Neutrino’s in SMEFT and the Neutrino Option

#SMEFT

M. Trott, Prospects in Neutrino Physics 2019
Two key ideas to transmit:

**Small neutrino mass can be linked to the smallness of the Higgs mass.** *(Small here compared to UV scales linked to Lepton number violation)*

**A consistent treatment of the seesaw model to one loop in SMEFT points to a possible origin for the SM Higgs potential and the EW scale.*

This idea is embedded in several overlapping expansions:

1) The SMEFT operator expansion.
2) A flavour space expansion for L5 operator due to seesaw.
3) The perturbative matching expansion of the seesaw model to the SMEFT.
The SMEFT expansion
Discovery of a (Higgs like) $J^P \sim 0^+$ particle in 2012
and a theory…the Standard Model EFT

- The SM, an SU(3) x SU(2) x U(1) gauge theory:

\[
\mathcal{L}_{\text{SM}} = -\frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) + \sum_{\psi = q,u,d,c,l,e} \bar{\psi} i \not{D} \psi \\
- \lambda \left( H^\dagger H - \frac{1}{2} v^2 \right)^2 - \left[ H^t j d Y_d q_j + \tilde{H}^t j u Y_u q_j + H^t j e Y_e l_j + h.c. \right],
\]

\[
\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^\delta L \neq 0} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \cdots
\]

- Weinberg 1979, Wilczek and Zee 1979
RunII and beyond: Resonance limits to local operators

### Masses of EW scale ($\sim g\nu$) states

$M_W, M_Z, M_t, M_h$
RunII and beyond: Resonance limits to local operators

Now that these bounds have been pushed away from $v$

USE that $v/M < 1$

to simplify/for more powerful conclusions:

- bound many models at once
- bound multiple resonances at same time

Deviations then look like local contact operator effects in EFT
When you do measurements below a particle threshold

- Observable is a function of the Lorentz invariants:
  \[ f(s, t, u) \]

- Generally an analytic function of these invariants, except in special regions of phase space, ex. where an internal state goes on-shell.

\[ \frac{1}{s - m^2 + i\Gamma(s)m} \]

- If the collision probe does not reach, observable’s dependence on that scale simplified

- You can Taylor expand in LOCAL functions (operators)

\[ \langle \rangle \sim O_{SM}^0 + \frac{f_1(s, t, u)}{M_{heavy}^2} + \frac{f_2(s, t, u)}{M_{heavy}^4} + \cdots \]

This is the core idea of EFT interpretations of the data.

EFT approach not a guess.

General approach based on S matrix theory and motivated by experimental situation.
General “BSM heavy” approach is SMEFT/HEFT

No BSM resonance seen

Decoupling
\[ \frac{v}{M} < 1 \]

VERY! Efficient to constrain BSM/interpret the data in EFT

no other (hidden) light states.

SMEFT observed scalar in doublet

HEFT observed scalar not in doublet

Basics of the SMEFT formulation:

IR operator form

\[ \mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \ldots, \quad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4, \]

UV dependent Wilson coefficient and suppression scale
SMEFT - built of H doublet + higher D ops

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_5 + \frac{1}{\Lambda^2_{\delta B = 0}} \mathcal{L}_6 + \frac{1}{\Lambda^2_{\delta B \neq 0}} \mathcal{L}_6' + \frac{1}{\Lambda^3_{\delta L \neq 0}} \mathcal{L}_7 + \frac{1}{\Lambda^4} \mathcal{L}_8 + \cdots \]

- Weinberg 1979, Wilczek and Zee 1979
- Weinberg 1979, Abbott Wise 1980
- Lehman 1410.4193, Henning et al. 1512.03433
- Lehman, Martin 1510.00372, Henning et al. 1512.03433

Q: “Are any of these damn Wilson coefficients in the SMEFT not 0?”
A: “Yes.” — Motivation for this neutrino work.

Are any Wilson coefficients not 0?

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L\neq 0}} \mathcal{L}_5 + \frac{1}{\Lambda_{\delta B=0}^2} \mathcal{L}_6 + \frac{1}{\Lambda_{\delta B\neq 0}^2} \mathcal{L}'_6 + \frac{1}{\Lambda_{\delta L\neq 0}^3} \mathcal{L}_7 + \frac{1}{\Lambda^4} \mathcal{L}_8 + \cdots \]

- \( C^5 \) seems to be non zero.

- Working in dirac spinors causes a bit of pain as we define \( \psi^c = (-i \gamma_2 \gamma_0) \bar{\psi}^T \)

- Introduce singlet right handed fields with majorana mass terms as

- Shift phases to couplings defining a field that is not a chiral eigenstate that satisfies Majorana condition (Broncano et al. hep-ph/0406019)

- Obtaining the Standard (type I) seesaw (Minkowski 77, Gell Mann et al 79, Yanagida 79, Mohapatra at al 79...)

\[ 2 \mathcal{L}_{N_p} = \bar{N}_p(i\phi - m_p)N_p - \ell_L^\beta \tilde{H} \omega_\beta^p \bar{T} N_p - \bar{\ell}_L^\beta \tilde{H}^* \omega_\beta^p \bar{T} N_p - \bar{N}_p \omega_\beta^p \tilde{H}^T \ell_L^\beta - \bar{N}_p \omega_\beta^p \tilde{H}^T \ell_L^\beta. \]
Integrating out the seesaw at tree level. Matching now done out to L7

\[ \left( \frac{s + m_p}{m_p^2} \right) \frac{1}{1 - \frac{s^2}{m_p^2}} = -\frac{1}{m_p} - \frac{s}{m_p^2} - \frac{s^2}{m_p^3} + \cdots \]

- MATCH!

Extremely well known result

\[ \mathcal{L}^{(5)} = \frac{c_\beta \kappa}{2} Q_5^{\beta \kappa} + h.c. \]

Here the \( \omega_p^\beta \) are complex vectors in flavour space.

To proceed with further matching we can perform a flavour space expansion

\[ x, y \in \mathbb{C}^3. \]

\[ x \cdot y = x_i^* y_i, \quad \|x\| = \sqrt{x \cdot x}, \quad x \times y = ((x \times y)_R)^* \]
L6 SMEFT matching

- At $\mathcal{L}_6$ the fun begins:

$$\mathcal{L}^{(6)} = \frac{(\omega_\beta^p)^\dagger \omega_\kappa^p}{2m_p^2} \left( Q_{H\ell}^{(1)} - Q_{H\ell}^{(3)} \right)$$

Can compare to Broncano et al. hep-ph/0406019

- But the N are integrated out in sequence so you also get:

$$\frac{1}{2} \mathcal{L}_{N_{2,3}}^{(6)} \supset \begin{align*}
\text{Re} \left[ \frac{x_\beta^1 x^* \cdot y^\dagger}{4m_1^2} \right] (Q_\beta^{N_2} - Q_\beta^{N_2}) &+ \frac{i \text{Im} \left[ x_\beta^1 x^* \cdot y^\dagger \right]}{4m_1^2} (Q_\beta^{N_2} + Q_\beta^{N_2}) \\
\text{Re} \left[ \frac{x_\beta^2 x^* \cdot z^\dagger}{4m_2^2} \right] (Q_\beta^{N_2} - Q_\beta^{N_3}) &+ \frac{i \text{Im} \left[ x_\beta^2 x^* \cdot z^\dagger \right]}{4m_2^2} (Q_\beta^{N_2} + Q_\beta^{N_3}) \\
\text{Re} \left[ \frac{y_\beta^3 y^* \cdot z^\dagger}{4m_2^2} \right] (Q_\beta^{N_3} - Q_\beta^{N_3}) &+ \frac{i \text{Im} \left[ y_\beta^3 y^* \cdot z^\dagger \right]}{4m_2^2} (Q_\beta^{N_3} + Q_\beta^{N_3})
\end{align*}$$

$$Q_\beta^{N_p} = (H^\dagger H) (\bar{\ell}_{\beta}^L \tilde{H}) N_p$$
L6 SMEFT matching

- At $\mathcal{L}_6$, the fun begins:

$$\mathcal{L}^{(6)} = \frac{(\omega^{\beta}_{\beta})^{\dagger} \omega^{\kappa}_{\kappa}}{2 m_p^2} \left( Q^{(1)}_{H^\ell} - Q^{(3)}_{H^\ell} \right)$$

Can compare to Broncano et al. hep-ph/0406019 (SU(2) diff)

- As a Majorana scale in the EOM:

$$\phi N_p = -i \left( m_p N_p + w^{p,*}_\beta H^T \ell^c_{L \beta} + w^{p}_\beta H^\dagger \ell^\beta_L \right)$$

which gives the extra matching contributions

$$\frac{1}{2} \mathcal{L}^{(6)}_{N_{2,3}} \supset \frac{(x_\beta)^T x^* \cdot y^T m_2}{4 m_1^3} \left[ \ell^c_{L \beta} H^* N_2 \right] (H^\dagger H) + \frac{(x_\beta)^T x^* \cdot z^T m_3}{4 m_1^3} \left[ \ell^c_{L \beta} H^* N_3 \right] (H^\dagger H) + (y_\beta)^T y^* \cdot z^T m_3 \frac{4 m_2^3}{4 m_2^3} \left[ \ell^c_{L \beta} H^* N_3 \right] (H^\dagger H) + h.c.$$
L7 SMEFT matching

- Summary of dim 7 results:

Basis of Lehman 1410.4193
Summary of dim 7 results: 

L7 SMEFT matching

Tree level matching contributions

Tree level matching onto ops with field strengths, from a weakly coupled renormalizable model.

Basis of Lehman 1410.4193
Linking the Higgs potential with Neutrino mass generation.
Strangeness of the Higgs potential

- Reminder: Why is the Higgs mechanism and classical potential curious?

\[
S_H = \int d^4x \left( |D_{\mu}H|^2 - \lambda \left( H^\dagger H - \frac{1}{2}v^2 \right)^2 \right),
\]

Partial Higgs action

\[
LG(s) = \int_{\mathbb{R}^3} dx^3 \left[ \frac{1}{2} |(d - 2 i e A)s|^2 + \frac{\gamma}{2} (|s|^2 - a^2) \right],
\]

Landau-Ginzberg actional, parameterization of Superconductivity

\[m_{W/Z} = 0 \text{ field config. energetically excluded (i.e. spon. sym breaking)}\]

Magnetic field energetically excluded from interior of SC

E. Witten, From superconductors and four-manifolds to weak interactions,
Challenge of constructing potential

- It would make sense for the Higgs mechanism to just parameterize symmetry breaking. To do better we can try and construct the Higgs potential with QFT

\[ \langle \Psi |\Psi \rangle \]

\[ H \]

- Naturally leads to the idea of composite Higgs field due to some new strong interaction

Kaplan, Georgi, Dimopoulos, Dugan 84-85

- Initial efforts studied the induced potential of a scalar and used vacuum misalignment to get

\[ SU(2)_L \times U(1)_Y \rightarrow U(1)_Q \]

\[ \Sigma(x) = e^{i\Pi_a T^a / f} U e^{i\Pi_a T^a / f} \]

\[ \mathcal{L} = \frac{f^2}{4} (D_\mu \Sigma)(D^\mu \Sigma)^T + \Lambda^4 \text{Tr} \left[ T^a \Sigma (T^a)^T \Sigma^\dagger \right] + \ldots \]

- Group theory embedding of SM into larger global sym groups exhaustively studied
It would make sense for the Higgs mechanism to just parameterize symmetry breaking. To do better we should construct the Higgs potential

\[ V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \]

- Muon decay: \( v = 246 \) GeV  
  Higgs mass: \( m_h = 125 \) GeV  
  \( \lambda = 0.13 \)

- Composite models (nobly) try to construct the Higgs potential:

\[ V(H) \simeq \frac{g_{SM}^2 \Lambda^2}{16\pi^2} \left( -2aH^\dagger H + 2b \frac{(H^\dagger H)^2}{f^2} \right) \]

- Can get the quartic to work: \( \sim 0.1 \left( \frac{g_{SM}}{N_c y_t} \right)^2 \left( \frac{\Lambda}{2f} \right)^2 \) for \( \Lambda/f \ll 4\pi \)

The problem.
It would make sense for the Higgs mechanism to just parameterize symmetry breaking. To do better we should construct the Higgs potential

\[ V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \]

Higgs coupling deviations scale as \( \sim 1 - \frac{v^2}{f^2} \) but pheno studies imply \( f \gtrsim \text{TeV} \)

Where are the new states at a weakly coupled mass scale below the full cut off?

Extensive tuning in these models: see 1401.2457 Bellazzini et al,

This problem killed the initial composite idea initially (Georgi-Kaplan 80’s), Modern models introduce tunings and constructed to avoid this. Generic feature - tev or below states to construct potential.
We know more about the potential now

- Due to the improved knowledge of the top and Higgs mass:
  
  $1205.6497$ Degrassi et al, $1112.3022$ Elias-Miro et al.

- What does this mean?

- For fate of the universe considerations see $1205.6497$ Degrassi et al.
  $1505.04825$ Espinosa et al.

- This might be a different message.

- Build the Higgs potential in the UV, as there $\lambda \sim 0$

An interesting mass scale is $10-100$ PeV (or $10^7 - 10^8$ GeV)

Unexplored compared to the fate of the universe issues.
A set of clues?

- Observed mass spectrum is such that the running of the quartic does something interesting

- Neutrino's have mass

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass"
Idea is use the non trivial spectrum we have

- Still need a non trivial spectrum with lots of interactions (and some weird couplings) to generate a non trivial potential. But let's use the weird spectrum we have.

\[ L_{\text{SM}} = -\frac{1}{4} G^A_{\mu \nu} G^{A \mu \nu} - \frac{1}{4} W^I_{\mu \nu} W^{I \mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} + \sum_{\psi = q, u, d, l, e} \bar{\psi} i D \psi \]

\[ + (D_{\mu} H)^\dagger (D^{\mu} H) - \lambda \left( H^\dagger H - \frac{1}{2} v^2 \right)^2 - \left[ H^\dagger t Y_d q_j + \tilde{H}^\dagger u Y_u q_j + H^\dagger e Y_e \ell_j + h.c. \right], \]

<table>
<thead>
<tr>
<th>Field</th>
<th>SU(3)</th>
<th>SU(2)</th>
<th>U_Y(1)</th>
<th>SO^+(3, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>q_L = (u_L, d_L)^T</td>
<td>3</td>
<td>2</td>
<td>1/6</td>
<td>(1/2, 0)</td>
</tr>
<tr>
<td>u_L = {u_R, c_R, t_R}</td>
<td>3</td>
<td>1</td>
<td>2/3</td>
<td>(0, 1/2)</td>
</tr>
<tr>
<td>d_L = {d_R, s_R, b_R}</td>
<td>3</td>
<td>1</td>
<td>-1/3</td>
<td>(0, 1/2)</td>
</tr>
<tr>
<td>\ell_L = (\nu_L, e_L)^T</td>
<td>1</td>
<td>2</td>
<td>-1/2</td>
<td>(1/2, 0)</td>
</tr>
<tr>
<td>e_L = {e_R, \mu_R, \tau_R}</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>(0, 1/2)</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>2</td>
<td>1/2</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>
Turns out the SM can do it.

- Lets use the weird spectrum we have expressed through the SM RGE's

\[
\begin{align*}
\beta(g_Y^2) &= g_Y^4 \frac{41}{6}, \quad \beta(g_2^2) = g_2^4 \left( -\frac{19}{6} \right), \quad \beta(g_3^2) = g_3^4 (-7), \\
\beta(\lambda) &= \left[ \lambda \left( 12\lambda + 6Y_t^2 - \frac{9}{10} (5g_2^2 + \frac{5g_Y^2}{3}) \right) \right. \\
&\quad - 3Y_t^4 + \frac{9}{16} g_2^4 + \frac{3}{16} g_Y^4 + \frac{3}{8} g_Y^2 g_2^2 \bigg], \\
\beta(m^2) &= m^2 \left[ 6\lambda + 3Y_t^2 - \frac{9}{20} (5g_2^2 + \frac{5g_Y^2}{3}) \right], \\
\beta(Y_t^2) &= Y_t^2 \left[ \frac{9}{2} Y_t^2 - 8g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_Y^2 \right].
\end{align*}
\]
Necessarily one loop results coming with tree level matchings:

\[ V(H^\dagger H) = -\frac{m_0^2 R_H + \Delta m^2}{2} (H^\dagger H) + (\lambda_0 R_H^2 + \Delta \lambda) (H^\dagger H)^2 + \cdots \]

Threshold matchings:

\[ \Delta \lambda = -\frac{5}{32\pi^2} \left[ |\omega_1|^4 + |\omega_2|^4 + |\omega_1 \omega_2|^2 \right] \left( 1 + \frac{2 M_1}{M_1 - M_2} \log \frac{M_2}{M_1} \right), \]
\[ \Delta m^2 = \frac{5}{16\pi^2} \left[ \text{Re}(\omega_1 \omega_2)^2 \frac{M_1 M_2}{M_1^2 - M_2^2} \log \frac{M_1^2}{M_2^2} \right], \]

here choose \( \mu = M e^{-3/4} \)

to be consistent with CW threshold correction


THE SIGN WORKS OUT due to FERMI statistics

If you assume a seesaw model for neutrino mass generation - this is a “known unknown”.
This threshold matching can be done to CW

\[ \Delta V_{CW} = -\frac{1}{32\pi^2} \left[ (m_{\nu_i}^2 (H^\dagger H))^4 \log \frac{m_{\nu_i}^2 (H^\dagger H)}{\mu^2} \right] \]

\[ m_{\nu_i}^i (H^\dagger H) = \frac{1}{2} (M \mp \sqrt{M^2 + 2 |\omega_p|^2 (H^\dagger H)}) \]

\[ \mu = Me^{-3/4} \]

- Coleman-Weinberg potential:

- If \( \frac{|\omega_p|^2 m_p^2}{16\pi^2} \gg v_0, \Lambda_{QCD}^2 \) such a threshold matching can dominate the potential and give low scale pheno that is the SM. IR scales are:
  - \( v_0 \)
    - Can be small
    - Doesn’t have to be 0.
  - \( \Lambda_{QCD} \)
    - Known to be smaller than induced vev.
  - \( \mu_{CW} \)
    - Exponentially separated due to asy nature of pert theory.


- Can one go the full way of dominantly generating the EW scale in this manner?  \( \times \) ?  arXiv:1703.10924 Neutrino Option  Ilaria Brivio, MT
Can the Neutrino Option work?

- Use the RGE (1205.6497 Degrassi et al, 1112.3022 Elias-Miro et al..) to run down the threshold matching corrections

  arXiv:1703.10924  Neutrino Option  Ilaria Brivio, MT

- Can get the troublesome \( \lambda \sim 0.13 \)

- This essentially fixes the mass scale and couplings (large uncertainties)
  \[
  m_p \sim 10^7 \text{GeV} \\
  |\omega| \sim 10^{-5}
  \]

- Expand around the classically scaleless limit of the SM. Punch the potential with threshold matching you kick off low scale EW sym. breaking?
The EW potential does get constructed correctly running down in a non-trivial manner.

In a non-trivial manner - and the right neutrino mass scale (diff) can result.

\[ \Delta m_\nu (\text{eV}) \]

\[ \Delta m_{21}^2 / 10^{-5} \text{eV}^2 = 6.93 - 7.97, \]
\[ \Delta m^2 / 10^{-3} \text{eV}^2 = 2.37 - 2.63 (2.33 - 2.60) \]
Neutrino option: the bad

- Very significant numerical uncertainties - top quark mass driven

- This is NOT a total solution to the Hierarchy problem. As there is no symmetry protection mechanism against other threshold corrections.

- No non-resonant leptogenesis in this parameter space 1404.6260 Davoudias, Lewis

  Resonant leptogenesis can work here (S. Petcov - private communication)

- No dynamical origin of the Majorana scale supplied. So the IR limit taken is not clearly self consistent.

"unburied body" plot
Improving numerical stability

- Severe upgrade in rigor of one loop calc and one loop running of $C^5$
- Consistency test reformulated to avoid asymptotic numerical sensitivity to $\lambda$
- Scan regions defined by first fitting Neutrino global data

Minimal case with two heavy neutrino’s.

$\hat{m}_t = 173.2$ GeV, NH

$\mu = M_1$ [GeV]

$m^2$ [GeV$^2$]

$\Delta m^2(M_1)$, $M_2 = M_1$

$\Delta m^2(M_1)$, $M_2 = 10 M_1$
Improving numerical stability

- Beyond one loop need a bare $\lambda$ OR other threshold corrections

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I809.03450 Brivio, Trott

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IPMU, Prospects for Neutrino Physics, April 11th 2019
Beyond one loop need a bare $\lambda$ OR other threshold corrections

An interpretation:

A consistent treatment of the seesaw model to one loop in SMEFT points to a possible origin for the SM Higgs potential and the EW scale.

What “breaks” EW symmetry in the Neutrino Option?

Fermi statistics + Majorana scale in the UV + SM state spectrum for RGE.
Conclusions/summary

• SMEFT is a theory defined by field redefinitions leading to local operators. Neutrino’s with mass embedded.

• Combined global studies are key to interpretation

• Severe care required in formulating the SMEFT (TH job) and in combining the data (EXP job)

• Seesaw model supplies an option for low energy pheno of the SM With the Higgs potential having an interesting UV boundary condition

\[ m_\nu \sim \frac{\omega^2 \overline{\nu}_T^2}{M}, \quad m_h \sim \frac{\omega M}{4\pi}, \quad \overline{\nu}_T \sim \frac{\omega M}{4\sqrt{2}\pi\sqrt{\lambda}}, \quad m_p \sim 10^7\text{GeV} \quad |\omega| \sim 10^{-5} \]

• This is a “self seesaw” with only one scale, the EW scale is a loop down from the Majorana scale. We don’t see new states at LHC due to a stabilizing symmetry consistent with this.
Can you build a UV completion that generates the majorana scale in a manner that does not induce other threshold corrections?

- 1807.11490 Brdar et al. Conformal UV completion of Neutrino Option
- ?

IF this was true what is the right experimental approach to probe

\[ m_p \sim 10^7 \text{GeV} \quad |\omega| \sim 10^{-5} \]

- 1810.12306 Brdar et al. Gravitational Waves are potentially significant
- ?
Backup
Flavour space expansion

- Summary of dim 7 results its VERY small, down by $\mathcal{O}(v^2/M_p^2)$ and interesting!

- Far bigger effect is how the expansion of

$$\mathcal{L}^{(5)} = \frac{c_\beta k}{2} Q_5^\beta k + \text{h.c.}$$

is perturbed as the N states are integrated out in sequence.

no known quantum numbers

expected to be uniform in interaction eigenbasis, once diagonalized expect

$$\|x\| \sim \|y\| \sim \|z\|$$
Flavour space expansion

- Lightest singlet state dominates the neutrino mass matrix, heavier singlet states then perturb the mass spectrum and eigenstate spectrum.

\[ M_{\nu\nu}^\beta\alpha (M_{\nu\nu}^\kappa\nu) \dagger \approx \frac{\|z^* \cdot z\|}{m_3^2} \left[ z_\beta^T z_\kappa + \frac{z^* \cdot y^\dagger}{\|z^* \cdot z\|} \frac{m_3}{m_2} z_\beta^T y_\kappa^* + \frac{y^* \cdot z^\dagger}{\|z^* \cdot z\|} \frac{m_3}{m_2} y_\beta^T z_\kappa^* + \cdots \right]. \]

< 1 by construction

use complex Cauchy-Schwarz

\[ a \cdot b = \|a\| \|b\| \Delta_{ab} \]

< 1 by construction again

- If it is true that

\[ \|y\|_{\Delta_{y^t z}} < m_2/m_3, \quad \|y\|_{\Delta_{yz^t}} < m_2/m_3. \]

another expansion to exploit - a flavour space expansion. 1203.4410 Grinstein, MT
Old school Perturbation theory

- Define eigenvectors that correspond to the mass eigenvalues of the $C^5$ matrix

\[
M_{\nu \nu} \hat{\rho}_p = m_p \hat{\rho}_p,
\]

- Construct the orthonormal set as eigenvectors in flavour space

\[
\hat{\rho}_a = \frac{z}{||z||}, \quad \hat{\rho}_b = \frac{z^* \times (\bar{y} \times \bar{z})}{||z|| ||\bar{z} \times \bar{y}||}, \quad \hat{\rho}_c = \frac{\bar{y}^* \times \bar{z}^*}{||\bar{z} \times \bar{y}||}.
\]

- Can systematically develop perturbations of the eigenvectors and eigenvalues

\[
\delta \rho_j = \sum_{i \neq j} \frac{\langle \hat{\rho}_i | M \delta M^\dagger + \delta M M^\dagger | \hat{\rho}_j \rangle}{m_j^2 - m_i^2} \hat{\rho}_i,
\]

\[
\delta m_i^2 = \langle \hat{\rho}_i | M \delta M^\dagger + \delta M M^\dagger + \delta M \delta M^\dagger | \hat{\rho}_i \rangle.
\]
What is the benefit of this approach?

- The only matrix involved in the neutrino mass spectrum.

\[
\langle c_{\beta\kappa} Q_5^{\beta\kappa} \rangle = -\frac{v^2}{2} \left[ U^T (\nu, L) \hat{\sigma}_\kappa \mathcal{U} (\nu, L) \right] (\nu'_L)^T \rho (\nu'_L)^r
\]

What is the benefit of this approach?

\[
\mathcal{U}(\nu, L) = \mathcal{U}(e, L) \mathcal{U}_{PNMS}^{\delta_{ij}}
\]

\[
\mathcal{U}^\dagger (e, L) = (\bar{\sigma}_1^*, \bar{\sigma}_2^*, \bar{\sigma}_3^*)^T
\]

\[
\tilde{\rho}_c = (s_{23} - c_{23} c_{12} s_{13} e^{i\delta}) \bar{\sigma}_3 + (-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta}) \bar{\sigma}_2 + c_{12} c_{13} \bar{\sigma}_1,
\]

\[
\tilde{\rho}_b e^{-i\frac{\theta_{21}}{2}} = (-c_{12} s_{23} - c_{23} s_{12} s_{13} e^{i\delta}) \bar{\sigma}_3 + (c_{12} c_{23} - s_{23} s_{12} s_{13} e^{i\delta}) \bar{\sigma}_2 + c_{13} s_{12} \bar{\sigma}_1,
\]

\[
\tilde{\rho}_a e^{-i\frac{\theta_{31}}{2}} = c_{13} c_{23} \bar{\sigma}_3 + c_{13} s_{23} \bar{\sigma}_2 + e^{-i\delta} s_{13} \bar{\sigma}_1.
\]

This is where Ben and I hit the wall in 1203.4410.
Just perturb in the unknown

- Although the $\sigma_i$ are unknown we do know one thing

\[ \begin{array}{c}
N_1 \\
N_2 \\
N_3
\end{array} \]

Hermitian positive mass matrix defined over field $\mathbb{C}^3$.

As $\mathcal{U}(e, L)$ diagonalizes a Hermitian positive mass matrix the $\sigma_i$ form a basis.

- So expand all the complex $\omega_i = A_i \sigma_1 + B_i \sigma_2 + C_i \sigma_3$

- Use the algebra properties

\[ \begin{align*}
\vec{\sigma}_i \times \vec{\sigma}_j &= \epsilon_{ijk} \vec{\sigma}_k \\
\vec{\sigma}_i^* \times \vec{\sigma}_j^* &= \epsilon_{ijk} \vec{\sigma}_k^*
\end{align*} \]

- This way we have a systematically improvable basis independent link between the neutrino mass spectrum and the PMNS. Might be useful long term.
Neutrino Option Numerics

<table>
<thead>
<tr>
<th></th>
<th>Normal Hierarchy</th>
<th>Inverted Hierarchy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>best fit</td>
<td>3σ range</td>
</tr>
<tr>
<td>$s_1^2$</td>
<td>0.441</td>
<td>0.385 – 0.635</td>
</tr>
<tr>
<td>$s_2^2$</td>
<td>0.02166</td>
<td>0.01934 – 0.02392</td>
</tr>
<tr>
<td>$s_3^2$</td>
<td>0.306</td>
<td>0.271 – 0.345</td>
</tr>
<tr>
<td>$\delta^\circ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta m^2_{21}$ (10^{-5} eV^2)</td>
<td>7.50</td>
<td>7.03 – 8.09</td>
</tr>
<tr>
<td>$\Delta m^2_{32}$ (10^{-3} eV^2)</td>
<td>2.524</td>
<td>2.407 – 2.643</td>
</tr>
</tbody>
</table>

Table 1: Best fit values of neutrino parameters taken from the global fit in Ref. [35].

SMEFT up to sub-leading order ($\mathcal{L}^{(7)}$ corrections) but we restrict our attention to the matching onto $\mathcal{L}^{(5)}$ in this work.

<table>
<thead>
<tr>
<th></th>
<th>best fit</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_F$ [GeV^{-2}]</td>
<td>1.1663787 \cdot 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>$\hat{s}_W$ [GeV]</td>
<td>91.1875</td>
<td></td>
</tr>
<tr>
<td>$\hat{m}_W$ [GeV]</td>
<td>80.387</td>
<td></td>
</tr>
<tr>
<td>$\hat{m}_h$ [GeV]</td>
<td>125.09</td>
<td></td>
</tr>
<tr>
<td>$\hat{m}_t$ [GeV]</td>
<td>173.2</td>
<td>171 – 175</td>
</tr>
<tr>
<td>$\hat{m}_b$ [GeV]</td>
<td>4.18</td>
<td></td>
</tr>
<tr>
<td>$\hat{m}_r$ [GeV]</td>
<td>1.776</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>tree</th>
<th>1-loop</th>
<th>2-loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\lambda}$</td>
<td>0.1291</td>
<td>0.1276</td>
<td>0.1258</td>
</tr>
<tr>
<td>$\hat{m}$ [GeV]</td>
<td>125.09</td>
<td>132.288</td>
<td>131.431</td>
</tr>
<tr>
<td>$\hat{g}_1$</td>
<td>0.451</td>
<td>0.463</td>
<td>0.461</td>
</tr>
<tr>
<td>$\hat{g}_2$</td>
<td>0.653</td>
<td>0.6435</td>
<td>0.644</td>
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<tr>
<td>$\hat{g}_3$</td>
<td></td>
<td>1.22029</td>
<td></td>
</tr>
<tr>
<td>$\hat{y}_t$</td>
<td>0.995</td>
<td>0.946</td>
<td>0.933</td>
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<tr>
<td>$\hat{y}_b$</td>
<td>0.024</td>
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<tr>
<td>$\hat{y}_r$</td>
<td>0.0102</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Left table: best fit values of the quantities used as inputs in the numerical analysis, while $m_t$ is varied in the range specified. Right table: matching values for the SM parameters at $\mu = m_t$ obtained from the expressions in Appendix A in Ref. [42] with the inputs on the left when $m_t = 173.2$ GeV.