Pure Natural Inflation

C01

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Cosmic inflation

— Homogeneity

— Flatness

— Heavy relics (e.g. monopoles)

— Origin of the structure (fluctuation)

— Beginning of spacetime

— …

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Inflation as a phenomenon

It is even occurring in our universe now!

… seems to be a rather ubiquitous phenomenon

→ It could have happened many times, play many different “roles.”

Cosmic inflation in the early stage of our universe

- — Homogeneity **?**
- — Flatness ←
- — Heavy relics (e.g. monopoles) **?**
- — Origin of the structure (fluctuation) ←
- — Beginning of spacetime **?**
- — …

Inflation as a phenomenon

It is even occurring in our universe now!

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→ It could have happened many times, play many different "roles."

A modern view

Guth, Kaiser, Y.N., Phys. Lett. **B733** (2014) 112

"Observable inflation" as a **specific** occurrence of the phenomenon

… important in shaping our **own** universe (in the multiverse)

Shocking discovery in 1998

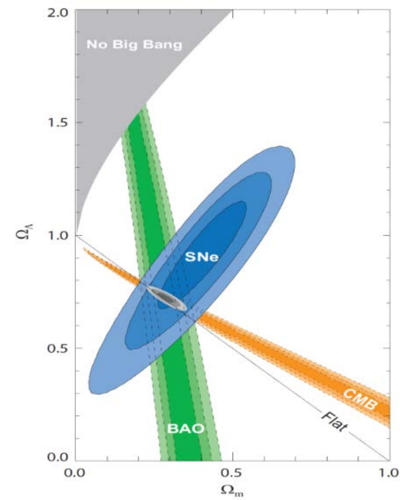
Supernova cosmology project; Supernova search team

Expansion of the Universe is accelerating!

$$\Lambda \neq 0\ !$$

Observationally,

$$\rho_\Lambda \sim (10^{-3}\ \mathrm{eV})^4$$



Particle Data Group (2010)

Its smallness is already hard to understand

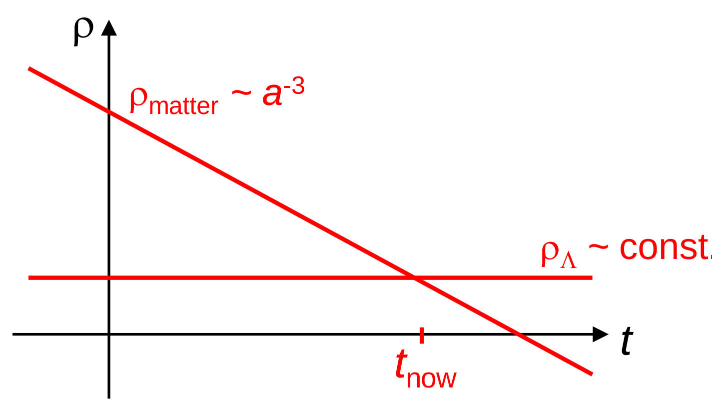
… natural size of $\rho_\Lambda \equiv \Lambda^2 M_{\rm Pl}^2 \sim M_{\rm Pl}^4$ (at the very least ~ TeV^4)

… Naïve estimate is $O(10^{120})$ too large

Moreover

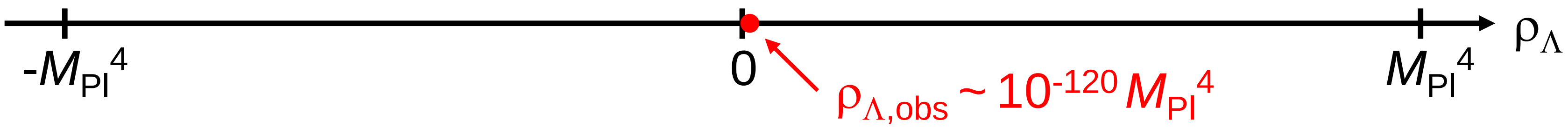
$$\rho_\Lambda \sim \rho_{\rm matter}$$

— Why now?



Nonzero value completely changes the view!

Natural size for vacuum energy $\rho_\Lambda \sim M_{Pl}^4$



Unnatural (Note: $\rho_\Lambda = 0$ is NOT special from theoretical point of view)

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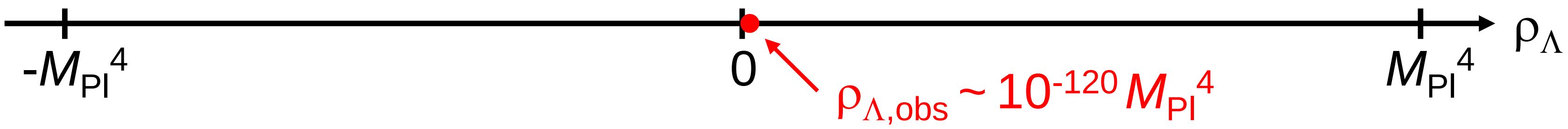
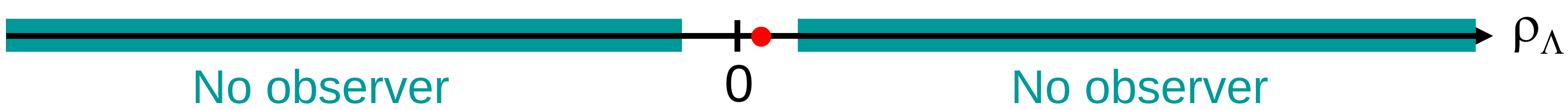


Unnatural (Note: $\rho_\Lambda = 0$ is NOT special from theoretical point of view)

$\Longrightarrow$ Wait!

Is it really unnatural to *observe* this value?



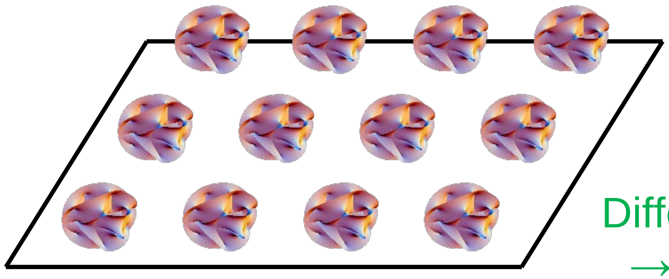
It is natural to observe $\rho_{\Lambda,obs}$,

as long as different values of ρ_Λ are "sampled" Weinberg ('87); also Banks, Linde, …

Theory also suggests:

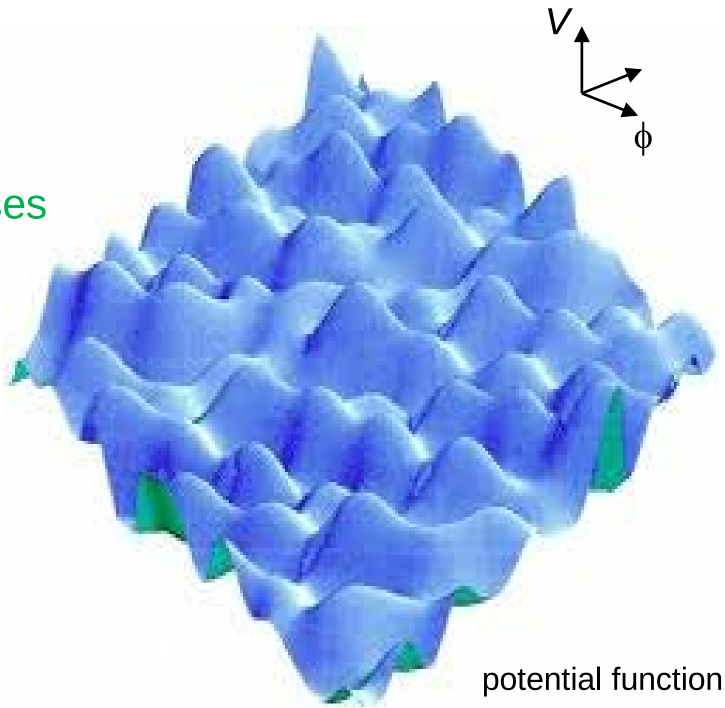
- String theory

  … existence of extra dimensions



https://commons.wikimedia.org/wiki/File:Calabi-Yau-alternate.png

Different solutions
→ Different universes



http://journalofcosmology.com/Multiverse9.html

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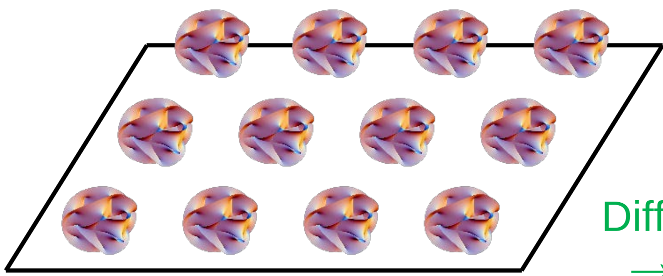

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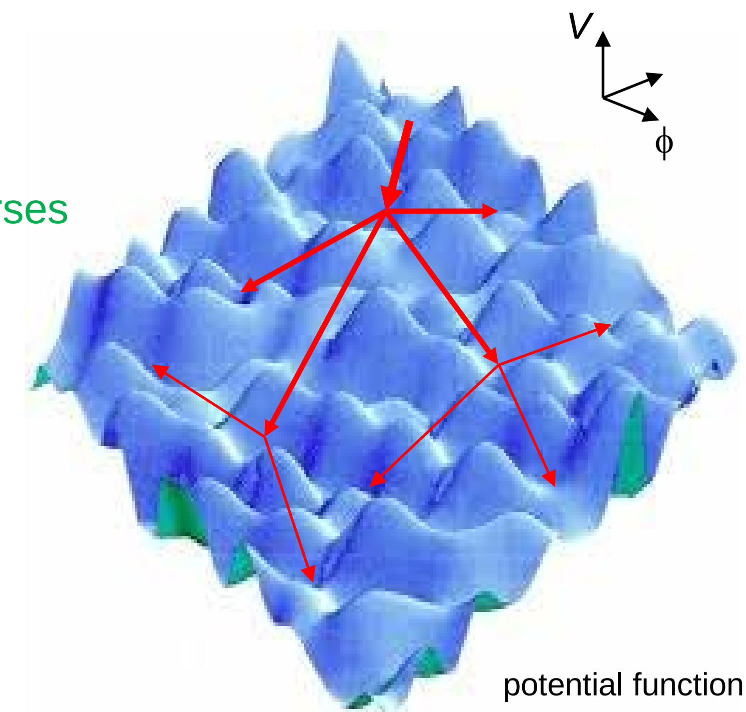

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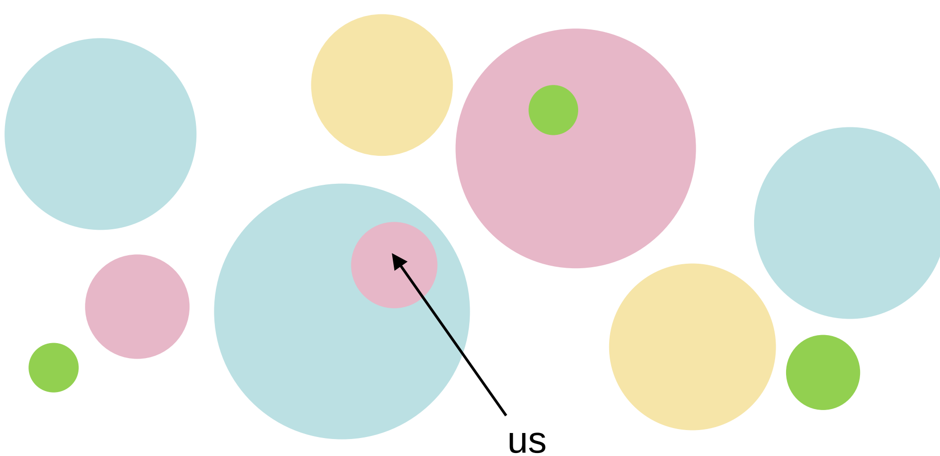
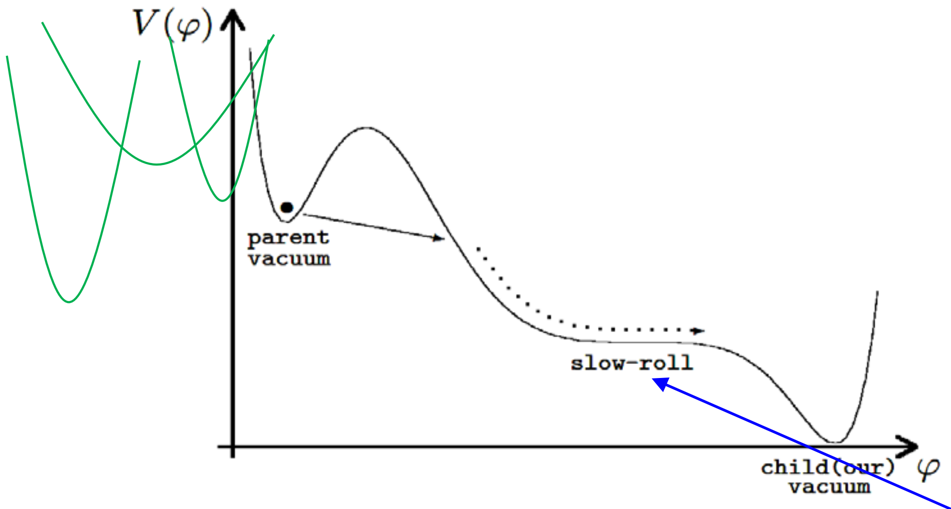
- Inflation

  … eternal to the future



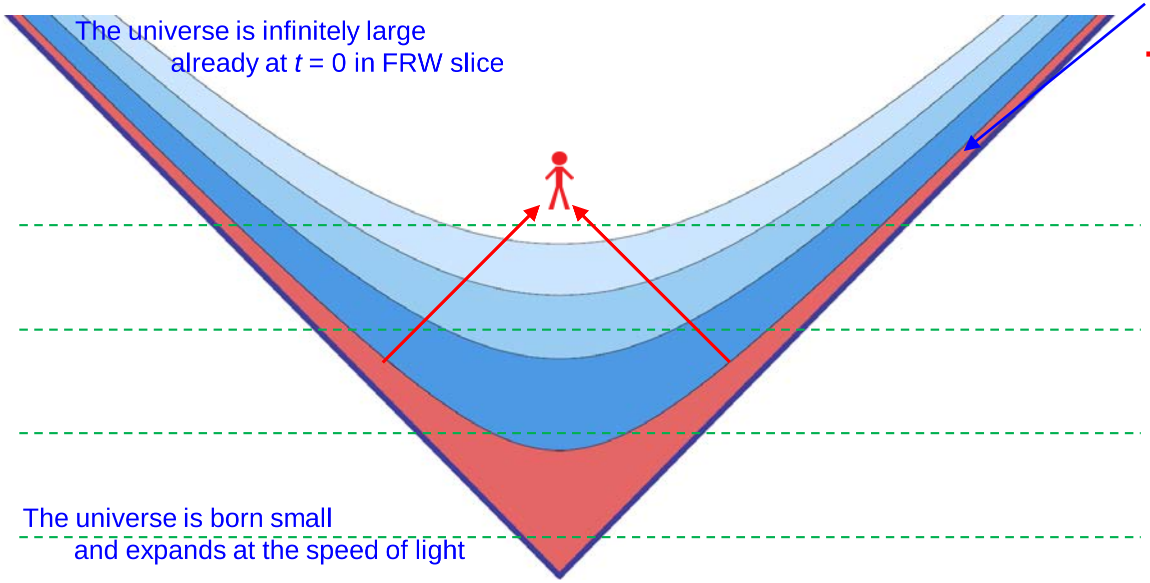
… keep forming new “bubbles”

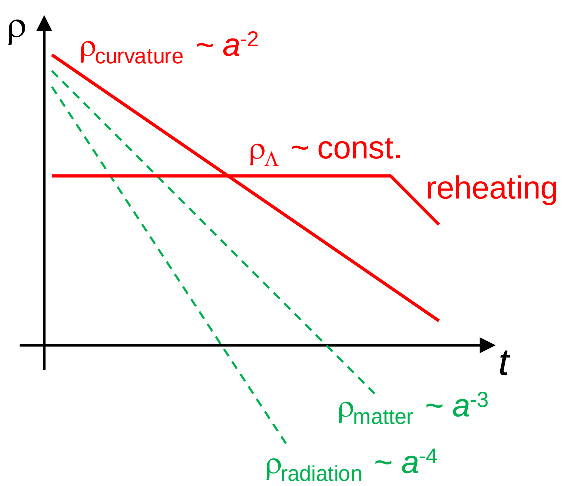
Our universe is a “bubble” inside a larger structure!



Observable inflation

… **needed** to dilute curvature





Observable (slow-roll) inflation — status

We are concerned about the predictions for density fluctuation.

→ Era of precision data

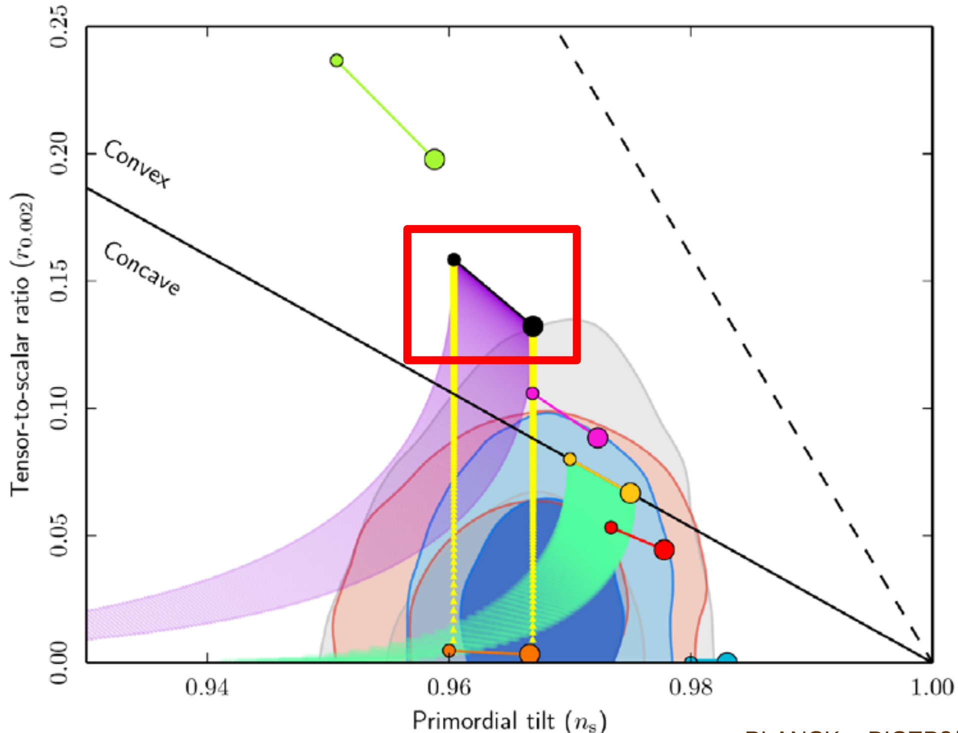
The "simplest" model

$$V(\phi) = \frac{m^2}{2}\phi^2$$

– makes a good prediction

$$n_s \simeq 0.96$$

– excluded by *r* …



PLANCK + BICEP2/Keck Array ('15)

- Does the model of inflation need to be significantly complicated?
- Is the agreement of n_s of the ϕ^2 potential with the data "accidental"?

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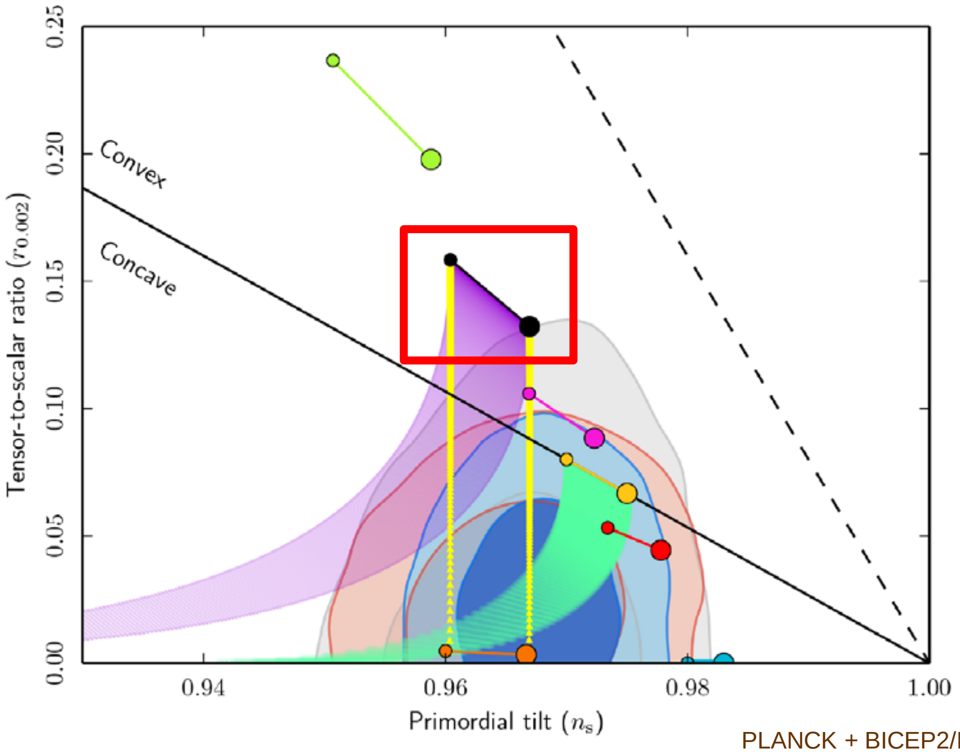

PLANCK + BICEP2/Keck Array ('15)

- Does the model of inflation need to be significantly complicated?
- Is the agreement of n_s of the ϕ^2 potential with the data "accidental"?

⟹ No

Pure Natural Inflation

Y.N., Watari, Yamazaki, Phys. Lett. **B776** (2018) 227

Consider

axionic (pseudo Nambu-Goldstone) inflaton

$$\mathcal{L} = \frac{1}{32\pi^2} \frac{\phi}{f} \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr}\, F_{\mu\nu} F_{\rho\sigma}$$

Physics is invariant under $\theta \equiv \dfrac{\phi}{f} \to \theta + 2\pi$

Instanton induced potential

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]$$

… "Natural" inflation

Freese, Frieman, Olinto ('90)

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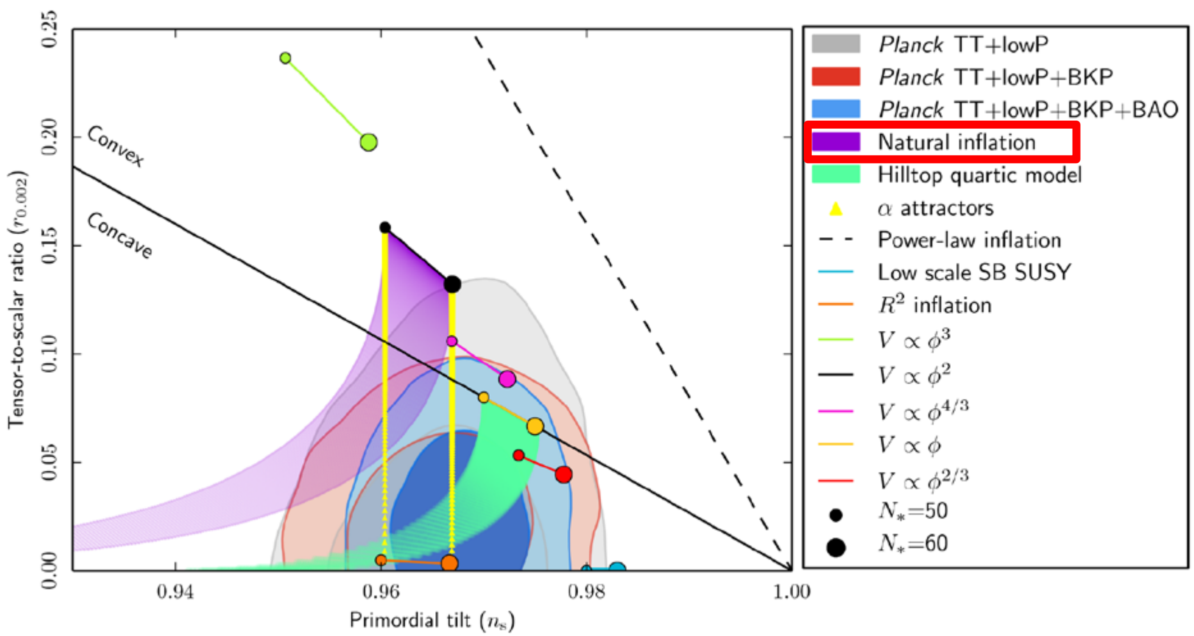

Simple, but facing challenges from the data

Consider even simpler !

For pure Yang-Milles theory

$$\mathcal{L} = \frac{1}{32\pi^2}\frac{\phi}{f}\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,F_{\mu\nu}F_{\rho\sigma}$$

instanton induced potential is **not** the right form Witten ('79, '80)

How can it be while respecting invariance under $\phi \to \phi + 2\pi f$?

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How can it be while respecting invariance under $\phi \to \phi + 2\pi f$?

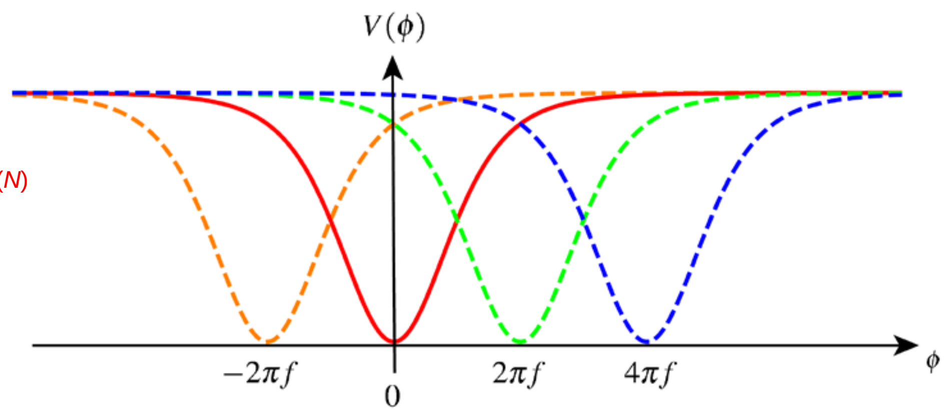
→ the **multi-valued** potential

The potential in a single branch

$$V(\phi) = N^2\Lambda^4\,\mathcal{V}\left(\frac{\lambda\phi}{8\pi^2 N f}\right)$$

N: the size of the gauge group, e.g. SU(*N*)

need not respect the periodicity



The expected form of the potential

(CP invariance $\phi \to -\phi$, high energy behavior $\mathcal{V}(x) \sim 1/(x^2)^p\ (p > 0)$, …)

$$V(x) = M^4\left[1 - \frac{1}{(1+cx^2)^p}\right] \quad (p>0)$$

$M \sim \sqrt{N}\Lambda \qquad c \approx O(1) > 0 \qquad x \equiv \frac{\lambda\phi}{8\pi^2 N f}$

… A holographic computation indeed gives this form with *p* = 4

Dubovsky, Lawrence, Roberts ('11)

We can parameterize this potential as

$$\boxed{V(\phi) = M^4 \left[1 - \frac{1}{\left(1 + \left(\frac{\phi}{F}\right)^2\right)^p} \right]}$$

where

dynamical scale

$$M \approx \sqrt{N}\Lambda, \qquad F \approx Nf$$

… very difference from the cosine potential!

Expanding as

$$V(\phi) = \sum_{n=1}^{\infty} b_{2n} \left(\frac{\phi}{F}\right)^{2n}$$

this potential gives

$$\mathrm{sgn}(b_{2n}) = (-1)^{n-1}$$

$$\frac{\frac{b_6}{b_4}}{\frac{b_4}{b_2}} = \frac{2(p+2)}{3(p+1)}, \quad \cdots, \quad \frac{\frac{b_{2n+4}}{b_{2n+2}}}{\frac{b_{2n+2}}{b_{2n}}} = \frac{(n+1)(p+n+1)}{(n+2)(p+n)}, \quad \cdots$$

double ratios of the coefficients:
relevant for the predictions

while the cosine potential leads to

$$\mathrm{sgn}(b_{2n}) = (-1)^{n-1}$$

$$\frac{\frac{b_6}{b_4}}{\frac{b_4}{b_2}} = \frac{2}{5}, \qquad \frac{\frac{b_8}{b_6}}{\frac{b_6}{b_4}} = \frac{15}{28}, \qquad \cdots$$

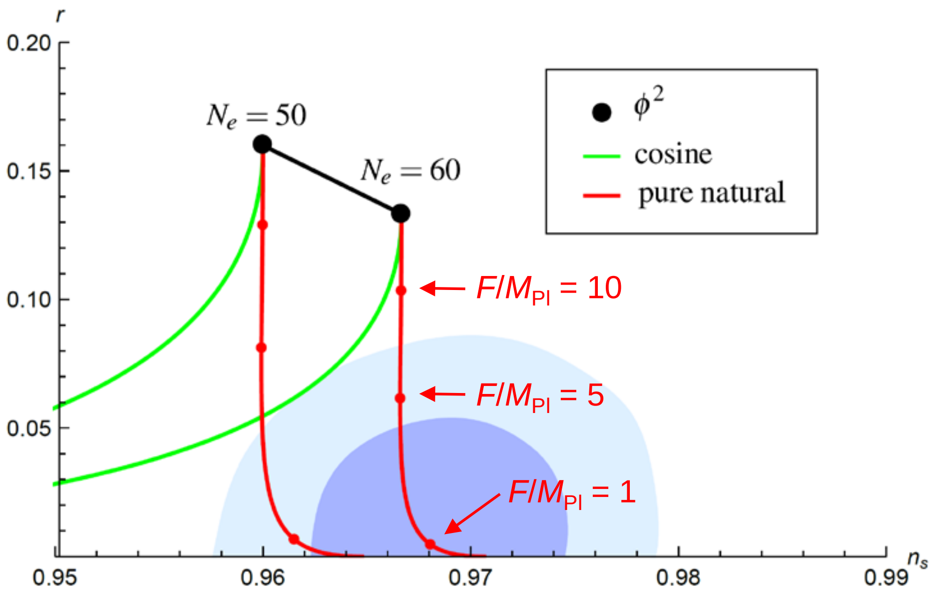
e.g., by equating $(b_6/b_4)/(b_4/b_2)$, we get $p = -7/2 < 0$

Prediction

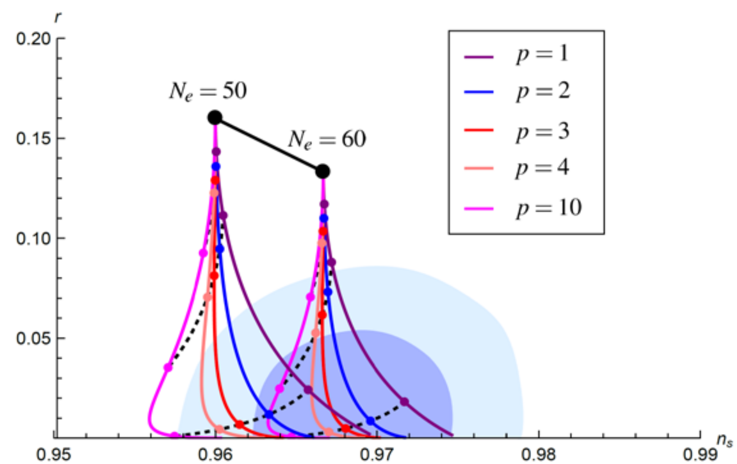
... determined by p and F/M_{Pl}

(M is determined by the amplitude: $M \sim 10^{16}$ GeV)

- p = 4 (holographic value)



- robust with respect to the variation of p



... Consistent at the 95% (68%) CL for

$$\frac{F}{M_{\rm Pl}} \lesssim \begin{cases} 3.3\ (0.7) \\ 6.8\ (4.4) \end{cases} \text{for } N_e = \begin{cases} 50 \\ 60 \end{cases}$$

For $F \gg M_{Pl}$, the prediction reduces to that of the ϕ^2 potential (as it must be)

We, however, expect deviations from the ϕ^2 point:

$F \approx Nf$ $\rightarrow$ Does this mean that F can be arbitrary large for $N \gg 1$?

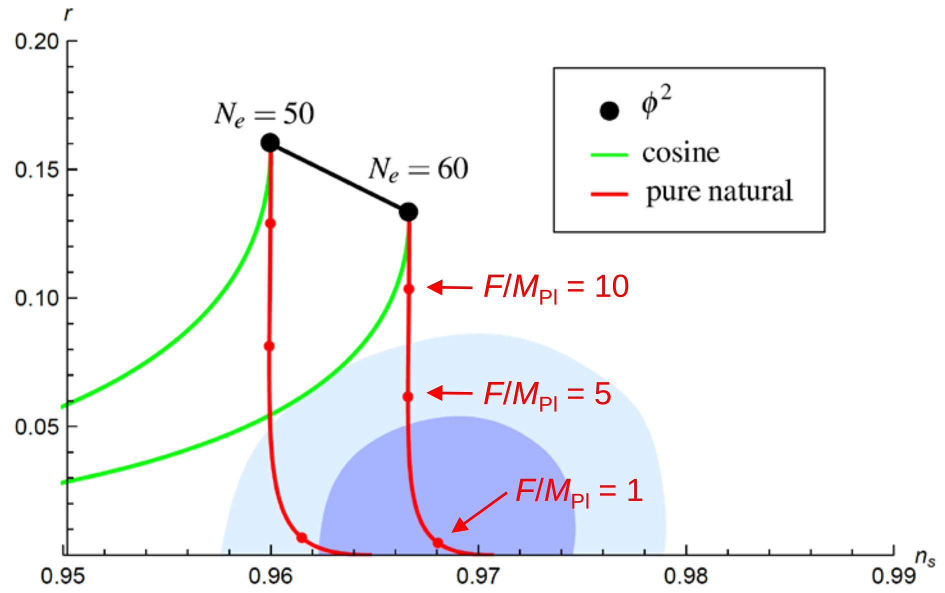
No.

cutoff (string) scale

$f \lesssim M_*$

$M_{\rm Pl}^2 \sim N^2 M_*^2$ In any (reasonable) quantum gravity

$$\Longrightarrow \boxed{F \lesssim O(M_{\rm Pl})}$$



… complete agreement with the current data!

Large N, however, does help reheating

$$f \approx \frac{F}{N}$$

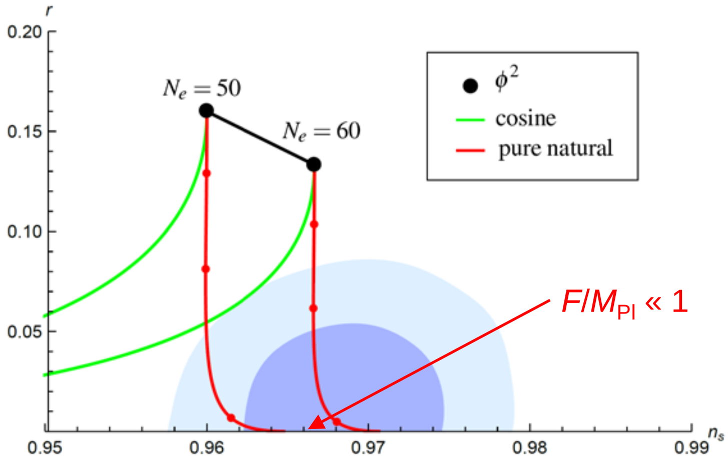
For example, $\mathcal{L} = \frac{1}{32\pi^2}\frac{\phi}{f}\epsilon_{\mu\nu\rho\sigma}\,{\rm Tr}\,F_{\rm SM}^{\mu\nu}F_{\rm SM}^{\rho\sigma}$

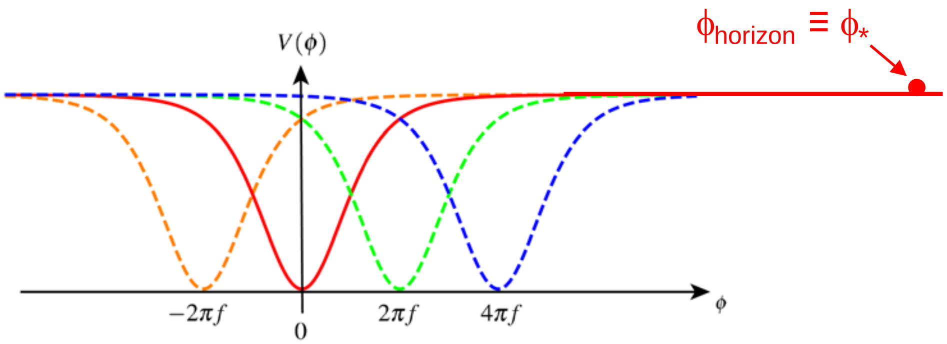
$$\Longrightarrow T_R \sim 10^9\ {\rm GeV}\left(\frac{N}{10}\right)\left(\frac{0.5}{F/M_{\rm Pl}}\right)^{5/2}$$

Tensor modes in pure natural inflation

Y.N., Yamazaki, Phys. Lett. **B780** (2018) 106

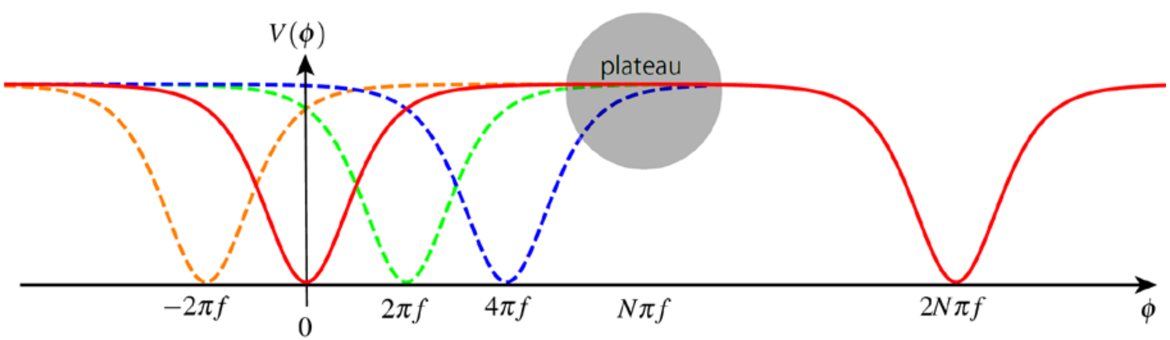
There are natural lower bounds on *r*





Effect of finite *N*

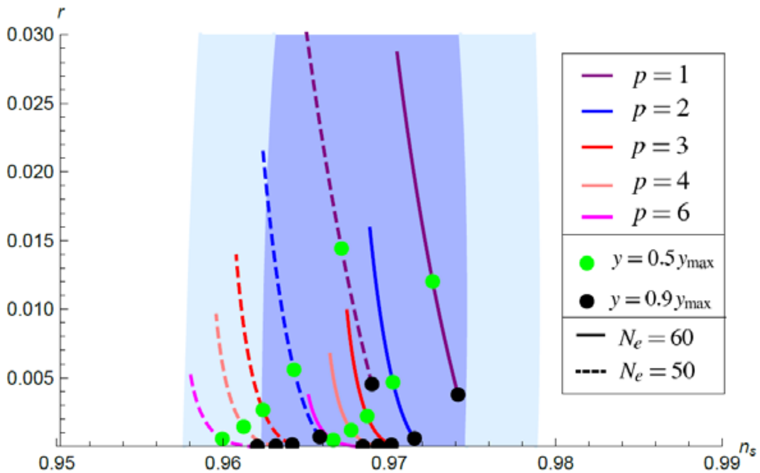
$$V(\phi + 2\pi N f) = V(\phi)$$

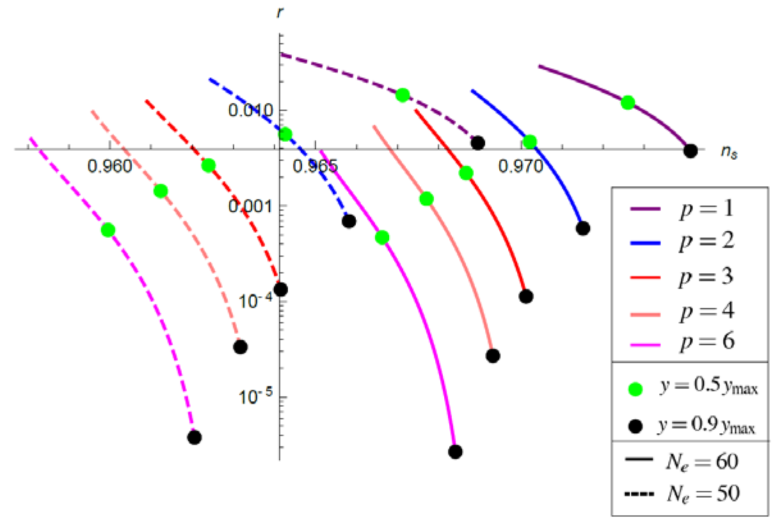


… infinitely long plateau **not** available

We expect $y \equiv \dfrac{\phi_*}{F}$ to take a generic value in

$$0 \leq y \leq y_{\max}$$





… Interesting parameter regions can be probed ($r > 10^{-3}$)

A major uncertainty — the value of *p*

… may be determined / constrained by future lattice computations

$$\bar{b}_4 = \frac{2(p+2)}{3(p+1)}\bar{b}_2^2 \simeq \frac{p+2}{p+1} \times 3.5 \times 10^{-2}$$

where

$$V(\theta) = \frac{1}{2}\chi\theta^2\left(1 + \sum_{n=1}^{\infty} b_{2n}\theta^{2n}\right)$$

$$b_{2n} = \frac{\bar{b}_{2n}}{N^{2n}}\left(1 + O\left(\frac{1}{N^2}\right)\right)$$

currently $\bar{b}_2 = -0.23(3), \quad \bar{b}_4 \lesssim 0.1$ Bonati, D'Elia, Rossi, Vicari ('16)

… An interesting interplay between fundamental theory and cosmology

Summary

Inflation (accelerating expansion)

Ubiquitous phenomenon

… occurs multiple times throughout the cosmic history

Observable (slow-roll) inflation

Important in shaping our own universe

… small curvature, the origin of structure

The era of precision measurement

… the simple ϕ^2 potential strongly disfavored

→ Does the inflation model necessarily complicated?

→ Is the success of n_s prediction from the ϕ^2 potential accidental?

Pure natural inflation

Simple model in complete agreement with the current data

– Implications for r

– An interplay between fundamental physics and cosmology