Prospects of Neutrino Physics

Predictions of Neutrino Mass Models for Charged Lepton Flavour Violation

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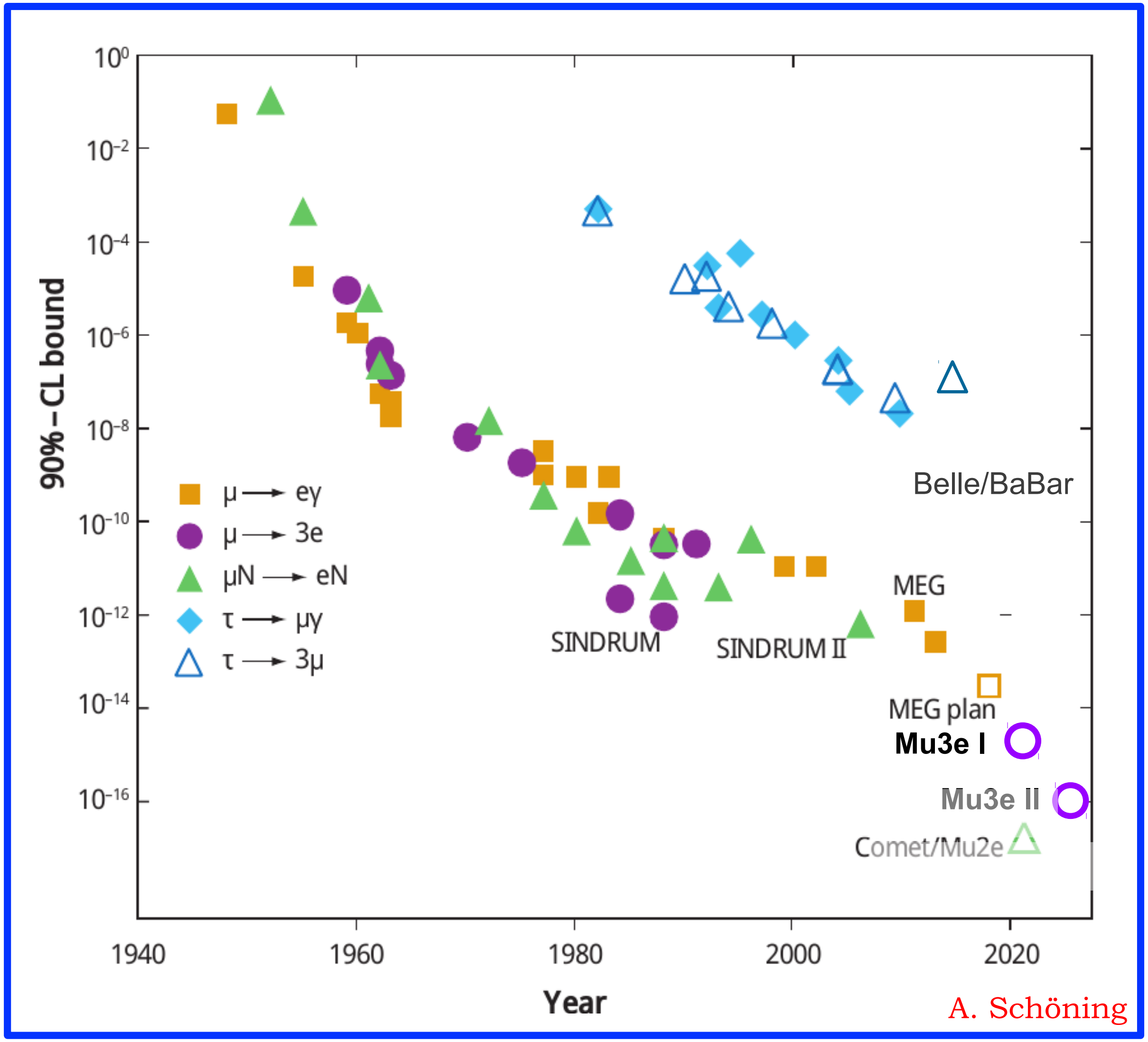
Kavli IPMU, April 11th 2019

CLFV has been sought for more than 70 years...



→ Kuno-san talk

Neutrino masses/oscillations $\iff \not{L}_e, \not{L}_\mu, \not{L}_\tau$

Lepton family numbers are not conserved:

why not $\mu \to e\gamma, \;\; \tau \to \mu\gamma, \;\; \mu \to eee, \;$ etc.?

Why no CLFV in the Standard Model?

In the SM fermion masses, thus the *flavour sector*,
stems from the Yukawa interactions:

$$-\mathcal{L}_Y = (Y_u)_{ij}\ \overline{Q}_{L\,i}\, u_{R\,j}\, \widetilde{\Phi} + (Y_d)_{ij}\ \overline{Q}_{L\,i}\, d_{R\,j}\, \Phi + (Y_e)_{ij}\ \overline{L}_{L\,i}\, e_{R\,j}\, \Phi + h.c.$$

Rotations to the fermion mass basis: $Y_f = V_f \hat{Y}_f W_f^\dagger, \qquad f = u, d, e$

Unitary rotation matrices, couplings to photon and Z remain flavour-diagonal:

$$e\ \bar{f}\gamma_\mu f A^\mu \qquad\qquad (g_L\ \bar{f}_L\gamma_\mu f_L + g_R\ \bar{f}_R\gamma_\mu f_R)Z^\mu$$

Couplings to the Higgs are also flavour-conserving (aligned to the mass matrix):

$$\frac{m_f}{v}\ \bar{f}_L f_R\, h$$

No (tree-level) flavour-changing neutral currents

Why no CLFV in the Standard Model?

In the SM fermion masses, thus the *flavour sector*, stems from the Yukawa interactions:

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Rotations to the fermion mass basis: $Y_f = V_f \hat{Y}_f W_f^\dagger, \qquad f = u, d, e$

Flavour violation occurs in charged currents only:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left(\overline{u}_L \gamma^\mu (V_u^\dagger V_d) d_L + \overline{\nu}_L \gamma^\mu (V_\nu^\dagger V_e) e_L\right) W_\mu^+ + h.c.$$

$$V_{\rm CKM} \equiv V_u^\dagger V_d \qquad\qquad U_{\rm PMNS} \equiv V_\nu^\dagger V_e$$

However, if neutrinos are massless, we can choose:

$$V_\nu = V_e$$

No LFV (Y_e only ‘direction’ in the leptonic flavour space)

So why are we searching for CLFV?

- Neutrinos oscillate → we have to introduce neutrino mass terms:

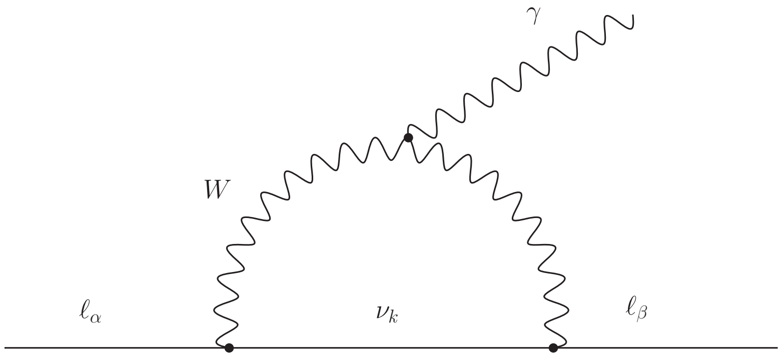
Dirac: $$\mathcal{L}_D = -(Y_\nu)_{ij}\, \overline{\nu}_{R\,i}\, \widetilde{\Phi}^\dagger\, L_{L\,j} + \text{h.c.} \quad \Longrightarrow \quad (m_\nu^D)_{ij} = \frac{v}{\sqrt{2}}(Y_\nu)_{ij}.$$

or Majorana: $$\mathcal{L} \supset \frac{C_{ij}}{\Lambda}\left(\overline{L^c_{L\,i}}\,\tau_2\Phi\right)\left(\Phi^T\tau_2 L_{L\,j}\right) + \text{h.c.} \quad \Longrightarrow \quad (m_\nu^M)_{ij} = \frac{C_{ij}v^2}{\Lambda}$$

- PMNS becomes 'physical': neutrino mass eigenstates couple to charged leptons of different flavours

- In the SM + massive neutrinos:

$$\frac{\Gamma(\ell_\alpha \to \ell_\beta\gamma)}{\Gamma(\ell_\alpha \to \ell_\beta\nu\bar{\nu})} = \frac{3\alpha}{32\pi}\left|\sum_{k=1,3} U_{\alpha k}U^*_{\beta k}\frac{m^2_{\nu_k}}{M_W^2}\right|^2$$



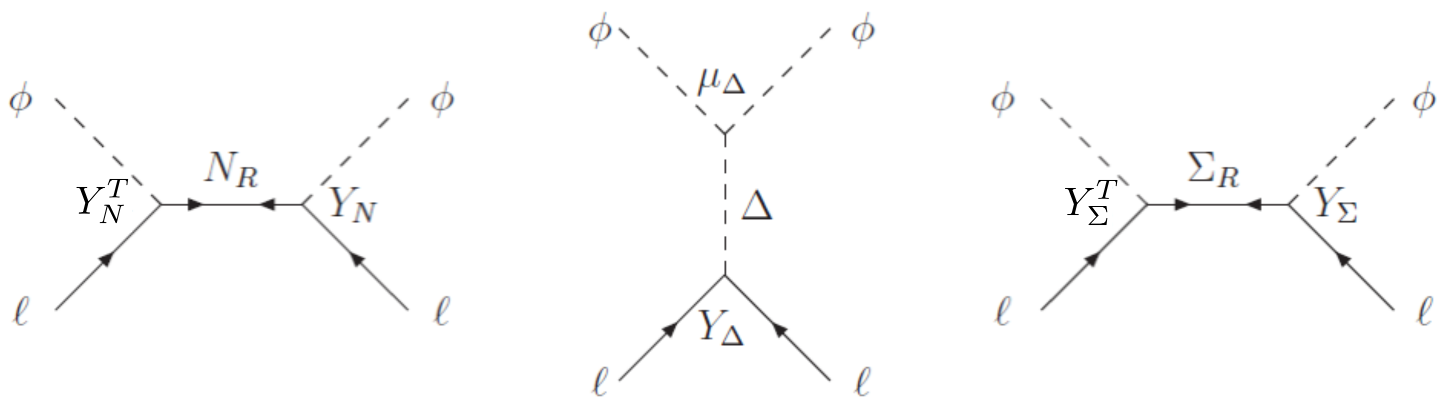
Petcov '77; Cheng Li '77, '80

⇨ $$\text{BR}(\mu\to e\gamma) \approx \text{BR}(\tau\to e\gamma) \approx \text{BR}(\tau\to\mu\gamma) = 10^{-55} \div 10^{-54}$$

Large mixing, but huge 'accidental' (?) suppression due to small neutrino masses

⇨ In presence of 'low-scale' NP, we can expect large effects

Three ways of generating the Weinberg operator at the tree level:



Type I

Heavy fermionic singlets (RH neutrinos)

Minkowski, Gell-Mann, Ramond, Slansky, Yanagida, Glashow, Mohapatra, Senjanovic, ...

Type II

Heavy scalar triplet

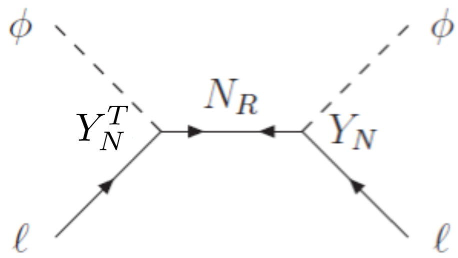
Magg, Wetterich, Lazarides, Shafi, Mohapatra, Senjanovic, Schecter, Valle, ...

Type III

Heavy fermionic triplets

Foot, Lew, He, Joshi, Ma, Roy, Hambye et al., Bajc et al., Dorsner, Fileviez-Perez, ...

Three ways of generating the Weinberg operator at the tree level:



Type I
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Type II
Heavy scalar triplet

Type III
Heavy fermionic triplets

Type I seesaw

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}\partial\!\!\!/ N - \left(Y_N \overline{N}\widetilde{\Phi}^\dagger L + \frac{1}{2} M_N \overline{N} N^c + \text{h.c.}\right)$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & Y_N^T v/\sqrt{2} \\ Y_N v/\sqrt{2} & M_N \end{pmatrix} \Longrightarrow m_\nu = -\frac{v^2}{2} Y_N^T M_N^{-1} Y_N$$

New contributions to CLFV processes:

- Light neutrinos contribution (non-unitary PMNS) $\mathcal{U} = \left(1 - \frac{v^2}{2} Y_N^\dagger M_N^{-2} Y_N\right) U$

$$\frac{\Gamma(\ell_i \to \ell_j \gamma)}{\Gamma(\ell_i \to \ell_j \nu_i \bar{\nu}_j)} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} \mathcal{U}_{ik} \mathcal{U}^\dagger_{kj} F\left(\frac{m^2_{\nu_k}}{M_W^2}\right) + \sum_{k=1,N} U^{\nu N}_{ik} U^{\nu N \dagger}_{kj} F\left(\frac{M_k^2}{M_W^2}\right)\right|^2$$

$$F(x) = \frac{10}{3} - x + \mathcal{O}(x^2) \qquad F(x) \simeq \frac{1}{2}, \quad x \gg 1$$

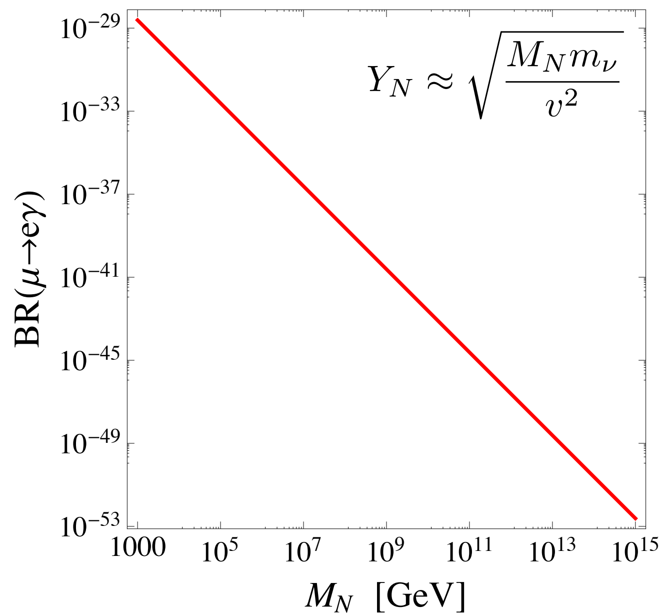
- Heavy neutrinos in the loop $U^{\nu N} = \frac{v}{\sqrt{2}} Y_N^\dagger M_N^{-1}$

Ilakovac Pilaftsis '94

Can we have large CLFV rates fulfilling with $m_{\nu_i} \lesssim 0.1$ eV ?

Type I seesaw

Naive expectation for RH neutrinos at the same scale:



But that's not (necessarily) the end of the story:

- Neutrino masses controlled by *L*-breaking dim-5 operator: $Y_N^T M_N^{-1} Y_N$
- CLFV controlled by *L*-conserving dim-6 operator: $Y_N^\dagger M_N^{-2} Y_N$

Broncano Gavela Jenkins '02

Can the dim-5 coefficient be small while the dim-6 one is large?
Yes! If the lepton number is approximately conserved…

Type I seesaw

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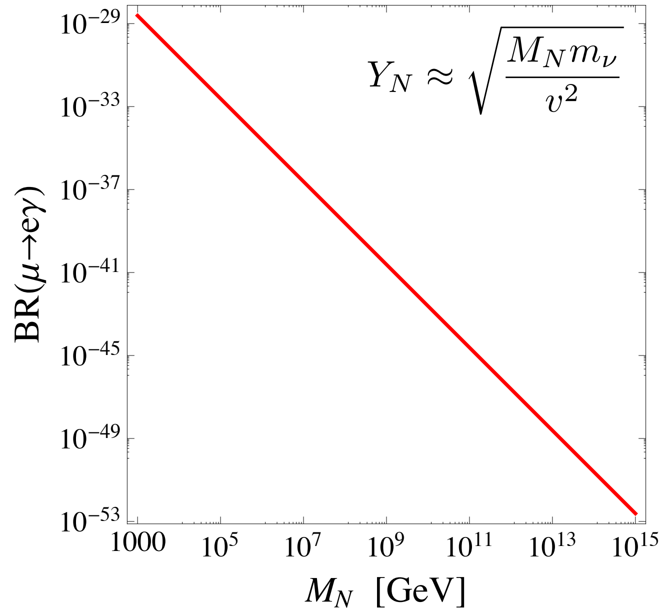


Observable effects possible for small breaking of lepton number, e.g.:

- Two almost degenerate RH neutrinos (pseudo-Dirac pair) — Ibarra Molinaro Petcov '11
- Extended mass matrix (inverse seesaw, linear seesaw...)

$$\begin{matrix} \nu_L & N\ (L=1) & S\ (L=-1) \end{matrix}$$

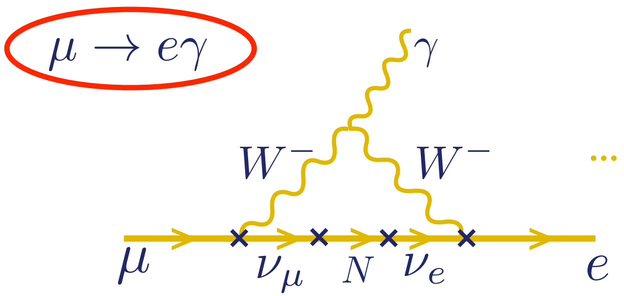
$$\mathcal{M}_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y_N & 0 \\ \frac{v}{\sqrt{2}} Y_N & 0 & M_N \\ 0 & M_N & \mu \end{pmatrix} \implies m_\nu = \frac{v^2}{2} Y_N^T \frac{\mu}{M_N^2} Y_N$$

$\mu \ll M_N$

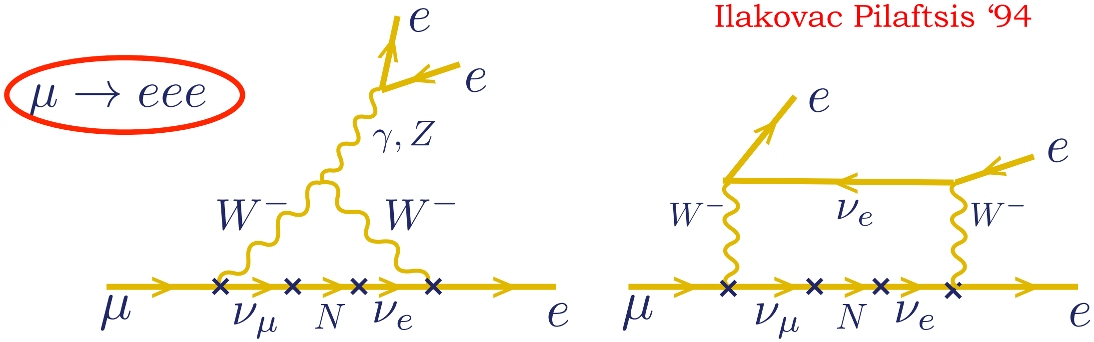
Mohapatra Valle '86

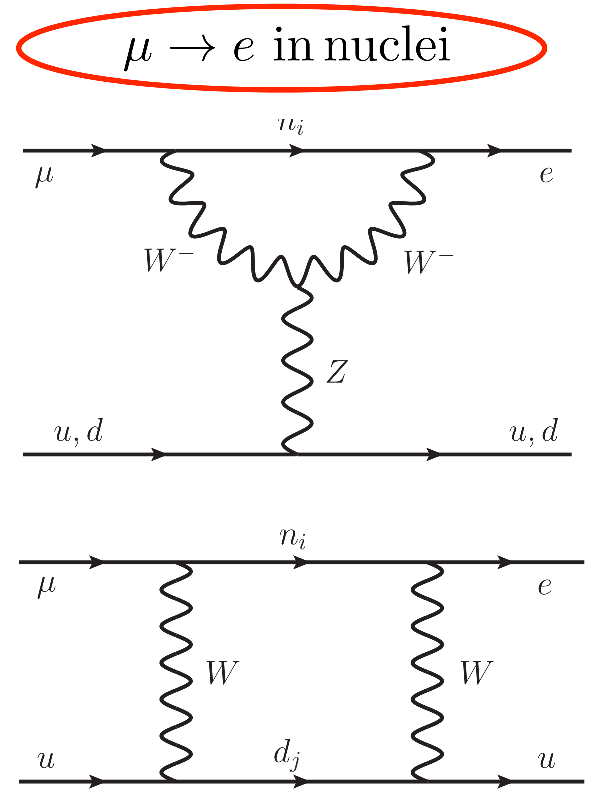
In type I seesaw, flavour-mixing occur only for neutral states

⇨ all CLFV modes arise at the loop level:



Ilakovac Pilaftsis '94





Dinh Ibarra Molinaro Petcov '12
Alonso Dhen Gavela Hambye '12

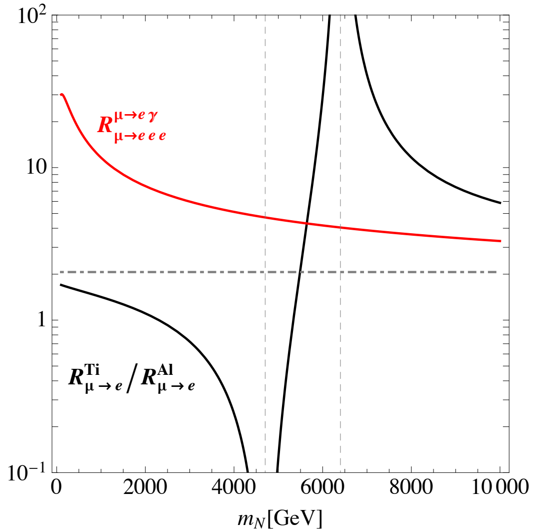
Type I

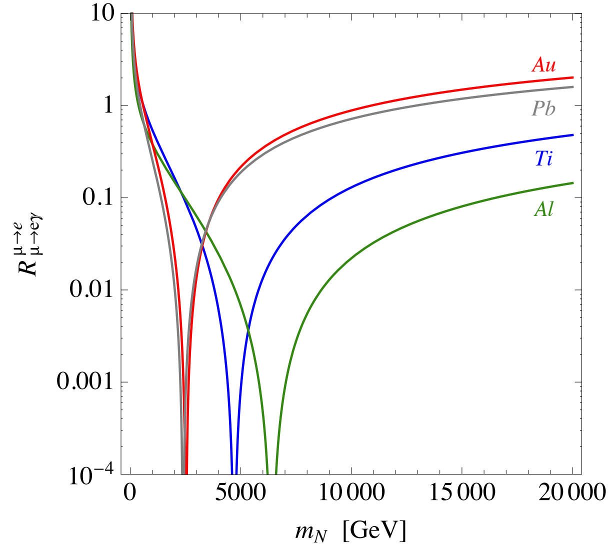
Chu Dhen Hambye '11
Dinh et al. '12, Alonso et al. '12

Quasi-degenerate RH neutrinos

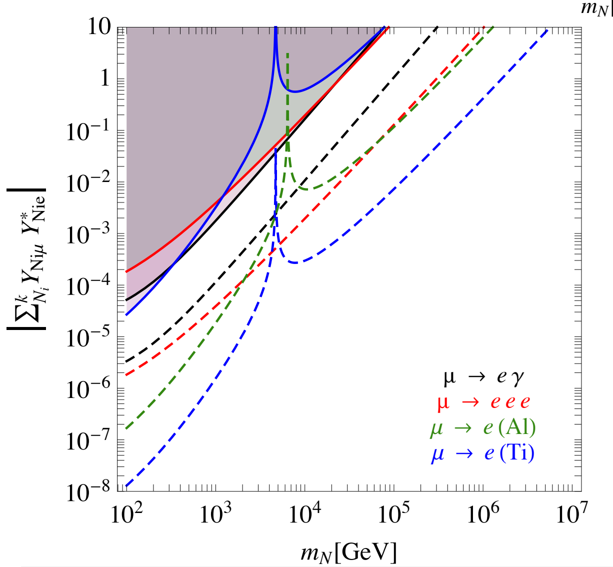
dependence on Yukawa couplings drops in ratios of BRs:

$$\frac{\mathrm{BR}(\mu \to e\gamma)}{\mathrm{BR}(\mu \to eee)}$$





$$\frac{\mathrm{CR}(\mu N \to eN)}{\mathrm{BR}(\mu \to e\gamma)}$$



Future experiments can test RH neutrino masses up to:

$$m_N \approx 300\ \mathrm{TeV} \times \left(\frac{10^{-14}}{\mathrm{BR}(\mu\to e\gamma)}\right)^{\frac{1}{4}},\ m_N \approx 1000\ \mathrm{TeV} \times \left(\frac{10^{-16}}{\mathrm{BR}(\mu\to eee)}\right)^{\frac{1}{4}},$$

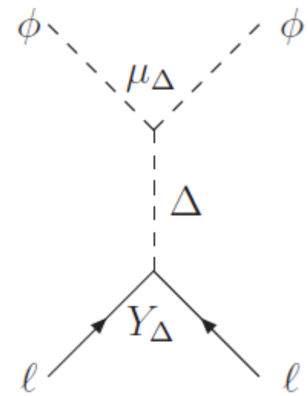
$$m_N \approx 1000\ \mathrm{TeV} \times \left(\frac{10^{-16}}{\mathrm{CR}(\mu\,\mathrm{Al}\to e\,\mathrm{Al})}\right)^{\frac{1}{4}},\ m_N \approx 6000\ \mathrm{TeV} \times \left(\frac{10^{-18}}{\mathrm{CR}(\mu\,\mathrm{Ti}\to e\,\mathrm{Ti})}\right)^{\frac{1}{4}}$$

Alonso Dhen Gavela Hambye '12

Three ways of generating the Weinberg operator at the tree level:

Type I

Heavy fermionic singlets
(RH neutrinos)



Type II

Heavy scalar triplet

Type III

Heavy fermionic
triplets

Hypercharge Y=1

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

Type II seesaw

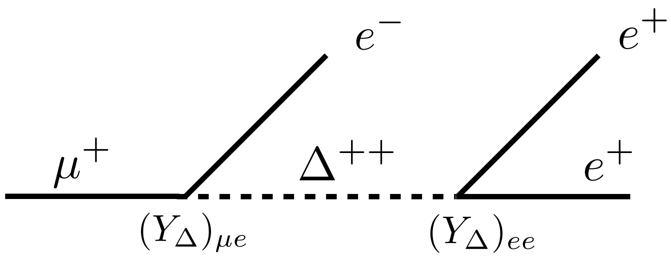
Type II

Scalar SU(2) triplet (Y=1): $\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$

L-breaking term

$$\mathcal{L} = \mathcal{L}_{\rm SM} + {\rm Tr}\left(D_\mu \Delta^\dagger\right)(D^\mu \Delta) - M_\Delta^2 {\rm Tr}\Delta^\dagger\Delta - \left(Y_\Delta L^T i\tau_2 \Delta L + \mu_\Delta \widetilde{\Phi}^T i\tau_2 \Delta \widetilde{\Phi} + {\rm h.c.}\right) \Rightarrow m_\nu = -2Y_\Delta \frac{v^2 \mu_\Delta}{M_\Delta^2}$$

Concerning CLFV, the main difference wrt Type is $\mu \to eee$ at the tree level:



$$\mathrm{BR}(\mu \to eee) = \frac{|(Y_\Delta)_{\mu e}|^2 |(Y_\Delta)_{ee}|^2}{M_\Delta^4 G_F^2}$$

Abada et al. '07, '08

Present bound: $Y_\Delta = \mathcal{O}(1) \implies M_\Delta \gtrsim 300$ TeV

Mu3e sensitivity: $\mathrm{BR}(\mu^+ \to e^+ e^+ e^-) \simeq 10^{-16} \Rightarrow M_\Delta \approx 3000$ TeV

Seesaw Mechanism(s)

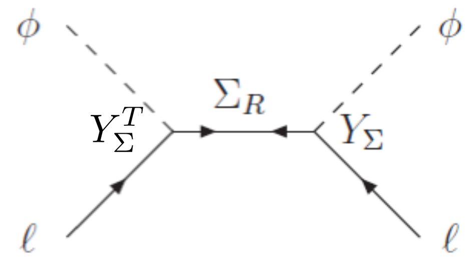
Three ways of generating the Weinberg operator at the tree level:

Type I

Heavy fermionic singlets (RH neutrinos)

Type II

Heavy scalar triplet



Type III

Heavy fermionic triplets

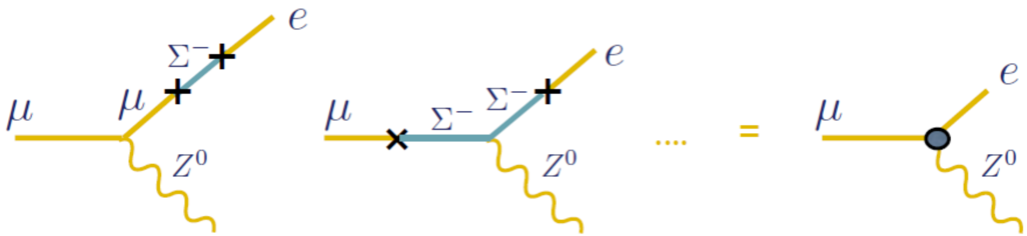
Hypercharge Y=0

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$$

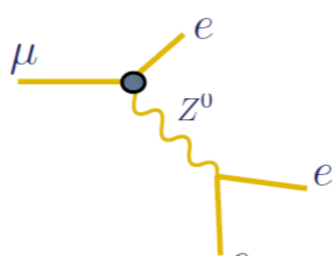
Type III seesaw

Type III

crucial property for CLFV in type-III seesaw: flavour mixing directly at the level of charged states



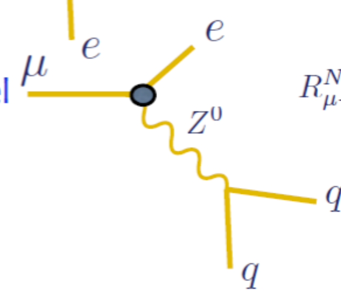
⇒ $\mu \to eee$: tree level



$$\Gamma(\mu \to eee) = \sum_{\Sigma_i} \frac{|Y_{\Sigma_{ie}} Y^\dagger_{\Sigma_{i\mu}}|^2}{m^4_{\Sigma_i}} \cdot d^2$$

Abada, Biggio, Bonnet, Gavela, TH 07', 08'

$\mu \to e$ conversion : tree level



$$R^N_{\mu\to e} = \sum_{\Sigma_i} \frac{|Y_{\Sigma_{ie}} Y^\dagger_{\Sigma_{i\mu}}|^2}{m^4_{\Sigma_i}} \cdot (b^N)^2$$

$\mu \to e\gamma$: still at one loop

⇒ ratios of 2 processes with same flavour transition: totally fixed!

$$\mathrm{BR}(\mu \to e\gamma) = 1.3 \times 10^{-3} \times \mathrm{BR}(\mu \to eee) = 3.1 \times 10^{-4} \times \mathrm{CR}(\mu\,\mathrm{Ti} \to e\,\mathrm{Ti}),$$
$$\mathrm{BR}(\tau \to \mu\gamma) = 1.3 \times 10^{-3} \times \mathrm{BR}(\tau \to \mu\mu\mu) = 2.1 \times 10^{-3} \times \mathrm{BR}(\tau^- \to \mu^- e^+ e^-),$$
$$\mathrm{BR}(\tau \to e\gamma) = 1.3 \times 10^{-3} \times \mathrm{BR}(\tau \to eee) = 2.1 \times 10^{-3} \times \mathrm{BR}(\tau^- \to e^- \mu^+ \mu^-).$$

borrowed from T. Hambye

Searches for the different $\mu \to e$ modes are highly complementary in terms of model discrimination:

TABLE VII. – *Pattern of the relative predictions for the $\mu \to e$ processes as predicted in several models (see the text for details). Whether the dominant contributions to $\mu \to eee$ and $\mu \to e$ conversion are at the tree or at the loop level is indicated; Loop* indicates that there are contributions that dominate over the dipole one, typically giving an enhancement compared to eqs.* (40), (41).

Model	$\mu \to eee$	$\mu N \to eN$	$\frac{\mathrm{BR}(\mu\to eee)}{\mathrm{BR}(\mu\to e\gamma)}$	$\frac{\mathrm{CR}(\mu N\to eN)}{\mathrm{BR}(\mu\to e\gamma)}$
MSSM	Loop	Loop	$\approx 6 \times 10^{-3}$	10^{-3}–10^{-2}
Type-I seesaw	Loop*	Loop*	3×10^{-3}–0.3	0.1–10
Type-II seesaw	Tree	Loop	$(0.1$–$3) \times 10^{3}$	$\mathcal{O}(10^{-2})$
Type-III seesaw	Tree	Tree	$\approx 10^{3}$	$\mathcal{O}(10^{3})$
LFV Higgs	Loop$^{(a)}$	Loop$^{*(a)}$	$\approx 10^{-2}$	$\mathcal{O}(0.1)$
Composite Higgs	Loop*	Loop*	0.05–0.5	2–20

(a) A tree-level contribution to this process exists but it is subdominant.

LC Signorelli '17

If dipole operator dominates (e.g. as in R-parity conserving SUSY):

$$\mathrm{BR}(\mu \to eee) \simeq \frac{\alpha}{3\pi}\left(\log\frac{m_\mu^2}{m_e^2} - 3\right) \times \mathrm{BR}(\mu \to e\gamma),$$

$$\mathrm{CR}(\mu\,\mathrm{N} \to e\,\mathrm{N}) \simeq \alpha \times \mathrm{BR}(\mu \to e\gamma).$$

Hisano et al. '95

Radiative neutrino mass models

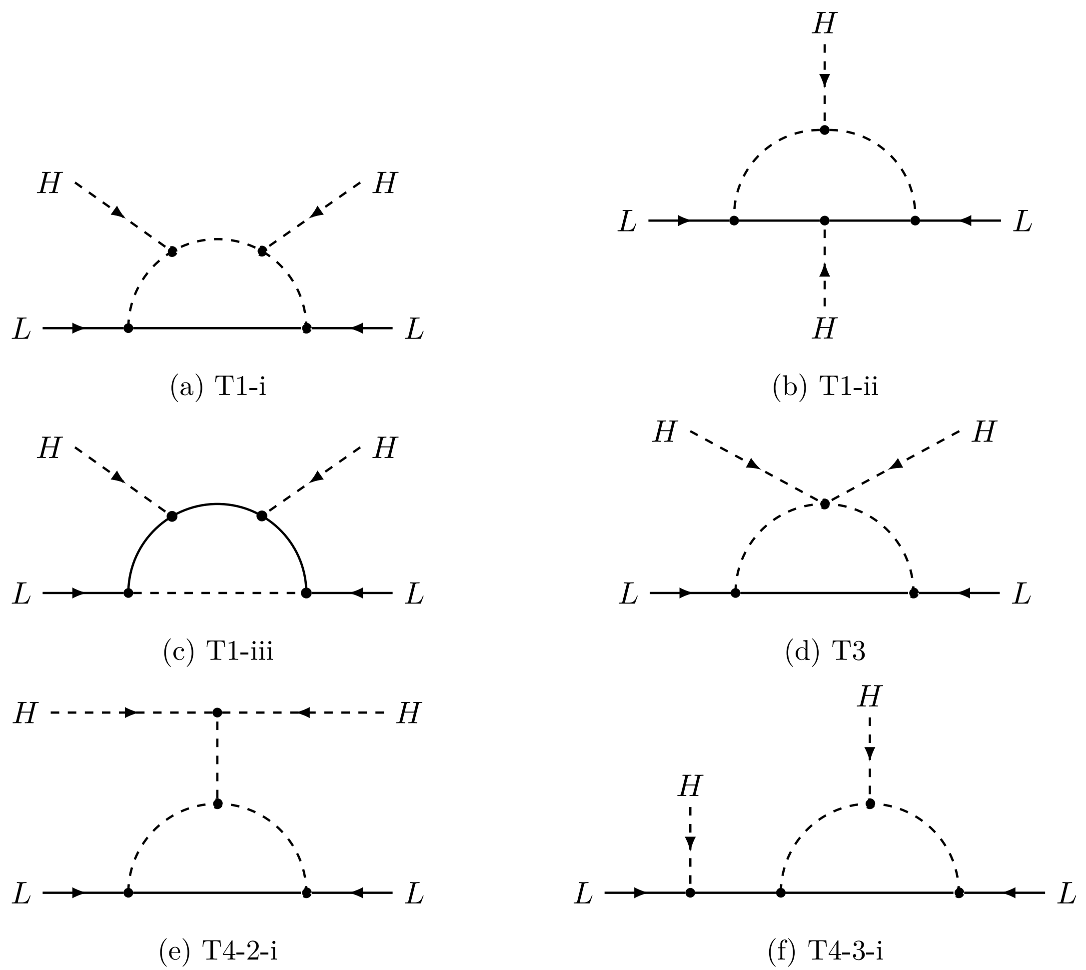
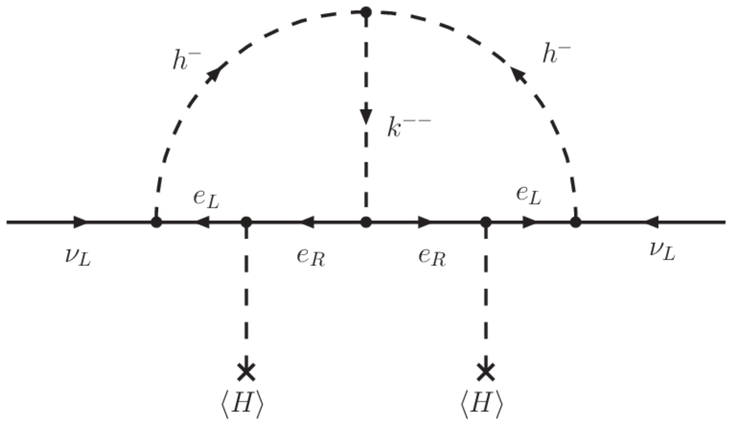


Figure 3: Feynman diagram topologies for 1-loop radiative neutrino mass generation with the Weinberg operator $O_1 = LLHH$. Dashed lines could be scalars or gauge bosons if allowed.

Review by Cai et al. 1706.08524

Zee-Babu model

Zee '86, Babu '88



$$\mathcal{L}_Y = \overline{L_L}\, Y e H + \overline{\tilde{L}_L} f \ell h^+ + \overline{e^c} g\, e\, k^{++} + \text{h.c.}$$

Three-body CLFV decays mediated at the tree level by k^{--} :

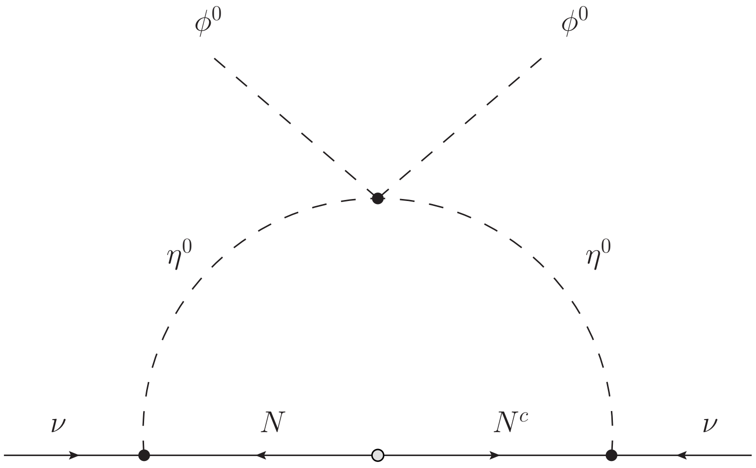
Process	Experiment (90% CL)	Bound (90% CL)
$\mu^- \to e^+e^-e^-$	BR$< 1.0 \times 10^{-12}$	$\lvert g_{e\mu} g_{ee}^* \rvert < 2.3 \times 10^{-5} \left(\frac{m_k}{\text{TeV}}\right)^2$
$\tau^- \to e^+e^-e^-$	BR$< 2.7 \times 10^{-8}$	$\lvert g_{e\tau} g_{ee}^* \rvert < 0.009 \left(\frac{m_k}{\text{TeV}}\right)^2$
$\tau^- \to e^+e^-\mu^-$	BR$< 1.8 \times 10^{-8}$	$\lvert g_{e\tau} g_{e\mu}^* \rvert < 0.005 \left(\frac{m_k}{\text{TeV}}\right)^2$
$\tau^- \to e^+\mu^-\mu^-$	BR$< 1.7 \times 10^{-8}$	$\lvert g_{e\tau} g_{\mu\mu}^* \rvert < 0.007 \left(\frac{m_k}{\text{TeV}}\right)^2$
$\tau^- \to \mu^+e^-e^-$	BR$< 1.5 \times 10^{-8}$	$\lvert g_{\mu\tau} g_{ee}^* \rvert < 0.007 \left(\frac{m_k}{\text{TeV}}\right)^2$
$\tau^- \to \mu^+e^-\mu^-$	BR$< 2.7 \times 10^{-8}$	$\lvert g_{\mu\tau} g_{e\mu}^* \rvert < 0.007 \left(\frac{m_k}{\text{TeV}}\right)^2$
$\tau^- \to \mu^+\mu^-\mu^-$	BR$< 2.1 \times 10^{-8}$	$\lvert g_{\mu\tau} g_{\mu\mu}^* \rvert < 0.008 \left(\frac{m_k}{\text{TeV}}\right)^2$
$\mu^+e^- \to \mu^-e^+$	$G_{M\bar{M}} < 0.003 G_F$	$\lvert g_{ee} g_{\mu\mu}^* \rvert < 0.2 \left(\frac{m_k}{\text{TeV}}\right)^2$

Herrero-Garcia et al. '14

Table I: Constraints from tree-level lepton flavour violating decays [3].

Scotogenic model

E. Ma '06

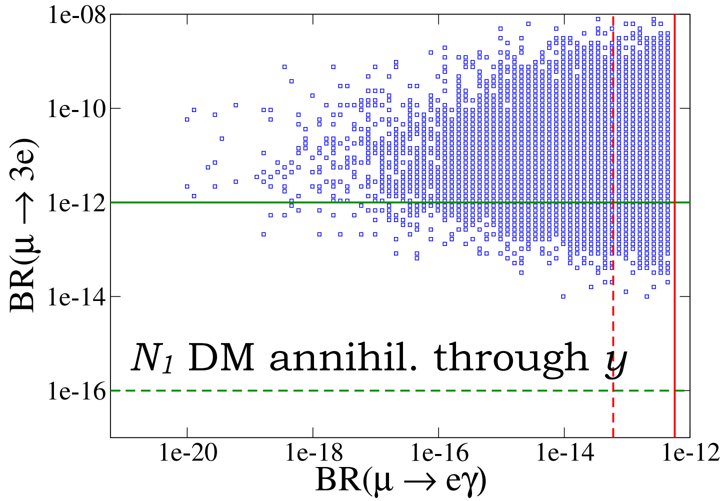


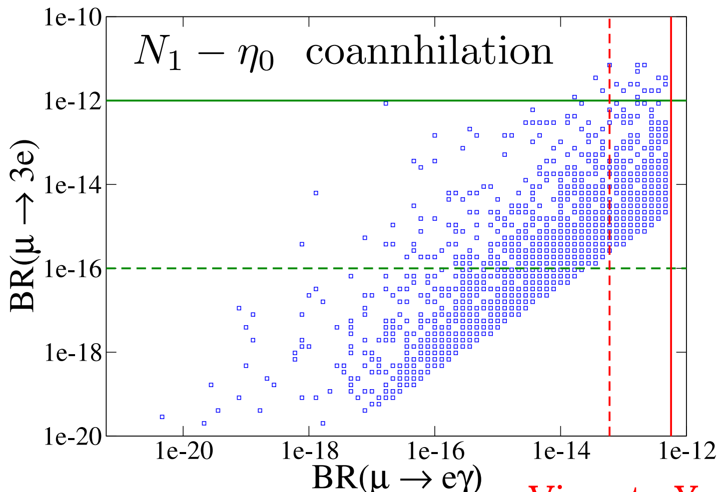
Z_2-odd singlet fermions and a scalar doublet are introduced:

$$\mathcal{L}_N = \overline{N_i}\partial\!\!\!/ N_i - \frac{M_{N_i}}{2}\overline{N_i^c}P_R N_i + y_{i\alpha}\eta\overline{N_i}P_L\ell_\alpha + \text{h.c.}\,,$$

$\rightarrow\ \eta_0,\ N_1$ DM candidates!

Observed DM abundance requires not too heavy masses / too small couplings, large CLFV effects are expected:



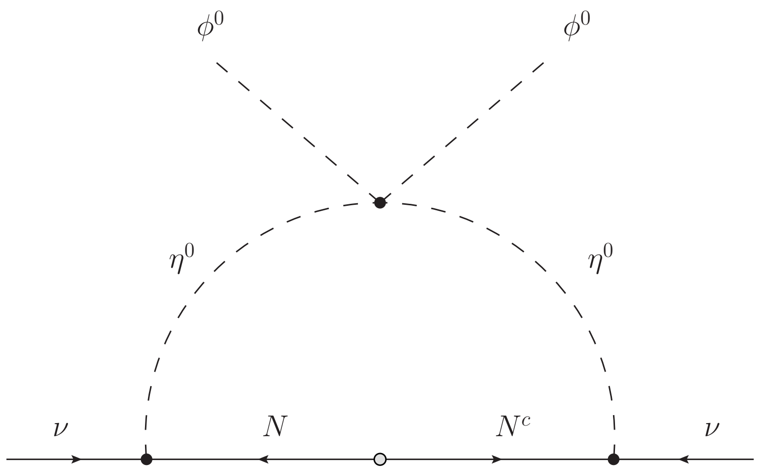


Vicente Yaguna '14

Mu3e and COMET/Mu2e can test most of the thermal DM parameter space!

Scotogenic model

E. Ma '06



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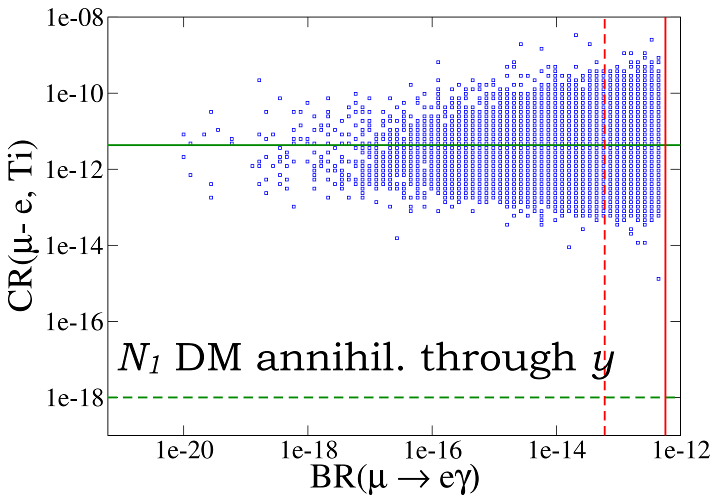
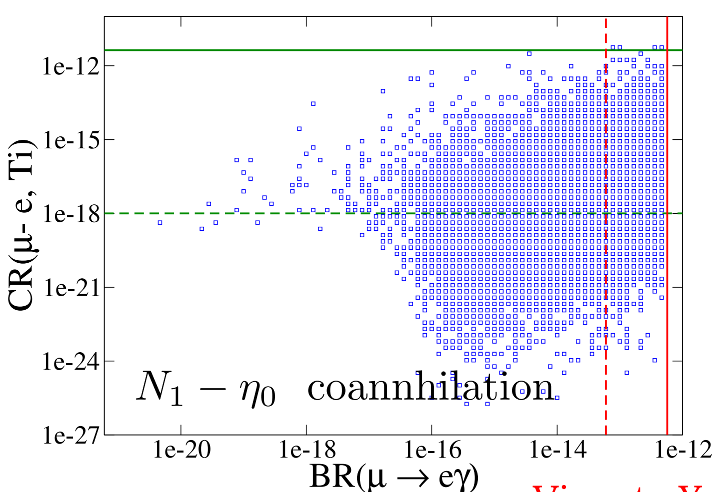




Vicente Yaguna '14

Mu3e and COMET/Mu2e can test most of the thermal DM parameter space!

Flavour models

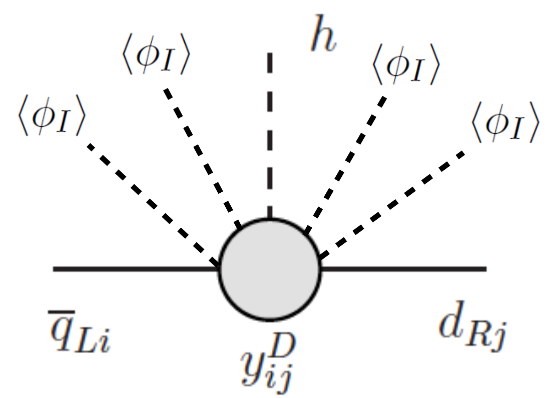
→ Everett talk

- SM fermions charged under a new horizontal symmetry G_F
- G_F forbids Yukawa couplings at the renormalisable level
- G_F spontaneously broken by the vev(s) of one or more scalars (the "flavons")
- Yukawas arise as higher dimensional operators

Froggatt Nielsen '79
Leurer Seiberg Nir '92, '93
...

$$\mathcal{L}_{yuk} = y_{ij}^{U}\, \overline{q}_{Li} u_{Rj}\, \tilde{h} + y_{ij}^{D}\, \overline{q}_{Li} d_{Rj}\, h + \text{h.c.}$$

$$y_{ij}^{U,D} \sim \prod_I \left(\frac{\langle\phi_I\rangle}{M}\right)^{n_{I,ij}^{U,D}}$$



$\phi_I < M \Longrightarrow \epsilon_I \equiv \langle\phi_I\rangle/M$ small expansion parameter (M=UV scale)

$n_{I,ij}^{U,D}$ dictated by the symmetry

G_F could abelian or non-abelian, continuous or discrete, global or local…

Which group for G_F?

U(1), U(1)xU(1), SU(2), SU(3), SO(3), A_4 ...

Froggatt Nielsen '79; Leurer Seiberg Nir '92, '93; Ibanez Ross '94; Dudas Pokorski Savoy '95; Binetruy Lavignac Ramond '96; Barbieri Dvali Hall '95; Pomarol Tommasini '95; Berezhiani Rossi '98; King Ross '01; Ma '02; Altarelli Feruglio '05 ... ...

If you have a model of flavour, in particular for the PMNS, prediction for CLFV processes can be affected in several ways:

- The symmetry shapes the flavour structure of effective operators (Studied especially in the context of discrete symmetries)

  Feruglio Hagedorn Lin Merlo '08, Altarelli Feruglio '10, Deppisch '12

- In SUSY models, slepton mass matrices determined by the flavour symmetry: same dynamics accounts for fermion masses and mixing and controls LFV. Examples:

  LC Lalak Pokorski Ziegler '12

SU(3)	$U(2)_l$x$U(2)_e$	A_4
LC Jones Vives '07; LC Jones Masiero Park Vives '09;	Blankenburg Isidori Jones '12	Feruglio Hagedorn Lin Merlo '08, '09; Hagedorn Molinaro Petcov '09 Altarelli Feruglio Merlo Stamou '12

- In general this works for the flavour structure of any NP couplings

Froggatt-Nielsen U(1)

Quark sector

	ϕ	$\bar{q}_i$	u_i	d_i	h
U(1)	-1	$[q]_i$	$[u]_i$	$[d]_i$	0

$$y^u_{ij} = a^u_{ij}\,\epsilon^{[q]_i+[u]_j}$$
$$y^d_{ij} = a^d_{ij}\,\epsilon^{[q]_i+[d]_j}$$

Rotation matrices $V_L^{f\dagger} Y^f V_R^f = Y^f_{diag}$ $\Rightarrow$ $(V_L^{u,d})_{ij} \approx \epsilon^{|[q]_i-[q]_j|}$ $(V_R^{u,d})_{ij} \approx \epsilon^{|[u,d]_i-[u,d]_j|}$

Successful predictions for $V^{\rm CKM} = V^u V^{d\dagger}$:

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx 1 \qquad V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

(independent of charge assignment)

Example:

$([q]_1,\, [q]_2,\, [q]_3) = (3,2,0)$ $([u]_1,\, [u]_2,\, [u]_3) = (5,2,0)$ $([d]_1,\, [d]_2,\, [d]_3) = (4,2,2)$

$$Y_u \sim \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix} \qquad Y_d \sim \begin{pmatrix} \epsilon^7 & \epsilon^5 & \epsilon^5 \\ \epsilon^6 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \end{pmatrix}$$

$\epsilon = \langle\phi\rangle/M \approx 0.23$

Froggatt-Nielsen U(1)

Lepton sector

	ϕ	$\bar{L}_i$	e_i	h
U(1)	-1	$[L]_i$	$[e]_i$	0

$\Rightarrow$

$$y^e_{ij} = a^e_{ij}\epsilon^{[L]_i+[e]_j}$$

$$(m_\nu)_{ij} \sim \frac{v^2}{\Lambda}\epsilon^{[L]_i+[L]_j}$$

LH charges can chosen to give a (quasi-)anarchical PMNS

RH charges then responsible for charged leptons hierarchy

Examples:

Altarelli Feruglio Masina Merlo '12

- Anarchy $([L]_1,\,[L]_2,\,[L]_3) = ([L],[L],[L])$
- Mu-tau anarchy $([L]_1,\,[L]_2,\,[L]_3) = ([L]+1,[L],[L])$
- Hierarchy $([L]_1,\,[L]_2,\,[L]_3) = ([L]+2,[L]+1,[L])$

Charged lepton hierarchy, e.g. : $([e]_1,\,[e]_2,\,[e]_3) = (8-[L]_1, 4-[L]_2, 2-[L]_3)$
(with $\epsilon \approx 0.2$)

Lepton sector

	ϕ	$\bar{L}_i$	e_i	h
U(1)	-1	$[L]_i$	$[e]_i$	0

$\Rightarrow$

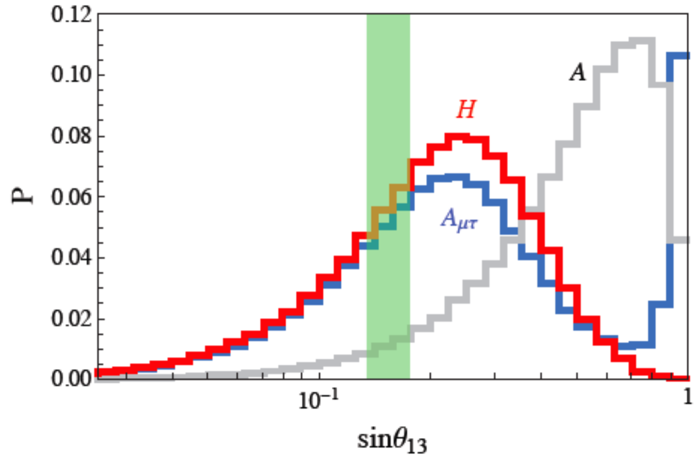
$$y^e_{ij} = a^e_{ij}\epsilon^{[L]_i+[e]_j}$$

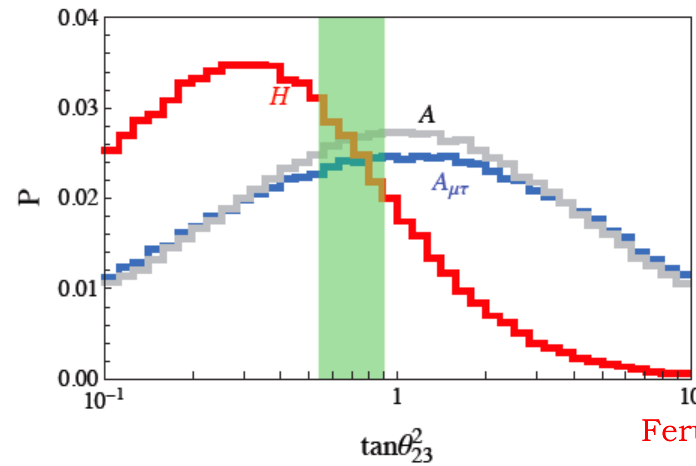
$$(m_\nu)_{ij} \sim \frac{v^2}{\Lambda}\epsilon^{[L]_i+[L]_j}$$

LH charges can chosen to give a (quasi-)anarchical PMNS

RH charges then responsible for charged leptons hierarchy

Examples:





Feruglio '17

PNGB of a spontaneously-broken leptonic FN U(1)

$$\phi = \frac{1}{\sqrt{2}}(f + \rho_\phi)e^{ia/f} \Rightarrow \mathcal{L}_{aff} = \frac{\partial^\mu a}{f}\left(C_V^{ij}\,\bar{\ell}_i\gamma_\mu\ell_j + C_A^{ij}\,\bar{\ell}_i\gamma_\mu\gamma_5\ell_j\right)$$

$$C_{V/A} = V_R^\dagger X_R V_R \pm V_L^\dagger X_L V_L \qquad X_L = \begin{pmatrix} [L]_1 & & \\ & [L]_2 & \\ & & [L]_3 \end{pmatrix} \quad X_R = \begin{pmatrix} [e]_1 & & \\ & [e]_2 & \\ & & [e]_3 \end{pmatrix}$$

flavour non-universal charges
→ flavour-violating couplings

$$(V_L)_{ij} \approx \epsilon^{|[L]_i - [L]_j|}, \qquad (V_R)_{ij} \approx \epsilon^{|[e]_i - [e]_j|}$$

Lepton-flavour-violating decays into an (invisible) PNGB:

$$\mu \to ea \qquad \tau \to ea \qquad \tau \to \mu a$$

$$\Gamma(\ell_i \to \ell_j a) = \frac{1}{16\pi}\frac{m_{\ell_i}^3}{f^2}\left(|C_V^{ij}|^2 + |C_A^{ij}|^2\right)\left(1 - \frac{m_a^2}{m_{\ell_i}^2}\right)^2$$

Feng Moroi Murayama Schnapka '97
LC Redigolo Ziegler Zupan to appear

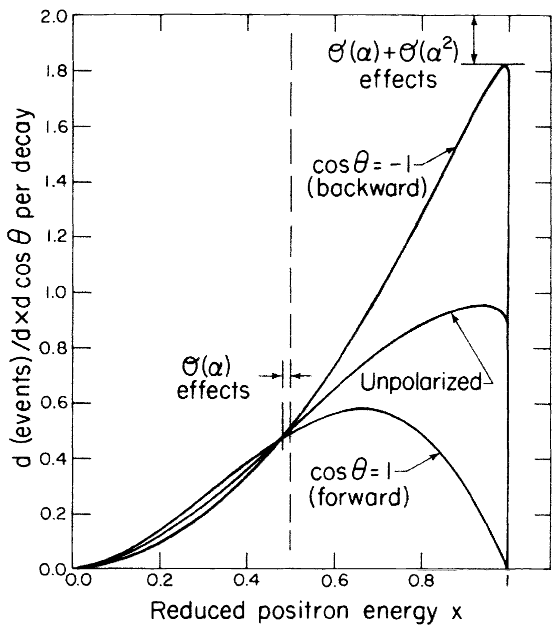
Bounds on LFV decays to a PNGB

Bound on isotropic $\mu \to ea$ using polarised anti-muons.

$\mu \to e\bar{\nu}\nu$ background:

$$\frac{d^2\Gamma}{dx\, d\cos\theta} = \Gamma_\mu \left((3-2x) - P(2x-1)\cos\theta\right) x^2$$

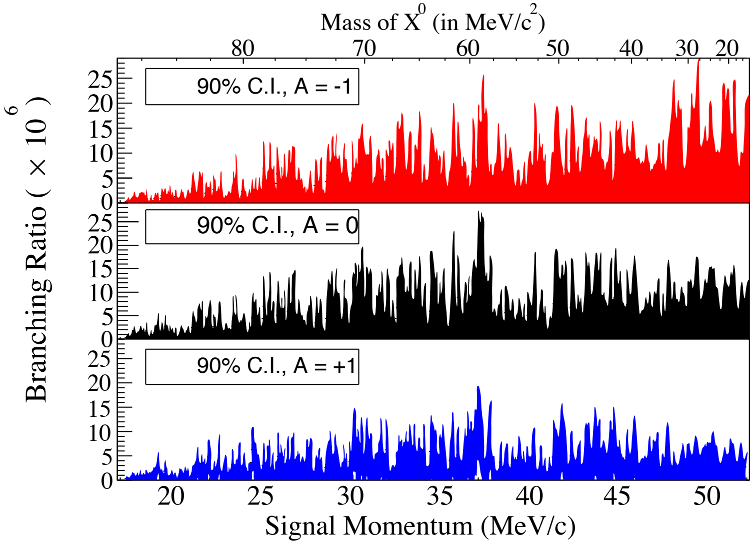
$$x = 2E_e/m_\mu$$



$$\Rightarrow \; \mathrm{BR}(\mu^+ \to e^+ a) < 2.6 \times 10^{-6}$$

Jodidio et al. (TRIUMF) ‘86

Considering anisotropy of the signal:



Decay Signal		90% C.L. (in ppm)	p-value
$A = 0$	Average	9	
	$p = 37.03$ MeV/c	26	0.66
	Endpoint	21	0.81
$A = -1$ (SM-like)	Average	10	
	$p = 37.28$ MeV/c	26	0.60
	Endpoint	58	0.80
$A = +1$	Average	6	
	$p = 19.13$ MeV/c	6	0.59
	Endpoint	10	0.90

$$\mathrm{BR}(\mu \to ea) < 5.8 \times 10^{-5}$$ TWIST 2014

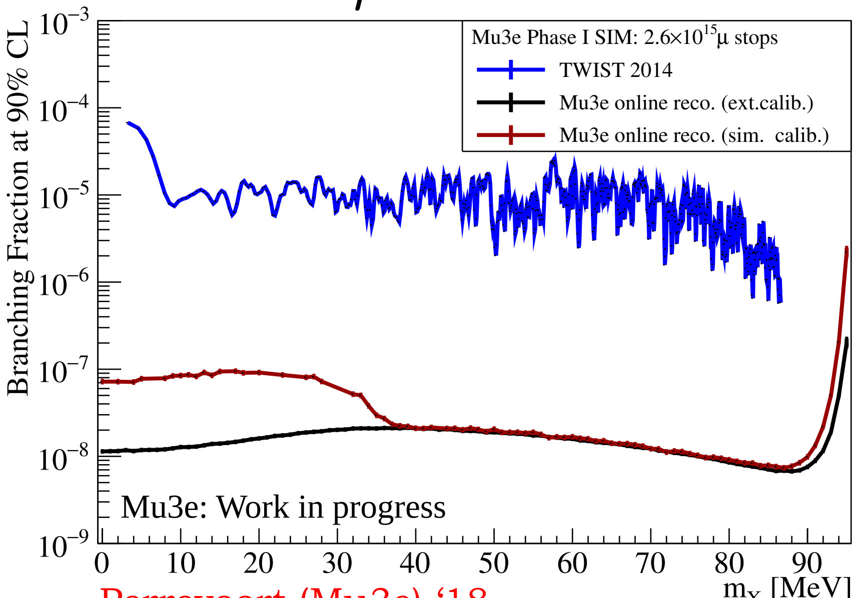
Bounds on LFV decays to a PNGB

Other modes

- $BR(\mu \to e\, a\, \gamma) < 1.1 \times 10^{-9}$ Crystal Box '88
- $BR(\tau \to e\, a) < 2.6 \times 10^{-3}$ ARGUS '95
- $BR(\tau \to \mu\, a) < 4.5 \times 10^{-3}$ ARGUS '95

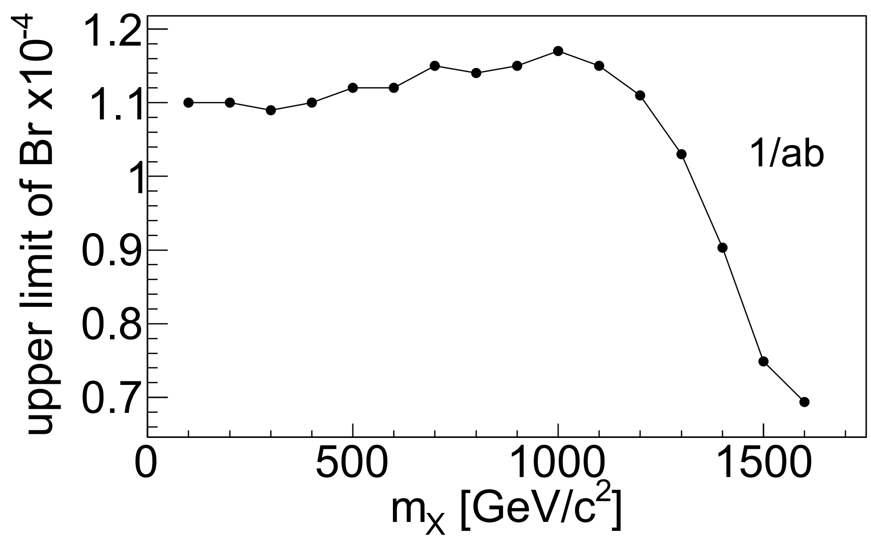
Future prospects

$\mu \to ea$



Perrevoort (Mu3e) '18

$\tau \to \mu a$



Yoshinobu Hayasaka (Belle simul.) '17

LFV decays into a leptonic familon

$\mu \to ea$ differential decay rate:

LC Redigolo Ziegler Zupan to appear

$$\frac{d\Gamma}{d\cos\theta} = \frac{1}{2\Gamma_{\mu\to ea}}\left[1 + 2P\cos\theta\frac{C_V^{\mu e}C_A^{\mu e}}{(C_V^{\mu e})^2 + (C_A^{\mu e})^2}\right]$$

Anisotropy (thus exp. bound) depends on the model:

$$C_{V/A} = V_R^\dagger X_R V_R \pm V_L^\dagger X_L V_L$$

Anarchical model

RH rotations dominate:

$$C_V^{ij} \approx C_A^{ij}$$

Stronger exp. limit applies:

$$\mathrm{BR}(\mu\to ea) < 2.6\times 10^{-6}$$

But suppressed rate:

$$\Gamma(\mu\to ea) \approx \frac{1}{16\pi}\frac{m_\mu^3}{f^2}|(V_R)_{12}|^2$$

$|(V_R)_{12}|^2$ → $\mathcal{O}(m_e/m_\mu)$

Hierarchical model

LH rotations dominate:

$$C_V^{ij} \approx -C_A^{ij}$$

Weaker exp. limit applies:

$$\mathrm{BR}(\mu\to ea) < 5.8\times 10^{-5}$$

But larger rate:

$$\Gamma(\mu\to ea) \approx \frac{1}{16\pi}\frac{m_\mu^3}{f^2}|(V_L)_{12}|^2$$

$|(V_L)_{12}|^2$ → $\mathcal{O}(\epsilon)$

Bounds on the flavour-breaking scale f

LC Redigolo Ziegler Zupan to appear

Present bounds

	Anarchical model	Hierarchical model
$\mu \to e\, a$	$f > 2 \times 10^7$ GeV	5×10^8 GeV
$\mu \to e\, a\, \gamma$	10^7 GeV	4×10^8 GeV
$\tau \to e\, a$	5×10^3 GeV	10^6 GeV
$\tau \to \mu\, a$	4×10^5 GeV	10^6 GeV

To be compared to the bound (from the coupling to electrons) from star cooling:

$$f > 2 \times 10^{10}\ \text{GeV}$$

LC Redigolo Ziegler Zupan to appear

Future sensitivity

	Anarchical model		Hierarchical model
$\mu \to e\, a$	$f > 8 \times 10^{8}$ GeV	Mu3e phase I	4×10^{10} GeV
$\mu \to e\, a\, \gamma$	?	MEG-II ? Mu3e?	?
$\tau \to e\, a$			
$\tau \to \mu\, a$	7×10^{6} GeV	Belle2 (50/ab)	2×10^{7} GeV

To be compared to the bound (from the coupling to electrons) from star cooling:

$$f > 2 \times 10^{10}\ \text{GeV}$$

Conclusions

CLFV observables among the cleanest and most stringent tests of physics beyond the Standard Model

CLFV processes are an unavoidable consequence of any physics that may be behind neutrino masses and mixing (At observable levels? This depends on the scale of NP…)

Different muon and tau CLFV modes nicely complementary as model discriminators

Simple abelian flavour models can be tested by CLFV decays into a light invisible pseudoscalar ‘familon’. Future limits can supersede stellar bounds.

If violation of lepton universality in B decays is confirmed, we expect observable tau CLFV at Belle-II!

ありがとうございました!

謝 謝！

Additional slides

Dimension-6 effective operators that can induce CLFV

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\sum_a C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2}\sum_a C_a^{(6)} Q_a^{(6)} + \ldots$$

Grzadkowski et al. '10

4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q_{\ell q}^{(1)}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q_{\ell q}^{(3)}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}_L^a e_R)(\bar{d}_R Q_L^a)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q_{\ell equ}^{(1)}$	$(\bar{L}_L^a e_R)\epsilon_{ab}(\bar{Q}_L^b u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q_{\ell equ}^{(3)}$	$(\bar{L}_i^a\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}_L^b\sigma^{\mu\nu}u_R)$
Lepton-Higgs operators			
$Q_{\Phi\ell}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q_{\Phi\ell}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}^I_\mu \Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

Crivellin Najjari Rosiek '13

Probing high energy scales

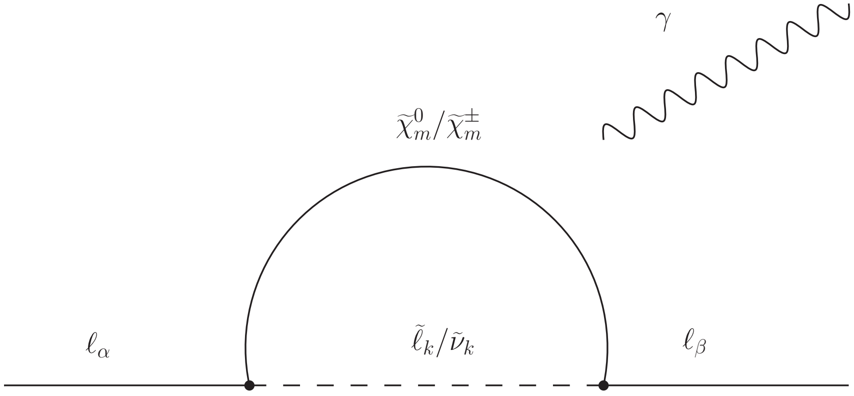
$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\sum_a C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2}\sum_a C_a^{(6)} Q_a^{(6)} + \ldots$$

	$\lvert C_a\rvert$ [$\Lambda = 1$ TeV]	Λ (TeV) [$\lvert C_a\rvert = 1$]	CLFV Process
$C_{e\gamma}^{\mu e}$	2.1×10^{-10}	6.8×10^4	$\mu \to e\gamma$
$C_{\ell e}^{\mu\mu\mu e, e\mu\mu\mu}$	1.8×10^{-4}	75	$\mu \to e\gamma$ [1-loop]
$C_{\ell e}^{\mu\tau\tau e, e\tau\tau\mu}$	1.0×10^{-5}	312	$\mu \to e\gamma$ [1-loop]
$C_{e\gamma}^{\mu e}$	4.0×10^{-9}	1.6×10^4	$\mu \to eee$
$C_{\ell\ell,ee}^{\mu eee}$	2.3×10^{-5}	207	$\mu \to eee$
$C_{\ell e}^{\mu eee, ee\mu e}$	3.3×10^{-5}	174	$\mu \to eee$
$C_{e\gamma}^{\mu e}$	5.2×10^{-9}	1.4×10^4	$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$
$C_{\ell q,\ell d, ed}^{e\mu}$	1.8×10^{-6}	745	$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$
$C_{eq}^{e\mu}$	9.2×10^{-7}	1.0×10^3	$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$
$C_{\ell u, eu}^{e\mu}$	2.0×10^{-6}	707	$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$
$C_{e\gamma}^{\tau\mu}$	2.7×10^{-6}	610	$\tau \to \mu\gamma$
$C_{e\gamma}^{\tau e}$	2.4×10^{-6}	650	$\tau \to e\gamma$
$C_{\ell\ell,ee}^{\mu\tau\mu\mu}$	7.8×10^{-3}	11.3	$\tau \to \mu\mu\mu$
$C_{\ell e}^{\mu\tau\mu\mu,\mu\mu\mu\tau}$	1.1×10^{-2}	9.5	$\tau \to \mu\mu\mu$
$C_{\ell\ell,ee}^{e\tau ee}$	9.2×10^{-3}	10.4	$\tau \to eee$
$C_{\ell e}^{e\tau ee, eee\tau}$	1.3×10^{-2}	8.8	$\tau \to eee$

Charged Lepton Flavour Violation in SUSY

Slepton mass matrix:

$$m^2_{\tilde{\ell}} = \begin{pmatrix} (\tilde{m}^2_L)_{ij} + (m^2_\ell)_{ij} - m^2_Z(\frac{1}{2} - \sin^2\theta_W)\delta_{ij} & A^{\ell *}_{ji} v_d - (m_\ell)_{ji}\mu \tan\beta \\ A^\ell_{ij} v_d - (m_\ell)_{ij}\mu^* \tan\beta & (\tilde{m}^2_E)_{ij} + (m^2_\ell)_{ij} - m^2_Z \sin^2\theta_W \delta_{ij} \end{pmatrix}$$



Flavour-conserving counterparts:

g-2, EDMs

$$\Rightarrow \quad BR(\ell_i \to \ell_j \gamma) = \frac{48\pi^3 \alpha_{\rm em}}{G_F^2}\left(|C^{ij}_L|^2 + |C^{ij}_R|^2\right) BR(\ell_i \to \ell_j \nu_i \bar{\nu}_j)$$

Hisano et al. ‘95

$$C^{ij}_L \sim \frac{g^2}{16\pi^2}\frac{(\tilde{m}^2_L)_{ij}}{\tilde{m}^4}\tan\beta$$

In SUSY, new fields interacting with the MSSM fields enter the radiative corrections of the sfermion masses

Hall Kostelecky Raby ‘86

⇨ This applies to the new seesaw interactions: generically induce LFV in the slepton mass matrix!

Borzumati Masiero ‘86

Type I

$$(\tilde{m}_L^2)_{ij} \propto m_0^2 \sum_k (\mathbf{Y}_N^*)_{ki} (\mathbf{Y}_N)_{kj} \ln\left(\frac{M_X}{M_{R_K}}\right)$$

Borzumati Masiero ‘86

Type II

$$(\tilde{m}_L^2)_{ij} \propto m_0^2 (\mathbf{Y}_\Delta{}^\dagger \mathbf{Y}_\Delta)_{ij} \ln\left(\frac{M_X}{M_\Delta}\right) \propto m_0^2 (\mathbf{m}_\nu{}^\dagger \mathbf{m}_\nu)_{ij} \ln\left(\frac{M_X}{M_\Delta}\right)$$

A. Rossi ‘02; Rossi Joaquim ‘06

Type III

Similar to type I

Biggio LC ‘10; Esteves et al. ‘10

Type I

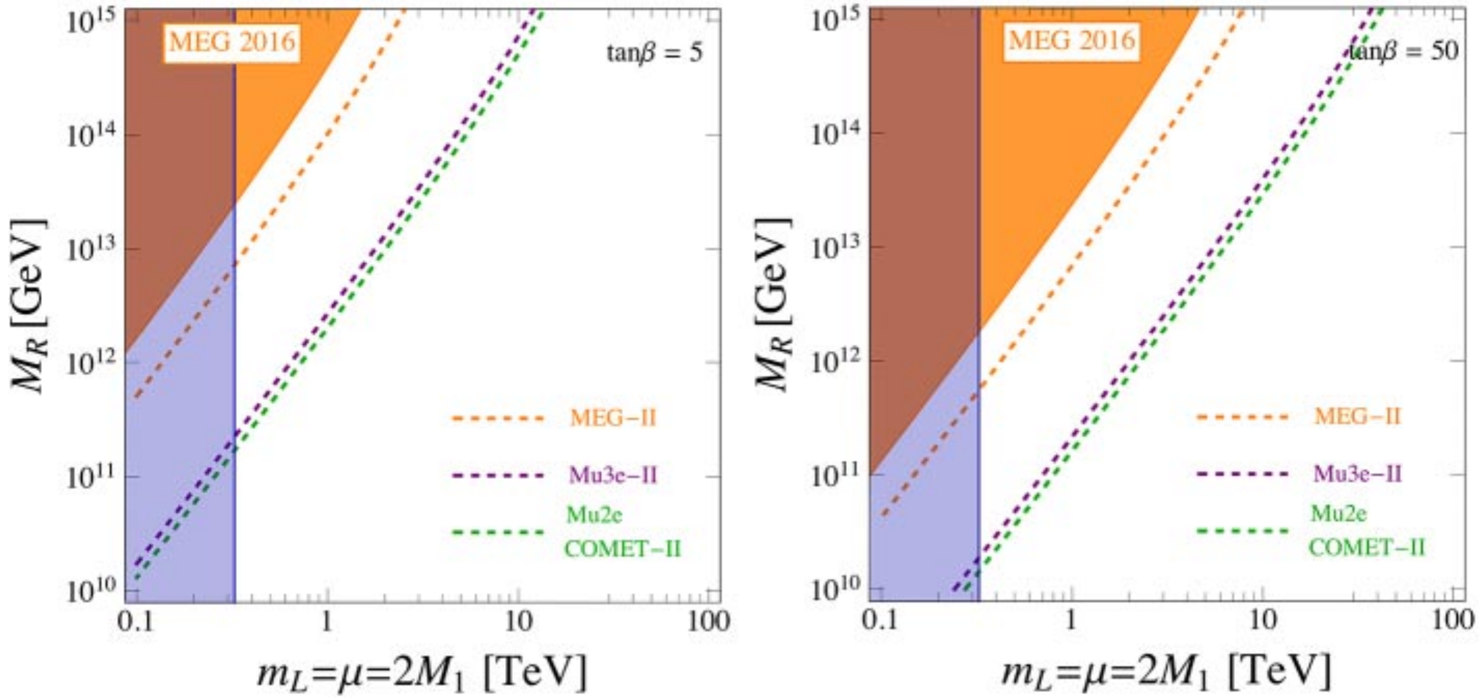


Fig. 12. – Bounds and prospects for a SUSY seesaw model with degenerate RH neutrinos, as in eq. (61), for $\tan\beta = 5$ (left) and 50 (right). The blue region is excluded by LHC searches for sleptons [137].

LC, Signorelli '17