２０１８年度研究報告

公募研究
「原始ブラックホール形成過程の精査と
その観測的検証」(18H04356)

計画研究A01
「インフレーション宇宙」(15H05888)
連携研究者

Shuichiro Yokoyama (KMI, Nagoya University)
(横山　修一郎 (KMI, 名古屋大))

List of submitted papers

- S. Saga, H. Tashiro, S. Yokoyama

  "Limits on primordial magnetic fields from direct detection experiments of gravitational background" , Phys. Rev. D 98, no.8, 083518(2018), 1807.00561

- T. Sekiguchi, T. Takahashi, H. Tashiro, S. Yokoyama

  "Probing primordial non-Gaussianity with 21cm fluctuations from minihalos", JCAP 1902 (2019) 033, 1807.02008

- T. Hiramatsu, S. Yokoyama, T. Fujita, I. Obata ➔ by Hiramatsu-san (Mar. 5th)

  "Hunting for statistical anisotropy in tensor modes with B-mode observations", Phys. Rev. D 98, no.8, 083522 (2018), 1808.08044

- H. Niikura, M. Takada, S. Yokoyama, T. Sumi, S. Masaki ➔ by Takada-san (Mar. 7th)

  "Earth-mass black holes? – Constraints on primordial black holes with 5-years OGLE microlensing events", accepted in PRD, 1901.07120

- S. Hirano, T. Kobayashi, D. Yamauchi, S. Yokoyama ➔ by Hirano-san (Mar. 8th)

  "Constraining DHOST theories with linear growth of matter density fluctuations", 1902.02946

Clustering of primordial BHs

Shuichiro Yokoyama（KMI, Nagoya Univ.）

with Teruaki Suyama (TITECH)

in preparation (arXiv: 1903.xxxxx?)

Brief intro. for PBH

Hawking (1971)
Carr and Hawking (1974), …

(also Zeldovich and Novikov (1967))

- Primordial Black Hole (PBH)
  - ✓ BHs formed in the early Universe (after inflation)
  - ✓ direct gravitational collapse of a overdense region
    (formation of a closed Universe) Sasaki-san's talk
  - ✓ mass of formed BH ~ Hubble horizon mass at the formation
    (We focus on the PBH formed in the radiation-dominated era)

$$M = \gamma M_{\rm PH} = \frac{4\pi}{3}\gamma \rho H^{-3} \approx 2.03 \times 10^5 \gamma \left(\frac{t}{1\ {\rm s}}\right) M_\odot.$$

$$t \approx 0.738 \left(\frac{g_*}{10.75}\right)^{-1/2} \left(\frac{T}{1\ {\rm MeV}}\right)^{-2}\ {\rm s},$$

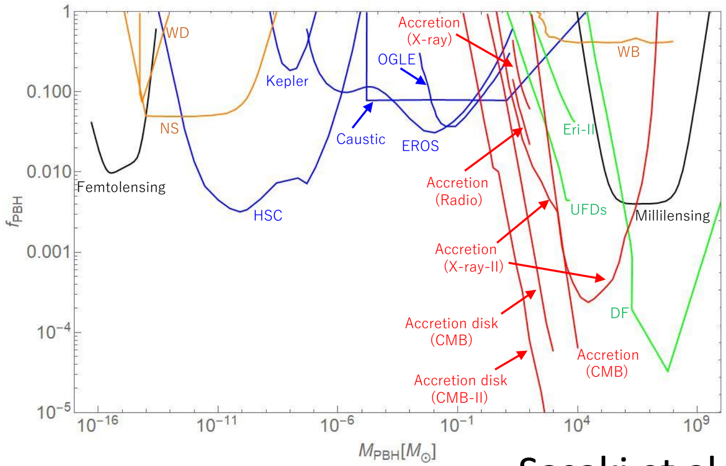
Various mass BHs could be formed.

(about PBH formation in matter dominated era, ➔ Kohri-san's talk)

Why PBH?

✓a candidate of dark matter

$$M > 10^{15}\,\mathrm{g}(\sim 10^{-18}\,M_\odot)$$



Sasaki et al. (2018)

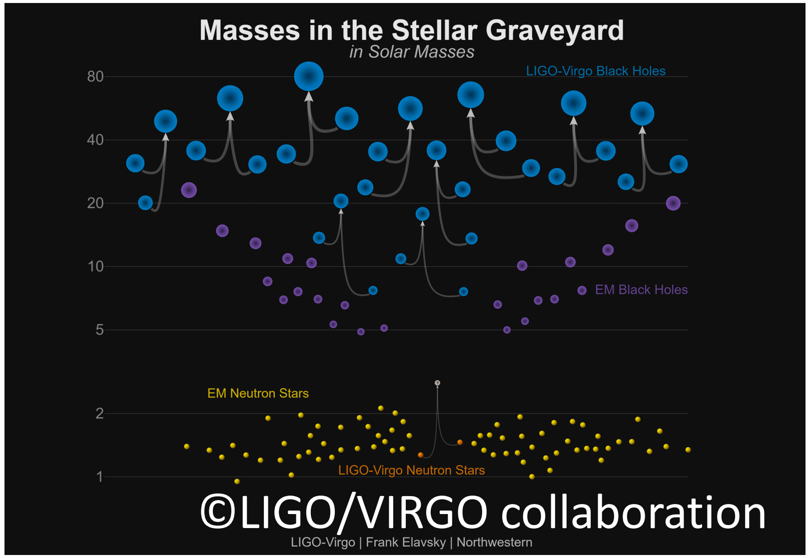
✓a "probe" of inflation model

➔ Tada-san's talk (Mar. 7th)

✓a source of LIGO events

$$M \sim 10\,M_\odot$$

Nakamura et al.(1997),
Sasaki et al. (2016), Bird et al. (2016), …



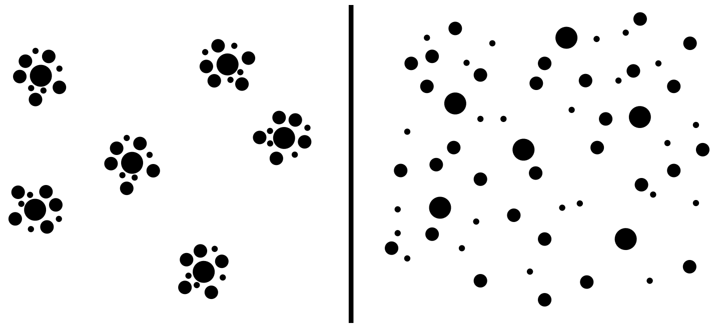
Why clustering?

- "clustering"

  = spatial distribution of PBHs

We focus on PBH formation during radiation dominated era, ..

➔ Spatial distribution of PBHs on super-Hubble scales at the formation



Ali-Haimoud (2018)

✓ DM isocurvature fluctuations

Tada, SY (2015), Young, Byrnes (2015), ...

✓ Event rate of PBH binary mergers

Raidal et al. (2017), Bringmann et al. (2018), ...

✓ (additional adiabatic pert.??)

related to the Hawking radiation...

This work

✓ super-Hubble spatial distribution of PBHs

➔ 2-point correlation function / power spectrum of PBH distribution

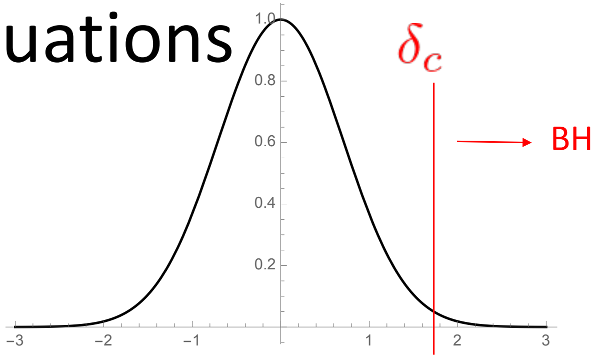
$\xi_{\mathrm{PBH}}(\boldsymbol{x}_1, \boldsymbol{x}_2)$ $\qquad$ $P_{\mathrm{PBH}}(k)$

for $|\boldsymbol{x}_1 - \boldsymbol{x}_2| \gg R$ $\qquad$ for $k\, R \ll 1$

R; comoving Hubble scale at the formation

✓ statistical property of primordial fluctuations

Gaussian or non-Gaussian?

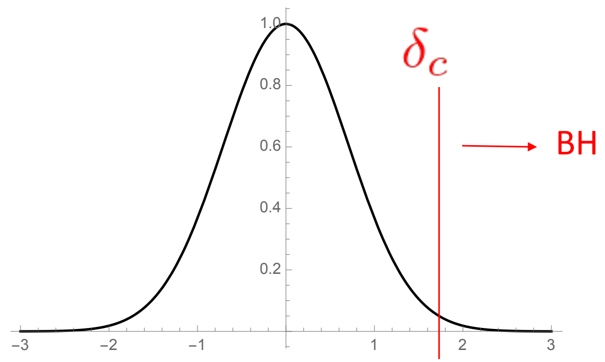


We employ "functional integration approach" (or peak formalism).

See, e.g. Matarrese, Luccin, Bonometto (1986) in the context of halo formation

Chisholm (2006), Ali-Haimoud (2018), Franciolini et al. (2018), … for PBH

Formulation 1

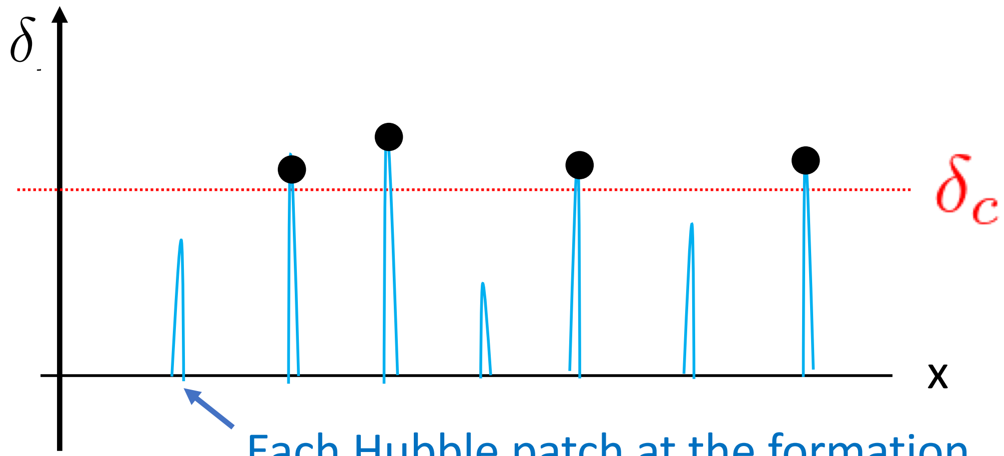


- Probability that a point (region) "x" becomes PBH

$$P_1(\boldsymbol{x}) = \int [D\delta] P[\delta] \int_{\delta_c}^{\infty} d\alpha\, \delta_D(\delta_{\rm local}(\boldsymbol{x}) - \alpha)$$

Probability Distribution Function of primordial fluctuations, $\delta(\boldsymbol{x})$

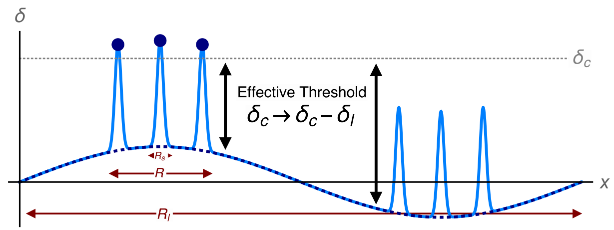
"local" smoothed fluctuations



"separate Universe picture"

see e.g., Young, Byrnes, Sasaki (2014)

cf. Halo case -> linear bias



Formulation 2

- Probability that two points (regions) “x1” and “x2” become PBHs

$$P_2(\boldsymbol{x}_1,\,\boldsymbol{x}_2) = \int [D\delta] P[\delta] \int_{\delta_c}^{\infty} d\alpha_1\, \delta_D(\delta_{\rm local}(\boldsymbol{x}) - \alpha_1) \int_{\delta_c}^{\infty} d\alpha_2\, \delta_D(\delta_{\rm local}(\boldsymbol{x}) - \alpha_2)$$

➔ 2 point correlation function;

$$\xi_{\rm PBH}(\boldsymbol{x}_1, \boldsymbol{x}_2) := \frac{P_2(\boldsymbol{x}_1, \boldsymbol{x}_2)}{P_1^2} - 1$$

Roughly, 1

$$\delta_{\rm local}(\boldsymbol{x}) = \int d^3y W_{\rm local}(\boldsymbol{x}-\boldsymbol{y})\delta(\boldsymbol{y})$$

$$P_1(\boldsymbol{x}) = \int [D\delta]P[\delta] \int_{\delta_c}^{\infty} d\alpha\, \delta_D(\delta_{\rm local}(\boldsymbol{x}) - \alpha)$$

$$\delta_D(x) = \int \frac{d\phi}{2\pi} e^{i\phi x},$$

$$P_1(\boldsymbol{x}) = \int [D\delta]P[\delta] \int_{\delta_c}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp\left[i\phi \int d^3y W_{\rm local}(\boldsymbol{x}-\boldsymbol{y})\delta(\boldsymbol{y}) - i\phi\,\alpha\right]$$

$$Z[J] := \int [D\delta]P[\delta] \exp\left[i\int d^3y J(\boldsymbol{y})\delta(\boldsymbol{y})\right] = \langle \exp\left[i\int d^3y J(\boldsymbol{y})\delta(\boldsymbol{y})\right]\rangle$$

$$\log Z[J] = \sum_{n=1}^{\infty} \frac{i^n}{n!} \int d^3y_1 d^3y_2 \cdots d^3y_n\, \xi_{\delta(c)}(\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_n)\, J(\boldsymbol{y}_1)J(\boldsymbol{y}_2)\cdots J(\boldsymbol{y}_n)$$

correlation function
of primordial fluctuations

$$P_1(\boldsymbol{x}) = \int_{\delta_c}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp\left[-i\phi\,\alpha\right] \exp\left[\sum_{n=2}^{\infty} \frac{i^n}{n!}\phi^n\, \xi^{(n)}_{{\rm local}(c)}\right]$$

moments

Roughly, 1

$$P_1(\boldsymbol{x}) = \int_{\delta_c}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp\left[-i\phi\,\alpha\right] \exp\left[\sum_{n=2}^{\infty} \frac{i^n}{n!} \phi^n \, \underline{\xi^{(n)}_{\mathrm{local}(c)}}\right]$$

moments

P_2 can be also reduced:

$$P_2(\boldsymbol{x}_1,\, \boldsymbol{x}_2) = \int_{\delta_c}^{\infty} d\alpha_1 \int_{\delta_c}^{\infty} d\alpha_2 \int_{-\infty}^{\infty} \frac{d\phi_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\phi_2}{2\pi} \exp\left[-i\phi_1\,\alpha_1 - i\phi_2\,\alpha_2\right]$$

$$\times \exp\left[\sum_{n=2}^{\infty} i^n \sum_{m=0}^{n} \frac{\phi_1^m \phi_2^{n-m}}{m!(n-m)!} \underline{\xi^{(n)}_{\mathrm{local}(c)}(\underbrace{\boldsymbol{x}_1, \boldsymbol{x}_1, \cdots, \boldsymbol{x}_1}_{\text{total } m}, \underbrace{\boldsymbol{x}_2, \boldsymbol{x}_2, \cdots \boldsymbol{x}_2}_{\text{total } n-m})}\right]$$

-

➔ Two-point correlation between moments

variance, skewness, kurtosis, …

Roughly, 2

$$P_1(\boldsymbol{x}) = \int_{\delta_c}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp\left[-i\phi\,\alpha\right] \exp\left[\sum_{n=2}^{\infty} \frac{i^n}{n!} \phi^n\, \xi^{(n)}_{\mathrm{local}(c)}\right]$$

integration with high-peak approx.   expand with weak non-Gaussianity approx.

$\nu := \delta_c/\sigma_{\mathrm{local}} \gg 1$ where $\sigma^2_{\mathrm{local}} := \xi^{(2)}_{\mathrm{local}(c)}$

$$\rightarrow \quad P_1 \approx \frac{e^{-\nu^2/2}}{\sqrt{2\pi}\nu}\left[1 + \sum_{n=3}^{\infty} \frac{1}{2^{n/2} n!} \frac{\xi^{(n)}_{\mathrm{local}(c)}}{\sigma^n_{\mathrm{local}}} H_n\left(\frac{\nu}{\sqrt{2}}\right)\right]$$

Hermite polynomials

P_2 can be reduced in the same way..

Finally,

$$\xi_{\mathrm{PBH}}(\boldsymbol{x}_1, \boldsymbol{x}_2) := \frac{P_2(\boldsymbol{x}_1, \boldsymbol{x}_2)}{P_1^2} - 1$$

$$\sim \frac{\nu^2}{\sigma^2_{\mathrm{local}}} \xi^{(2)}_{\mathrm{local}(c)}(\boldsymbol{x}_1, \boldsymbol{x}_2) + \frac{1}{2}\frac{\nu^3}{\sigma^3_{\mathrm{local}}}\left(\xi^{(3)}_{\mathrm{local}(c)}(\boldsymbol{x}_1, \boldsymbol{x}_1, \boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$

$$+ \frac{1}{4}\frac{\nu^4}{\sigma^4_{\mathrm{local}}} \xi^{(4)}_{\mathrm{local}(c)}(\boldsymbol{x}_1, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_2) + \frac{1}{6}\frac{\nu^4}{\sigma^4_{\mathrm{local}}}\left(\xi^{(4)}_{\mathrm{local}(c)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_2, \boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$

up to the 4-point, tree-level

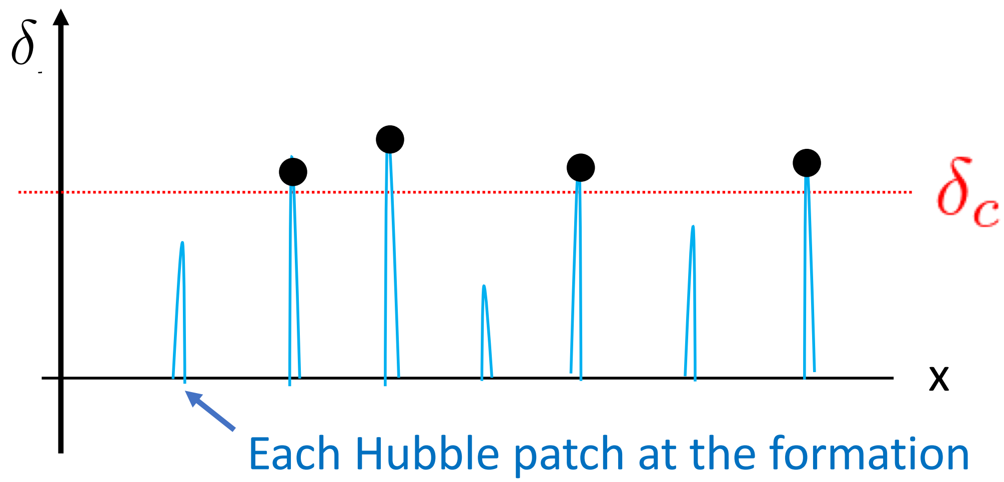
PBH correlation function

SY, Suyama in prep.

Up to the primordial 4-point corr.

$$\xi_{\rm PBH}(\boldsymbol{x}_1,\boldsymbol{x}_2) \sim \frac{\nu^2}{\sigma_{\rm local}^2}\xi^{(2)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2) + \frac{1}{2}\frac{\nu^3}{\sigma_{\rm local}^3}\left(\xi^{(3)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$
$$+\frac{1}{4}\frac{\nu^4}{\sigma_{\rm local}^4}\xi^{(4)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) + \frac{1}{6}\frac{\nu^4}{\sigma_{\rm local}^4}\left(\xi^{(4)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$

For $|\boldsymbol{x}_1 - \boldsymbol{x}_2| \gg R$



PBH correlation function

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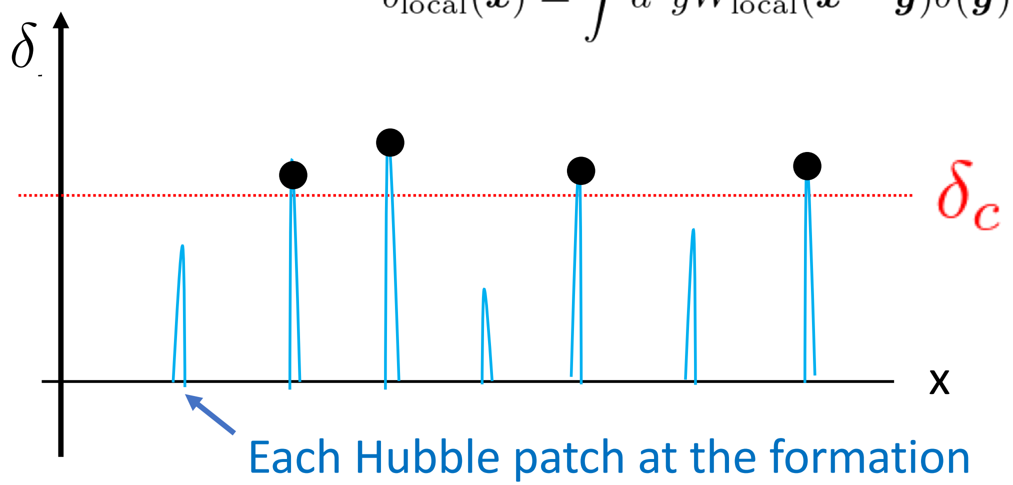
$$\xi_{\rm PBH}(\boldsymbol{x}_1,\boldsymbol{x}_2) \sim \frac{\nu^2}{\sigma_{\rm local}^2}\xi^{(2)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2) + \frac{1}{2}\frac{\nu^3}{\sigma_{\rm local}^3}\left(\xi^{(3)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$
$$+\frac{1}{4}\frac{\nu^4}{\sigma_{\rm local}^4}\xi^{(4)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) + \frac{1}{6}\frac{\nu^4}{\sigma_{\rm local}^4}\left(\xi^{(4)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$

For $|\boldsymbol{x}_1 - \boldsymbol{x}_2| \gg R$ → 0

$$\delta_{\rm local}(\boldsymbol{x}) = \int d^3y W_{\rm local}(\boldsymbol{x}-\boldsymbol{y})\delta(\boldsymbol{y})$$



For Gaussian fluctuations,

can never expect the PBH clustering !

Chisholm (2006),
Ali-Haimoud (2018),

PBH correlation function with NG

SY, Suyama in prep.

Up to the primordial 4-point corr.

$$\xi_{\rm PBH}(\boldsymbol{x}_1,\boldsymbol{x}_2) \sim \frac{\nu^2}{\sigma^2_{\rm local}}\xi^{(2)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2) + \frac{1}{2}\frac{\nu^3}{\sigma^3_{\rm local}}\left(\xi^{(3)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$
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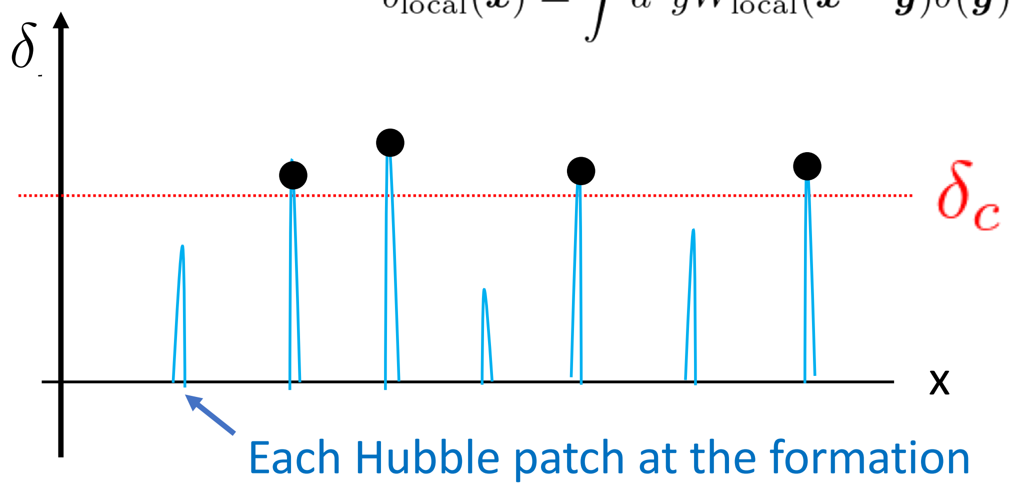
For $|\boldsymbol{x}_1 - \boldsymbol{x}_2| \gg R$

0

$$\langle \sigma^2_{\rm local}\delta_{\rm local}\rangle \to 0$$

$$\delta_{\rm local}(\boldsymbol{x}) = \int d^3y W_{\rm local}(\boldsymbol{x}-\boldsymbol{y})\delta(\boldsymbol{y})$$



PBH correlation function with NG

SY, Suyama in prep.

Up to the primordial 4-point corr.

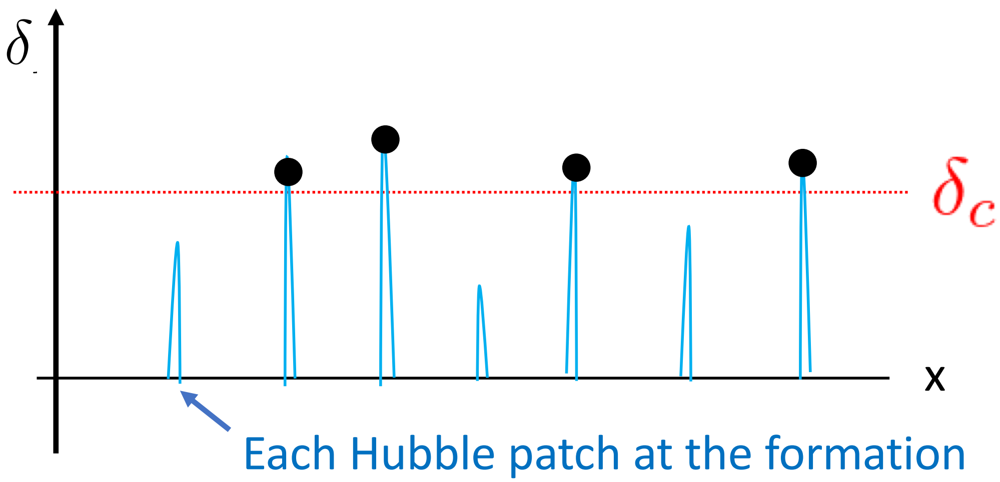
$$\xi_{\rm PBH}(\boldsymbol{x}_1,\boldsymbol{x}_2) \sim \frac{\nu^2}{\sigma^2_{\rm local}}\xi^{(2)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2) + \frac{1}{2}\frac{\nu^3}{\sigma^3_{\rm local}}\left(\xi^{(3)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$
$$+\frac{1}{4}\frac{\nu^4}{\sigma^4_{\rm local}}\xi^{(4)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) + \frac{1}{6}\frac{\nu^4}{\sigma^4_{\rm local}}\left(\xi^{(4)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$

For $|\boldsymbol{x}_1 - \boldsymbol{x}_2| \gg R$

0

$\langle \sigma^2_{\rm local}\delta_{\rm local}\rangle \to 0$

$\langle ({\rm skewness})_{\rm local}\,\delta_{\rm local}\rangle \to 0$



PBH correlation function

SY, Suyama in prep.

Up to the primordial 4-point corr.

$$\xi_{\rm PBH}(\boldsymbol{x}_1,\boldsymbol{x}_2) \sim \frac{\nu^2}{\sigma^2_{\rm local}}\xi^{(2)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2) + \frac{1}{2}\frac{\nu^3}{\sigma^3_{\rm local}}\left(\xi^{(3)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$

$$+\frac{1}{4}\frac{\nu^4}{\sigma^4_{\rm local}}\xi^{(4)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) + \frac{1}{6}\frac{\nu^4}{\sigma^4_{\rm local}}\left(\xi^{(4)}_{{\rm local}(c)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2,\boldsymbol{x}_2) + (\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2)\right)$$

For $|\boldsymbol{x}_1 - \boldsymbol{x}_2| \gg R$

0

$\langle \sigma^2_{\rm local}\delta_{\rm local}\rangle \to 0$

$\langle ({\rm skewness})_{\rm local}\,\delta_{\rm local}\rangle \to 0$

$\langle \sigma^2_{\rm local}\,\sigma^2_{\rm local}\rangle \to\ ?$

Can we realize non-zero value??

2-point correlation of $\sigma^2_{\rm local}$

As primordial fluctuations, $\delta(\boldsymbol{x})$,

comoving curvature perturbations

comoving density fluctuations: $\delta(\boldsymbol{x}) = \Delta(\boldsymbol{x}) = -\frac{4}{9} R^2 \nabla^2 \mathcal{R}_c(\boldsymbol{x})$

comoving Hubble at the formation

Assuming "local-type" non-Gaussianity,

$$\mathcal{R}_c(\boldsymbol{x}) = \mathcal{R}_{c,G}(\boldsymbol{x}) + \frac{3}{5} f_{\rm NL} \left(\mathcal{R}_{c,G}(\boldsymbol{x})^2 - \langle \mathcal{R}^2_{c,G} \rangle \right)$$

$$\Rightarrow \quad \Delta(\boldsymbol{x}) \simeq \left(1 + \frac{6}{5} f_{\rm NL} \mathcal{R}_{c,G}(\boldsymbol{x}) \right) \Delta_G(\boldsymbol{x}),$$

$$\Rightarrow \quad \sigma^2_{\rm local}(\boldsymbol{x}) = \left(1 + \frac{6}{5} f_{\rm NL} \mathcal{R}_c(\boldsymbol{x}) \right)^2 \sigma^2_{\rm local}$$

large scale fluctuations of $\sigma^2_{\rm local}$

2-point correlation of $\sigma^2_{\rm local}$

As primordial fluctuations, $\delta(\boldsymbol{x})$,

comoving curvature perturbations

comoving density fluctuations: $\delta(\boldsymbol{x}) = \Delta(\boldsymbol{x}) = -\frac{4}{9} R^2 \nabla^2 \mathcal{R}_c(\boldsymbol{x})$

comoving Hubble at the formation

Assuming "local-type" non-Gaussianity,

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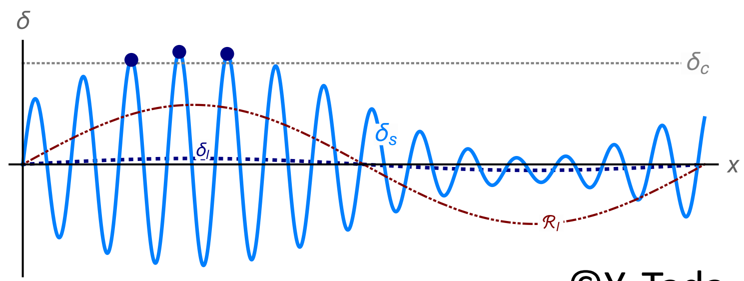
➔ $$\Delta(\boldsymbol{x}) \simeq \left(1 + \frac{6}{5} f_{\rm NL} \mathcal{R}_{c,G}(\boldsymbol{x}) \right) \Delta_G(\boldsymbol{x}).$$

➔ $$\sigma^2_{\rm local}(\boldsymbol{x}) = \left(1 + \frac{6}{5} f_{\rm NL} \mathcal{R}_c(\boldsymbol{x}) \right)^2 \sigma^2_{\rm local}$$

large scale fluctuations of $\sigma^2_{\rm local}$



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Primordial 4-point → PBH clustering

For local type non-Gaussianity,

$$\xi_{\rm PBH}(\boldsymbol{x}_1, \boldsymbol{x}_2) \approx \nu^4 \left(\frac{6}{5} f_{\rm NL} \right)^2 \xi_{\mathcal{R}_{c,G}}(\boldsymbol{x}_1, \boldsymbol{x}_2)$$

more general $= \nu^4 \, \tau_{\rm NL} \, \xi_{\mathcal{R}_c}(\boldsymbol{x}_1, \boldsymbol{x}_2)$

In Fourier space, $P_{\rm PBH}(k) \approx \nu^4 \, \tau_{\rm NL} \, P_{\mathcal{R}_c}(k)$

$$= 10^7 \times \left(\frac{\nu}{10} \right)^4 \left(\frac{\tau_{\rm NL}}{10^3} \right) P_{\mathcal{R}_c}(k) \quad \text{for} \quad k\,R \ll 1$$

- ✓ large DM isocurvature perturbations on CMB scales !?

  Tada, SY (2015), Young, Byrnes (2015), …

- ✓ large modification of event rate for PBH binary mergers !?

  Raidal et al. (2017), Bringmann et al. (2018), …

Summary

- We investigate the clustering of PBHs
- derive PBH 2-point correl. func. on large scales
- clustering of PBHs could be never induced for Gaussian fluctuations
- Trispectrum (4-point correlation) should be important !

➔ future issues…

- ✓ How about the primordial non-Gaussianity
in the inflationary models which can generate PBHs ??
- ✓ Effect on the event rate estimation of PBH binary mergers