新学術・加速膨張宇宙, YITP 2019.3.3

Detectability of gravitons with HBT interferometers

Sugumi Kanno & J.S., arXiv:1810.07604 [hep-th]

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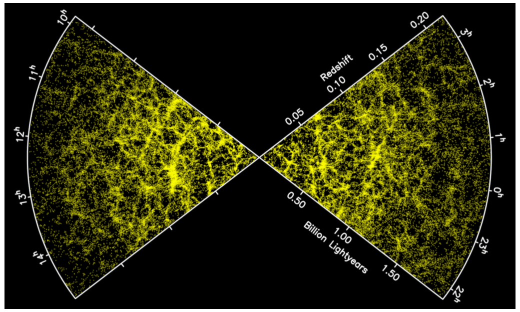
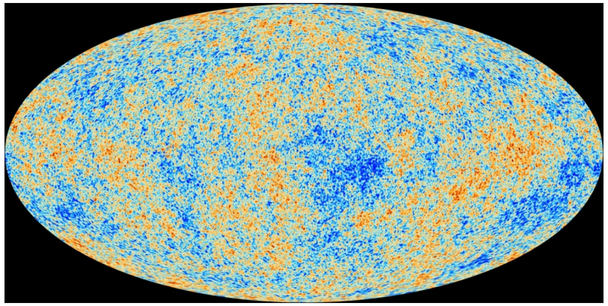
Main question

The large scale structure of the universe can be explained by inflation.





The most important prediction of inflation is that
the primordial fluctuations stem from quantum fluctuations.

In particular, PGWs originate from quantum fluctuations of space-time.

The natural question is if we can prove this prediction.

The answer could be positive

In 2015, gravitational waves are finally detected!

This has boosted the space interferometer project,
where an important scientific target is to detect PGWs.

Since GWs have very weak interaction with matter,
we can expect PGWs can carry the information during inflation.

We need to prove quantum nature of PGWs.
However that does not mean PGWs should be quantum at present.

We need a remnant of quantum fluctuations.

We focus on graviton statistics and connect it to HBT interferometory.

Possible impact

It has been believed that the key for proving inflation is to observe PGWs.
However, we have shown that even Ekpyrotic model can produce classical PGWs.

Ito & Soda 2017

We can prove inflation if we detected quantum nature of PGWs.

Nonclassicality of PGWs can be used to discriminate inflation models.

Detection of nonclassical GWs implies discovery of gravitons.

It provides a hint of quantum gravity.

Basics of photon statistics

Coherent state

Glauber 1963

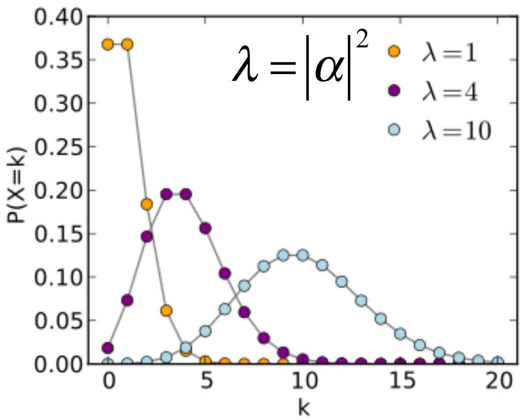
coherent state $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ $\qquad |\alpha\rangle = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle \equiv D(\alpha)|0\rangle$

statistics $\langle\alpha|\hat{n}|\alpha\rangle = |\alpha|^2$

$$(\Delta n)^2 = \langle\alpha|\hat{n}^2|\alpha\rangle - \langle\alpha|\hat{n}|\alpha\rangle^2 = |\alpha|^2$$

Poisson distribution $\left|\langle k|\alpha\rangle\right|^2 = \exp\left(-|\alpha|^2\right)\frac{|\alpha|^{2k}}{k!}$

$$\langle\alpha|\hat{n}|\alpha\rangle = (\Delta n)^2$$



Note that, in case of photons, the coherent state can be produced by the current

$$|\alpha\rangle = \exp\left[-ie\int d^3x\, j_i A^i\right]|0\rangle$$

Squeezed state

squeezing operator

$$S(\zeta) = e^{\left(\zeta^* \hat{a}^2 - \zeta \hat{a}^{\dagger 2}\right)/2} \qquad \zeta = r\, e^{i\varphi}$$

$$S(\zeta)\hat{a}\, S^\dagger(\zeta) = \hat{a}\cosh r + \hat{a}^\dagger e^{i\varphi} \sinh r$$

This is nothing but Bogoliubov transformation.

squeezed state

$$|\zeta\rangle = S(\zeta)|0\rangle$$

super-Poisson distribution

$$\langle \zeta | \hat{n} | \zeta \rangle = \sinh^2 r$$

$$(\Delta n)^2 = 2\sinh^2 r + 2\sinh^4 r > \langle n \rangle$$

We can prove that number distribution is always super-Poisson in classical theory.
In other words, sub-Poisson number distribution $(\Delta n)^2 < \langle \alpha | \hat{n} | \alpha \rangle$ implies quantumness.
But, the above result tells us that the opposite is not true.

Squeezed coherent state

squeezed coherent state $$|\zeta,\alpha\rangle = S(\zeta)|\alpha\rangle = S(\zeta)D(\alpha)|0\rangle$$

$$\langle\zeta,\alpha|\hat{n}|\zeta,\alpha\rangle = |\alpha|^2\left[e^{-2r}\cos^2\left(\theta-\frac{\varphi}{2}\right)+e^{2r}\sin^2\left(\theta-\frac{\varphi}{2}\right)\right]+\sinh^2 r \qquad \alpha=|\alpha|e^{i\theta}$$

$$(\Delta n)^2 = |\alpha|^2\left[e^{-4r}\cos^2\left(\theta-\frac{\varphi}{2}\right)+e^{4r}\sin^2\left(\theta-\frac{\varphi}{2}\right)\right]+2\sinh^2 r+2\sinh^4 r$$

Let us consider the simplest case $\theta-\frac{\varphi}{2}=0$

$$\langle\zeta,\alpha|\hat{n}|\zeta,\alpha\rangle = |\alpha|^2 e^{-2r}+\sinh^2 r$$

$$(\Delta n)^2 = |\alpha|^2 e^{-4r}+2\sinh^2 r+2\sinh^4 r$$

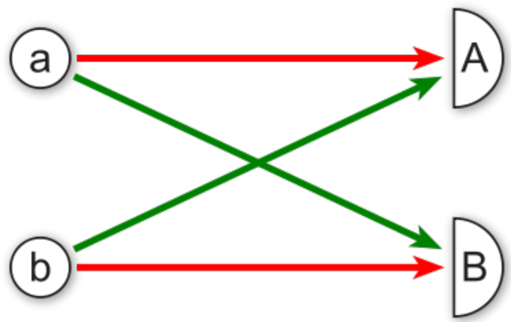
sub-Poisson distribution $\longrightarrow$ $$|\alpha|^2\left(e^{-2r}-e^{-4r}\right) > \sinh^2 r+2\sinh^4 r$$

We should look for this feature in PGWs!!

HBT interferometry

Hanbury Brown − Twiss interferometry

$$E_A^{\ +} = i\varepsilon\left(a_a e^{i\mathbf{k}_{aA}\cdot\mathbf{r}_A} + a_b e^{i\mathbf{k}_{bA}\cdot\mathbf{r}_A}\right) \qquad E_B^{\ +} = i\varepsilon\left(a_a e^{i\mathbf{k}_{aB}\cdot\mathbf{r}_2} + a_b e^{i\mathbf{k}_{bB}\cdot\mathbf{r}_B}\right)$$



$$\left\langle E^-(\mathbf{r}_A,t_A)E^-(\mathbf{r}_B,t_B)E^+(\mathbf{r}_B,t_B)E^+(\mathbf{r}_A,t_A)\right\rangle$$

$$= \varepsilon^2\Big[a_a^\dagger a_a^\dagger a_a a_a + a_b^\dagger a_b^\dagger a_b a_b + a_a^\dagger a_b^\dagger a_b a_a$$

$$+ a_b^\dagger a_a^\dagger a_a a_b + a_a^\dagger a_b^\dagger a_a a_b e^{2\pi i(x_A - x_B)/L} + h.c.\Big]$$

Intensity correlation $$g^{(2)}(\tau) = \frac{\left\langle E^-(t)E^-(t+\tau)E^+(t+\tau)E^+(t)\right\rangle}{\sqrt{\left\langle E^-(t)E^+(t)\right\rangle\left\langle E^-(t+\tau)E^+(t+\tau)\right\rangle}} = \frac{\left\langle :I(t)I(t+\tau):\right\rangle}{\sqrt{\left\langle I(t)\right\rangle\left\langle I(t+\tau)\right\rangle}}$$

Remarkably, we have a relation $$g^{(2)}(0) = 1 + \frac{(\Delta n)^2 - \langle\alpha|\hat{n}|\alpha\rangle}{\langle\alpha|\hat{n}|\alpha\rangle^2}$$

Namely, sub-Poisson distribution $(\Delta n)^2 < \langle\alpha|\hat{n}|\alpha\rangle$ yields anti-bunching photons

Thus, we can detect nonclassicality with HBT ineterferometry.

Application to Gravitons

PGWs

Background spacetime

Inflation	$a(\eta) = -\dfrac{1}{H(\eta - 2\eta_e)}$	$-\infty < \eta < \eta_e$
Radiation dominant	$a(\eta) = \dfrac{\eta}{H\eta_e^2}$	$\eta_e < \eta$

GWs in the background

$$a(\eta)h_{ij}(\mathbf{x},\eta) = 2\kappa \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}} \sum_{s=\pm} \left[e_{ij}^{s}(\mathbf{k}) u_k(\eta) a_s(\mathbf{k}) + e_{ij}^{*s}(-\mathbf{k}) u^{*}{}_{k}(\eta) a^{\dagger}{}_{s}(-\mathbf{k}) \right]$$

$$\left(\frac{d^2}{d\eta^2} + k^2 - \frac{a''}{a} \right) u_k(\eta) = 0$$

In vacuum mode	$u_k^{in}(\eta) \xrightarrow[\eta \to -\infty]{} \dfrac{1}{\sqrt{2k}} \left(1 - \dfrac{i}{k(\eta - 2\eta_e)} \right) e^{-ik(\eta - 2\eta_e)}$
Out vacuum mode	$u_k^{out}(\eta) \xrightarrow[\eta_e < \eta]{} \dfrac{1}{\sqrt{2k}} e^{-ik\eta}$

Squeezed Coherent initial state for PGWs

Bogoliubov transformation $a_{\mathbf{k}}^{in} = \cosh r_k a_{\mathbf{k}}^{out} - \sinh r_k a_{-\mathbf{k}}^{out\dagger}$ $\qquad \sinh r_k = \dfrac{1}{2k^2\eta_e^2}$

Bunch-Davies vacuum $a_{\mathbf{k}}^{in}|0_{in}\rangle = 0$

$$|0_{in}\rangle \equiv |BD\rangle = \frac{1}{\cosh r_k}\prod_{\mathbf{k}} e^{\frac{1}{2}\tanh r_k a_{\mathbf{k}}^{out\dagger} a_{-\mathbf{k}}^{out\dagger}}|0_{out}\rangle \qquad a_{\mathbf{k}}^{out}|0_{out}\rangle = 0$$

Coherent state can be generated as

$$|h^{cl}\rangle = \exp\left[-i\kappa\int d^3x\, h_{ij}T^{ij}\right]|BD\rangle = \exp\left[\int\frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}\sum_{s=\pm}\left[-\alpha_{\mathbf{k}}^{s*}\hat{a}_s(\mathbf{k}) + \alpha_{\mathbf{k}}^{s}\hat{a}_s^{\dagger}(-\mathbf{k})\right]\right]|BD\rangle$$

$$\alpha_{\mathbf{k}}^{s} = 2i\kappa^2 e_{ij}^{*s}(-\mathbf{k})u_k^*(\eta)T^{ij}(\mathbf{k})$$

The graviton state can be represented by the squeezed coherent state

$$|h^{cl}\rangle = \exp\left[\int\frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}\sum_{s=\pm}\left[-\alpha_{\mathbf{k}}^{s*}\hat{a}_s^{in}(\mathbf{k}) + \alpha_{\mathbf{k}}^{s}\hat{a}_s^{in\dagger}(-\mathbf{k})\right]\right]|BD\rangle \quad = S(\zeta)D(\alpha)|0_{out}\rangle$$

Criterion for nonclassicality

The condition for nonclassicality is given by

$$\left|\alpha\right|^2\left(e^{-2r}-e^{-4r}\right)>\sinh^2 r+2\sinh^4 r$$

The Bogoliubov coefficient is $$\sinh r=\frac{1}{2k^2\eta_1^2}\gg 1$$

The cutoff scale is given by

$$k\left|\eta_1\right|=\frac{f}{f_c} \qquad f_c=10^9\sqrt{\frac{H}{10^{-4}M_p}}\ \mathrm{Hz}$$

The condition can be written as

$$f>10^9\left|\alpha\right|^{-\frac{1}{6}}\left(\frac{H}{10^{-4}M_p}\right)^{\frac{1}{2}}\ \mathrm{Hz} \qquad \alpha_{\mathbf{k}}^s=2i\kappa^2\, e_{ij}^{*s}(-\mathbf{k})u_k^*(\eta)T^{ij}\left(\mathbf{k}\right)$$

For high frequency GWs, we may have a chance …

Examples

Case 1: Anisotropic inflation

$$S=\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R-\frac{1}{2}\left(\partial_\mu\phi\right)^2-V(\phi)-\frac{1}{4}f^2(\phi)\,F_{\mu\nu}F^{\mu\nu}\right] \qquad f=\exp\int d\phi\frac{qV}{M_p^2V_\phi}$$

The criterion gives rise to

$$f>10^{8.1}e^{-\frac{4}{17}\nu N_g}\left(\frac{H}{10^{-4}M_p}\right)^{\frac{6}{17}}\ \mathrm{Hz} \qquad \nu=\frac{qV}{M_pV_\phi}$$

N_g is the number of e-foldings from the horizon exit to the end of inflation

For $\nu N_g\approx 30,\qquad f>100\,\mathrm{kHz}$

Case 2: Axion inflation

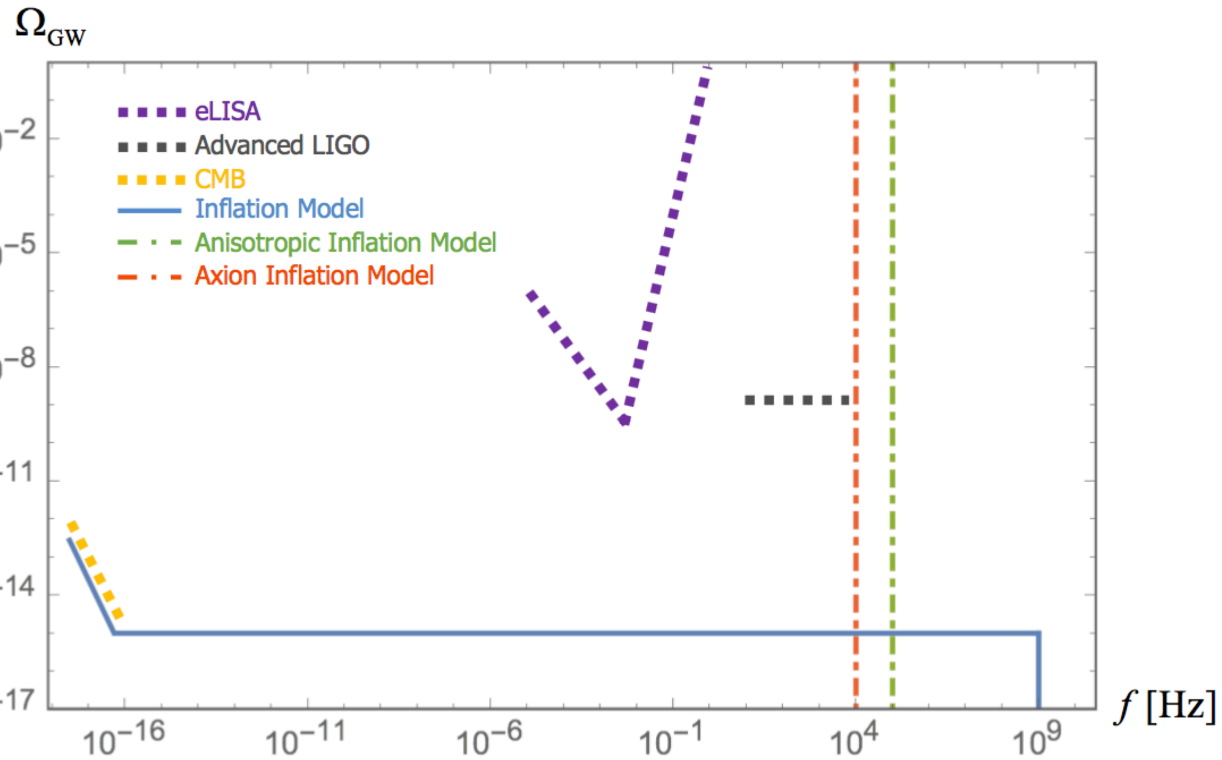
$$S=\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R-\frac{1}{2}\left(\partial_\mu\phi\right)^2-V(\phi)-\frac{1}{8}\frac{\alpha}{f}\phi\,\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\right]$$

The gauge field undergoes the instability

$$f>10^{7.9}e^{-\frac{2\pi}{7}\xi}\xi^{\frac{1}{14}}\left(\frac{H}{10^{-4}M_p}\right)^{\frac{9}{28}}\ \mathrm{Hz} \qquad \xi\approx\frac{\alpha\phi'}{2afH}$$

For $\xi\approx 10,\qquad f>10\,\mathrm{kHz}$

Summary



We need to invent a completely new detector for detecting GHz GWs.