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# Extended Cuscuton

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Based on: Aya Iyonaga (彌永亜矢), Kazufumi Takahashi (高橋一史) , and TK
JCAP 1812 (2018) no.12, 002 [arXiv:1809.10935]

# Motivation

- Modifying gravity = Adding new degrees of freedom
  - General Relativity:
    **2** ( $h_{ij}^{+}, h_{ij}^{\times}$ ) DOFs
  - Scalar-tensor theories:
    **2** ( $h_{ij}^{+}, h_{ij}^{\times}$ ) + **1** ($\phi$) DOFs
- ***Modified gravity <u>without</u> extra DOFs?***
  - "Minimal" modification of gravity breaking 4D diffeo.

Lin & Mukohyama 1708.03757; Aoki & Mukohyama 1804.03902; Aoki *et al.* 1810.01047

  - *Cuscuton* – scalar field having preferred foliation

Afshordi *et al.* hep-th/0609150

# Talk Plan

- Motivation
- What is *Cuscuton*?
- Extended Cuscuton
- Perturbations in the presence of matter
- Summary

# What is Cuscuton?

Afshordi *et al.* hep-th/0609150

$$\mathcal{L} = \frac{\mathcal{R}}{2} + P(\phi, X), \quad P = \mu^2 \sqrt{-(\partial\phi)^2} - V(\phi)$$

— $c_s^2 \to \infty$ limit of *k*-essence

- $\phi$'s EOM in FRW universe ⟶ First-order equation

$$3\mu^2 H + V_{,\phi} = 0$$

- Cosmological perturbations (without any other matter)
  - Tensor $\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\Delta h_{ij} = 0$
  - Scalar $\Delta\zeta = 0$ Scalar mode is non-dynamical
- Gravity is modified in the presence of other matter

Afshordi *et al.* astro-ph/0702002

# What is Cuscuton?

Afshordi *et al.* hep-th/0609150

$$\mathcal{L} = \frac{\mathcal{R}}{2} + P(\phi, X), \quad P = \mu^2 \sqrt{-(\partial\phi)^2} - V(\phi)$$

— $c_s^2 \to \infty$ limit of *k*-essence

- Hamiltonian analysis in the unitary gauge ($\phi = \phi(t)$)
  ($\partial_\mu \phi$ is forced to be timelike!)

  ⟶ Cuscuton field is non-dynamical

  Gomes & Guariento 1703.08226
  Lin & Mukohyama 1708.03757

- Something similar to "U-degenerate" theories

  ⟶ appear to have extra mode in arbitrary gauge,
  but it does not propagate

  De Felice *et al.* 1803.0624

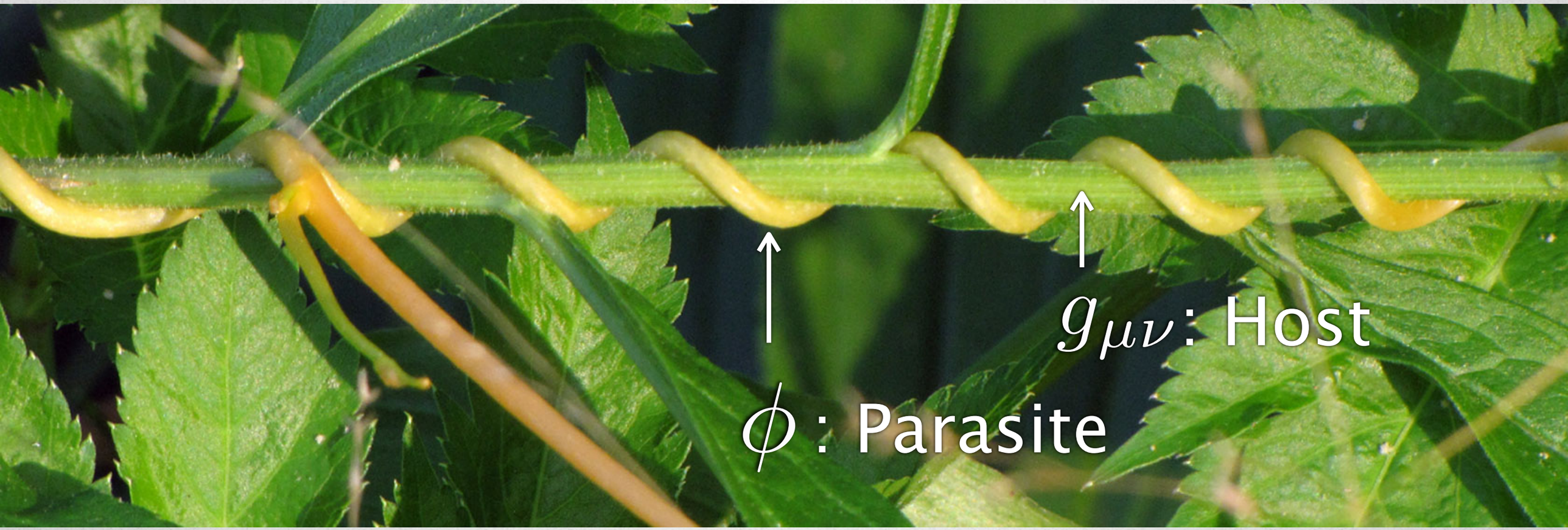
- Something similar to Horava-Lifshitz gravity

  Afshordi 0907.5201
  Bhattacharyya *et al.* 1612.01824

# *Cuscuton* is named after *Cuscuta* (parasitic plants)

Afshordi *et al*. hep-th/0609150

ネナシカズラ
[つる性の寄生植物]



Iyonaga, Takahashi, TK 1809.10935

# Extending Cuscuton

- Cuscuton-like theories from Horndeski/beyond Horndeski
  - Starting point: "beyond Horndeski" scalar-tensor theories

$$\mathcal{L} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)\mathcal{R} + G_{4,X}\left[(\Box\phi)^2 - \cdots\right] + G_5(\phi, X)\cdots$$
$$+ F_4(\phi, X)\left\{-2X\left[(\Box\phi)^2 - \cdots\right] + \cdots\right\} + F_5(\phi, X)\cdots$$

**6 free functions**

$X := -(\partial\phi)^2/2$

Horndeski (1974); Gleyzes *et al.* 1404.6495

## 3-step construction

- 1. Determine possible form of functions by inspecting the structure of dynamical equations for FRW background
- 2. Check that $\zeta$ (scalar perturbation) is non-dynamical
- 3. Check that Hamiltonian analysis in unitary gauge gives dynamical DOFs less than 3

# Step 1: Dynamical equations for FRW

- *ij*-component of gravitational field equations:

$$A(\phi, \dot{\phi}, H)\dot{H} + B(\phi, \dot{\phi}, H)\ddot{\phi} + \cdots = 0$$

- Scalar-field equation:

$$C(\phi, \dot{\phi}, H)\dot{H} + D(\phi, \dot{\phi}, H)\ddot{\phi} + \cdots = 0$$

- # of DOFs reduces if these equations are *degenerate*:

$$AD - BC = 0$$

spatial curvature

$$= \sum_{n=0}^{4} a_n(\phi, X)H^n + a_5(\phi, X)\frac{\mathcal{K}}{a^2} + a_6(\phi, X)\frac{\mathcal{K}^2}{a^4} + a_7(\phi, X)H\frac{\mathcal{K}}{a^2} + a_8(\phi, X)H^2\frac{\mathcal{K}}{a^2}$$

⟶ 8 differential equations w.r.t. $X$

$$a_n(\phi, X) = 0, \quad a_n \supset G_2, G_{2,X}, \cdots$$

# Remaining checkpoints: Steps 2 & 3

- Extended Cuscuton:

$$\mathcal{L} = -V(\phi) + U(\phi)\sqrt{X} + \cdots + \left[f(\phi) + h(\phi)\sqrt{X}\right]\mathcal{R} + \cdots$$

  - Several free functions of $\phi$
  - Much more compact expression in ADM language

**Can check that**

- ☑ $\zeta$ is non-dynamical: $\Delta\zeta = 0$
- ☑ Hamiltonian analysis in unitary gauge:
  No 3rd DOF even away from FRW

# Extended Cuscuton – Example

- The most general Lagrangian for extended cuscuton satisfying $c_{\rm GW}^2 = 1$ (c.f. GW170817)

$$\mathcal{L} = f(\phi)\mathcal{R} - V(\phi) + U(\phi)\sqrt{X} - 4\left(g' + f'' + \frac{3g^2}{4f}\right)X$$
$$+ 2(g' + f'')X\ln X - [g(\phi) + f']\ln X\Box\phi$$

# Perturbations in the presence of matter

Iyonaga, Takahashi, TK 1809.10935

- $\mathcal{L} = \mathcal{L}_{\rm cuscuton} + P(Y), \quad Y := -\frac{1}{2}(\partial\chi)^2$ $\qquad g_{ij} = a^2 e^{2\psi}\delta_{ij}$
  - *k*-essence as perfect fluid
- Quadratic Lagrangian for $\zeta := \psi - \left(\frac{G_4 H + \cdots}{G_4 - 2XG_{4,X} + \cdots}\right)\frac{\delta\chi}{\dot\chi}$

$$\mathcal{L}^{(2)}_\zeta = a^3\left[\mathcal{A}\dot\zeta^2 - \mathcal{B}\frac{(\partial\zeta)^2}{a^2}\right]$$

$$\mathcal{A} := (\cdots)\frac{\Delta/a^2 - \alpha_1}{\Delta/a^2 - \alpha_2}, \quad \mathcal{B} := (\cdots)\frac{\Delta^2/a^4 - \beta_1\Delta/a^2 + \beta_2}{(\Delta/a^2 - \alpha_2)^2}$$

- Non-standard dispersion relation may lead to interesting cosmology

Boruah *et al.* 1704.01131; Boruah *et al.* 1802.06818

# Summary

- *Cuscuton* is an extreme case of (Einstein-Hilbert +) k-essence having 2 DOFs

- *Extended Cuscuton*: extreme case of Horndeski/beyond Horndeski theories having 2 DOFs
  - (Would-be) dynamical equations for scalar and metric are degenerate
  - Gravity is modified (non-standard dispersion relation etc.)
  - May lead to interesting cosmology (currently ongoing)

  See also Ito, Iyonaga, Kim & Soda 1902.08663