< L <$100 km at 1 MeV$< E <$8 MeV.

  S.F.Ge, KH, N.Okamura, Y.Takaesu, JHEP 1305(2013)131 [arXiv:1210.8241]

- Accelerator neutrino oscillation experiments at two long baselines,
  T2K ($L =$295 km)
  $+$ Tokai-to-Oki ($L =$653 km) or Tokai-to-Korea ($L =$1000 km)
  at 0.5 GeV$< E <$2 GeV.

  KH, T.Kiwanami, N.Okamura, K.Senda, JHEP 1306(2013)036 [arXiv:1209.2763]
  KH, P.Ko, N.Okamura, Y.Takaesu, in preparation.

- Atmospheric neutrino oscillation experments with a huge detector such as PINGU in IceCube,
  2000 km$< L <$13000 km at 2 GeV$< E <$20 GeV.

  S.F.Ge, KH, C.Rott, arXiv:1309.3176, and in preparation.

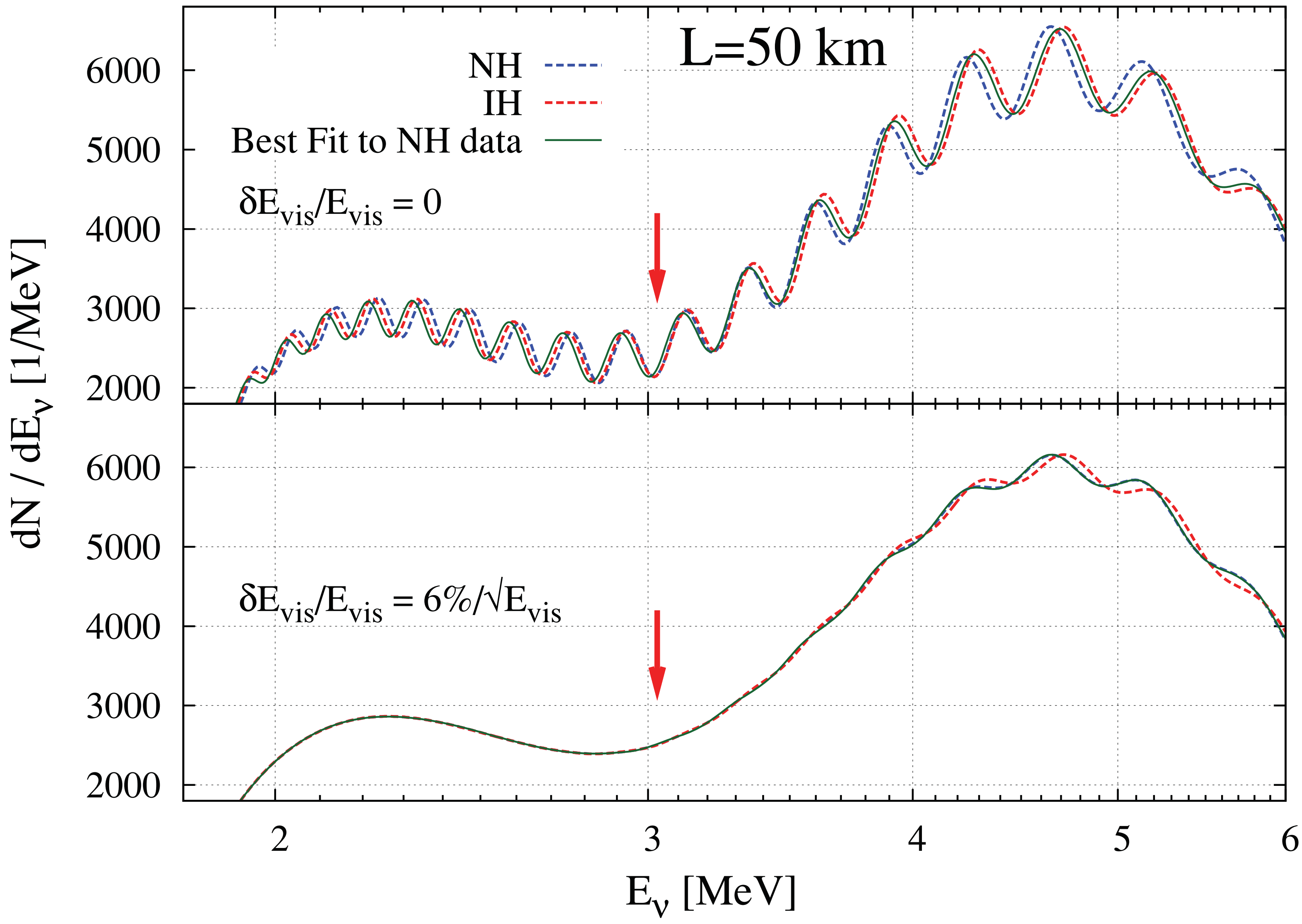


Figure 3. Same as figure 2 but with baseline length $L = 50$ km.

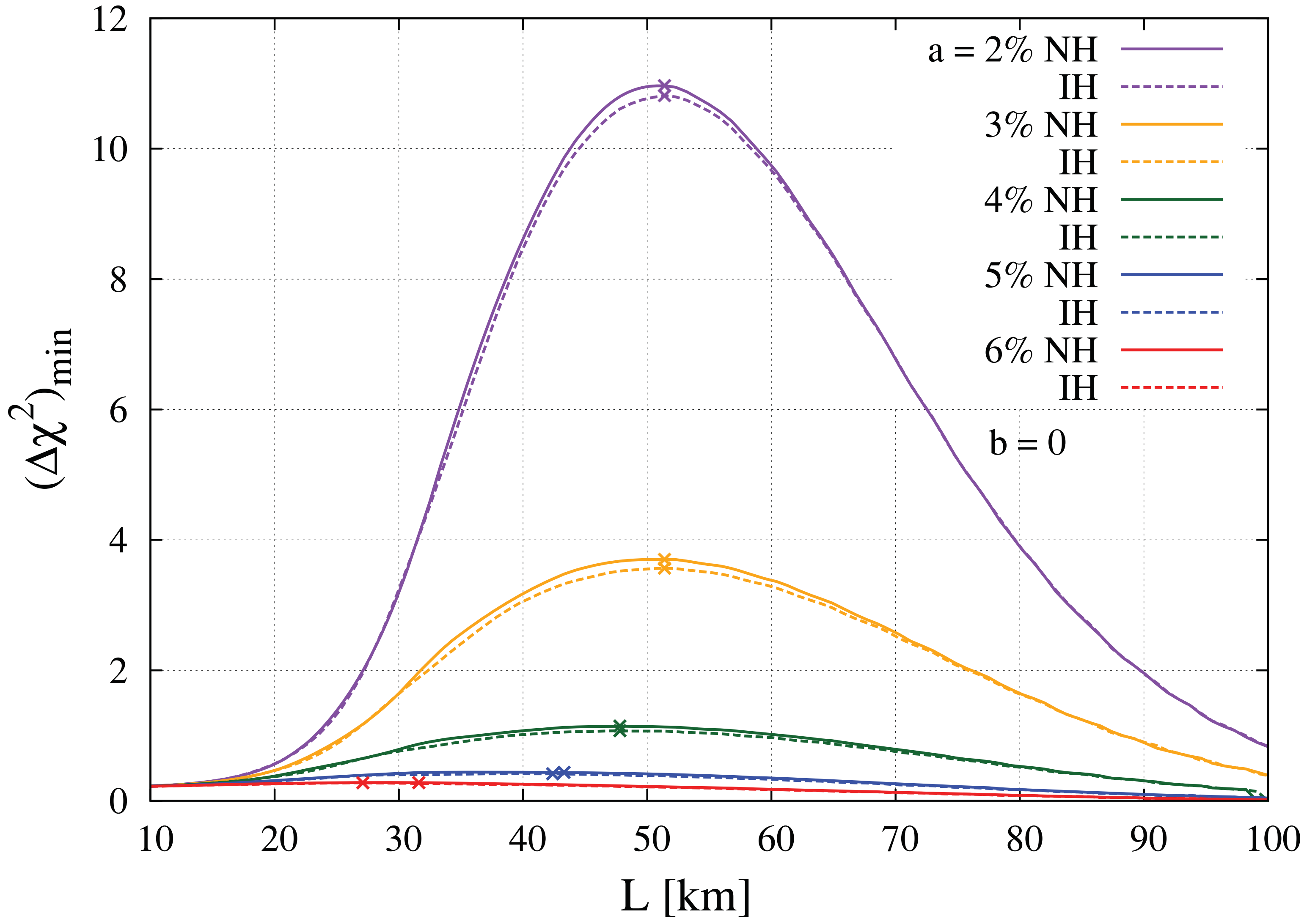


Figure 5. $(\Delta\chi^2)_{\rm min}$ for mass hierarchy discrimination shown as a function of the baseline length L, when the energy resolution in eq. (2.13) is varied with $a = 2$ to 6% and $b = 0$, from the top to the bottom. The results for $20\,{\rm GW_{th}}\cdot 5{\rm kt}$ (12% free-proton weight fraction)$\cdot$5yrs exposure are represented by solid curves for NH, and by dashed curves for IH. The cross symbols mark the optimal baseline lengths.

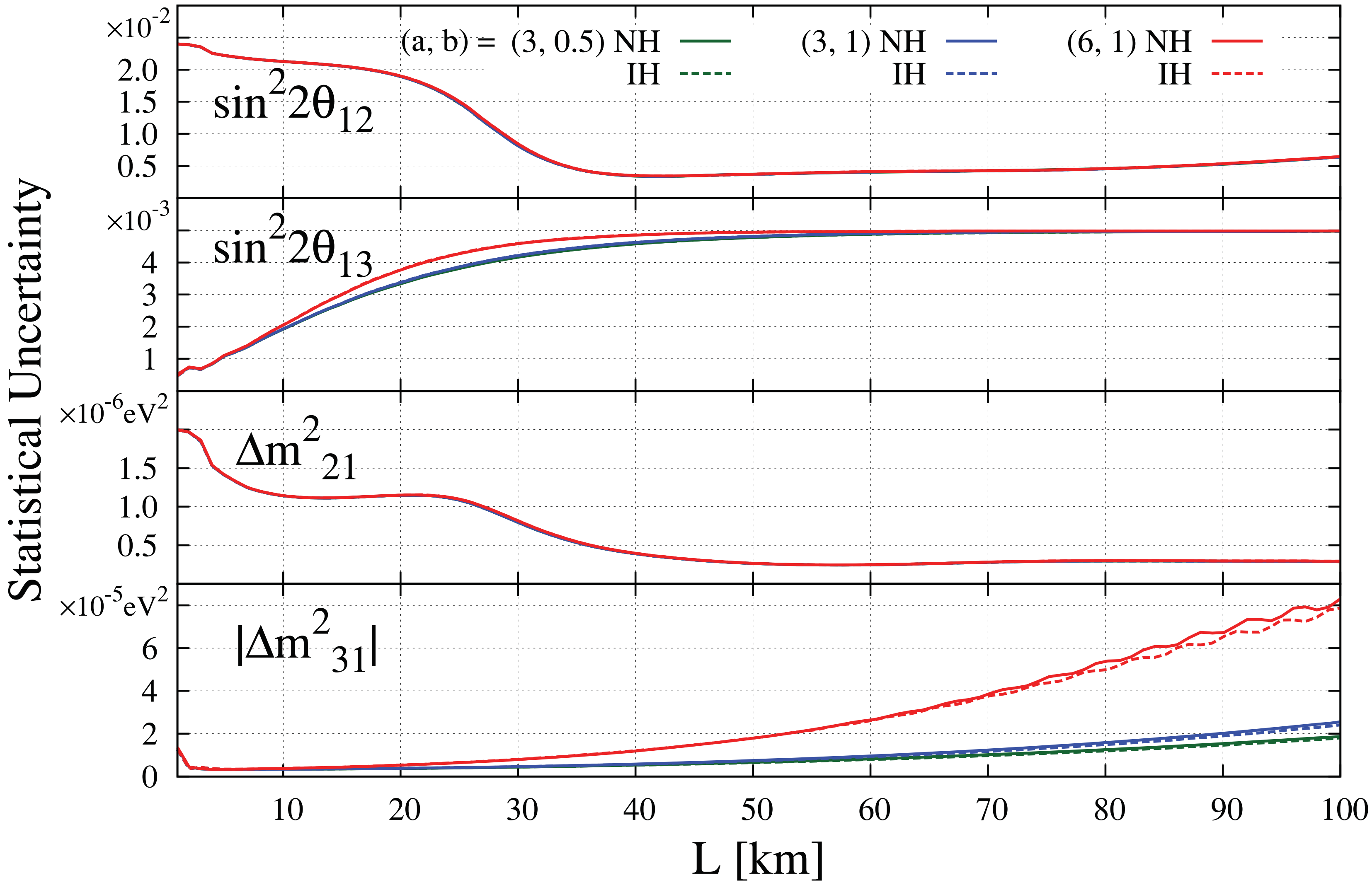


Figure 8. The statistical uncertainties of the neutrino model parameters measured by this experiment as functions of the baseline length L after $20\,\mathrm{GW_{th}}$·5kt (12% free-proton weight fraction)·5yrs exposure. The results for both hierarchy (NH by solid and IH by dashed curves) and for the energy resolution of eq. (2.13) with $(a, b) = (3, 0.5), (3, 1)$ and $(6, 1)\%$ are shown.

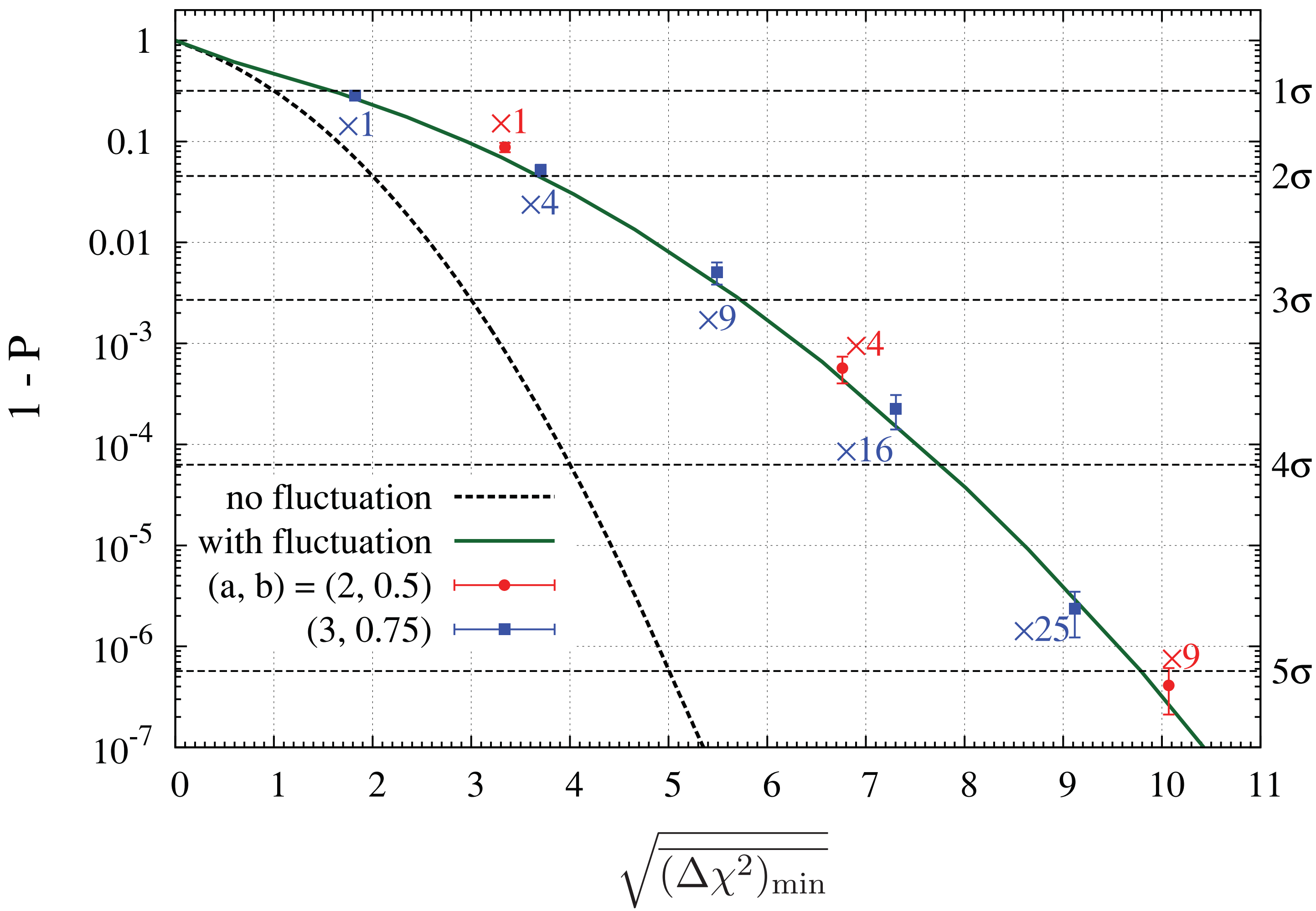


Figure 9. The probability for an experiment to determine the right mass hierarchy as a function of the mean sensitivity, $\sqrt{\overline{(\Delta\chi^2)_{\rm min}}}$, which is calculated by ignoring fluctuation in the data. The solid curve is obtained by considering fluctuations of data using our method, while the dashed curve shows the simple Gaussian interpretation of the $\overline{(\Delta\chi^2)_{\rm min}}$ as a reference. Points with error bars show the probability obtained with the MC method, which performs 1,000 pseudo-experiments for each points. The circle points correspond to experiments with the exposures of 20 $\mathrm{GW_{th}}$·5kt (12% free-proton weight fraction)·5yrs $\times1$, $\times4$ and $\times9$ for $(a, b) = (2, 0.5)$% energy resolution in eq. (2.13), while the rectangular ones correspond to experiments with the exposures of $\times1, \cdots, \times25$ for $(3, 0.75)$% resolution.

KEK-TH-1568

Physics potential of neutrino oscillation experiment with a far detector in Oki Island along the T2K baseline

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Abstract

Recently another new reactor experiment, the DayaBay experiment [17], announced that they have measured the neutrino mixing angle as

$$\sin^2 2\theta_{\rm RCT} = 0.092 \pm 0.016 \text{ (stat.) } \pm 0.005 \text{ (syst.)}\,, \tag{7}$$

which is more than 5σ away from zero. The RENO collaboration, which also measure the reactor $\bar{\nu}_e$ survival probability, shows the evidence of the non-zero mixing angle;

$$\sin^2 2\theta_{\rm RCT} = 0.113 \pm 0.013 \text{ (stat.) } \pm 0.019 \text{ (syst.)}\,, \tag{8}$$

from a rate-only analysis, which is 4.9σ away from zero.

Since the MiniBooNE experiment [12] did not confirm the LSND observation of rapid $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation [18], there is no clear indication of experimental data which suggests more than three neutrinos. Therefore the $\nu_\mu \to \nu_e$ appearance analysis of T2K [14] and MINOS[15] presented above assume the 3 neutrino model, with the 3×3 flavor mixing, the MNS (Maki-Nakagawa-Sakata) matrix [19]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}\,, \tag{9}$$

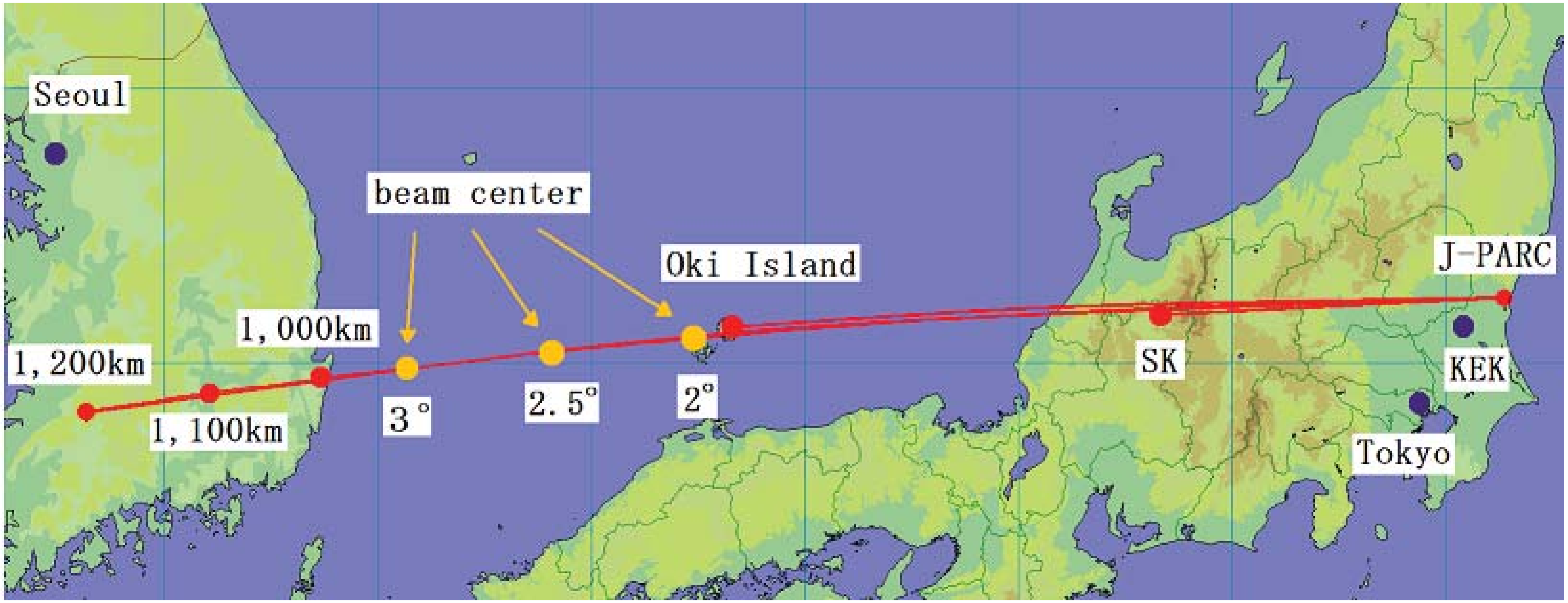


Figure 1: The surface map of the T2K, T2KO, and T2KK experiment. The yellow blobs show the center of the neutrino beam for the T2K experiment at the sea level, where the number in the white box is the off-axis angle at SK.

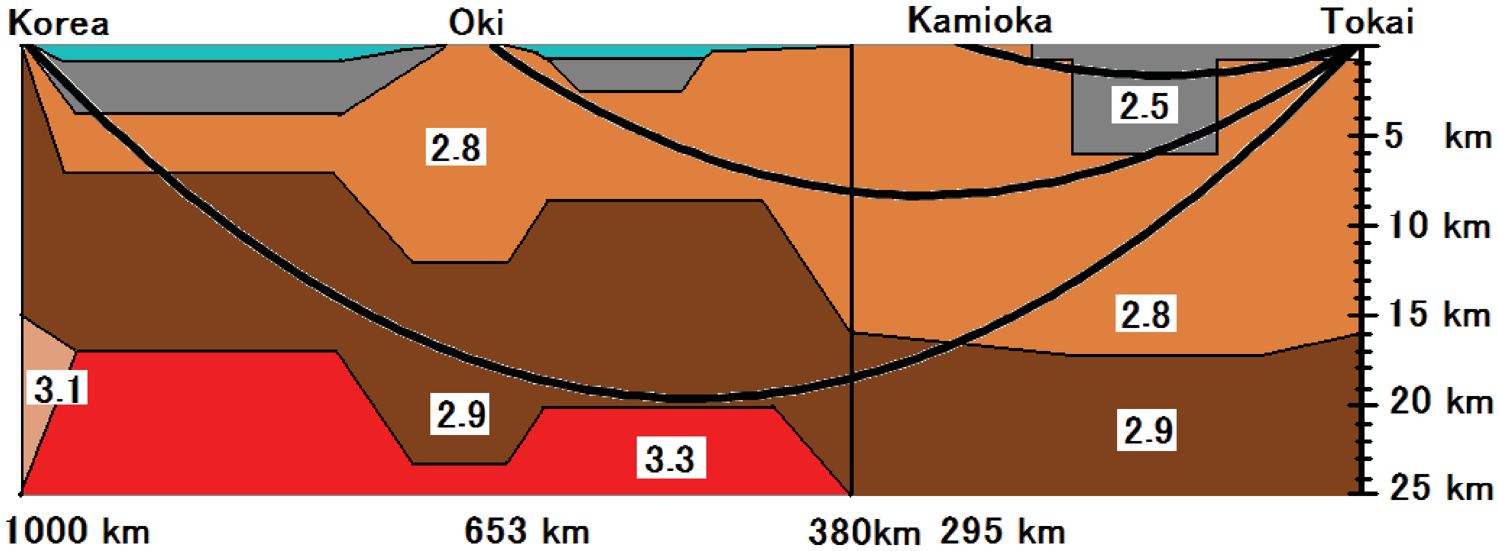


Figure 2: The cross section view of the T2K, T2KO, and T2KK experiments along the baselines, which are shown by the three curves. The horizontal scale gives the distance from J-PARC along the arc of the earth surface and the vertical scale measures the depth of the baseline below the sea level. The numbers in the white boxes are the average matter density in units of g/cm^3 [35]-[42].

Under the same conditions that give eq. (17) for the ν_μ survival probability, the ν_e appearance probability can be approximated as [30]

$$P_{\nu_\mu \to \nu_e} \;=\; 4 \sin^2 \theta_{\rm ATM} \sin^2 \theta_{\rm RCT} \left\{ (1 + A^e) \sin^2 \left(\frac{\Delta_{13}}{2} \right) + \frac{B^e}{2} \sin \Delta_{13} \right\} + C^e \,, \qquad (22)$$

where we retain both linear and quadratic terms of Δ_{12} and a_0. The analytic expressions for the correction terms A^e, B^e and C^e are found in Ref.[30]. For our semi-quantitative discussion below, the following numerical estimates [30] for $\sin^2 2\theta_{\rm ATM} = 1$ and $\sin^2 2\theta_{\rm SOL} = 0.852$ suffice:

$$A^e \;\simeq\; 0.37 \frac{\bar{\rho}}{3{\rm g/cm}^3} \frac{L}{1000{\rm km}} \frac{\pi}{\Delta_{13}} \left(1 - \frac{\sin^2 2\theta_{\rm RCT}}{2} \right)$$
$$-0.29 \left| \frac{\Delta_{13}}{\pi} \right| \sqrt{\frac{0.1}{\sin^2 2\theta_{\rm RCT}}} \left[1 + 0.18 \frac{\bar{\rho}}{3{\rm g/cm}^3} \frac{L}{1000{\rm km}} \frac{\pi}{\Delta_{13}} \right] \sin \delta_{\rm MNS} \,, \qquad (23a)$$

$$B^e \;\simeq\; -0.58 \frac{\bar{\rho}}{3{\rm g/cm}^3} \frac{L}{1000{\rm km}} \left(1 - \frac{\sin^2 2\theta_{\rm RCT}}{2} \right)$$
$$+0.30 \left| \frac{\Delta_{13}}{\pi} \right| \left[\sqrt{\frac{0.1}{\sin^2 2\theta_{\rm RCT}}} \cos \delta_{\rm MNS} - 0.11 \right] \left[1 + 0.18 \frac{\bar{\rho}}{3{\rm g/cm}^3} \frac{L}{1000{\rm km}} \frac{\pi}{\Delta_{13}} \right] . \qquad (23b)$$

The first term in A^e in eq. (23a) is sensitive not only to the matter effect but also to the mass hierarchy pattern, since $\Delta_{13} \sim \pi$ $(-\pi)$ for the normal (inverted) hierarchy around the oscillation maximum $|\Delta_{13}| \sim \pi$. For the normal (inverted) hierarchy, the magnitude of the $\nu_\mu \to \nu_e$ transition probability is enhanced (suppressed) by about 10% at Kamioka, 24% at Oki Island, and 37% at $L \sim 1000$ km in Korea, around the first oscillation maximum, $|\Delta_{13}| \sim \pi$. When L/E_ν is fixed at $|\Delta_{13}| \sim \pi$, the difference between the two hierarchy cases grows with L, because the matter effect grows with E_ν. Within the allowed range of the model parameters, the difference of the A^e between SK and a far detector at Oki or Korea becomes

$$A^e_{\rm peak}(L = 653{\rm km}) - A^e_{\rm peak}(L = 295{\rm km}) \simeq \pm 0.13\,, \tag{24a}$$

$$A^e_{\rm peak}(L \sim 1000{\rm km}) - A^e_{\rm peak}(L = 295{\rm km}) \simeq \pm 0.26\,, \tag{24b}$$

where the upper sign corresponds to the normal, and the lower sign for the inverted hierarchy. The hierarchy pattern can hence be determined by comparing $P_{\nu_\mu \to \nu_e}$ near the oscillation maximum $|\Delta_{13}| \simeq \pi$ at two vastly different baseline lengths [26]-[30], **independently** of the sign and magnitude of $\sin\delta_{\rm MNS}$.

In eq. (23b), it is also found that the first term in B^e, which shifts the oscillation phase from $|\Delta_{13}|$ to $|\Delta_{13} + B^e| = |\Delta_{13}| \pm B^e$, is also sensitive to the mass hierarchy pattern. As in the case for A^e, the difference in B^e between SK and a far detectors is found

$$B^e_{\rm peak}(L = 653{\rm km}) - B^e_{\rm peak}(L = 295{\rm km}) \simeq \mp 0.10\,, \tag{25a}$$

$$B^e_{\rm peak}(L \sim 1000{\rm km}) - B^e_{\rm peak}(L = 295{\rm km}) \simeq \mp 0.20\,, \tag{25b}$$

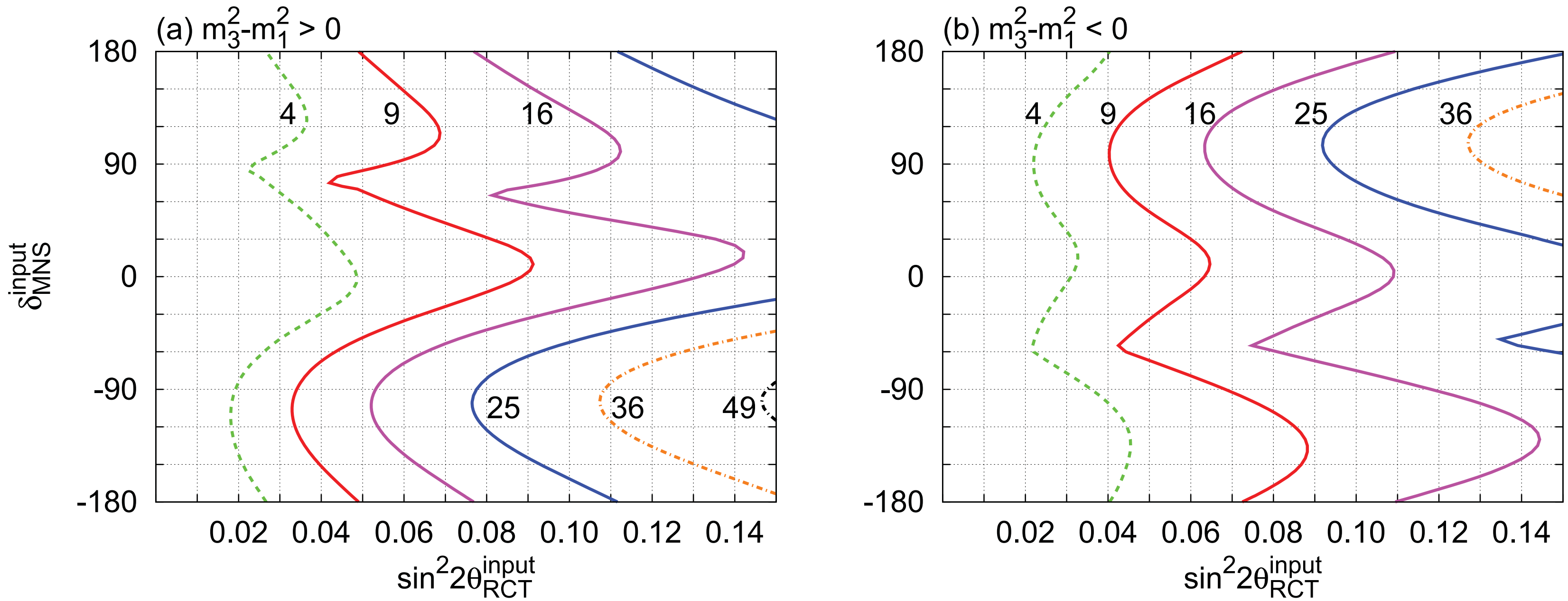


Figure 10: The $\Delta\chi^2_{\rm min}$ contour plot for the T2KO experiment to exclude the wrong mass hierarchy in the plane of $\sin^2 2\theta_{\rm RCT}$ and $\delta_{\rm MNS}$. The left figure is for the normal hierarchy and the right one is for the inverted hierarchy. The OAB combination for both figures is 3.0° at SK and 1.4° at Oki Island with 2.5×10^{21} POT for both ν_μ and $\bar{\nu}_\mu$ focusing beams. Contours for $\Delta\chi^2_{\rm min} = 4, 9, 16, 25, 36, 49$ are shown. All the input parameters other than $\sin^2 2\theta_{\rm RCT}$ and $\delta_{\rm MNS}$ are shown in eqs. (28) and (29).

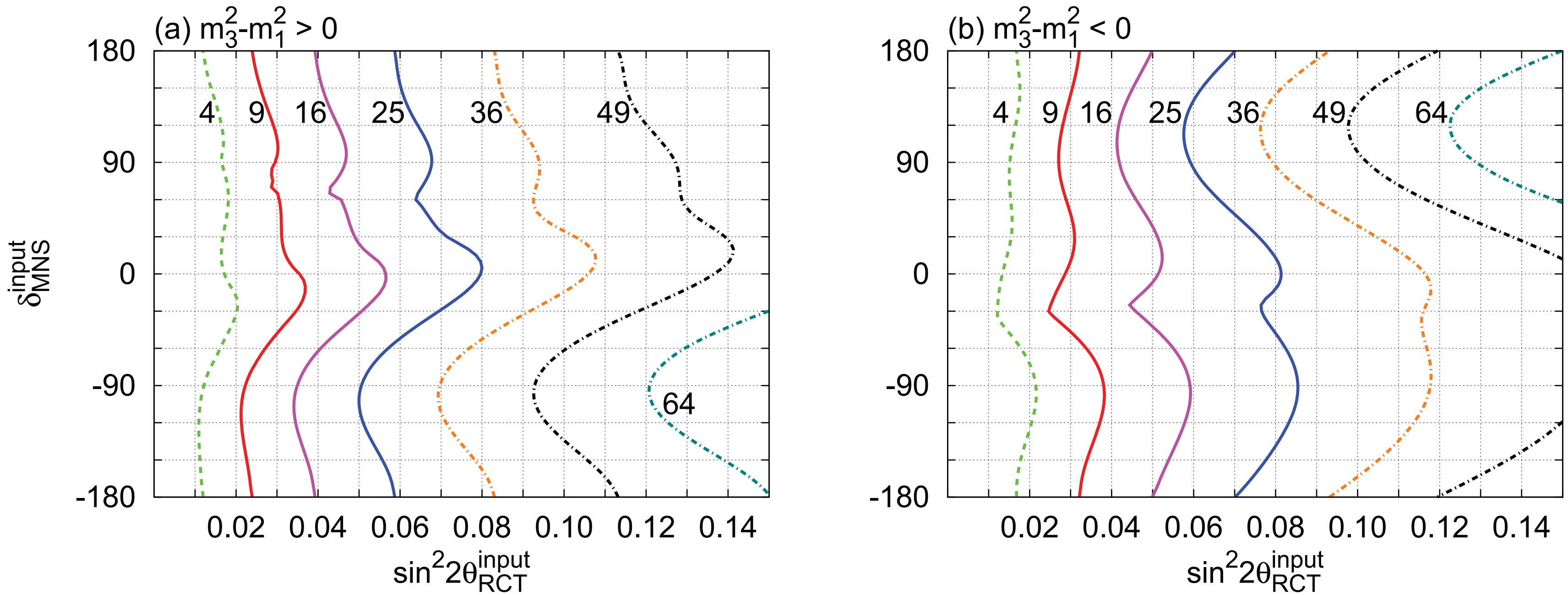


Figure 11: The same as Fig. 10, but for T2KK experiment with the optimum OAB combination, 3.0° OAB at SK and 0.5° OAB at $L = 1000$km. $\Delta\chi^2_{\rm min}$ values are given along the contours.

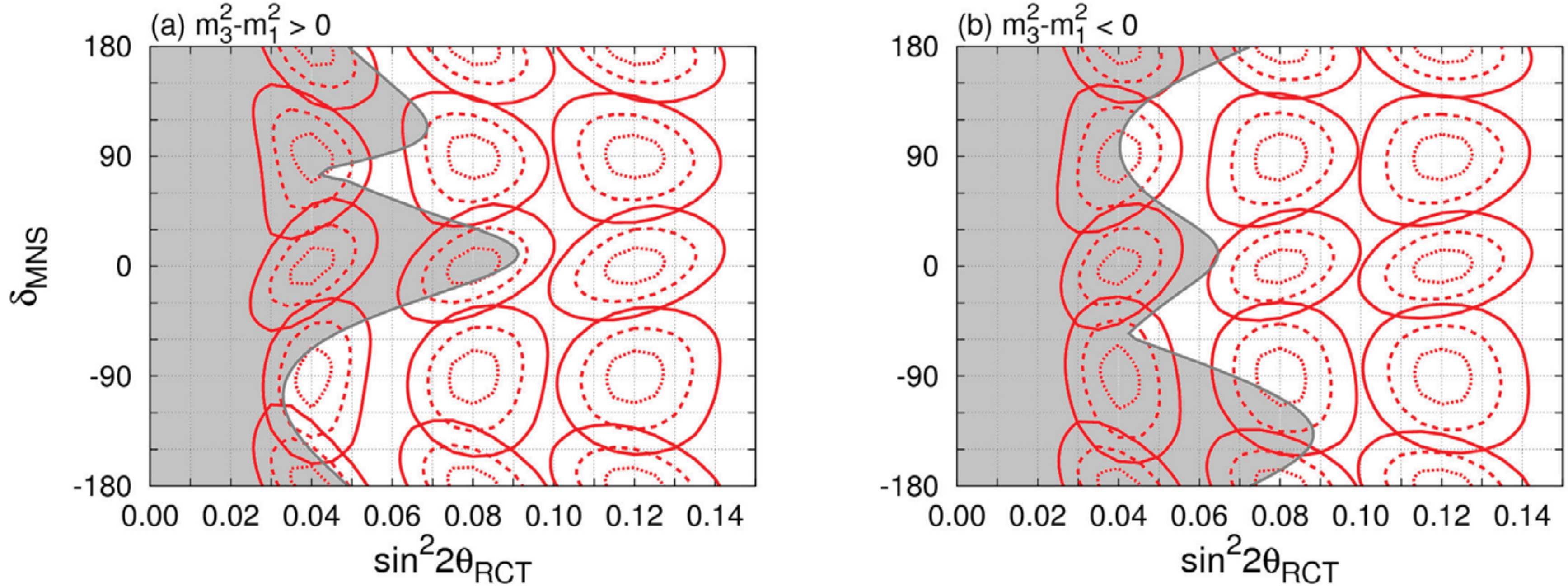


Figure 12: The $\Delta\chi^2$ contour plot for the T2KO experiment in the plane of $\sin^2 2\theta_{\rm RCT}$ and $\delta_{\rm MNS}$ when the mass hierarchy is assumed to be normal (left) or inverted (right). Allowed regions in the plane of $\sin^2 2\theta_{\rm RCT}$ and $\delta_{\rm MNS}$ are shown for the combination of 3.0° OAB at SK and 1.4° at Oki Island with 2.5×10^{21} POT each for ν_μ and $\bar{\nu}_\mu$ focusing beams. The input values of $\sin^2 2\theta_{\rm RCT}$ is 0.04, 0.08, and 0.12 and $\delta_{\rm MNS}$ is 0°, 90°, 180°, and 270°. The other input parameters are given in eqs. (28) and (29). The dotted-lines, dashed-lines, and solid-lines show $\Delta\chi^2_{\rm min} = 1$, 4, and 9 respectively. The blue shaded region has “mirror” solutions for the wrong mass hierarchy giving $\Delta\chi^2_{\rm min} < 9$.

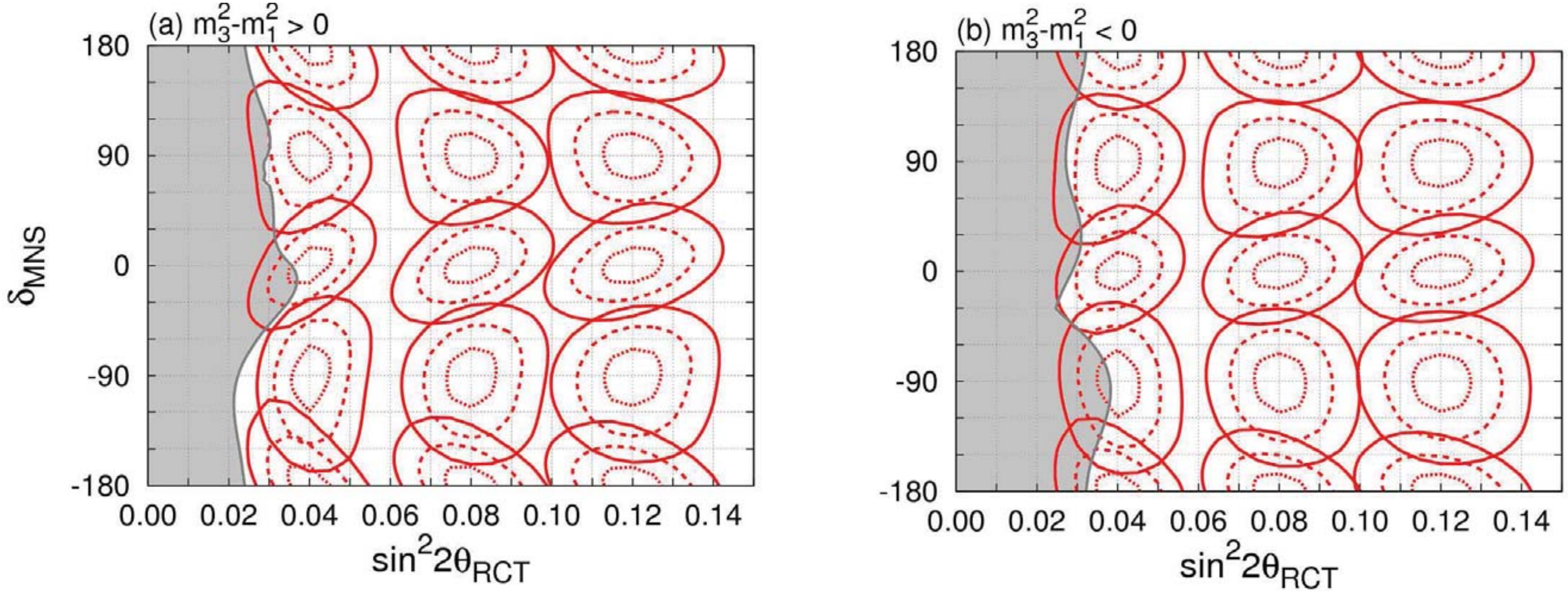


Figure 13: The same as Fig. 12, but for T2KK experiment with 3.0° OAB at SK and 0.5° OAB at $L = 1000$km.

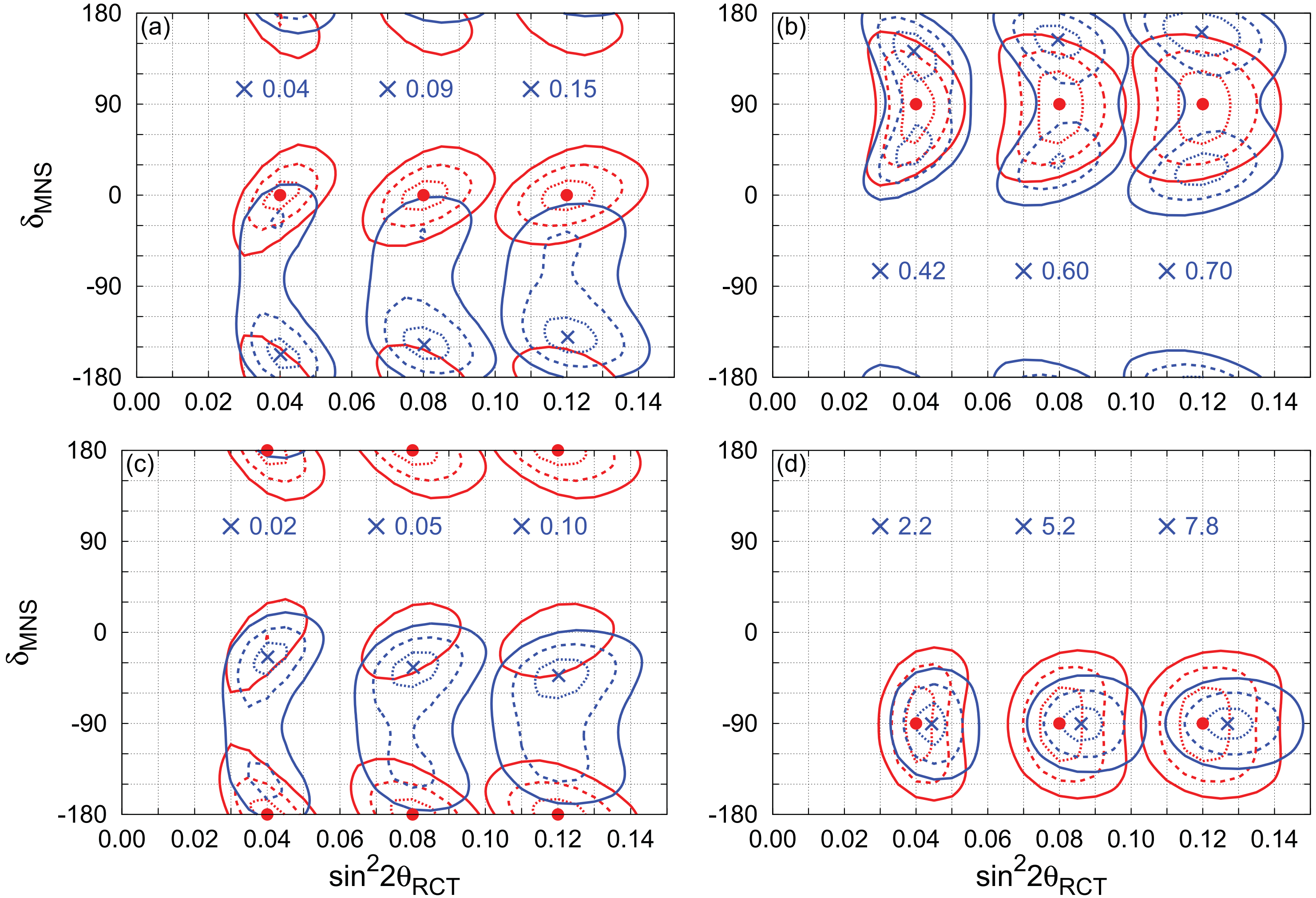


Figure 14: The $\Delta\chi^2_{\rm min}$ contour plot for the T2K$_{122}$ experiment in the plane of $\sin^2 2\theta_{\rm RCT}$ and $\delta_{\rm MNS}$ when the mass hierarchy is assumed to be normal ($m_3^2 - m_1^2 > 0$). Allowed regions in the plane of $\sin^2 2\theta_{\rm RCT}$ and $\delta_{\rm MNS}$ are shown for experiments with 2.5×10^{21} POT each for ν_μ and $\bar{\nu}_\mu$ focusing beam at 3.0° off-axis angle. The input values of $\sin^2 2\theta_{\rm RCT}$ are 0.04, 0.08, and 0.12 and $\delta_{\rm MNS}$ are 0° (a), 90° (b), 180° (c), and 270° (d). The other input parameters are listed in eqs. (29) and (28). The red dotted-lines, dashed-lines, and solid-lines show $\Delta\chi^2_{\rm min} = 1$, 4, and 9 contours, respectively, when the right mass hierarchy is assumed in the fit, whereas the blue contours give $\Delta\chi^2_{\rm min}$ measured from the local minimum value (shown besides the × symbol) at the cross point when the wrong hierarchy is assumed in the fit.

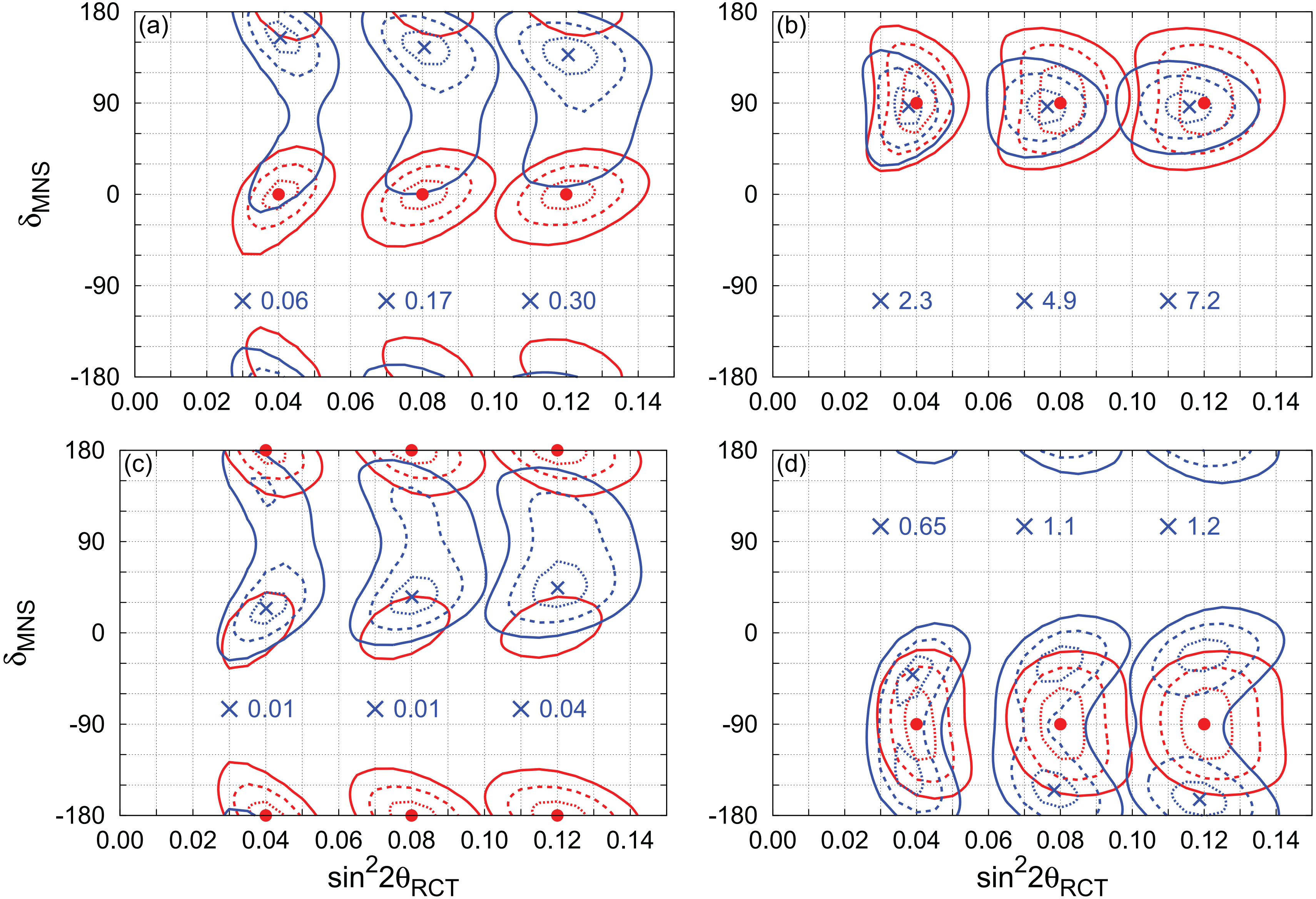
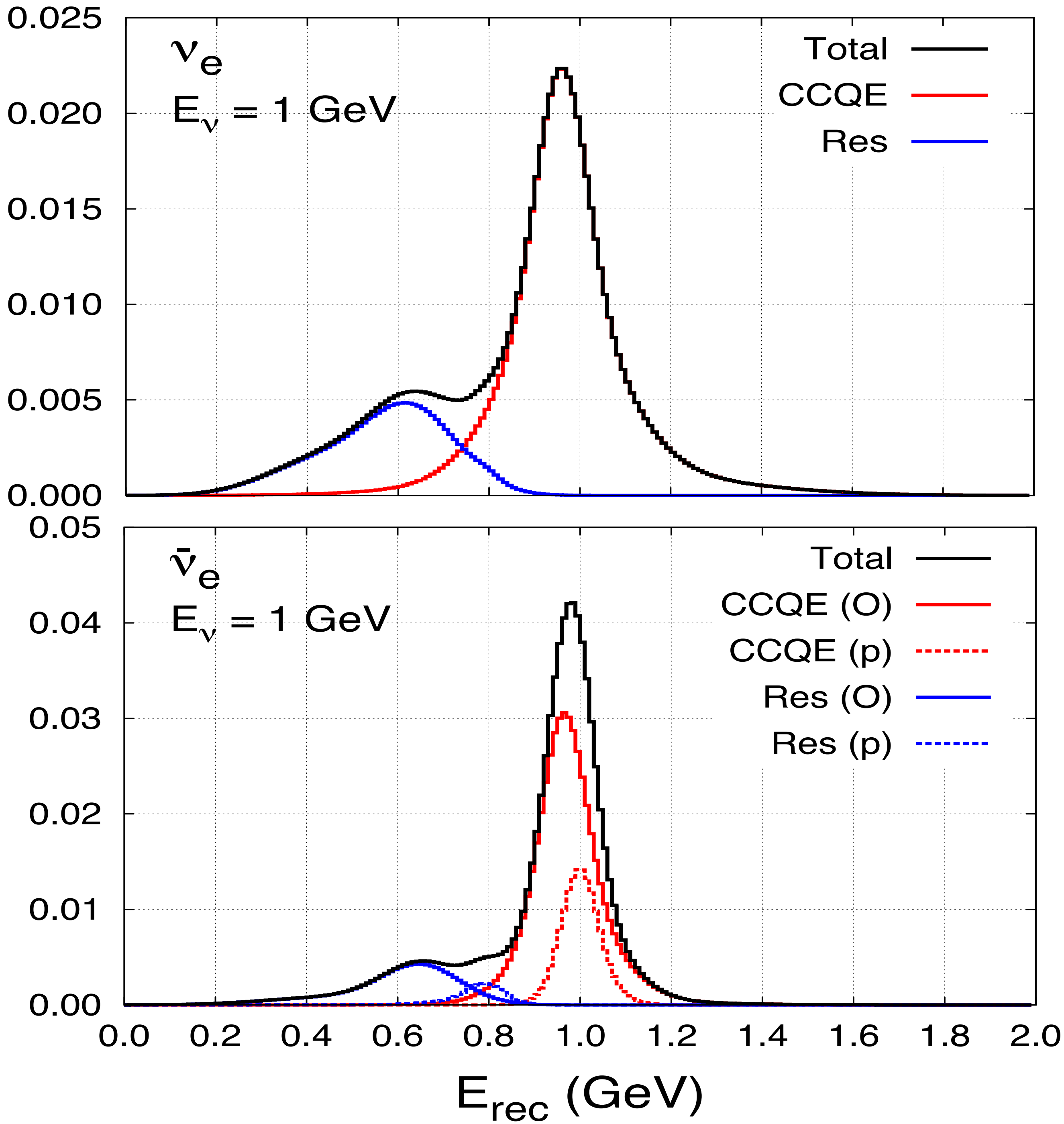
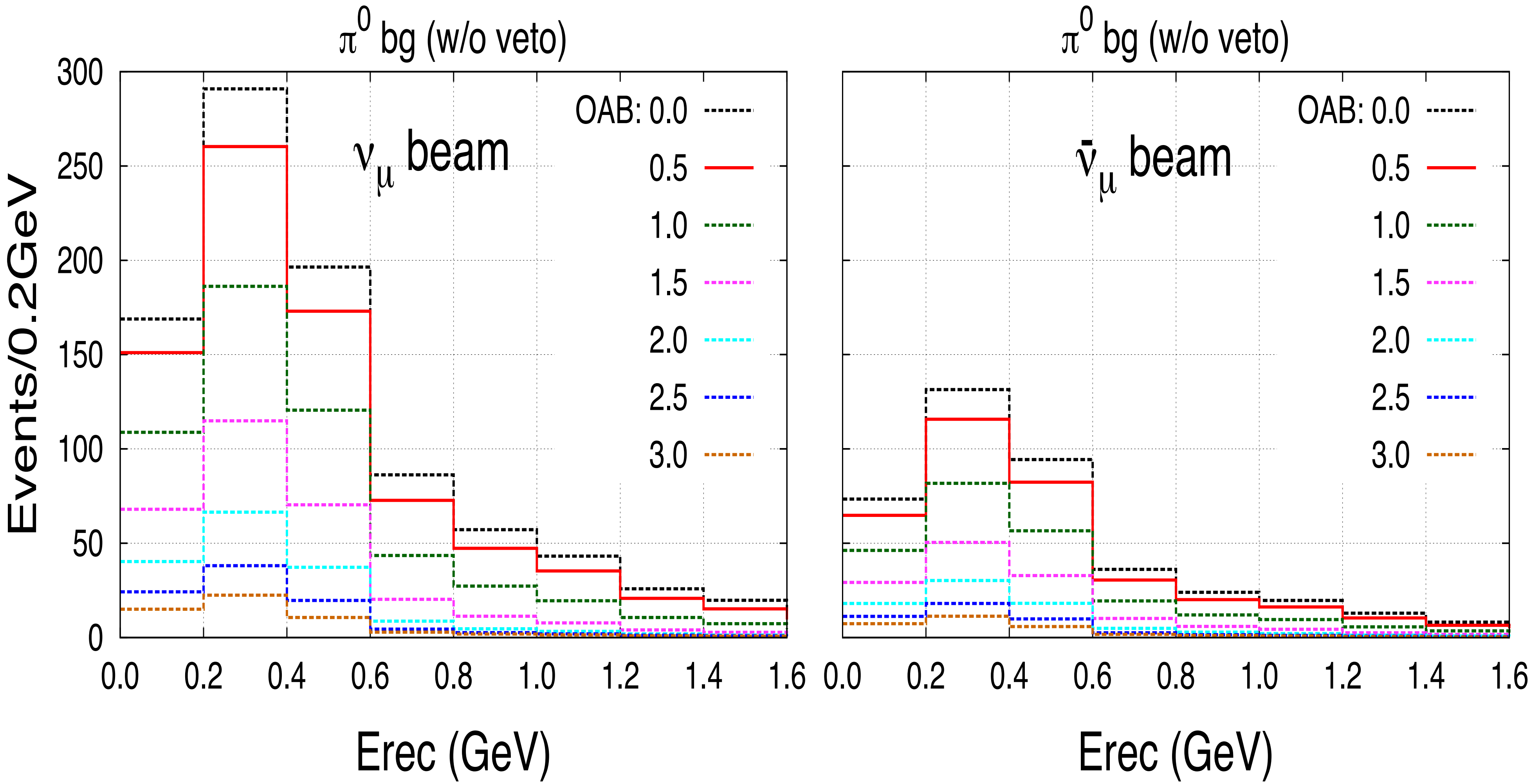


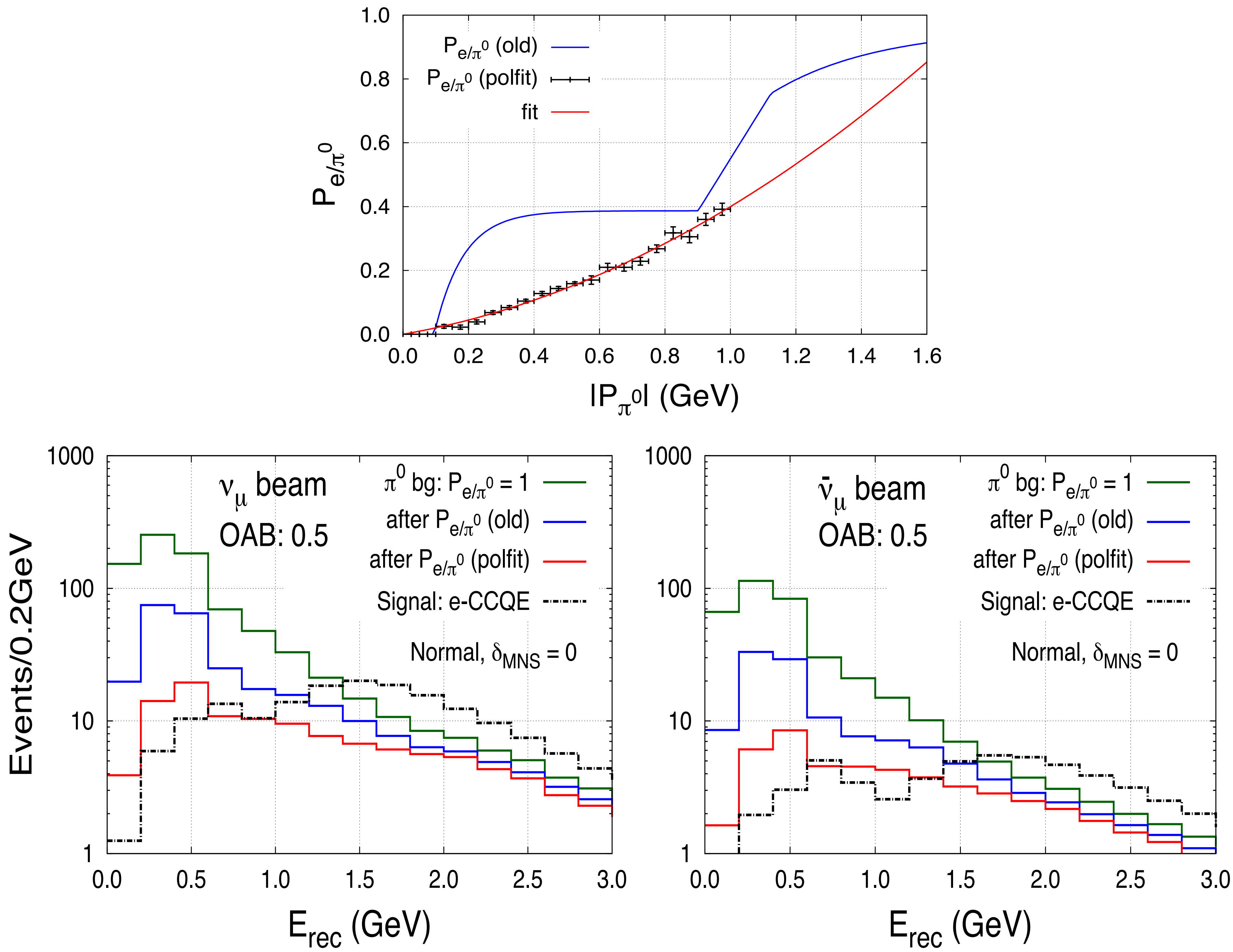
Figure 15: The same as Fig. 14, but for the inverted mass hierarchy ($m_3^2 - m_1^2 < 0$).



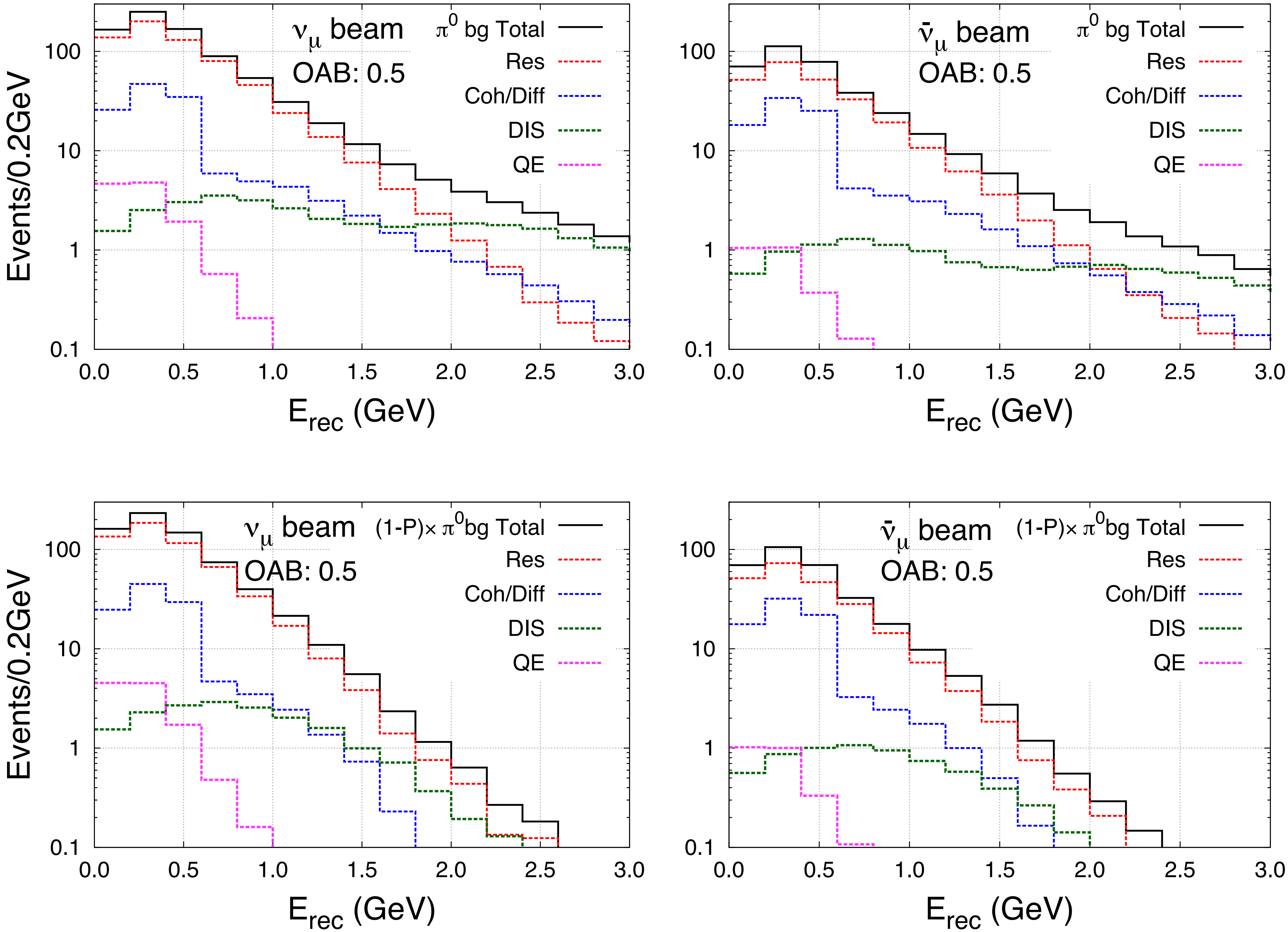
0.025
0.020
0.015
0.010
0.005
0.000
ν_e
E_ν = 1 GeV
Total
CCQE
Res
0.05
0.04
0.03
0.02
0.01
0.00
$\bar{\nu}_e$
E_ν = 1 GeV
Total
CCQE (O)
CCQE (p)
Res (O)
Res (p)
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
E_{rec} (GeV)



π^0 bg (w/o veto)
ν_μ beam
OAB: 0.0
0.5
1.0
1.5
2.0
2.5
3.0
Events/0.2GeV
Erec (GeV)
π^0 bg (w/o veto)
$\bar{\nu}_\mu$ beam
OAB: 0.0
0.5
1.0
1.5
2.0
2.5
3.0
Erec (GeV)



P_{e/π^0} (old)
P_{e/π^0} (polfit)
fit
P_{e/π^0}
$|P_{\pi^0}|$ (GeV)
ν_μ beam
OAB: 0.5
π^0 bg: $P_{e/\pi^0} = 1$
after P_{e/π^0} (old)
after P_{e/π^0} (polfit)
Signal: e-CCQE
Normal, $\delta_{MNS} = 0$
Events/0.2GeV
E_{rec} (GeV)
$\bar{\nu}_\mu$ beam
OAB: 0.5
π^0 bg: $P_{e/\pi^0} = 1$
after P_{e/π^0} (old)
after P_{e/π^0} (polfit)
Signal: e-CCQE
Normal, $\delta_{MNS} = 0$
E_{rec} (GeV)


νμ beam
OAB: 0.5
π0 bg Total
Res
Coh/Diff
DIS
QE
Events/0.2GeV
Erec (GeV)
ν̄μ beam
OAB: 0.5
(1-P)× π0bg Total

Phenomenology of Atmospheric ν Oscillation @ PINGU

Shao-Feng Ge

(gesf02@gmail.com)

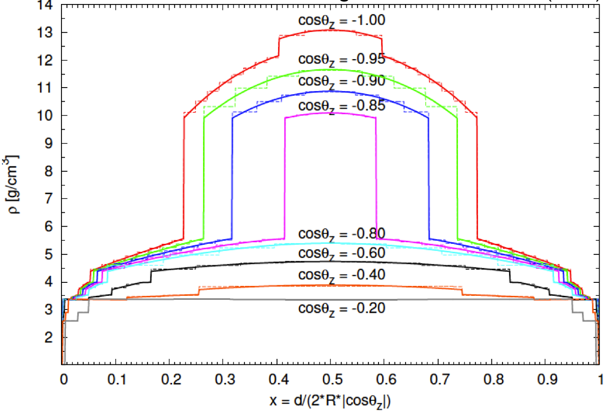
KEK, Tsukuba

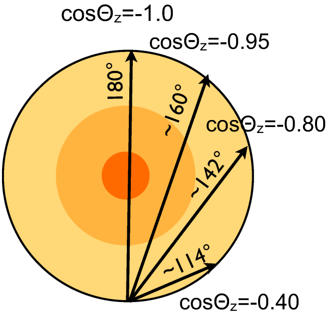
2013-11-14

In collaboration with **K. Hagiwara, C. Rott** [1309.3176, 1312.xxxx]

PREM

Ge, Hagiwara, Okamura, Rott (2012)

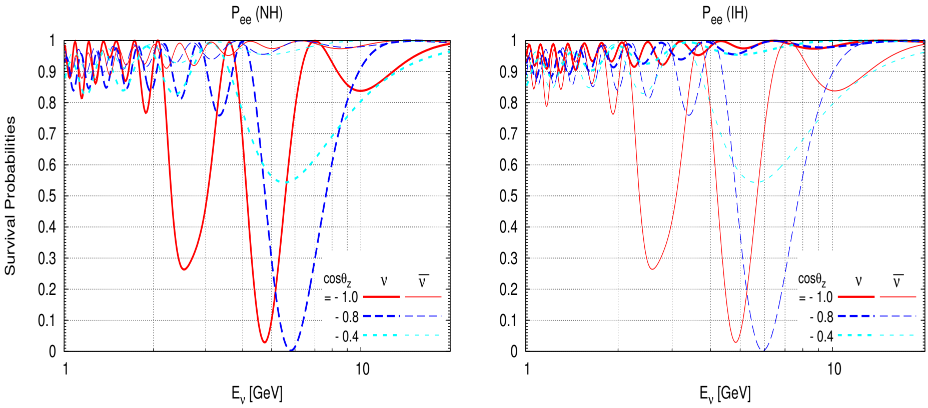




- The PREM - Preliminary Reference Earth Model is based on a paper by Dziewonski and Anderson in 1981. It still still represents the standard framework for interpretation of seismological data

Sensitivity of MH enhanced by MSW & Parametric Resonances

$$\sin 2\widetilde{\theta} = \frac{\sin 2\theta}{\sqrt{\sin^2 2\theta + (\cos 2\theta - 2EV/\delta m^2)^2}} .$$



(Semi)-Analytical Way?

3.1. Propagation Basis

$$\mathcal{H} = \frac{1}{2E_\nu}\left[U\begin{pmatrix}0 & & \\ & \delta m_s^2 & \\ & & \delta m_a^2\end{pmatrix}U^\dagger + \begin{pmatrix}a(x) & & \\ & 0 & \\ & & 0\end{pmatrix}\right], \tag{3.1}$$

where $a(x) \equiv 2E_\nu V(x) = 2\sqrt{2}E_\nu G_F N_e(x)$ characterizes the matter effect, $\delta m_s^2 \equiv \delta m_{12}^2$ & $\delta m_a^2 \equiv \delta m_{13}^2$,

$$U \equiv O_{23}(\theta_a)P_\delta O_{13}(\theta_r)P_\delta^\dagger O_{12}(\theta_s) = \begin{pmatrix}1 & & \\ & c_a & s_a \\ & -s_a & c_a\end{pmatrix}\begin{pmatrix}1 & & \\ & 1 & \\ & & e^{i\delta}\end{pmatrix}\begin{pmatrix}c_r & & s_r \\ & 1 & \\ -s_r & & c_r\end{pmatrix}\begin{pmatrix}1 & & \\ & 1 & \\ & & e^{-i\delta}\end{pmatrix}\begin{pmatrix}c_s & s_s & \\ -s_s & c_s & \\ & & 1\end{pmatrix},$$

where $c_\alpha \equiv \cos\theta_\alpha$ and $s_\alpha \equiv \sin\theta_\alpha$ with $(s,a,r) \equiv (12, 23, 13)$. Note that in this basis O_{23} and P_δ can be extracted out as overall matrices [42],

$$\mathcal{H} = \frac{1}{2E_\nu}(O_{23}P_\delta)\left[(O_{13}O_{12})\begin{pmatrix}0 & & \\ & \delta m_s^2 & \\ & & \delta m_a^2\end{pmatrix}(O_{13}O_{12})^\dagger + \begin{pmatrix}a(x) & & \\ & 0 & \\ & & 0\end{pmatrix}\right](O_{23}P_\delta)^\dagger. \tag{3.2}$$

This is a huge simplification,

$$\mathcal{H}' = \frac{1}{2E_\nu}\left[(O_{13}O_{12})\begin{pmatrix}0 & & \\ & \delta m_s^2 & \\ & & \delta m_a^2\end{pmatrix}(O_{13}O_{12})^\dagger + \begin{pmatrix}a(x) & & \\ & 0 & \\ & & 0\end{pmatrix}\right] = (O_{23}P_\delta)^\dagger \mathcal{H}(O_{23}P_\delta). \tag{3.3}$$

Propagation Basis

$$\nu_\alpha = [O_{23}(\theta_a)P_\delta]_{\alpha i}\nu'_i. \tag{3.4}$$

$$S = (O_{23}P_\delta)S'(O_{23}P_\delta)^\dagger \equiv (O_{23}P_\delta)\begin{pmatrix}S'_{11} & S'_{12} & S'_{13} \\ S'_{21} & S'_{22} & S'_{23} \\ S'_{31} & S'_{32} & S'_{33}\end{pmatrix}(O_{23}P_\delta)^\dagger, \tag{3.5}$$

with $S'_{ij} \equiv \langle\nu'_j|S'|\nu'_i\rangle$ and $S_{\beta\alpha} \equiv \langle\nu_\beta|S|\nu_\alpha\rangle$.

3.2. Oscillation Probabilities

$$S_{ee} = S'_{11}\,, \tag{3.14a}$$
$$S_{e\mu} = c_a S'_{12} + s_a e^{-i\delta} S'_{13}\,, \tag{3.14b}$$
$$S_{\mu e} = c_a S'_{21} + s_a e^{+i\delta} S'_{31}\,, \tag{3.14c}$$
$$S_{\mu\mu} = c_a^2 S'_{22} + c_a s_a (e^{-i\delta} S'_{23} + e^{+i\delta} S'_{32}) + s_a^2 S'_{33}\,. \tag{3.14d}$$

Note that only the elements among e and μ flavors are shown since they are sufficient to derive all the flavor basis oscillation probabilities, $P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | S | \nu_\alpha \rangle|^2 = |S_{\beta\alpha}|^2$,

$$P_{ee} \equiv |S_{ee}|^2 = |S'_{11}|^2\,, \tag{3.15a}$$
$$P_{e\mu} \equiv |S_{\mu e}|^2 = c_a^2 |S'_{12}|^2 + s_a^2 |S'_{13}|^2 + 2 c_a s_a (\cos\delta\,\mathbb{R} + \sin\delta\,\mathbb{I})(S'_{12} S'^*_{13})\,, \tag{3.15b}$$
$$P_{\mu e} \equiv |S_{e\mu}|^2 = c_a^2 |S'_{21}|^2 + s_a^2 |S'_{31}|^2 + 2 c_a s_a (\cos\delta\,\mathbb{R} - \sin\delta\,\mathbb{I})(S'_{21} S'^*_{31})\,, \tag{3.15c}$$
$$\begin{aligned} P_{\mu\mu} \equiv |S_{\mu\mu}|^2 &= c_a^4 |S'_{22}|^2 + s_a^4 |S'_{33}|^2 + 4 c_a^2 s_a^2 \mathbb{R}(S'_{22} S'^*_{33}) \\ &+ c_a^2 s_a^2 \left[|S'_{23}|^2 + 2(\cos 2\delta\,\mathbb{R} + \sin 2\delta\,\mathbb{I})(S'_{23} S'^*_{32}) + |S'_{32}|^2 \right] \\ &+ 2\, c_a\, s_a \cos\delta\,\mathbb{R}[(c_a^2 S'_{22} + s_a^2 S'_{33})(S'_{23} + S'_{32})^*] \\ &+ 2\, c_a\, s_a \sin\delta\,\mathbb{I}[(c_a^2 S'_{22} + s_a^2 S'_{33})(S'_{32} - S'_{23})^*]\,, \end{aligned} \tag{3.15d}$$

where $\mathbb{R}$ and $\mathbb{I}$ gives the real and imaginary parts, respectively. The transition probability into ν_τ are then obtained by unitarity conditions, $P_{e\tau} = 1 - P_{ee} - P_{e\mu}$, $P_{\mu\tau} = 1 - P_{\mu e} - P_{\mu\mu}$, $\overline{P}_{\alpha\beta} \equiv P(\bar\nu_\alpha \to \bar\nu_\beta) = P_{\alpha\beta}(a(x) \to -a(x), \delta \to -\delta)$,

3.3. Simplifications with Symmetric Matter Profile

The oscillation amplitude matrix after experiencing a reversible matter profile is symmetric in the absence of CP violation [45].

$$S'_{ij} = S'_{ji}\,. \tag{3.16}$$

3.4. Expansion of Oscillation Probabilities with respect to $x_a = \cos 2\theta_a$ and δm^2_s

The deviation of θ_a from its maximal value $\theta_a \approx \frac{\pi}{4}$ can be explored analytically,

$$c^2_a = \frac{1}{2}(1 + x_a)\,, \qquad s^2_a = \frac{1}{2}(1 - x_a)\,, \qquad c^2_a s^2_a = \frac{1}{4}(1 - x^2_a)\,, \tag{3.23}$$

Then

$$P_{\alpha\beta} \equiv P^{(0)}_{\alpha\beta} + P^{(1)}_{\alpha\beta} x_a + P^{(2)}_{\alpha\beta} \cos\delta' + P^{(3)}_{\alpha\beta} \sin\delta' + P^{(4)}_{\alpha\beta} x_a \cos\delta' + P^{(5)}_{\alpha\beta} x^2_a + P^{(6)}_{\alpha\beta} \cos^2\delta'\,, \tag{3.24a}$$

$$\overline{P}_{\alpha\beta} \equiv \overline{P}^{(0)}_{\alpha\beta} + \overline{P}^{(1)}_{\alpha\beta} x_a + \overline{P}^{(2)}_{\alpha\beta} \cos\delta' + \overline{P}^{(3)}_{\alpha\beta} \sin\delta' + \overline{P}^{(4)}_{\alpha\beta} x_a \cos\delta' + \overline{P}^{(5)}_{\alpha\beta} x^2_a + \overline{P}^{(6)}_{\alpha\beta} \cos^2\delta'\,, \tag{3.24b}$$

with $\cos\delta' \equiv 2c_a s_a \cos\delta \approx \sqrt{1 - x^2_a}\cos\delta\,, \quad \sin\delta' \equiv 2c_a s_a \sin\delta \approx \sqrt{1 - x^2_a}\sin\delta$ and

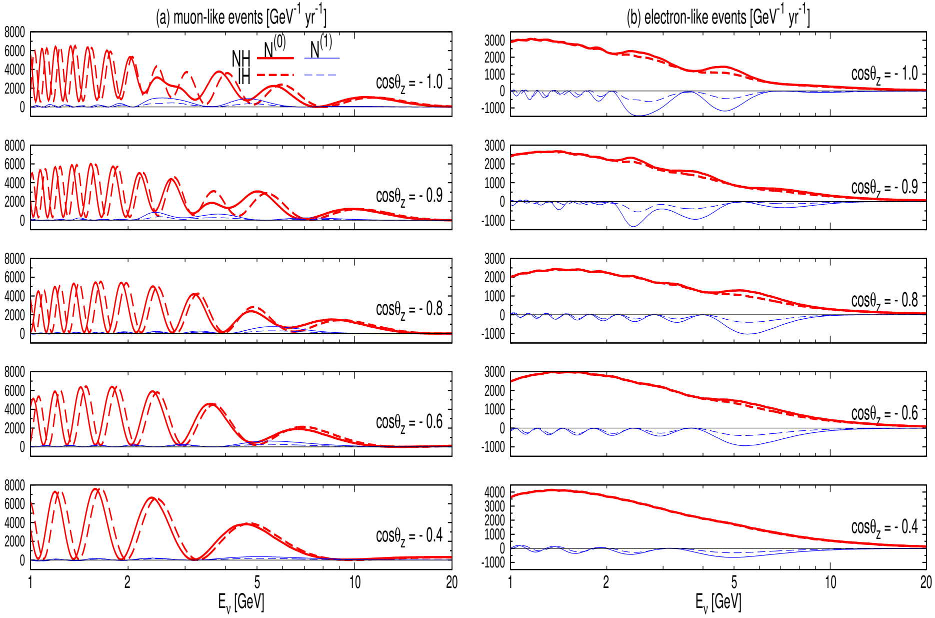
	$P^{(k)}_{ee}$	$P^{(k)}_{e\mu}$	$P^{(k)}_{\mu e}$	$P^{(k)}_{\mu\mu}$
(0)	$\lvert S'_{11}\rvert$	$\frac{1}{2}(1 - \lvert S'_{11}\rvert^2)$	$\frac{1}{2}(1 - \lvert S'_{11}\rvert^2)$	$\frac{1}{4}\lvert S'_{22} + S'_{33}\rvert^2$
(1)	0	$\frac{1}{2}(\lvert S'_{12}\rvert^2 - \lvert S'_{13}\rvert^2)$	$\frac{1}{2}(\lvert S'_{12}\rvert^2 - \lvert S'_{13}\rvert^2)$	$\frac{1}{2}(\lvert S'_{22}\rvert^2 - \lvert S'_{33}\rvert^2)$
(2)	0	$\mathbb{R}(S'_{12} S'^*_{13})$	$\mathbb{R}(S'_{12} S'^*_{13})$	$\mathbb{R}[S'_{23}(S'_{22} + S'_{33})^*]$
(3)	0	$\mathbb{I}(S'_{12} S'^*_{13})$	$-\mathbb{I}(S'_{12} S'^*_{13})$	0
(4)	0	0	0	$\mathbb{R}[S'_{23}(S'_{22} - S'_{33})^*]$
(5)	0	0	0	$\frac{1}{4}\lvert S'_{22} - S'_{33}\rvert^2$
(6)	0	0	0	$\lvert S'_{23}\rvert^2$

(3.25)

$$S'_{12} \sim S'_{23} \sim \delta m^2_s / \delta m^2_a \sim 3\%$$

$$\frac{dN_\alpha}{dE_\nu\, d\cos\theta_z} \equiv N^{(0)}_\alpha + N^{(1)}_\alpha x_a + N^{(2)}_\alpha \cos\delta' + N^{(3)}_\alpha \sin\delta' + N^{(4)}_\alpha x_a \cos\delta' + N^{(5)}_\alpha x^2_a + N^{(6)}_\alpha \cos^2\delta'\,. \tag{4.2}$$

Neutrinos arriving @ PINGU 1309.3176

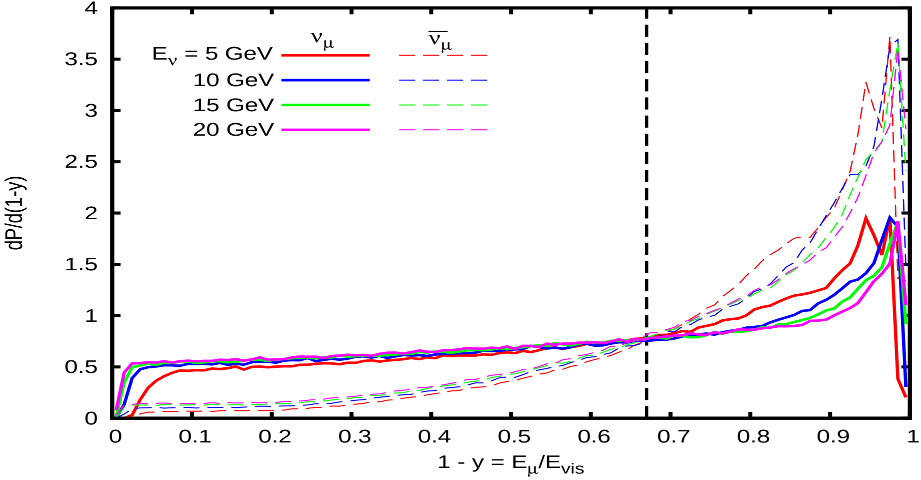


Enhancing the MH sensitivity by splitting the μ events

$$\frac{d\sigma_{\nu}^{CC}}{d(1-y)} = \left[0.72 + 0.06(1-y)^2\right] 10^{-38}\text{cm}^2 \frac{E_\nu}{1GeV}$$

$$\frac{d\sigma_{\bar{\nu}}^{CC}}{d(1-y)} = \left[0.09 + 0.69(1-y)^2\right] 10^{-38}\text{cm}^2 \frac{E_{\bar{\nu}}}{1GeV}$$

1303.0758



Ability of Distinguishing ν & $\overline{\nu}$

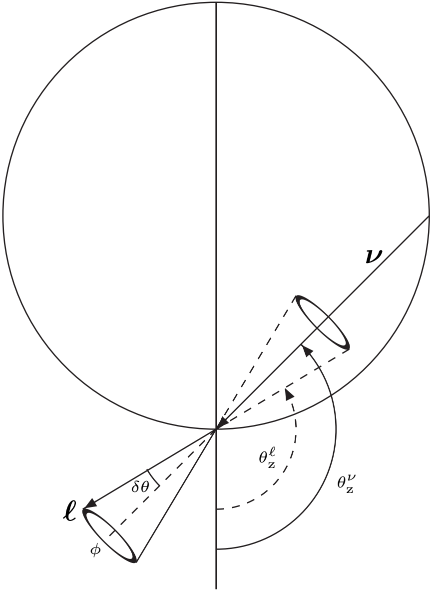
- **HK** 1109.3262
  - μ – NO
  - e (multi-GeV, non-QE) – YES

$$\begin{cases} \text{single-ring} \begin{cases} \nu_e \to e^- + (\pi^+ \to \mu^+ \to e^+, \text{delayed signal}) \\ \bar{\nu}_e \to e^+ + (\pi^- \text{absorbed by water}) \end{cases} \\ \text{multi-ring} \begin{cases} \nu_e & \text{flat distribution of 1-y} \\ \bar{\nu}_e & \text{tend to have large 1-y} \end{cases} \end{cases}$$

- **INO** 1212.1305
  - μ – YES, by Magnetic Field
  - e – NO
- **Liquid Argon** hep-ph/0510131
  - μ – YES, by Magnetic Field (?)
  - e – YES, like HK (?)
- **PINGU** 1205.4965
  - μ – YES, by estimating 1-y $\begin{cases} \nu_\mu & \text{flat distribution of 1-y} \\ \bar{\nu}_\mu & \text{tend to have large 1-y} \end{cases}$
  - e – NO

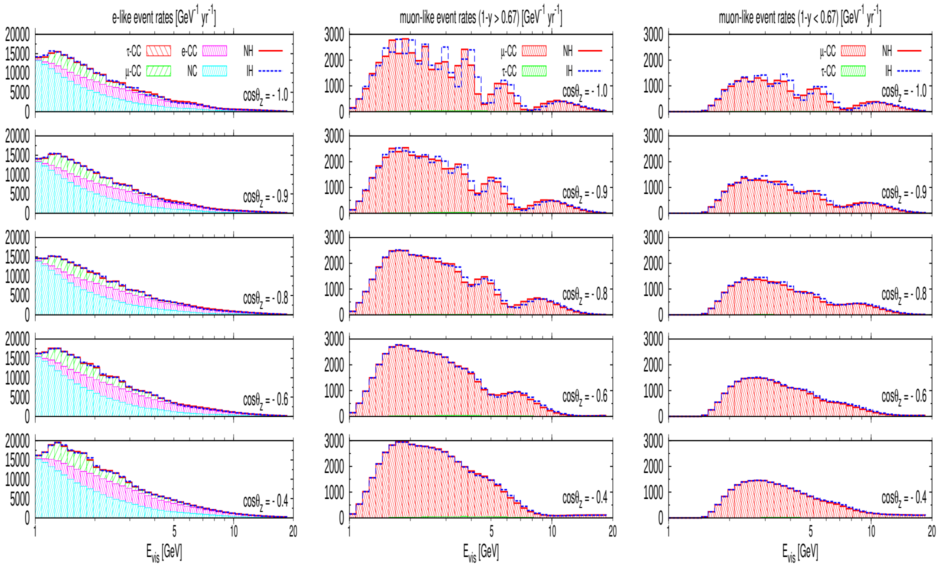
Detector Smearing

- **Energy** Reconstruction & Smearing
  - $E_{\rm vis} = \begin{cases} E_\mu + E_{\rm cas} & \mu(E_\mu > 1GeV \text{ \& } 1-y \equiv \frac{E_\mu}{E_{\rm vis}} > 0.2) \\ E_\ell + \frac{E_{\rm cas}}{0.8} & \mu(E_\mu < 1GeV \text{ or } 1-y < 0.2), e \text{ \& } NC \end{cases}$
  - $\Delta E = 0.2\sqrt{E}$ for E_ℓ & $E_{\rm cas}(\equiv E_\nu - E_\ell - E_{\nu'})$.
- **Zenith Angle** Reconstruction & Smearing
  - $\vec{P}_{\rm vis} = \begin{cases} \vec{P}_\ell, & E_\ell > 1GeV \\ \vec{P}_\nu - \vec{P}_{\nu'}, & E_\ell < 1GeV \text{ \& } NC \end{cases} \equiv |P_{\rm vis}| \begin{pmatrix} \sin\theta_\ell \cos\phi_\ell \\ \sin\theta_\ell \sin\phi_\ell \\ \cos\theta_\ell \end{pmatrix}$
  - $\Delta\theta = \begin{cases} 1.0 \times 15^\circ E_\mu^{-0.6}, & \mu \text{ with } E_\mu > 1GeV, 1-y > 0.2 \\ 1.5 \times 15^\circ E_\mu^{-0.6}, & \mu \text{ with } E_\mu > 1GeV, 1-y < 0.2 \\ 2.0 \times 15^\circ E_e^{-0.6}, & e \text{ with } E_e > 1GeV, 1-y > 0.4 \\ 3.0 \times 15^\circ E_e^{-0.6}, & e \text{ with } E_e > 1GeV, 1-y < 0.4 \\ P(\theta_\ell)|_{E_\ell < 1GeV}, & E_\ell < 1GeV, 1-y_\mu < 0.2, 1-y_e < 0.4 \text{ \& } NC \end{cases}$

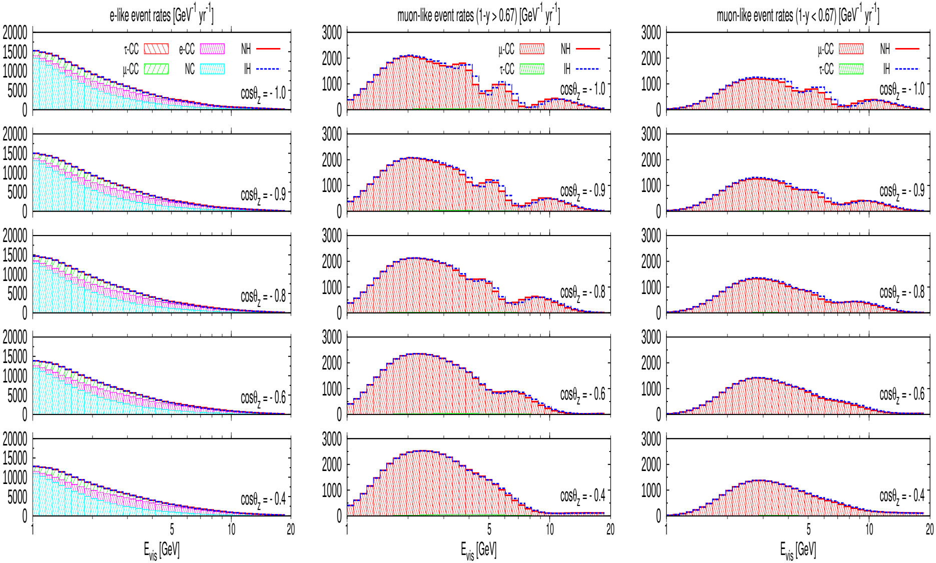


$$\cos\theta_z^\ell = \cos\theta_z^\nu \cos\delta\theta - \sin\theta_z^\nu \sin\delta\theta \cos\phi\,,$$
$$\min(0, \theta_z^\nu - \delta\theta) \le \theta_z^\ell \le \max(\pi, \theta_z^\nu + \delta\theta)\,.$$

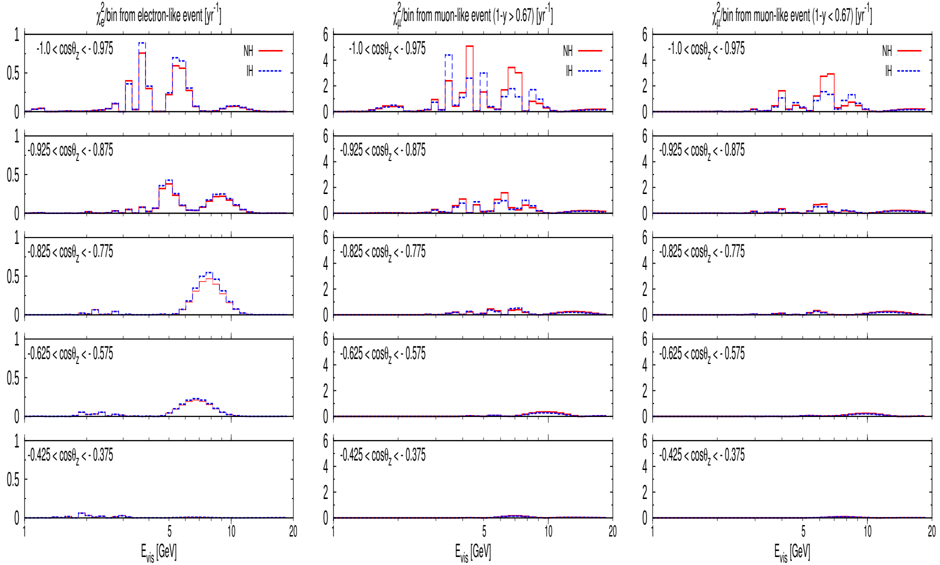
Events Rates with Scattering & Splitting μ Events



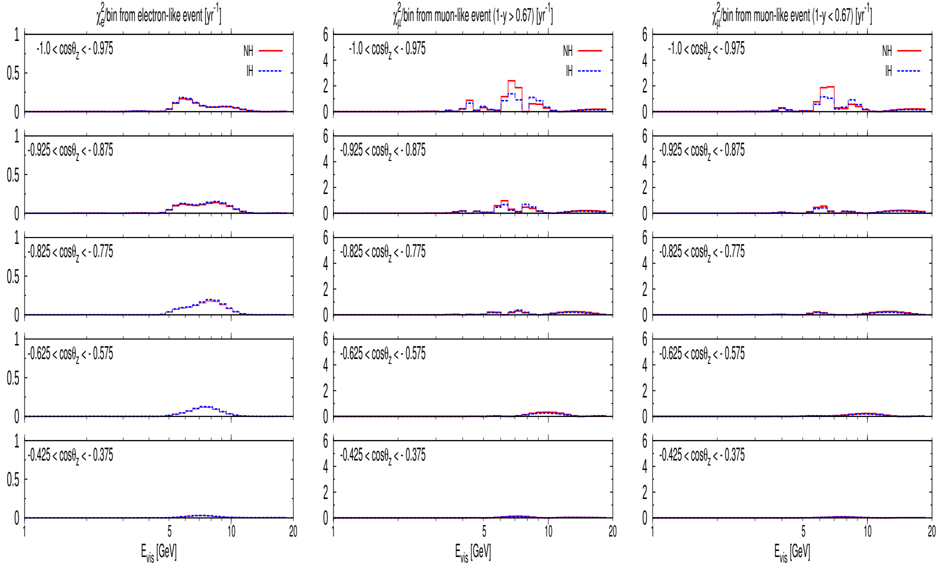
Events Rates with Detector Smearing



MH Sensitivity Distribution with Scattering



MH Sensitivity Distribution with Smearing



MH Sensitivity with μ or μ+e ($E > 4GeV$ & $\cos\theta_z < -0.4$)

$\Delta\chi^2_{\min}$	NH (true)			IH (true)		
x_a (true)	-0.2	0	$+0.2$	-0.2	0	$+0.2$
ν	166 257	175 215	142 169	101 144	110 141	102 120
Scattering	26.4 67.3	13.2 32.2	10.2 21.3	14.9 21.2	12.7 27.9	10.1 21.4
Split μ ($1 - y \gtrsim 0.67$)	35.9 76.6	19.4 38.6	13.7 24.8	22.4 28.7	18.4 33.7	13.4 24.8
Smearing	29.4 54.7	14.6 23.5	10.0 12.7	15.8 16.0	13.3 20.2	9.7 13.7
$\Delta E = (0.2 \pm 0.03)\sqrt{E}$	28.9 52.4	14.2 23.1	9.7 12.6	15.8 16.0	13.3 19.9	9.7 13.5
$\Delta\theta = (15^\circ \pm 3^\circ)E^{-0.6}$	29.2 54.4	14.5 23.4	9.9 12.7	15.8 16.0	13.1 20.2	9.6 13.7
μ mis-ID ($10\% \pm 2\%$)	19.0 45.1	14.4 22.1	8.1 11.1	15.8 15.9	13.3 17.2	8.3 9.8
Normalization (1 ± 0.05)	16.8 54.6	10.4 23.5	7.6 12.7	14.6 16.0	10.9 19.8	7.9 13.7

$\mathbf{x_a}$ Uncertainty with μ or μ+e ($E > 4GeV$ & $\cos\theta_z < -0.4$)

$\Delta(x_a)$	NH (true)			IH (true)		
x_a (true)	-0.2	0	$+0.2$	-0.2	0	$+0.2$
ν	0.014 0.012	0.036 0.025	0.011 0.010	0.014 0.012	0.034 0.033	0.011 0.011
Scattering	0.023 0.019	0.051 0.037	0.015 0.014	0.023 0.021	0.068 0.059	0.017 0.016
Split μ ($1 - y \gtrsim 0.67$)	0.022 0.018	0.050 0.037	0.015 0.014	0.022 0.020	0.065 0.057	0.016 0.016
Smearing	0.024 0.020	0.054 0.042	0.016 0.015	0.024 0.023	0.071 0.062	0.018 0.018
$\Delta E = (0.2 \pm 0.03)\sqrt{E}$	0.025 0.020	0.054 0.042	0.016 0.016	0.025 0.023	0.071 0.063	0.018 0.018
$\Delta\theta = (15^\circ \pm 3^\circ)E^{-0.6}$	0.025 0.020	0.055 0.042	0.016 0.015	0.024 0.023	0.071 0.063	0.018 0.018
μ mis-ID ($10\% \pm 2\%$)	0.025 0.022	0.054 0.042	0.016 0.015	0.026 0.023	0.072 0.063	0.018 0.018
Normalization (1 ± 0.05)	0.024 0.021	0.078 0.042	0.022 0.016	0.025 0.024	0.079 0.071	0.023 0.018

Octant Sensitivity with μ or μ+e ($E > 4GeV$ & $\cos\theta_z < -0.4$)

$\Delta\chi^2_{\min}$	NH (true)				IH (true)			
x_a (true)	-0.2	-0.1	$+0.1$	$+0.2$	-0.2	-0.1	$+0.1$	$+0.2$
ν	25.3 / 63.3	5.2 / 9.2	9.4 / 18.9	29.7 / 94.7	5.4 / 16.7	1.4 / 3.8	2.0 / 6.2	6.3 / 23.5
Scattering	24.4 / 36.9	3.1 / 6.2	9.6 / 13.2	47.7 / 103.0	7.5 / 15.6	1.8 / 3.3	2.8 / 5.5	9.9 / 21.4
Split μ ($1-y \gtrless 0.67$)	25.9 / 38.7	3.4 / 6.4	10.2 / 13.8	49.5 / 105.4	8.9 / 16.9	2.1 / 3.3	3.3 / 6.0	11.3 / 22.9
Smearing	21.6 / 31.1	2.8 / 4.9	8.5 / 10.8	42.3 / 80.7	7.2 / 12.9	1.7 / 2.7	2.7 / 4.5	9.4 / 17.2
$\Delta E = (0.2 \pm 0.03)\sqrt{E}$	21.6 / 31.1	2.8 / 4.9	8.3 / 10.4	41.6 / 77.3	7.2 / 12.8	1.7 / 2.7	2.6 / 4.4	9.3 / 16.7
$\Delta\theta = (15^\circ \pm 3^\circ)E^{-0.6}$	21.5 / 31.0	2.7 / 4.9	8.3 / 10.7	41.3 / 80.6	7.2 / 12.9	1.7 / 2.7	2.6 / 4.5	9.2 / 17.2
μ mis-ID (10% $\pm$ 2%)	21.6 / 31.1	2.8 / 4.9	5.8 / 9.9	42.3 / 80.7	5.1 / 10.2	1.7 / 2.7	1.8 / 1.6	9.4 / 17.2
Normalization (1 ± 0.05)	11.8 / 30.7	2.2 / 4.9	3.7 / 10.6	15.3 / 80.7	4.1 / 12.9	1.0 / 2.7	1.1 / 4.5	4.2 / 17.2

Closing:

- Proton decay remains the most definitive test of GUT, the unification of quarks and leptons.

  $\rightarrow$ Tera-scale SUSY predicts GUT gauge-boson mediated proton lifetime of 10^{37} years.
  $\rightarrow$ Any life time between the present bound and 10^{37} years is possible in SUSY GUT, depending on the doublet-triplet splitting mechanisum.
  $\rightarrow$ In scenarios without Tera-scale SUSY, the proton can still decay, but its lifetime may be much longer.

- Majorana neutrino mass via the dimension 5 operator can be the first definitive physics beyond the SM, which may tell us the physics scale of the singlet sector below the GUT scale.

  $\rightarrow$ Precision measurements of $|m_{\beta\beta}|$ can reveal CP violation in the lepton-number violating sector, for which the precision measurements of all the neutrino oscillation parameters, the mass hierarchy, the mixing angles, and the CP phase δ are required, in addition to the independent measurement of the sum $m_1+m_2+m_3$ from Cosmology.

Closing (continued): I introduced three recent works on possible neutrino oscillation experiments in the near future.

- Intermediate baseline reactor anti-neutrino oscillation experiments like DayaBay2 and RENO2 can
  (1) measure $\sin^2\theta_{12}$, $m_2^2 - m_1^2$, and $|m_3^2 - m_1^2|$ very accurately.
  (2) may determine the mass hierarchy with fine energy resolution $(dE/E)^2 \lessapprox (0.03/\sqrt{E/\mathrm{MeV}})^2 + (0.0075)^2$

- T2K+Korea and/or Oki is a very cost effective one-beam two-detector LBL neutrino oscillation experiment, which can
  (1) determine the mass hierarchy
  (2) measure δ_{MNS}
  (3) and may resolve the octant $\cos^2\theta_{23} - \sin^2\theta_{23} > 0$ vs < 0

- Atmospheric neutrinos produced in the other side of the earth can be studied in detail at a huge underground detector such as PINGU in ICECUBE , which can
  (1) determine the mass hierarchy
  (2) resolve the octant $\cos^2\theta_{23} - \sin^2\theta_{23} > 0$ vs < 0
  (3) but to measure δ_{MNS} may be challenging.