

Review of Theoretical Ideas on Neutrino Masses and Mixing

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"Prospects of Neutrino Physics," IPMU, April 2019



A wealth of discoveries in neutrino physics since 1998...





(image credit: C. Wiens)

See Gonzalez-Garcia's very nice summary of recent history (1902.04583)

Some highlights:

- 1998: atmospheric ν_{μ} disappearance (SK)
- 2002: solar ν_e disappearance (SK)
- 2002: solar ν_e appear as ν_μ, ν_τ (SNO)
- 2004: reactor $\overline{\nu}_e$ oscillations (KamLAND)
- 2004: accelerator ν_{μ} disappearance (K2K)
- 2006: accelerator ν_{μ} disappearance (MINOS)

- 2011: accelerator ν_{μ} appear as ν_{e} (T2K,MINOS)
- 2012: reactor $\overline{\nu}_e$ disappear (Daya Bay, RENO) reactor angle measured!
- (T2K) 2014: hint for CP violation?
- 2015: hints for normal hierarchy? (SK, T2K, NOvA)
- 2016: hint for non-maximal atm mixing? (NOvA)
- 2018: trivial Dirac phase disfavored at 2σ (T2K)

Signals physics beyond the Standard Model (SM)!

The emergent picture... a (seemingly) robust 3-neutrino mixing scheme





Global Fits:

Forero et al., '17 Capozzi et al.,'18 Gonzalez-Garcia et al., (<u>www.nu-fit.org</u>)

			Normal Ordering (best fit)		Inverted Ordering ($\Delta \chi^2 = 4.7$)		
				bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
			$\sin^2 heta_{12}$	$0.310\substack{+0.013\\-0.012}$	$0.275 \rightarrow 0.350$	$0.310\substack{+0.013\\-0.012}$	$0.275 \rightarrow 0.350$
	- 0		$\theta_{12}/^{\circ}$	$33.82\substack{+0.78\\-0.76}$	$31.61 \rightarrow 36.27$	$33.82\substack{+0.78\\-0.76}$	$31.61 \rightarrow 36.27$
archy)			$\sin^2 heta_{23}$	$0.580\substack{+0.017\\-0.021}$	$0.418 \rightarrow 0.627$	$0.584^{+0.016}_{-0.020}$	$0.423 \rightarrow 0.629$
			$\theta_{23}/^{\circ}$	$49.6^{+1.0}_{-1.2}$	$40.3 \rightarrow 52.4$	$49.8^{+1.0}_{-1.1}$	$40.6 \rightarrow 52.5$
			$\sin^2 heta_{13}$	$0.02241\substack{+0.00065\\-0.00065}$	0.02045 ightarrow 0.02439	$0.02264\substack{+0.00066\\-0.00066}$	$0.02068 \to 0.02463$
		-atm	$\theta_{13}/^{\circ}$	$8.61^{+0.13}_{-0.13}$	$8.22 \rightarrow 8.99$	$8.65\substack{+0.13\\-0.13}$	$8.27 \rightarrow 9.03$
		tt SK-	$\delta_{ m CP}/^{\circ}$	215^{+40}_{-29}	$125 \rightarrow 392$	284^{+27}_{-29}	$196 \to 360$
		withou	$\frac{\Delta m^2_{21}}{10^{-5}~{\rm eV^2}}$	$7.39\substack{+0.21 \\ -0.20}$	$6.79 \rightarrow 8.01$	$7.39\substack{+0.21 \\ -0.20}$	$6.79 \rightarrow 8.01$
NuFit, M	Nov 2018		$\frac{\Delta m^2_{3\ell}}{10^{-3}~{\rm eV}^2}$	$+2.525^{+0.033}_{-0.032}$	$+2.427 \rightarrow +2.625$	$-2.512\substack{+0.034\\-0.032}$	-2.611 ightarrow -2.412

Caveat: sterile neutrino(s)?

(image credit: ParticleBites)

Anomalies in the data:

1995: $\overline{\nu}_e$ appearance (LSND)2007: $\overline{\nu}_e$ appearance (MiniBooNE)2012: ν_e appearance (MiniBooNE)1995: ν_e disappearance (Gallium)2011: ν_e disappearance (Reactor)

[lots of results, investigation in the interim...]

[well-documented tension between appearance and disappearance data]



See Huber's IPA 2017 talk for "scorecard" Maltoni's talk at Neutrino 2018 Talks at this workshop...

For this talk, focus on 3 active families only...

New questions, excitement for BSM physics!



SM $\rightarrow \nu$ SM

Implications for the SM flavor puzzle:

what is the origin of the quark and lepton masses and mixings?

Goal: a satisfactory and credible theory of flavor *(very difficult!)*

Many questions:

Majorana or Dirac neutrinos? Nature of neutrino mass suppression? Mass hierarchy? Lepton mixing angle pattern? CP violation? Implications for BSM paradigms? Connections to other new physics (NP)?

Mass Generation

Quarks, Charged Leptons

"natural" mass scale tied to electroweak scale

Dirac mass terms, parametrized by Yukawa couplings



$$Y_{ij}H \cdot \bar{\psi}_{Li}\psi_{Rj} \longrightarrow \mathcal{M}_u, \mathcal{M}_d, \mathcal{M}_e$$

top quark: O(1) Yukawa coupling rest: suppression (flavor symmetry)

Neutrinos

Main question: origin of neutrino mass suppression

Options: Dirac

 $\Delta L = 0$





Majorana first: $\Delta L = 2$

advantages: naturalness, leptogenesis, $0\nu\beta\beta$

SM at NR level: Weinberg dimension 5 operator

 $\frac{\lambda_{ij}}{-}L_iHL_iH$

if $\lambda \sim O(1)$ $\Lambda \gg m \sim O(100 \,\text{GeV})$ (but wide range possible)

Underlying mechanism:



- a. Type I seesaw ν_R (fermion singlet)
- **b. Type II seesa** Δ (scalar triplet)
- **c. Type III seesaw** \sum (fermion triplet)



³ tree-level options

Prototype: Type I seesaw

Type I: Minkowski; Yanagida; Gell-Mann, Ramond, Slansky; Mohapatra, Senjanovic;...

Right-handed neutrinos:



(image credit: T. Ohlsson et al., Nat. Comm.)

$$Y_{ij}L_i\nu_{Rj}H + M_{R\,ij}\nu_{Ri}\nu_{Rj}^c$$
$$\mathcal{M}_{\nu} \sim \langle H \rangle^2 Y M_R^{-1}Y^T$$
$$\mathcal{M}_{\nu} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \qquad \begin{array}{l} m \sim \mathcal{O}(100 \,\text{GeV}) \\ M \gg m \end{array}$$

$$m_1 \sim \frac{m^2}{M}$$
 $m_2 \sim M \gg m_1$ $\nu_{1,2} \sim \nu_{L,R} + \frac{m}{M} \nu_{R,L}$

•

advantages: naturalness, connection to grand unification, leptogenesis,... disadvantage: testability without model assumptions

Other tree-level seesaws

(image credits: T. Ohlsson et al., Nat. Comm.)



advantage: testability usually accompanied by new EW charged states — visible at LHC? disadvantages: naturalness, economy (subjective)

Type II: Konetchsy, Kummer; Cheng, Li; Lazarides, Shafi, Wetterich; Schecter, Valle; Mohapatra et al,; Ma;... Type III: Foot, He, Joshi; Ma;...

Zee; Babu; Ma; Gustafsson, No, Rivera;...

Radiative neutrino mass generation:

complete Weinberg operator via loops

(loop suppression factor aids in overall mass suppression)

A canonical example: "scotogenic" model



(image credit: T. Ohlsson et al., Nat. Comm.)

introduce new electroweak doublet(s) and right-handed neutrinos

$$\mathcal{M}_{\nu} \sim \lambda \frac{\langle H \rangle^2}{16\pi^2} Y M_R^{-1} Y^T$$

(e.g. new states can be DM candidates)

Generic advantage of radiative models: testability

Radiative neutrino mass generation:

can have other NR operators in SM with $\Delta L = 2$

Babu and Leung '01 de Gouvea and Jenkins '07

(odd mass dimension d>5)

d=7	d=9
$LLLe^{c}H$	$LLLe^{c}Le^{c}$ (Zee, Babu)
$LLQd^{c}H$	$LLQd^cQd^c$
$LL\overline{Q}\overline{u}^{c}H$	+ many others
$L\overline{e}^{c}\overline{u}^{c}d^{c}H$	NP scale can be accessible at LHC (subject to LFV bounds)

Possible connection with flavor physics anomalies...

One way leptoquarks can manifest themselves:

Päs and Schumacher, '15 Deppisch et al., '16 ...

<u>A two-loop example:</u>

scalar leptoquark $\phi \sim (\mathbf{3}, \mathbf{1}, -1/3)$ + octet fermion $f \sim (\mathbf{8}, \mathbf{1}, 0)$



Cai, Gargalones, Schmidt, Volkas '17

Many other ideas for Majorana neutrino masses...



more seesaws (double, inverse,...), SUSY with R-parity violation, RS models...

lepton number violation

Majorana ν masses

Dirac neutrino masses:

Require strong suppression $Y_{\nu} \sim 10^{-14}$

Less intuitive, but mechanisms exist...

radiative masses, extra dimensions, extended gauge sectors (non-singlet ν_R), SUSY breaking, string instanton effects,...



See e.g. Hagedorn and Rodejohann, '05 Many other studies: Ma; Mohapatra, Senjanovic,....

General themes:

Much richer than quark and charged lepton sectors. Trade-off between naturalness and testability.

Lepton mixings

diagonal phase matrix (Majorana neutrinos)

$ u_i$	$\mathcal{U}_{\mathrm{MNSP}}$	=
	$(\mathcal{U}_{\mathrm{MNSP}})_{ij}$	
	$\searrow \bigvee \bigvee W^{\pm}$	=
e_j	Pontecorvo; Maki, Nakagawa, Sakata	

_	\mathcal{R}_1	$(\theta_{23})\mathcal{R}_{23}$	$_{2}(heta_{13},\delta)$	$)\mathcal{R}_{3}(heta_{12})\mathcal{P}$				
_	$\left(\begin{array}{c}1\\0\\0\end{array}\right)$	$0 \\ \cos \theta_{23} \\ -\sin \theta_{23}$	$\begin{pmatrix} 0\\ \sin\theta_{23}\\ \cos\theta_{23} \end{pmatrix}$	$ \begin{pmatrix} \cos\theta_{13} \\ 0 \\ -\sin\theta_{13}e^{-i\delta} \end{cases} $	$\begin{array}{c} 0 \\ 1 \\ 0 \end{array}$	$ \begin{array}{c} \sin\theta_{13}e^{i\delta} \\ 0 \\ \cos\theta_{13} \end{array} \right) $	$ \begin{pmatrix} \cos \theta_{12} \\ -\sin \theta_{12} \\ 0 \end{pmatrix} $	$\left(\begin{array}{c}0\\0\\1\end{array}\right)\mathcal{P}$

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NuFit, Nov 2018

Compare quarks:

$$u_{i} \qquad (\mathcal{U}_{\text{CKM}})_{ij} \qquad \mathcal{U}_{\text{CKM}} = \mathcal{R}_{1}(\theta_{23}^{\text{CKM}})\mathcal{R}_{2}(\theta_{13}^{\text{CKM}}, \delta_{\text{CKM}})\mathcal{R}_{3}(\theta_{12}^{\text{CKM}})$$

$$d_{j} \qquad \qquad \mathcal{U}_{\text{CKM}} = 13.0^{\circ} \pm 0.1^{\circ} = \theta_{C} \qquad \text{(Cabibbo angle)}$$

$$\theta_{23}^{\text{CKM}} = 2.4^{\circ} \pm 0.1^{\circ}$$

$$\theta_{13}^{\text{CKM}} = 0.2^{\circ} \pm 0.1^{\circ} \qquad \textbf{3 "small" angles, 1 O(1) phase}$$

$$\delta_{\text{CKM}} = 60^{\circ} \pm 14^{\circ}$$

Lepton mixings

$\mathcal{U}_{\mathrm{MNSP}} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13},\delta)\mathcal{R}_3(\theta_{12})\mathcal{P}$

Certainly two large mixing angles: θ_{23} , θ_{12}

Dirac phase: too soon to say, but intriguing hints... **Majorana phases**: unlikely to know anytime soon

A basic question: is θ_{13} "large" or "small"?



Neutrino anarchy

 \mathcal{U}_{ν} from a random draw of unbiased distribution of 3x3 unitary matrices statistical tests: lower bound on $|\mathcal{U}_{e3}|^2$

Basis independence:

distribution invariant under unitary transformations

flat in Haar measure

Haba and Murayama '00

Post-reactor angle measurement: renewed focus

de Gouvea and Murayama '12 Altarelli et al. '12, Bai and Torroba '12,...

Some recent highlights:

RG analysis Brdar, Konig, Kopp '15

Model-building + quark sector Babu et al. '16,...

Fortin et al. '17

Anarchy hypothesis alone does not provide information on Δm^2



(character: Watterson)

Family symmetries (structure)

Idea: postulate family (horizontal) symmetry G_f

(image credit: T. Ohlsson, KTH)

Usual paradigm:



different subgroups preserved in neutrino and charged lepton sectors



Focus here on this case.

But see recent interesting work on lepton sector in symmetric limit

Reyiumaji and Romanino, '18

Spontaneously broken family symmetries



Quarks:

small mixings and hierarchical masses:

continuous family symmetry

both Abelian and non-Abelian: many examples!

 $\mathcal{M}_u, \mathcal{M}_d$ approx diagonalized by same unitary transformation (can choose basis w/both approx diagonal)

$$\mathcal{U}_{\text{CKM}} = \mathcal{U}_u \mathcal{U}_d^{\dagger} \sim 1 + \mathcal{O}(\lambda) \qquad \lambda \sim \frac{\varphi}{M}$$

Wolfenstein parametrization: $\lambda \equiv \sin \theta_c = 0.22$

suggests Cabibbo angle (or some power) as a flavor expansion parameter

Leptons:

charged leptons: hierarchical masses — *imilar strategy*? But now, in basis where \mathcal{M}_e is diagonal, \mathcal{M}_{ν} is not diagonal: \mathcal{M}_{ν} diagonalization requires 1 small, 2 large mixing angles!

Arguably the **most challenging*** pattern:

(* for three families)

relatively straightforward

at leading order

- 3 small angles $\longrightarrow \sim \text{diagonal } \mathcal{M}_{\nu}$ 1 large, 2 small $\longrightarrow \sim \text{Rank}\mathcal{M}_{\nu} < 3$
- 3 large angles \longrightarrow anarchical \mathcal{M}_{ν}

1 small, 2 large \longrightarrow fine-tuning, non-Abelian



A model-building opportunity!

Lepton mixings:

No unique theoretical starting point for the flavor expansion!

$$\begin{aligned} \mathcal{U}_{\mathrm{MNSP}} \sim \mathcal{W} + O(\lambda') & \text{flavor expansion} \\ & & \text{mixing angles } (\theta_{12}^{\nu}, \theta_{23}^{\nu}, \theta_{13}^{\nu}) & \text{parameter} \\ & & \text{(diagonal charged lepton basis)} \end{aligned}$$

"Bare" mixing angles generically shift due to $O(\lambda')$ corrections

A priori, expansions in quark and lepton sectors unrelated.

Unification paradigm (broad sense): set $\lambda' = \lambda_C$

ideas of quark-lepton complementarity and "Cabibbo haze"

Raidal '04, Minakata+Smirnov '04, many others... ("haze" terminology from Datta, L.E., Ramond '05)

Pre-measurement, speculation that reactor angle is a Cabibbo effect

Vissiani '98, '01 Ramond '04

$$\theta_{13}^{\nu} = 0 \qquad \qquad \theta_{13} \sim \frac{\lambda_C}{\sqrt{2}}$$

Possible starting points:

Most studied: maximal atmospheric, zero reactor $\theta_{23}^{\nu} = \frac{\pi}{4}$ $\theta_{13}^{\nu} = 0$

classify scenarios by bare solar angle

tri-bimaximal mixing: $\sin^2 \theta_{12}^{\nu} = 1/3$ Harrison, Perkins, Scott '02; Xing '02; He, Zee '02; Ma '03... bimaximal mixing: $\sin^2 \theta_{12}^{\nu} = 1/2$ Vissiani '97; Barger et al. '98; Baltz, A. Goldhaber, M. Goldhaber '98;... golden ratio (A) mixing: $\sin^2 \theta_{12}^{\nu} = 1/(2+r) \sim 0.276$ Datta, Ling, Ramond '03; Kajiyama, Raidal, Strumia '08;... $r = (1 + \sqrt{5})/2$ $\sin^2 \theta_{12}^{\nu} = (3-r)/4 \sim 0.345$ golden ratio (B) mixing: Rodejohann '09,... $\sin^2 \theta_{12}^{\nu} = 1/4$ hexagonal mixing: Albright, Duecht, Rodejohann '10, Kimand, Seo '11,...

Also can study scenarios without $\theta_{13}^{\nu} = 0$ Lam '13; Holthausen et al. '12; Hagendorn...

many others...

All can be obtained via discrete non-Abelian family symmetries (spontaneously broken to specific subgroups)

Model-building approach

Choose a discrete non-Abelian group for family symmetry

Options: SU(3), SO(3) subgroups: $\mathcal{A}_4 \quad \mathcal{S}_4 \quad \mathcal{A}_5 \quad \Delta(3n^2) \quad \Delta(6n^2) \quad \mathcal{D}_n \quad \mathcal{T}' \quad \mathcal{I}' \quad \dots$



corrections in flavor expansion: (i) NLO in flavons, (ii) "charged lepton"/kinetic/RG...

Many papers and authors! Some authors (not comprehensive):

Altarelli, Babu, Chen, Ding, L.E., Feruglio, Hagedorn, King, Lam, Luhn, Ma, Merle, Ohlsson, Rodejohann, Stuart,...

Example: tri-bimaximal mixing (TBM/HPS)

(Majorana neutrinos, Type I seesaw)

Can further break down Klein symmetry:

1 column only of HPS matrix preserved: TM1, TM2 + corrections

see e.g. King '17 for review

Example: tri-bimaximal mixing (TBM/HPS)

Bottom-up approach: get needed corrections through "Cabibbo Haze"

Interesting recent example:

asymmetric charged lepton corrections to TBM/HPS Rahat, Ramond, Xu '18 (with a dash of grand unification)

SU(5), SO(10) GUT-inspired relations:

symmetric Yukawas \longrightarrow insufficient corrections to θ_{13} Kile, Perez, Ramond, Zhang '14 asymmetric Yukawas \longrightarrow possible for specific $O(\lambda_C)$ corrections to Y_e (via $Y_{\overline{5}}$)

Notable feature:

phase required in $U_{\nu} \sim U^{(\text{HPS})}$ for consistency with mixing angle data numerical example: $\delta \simeq \pm 1.3\pi$, $J \simeq \mp 0.03$

CP Violation

Consider case of spontaneous CP violation — calculable phases.

Idea of generalized CP: $X^T \mathcal{M}_{\nu} X = \mathcal{M}_{\nu}^* \quad Y^{\dagger} \mathcal{M}_e \mathcal{M}_e^{\dagger} Y = (\mathcal{M}_e \mathcal{M}_e^{\dagger})^*$ "ordinary" CP has X = Y = 1

Branco, Lavoura, Rebelo '86...

Grimus, Rebelo '95

automorphisms of discrete family symmetry:

$$X\rho(g)^*X^{-1} = \rho(g')$$
 (consistency condition)

Holthausen et al. '12; Feruglio et al. '12; Chen et al. '14; Ding et al. '14; Branco et al. '15; ...

family symmetry



many recent papers! see King '17 for review

Residual/generalized CP symmetries

existence of "CP basis" group classification

bottom-up approach (Klein symm preserved)

. . .

Holthausen, Lindner, Schmidt '12,... Chen et al. '14,...

Feruglio et al. '12

L.E., Garon, Stuart '15 L.E., Stuart '16

SUSY GUTs and String Models: Top-Down

SUSY GUTs: explicit realizations of these scenarios (+ quark sector)



recent example:SUSY Pati-SalamPoh, Raby, Wang '17 $SU(4)_C \times SU(2)_L \times SU(2)_R$ $\mathcal{D}_3 \times U(1) \times \mathcal{Z}_2 \times \mathcal{Z}_3$

can achieve consistency with LHC, neutrino data (26-parameter fit)

String Models:

variety of possibilities, not necessarily just minimal Type I seesaw

 $\rightarrow \nu_R$ candidates often not pure gauge singlets

explorations of Type I seesaw in heterotic orbifolds Giedt et al.; Buchmuller et al.;...

braneworlds: exponentially suppressed Yukawas

see e.g. Langacker for reviews

"Mixed" scenarios with seesaw and R-parity violation

e.g. G2 models Acharya et al. '16;...

Conclusions

Neutrino data has led to a renaissance for SM flavor puzzle

starting point: are neutrinos Dirac or Majorana?

Many mechanisms for suppressing the neutrino mass scale

often a tradeoff between minimality/naturalness and testability

For 3 active neutrinos only:

mixings: anarchy or symmetry

(spontaneously broken) symmetries: discrete non-Abelian groups many examples (top-down and bottom-up)

but still seeking compelling, complete, testable theories

More data (atmospheric angle, Dirac CP phase,...) will help enormously

If sterile neutrinos confirmed: paradigm shifts again!

Stay tuned!