Review of Theoretical Ideas on Neutrino Masses and Mixing

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“Prospects of Neutrino Physics,” IPMU, April 2019

(image credits: Wikipedia, E. Palti)
A wealth of discoveries in neutrino physics since 1998...

Some highlights:

1998: atmospheric $\nu_\mu$ disappearance (SK)
2002: solar $\nu_e$ disappearance (SK)
2002: solar $\nu_e$ appear as $\nu_\mu, \nu_\tau$ (SNO)
2004: reactor $\bar{\nu}_e$ oscillations (KamLAND)
2004: accelerator $\nu_\mu$ disappearance (K2K)
2006: accelerator $\nu_\mu$ disappearance (MINOS)
2011: accelerator $\nu_\mu$ appear as $\nu_e$ (T2K,MINOS)
2012: reactor $\bar{\nu}_e$ disappear (Daya Bay, RENO)
2011: reactor angle measured!
2014: hint for CP violation? (T2K)
2015: hints for normal hierarchy? (SK, T2K, NOvA)
2016: hint for non-maximal atm mixing? (NOvA)
2018: trivial Dirac phase disfavored at $2\sigma$ (T2K)

Signals physics beyond the Standard Model (SM)!

See Gonzalez-Garcia’s very nice summary of recent history (1902.04583)
The emergent picture... a (seemingly) robust 3-neutrino mixing scheme

Global Fits:
- Forero et al., '17
- Capozzi et al., '18
- Gonzalez-Garcia et al., [www.nu-fit.org](http://www.nu-fit.org)

NuFit, Nov 2018
Caveat: sterile neutrino(s)?

Anomalies in the data:

1995: $\bar{\nu}_e$ appearance (LSND)
2007: $\bar{\nu}_e$ appearance (MiniBooNE)
2012: $\nu_e$ appearance (MiniBooNE)
1995: $\nu_e$ disappearance (Gallium)
2011: $\nu_e$ disappearance (Reactor)

[lots of results, investigation in the interim…]

[well-documented tension between appearance and disappearance data]

See Huber’s IPA 2017 talk for “scorecard”
Maltoni’s talk at Neutrino 2018
Talks at this workshop…

For this talk, focus on 3 active families only…
Implications for the SM flavor puzzle:

what is the origin of the quark and lepton masses and mixings?

Goal: a satisfactory and credible theory of flavor *(very difficult!)*

Many questions:

- Majorana or Dirac neutrinos?
- Nature of neutrino mass suppression?
- Mass hierarchy?
- Lepton mixing angle pattern?
- CP violation?
- Implications for BSM paradigms?
- Connections to other new physics (NP)?
Mass Generation

Quarks, Charged Leptons

“natural” mass scale tied to electroweak scale

Dirac mass terms, parametrized by Yukawa couplings

\[ Y_{ij} H \cdot \bar{\psi}_L \psi_R \rightarrow M_u, M_d, M_e \]

top quark: O(1) Yukawa coupling

rest: suppression (flavor symmetry)

Neutrinos

Main question: origin of neutrino mass suppression

Options: Dirac \[ \Delta L = 0 \]

Majorana \[ \Delta L = 2 \]
Majorana first: \( \Delta L = 2 \)

Advantages: naturalness, leptogenesis, \( 0\nu\beta\beta \)

SM at NR level: Weinberg dimension 5 operator

\[
\frac{\lambda_{ij}}{\Lambda} L_i H L_j H
\]

if \( \lambda \sim O(1) \), \( \Lambda \gg m \sim O(100 \text{ GeV}) \) (but wide range possible)

Underlying mechanism: 3 tree-level options

a. Type I seesaw
b. Type II seesaw
c. Type III seesaw

(fermion singlet)
(scalar triplet)
(fermion triplet)

(image credit: Dinh et al.)
Prototype: Type I seesaw

Type I: Minkowski; Yanagida; Gell-Mann, Ramond, Slansky; Mohapatra, Senjanovic;...

Right-handed neutrinos:

\[ Y_{ij} L_i \nu_{Rj} H + M_{Rij} \nu_{Ri} \nu_{Rj}^c \]

\[ \mathcal{M}_\nu \sim \langle H \rangle^2 Y M^{-1} R Y^T \]

\[ \mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \]

\[ m \sim \mathcal{O}(100 \text{ GeV}) \]

\[ M \gg m \]

\[ m_1 \sim \frac{m^2}{M} \quad m_2 \sim M \gg m_1 \quad \nu_{1,2} \sim \nu_{L,R} + \frac{m}{M} \nu_{R,L} \]

advantages: naturalness, connection to grand unification, leptogenesis,...

disadvantage: testability without model assumptions
Other tree-level seesaws

Type II

\[ M_\nu \sim \langle H \rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2 \]

Type III

\[ M_\nu \sim \langle H \rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T \]

**advantage**: testability
usually accompanied by new EW charged states — visible at LHC?

**disadvantages**: naturalness, economy (subjective)

Type II: Konetchsy, Kummer; Cheng, Li; Lazarides, Shafi, Wetterich; Schecter, Valle; Mohapatra et al.; Ma;...

Type III: Foot, He, Joshi; Ma;...
Radiative neutrino mass generation:

complete Weinberg operator via loops

(loop suppression factor aids in overall mass suppression)

A canonical example: “scotogenic” model

introduce new electroweak doublet(s) and right-handed neutrinos

\[ \mathcal{M}_\nu \sim \lambda \frac{\langle H \rangle^2}{16\pi^2} Y M_R^{-1} Y^T \]

(e.g. new states can be DM candidates)

Generic advantage of radiative models: **testability**
Radiative neutrino mass generation:

can have other NR operators in SM with $\Delta L = 2$

(odd mass dimension $d>5$)

Babu and Leung ’01

d=7

\[
\begin{array}{l}
LLLe^c H \\
LLQd^c H \\
LLQ\bar{u}^c H \\
L\bar{e}^c \bar{u}^c d^c H
\end{array}
\]

\[
\begin{array}{l}
\text{d=9}
\end{array}
\]

\[
\begin{array}{l}
LLLe^c L^c e^c \\
LLQd^c Qd^c
\end{array}
\]

+ many others…

NP scale can be accessible at LHC (subject to LFV bounds)

Possible connection with flavor physics anomalies…

One way **leptoquarks** can manifest themselves:

A two-loop example:

scalar leptoquark \( \phi \sim (3, 1, -1/3) \)

+ octet fermion \( f \sim (8, 1, 0) \)

Cai, Gargalones, Schmidt, Volkas ’17

Babu and Leung ’01

de Gouvea and Jenkins ’07

Päs and Schumacher, ’15

Deppisch et al., ’16 …
Many other ideas for Majorana neutrino masses...

more seesaws (double, inverse,...),
SUSY with R-parity violation, RS models...

lepton number violation  \[ \rightarrow \]  Majorana \( \nu \) masses

Dirac neutrino masses:

Require strong suppression \[ Y_\nu \sim 10^{-14} \]
Less intuitive, but mechanisms exist...
radiative masses, extra dimensions, extended gauge sectors
(non-singlet \( \nu_R \)), SUSY breaking, string instanton effects,...

See e.g. Hagedorn and Rodejohann, ’05
Many other studies: Ma; Mohapatra, Senjanovic,...

General themes:

Much richer than quark and charged lepton sectors.
Trade-off between naturalness and testability.
Lepton mixings

\[ \mathcal{U}_{\text{MNSP}} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13}, \delta)\mathcal{R}_3(\theta_{12}) \mathcal{P} \]

Pontecorvo; Maki, Nakagawa, Sakata

Diagonal phase matrix (Majorana neutrinos)

\[
\left( \begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \theta_{23} & \sin \theta_{23} \\
0 & -\sin \theta_{23} & \cos \theta_{23}
\end{array} \right)
\left( \begin{array}{ccc}
\cos \theta_{13} & 0 & \sin \theta_{13}\text{e}^{i\delta} \\
0 & 1 & 0 \\
-\sin \theta_{13}\text{e}^{-i\delta} & 0 & \cos \theta_{13}
\end{array} \right)
\left( \begin{array}{ccc}
\cos \theta_{12} & \sin \theta_{12} & 0 \\
-\sin \theta_{12} & \cos \theta_{12} & 0 \\
0 & 0 & 1
\end{array} \right) \mathcal{P}
\]

Compare quarks:

\[ \mathcal{U}_{\text{CKM}} = \mathcal{R}_1(\theta_{23}^{\text{CKM}})\mathcal{R}_2(\theta_{13}^{\text{CKM}}, \delta_{\text{CKM}})\mathcal{R}_3(\theta_{12}^{\text{CKM}}) \]

Cabibbo

Kobayashi, Maskawa

NuFit, Nov 2018

\begin{tabular}{|c|c|c|c|c|}
\hline
 & Normal Ordering (best fit) & Inverted Ordering (\(\Delta \chi^2 = 4.7\)) & \\
 & bfp \pm 1\sigma & 3\sigma \text{ range} & bfp \pm 1\sigma & 3\sigma \text{ range} \\
\hline
\sin^2 \theta_{12} & 0.310^{+0.013}_{-0.012} & 0.275 \rightarrow 0.350 & 0.310^{+0.013}_{-0.012} & 0.275 \rightarrow 0.350 \\
\theta_{12}^{\text{CKM}} & 33.82^{+0.75}_{-0.76} & 31.61 \rightarrow 36.27 & 33.82^{+0.75}_{-0.76} & 31.61 \rightarrow 36.27 \\
\sin^2 \theta_{23} & 0.580^{+0.017}_{-0.021} & 0.418 \rightarrow 0.627 & 0.584^{+0.015}_{-0.020} & 0.423 \rightarrow 0.629 \\
\theta_{23}^{\text{CKM}} & 49.6^{+1.0}_{-1.2} & 40.3 \rightarrow 52.4 & 49.8^{+1.0}_{-1.1} & 40.6 \rightarrow 52.5 \\
\sin^2 \theta_{13} & 0.02241^{+0.00065}_{-0.00065} & 0.02045 \rightarrow 0.02439 & 0.02264^{+0.00066}_{-0.00066} & 0.02068 \rightarrow 0.02463 \\
\theta_{13}^{\text{CKM}} & 8.61^{+0.13}_{-0.13} & 8.22 \rightarrow 8.99 & 8.65^{+0.13}_{-0.13} & 8.27 \rightarrow 9.03 \\
\delta_{\text{CP}}^{\text{CKM}} & 215^{+40}_{-29} & 125 \rightarrow 392 & 284^{+27}_{-27} & 196 \rightarrow 360 \\
\Delta m^2_{21} & 7.39^{+0.21}_{-0.20} & 6.79 \rightarrow 8.01 & 7.39^{+0.21}_{-0.20} & 6.79 \rightarrow 8.01 \\
\Delta m^2_{32} & +2.526^{+0.032}_{-0.032} & +2.427 \rightarrow +2.625 & -2.512^{+0.034}_{-0.032} & -2.611 \rightarrow -2.412 \\
\hline
\end{tabular}

3 “small” angles, 1 \(O(1)\) phase
Lepton mixings

\[ U_{\text{MNSP}} = R_1(\theta_{23})R_2(\theta_{13}, \delta)R_3(\theta_{12})P \]

Certainly **two large mixing angles**: \( \theta_{23}, \theta_{12} \)

**Dirac phase**: too soon to say, but intriguing hints…

**Majorana phases**: unlikely to know anytime soon

**A basic question**: is \( \theta_{13} \) “large” or “small”?

**Large reactor angle**: vs. **Small reactor angle**:

the case for **anarchy**

the case for **symmetry**
Neutrino anarchy

$U_{\nu}$ from a random draw of unbiased distribution of 3x3 unitary matrices

statistical tests: lower bound on $|U_{e3}|^2$

Basis independence:
distribution invariant under unitary transformations

flat in Haar measure

Haba and Murayama ’00

Post-reactor angle measurement: renewed focus

de Gouvea and Murayama ’12, Altarelli et al. ’12, Bai and Torroba ’12, ...

Some recent highlights:

RG analysis

Brdar, Konig, Kopp ’15

Model-building + quark sector

Babu et al. ’16, ...

Fortin et al. ’17

Anarchy hypothesis alone does not provide information on $\Delta m^2$
Family symmetries (structure)

Idea: postulate family (horizontal) symmetry $G_f$

Usual paradigm:

$G_f$ spontaneously broken at scale $M$

$$Y_{ij} H \cdot \bar{\psi}_L \psi_R \rightarrow \left( \frac{\varphi}{M} \right)^{n_{ij}} H \cdot \bar{\psi}_L \psi_R$$

Froggatt, Nielsen

$\varphi = \text{“flavon”}$

Different subgroups preserved in neutrino and charged lepton sectors

Large mixing angles

Focus here on this case.

But see recent interesting work on lepton sector in symmetric limit

Reyiumaji and Romanino, ’18
Spontaneously broken family symmetries

Quarks:

small mixings and hierarchical masses:

→ continuous family symmetry

both Abelian and non-Abelian: many examples!

\[ \mathcal{M}_u, \mathcal{M}_d \text{ approx diagonalized by same unitary transformation} \]

\[ (\text{can choose basis w/both approx diagonal}) \]

\[ \mathcal{U}_{\text{CKM}} = \mathcal{U}_u \mathcal{U}_d^\dagger \sim 1 + \mathcal{O}(\lambda) \]

\[ \lambda \sim \frac{\varphi}{M} \]

Wolfenstein parametrization:

\[ \lambda \equiv \sin \theta_c = 0.22 \]

suggests Cabibbo angle (or some power) as a flavor expansion parameter
Leptons:

**charged leptons**: hierarchical masses → similar strategy?

But now, in basis where $M_e$ is diagonal, $M_\nu$ is not diagonal:

$M_\nu$ diagonalization requires 1 small, 2 large mixing angles!

Arguably the **most challenging** * pattern: (* for three families)

- 3 small angles → $\sim$ diagonal $M_\nu$
- 1 large, 2 small → $\sim$ Rank$M_\nu < 3$
- 3 large angles → anarchical $M_\nu$
- 1 small, 2 large → fine-tuning, non-Abelian

Given these patterns, it is relatively straightforward at leading order

A model-building opportunity!
Lepton mixings:

No unique theoretical starting point for the flavor expansion!

\[ \mathcal{U}_{\text{MNSP}} \sim \mathcal{W} + O(\lambda') \]

flavor expansion parameter

mixing angles \((\theta_{12}^{\nu}, \theta_{23}^{\nu}, \theta_{13}^{\nu})\)
(diagonal charged lepton basis)

“Bare” mixing angles generically shift due to \(O(\lambda')\) corrections

A priori, expansions in quark and lepton sectors unrelated.

Unification paradigm (broad sense): set \(\lambda' = \lambda_C\)

ideas of quark-lepton complementarity and “Cabibbo haze”

Raidal ’04, Minakata+Smirnov ’04, many others...
(“haze” terminology from Datta, L.E., Ramond ’05)

Pre-measurement, speculation that reactor angle is a Cabibbo effect

\[ \theta_{13}^{\nu} = 0 \quad \theta_{13} \sim \frac{\lambda_C}{\sqrt{2}} \]

Raidal ’04, Vissiani ’98, ’01
Ramond ’04
Possible starting points:

Most studied: maximal atmospheric, zero reactor \[ \theta_{23} = \frac{\pi}{4}, \quad \theta_{13} = 0 \]

classify scenarios by bare solar angle

**tri-bimaximal mixing:** \[ \sin^2 \theta_{12} = \frac{1}{3} \]

**bimaximal mixing:** \[ \sin^2 \theta_{12} = \frac{1}{2} \]

**golden ratio (A) mixing:** \[ \sin^2 \theta_{12} = \frac{1}{(2 + r)} \sim 0.276 \]
\[ r = \frac{(1 + \sqrt{5})}{2} \]

**golden ratio (B) mixing:** \[ \sin^2 \theta_{12} = \frac{(3 - r)}{4} \sim 0.345 \]

**hexagonal mixing:** \[ \sin^2 \theta_{12} = \frac{1}{4} \]

Also can study scenarios without \[ \theta_{13} = 0 \]

All can be obtained via discrete non-Abelian family symmetries (spontaneously broken to specific subgroups)
Model-building approach

Choose a discrete non-Abelian group for family symmetry

Options: $SU(3)$, $SO(3)$ subgroups:

$$A_4 \quad S_4 \quad A_5 \quad \Delta(3n^2) \quad \Delta(6n^2) \quad D_n \quad T' \quad T'' \ldots$$

Example (Majorana $\nu$):

Flavons:

$$\phi^l, \phi^\nu$$

Residual symmetries:

$$T\langle \phi^l \rangle \approx \langle \phi^l \rangle$$

$$S, U \langle \phi^\nu \rangle \approx \langle \phi^\nu \rangle$$

(or broken further, e.g. only $S$ or $U$ unbroken)

corrections in flavor expansion: (i) NLO in flavons, (ii) “charged lepton”/kinetic/RG...

Many papers and authors! Some authors (not comprehensive):

Altarelli, Babu, Chen, Ding, L.E., Feruglio, Hagedorn, King, Lam, Luhn, Ma, Merle, Ohlsson, Rodejohann, Stuart,...
Example: tri-bimaximal mixing (TBM/HPS) [Majorana neutrinos, Type I seesaw]

$\mathcal{U}_{\text{MNSP}}^{(\text{HPS})} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}$ (~Clebsch-Gordan coeffs!)

Meshkov, Zee...

Many models pre-dated reactor angle measurements

$\rightarrow$ current data requires Cabibbo-sized corrections

Prototypical scenarios: $\mathcal{A}_4 \quad S_4 \quad T'$ (typically SUSY/SUSY-GUT)

Many, many authors!!

Ma et al.; Altarelli, Feruglio; Carone et al.; Chen et al.; King et al.; Ding; Lam...

“minimal” flavor group
(contains $S, T, U$ generators)

Lam; Ding et al;...

Residual symmetries: $\mathbb{Z}_3 \sim T \quad \mathbb{Z}_2 \times \mathbb{Z}_2 \sim S, U, SU$ (Klein symmetry)

Can further break down Klein symmetry:

1 column only of HPS matrix preserved: TM1, TM2 + corrections

see e.g. King ’17 for review
Example: tri-bimaximal mixing (TBM/HPS)

**Bottom-up approach:** get needed corrections through “Cabibbo Haze”

**Interesting recent example:**

asymmetric charged lepton corrections to TBM/HPS
(with a dash of grand unification)

Rahat, Ramond, Xu ’18

SU(5), SO(10) GUT-inspired relations:

- **symmetric** Yukawas → insufficient corrections to $\theta_{13}$
  
  Kile, Perez, Ramond, Zhang ’14

- **asymmetric** Yukawas → possible for specific $O(\lambda_C)$
corrections to $Y_e$ (via $Y_5$)

**Notable feature:**

\[ \delta \simeq \pm 1.3\pi, \quad J \simeq \mp 0.03 \]
CP Violation

Consider case of spontaneous CP violation — calculable phases.

Idea of generalized CP:

\[ X^T M_\nu X = M_\nu^* \quad Y^\dagger M_e M_e^\dagger Y = (M_e M_e^\dagger)^* \]

“ordinary” CP has \( X = Y = 1 \)

\[ G \rtimes H_{CP} \]

family symmetry

Residual/generalized CP symmetries

existence of “CP basis”

\[ G^l \rtimes H_{CP}^l \quad G^\nu \rtimes H_{CP}^\nu \]

group classification

bottom-up approach

(Klein symm preserved)

many recent papers! see King ’17 for review
SUSY GUTs and String Models: Top-Down

**SUSY GUTs:** explicit realizations of these scenarios (+ quark sector)

recent example: **SUSY Pati-Salam**

\[ SU(4)_C \times SU(2)_L \times SU(2)_R \times D_3 \times U(1) \times Z_2 \times Z_3 \]

can achieve consistency with LHC, neutrino data

(26-parameter fit)

**String Models:**

variety of possibilities, not necessarily just minimal Type I seesaw

\[ \nu_R \] candidates often not pure gauge singlets

explorations of Type I seesaw in heterotic orbifolds

braneworlds: exponentially suppressed Yukawas

“Mixed” scenarios with seesaw and R-parity violation

see e.g. Langacker for reviews

e.g. G2 models

Acharya et al. '16;...
Conclusions

Neutrino data has led to a renaissance for SM flavor puzzle

**starting point:** are neutrinos Dirac or Majorana?

Many mechanisms for suppressing the neutrino mass scale
often a tradeoff between minimality/naturalness and testability

For 3 active neutrinos only:

**mixings:** anarchy or symmetry
(spontaneously broken) symmetries: discrete non-Abelian groups
many examples (top-down and bottom-up)
but still seeking **compelling, complete, testable** theories

More data (atmospheric angle, Dirac CP phase,…) will help enormously

If sterile neutrinos confirmed: paradigm shifts again!

Stay tuned!