

Weak Gravity Conjecture from Unitarity and Causality

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based on 1810.03637 w/Y. Hamada, G. Shiu

Introduction: Landscape & Swampland

various QFT models w/quantum gravity
ex. for particle physics and cosmology

QFT 1

QFT 2

QFT 3

QFT 4

Landscape : string theory has infinitely many vacua!
shape of extra dimensions, brane configurations, ...

string theory

= generator of QFT models w/quantum gravity

Q. every QFT model is realized in string theory?

A. NO!!!

no global symmetry in string theory

continuous symmetries in string theory are gauged!

- world sheet theory analysis [Banks-Dixon '88, ...]

conserved current \rightarrow gauge boson vertex operator

- if we assume AdS/CFT ...

conserved current J^μ in CFT \Leftrightarrow gauge field A_M in AdS

- BH thought experiments

holographic proof including discrete symmetries

[Harlow-Ooguri 18']

in this way,

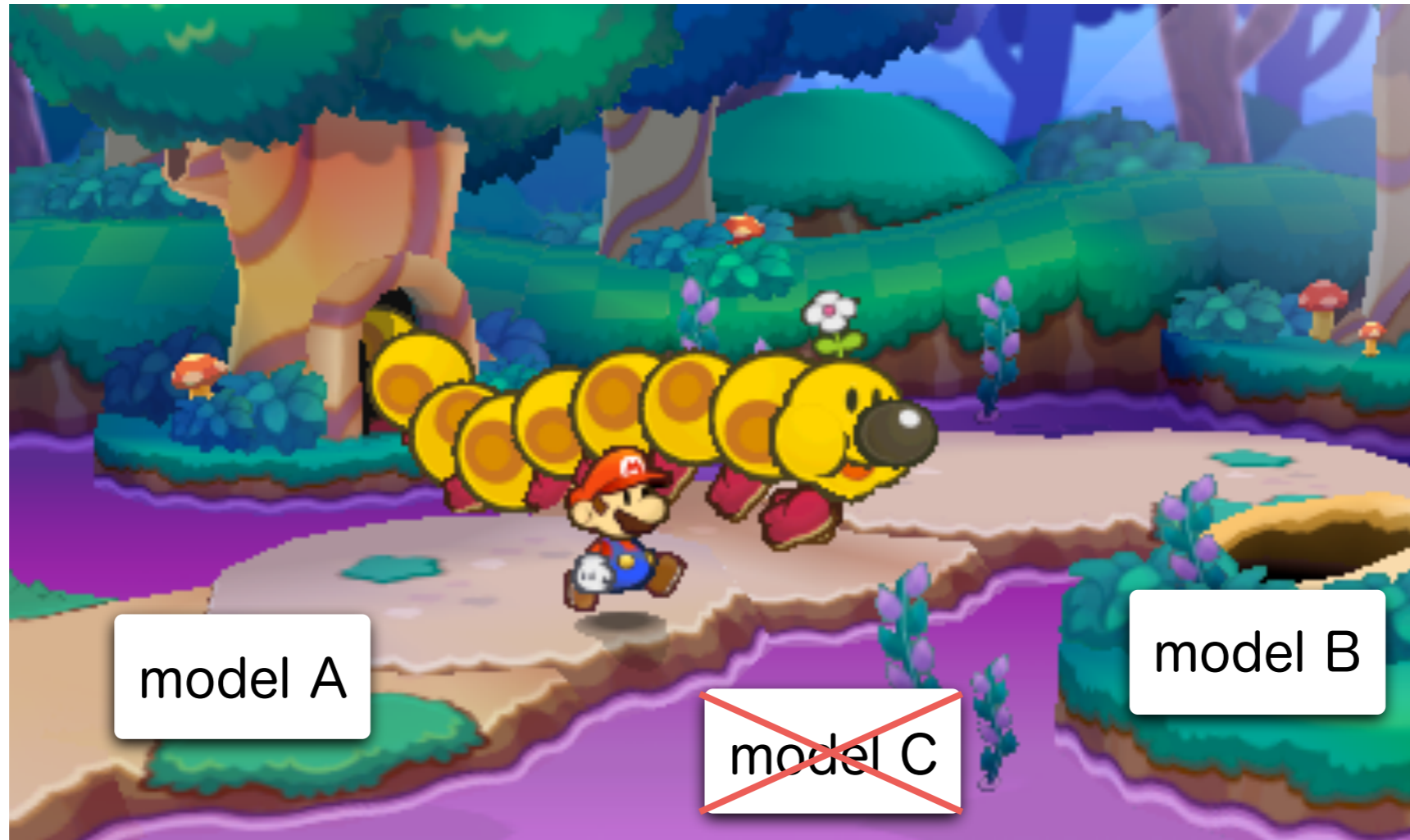
nontrivial constraints on symmetry & matter contents

in string theory (quantum gravity in more general)

→ Landscape & Swampland [Vafa '05, Ooguri-Vafa '06]

landscape :

QFT models consistent w/quantum gravity



swampland :

apparently consistent, but not UV
completable when coupled to gravity

- where is the boundary?
- phenomenological implications?

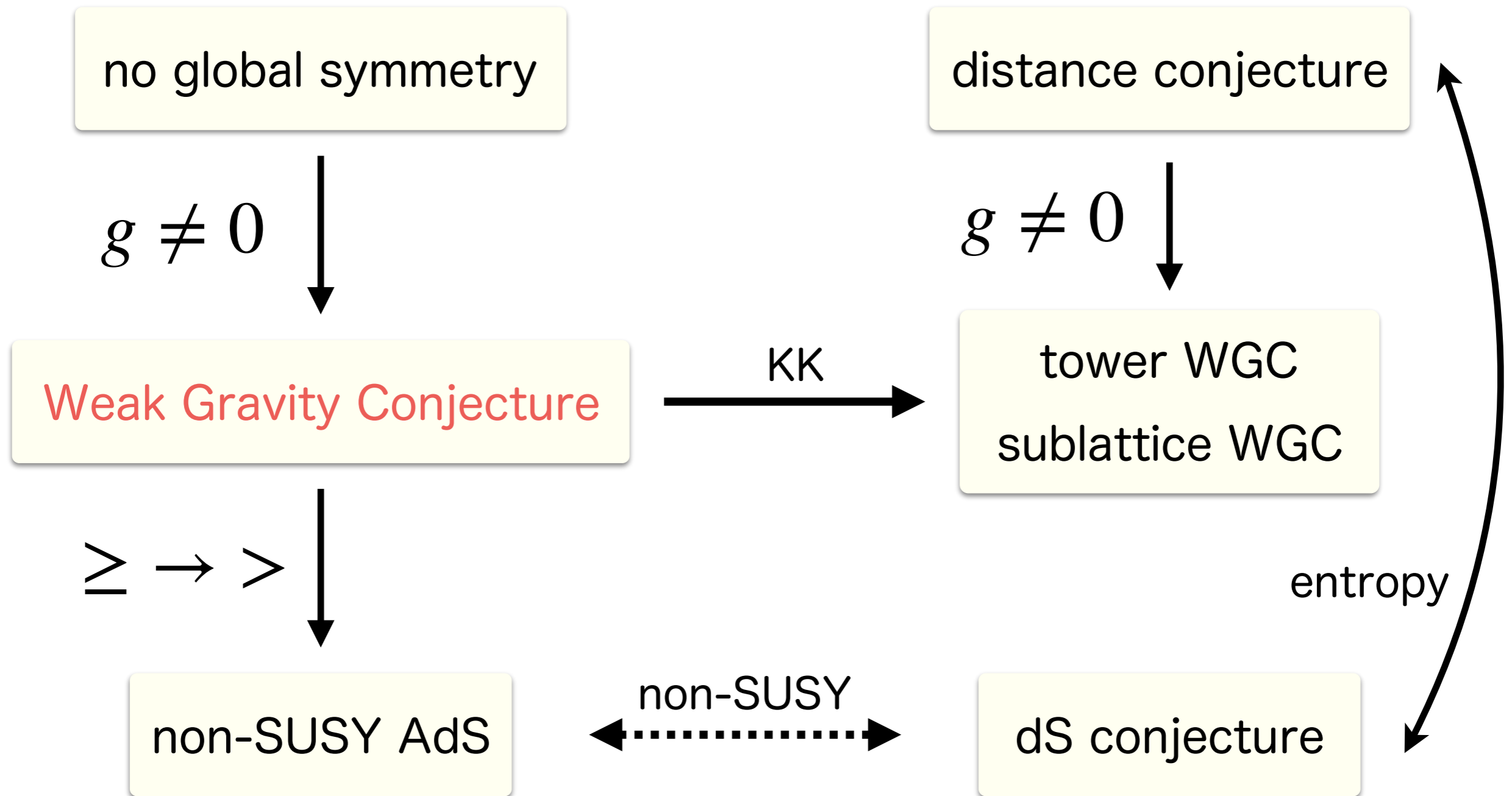




Weak Gravity Conjecture [Arkani-Hamed et al 06']

is a conjectured criterion defining the boundary

web of swampland conjectures



Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06']

Weak Gravity Conjecture

in quantum gravity,

\exists a charged state satisfying

$$g^2 q^2 \geq \frac{m^2}{2M_{\text{Pl}}^2}$$

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06']

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Weak Gravity Conjecture

in quantum gravity,

\exists a charged state satisfying

$$g^2 q^2 \geq \frac{m^2}{2M_{\text{Pl}}^2} \xrightarrow{M_{\text{Pl}} \rightarrow \infty} 0$$

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06']

Weak Gravity Conjecture

in quantum gravity,

\exists a charged state satisfying

$$0 \xleftarrow{g \rightarrow 0} g^2 q^2 \geq \frac{m^2}{2M_{\text{Pl}}^2}$$

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06']

- in QED, electron trivially satisfies the bound:

$$10^{-2} \sim g^2 q^2 \geq \frac{m^2}{2M_{\text{Pl}}^2} \sim 10^{-44}$$

- generalization to axion (0-form gauge field):

$$\frac{f}{M_{\text{Pl}}} \cdot S_{\text{inst}} \leq 1$$

relevant to inflation, dark matter, quintessence, ...

- neutrino mass from higher form WGC [Ooguri-Vafa '17]

very interesting if it is true

in [Hamada-TN-Shiu '18] we demonstrated
existence of a charged **black hole** satisfying

$$g^2 q^2 > \frac{m^2}{2M_{\text{Pl}}^2}$$

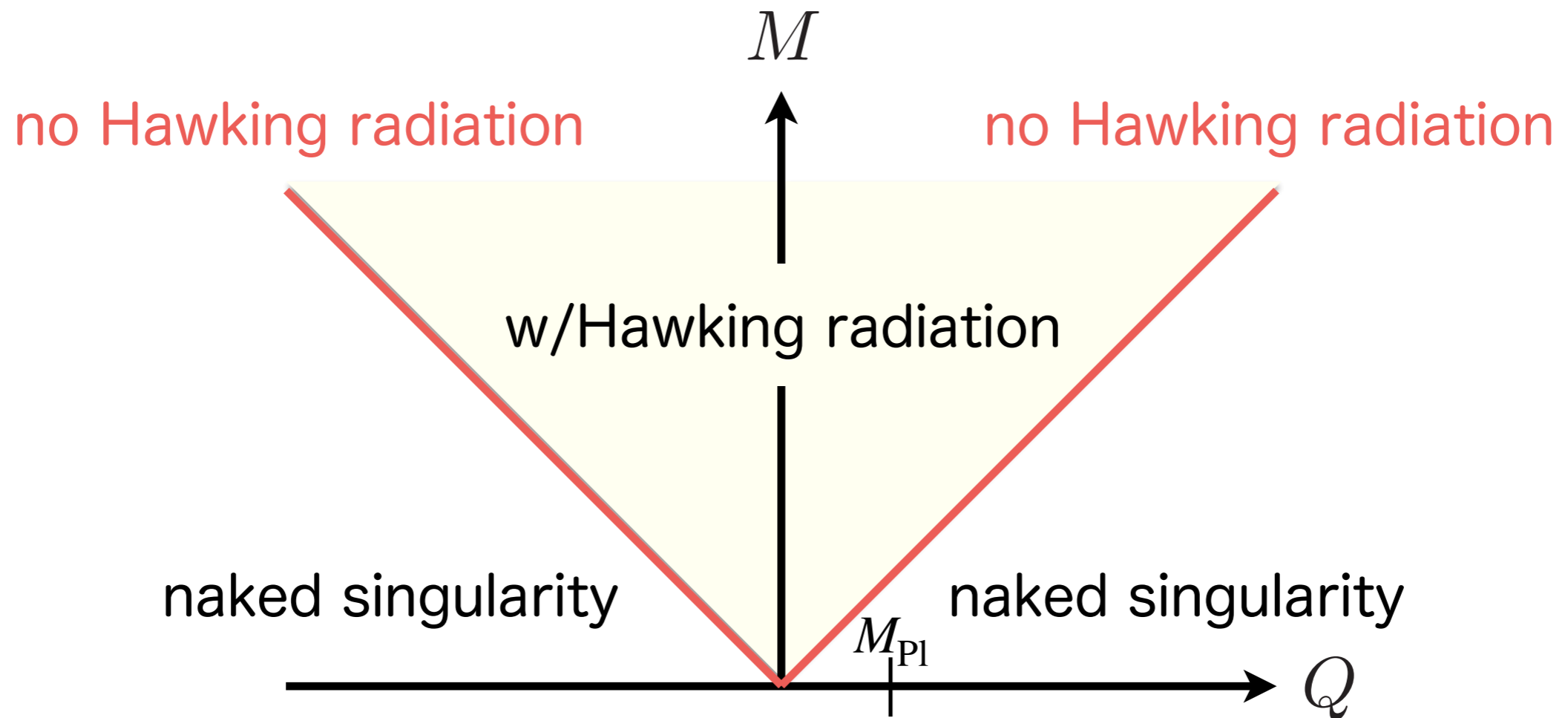
follows from unitarity & causality

in a wide class of theories

(ex. stringy setups w/dilaton or moduli stabilized below M_s)

WGC from unitarity and causality

[Hamada-TN-Shiu '18]

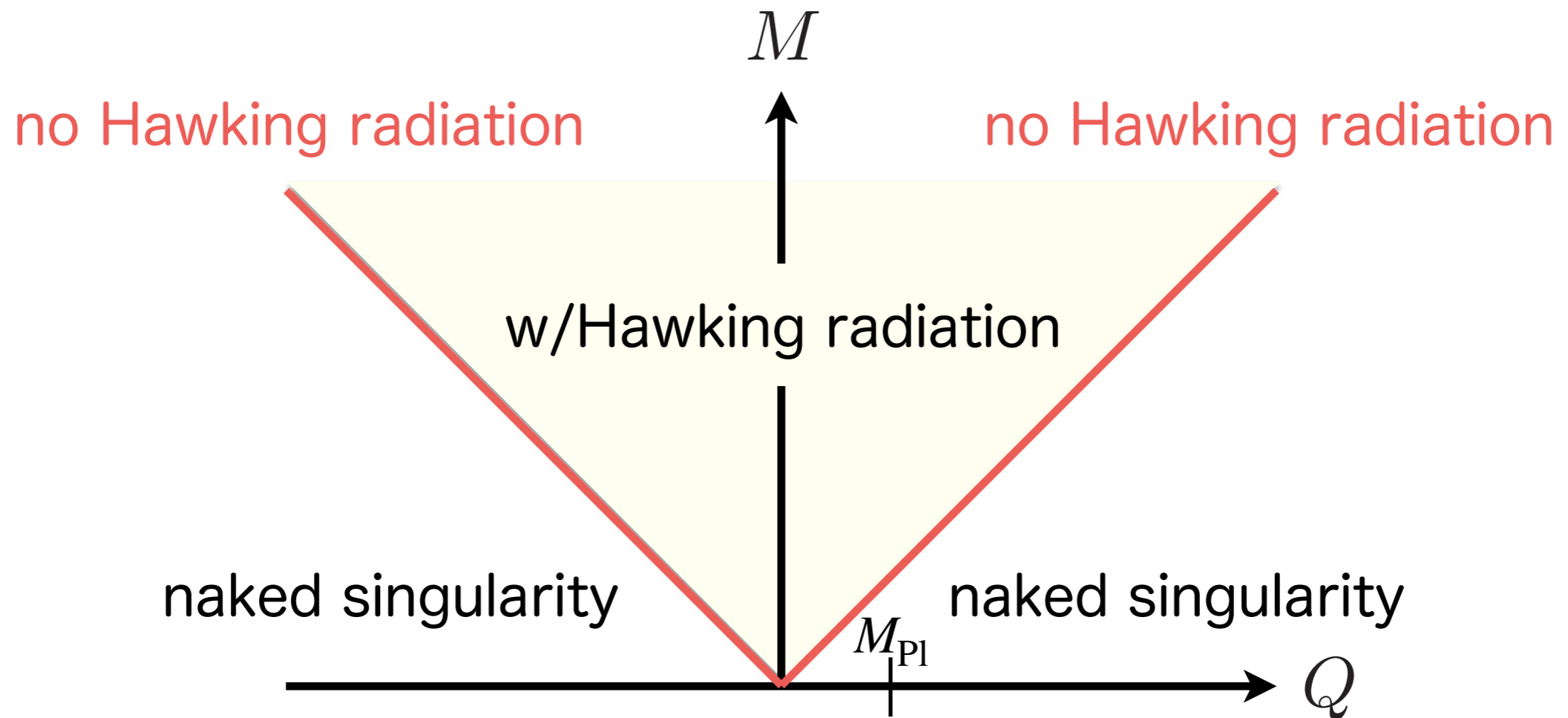


in the Einstein-Maxwell theory,

absence of naked singularity requires the extremal bound:

$$\frac{|Q|}{M} \leq 1 \quad \leftarrow \text{opposite to the WGC bound}$$

※ work in the unit $2M_{Pl}^2 = 1, g = 1$ in the following



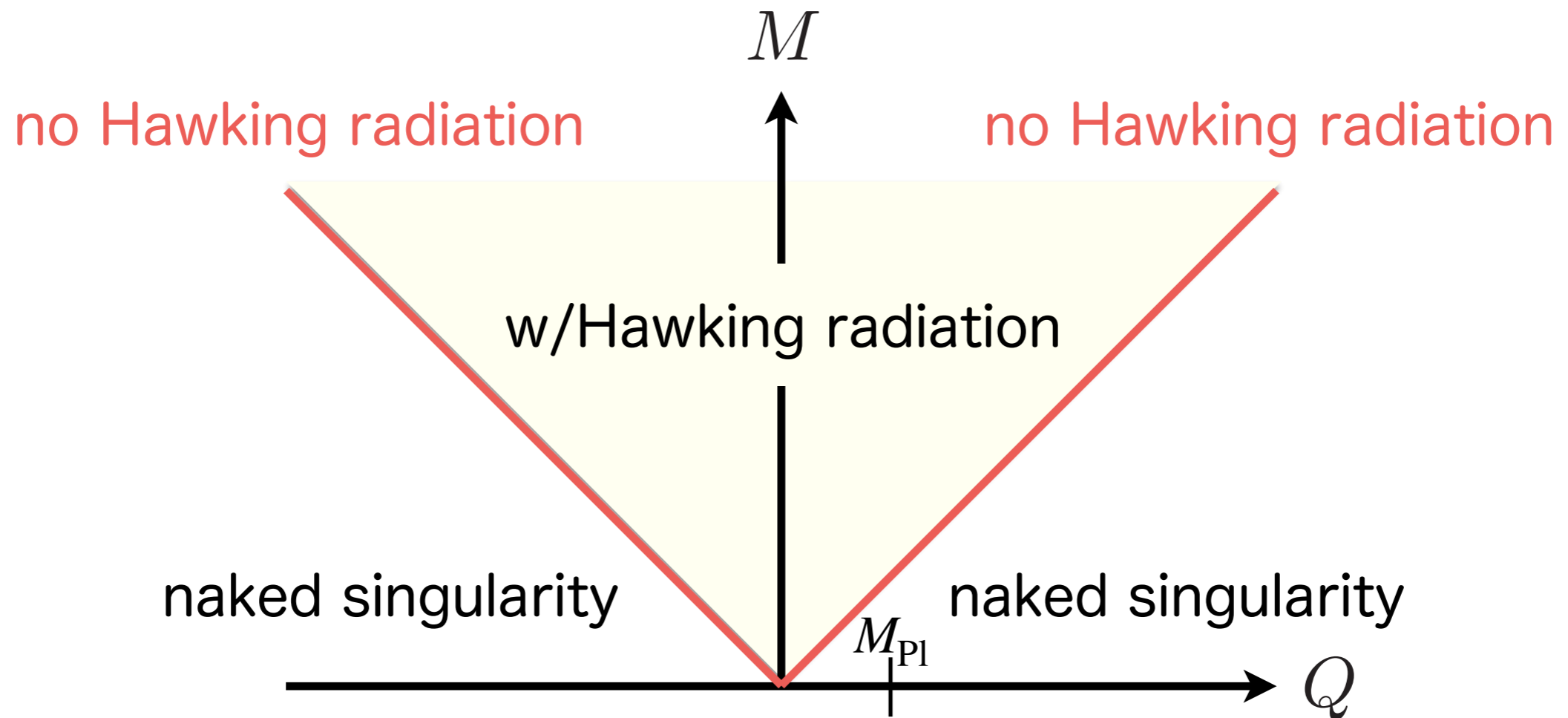
higher derivative corrections to Einstein-Maxwell theory

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{4} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \alpha_3 F_{\mu\nu} F_{\rho\sigma} W^{\mu\nu\rho\sigma} + \dots \right]$$

⊗ work in the unit $2M_{\text{Pl}}^2 = 1$, $g = 1$ in the following

⊗ higher order terms are negligible for heavy BHs

$$F^2 \sim R \sim 1/M^2$$

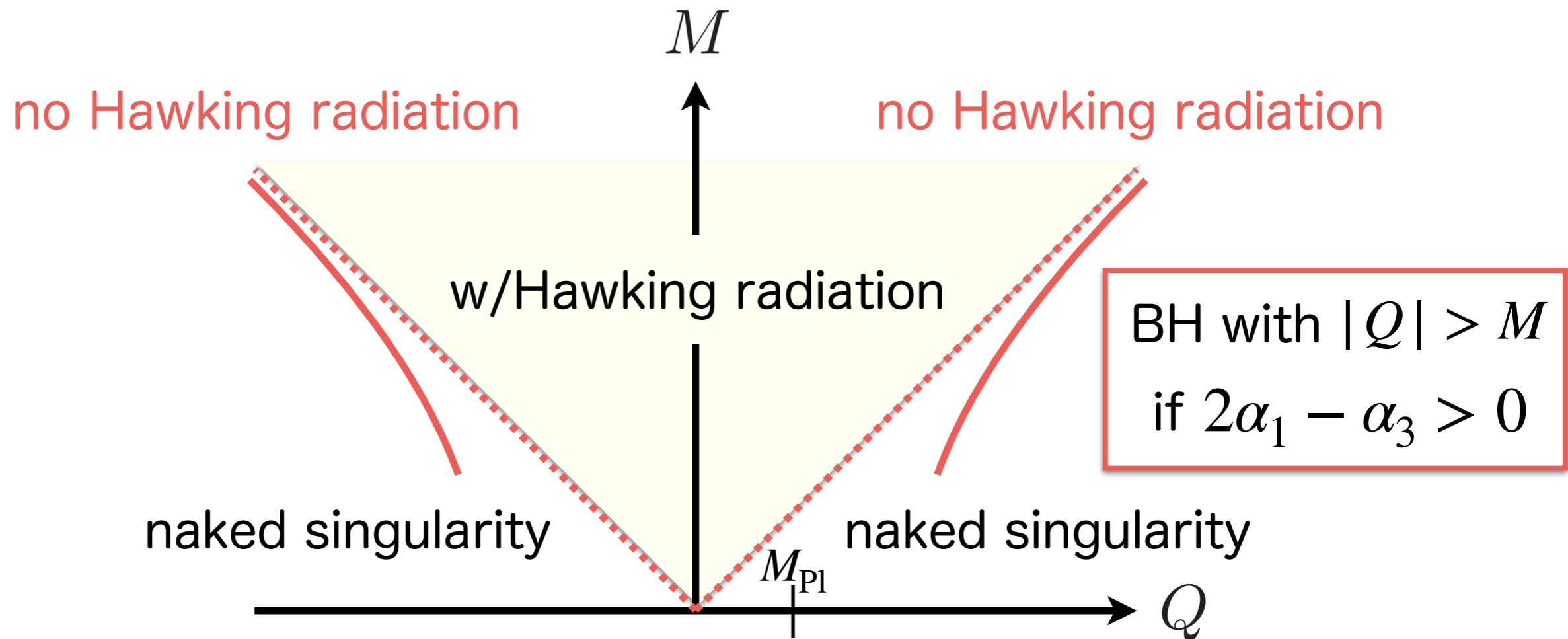


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→ modify BH solutions and horizon structure [Kats-Motl-Padi '06]

no naked singularity if $\frac{|Q|}{M} \leq 1 + \frac{2}{5} \frac{(4\pi)^2}{Q^2} (2\alpha_1 - \alpha_3) + \mathcal{O}(1/Q^4)$



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if we can derive $2\alpha_1 - \alpha_3 > 0$,

an existence proof of WGC (more precisely, mild version)

Evidence of WGC from unitarity and causality

[Hamada-TN-Shiu '18]

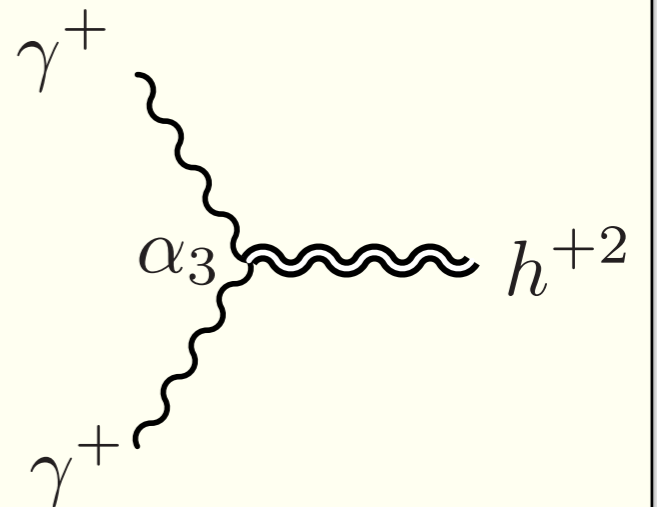
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① α_3 is highly constrained by symmetry causality

SUSY: $\alpha_3 = 0$, non-SUSY: $|\alpha_1|, |\alpha_2| \gg |\alpha_3|$

causality constraint

[cf. Camanho-Edelstein-Maldacena-Zhiboedov '14]



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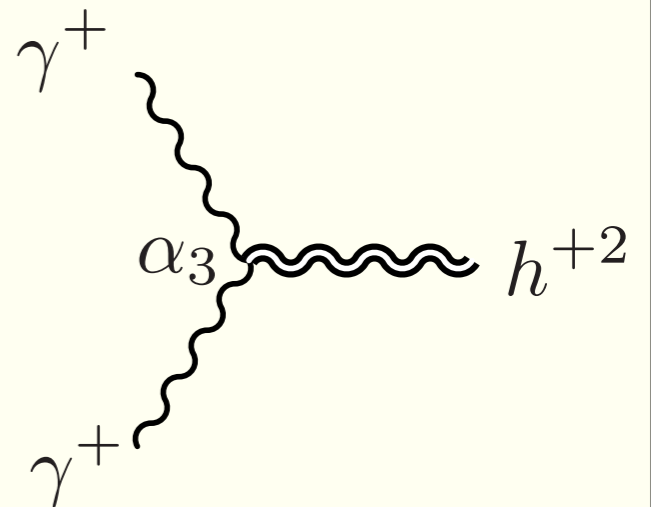
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② sign of α_1 is constrained by unitarity

ex. dilaton coupling

$$\frac{\phi}{f} F_{\mu\nu} F^{\mu\nu} \rightarrow \text{integrate out} \rightarrow \text{effective int. } \frac{1}{2m^2 f^2} (F_{\mu\nu} F^{\mu\nu})^2$$

even if there is no charged particle w/ $|q| \geq m$

dilaton, moduli etc may give $2\alpha_1 - \alpha_3 \simeq 2\alpha_1 > 0$

→ there exist BHs w/ $|Q| > M$ causality unitarity

※ existence proof of (the mild form of) WGC

Summary and prospects

Summary

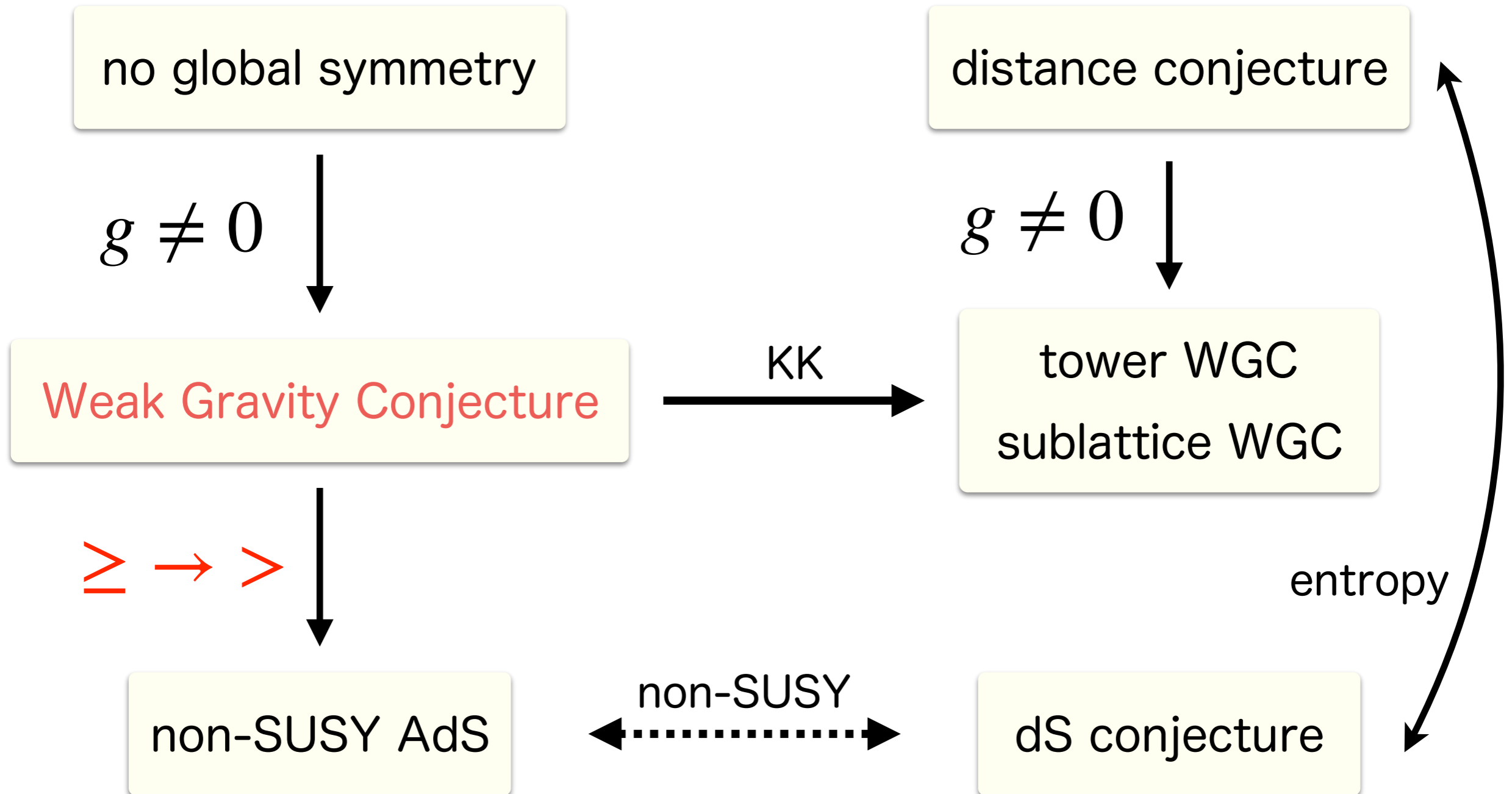
as a consequence of unitarity & causality,
heavy charged BH satisfies the WGC bound

$$g^2 q^2 > \frac{m^2}{2M_{\text{Pl}}^2}$$

- in theories w/light neutral particles (ex. dilaton, moduli)
- in open string theory type UV completion

- ✓ extension to higher dimension, multiple U(1)'s
- ✓ entropy correction is positive in these theories
(cf. [Cheung-Liu-Remmen 18'])

web of swampland conjectures



proof and implications of other conjectures!?

Thank you!