C01(公募研究)

Infrared universality *a la* Weinberg
: New insight on inflation

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Tanaka & Y.U. JCAP 1606 (16) 020

Tanaka & Y.U. JHEP 1710 (17)127

Bordin, Tanaka & Y.U. arXiv:1903.*****

with Lorenzo Bordin (Nottingham U.) and Takahiro Tanaka (Kyoto U.)

Weinberg's adiabatic mode (WAM)

Adiabatic Modes in Cosmology

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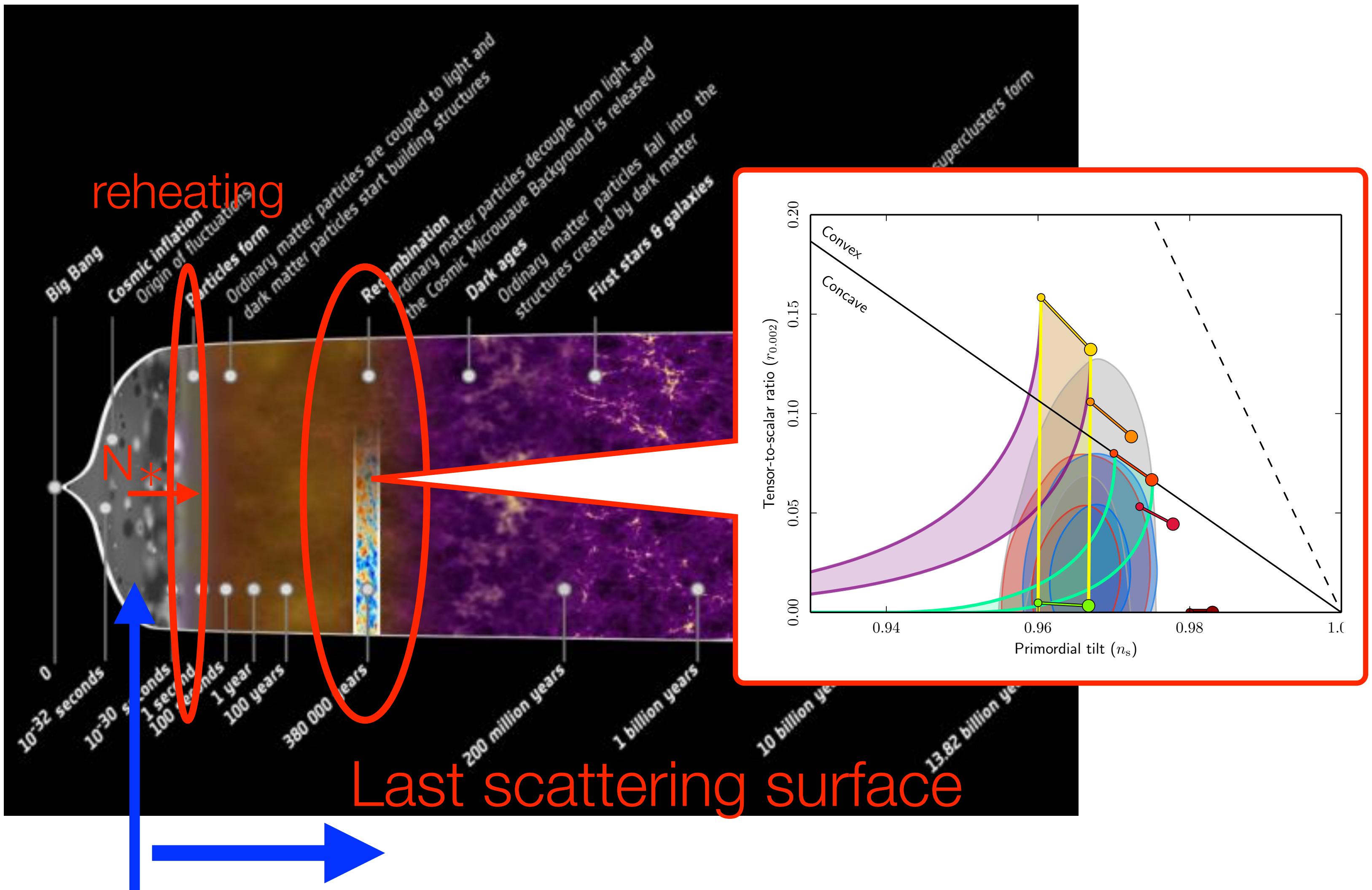
We show that the field equations for cosmological perturbations in Newtonian gauge always have an adiabatic solution, for which a quantity $\mathcal{R}$ is non-zero and constant in all eras in the limit of large wavelength, so that it can be used to connect observed cosmological fluctuations in this mode with those at very early times. There is also a second adiabatic mode, for which $\mathcal{R}$ vanishes for large wavelength, and in general there may be non-adiabatic modes as well. These conclusions apply in all eras and whatever the constituents of the universe, under only a mild technical assumption about the wavelength dependence of the field equations for large wave length. In the absence of anisotropic inertia, the perturbations in the adiabatic modes are given for large wavelength by universal formulas in terms of the Robertson–Walker scale factor. We discuss an apparent discrepancy between these results and what appears to be a conservation law in all modes found for large wavelength in synchronous gauge: it turns out that, although equivalent, synchronous and Newtonian gauges suggest inequivalent assumptions about the behavior of the perturbations for large wavelength.

Why important?



$\mathcal{P}_\zeta(k)|_{k/aH\sim 1}$

Large scale evolution

Claims

Tanaka & Y.U. (17), Bordin, Tanaka, & Y.U. (in prep.)

1) What's the condition for the existence of WAM?

- perturbed LFRW is enough? No, recall solid inflation Enlich et al. (11)

There exists WAM iff dilatation inv. + locality condition (LC) holds.

2) Inflation models in perturbed FLRW can be categorized into three classes:

Q. Whether LC holds or not?

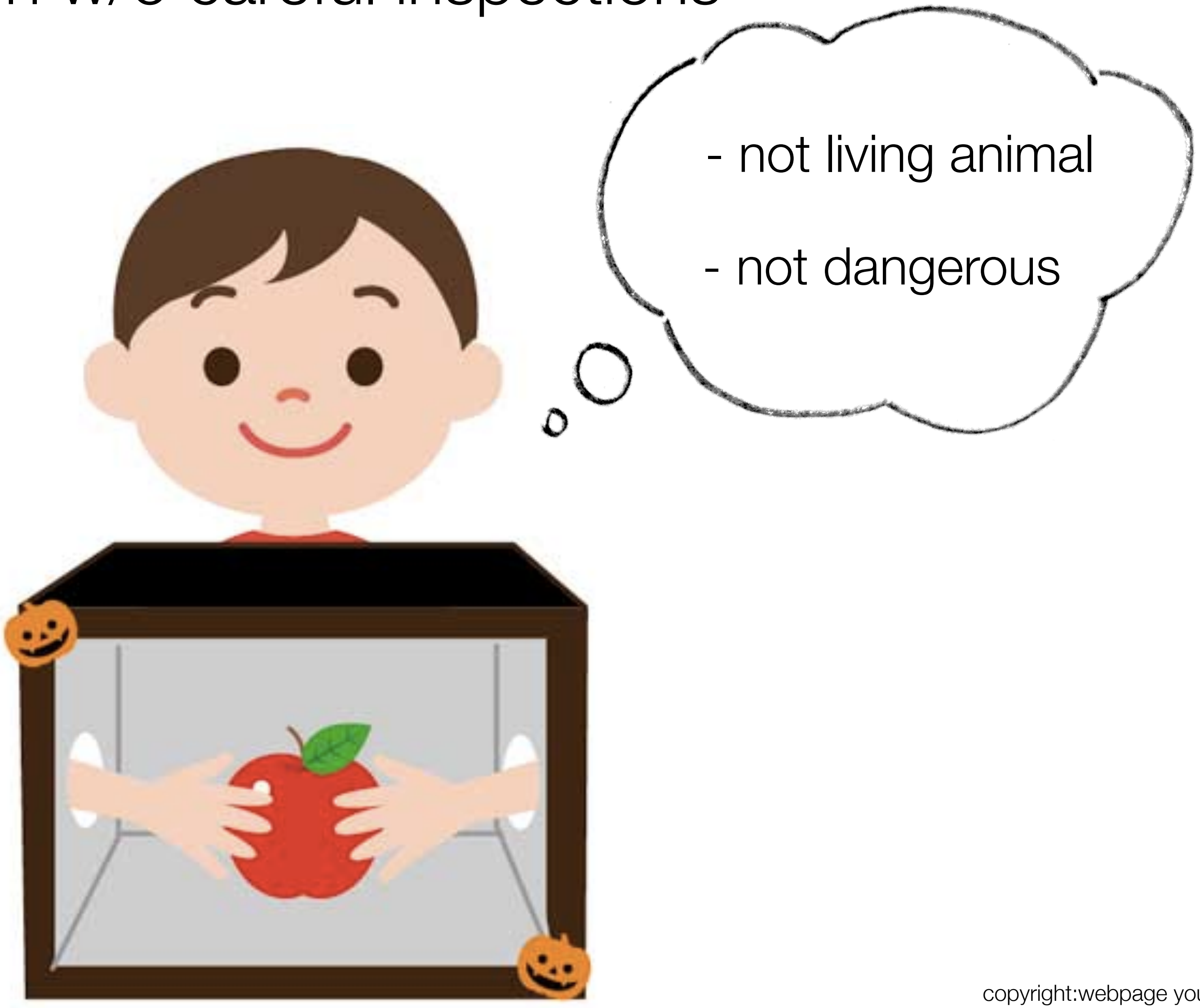
Type ME: Manifest existence of WAM → Universal IR structure (No outstanding signal)

Type HE: Hidden existence of WAM

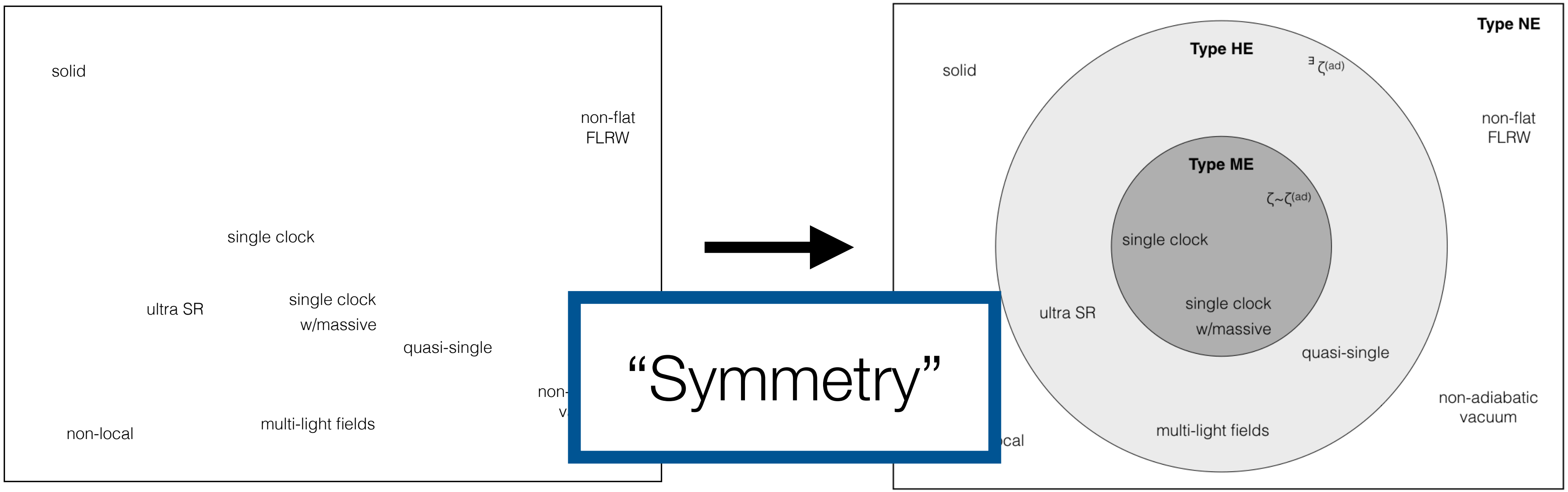
Type NE: Non-existence of WAM

Why useful?

Getting an intuition w/o careful inspections



Zoo of inflation models

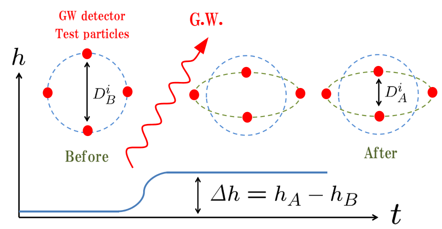


Infrared triangle

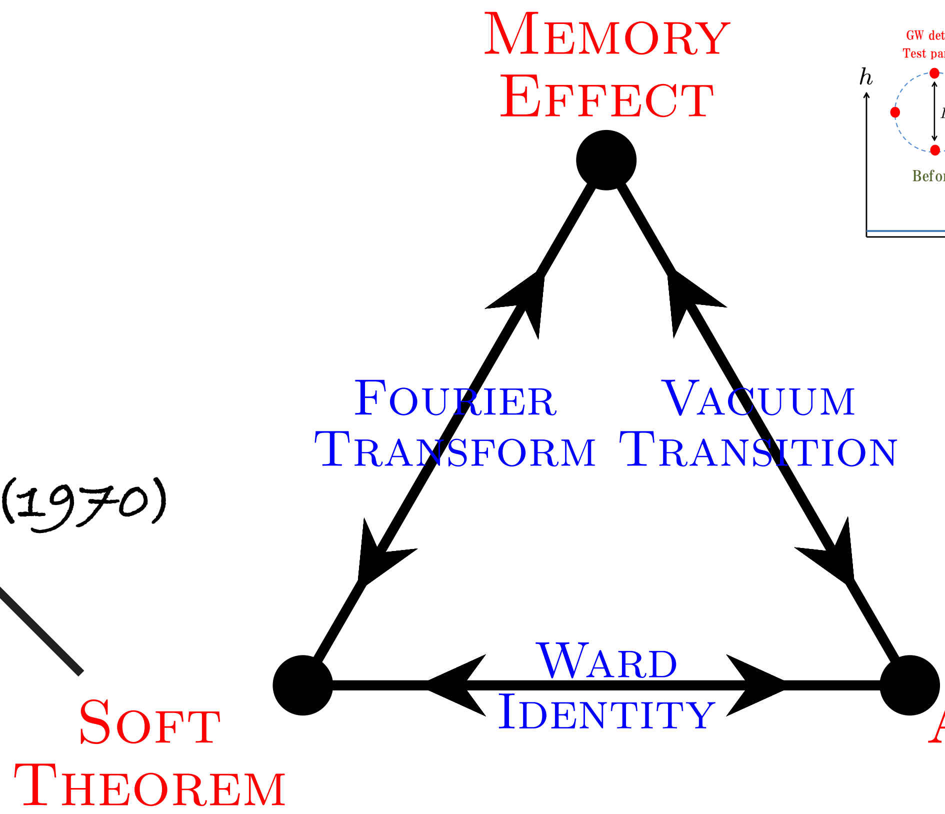
Strominger+ (13,14, ...)

review 1703.05448

Christodoulou (1991)



from talk slide of A.Ishibashi



Cancellation of IR divergence

Faddeev-Kulish (1970)

Bloch & Nordsieck (1937)

Weinberg (1965)

Bondi, Metzner, Sachs (1962)

Large gauge transformations ?

see, e.g., Harvey (96)

GTs
- Small GTs $\mathcal{G}_S$ $\quad g \to \mathbf{1}$ (in $|x| \to \infty$) $\quad g \in \mathcal{G}_S$
- Large GTs $\mathcal{G}_L$ $\quad g \not\to \mathbf{1}$ (in $|x| \to \infty$) $\quad g \in \mathcal{G}_L$

e.g. Large GTs in U(1) gauge theory

$$A^\mu(x) \to A'^\mu(x) = A^\mu(x) + \partial^\mu \lambda(x)$$

<u>Lorentz gauge</u> $\quad \partial_\mu A'^\mu(x) = 0$

- Small GTs fixed by $\quad \partial_\mu \partial^\mu \lambda(x) = -\partial_\mu A^\mu(x)$
- Large GTs $\quad \partial_\mu \partial^\mu \lambda(x) = 0$

$$\lambda(x) = \sum_{\mu_1 \cdots \mu_n} \underline{C_{\mu_1 \cdots \mu_n}}\, x^{\mu_1} \cdots x^{\mu_n}$$

symmetric traceless tensor

Large GTs in cosmology

Fixing small GTs, e.g., w/

- Time slicing $\qquad \phi(t, x) = \phi(t)$
- Spatial coordinates $\qquad \partial^i \gamma_{ij} = 0 \qquad h_{ij} = a^2 e^{2\zeta} \left[e^{\gamma}\right]_{ij}$

Yet, there are infinite number of large GTs Hinterbichler et al. (14)

e.g. Spatial dilatation $\qquad x_s^i \equiv e^s x^i \qquad$ s: constant

$$\frac{dl^2}{a^2} = e^{2\zeta(t,x)}[e^{\gamma(t,x)}]_{ij} dx^i dx^j = e^{2\zeta_s(t,x_s)+2s}[e^{\gamma_s(t,x_s)}]_{ij} dx^i dx^j$$

Weinberg's adiabatic mode (WAM)

Weinberg (03)

Extension of k=0 mode generated by large GTs to soft modes (k ≠ 0)

Dilatation $x^i \to e^s x^i$ ⟷ $\zeta_{k=0} \to \zeta_{k=0} - s$

s: const

Excitation of k=0 mode

Extension to soft mode $k_L \neq 0$

(inhomogeneous) Dilatation $x^i \to e^{s(k_L)} x^i$ ⟷ $\zeta_{k_L} \to \zeta_{k_L} - s(k_L)$

WAM

$s(k_L)$: time indep., varies in $1/k_L$

Excitation of k_L mode

* Similar argument is possible also for tensor mode.

WAM "generically" exists for perturbations around LFRW spacetime (linear classical theory). Weinberg (03)

WAM in single field model

Single field inflation model around LFRW w/4D Diff

$$S^{(2)} = \frac{1}{2}\int d\eta \int d^3\boldsymbol{x}\, z^2(\eta)\left[\zeta'^2 - c_s^2(\eta)(\partial_i\zeta)^2\right]$$

e.g., canonical scalar field $\quad z^2 = 2M_P^2 a^2 \varepsilon \quad$ $c_s=1$

$$\longrightarrow \quad \zeta_k \simeq \underbrace{c_1(k)}_{\text{WAM}} + c_2(k)\int \frac{d\eta}{z^2(\eta)} \qquad -c_s k\eta \ll 1$$

Bottom-line story

Bordin, Tanaka, & Y.U. (in preparation)

Existence of WAM = $\zeta^{(ad)}$ is a solution of quantized system

Dilatation inv.
Locality condition

$\Gamma[\zeta_k] = \Gamma[\zeta_k + \zeta^{(ad)}{}_k]$

Γ: effective action

Tanaka & Y.U. (17)

$Q_\zeta |\Psi\rangle = 0$

WT1(k=0)

WT2(others)

$x^i \to e^{s(k_L)} x^i$

Soft theorem
for $\zeta^{(ad)}$

Maldacena (02),
Creminelli & Zaldarriaga (04),.....

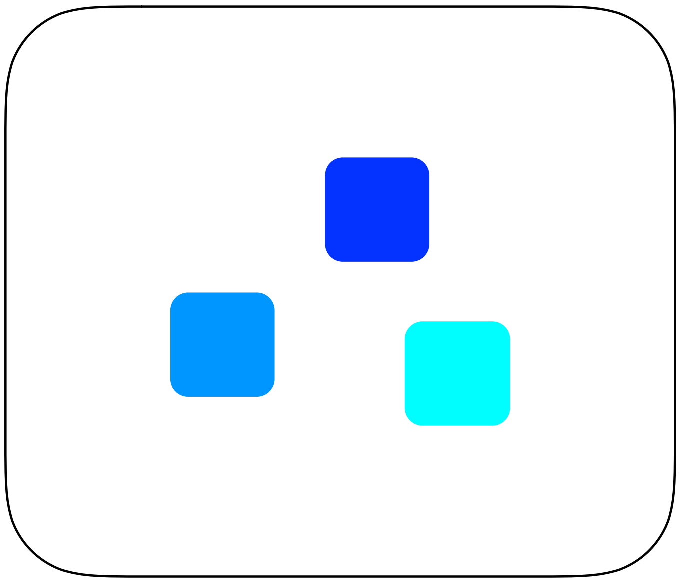
Cancellation of
IR div. of $\zeta^{(ad)}$

Extension of Weinberg's argument to non-linear quantum
Clarification of the underlined assumption

Locality condition?

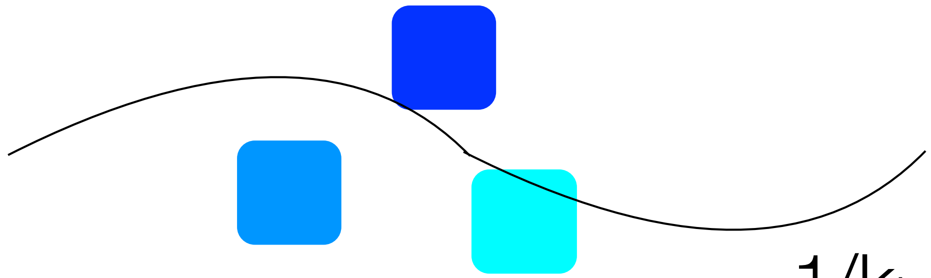
Tanaka & Y.U. (17)

$$iQ_{\zeta}^{W}(\boldsymbol{k}_L)|\Psi\rangle_{\zeta^c_{\boldsymbol{p}_L}} = s_{\boldsymbol{p}_L}\delta(\boldsymbol{k}_L+\boldsymbol{p}_L)\frac{\partial}{\partial\zeta^c_{\boldsymbol{p}_L}}|\Psi\rangle_{\zeta^c_{\boldsymbol{p}_L}}$$



1/kL

=



1/kL

1/kS

(kL << kS)

Change of the coordinates
in the local patch 1/kL

Change of the short modes
due to long mode

Just gauge effect locally

Bordin, Tanaka, & Y.U. (in preparation)

3 categories of “general” inflation models

(in FLRW)

WAM exits iff we choose a quantized model w/DI + LC

DI + LC?

Yes

Type ME (Manifest Existence):
WAM exists as the dominant mode of ζ_{kL}

Type HE (Hidden Existence):
WAM exists, but is hidden by other modes of ζ_{kL}

No

Type NE (Non Existence):
WAM does not exist.

Bordin, Tanaka, & Y.U. (in preparation)

Classification

(in FLRW)

Types	$\mathrm{DI} + \mathrm{LC}$	$\zeta_{\mathbf{k}_L} \simeq \zeta^{(\mathrm{ad})}_{\mathbf{k}_L}$	CRW	$\dot{\zeta}_{\mathbf{k}_L} \to 0$	Cancel. of IR div
Type ME	✔	✔	✔	✔	✔
Type HE	✔	x	✔	x	x (*)
Type NE	x	NA	NA	x	x

x: Leaving possibility that these can be ensured in a different way

Type ME

Tanaka & Y.U. (15, 17)

WAM exists & $\zeta_{\boldsymbol{k}} - \zeta_{\boldsymbol{k}}^{(ad)} = \mathcal{O}(1) \times \left(\frac{c_s k}{aH} \right)^p \zeta_{\boldsymbol{k}}^{(ad)}$ *p: natural number*

IR universality of WAM → CRW = CR of ζ_k

→ Time conservation of ζ_k

Ex 1) Single clock model

1 DOF in background phase space

Ex 2) Single clock model + Massive (HS) excitations

(M> H)

Type HE

WAM exists & $\zeta_{\boldsymbol{k}} - \zeta_{\boldsymbol{k}}^{(ad)} \neq \mathcal{O}(1) \times \left(\frac{c_s k}{aH}\right)^p \zeta_{\boldsymbol{k}}^{(ad)}$ *p: natural number*

IR universality of WAM → CRW, which does not describe correlation of soft mode.

$\not\rightarrow$ Time conservation of ζ_k

Ex 1) Multi-light field models (m<<H)

Ex 2) Non-attractor single field model Kinney (05),....

$$\zeta_k \simeq c_1(k) + c_2(k) \int \frac{d\eta}{z^2(\eta)}$$

second mode grows

Ex 3) Quasi-single field model Cheng & Wang (09)

Type NE

Dilatation inv. $Q_\zeta|\Psi\rangle = 0$

Locality condition $iQ_\zeta^W(\boldsymbol{k}_L)|\Psi\rangle_{\zeta^c_{\boldsymbol{p}_L}} = s_{\boldsymbol{p}_L}\delta(\boldsymbol{k}_L+\boldsymbol{p}_L)\frac{\partial}{\partial\zeta^c_{\boldsymbol{p}_L}}|\Psi\rangle_{\zeta^c_{\boldsymbol{p}_L}}$

Types	LC	Dilatation inv.
Type NE-d	x	✔
Type NE-l	x	x

Type NE-d

WAM does not exist, because the dilatation inv. is violated.

$$Q_\zeta | \Psi \rangle \neq 0$$

Ex 1) No spatil Diff.

Ex 2) Non-flat FLRW background

Spatial Diff. $\not\to$ Dilatation inv.

Ex 3) Solid inflation

Enlich, Nicholas and Wang (11, 12)

N=N[ζ, …] changes under the dilatation.

Dilatation $\neq$ Dilatation described by Q_ζ

Type NE-I

WAM does not exist, because the dilatation inv. holds, but the locality condition is violated.

Ex 1) Non-local theory

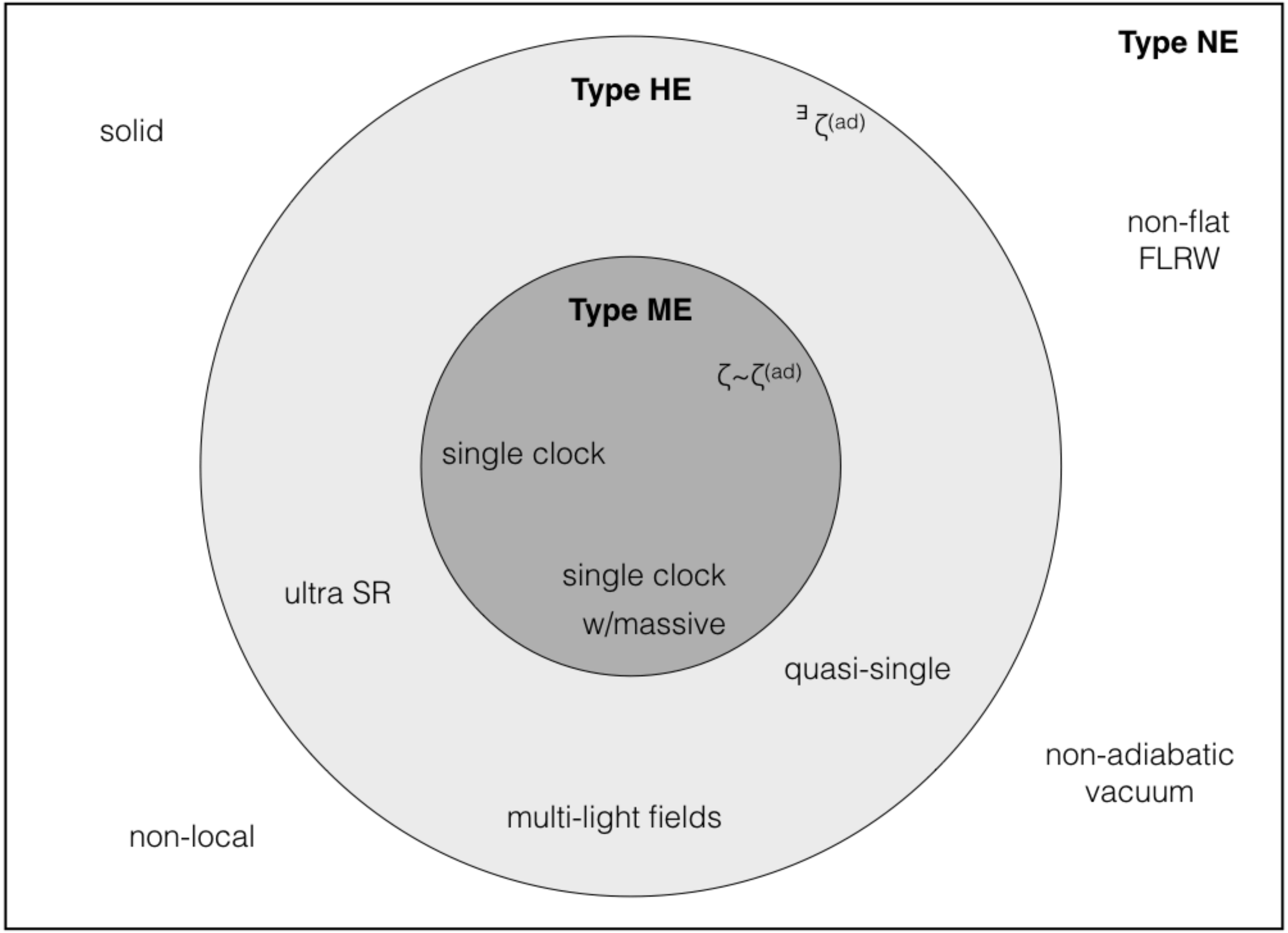
e.g., $c_s \gg 1$

Ex 2) Non-adiabatic vacuum

Gong & Sasaki (13)

Sorting out Zoo more tidily

Bordin, Tanaka, & Y.U. (in preparation)



Summary

- IR structure of ζ/γ_{ij} is much richer than the one for gauge theories in asymptotically spacetimes.

- WAM has IR universality: CRW, Cancellation of IR div.

- ”General" inflation models can be categorized into

  ME, HE, or NE