θ_{23} and δ can be measured accurately at the same time

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All the angles are measured !
lepton CP phase δ left

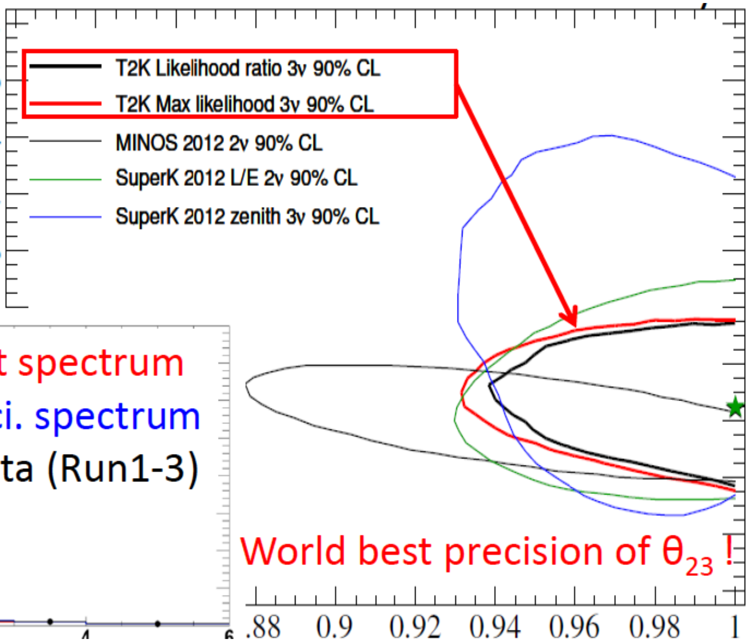
$\nu_\alpha = U_{\alpha i}\, \nu_i$

$$U_{MNS} = U_{23}U_{13}U_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{23} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

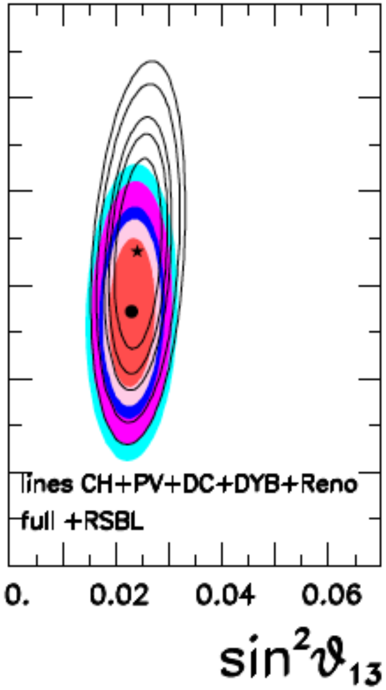
SK-atm+MINOS+T2K

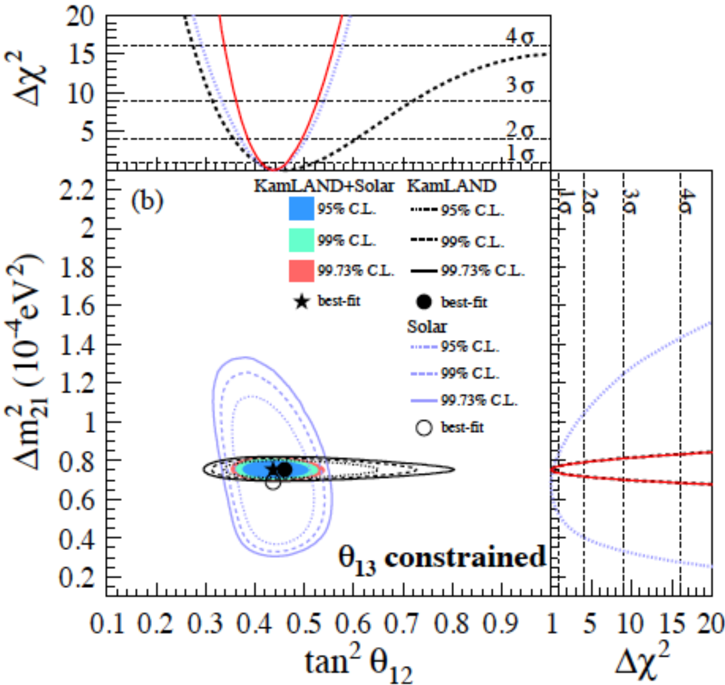
T2K-MINOS-DC-DB-RENO

solar+KamLAND



World best precision of θ_{23} !





θ_{13} constrained

θ_{12} and θ_{13}

θ_{12} and θ_{13} are already measured ~ rather accurately

$$\tan^2\theta_{12} = 0.436^{+0.029}_{-0.025},\ \Delta m^2_{21} = 7.53^{+0.18}_{-0.18}\times 10^{-5}\,\mathrm{eV}^2,$$

KamLAND Mar 2013

- $\sin^2\theta_{12}$ = 0.304 +/- 0.013
- Error of $\sin^2\theta_{12}$ = 4.3%
- Error of Δm^2_{21} = 2.4%

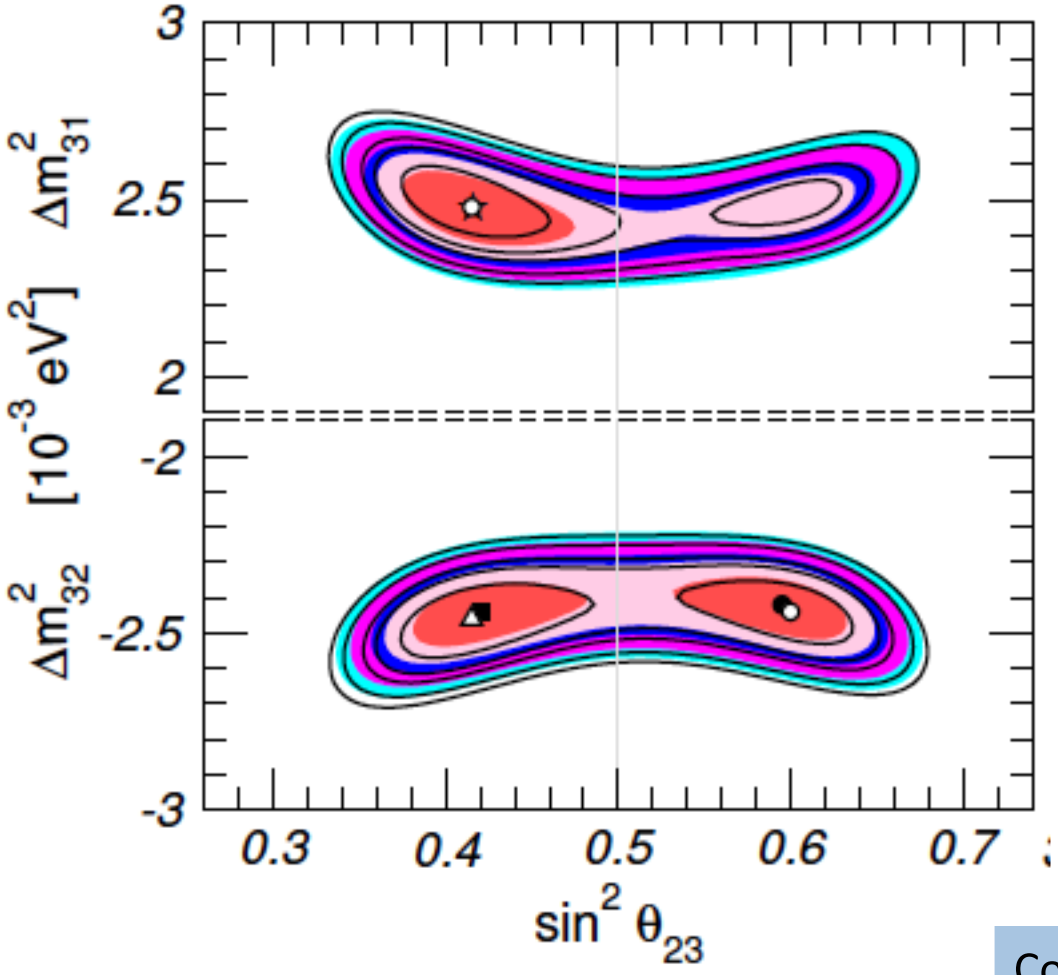
$$\boxed{\sin^2 2\theta_{13} = 0.089 \pm 0.010(\mathrm{stat.}) \pm 0.005(\mathrm{syst.})}$$

- If added quadrature, $\sin^2 2\theta_{13}$ = 0.089 +/- 0.011
- $\sin^2\theta_{13}$ = 0.0228 +/- 0.0029
- Error of $\sin^2\theta_{13}$ = 13% ➔ ~ 5% if syst. only

Daya Bay Nov. 2012

Accuracy of θ_{23} is not great

θ_{23}: mixing angle with the largest error of 10-20% level

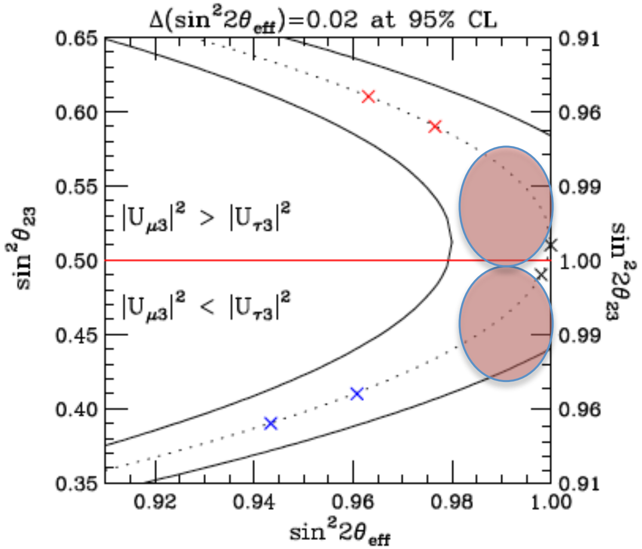


Concha etal. JHEP 2012

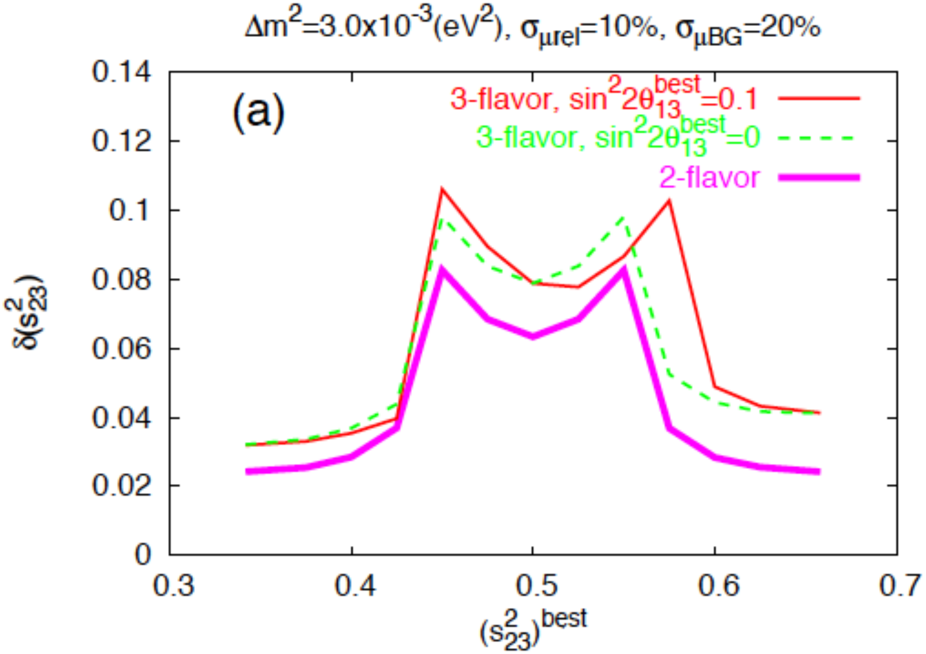
Large error of θ_{23} is robust because ..

- Jacobian effect:

$$\frac{\delta(s_{23}^2)}{\delta(\sin^2 2\theta_{23})} \simeq \frac{ds_{23}^2}{d\sin^2 2\theta_{23}} = \frac{1}{4\cos 2\theta_{23}},$$



- Octant degeneracy + exp. uncertainty ➔ merging of clone solution to true one

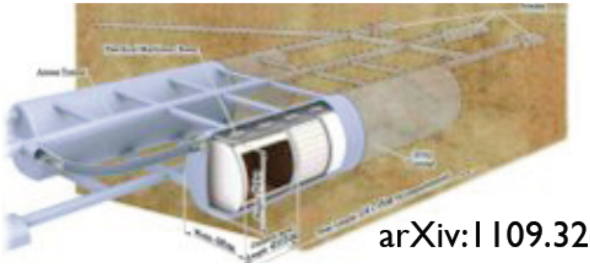


HM-Sonoyama-Sugiyama
PRD 2004 ➔

Hyper-K is of course for CP δ, but it is intimately connected to θ_{23}

HK seems champion about CP δ

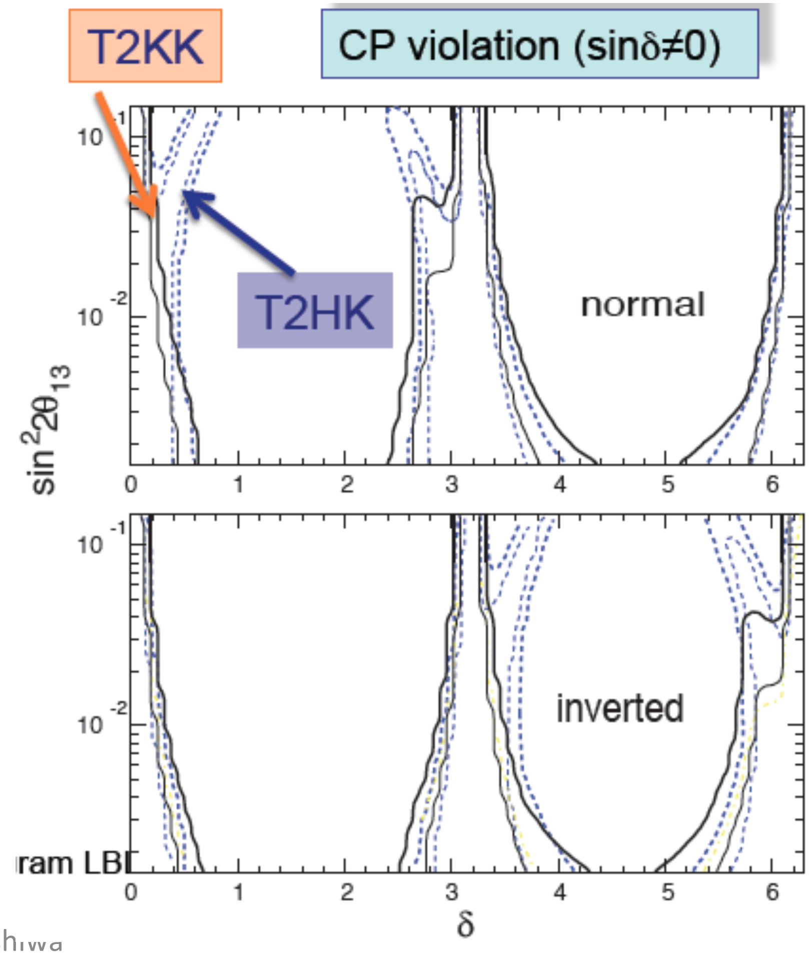
Hyper-Kamiokande (JP)

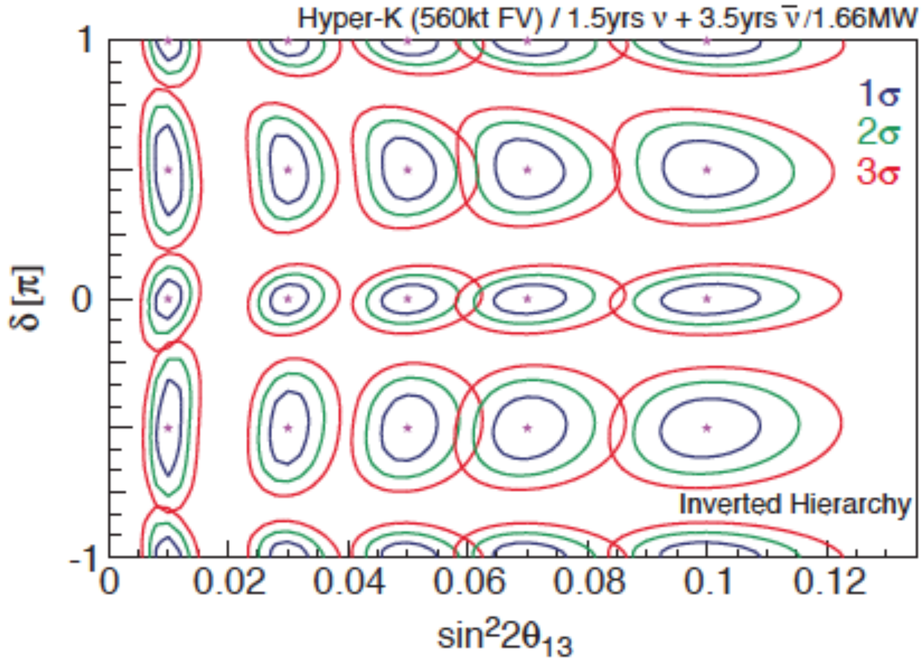


arXiv:1109.3262

- ν_μ superbeam appears to be the prime candidate
- T2KK CP sensitivity ~ T2HK CP sensitivity

Total mass of the detectors = 0.54 Mton fid. mass
4 years neutrino beam + 4 years anti-neutrino beam





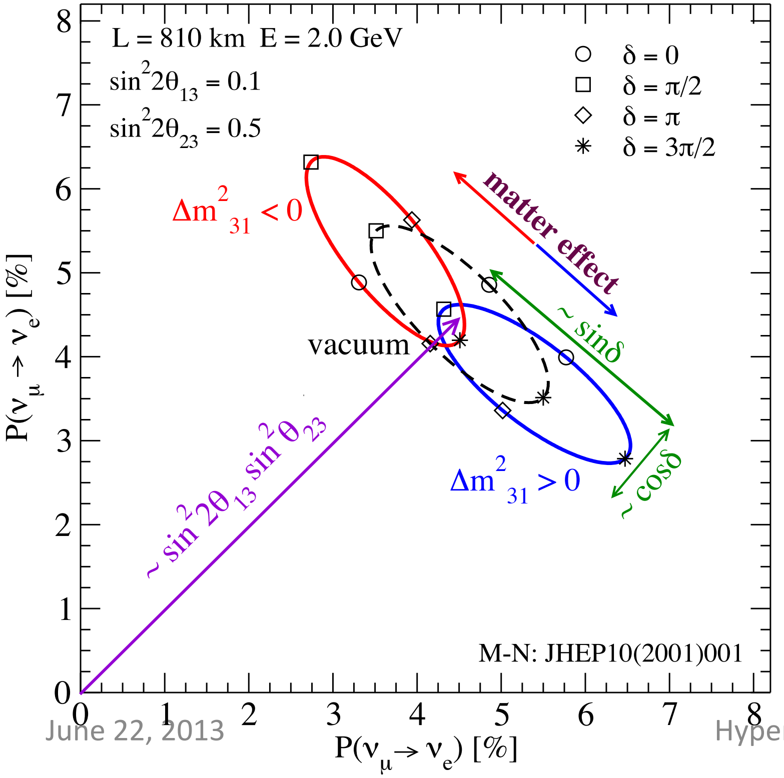
Why θ_{23} relevant for CP?

- In precision era for CP the errors of δ will be dominated by uncertainty of s^2_{23}
- Δ sinδ =
- Δ cosδ =

$$\sqrt{(\Delta P/P)^2 + (\Delta \bar{P}/\bar{P})^2 + \eta(\Delta s^2_{23}/s^2_{23})^2}$$

Error formula for CP δ
η = order unity

HM-Parke, ArXiv 1303.6178

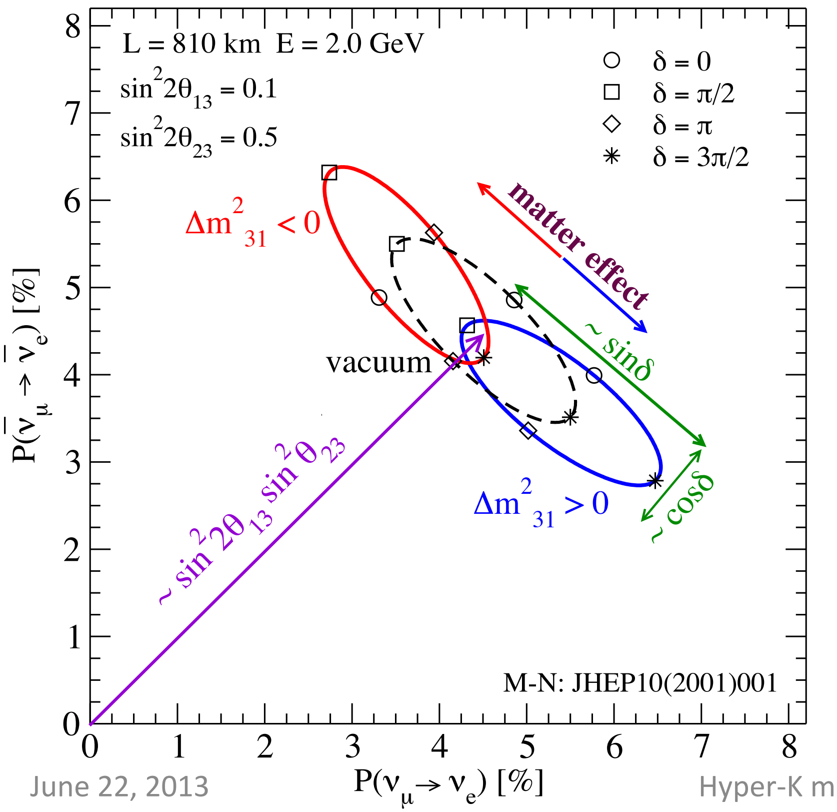


For a given set of (P, P-bar) with finite error, a change in s^2_{23} can be compensated by adjusting δ

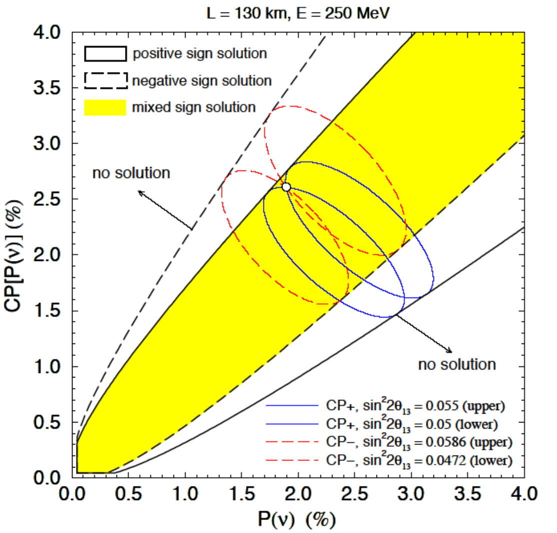
A new method for θ_{23} and δ: HK is the best machine for θ_{23}

Simultaneous θ_{23}– δ determination

What you need is to switch θ_{13} ➔ θ_{23}
everything goes through as in (θ_{13}–δ) case



For a given set of (P, P-bar) there are 2x2=4 solutions of θ_{23} and δ



1st Oscillation maximum

Things are much simpler in oscillation maximum

$$\begin{aligned} P_{\mu e} &= 2s_{23}^2 A_{\oplus}^2 - 2\epsilon \sin 2\theta_{23} A_{\oplus} A_{\odot} \sin\delta + 2c_{23}^2 \epsilon^2 A_{\odot}^2 \\ \bar{P}_{\mu e} &= 2s_{23}^2 \bar{A}_{\oplus}^2 + 2\epsilon \sin 2\theta_{23} \bar{A}_{\oplus} A_{\odot} \sin\delta + 2c_{23}^2 \epsilon^2 A_{\odot}^2 \end{aligned}$$

$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4E} \text{ and } \epsilon \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \simeq 0.03$$

$$\begin{aligned} A_{\oplus} &\equiv \sqrt{2} s_{13} c_{13} \left(\frac{\Delta m_{31}^2}{\Delta m_{31}^2 - a} \right) \sin\left(\frac{(\Delta m_{31}^2 - a)L}{4E} \right) \\ \bar{A}_{\oplus} &\equiv \sqrt{2} s_{13} c_{13} \left(\frac{\Delta m_{31}^2}{\Delta m_{31}^2 + a} \right) \sin\left(\frac{(\Delta m_{31}^2 + a)L}{4E} \right) \\ A_{\odot} &\equiv \sqrt{2} c_{12} s_{12} c_{13} \left(\frac{\Delta m_{31}^2}{a} \right) \sin\left(\frac{aL}{4E} \right) = \bar{A}_{\odot} \end{aligned}$$

1st Oscillation maximum (continued)

To first order in $\frac{a}{\Delta m^2_{31}}$,

$$\begin{aligned}(s^2_{23})_0 &= \frac{1}{8s^2_{13}}\left[(\bar{P}+P)+\frac{a}{\Delta m^2_{31}}(\bar{P}-P)\right]\\ \sin\delta_0 &= \frac{1}{8\epsilon J_r\pi}\left[(\bar{P}-P)+\frac{2a}{\Delta m^2_{31}}(\bar{P}+P)\right]\end{aligned}$$

the errors, $\Delta(s^2_{23})$ and $\Delta(\sin\delta)$, are given by

$$\begin{aligned}\Delta(s^2_{23}) &\approx \frac{1}{8s^2_{13}}\sqrt{(\Delta P)^2+(\Delta\bar{P})^2}\\ \Delta(\sin\delta) &\approx \frac{1}{8\epsilon J_r\pi}\sqrt{(\Delta P)^2+(\Delta\bar{P})^2}\end{aligned}$$

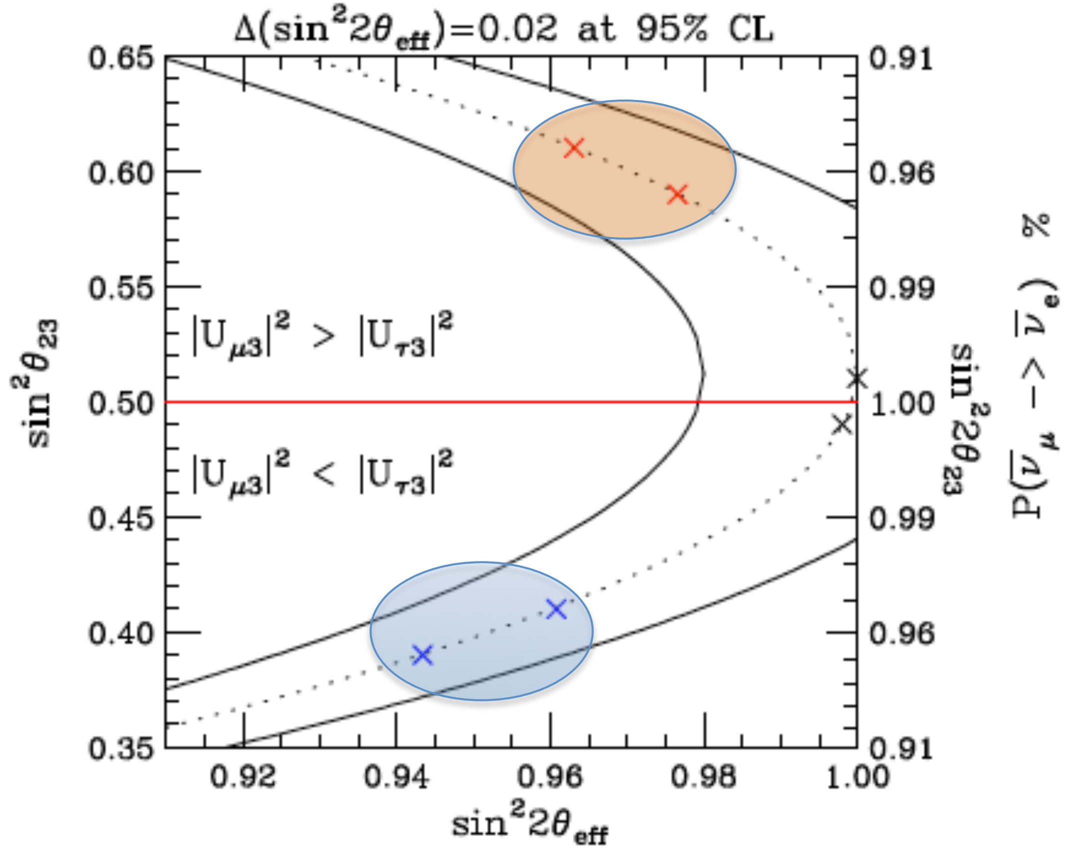
In near vacuum environment

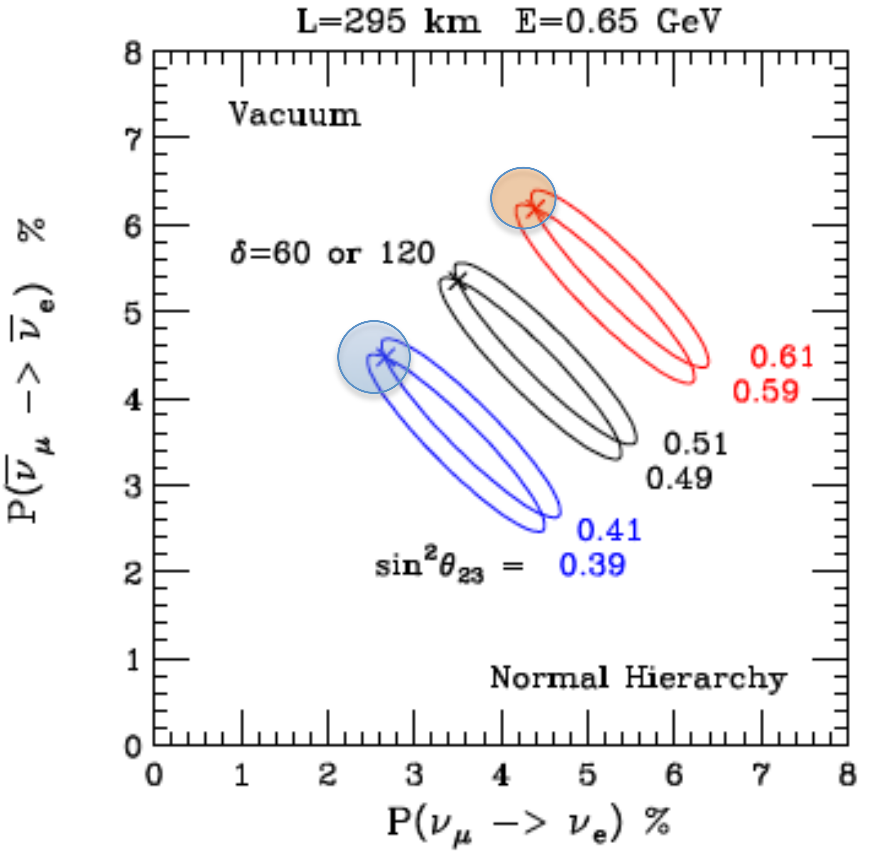
HM-Parke, ArXiv 1303.6178

$$\Delta(s^2_{23}) \approx \frac{\pi\epsilon J_r}{s^2_{13}}\Delta(\sin\delta) \approx \frac{1}{6.6}\Delta(\sin\delta)$$

$J_r \equiv c_{12}s_{12}c_{23}s_{23}s_{13} \simeq 0.034$, $\epsilon \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \simeq 0.033$ and $s^2_{13} \simeq 0.0228$.

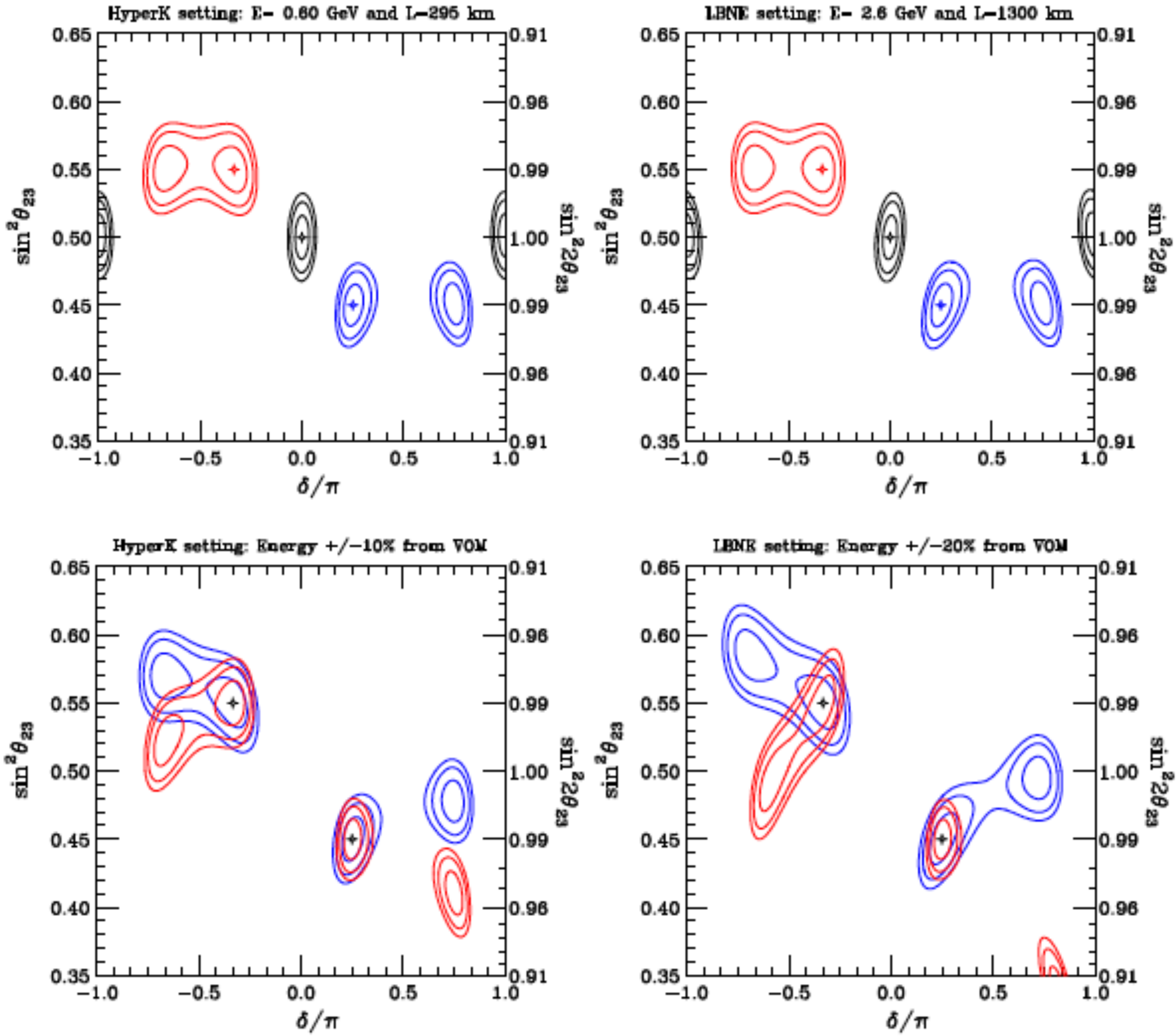
Use ν_e appearance channel





- simultaneous $\delta - \theta_{23}$ determination by ν_e and ν_e-bar appearance channels
- θ_{23} intrinsic degeneracy

Breaking θ_{23}– δ degeneracy



At 1st VOM

Spectrum information must solve the degeneracy!

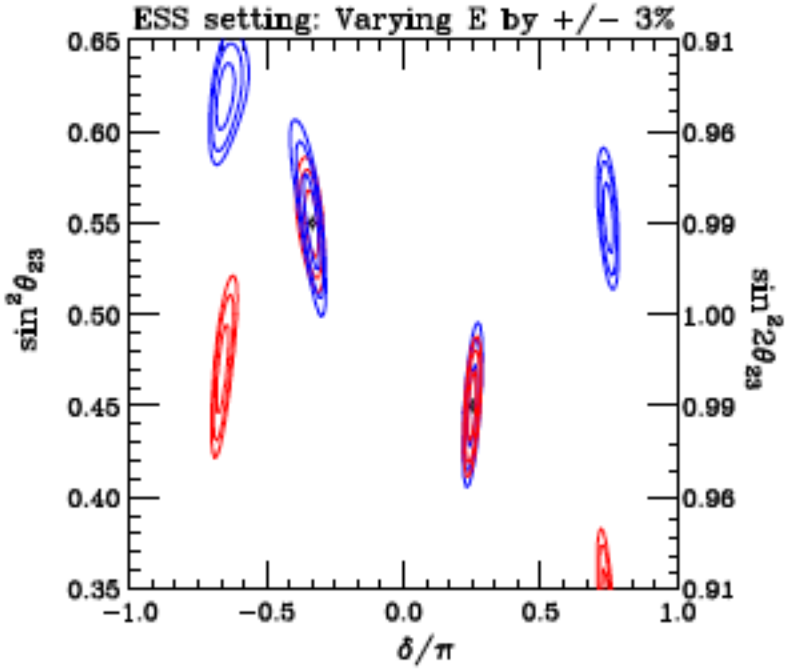
Off 1st VOM

Hyper-K: ΔE= +/- 10%:

LBNE: ΔE= +/- 20%:

2nd vacuum oscillation maximum

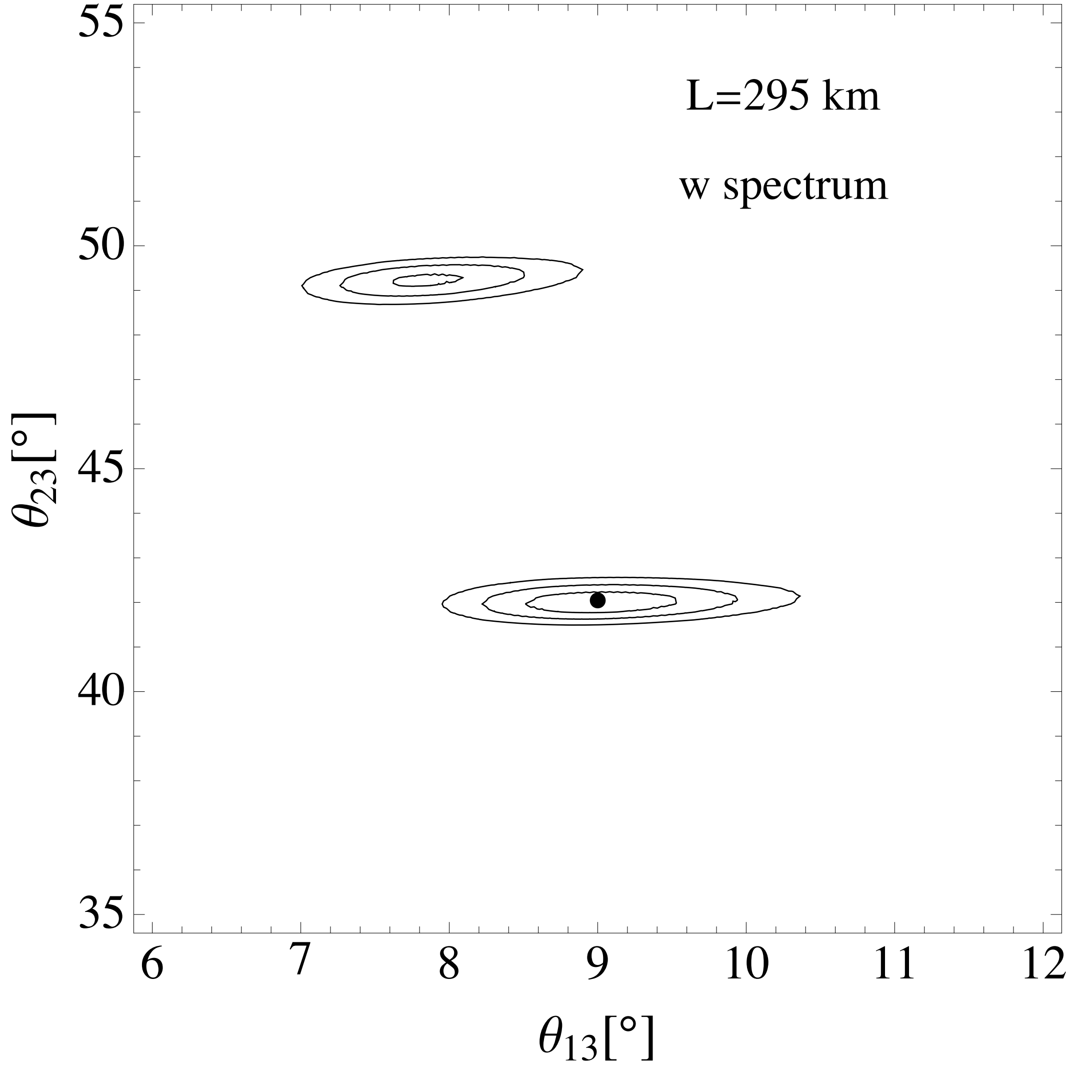
At 2nd VOM, Δ=3π/2



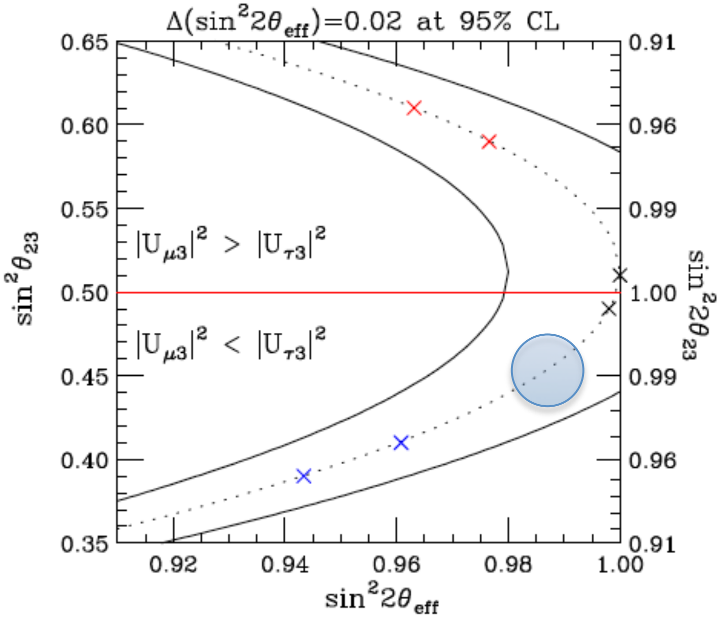
ΔE= +/- 3%: Movement of clone solution is more prominent at 2nd VOM

FIG. 6: Contours of allowed regions at 1σ, 2σ, and 3σ CL (2 d.o.f.) in δ/π vs. $\sin^2\theta_{23}$ space for ESS setting with $L = 540$ km. Two particular values of neutrino energies are taken for comparison, one at $E = 0.342$ GeV (shown in red) and $E = 0.364$ GeV (shown in blue), which amount to $\pm3\%$ of the one at 2nd VOM. The true values of (δ, $\sin^2\theta_{23}$) are taken as (45°, 0.45) and (−60° 0.55).

Hyper-K result (too good?)



Coloma-HM-Parke in progress



Conclusion

- mixing angles θ_{12} are already measured in a good precision
- θ_{13} precision will reach a comparable level soon
- It is very difficult to improve θ_{23} precision (T2K will not resolve this issue)
- In precision era for CP δ, things change completely: you automatically get θ_{23} precision!

PUC-Rio, where?

Maracana