Matter Effect in LBL Experiments

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Neutrino Oscillations in 3 Flavors

$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

θ_{23} : $P(\nu_\mu \to \nu_\mu)$ by Atoms. ν and ν beam

θ_{13}: $P(\nu_e \to \nu_e)$ by Reactor ν
θ_{13} & δ: $P(\nu_\mu \to \nu_e)$ by ν beam

θ_{12} : $P(\nu_e \to \nu_e)$ by Reactor and solar ν

Three mixing angles: θ_{23} , θ_{13} , θ_{12} and one CP violating (Dirac) phase δ_{CP}

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13}e^{-i\delta}$$

3 mixing angles simply related to flavor components of 3 mass eigenstates

Over a distance L, changes in the relative phases of the mass states may induce flavor change

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}[U^*_{\alpha i}U_{\alpha j}U_{\beta i}U^*_{\beta j}]\sin^2\Delta_{ij} - 2\sum_{i>j}\mathrm{Im}[U^*_{\alpha i}U_{\alpha j}U_{\beta i}U^*_{\beta j}]\sin 2\Delta_{ij}$$

$$\Delta_{ij} = \Delta m^2_{ij}L/4E_\nu$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

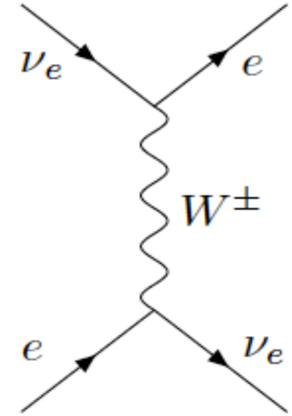
2 independent mass splittings Δm^2_{21} and Δm^2_{32}, for anti-neutrinos replace δ_{CP} by -δ_{CP}

Neutrino Oscillations in Matter

Neutrino propagation through matter modify the oscillations significantly

Coherent forward elastic scattering of neutrinos with matter particles

Charged current interaction of ν_e with electrons creates an extra potential for ν_e



MSW matter term: $\boxed{A = \pm 2\sqrt{2} G_F N_e E}$ or $\boxed{A(\mathrm{eV}^2) = 0.76 \times 10^{-4} \rho\ (\mathrm{g/cc}) E(\mathrm{GeV})}$

N_e = electron number density , + (-) for neutrinos (anti-neutrinos) , ρ = matter density in Earth

Matter term changes sign when we switch from neutrino mode to anti-neutrino mode

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq 0$$ → ***even if $\delta_{CP} = 0$, causes fake CP asymmetry***

Matter term modifies oscillation probability differently depending on the sign of Δm^2

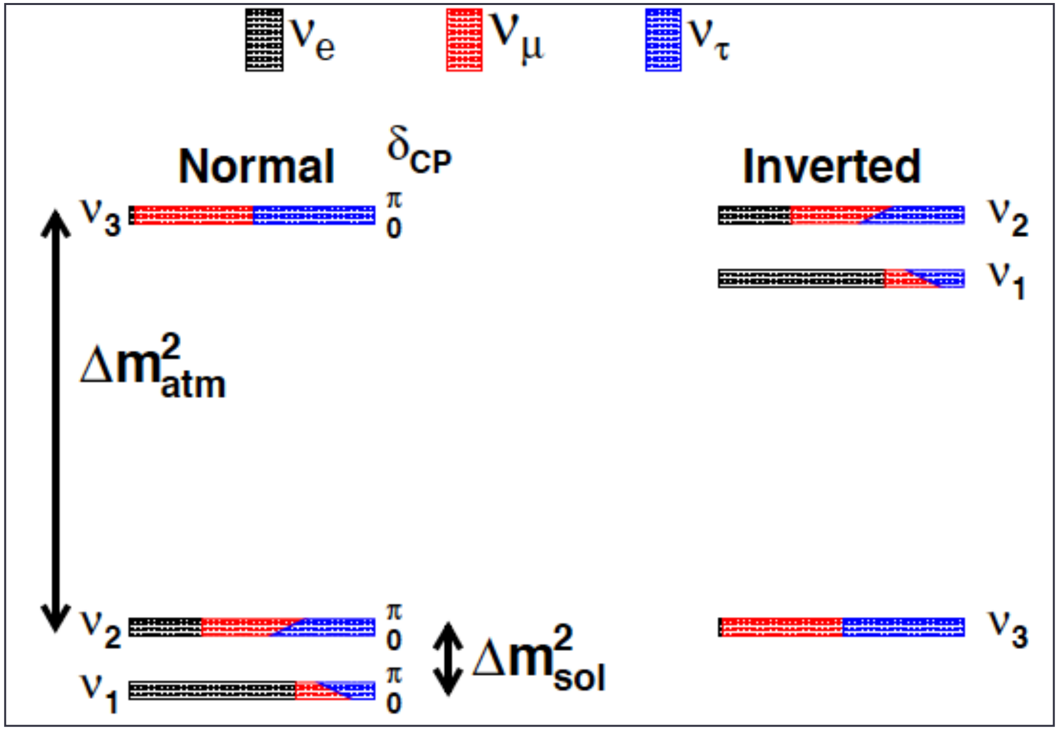
$$\Delta m^2 \simeq A \quad \Leftrightarrow \quad E_{\mathrm{res}}^{\mathrm{Earth}} = 6 - 8\,\mathrm{GeV}$$ → ***Resonant conversion – Matter effect***

	ν	$\bar{\nu}$
$\Delta m^2 > 0$	MSW	-
$\Delta m^2 < 0$	-	MSW

→ ***Resonance occurs for neutrinos (anti-neutrinos) if Δm^2 is positive (negative)***

Neutrino Mass Ordering: Important Open Question

The sign of Δm^2_{31} $(m_3^2 - m_1^2)$ is not known



Neutrino mass spectrum can be normal or inverted ordered

We only have a lower bound on the mass of the heaviest neutrino

$$\boxed{\sqrt{2.5 \cdot 10^{-3} \mathrm{eV}^2} \sim 0.05 \text{ eV}}$$

We currently do not know which neutrino is the heaviest

$$\boxed{|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2}$$

ν_e component of ν_1 > ν_e component of ν_2 > ν_e component of ν_3

Mass Ordering Discrimination : A Binary yes-or-no type question

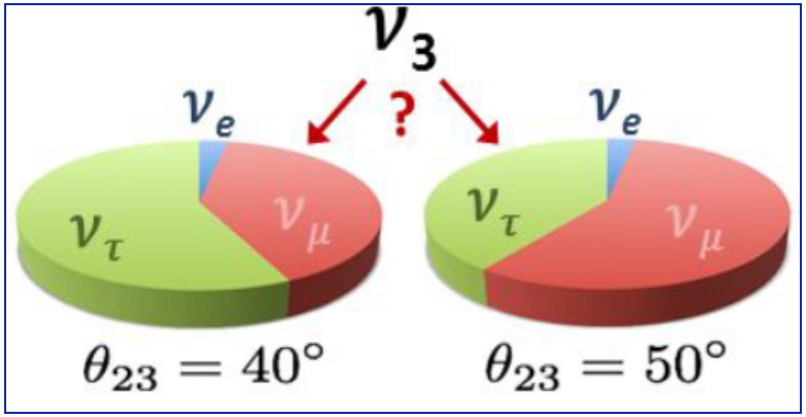
Octant of 2-3 Mixing Angle: Important Open Question

→ *In ν_μ survival probability, the dominant term is mainly sensitive to $\sin^2 2\theta_{23}$*

→ *If $\sin^2 2\theta_{23}$ differs from 1 (recent hints), we get two solutions for θ_{23}*

→ *One in lower octant (LO: θ_{23} < 45 degree)*

→ *Other in higher octant (HO: θ_{23} > 45 degree)*



Octant ambiguity of θ_{23}

Fogli and Lisi, hep-ph/9604415

ν_μ to ν_e oscillation channel can break this degeneracy
preferred value would depend on the choice of neutrino mass ordering

Leptonic CP-violation: Important Open Question

Is CP violated in the neutrino sector, as in the quark sector?

Mixing can cause CPV in ν sector, provided $\delta_{CP} \neq 0°$ and $180°$

Need to measure the CP-odd asymmetries:

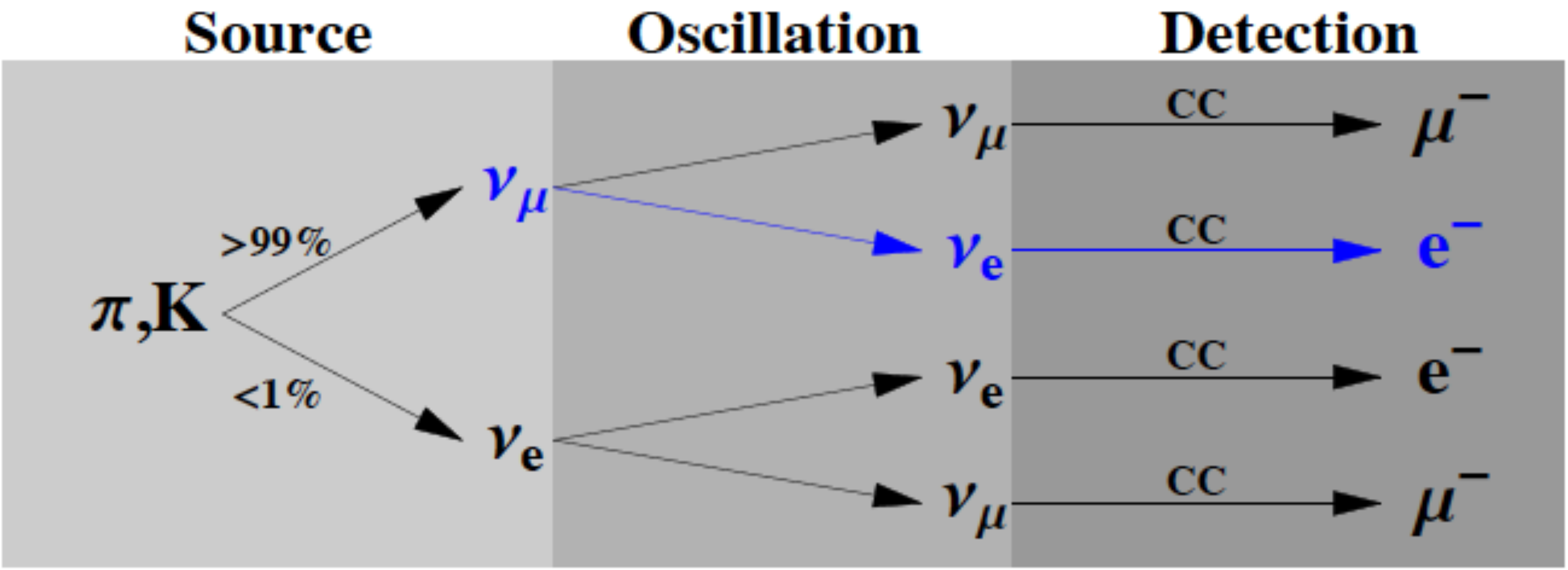
$$\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta;\, L) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta;\, L) \quad (\alpha \neq \beta)$$

$$\Delta P_{e\mu} = \Delta P_{\mu\tau} = \Delta P_{\tau e} = 4J_{CP} \times \left[\sin\left(\frac{\Delta m^2_{21}}{2E}L\right) + \sin\left(\frac{\Delta m^2_{32}}{2E}L\right) + \sin\left(\frac{\Delta m^2_{13}}{2E}L\right)\right]$$

Jarlskog CP-odd Invariant → $J_{CP} = \frac{1}{8}\cos\theta_{13}\,\sin 2\theta_{13}\,\sin 2\theta_{23}\,\sin 2\theta_{12}\,\sin\delta_{CP}$

Three-flavor effects are key for CPV, need to observe interference

Conditions for observing CPV:
1) *Non-degenerate masses* ✔
2) *Mixing angles $\neq 0°$ and $90°$* ✔
3) *$\delta_{CP} \neq 0°$ and $180°$ (Hints)*



Traditional approach: Neutrino beam from pion decay

Accelerator Long-baseline Neutrino Experiments

Appearance: ($\nu_\mu \rightarrow \nu_e$) and (anti-$\nu_\mu \rightarrow$ anti-ν_e)

(essential for MO, CPV, Octant)

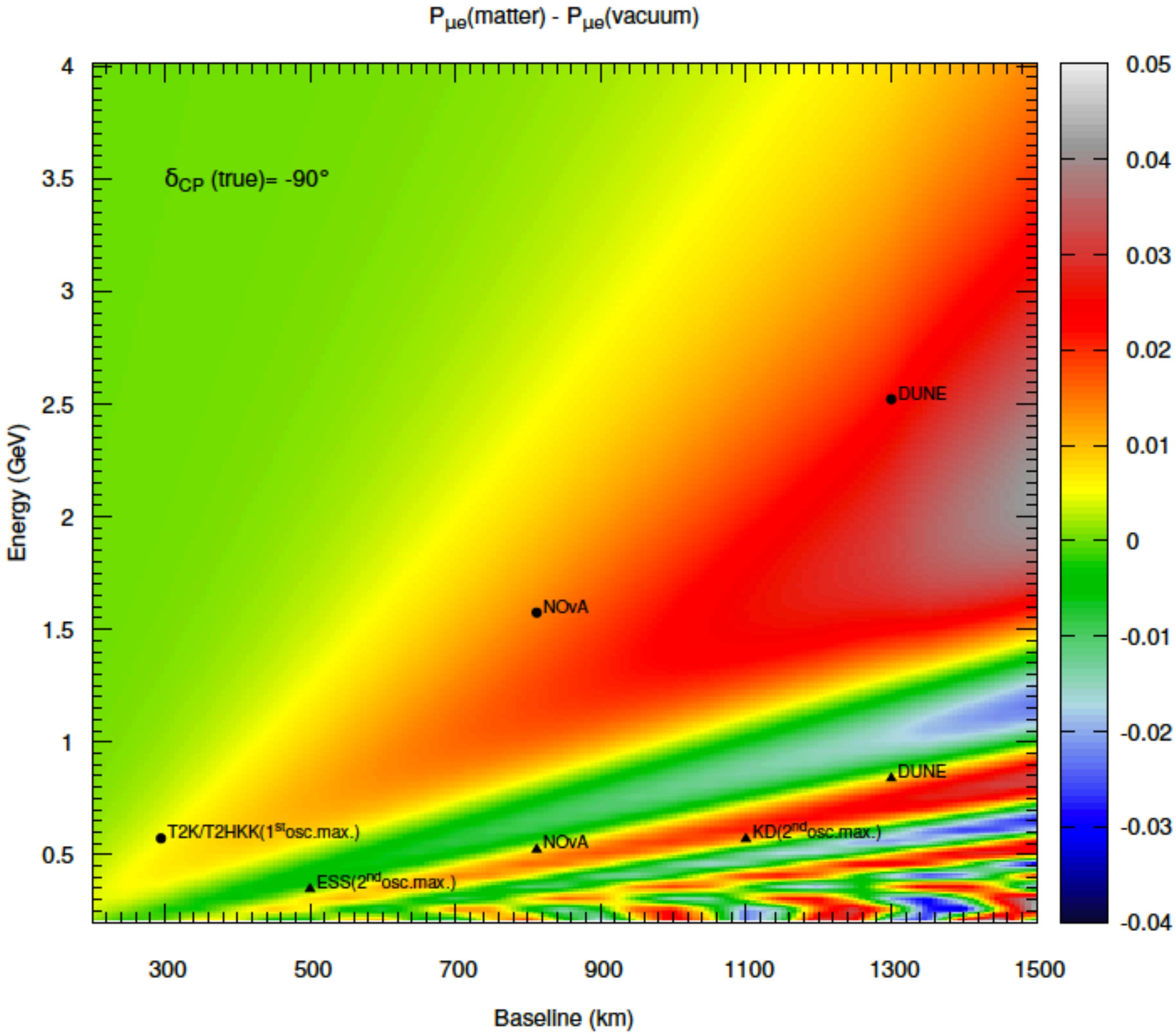
Disappearance: ($\nu_\mu \rightarrow \nu_\mu$) and (anti-$\nu_\mu \rightarrow$ anti-ν_μ)

(key for precise measurement of Δm^2_{32} and θ_{23})

Present: T2K & NOνA

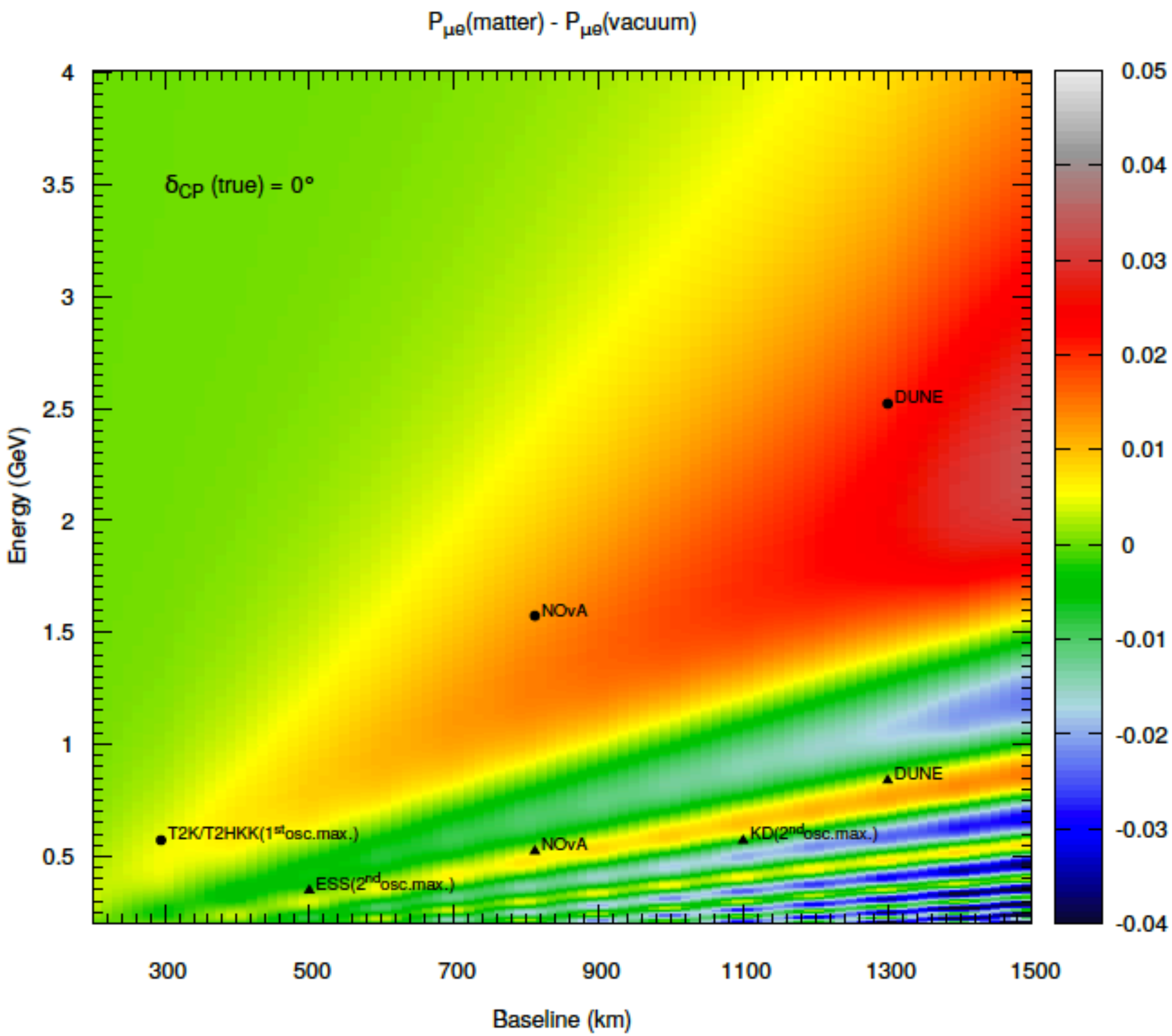
Future: DUNE, T2HK, T2HKK, ESSνSB

Matter Effect in LBL Experiments



SKA, Soumya C., Masoom Singh, in preparation

Matter Effect in LBL Experiments



SKA, Soumya C., Masoom Singh, in preparation

Three Flavor Effects in $\nu_\mu \rightarrow \nu_e$ oscillation probability

The appearance probability $(\nu_\mu \to \nu_e)$ in matter, upto second order in the small parameters $\alpha \equiv \Delta m^2_{21}/\Delta m^2_{31}$ and $\sin 2\theta_{13}$,

0.03 **0.3**

$$P_{\mu e} \simeq \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2}$$

0.09

$\Rightarrow$ θ_{13} Driven

Resolves octant

$$- \alpha \sin 2\theta_{13}\, \xi \sin\delta_{CP} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})}$$

0.009

$\Rightarrow$ CP odd

$$+ \alpha \sin 2\theta_{13}\, \xi \cos\delta_{CP} \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})}$$

$\Rightarrow$ CP even

$$+ \alpha^2 \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2};$$

0.0009

$\Rightarrow$ Solar Term

where $\Delta \equiv \Delta m^2_{31} L/(4E)$, $\xi \equiv \cos\theta_{13} \sin 2\theta_{21} \sin 2\theta_{23}$,
and $\hat{A} \equiv \pm(2\sqrt{2}G_F n_e E)/\Delta m^2_{31}$

changes sign with sgn(Δm^2_{31}) key to resolve hierarchy!

changes sign with polarity causes fake CP asymmetry!

Cervera etal., hep-ph/0002108
Freund, hep-ph/0103300

This channel suffers from: (Hierarchy – δ_{CP}) & (Octant – δ_{CP}) degeneracy! How can we break them?

JHEP

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Analytical approximation of the neutrino oscillation matter effects at large θ_{13}

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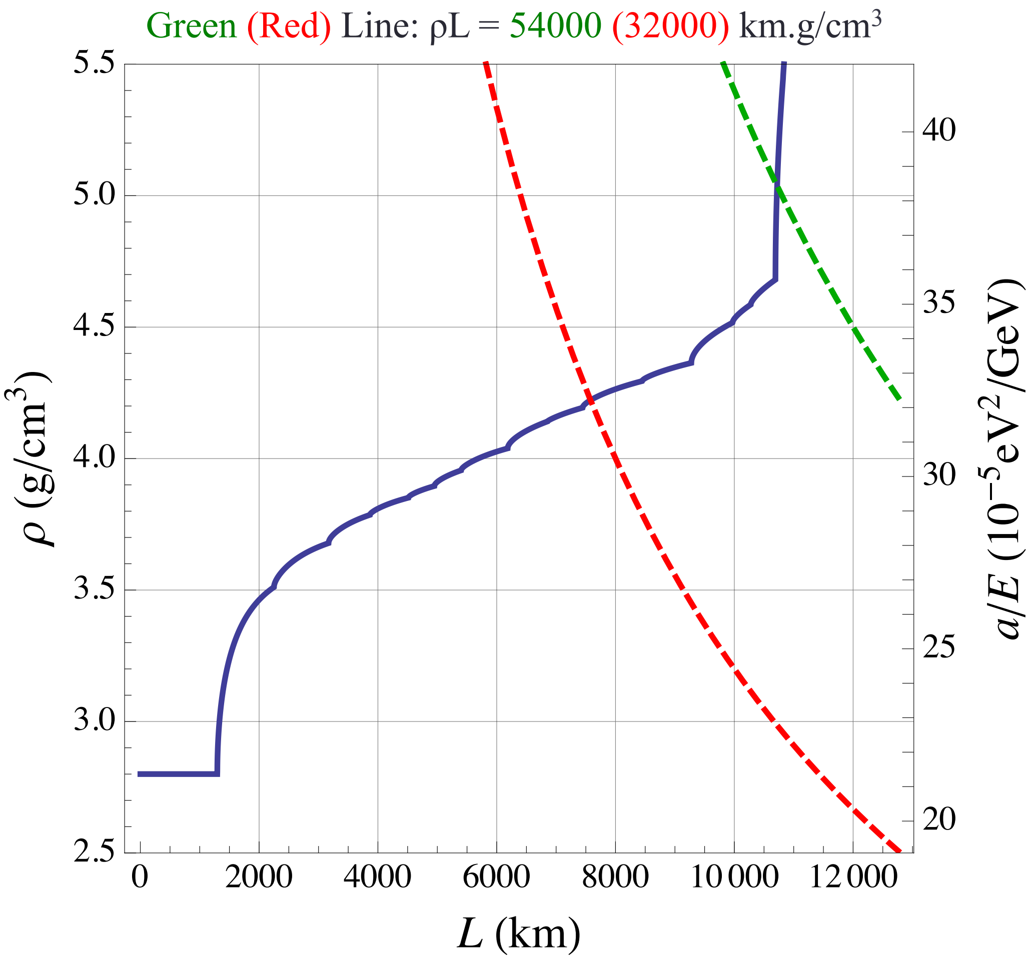
Abstract: We argue that the neutrino oscillation probabilities in matter are best understood by allowing the mixing angles and mass-squared differences in the standard parametrization to 'run' with the matter effect parameter $a = 2\sqrt{2}G_F N_e E$, where N_e is the electron density in matter and E is the neutrino energy. We present simple analytical approximations to these 'running' parameters. We show that for the moderately large

JHEP04(2014)047

Matter Effect Parameter a

$$a = 2\sqrt{2} G_F N_e E \quad = 7.63\times 10^{-5}(\text{eV}^2)\left(\frac{\rho}{\text{g/cm}^3}\right)\left(\frac{E}{\text{GeV}}\right)$$



Agarwalla, Kao, Takeuchi, JHEP 1404, 047 (2014)

- Matter effects play an important role
- Mixing angles and and mass-squared differences run with the matter effect parameter 'a'
- We present simple analytical approximations to these running parameters using the Jacobi method
- We show that for large θ_{13}, the running of θ_{23} and δ_{CP} can be neglected, simplifying the probability expression
- We need to rotate only θ_{12} and θ_{13}

First noticed by Krastev and Petcov
Phys.Lett. B205 (1988) 84-92

Our Approach

Use the expressions for the vacuum oscillation probabilities as it is, but make the following replacements:

$$\theta_{12} \to \theta'_{12}, \quad \theta_{13} \to \theta'_{13}, \quad \delta m^2_{jk} \to \lambda_j - \lambda_k$$

where

$$\tan 2\theta'_{12} = \frac{(\delta m^2_{21}/c^2_{13})\sin 2\theta_{12}}{(\delta m^2_{21}/c^2_{13})\cos 2\theta_{12} - a}, \qquad \tan 2\theta'_{13} = \frac{(\delta m^2_{31} - \delta m^2_{21} s^2_{12})\sin 2\theta_{13}}{(\delta m^2_{31} - \delta m^2_{21} s^2_{12})\cos 2\theta_{13} - a},$$

$$\lambda_1 = \lambda'_-$$
$$\lambda_2 = \lambda''_\mp$$
$$\lambda_3 = \lambda''_\pm$$

$$\lambda'_\pm = \frac{(\delta m^2_{21} + a c^2_{13}) \pm \sqrt{(\delta m^2_{21} - a c^2_{13})^2 + 4 a c^2_{13} s^2_{12} \delta m^2_{21}}}{2}$$

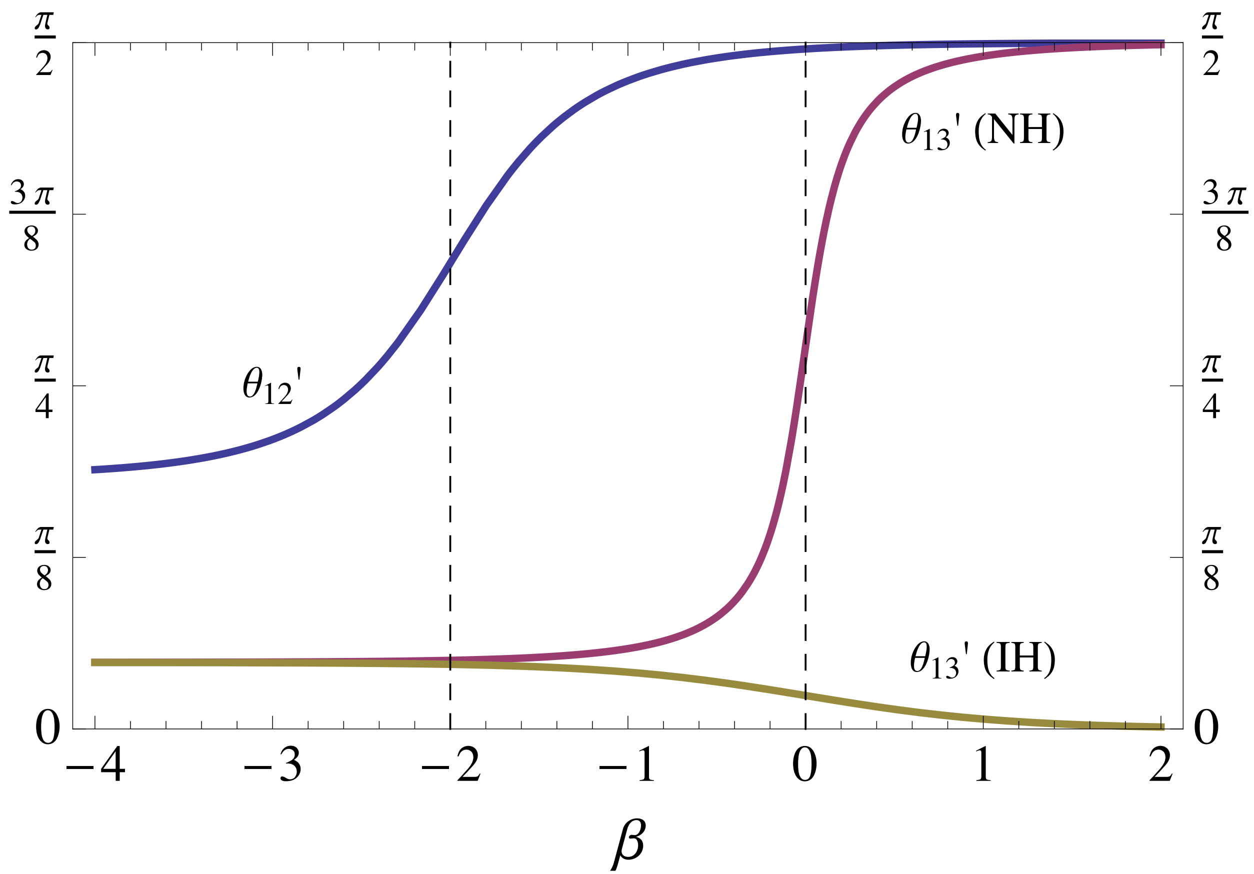
$$\lambda''_\pm = \frac{\left[\lambda'_+ + (\delta m^2_{31} + a s^2_{13})\right] \pm \sqrt{\left[\lambda'_+ - (\delta m^2_{31} + a s^2_{13})\right]^2 + 4 a^2 s'^2_{12} c^2_{13} s^2_{13}}}{2}$$

upper (lower) sign for NH (IH)

Approximation works when $\theta_{13} = O(\varepsilon)$, where $\varepsilon = \sqrt{\delta m^2_{21}/|\delta m^2_{31}|} = 0.17$

Agarwalla, Kao, Takeuchi, JHEP 1404, 047 (2014)

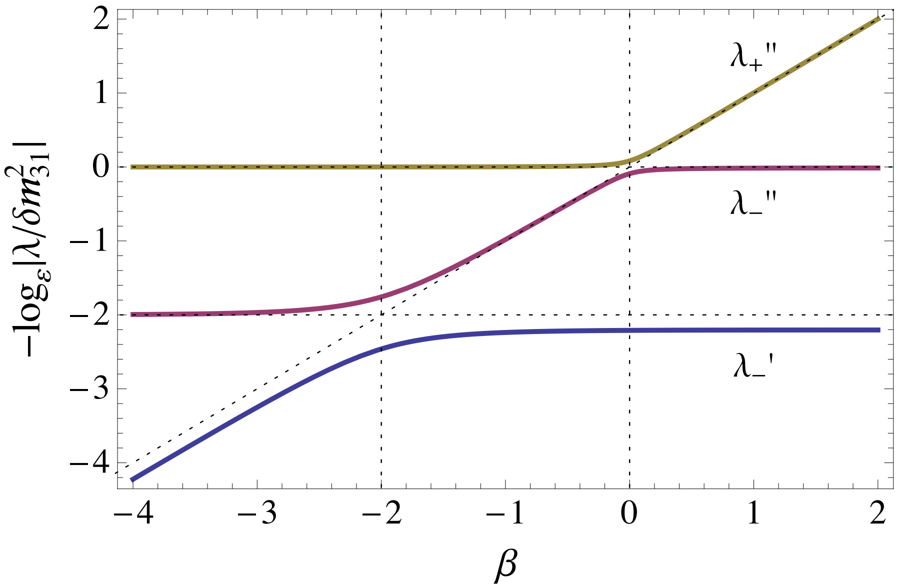
a-dependence of effective mixing angles



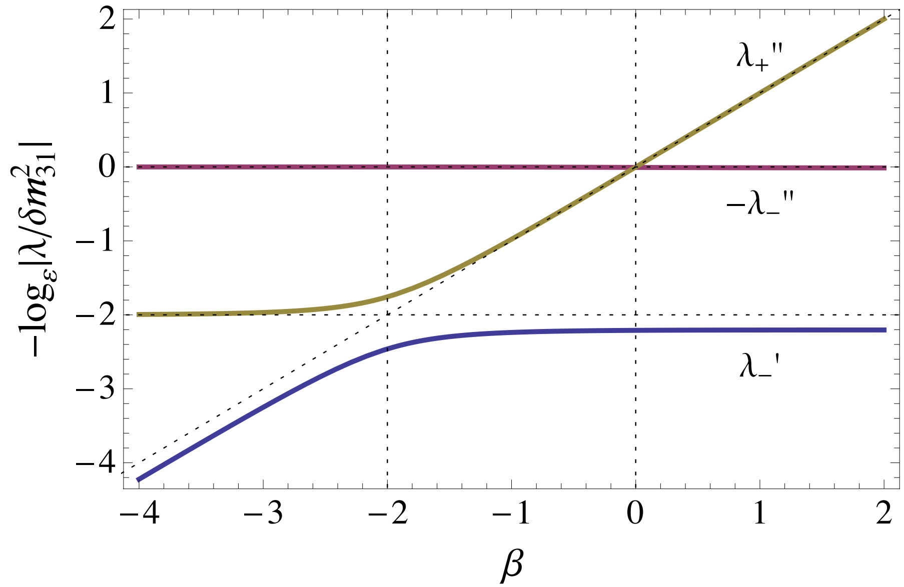
$$\frac{a}{|\delta m^2_{31}|} = \varepsilon^{-\beta}, \qquad \varepsilon = \sqrt{\frac{\delta m^2_{21}}{|\delta m^2_{31}|}} \approx 0.17$$

Agarwalla, Kao, Takeuchi, JHEP 1404, 047 (2014)

a-dependence of effective mass-squared differences



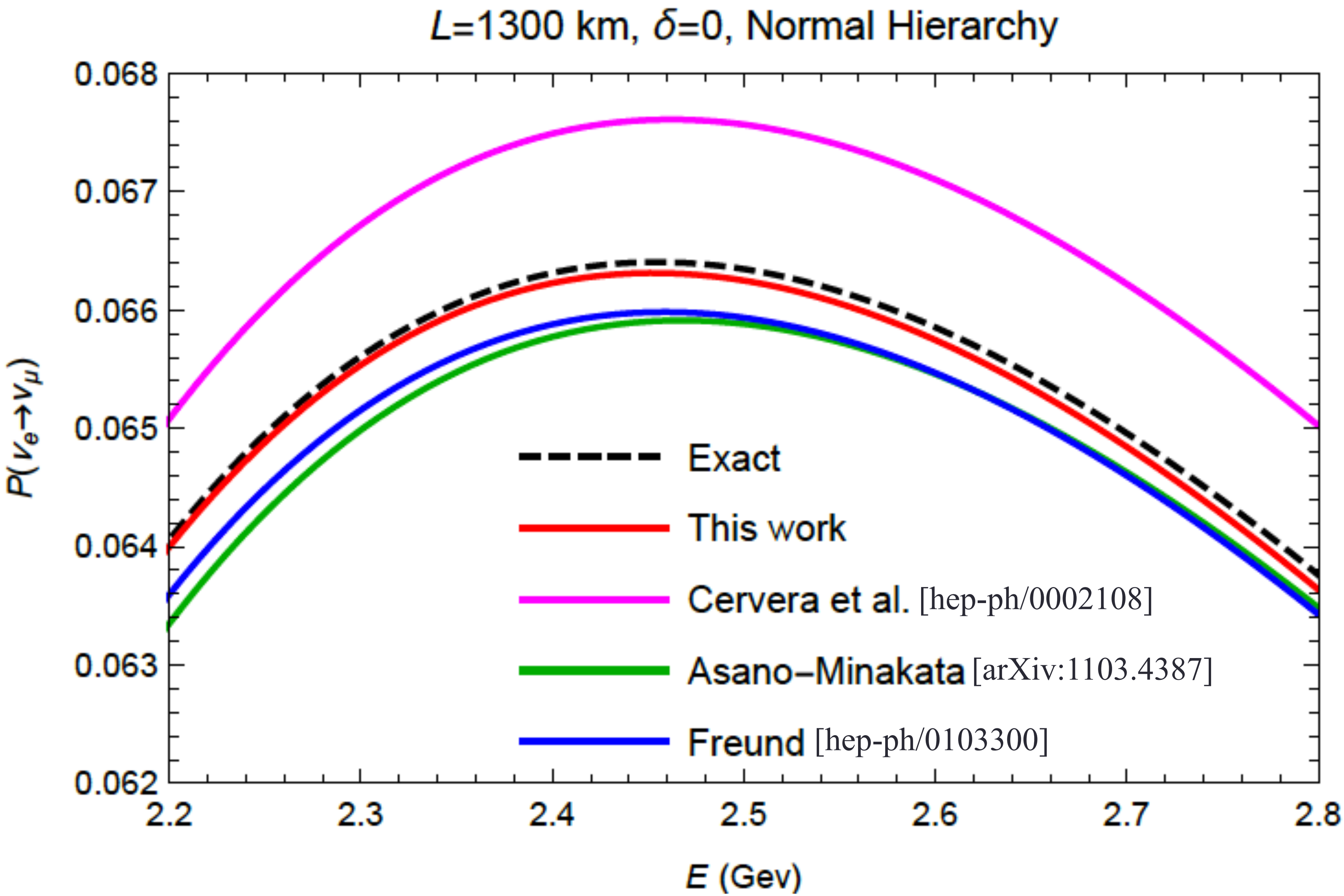
Normal Hierarchy



Inverted Hierarchy

Agarwalla, Kao, Takeuchi, JHEP 1404, 047 (2014)

Accuracy of Our Method and Comparison with Existing Literature



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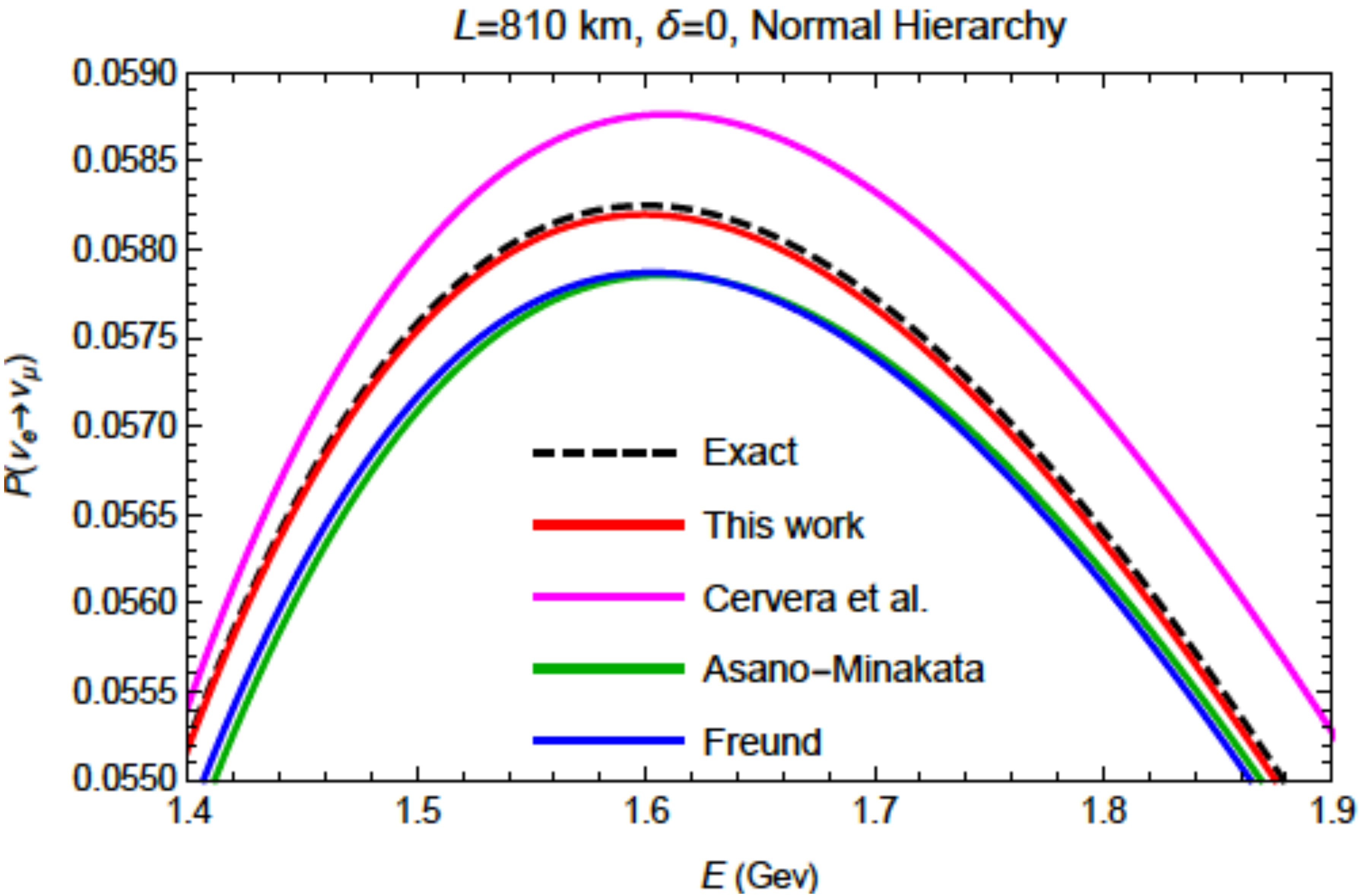

Agarwalla, Kao, Takeuchi, JHEP 1404, 047 (2014)

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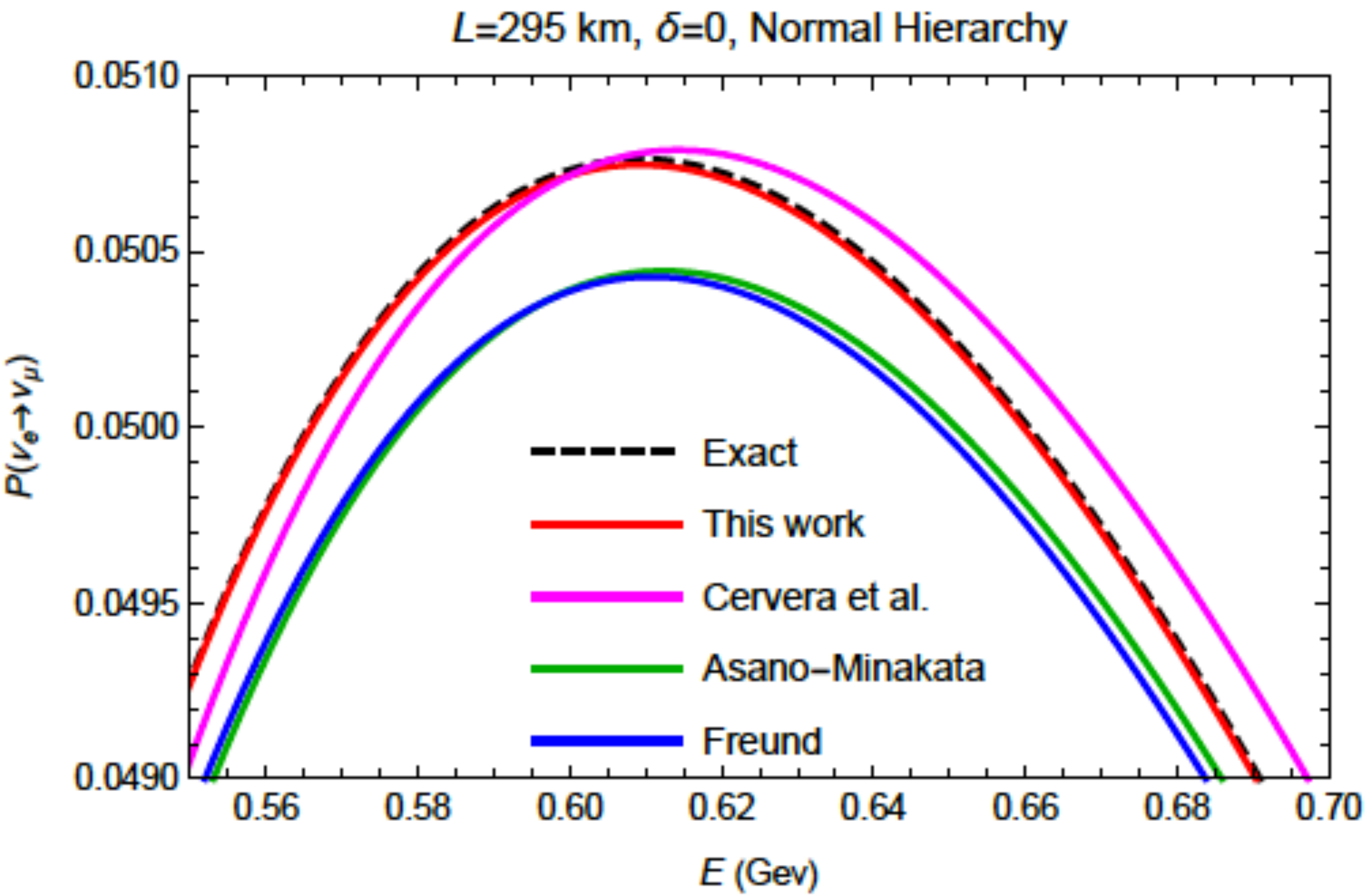

Agarwalla, Kao, Takeuchi, JHEP 1404, 047 (2014)

Other analytical expressions suffer in accuracy due to their reliance on expansion in θ_{13}, or in simplicity when higher order terms in θ_{13} included

Our method gives accurate probability for all channels, baselines, and energies arXiv:1604.08167v1 [hep-ph] 27 Apr 2016

Compact Perturbative Expressions For Neutrino Oscillations in Matter

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Abstract: We further develop and extend a recent perturbative framework for neutrino oscillations in uniform matter density so that the resulting oscillation probabilities are accurate for the complete matter potential versus baseline divided by neutrino energy plane. This extension also gives the *exact* oscillation probabilities in vacuum for all values of baseline divided by neutrino energy. The expansion parameter used is related to the ratio of the solar to the atmospheric Δm^2 scales but with a unique choice of the atmospheric Δm^2 such that certain first-order effects are taken into account in the zeroth-order Hamiltonian. Using a mixing matrix formulation, this framework has the exceptional feature that the neutrino oscillation probability in matter has the same structure as in vacuum, to all orders in the expansion parameter. It also contains all orders in the matter potential and $\sin\theta_{13}$. It facilitates immediate physical interpretation of the analytic results, and makes the expressions for the neutrino oscillation probabilities extremely compact and very accurate even at zeroth order in our perturbative expansion. The first and second order results are also given which improve the precision by approximately two or more orders of magnitude per perturbative order.

Similar treatment by Ioannisian & Pokorski, arXiv:1801.10488

Analytical Understanding of Neutrino Oscillation Probability

Neutrino oscillation probabilities through the looking glass

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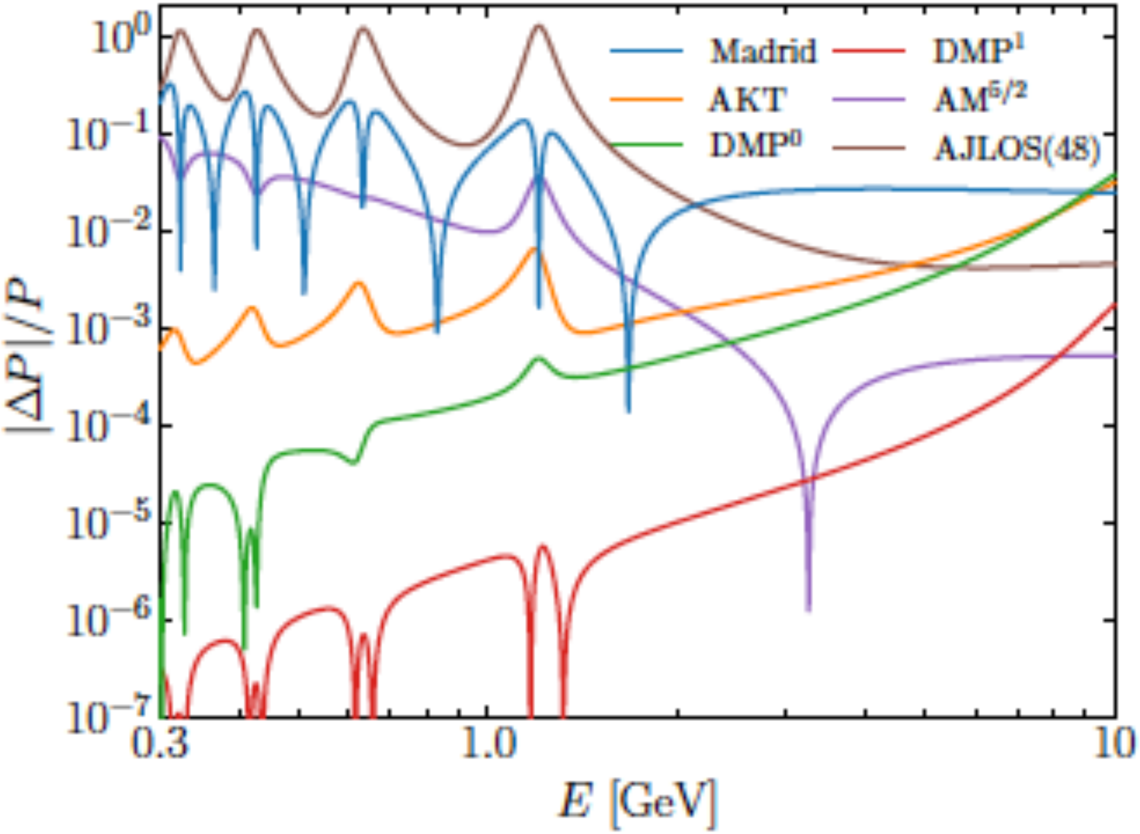
Abstract

In this paper we review different expansions for neutrino oscillation probabilities in matter in the context of long-baseline neutrino experiments. We examine the accuracy and computational efficiency of different exact and approximate expressions. We find that many of the expressions used in the literature are not precise enough for the next generation of long-baseline experiments, but several of them are while maintaining comparable simplicity. The results of this paper can be used as guidance to both phenomenologists and experimentalists when implementing the various oscillation expressions into their analysis tools.

Keywords: Neutrino physics, Neutrino oscillations in matter



	ϵ (ε)	s_{13}	$a/\Delta m^2_{31}$
Madrid(like)	✗	✗	✗
AKT	✓	✓	✓
MP	✓	✗	✗
DMP	✓	✓	✓
AKS	✗	✗	✗
MF	✓	✗	✗
AJLOS(48)	✓	✗	✗
AM	✗	✗	✗

ϵ	$\Delta m^2_{21}/\Delta m^2_{ee}$
ε	$\Delta m^2_{21}/\Delta m^2_{31}$

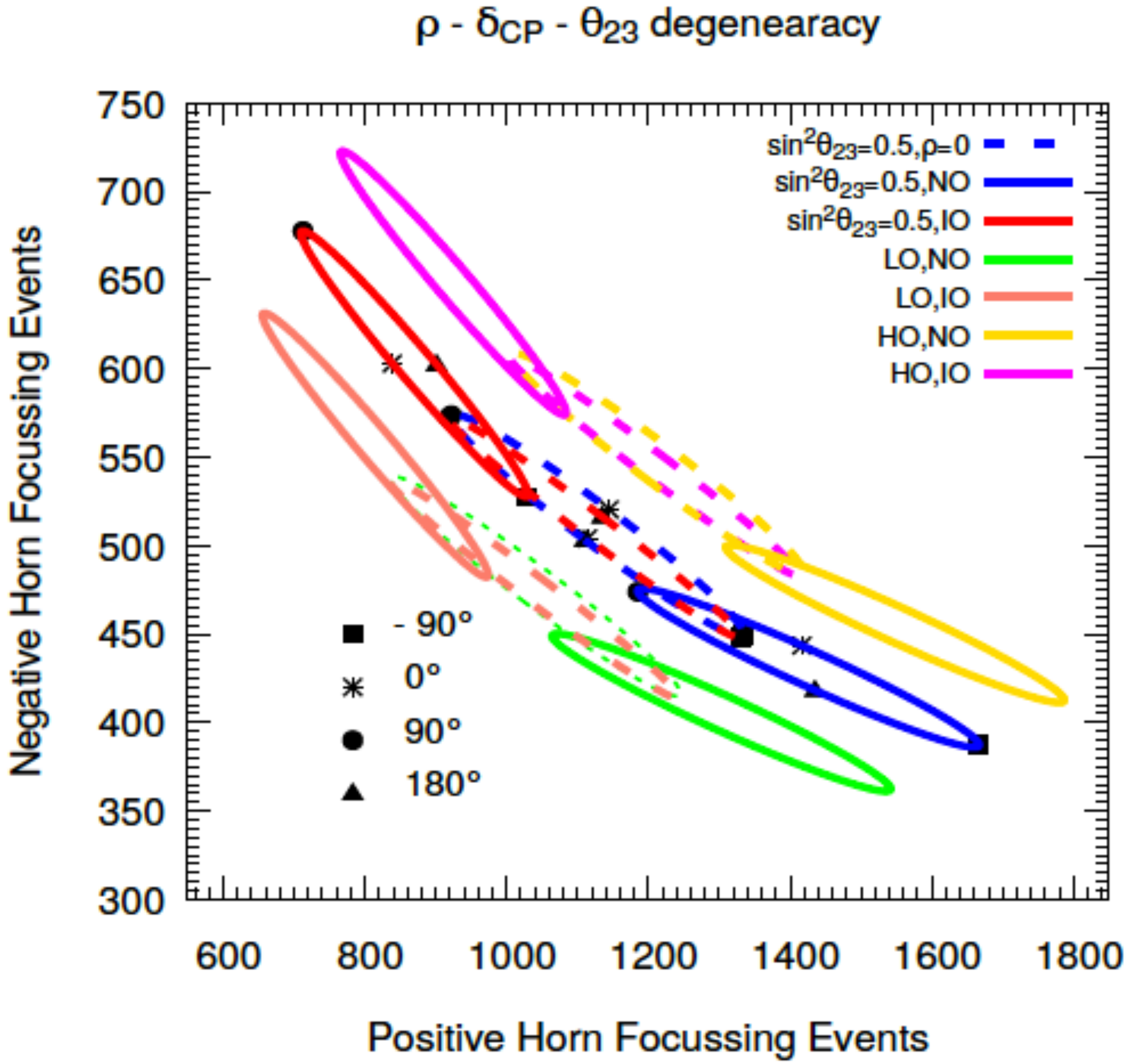
$$\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

In order to qualify as an expansion parameter, the authors require that the probability recovers the exact (to all orders) expression as that parameter goes to zero. That is, x is an expansion parameter if and only if

$$\boxed{\lim_{x\to 0} P_{\rm approx}(x) = P_{\rm exact}(x=0)}$$

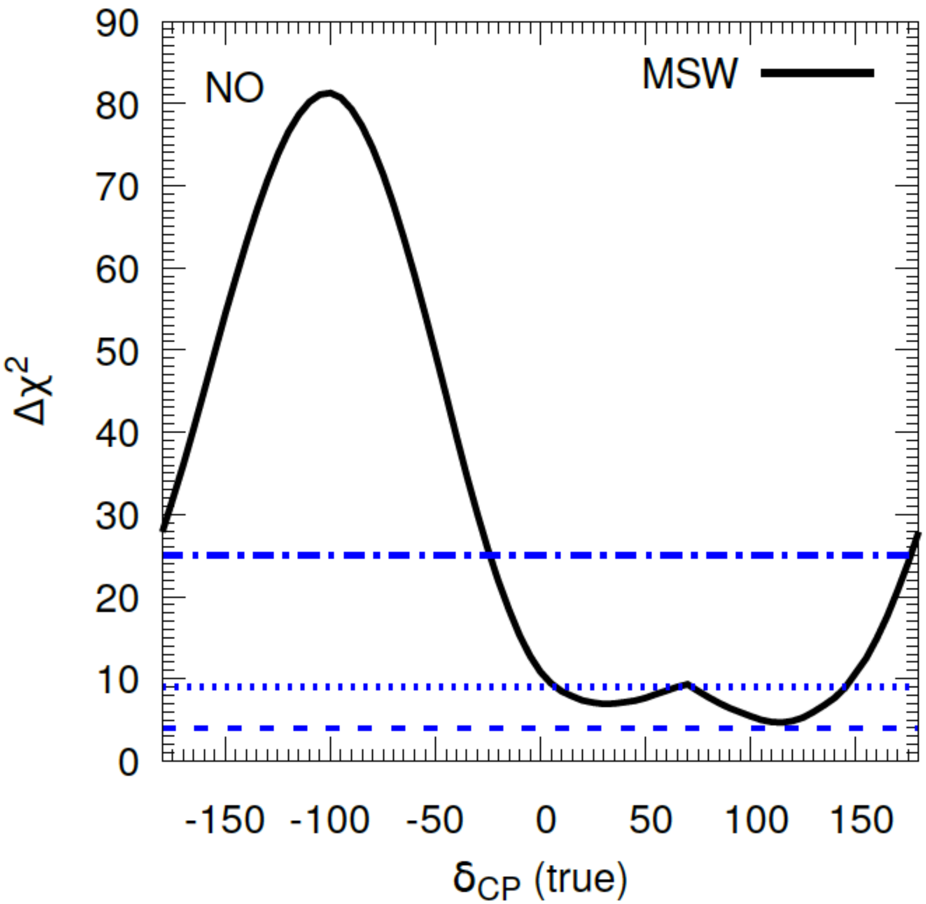
Berenboim, Denton, Parke, Ternes, arXiv:1902.00517

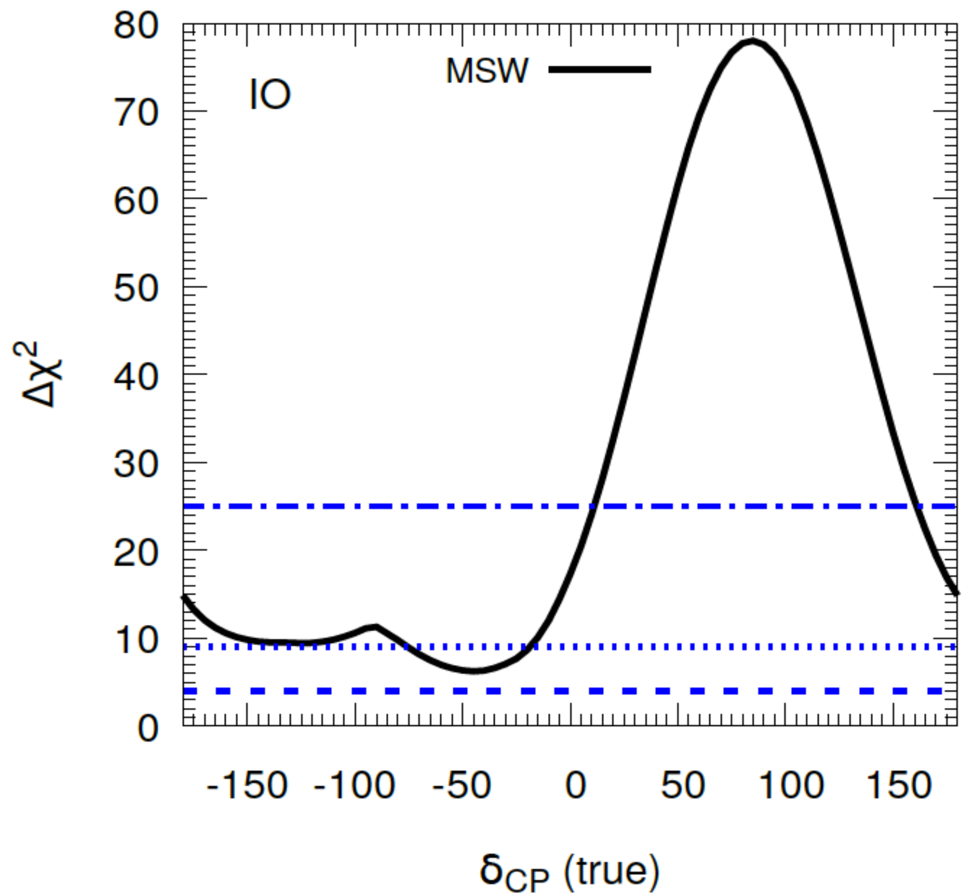
Bi-event Plot for DUNE



SKA, Soumya C., Masoom Singh, in preparation

DUNE Sensitivity to Reject Vacuum Solution





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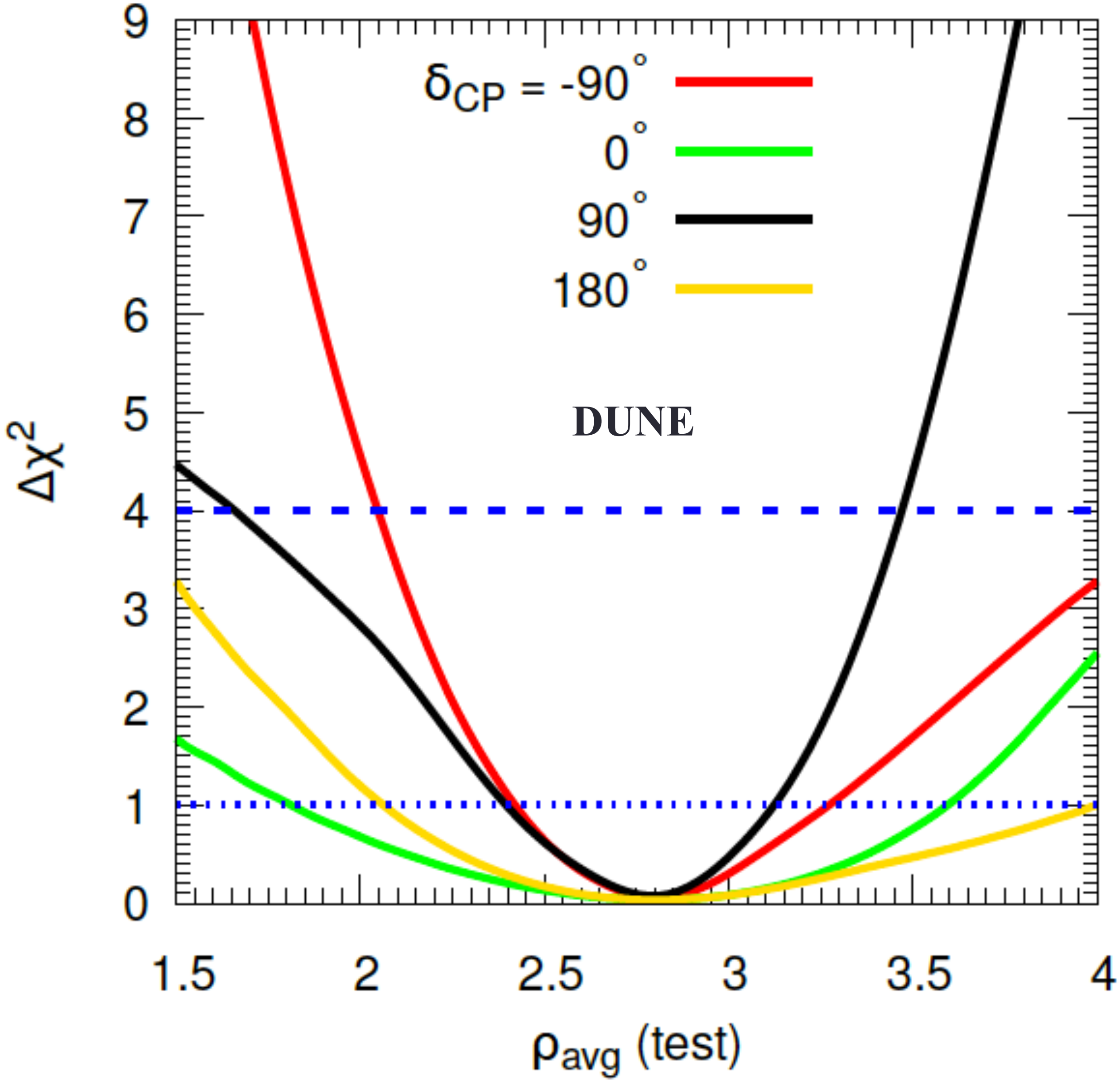
Precision in measuring Line-Averaged Constant Density



For δ_{CP} = - 90 degree
Relative 1σ precision
in ρ_{avg} is ~ 15%

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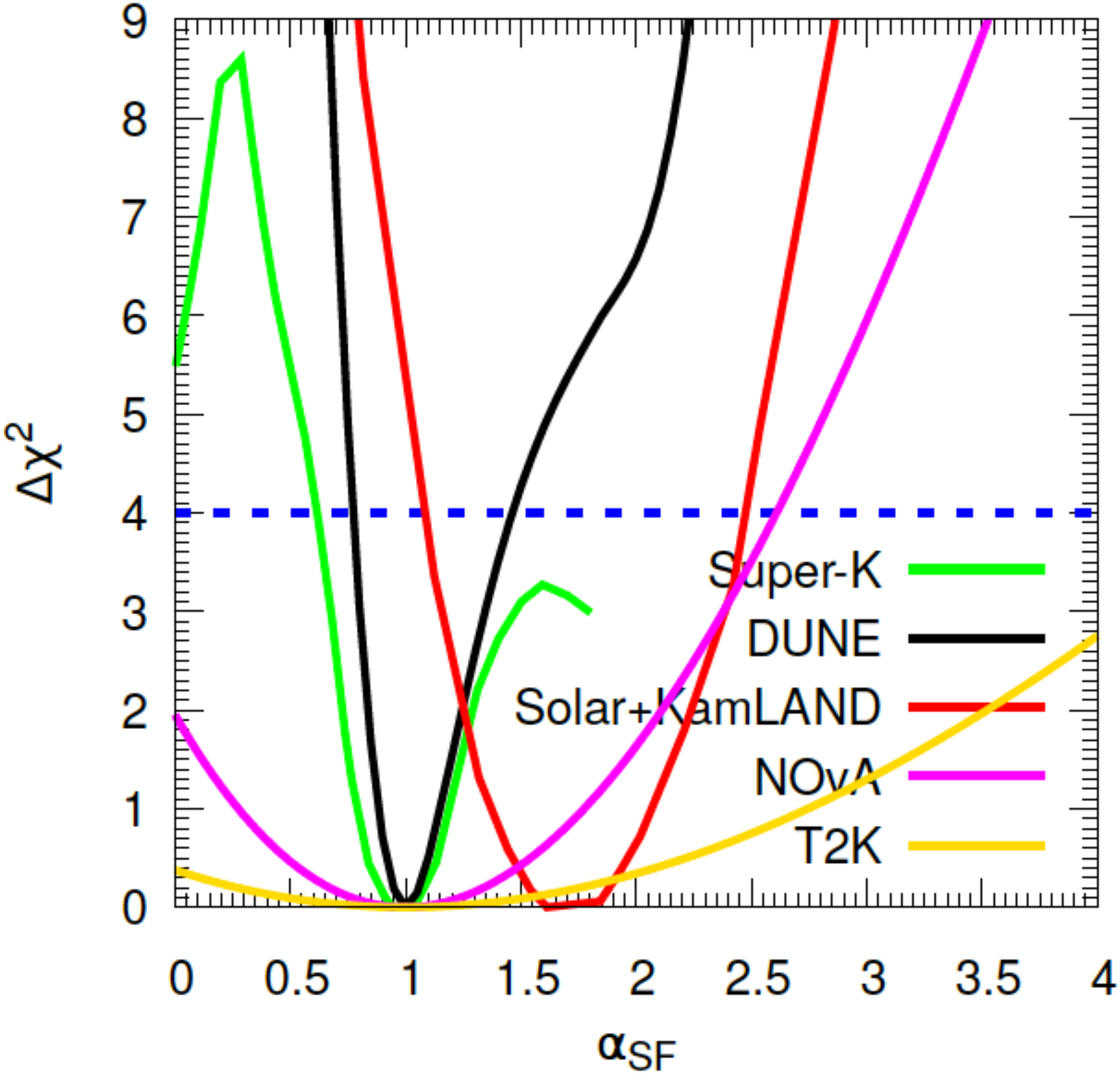
Testing Standard Matter Profile



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$\rho_{avg} \rightarrow \alpha_{SF} \times \rho_{avg}$

Vacuum: $\alpha_{SF} = 0$

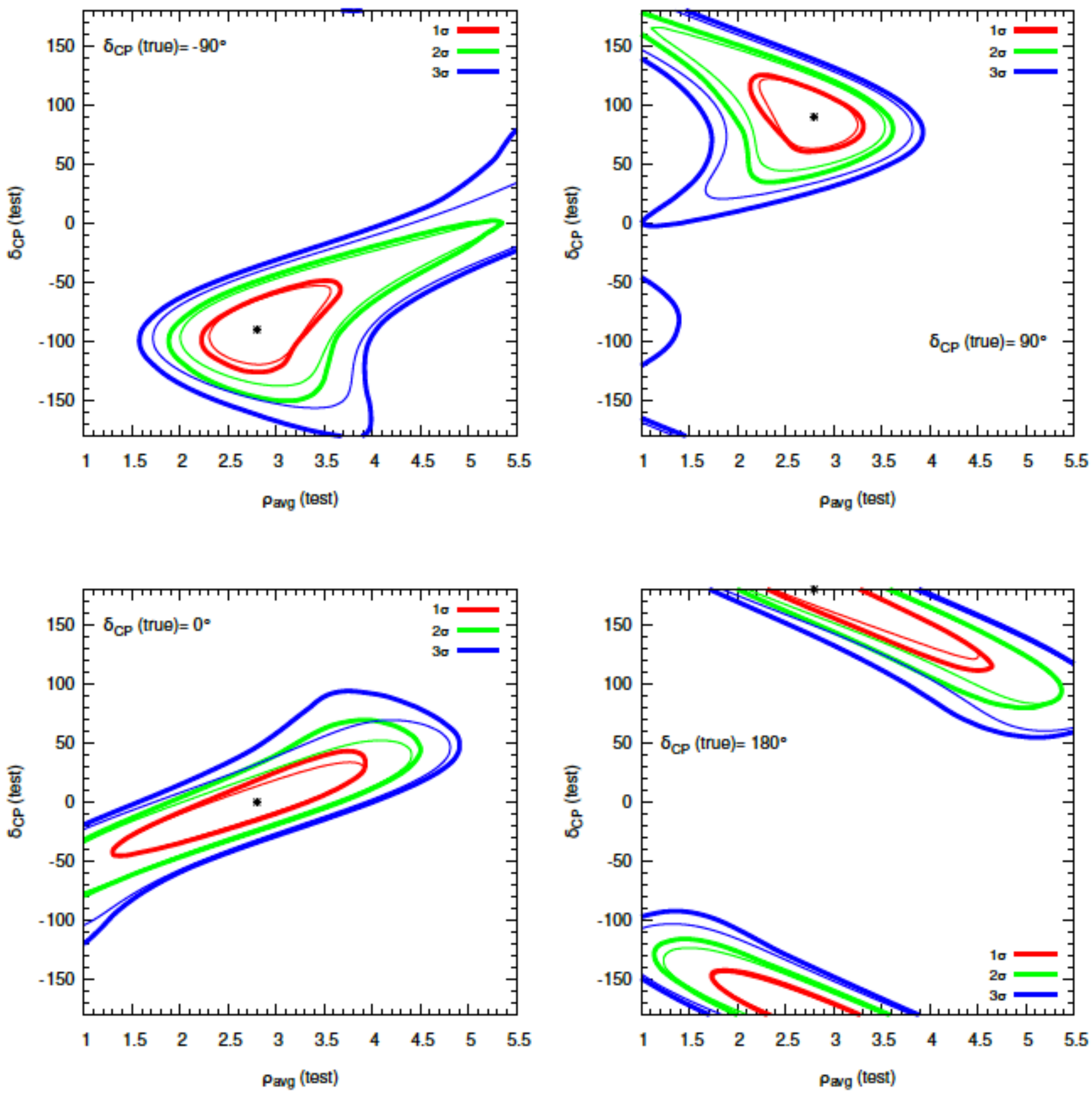
Standard Profile: $\alpha_{SF} = 1$

Super-K
arXiv:1710.09126

Solar+KamLAND
arXiv:1507.05287

Standard $\alpha_{SF} = 1$ disfavored by Solar + KamLAND data due to tension between them in measuring Δm^2_{21}

Degeneracies in $\rho_{avg} - \delta_{CP}$ Plane for DUNE



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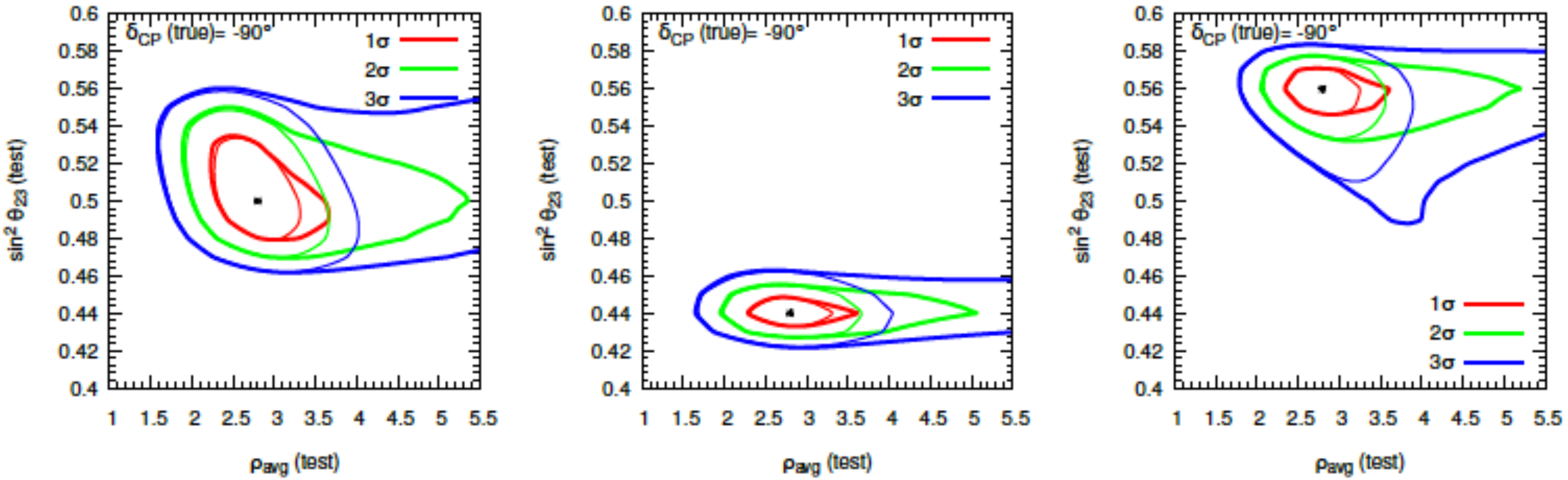
Degeneracies in $\rho_{avg} - \sin^2\theta_{23}$ Plane for DUNE



SKA, Soumya C., Masoom Singh, in preparation

Concluding Remarks

- ***Earth's Matter Effect plays an important role in present and future long-baseline neutrino oscillation experiments***

- ***Precise understanding of mixing angles and mass-squared differences in matter is important to explain the results***

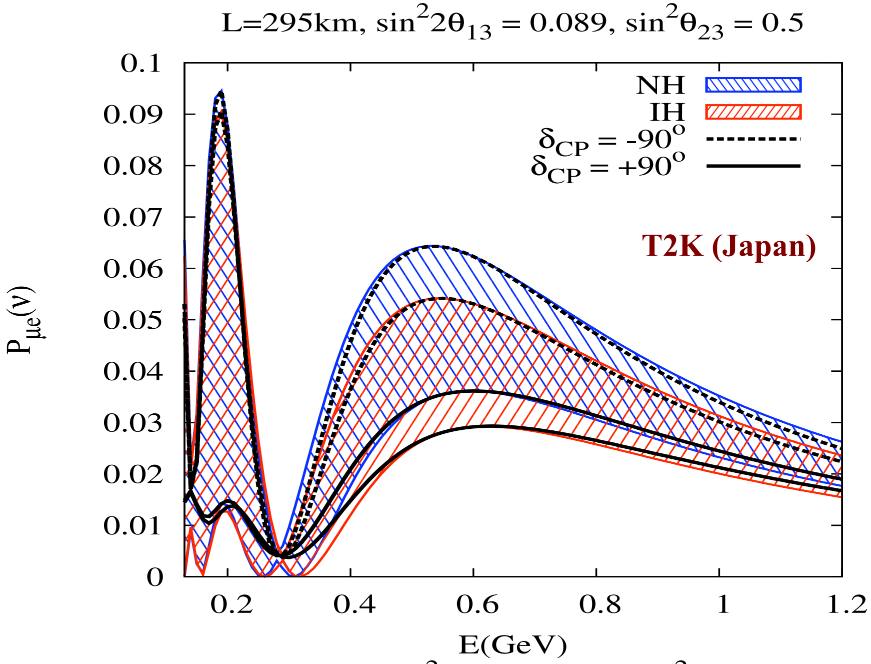
- ***Approximate analytical expressions of oscillation probability can help to understand various parameter degeneracies***

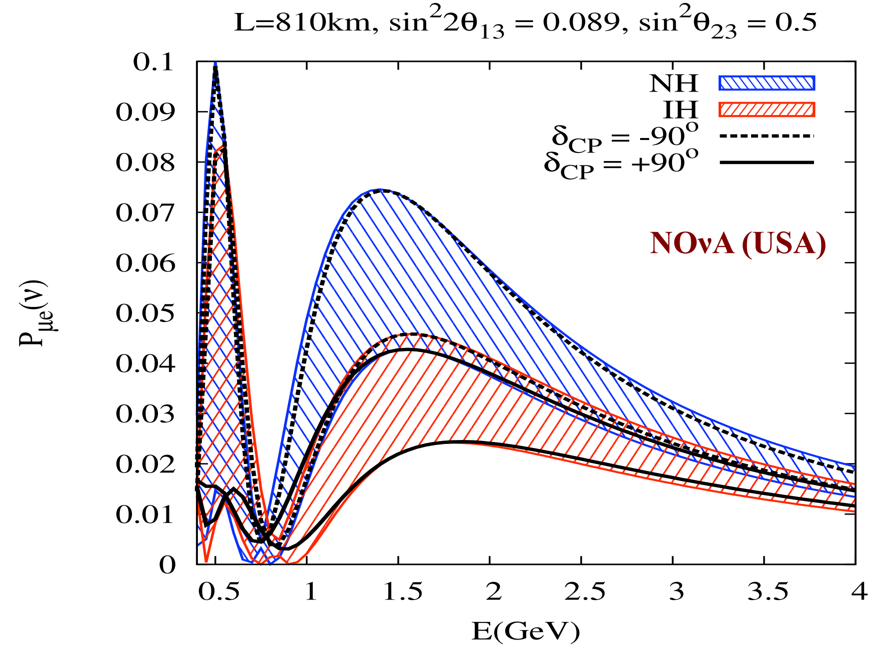
- ***Future goal is to have a robust test of three-flavor paradigm in presence of Earth Matter***

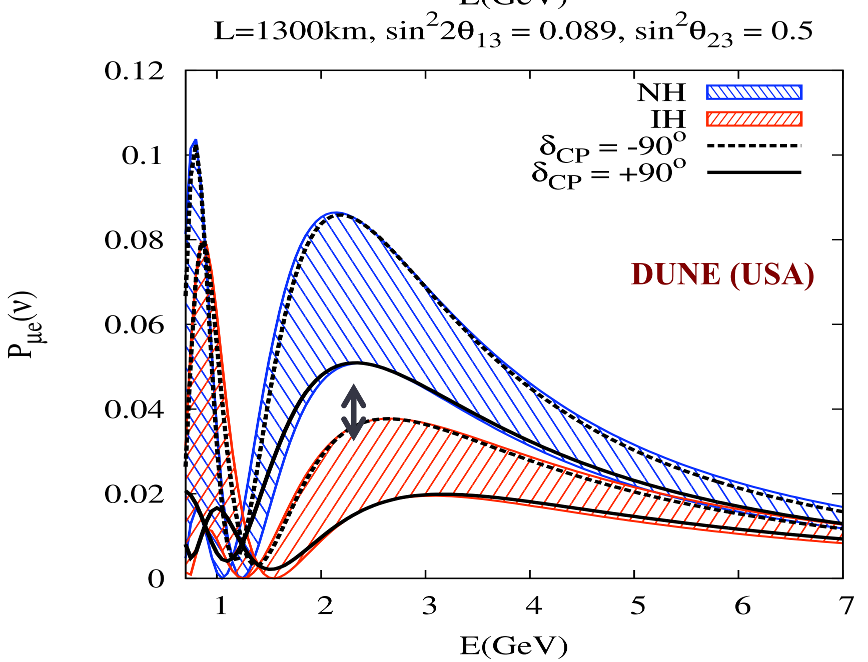
- ***Future large-scale oscillation facilities should measure Earth Matter Density and explore possible degeneracies among line-averaged constant density, CP Phase, and θ_{23}***

Thank you

Hierarchy – δ_{CP} degeneracy in $\nu_\mu \rightarrow \nu_e$ oscillation channel







For ν: Max: NH, -90° and Min: IH, 90°

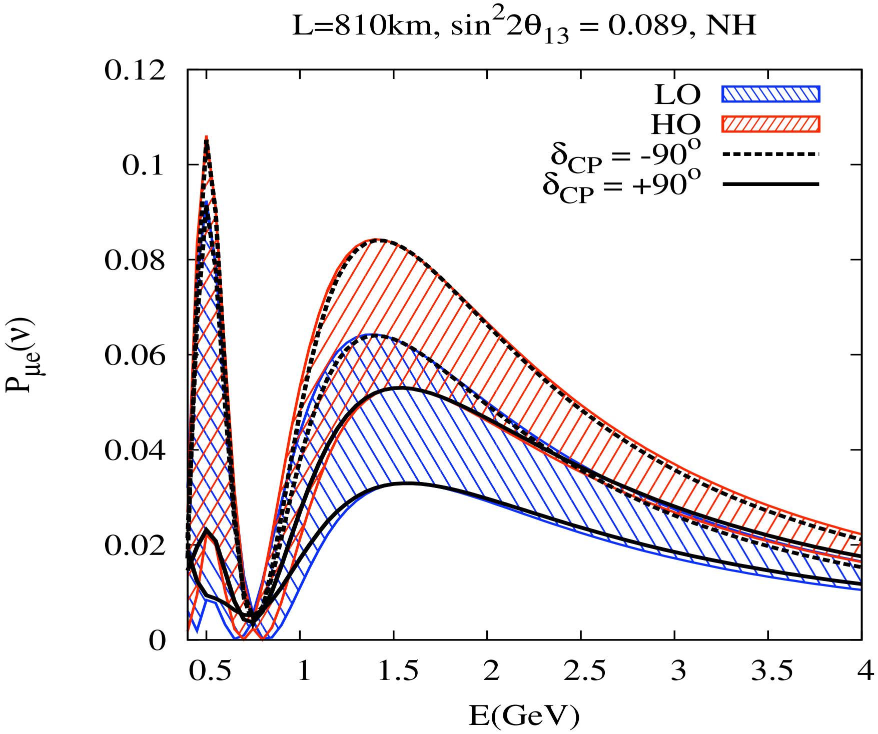
Favorable combinations
NH, LHP (-180° to 0°) and IH, UHP (0° to 180°)

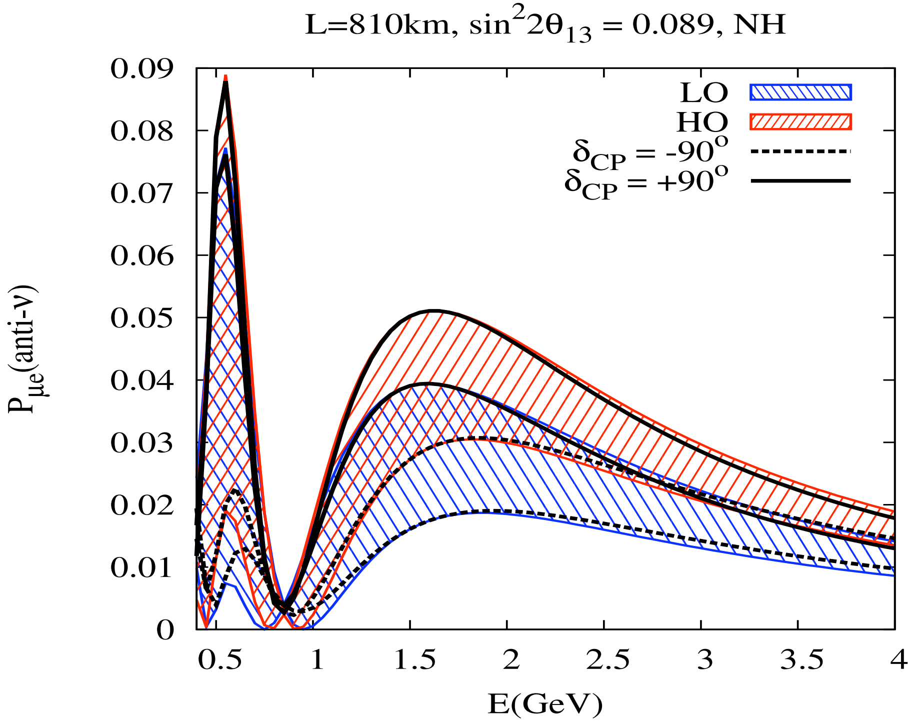
Degeneracy pattern different between T2K & NOνA

DUNE: Large Earth matter effects
Clear separation between NH and IH

Agarwalla, arXiv:1401.4705 [hep-ph]

Octant – δ_{CP} degeneracy in $\nu_\mu \rightarrow \nu_e$ oscillation channel





For neutrino:
Maximum: HO, -90°
Minimum: LO, 90°

LO: $\sin^2\theta_{23} = 0.41$
HO: $\sin^2\theta_{23} = 0.59$

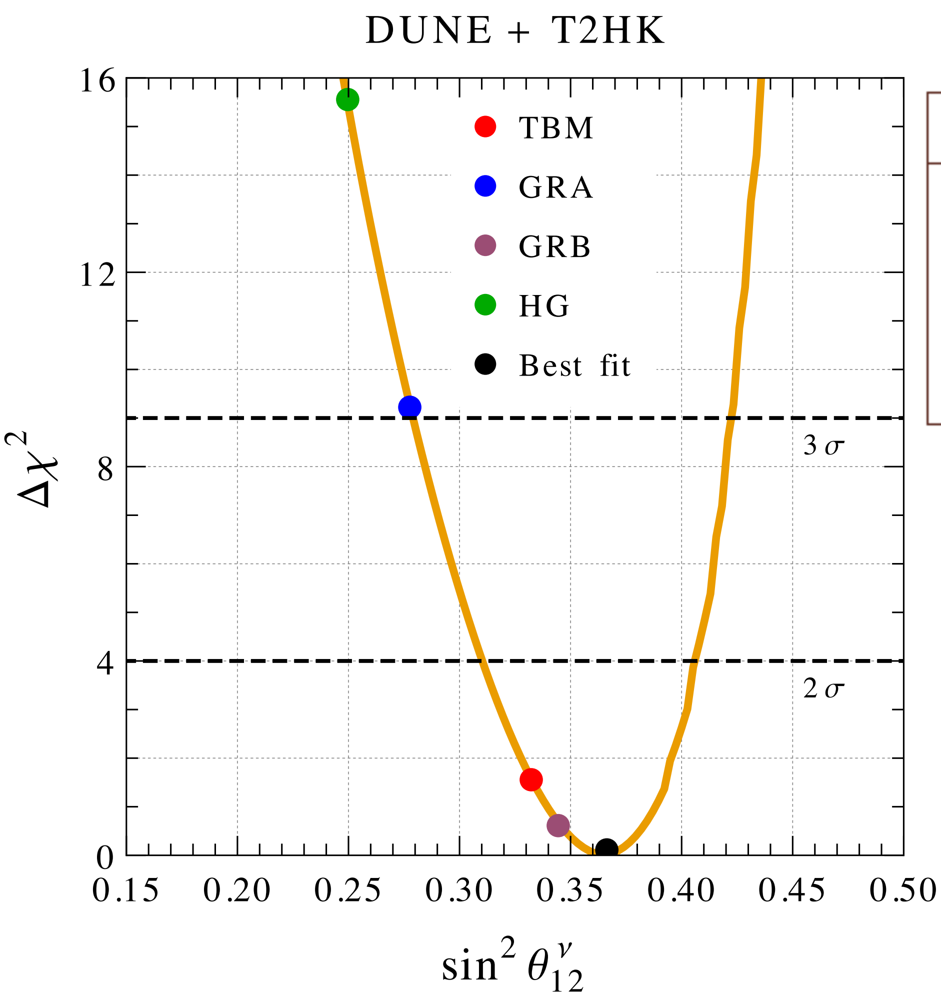
For anti-neutrino:
Maximum: HO, 90°
Minimum: LO, -90°

Unfavorable CP values for neutrino are favorable for anti-neutrino & vice-versa

Agarwalla, Prakash, Sankar, arXiv: 1301.2574

Oscillation Data and Neutrino Mixing Schemes

Sum Rule: $\cos\delta_{CP} = \dfrac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right]$



Symmetry form	θ^{ν}_{12} [°]	$\cos\delta_{CP}$	δ_{CP} [°]
BM	45	unphysical	unphysical
TBM	$\arcsin(1/\sqrt{3}) \approx 35$	-0.16	$99 \vee 261$
GRA	$\arctan(1/\phi) \approx 32$	0.21	$78 \vee 282$
GRB	$\arccos(\phi/2) = 36$	-0.24	$104 \vee 256$
HG	30	0.39	$67 \vee 293$

$U_{PMNS} = U_e^{\dagger} U_{\nu}$ golden ratio: $\phi = (1 + \sqrt{5})/2$

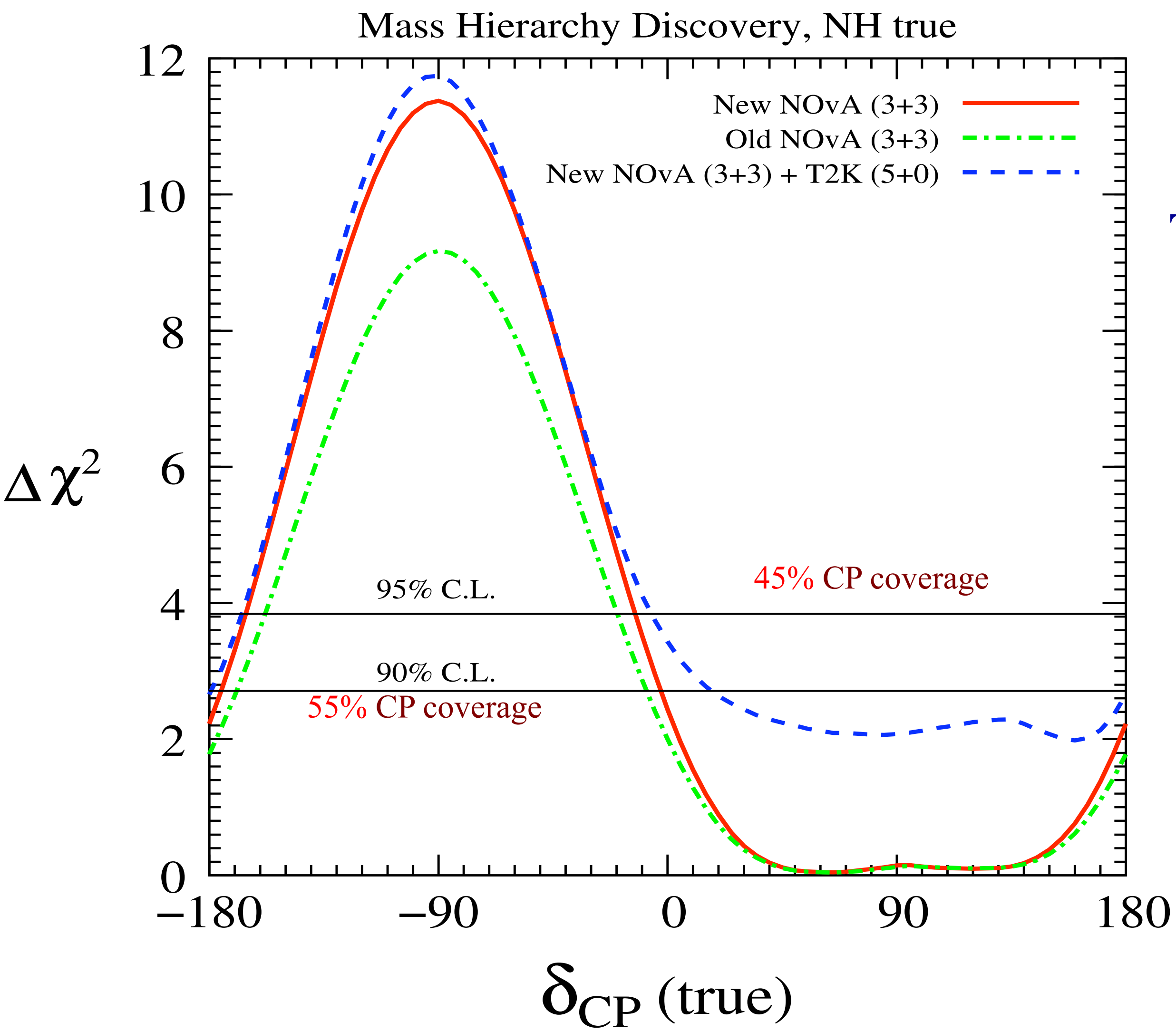
black dot: current best-fit value of $\delta_{CP} = 248°$ which means

$\sin^2\theta^{\nu}_{12} = 0.364$ $(\Delta\chi^2 = 0)$

Agarwalla, Chatterjee, Petcov, Titov, arXiv:1711.02107

the coloured dots corresponding to the values of $\sin^2\theta^{\nu}_{12}$ which characterise the GRB (violet), TBM (red), GRA (blue) and HG (green) symmetry forms.

Mass Hierarchy Discovery with T2K and NOvA



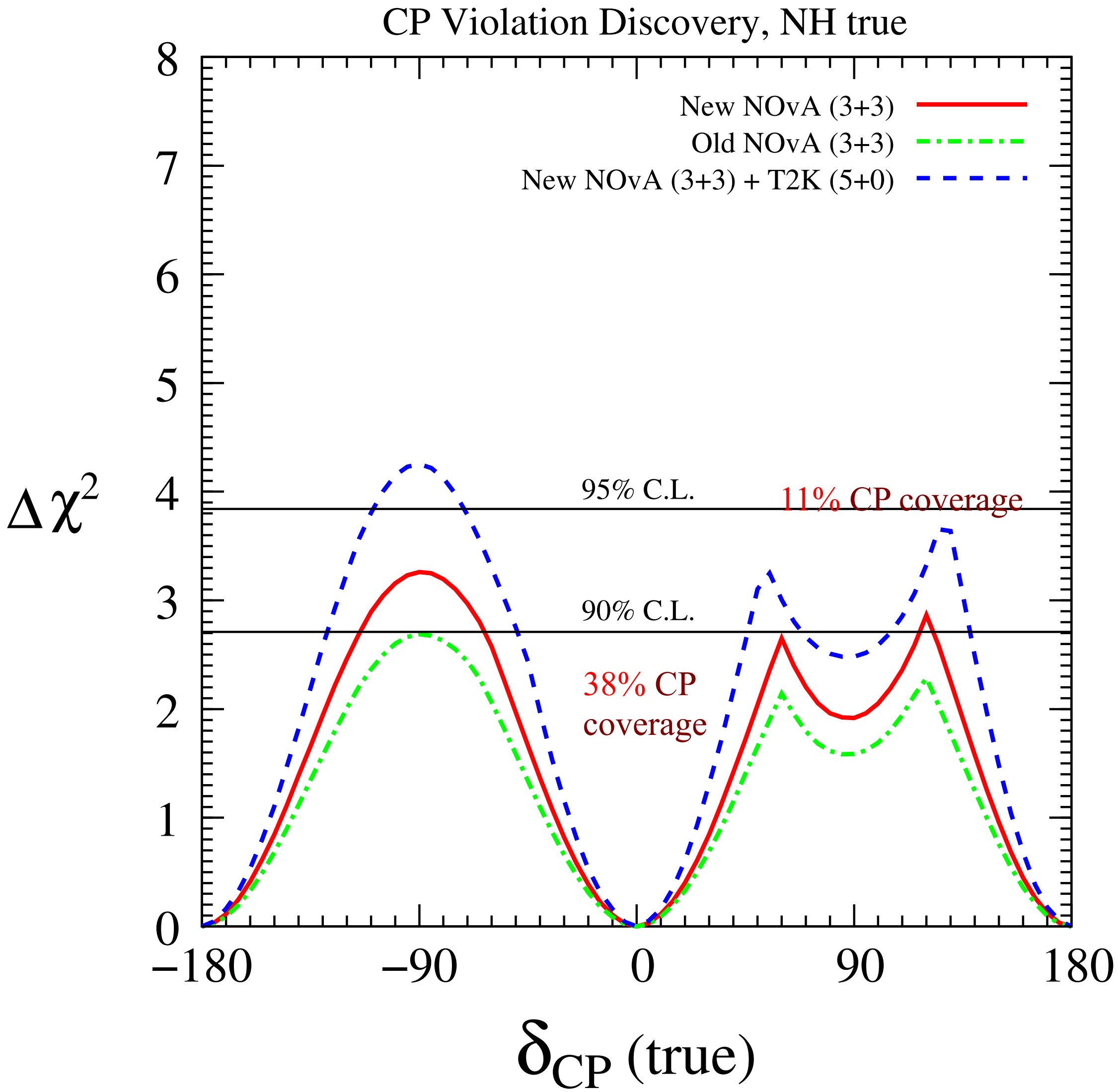
T2K: Total p.o.t.: 7.8×10^{21}

NOvA: Total p.o.t.: 3.6×10^{21}

Adding data from T2K and NOvA is useful to kill the intrinsic degeneracies

Agarwalla, Prakash, Raut, Sankar, arXiv: 1208.3644 [hep-ph]

CP-Violation Discovery with T2K and NOvA



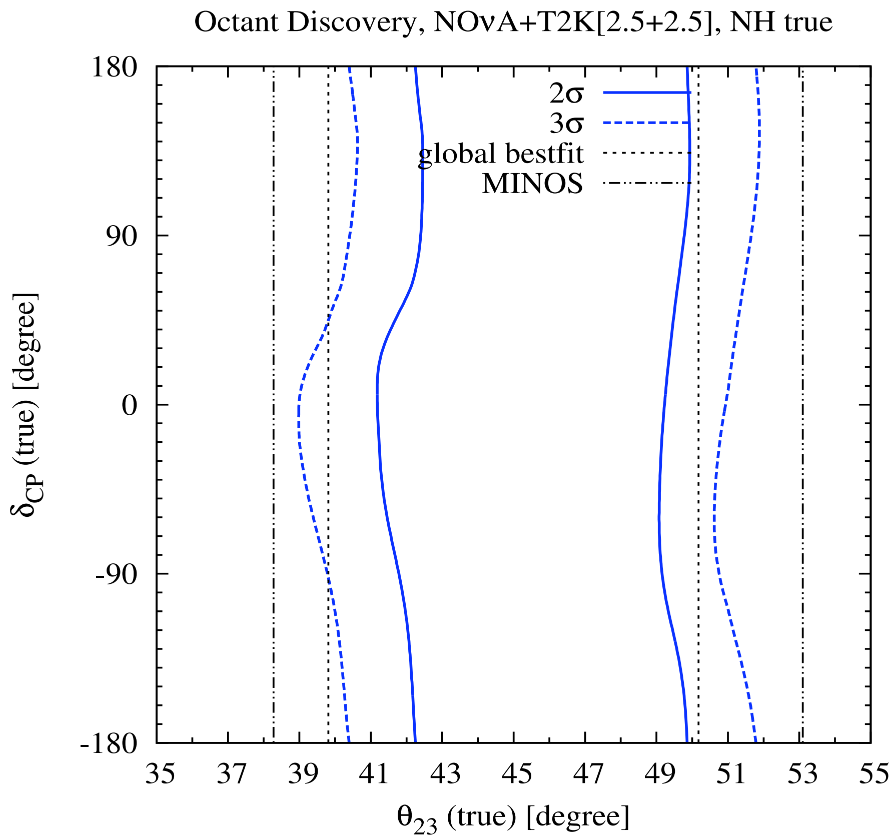
CP asymmetry $\propto 1/\sin 2\theta_{13}$

Large θ_{13} increases statistics but reduces asymmetry

Systematics are important

Agarwalla, Prakash, Raut, Sankar, arXiv: 1208.3644 [hep-ph]
Ghosh, Ghosal, Goswami, Raut, arXiv:1401.7243 [hep-ph]

Resolving Octant of θ_{23} with T2K and NOνA



Agarwalla, Prakash, Sankar, arXiv:1301.2574 [hep-ph]

If $\theta_{23} < 41°$ or $\theta_{23} > 50°$, we can resolve the octant issue at 2σ irrespective of δ_{CP}

If $\theta_{23} < 39°$ or $\theta_{23} > 52°$, we can resolve the octant issue at 3σ irrespective of δ_{CP}

Important message: T2K must run in anti-neutrino mode in future

Diagonalization of the Effective Hamiltonian

$$H_a = U\begin{bmatrix} 0 & 0 & 0 \\ 0 & \delta m_{21}^2 & 0 \\ 0 & 0 & \delta m_{31}^2 \end{bmatrix} U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \tilde{U}\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}\tilde{U}^\dagger,$$

$$H'_a = Q^\dagger U^\dagger H_a U Q$$

$$= Q^\dagger \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & \delta m_{21}^2 & 0 \\ 0 & 0 & \delta m_{31}^2 \end{bmatrix} + U^\dagger \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U \right\} Q$$

$$= \begin{bmatrix} a c_{12}^2 c_{13}^2 & a c_{12} s_{12} c_{13}^2 & a c_{12} c_{13} s_{13} \\ a c_{12} s_{12} c_{13}^2 & a s_{12}^2 c_{13}^2 + \delta m_{21}^2 & a s_{12} c_{13} s_{13} \\ a c_{12} c_{13} s_{13} & a s_{12} c_{13} s_{13} & a s_{13}^2 + \delta m_{31}^2 \end{bmatrix}, \qquad Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{bmatrix}.$$

Jacobi Method (1846)

- Carl Gustav Jacob Jacobi (1804-1851)
- "Über ein leichtes Verfahren die in der Theorie der Säkularstörungen vorkommenden Gleichungen numerisch aufzulösen,"
  Crelle's Journal **30** (1846) 51-94.

1st Rotation

$$H_a' = \begin{bmatrix} ac_{12}^2 c_{13}^2 & ac_{12}s_{12}c_{13}^2 & ac_{12}c_{13}s_{13} \\ ac_{12}s_{12}c_{13}^2 & as_{12}^2 c_{13}^2 + \delta m_{21}^2 & as_{12}c_{13}s_{13} \\ ac_{12}c_{13}s_{13} & as_{12}c_{13}s_{13} & as_{13}^2 + \delta m_{31}^2 \end{bmatrix}$$

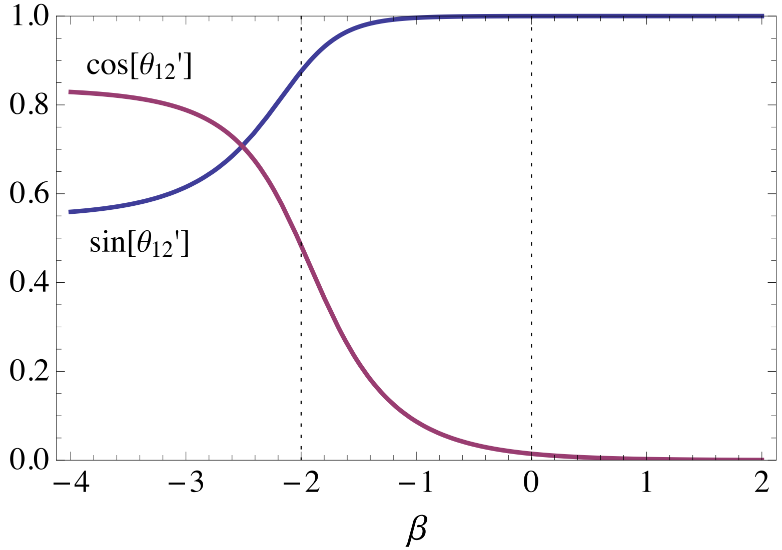
$$V = \begin{bmatrix} c_\varphi & s_\varphi & 0 \\ -s_\varphi & c_\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \tan 2\varphi = \frac{a \sin 2\theta_{12}}{(\delta m_{21}^2 / c_{13}^2) - a\cos 2\theta_{12}}$$

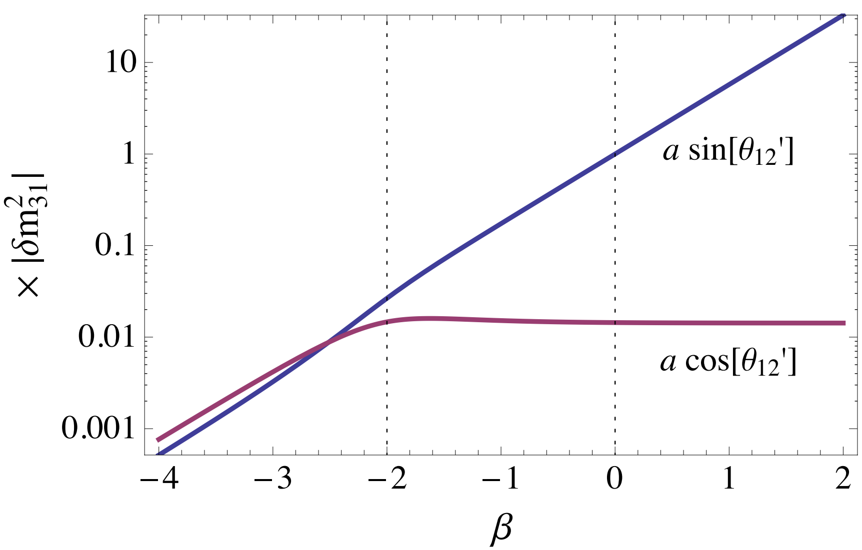
$$H_a'' = V^\dagger H_a' V = \begin{bmatrix} \lambda_-' & 0 & ac_{12}' c_{13}s_{13} \\ 0 & \lambda_+' & as_{12}' c_{13}s_{13} \\ ac_{12}' c_{13}s_{13} & as_{12}' c_{13}s_{13} & as_{13}^2 + \delta m_{31}^2 \end{bmatrix},$$

$$\theta_{12}' = \theta_{12} + \varphi, \qquad \lambda_\pm' = \frac{(\delta m_{21}^2 + ac_{13}^2) \pm \sqrt{(\delta m_{21}^2 - ac_{13}^2)^2 + 4ac_{13}^2 s_{12}^2 \delta m_{21}^2}}{2}$$

1st Rotation

$$\tan 2\theta'_{12} = \tan 2(\theta_{12} + \varphi) = \frac{(\delta m^2_{21} / c^2_{13}) \sin 2\theta_{12}}{(\delta m^2_{21} / c^2_{13}) \cos 2\theta_{12} - a}$$





2nd Rotation

$$H_a'' = \begin{bmatrix} \lambda_-' & 0 & ac_{12}'c_{13}s_{13} \\ 0 & \lambda_+' & as_{12}'c_{13}s_{13} \\ ac_{12}'c_{13}s_{13} & as_{12}'c_{13}s_{13} & as_{13}^2 + \delta m_{31}^2 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & -s_\phi & c_\phi \end{bmatrix}, \qquad \tan 2\phi = \frac{as_{12}' \sin 2\theta_{13}}{\delta m_{31}^2 + as_{13}^2 - \lambda_+'} \approx \frac{a \sin 2\theta_{13}}{(\delta m_{31}^2 - \delta m_{31}^2 s_{12}^2) - a\cos 2\theta_{13}}$$

$$H_a''' = W^\dagger H_a'' W = \begin{bmatrix} \lambda_-' & -ac_{12}'c_{13}s_{13}s_\phi & ac_{12}'c_{13}s_{13}c_\phi \\ -ac_{12}'c_{13}s_{13}s_\phi & \lambda_\mp'' & 0 \\ ac_{12}'c_{13}s_{13}c_\phi & 0 & \lambda_\pm'' \end{bmatrix},$$

$$\lambda_\pm'' = \frac{\left[\lambda_+' + (\delta m_{31}^2 + as_{13}^2)\right] \pm \sqrt{\left[\lambda_+' - (\delta m_{31}^2 + as_{13}^2)\right]^2 + 4a^2 s_{12}'^2 c_{13}^2 s_{13}^2}}{2}$$

Effective Mixing Matrix

$$
\begin{aligned}
\tilde{U} &= UQVW \\
&= R_{23}(\theta_{23},0)R_{13}(\theta_{13},\delta)R_{12}(\theta_{12},0)\,Q\,R_{12}(\varphi,0)R_{23}(\phi,0) \\
&= R_{23}(\theta_{23},0)\,Q\,R_{13}(\theta_{13},0)R_{12}(\theta_{12},0)R_{12}(\varphi,0)R_{23}(\phi,0) \\
&= R_{23}(\theta_{23},0)\,Q\,R_{13}(\theta_{13},0)R_{12}(\theta_{12}+\varphi,0)R_{23}(\phi,0) \\
&= R_{23}(\theta_{23},0)\,Q\,R_{13}(\theta_{13},0)R_{12}(\theta'_{12},0)R_{23}(\phi,0) \\
&\approx R_{23}(\theta_{23},0)\,Q\,R_{13}(\theta_{13},0)R_{13}(\phi,0)R_{12}(\theta'_{12},0) \\
&= R_{23}(\theta_{23},0)\,Q\,R_{13}(\theta_{13}+\phi,0)R_{12}(\theta'_{12},0) \\
&= R_{23}(\theta_{23},0)\,Q\,R_{13}(\theta'_{13},0)R_{12}(\theta'_{12},0) \\
&= R_{23}(\theta_{23},0)R_{13}(\theta'_{13},\delta)R_{12}(\theta'_{12},0)\,Q
\end{aligned}
$$

$$\theta'_{13} = \theta_{13} + \phi$$

Effective Mixing Matrix

$$R_{12}(\theta'_{12},0)R_{23}(\phi,0)$$

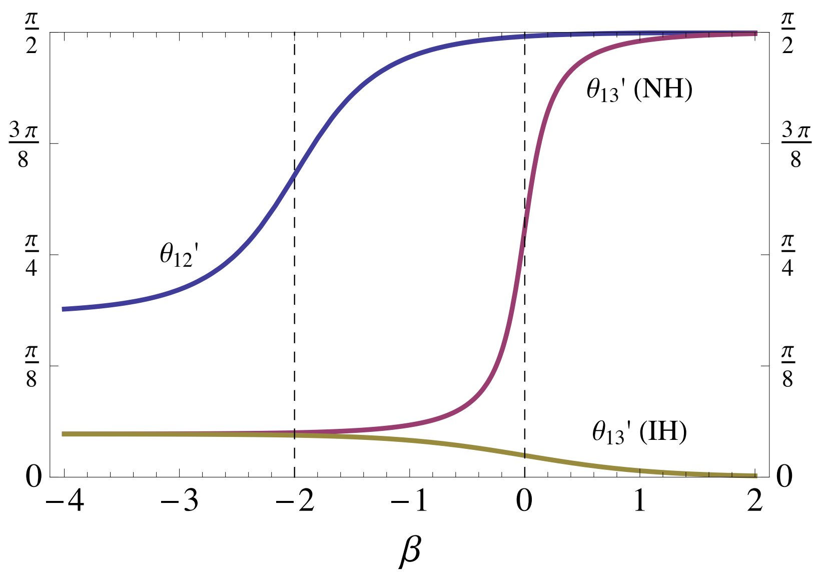
$$= \begin{bmatrix} c'_{12} & s'_{12} & 0 \\ -s'_{12} & c'_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & -s_\phi & c_\phi \end{bmatrix} \approx \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & -s_\phi & c_\phi \end{bmatrix}$$

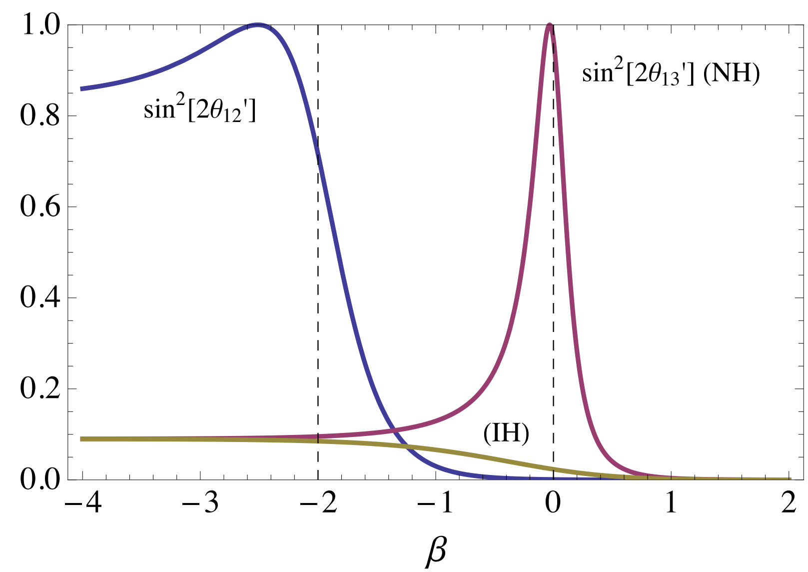
$$= \begin{bmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{bmatrix} \begin{bmatrix} c'_{12} & s'_{12} & 0 \\ -s'_{12} & c'_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= R_{13}(\phi,0)R_{12}(\theta'_{12},0)$$

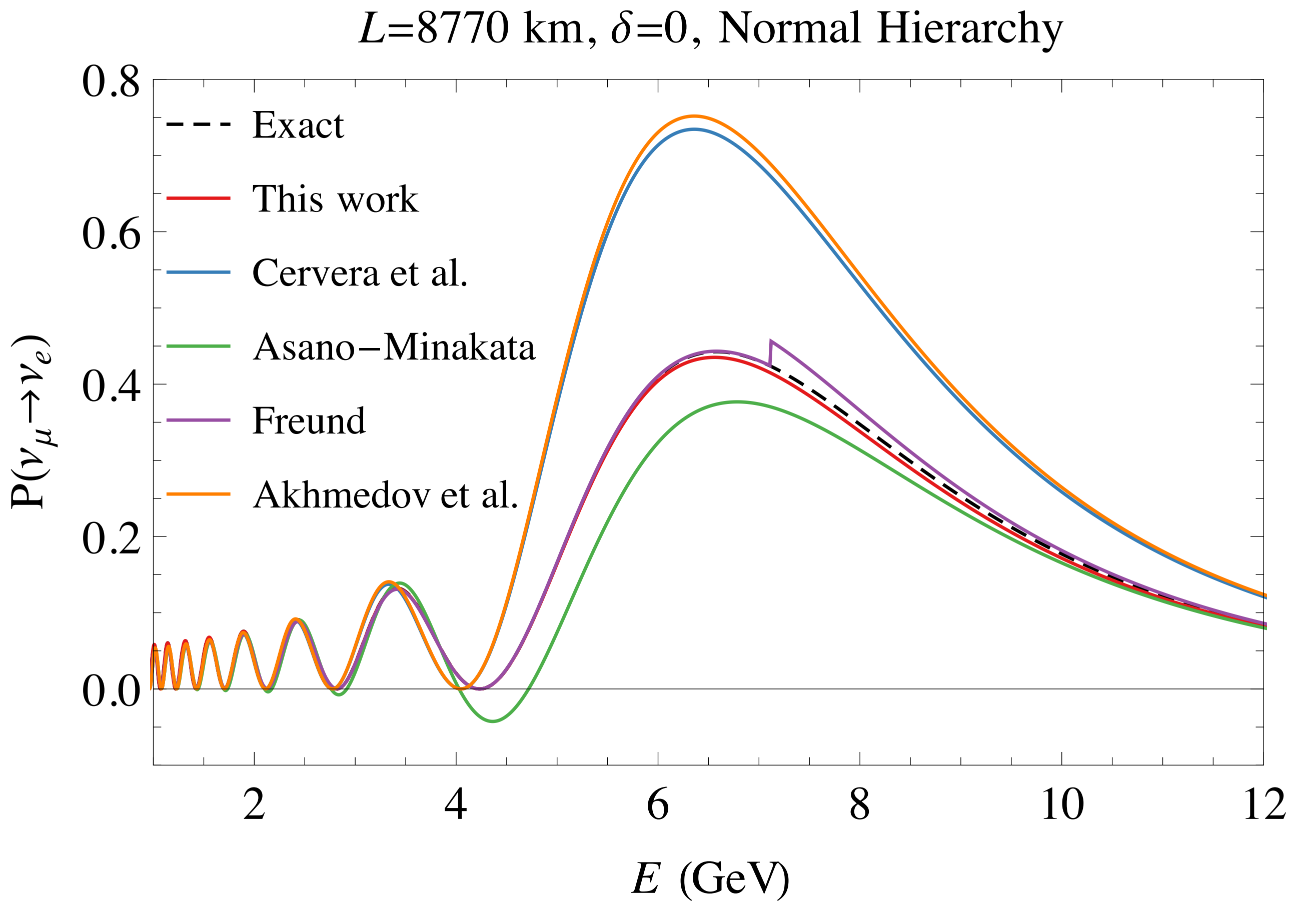
Effective Mixing Angles

$$\tan 2\theta'_{12} = \frac{(\delta m^2_{21}/c^2_{13})\sin 2\theta_{12}}{(\delta m^2_{21}/c^2_{13})\cos 2\theta_{12} - a}, \qquad \tan 2\theta'_{13} = \frac{(\delta m^2_{31} - \delta m^2_{21} s^2_{12})\sin 2\theta_{13}}{(\delta m^2_{31} - \delta m^2_{21} s^2_{12})\cos 2\theta_{13} - a},$$





Accuracy of Our Method and Comparison with Existing Literature

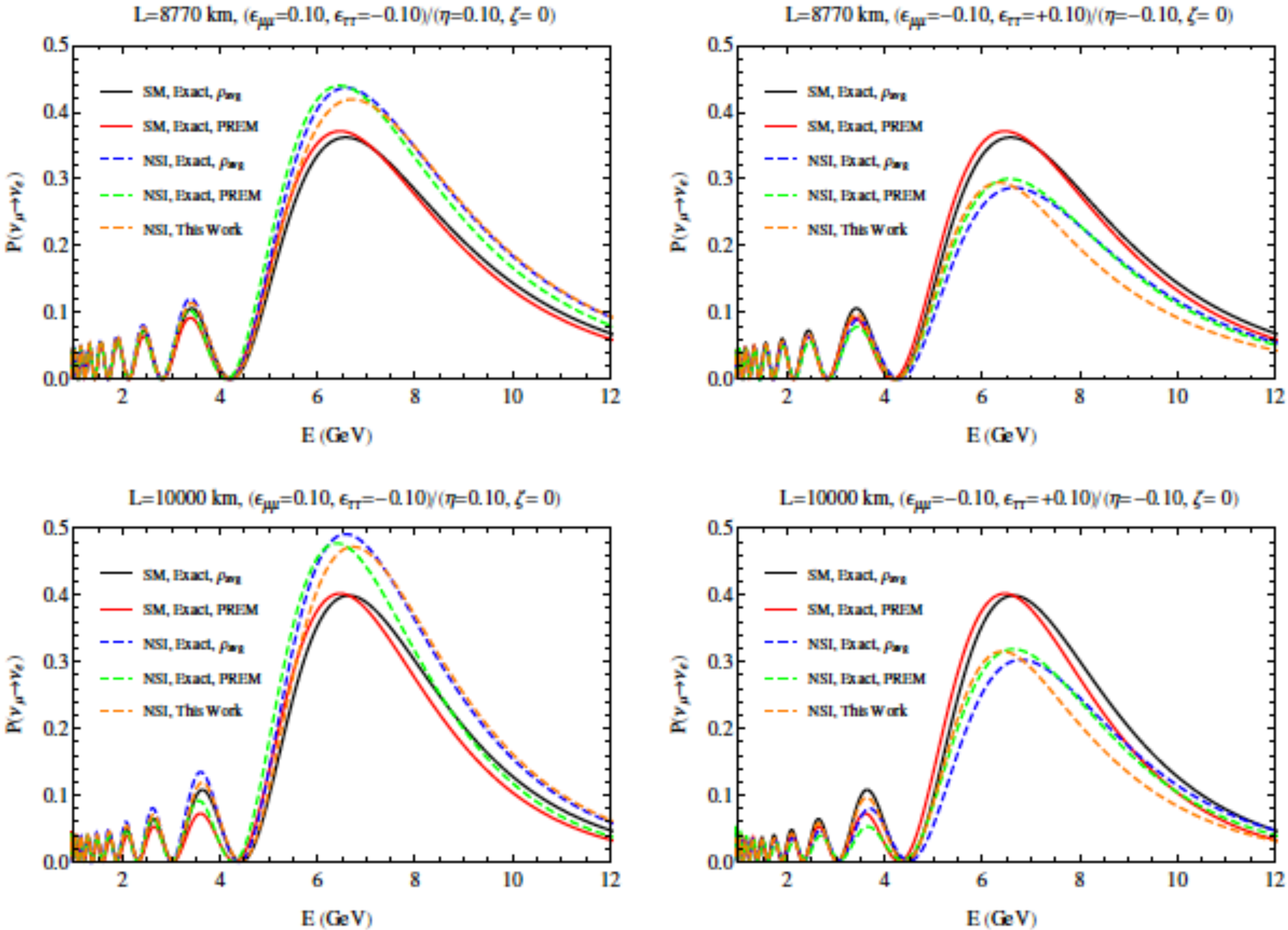


Agarwalla, Kao, Takeuchi, JHEP 1404, 047 (2014)

Other analytical expressions suffer in accuracy due to their reliance on expansion in θ_{13}, or in simplicity when higher order terms in θ_{13} included

Our method gives accurate probability for all channels, baselines, and energies

Comparison between Constant and Varying Earth Density Profile



Agarwalla, Kao, Saha, Takeuchi, JHEP 1511 (2015) 035

Three Flavor Effects in $\nu_\mu \rightarrow \nu_e$ oscillation probability

$$P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) =$$ ← Our measurement

$$\frac{16A}{\Delta m^2_{31}} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) c^2_{13} s^2_{13} s^2_{23} (1 - 2s^2_{13})$$ ← Matter Effects, small

$$-\frac{2AL}{E} \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) c^2_{13} s^2_{13} s^2_{23} (1 - 2s^2_{13})$$ ← Matter Effects, ∞ L

$$-8\frac{\Delta m^2_{21} L}{2E} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\delta\, s_{13} c^2_{13} c_{23} s_{23} c_{12} s_{12}$$ ← CPV, Our goal!

Here, $A = 2\sqrt{2} G_F n_e E = 7.6 \times 10^{-5} \mathrm{eV}^2 \cdot \frac{\rho}{\mathrm{g\,cm^{-3}}} \cdot \frac{E}{\mathrm{GeV}}$

How to proceed?

First possibility:

Choose small L (~ 200 km), so that matter effects are small

But, we want to work at oscillation maximum:

$$\frac{\Delta m_{31}^2 L}{4E} \sim \frac{\pi}{2} \quad \Rightarrow \quad E_\nu < 1 \text{ GeV}$$

Since, $\sigma \propto E_\nu$: we need a high flux at oscillation maximum

Off-axis beam: narrow range of neutrino energies

This is the working principle of Hyper-Kamiokande

How to proceed?

Second possibility:

Take large L (> 1000 km)

Estimate the matter effects, and settle the issue of Mass Hierarchy

But, we still want to work at oscillation maximum:

$$\frac{\Delta m_{31}^2 L}{4E} \sim \frac{\pi}{2} \Rightarrow E_\nu > 2\ \mathrm{GeV}$$

Unfold CP-violation from matter effects through energy dependence

On-axis beam: wide range of neutrino energies

This is the working principle of DUNE

Present Understanding of the 2-3 Mixing Angle

Information on θ_{23} comes from: a) atmospheric neutrinos and b) accelerator neutrinos

In two-flavor scenario: $$P_{\mu\mu} = 1 - \sin^2 2\theta_{\rm eff} \sin^2\left(\frac{\Delta m^2_{\rm eff} L}{4E}\right)$$

For accelerator neutrinos: relate effective 2-flavor parameters with 3-flavor parameters:

$$\boxed{\Delta m^2_{\rm eff} = \Delta m^2_{31} - \Delta m^2_{21}(\cos^2\theta_{12} - \cos\delta_{\rm CP}\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23})}$$

$$\boxed{\sin^2 2\theta_{\rm eff} = 4\cos^2\theta_{13}\sin^2\theta_{23}\left(1-\cos^2\theta_{13}\sin^2\theta_{23}\right)}$$ where $$\boxed{\frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2} = \tan^2\theta_{23}}$$

Nunokawa etal, hep-ph/0503283; A. de Gouvea etal, hep-ph/0503079

Combining bean and atmospheric data in MINOS, we have:

MINOS Collaboration: arXiv:1304.6335v2 [hep-ex]

$$\boxed{\sin^2 2\theta_{\rm eff} = 0.95^{+0.035}_{-0.036}\ (10.71\times 10^{21}\ \text{p.o.t})}$$ $$\boxed{\sin^2 2\bar{\theta}_{\rm eff} = 0.97^{+0.03}_{-0.08}\ (3.36\times 10^{21}\ \text{p.o.t})}$$

Atmospheric data, dominated by Super-Kamiokande, still prefers maximal value of $\sin^2 2\theta_{\rm eff} = 1$ (≥ 0.94 (90% C.L.))

Talk by Y. Itow in Neutrino 2012 conference, Kyoto, Japan

Bounds on θ_{23} from the global fits

	Forero etal	Fogli etal	Gonzalez-Garcia etal
$\sin^2\theta_{23}$ (NH)	$0.427^{+0.034}_{-0.027} \oplus 0.613^{+0.022}_{-0.040}$	$0.386^{+0.024}_{-0.021}$	$0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$
3σ range	$0.36 \to 0.68$	$0.331 \to 0.637$	$0.34 \to 0.67$
$\sin^2\theta_{23}$ (IH)	$0.600^{+0.026}_{-0.031}$	$0.392^{+0.039}_{-0.022}$	**Relative 1σ precision of 11%**
3σ range	$0.37 \to 0.67$	$0.335 \to 0.663$	

All the three global fits indicate for non-maximal 2-3 mixing!

In ν_μ survival probability, the dominant term is mainly sensitive to $\sin^2 2\theta_{23}$!

If $\sin^2 2\theta_{23}$ differs from 1 (as indicated by recent data), we get two solutions for θ_{23}:

one in lower octant (LO: θ_{23} < 45 degree), other in higher octant (HO: θ_{23} > 45 degree)

In other words, if (0.5 – $\sin^2\theta_{23}$) is +ve (-ve) then θ_{23} belongs to LO (HO)

This is known as the octant ambiguity of θ_{23} !

Fogli and Lisi, hep-ph/9604415

ν_μ to ν_e oscillation data can break this degeneracy!

The preferred value would depend on the choice of the neutrino mass hierarchy!

Octant – δ_{CP} degeneracy in $\nu_\mu \rightarrow \nu_e$ oscillation channel

$P_{\mu e} = \beta_1 \sin^2\theta_{23} + \beta_2 \cos(\hat{\Delta} + \delta_{\rm CP}) + \beta_3 \cos^2\theta_{23}$ (upto second order in $\alpha = \Delta_{21}/\Delta_{31}$ and $\sin 2\theta_{13}$)

$$\beta_1 = \sin^2 2\theta_{13} \frac{\sin^2 \hat{\Delta}(1-\hat{A})}{(1-\hat{A})^2}, \quad \beta_3 = \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{13} \frac{\sin^2 \hat{\Delta}\hat{A}}{\hat{A}^2}$$

$$\beta_2 = \alpha \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \frac{\sin \hat{\Delta}\hat{A}}{\hat{A}} \frac{\sin \hat{\Delta}(1-\hat{A})}{1-\hat{A}}$$

$$A(\mathrm{eV}^2) = 0.76 \times 10^{-4} \rho \ (\mathrm{g/cc}) E(\mathrm{GeV}) \qquad \hat{\Delta} = \Delta_{31} L/4E, \ \hat{A} = A/\Delta_{31}$$

Cervera etal, hep-ph/0002108; Freund etal, hep-ph/0105071

We demand that: $P_{\mu e}(\mathrm{LO}, \delta_{\rm CP}^{\rm LO}) = P_{\mu e}(\mathrm{HO}, \delta_{\rm CP}^{\rm HO})$

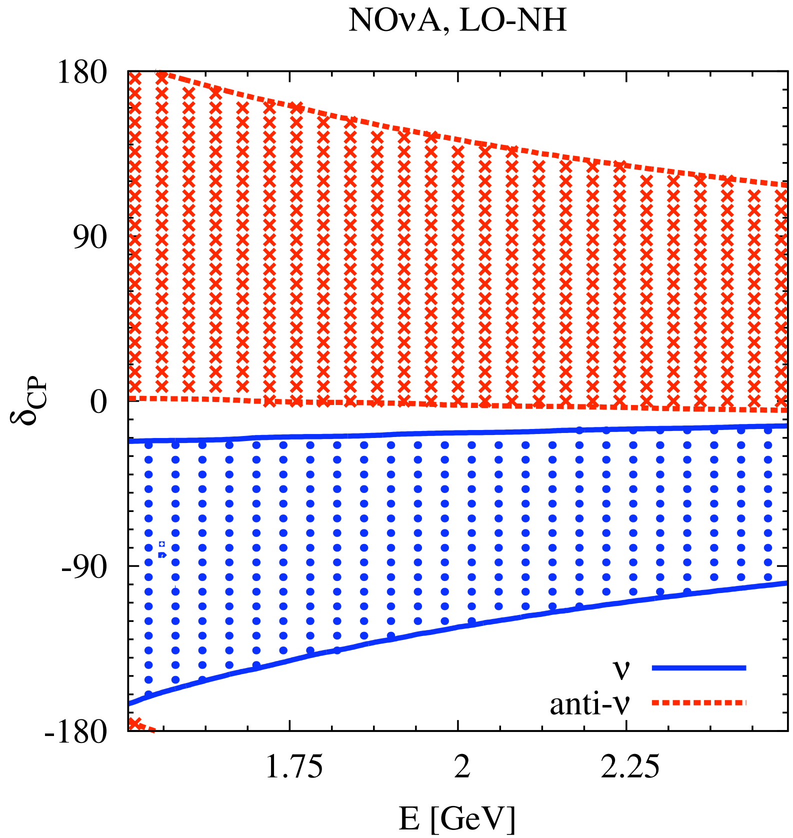
Above condition gives us: $\cos(\hat{\Delta} + \delta_{\rm CP}^{\rm LO}) - \cos(\hat{\Delta} + \delta_{\rm CP}^{\rm HO}) = \frac{\beta_1 - \beta_3}{\beta_2}(\sin^2\theta_{23}^{\rm HO} - \sin^2\theta_{23}^{\rm LO})$

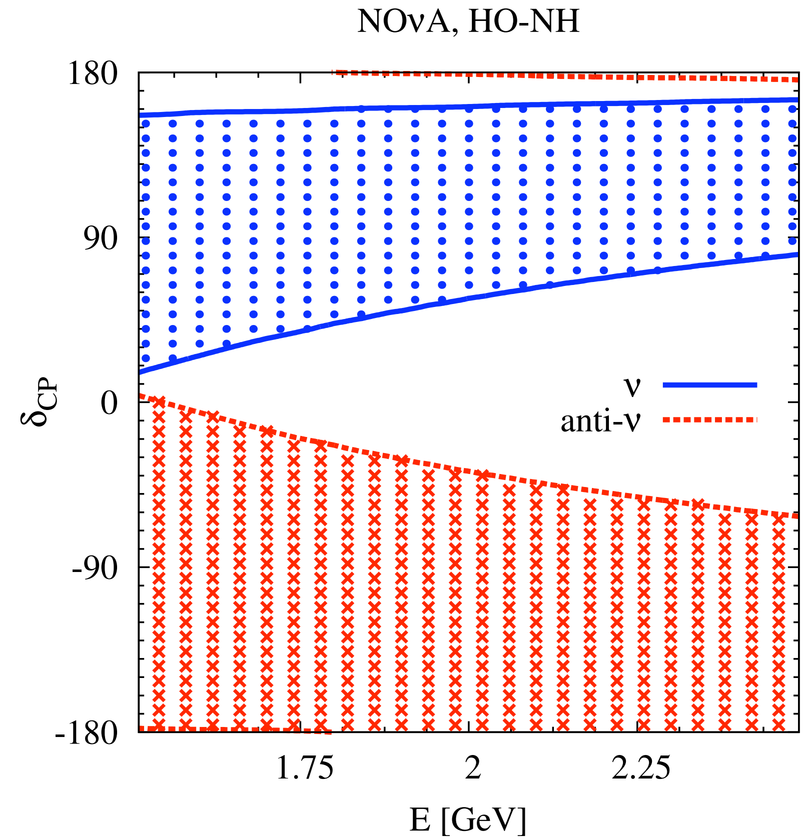
For L=810 km & E=2 GeV, we get for NH and neutrino: $\cos(\hat{\Delta} + \delta_{\rm CP}^{\rm LO}) - \cos(\hat{\Delta} + \delta_{\rm CP}^{\rm HO}) = 1.7$

$P_{\mu e}(\mathrm{LO}, -116^\circ \le \delta_{\rm CP} \le -26^\circ)$ is degenerate with $P_{\mu e}(\mathrm{HO}, 64^\circ \le \delta_{\rm CP} \le 161^\circ)$

Agarwalla, Prakash, Uma Sankar, arXiv:1301.2574

Octant – δ_{CP} degeneracy in $\nu_\mu \rightarrow \nu_e$ oscillation channel





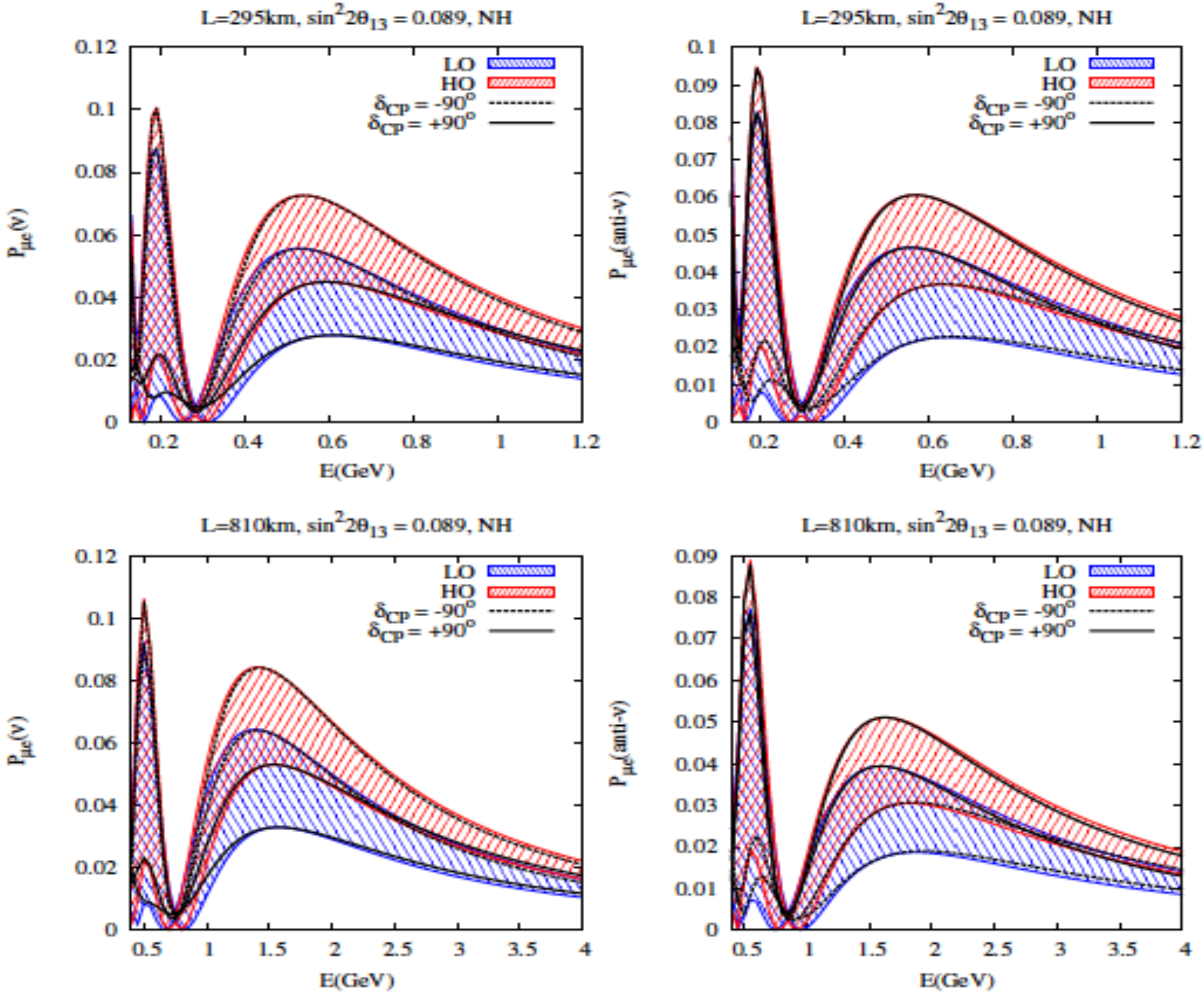
Agarwalla, Prakash, Sankar, arXiv:1301.2574 [hep-ph]

Octant – δ_{CP} degeneracy in $P_{\mu e}$ as a function of neutrino energy

At 2 GeV, $P_{\mu e}(\text{LO}, -116^\circ \leq \delta_{\text{CP}} \leq -26^\circ)$ is degenerate with $P_{\mu e}(\text{HO}, 64^\circ \leq \delta_{\text{CP}} \leq 161^\circ)$

As an example, $P_{\mu e}(\text{LO}, \delta_{\text{CP}} = -90^\circ)$ is degenerate with $P_{\mu e}(\text{HO}, \delta_{\text{CP}} \approx 66^\circ)$

Octant – δ_{CP} degeneracy in T2K and NOνA



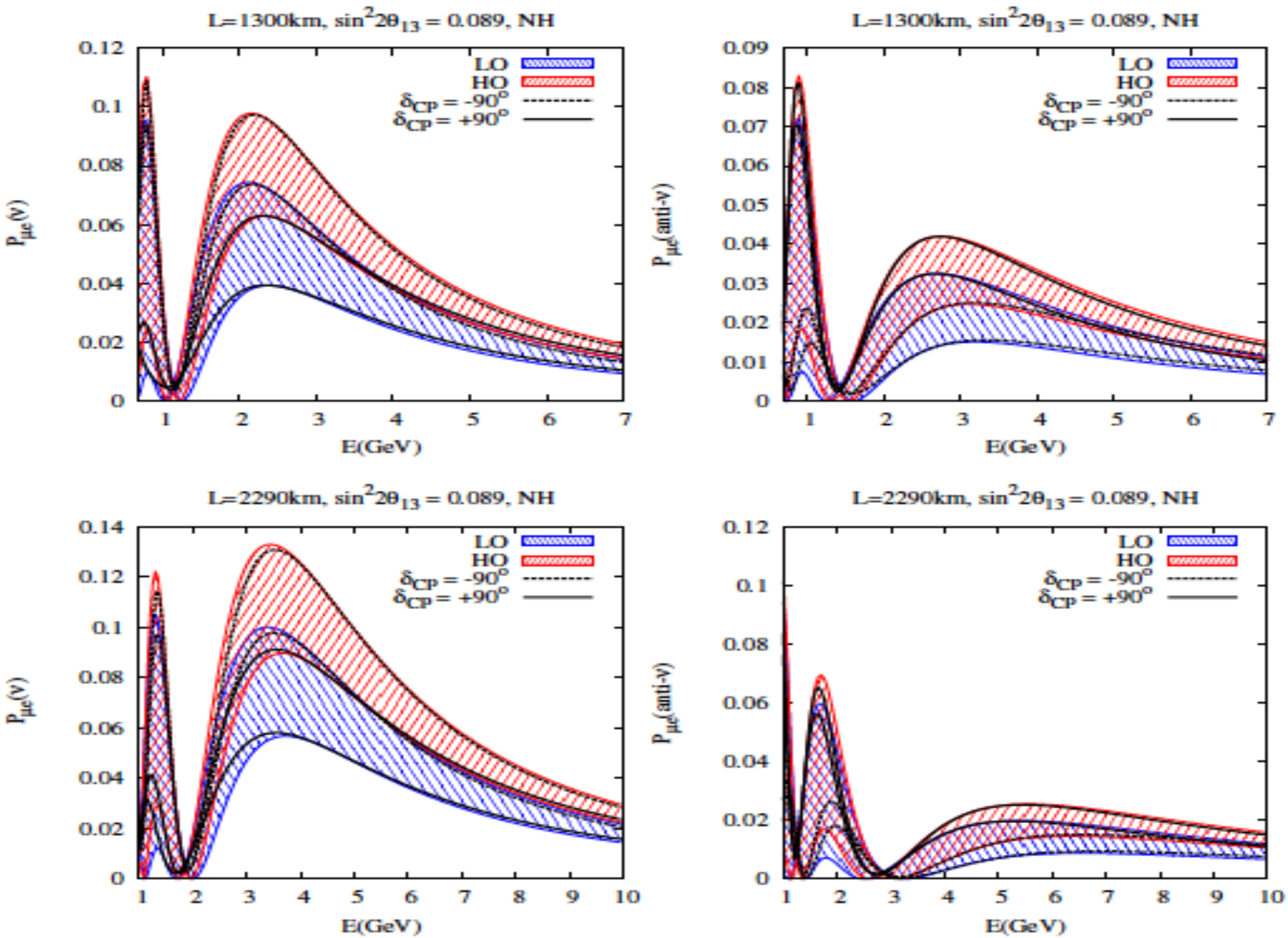
For neutrino: favorable combinations:
Max: HO, -90°
Min: LO, 90°

For anti-neutrino: favorable combinations:
Max: HO, 90°
Min: LO, -90°

Unfavorable CP values for neutrino are favorable for anti-neutrino and vice-versa!

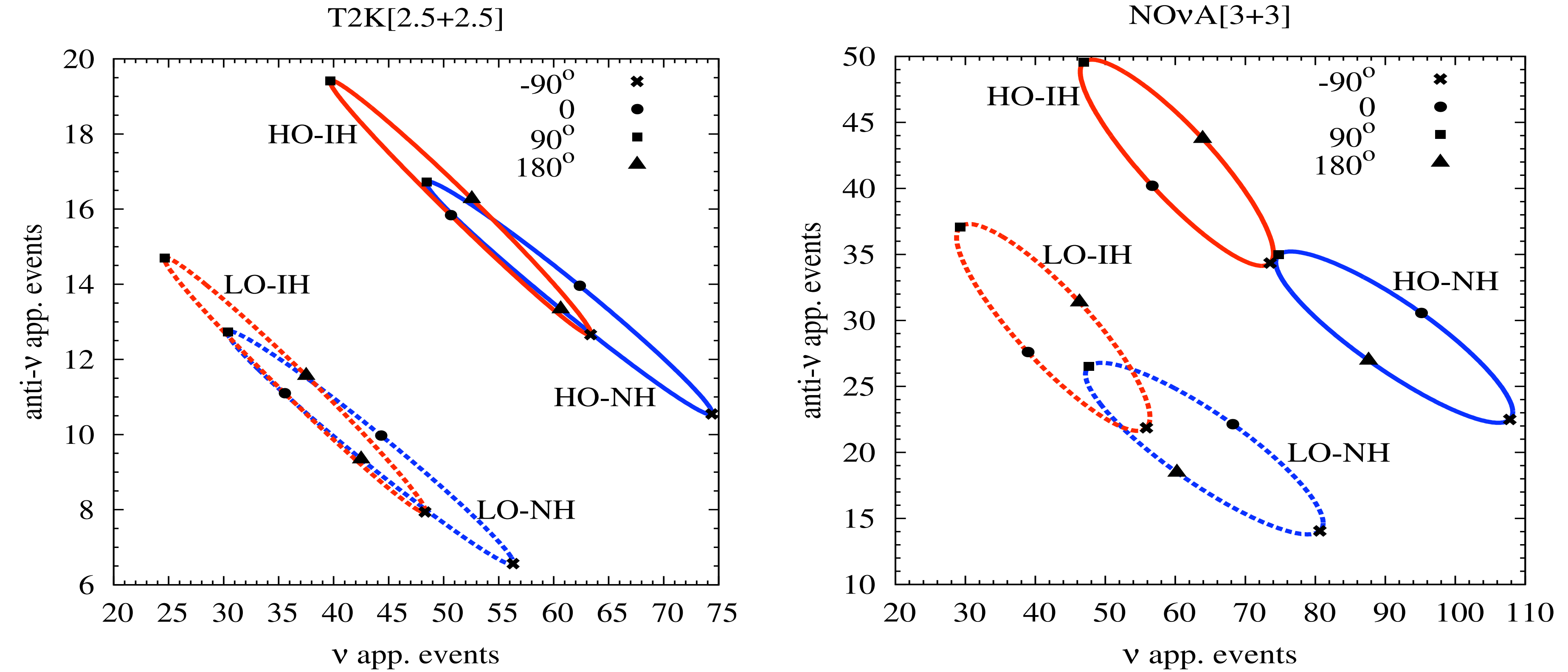
Agarwalla, Prakash, Uma Sankar, arXiv:1301.2574

Octant – δ_{CP} degeneracy in LBNE and LBNO



Agarwalla, Prakash, Sankar, arXiv:1304.3251 [hep-ph]

Bi-Event Plots for T2K and NOvA



Agarwalla, Prakash, Sankar, arXiv:1301.2574 [hep-ph]; see also the talk by T. Nakadaira in this workshop

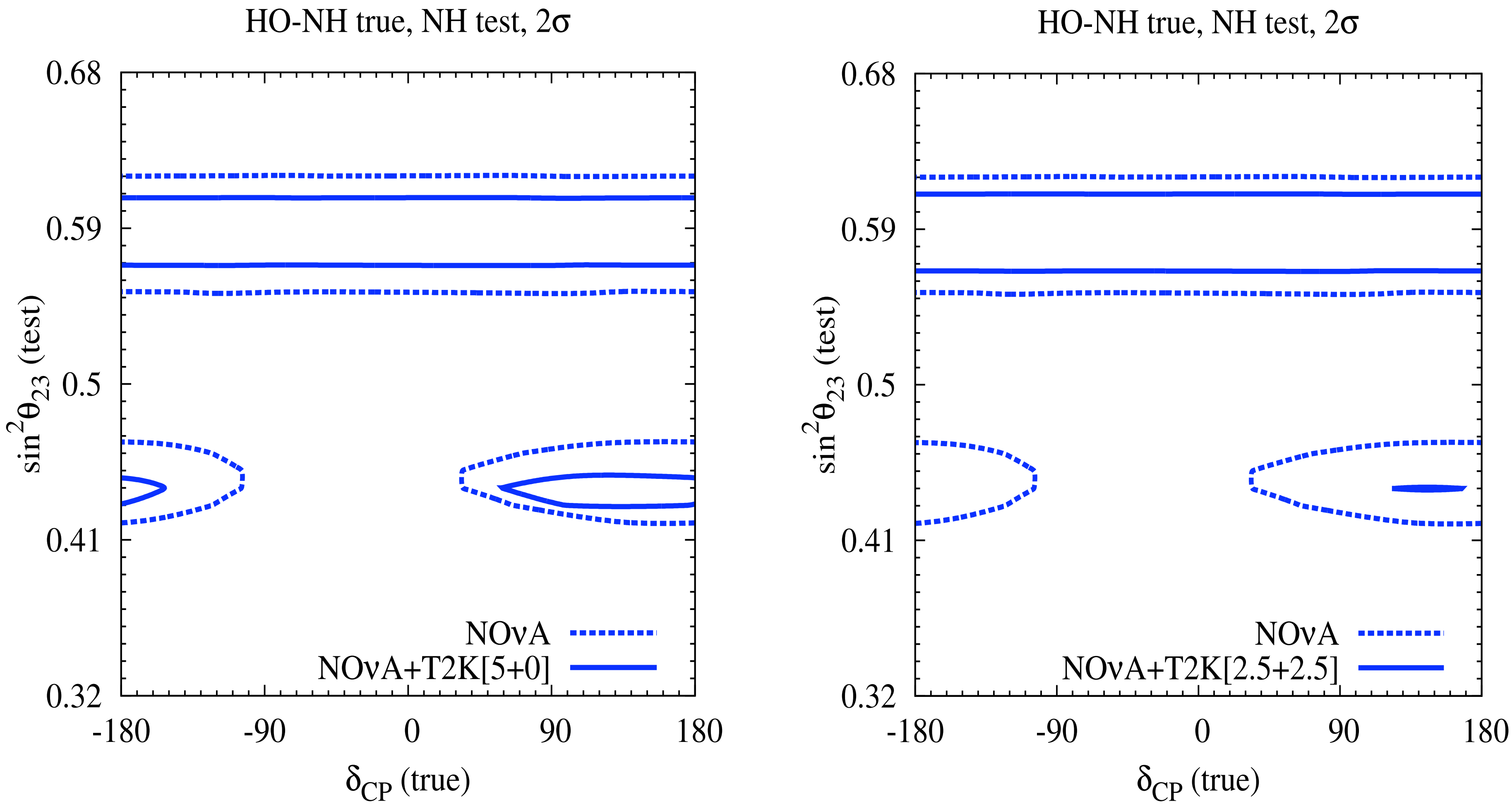
neutrino vs. anti-neutrino events for various octant-hierarchy combinations, ellipses due to varying δ_{CP}!

If δ_{CP} = -90° (90°), the asymmetry between ν and anti-ν events is largest for NH (IH)

For NOνA & T2K, the ellipses for the two hierarchies overlap whereas the ellipses of LO are well separated from those of HO, the same is true for T2K as well!

Octant discovery: balanced neutrino & anti-neutrino runs needed in each experiment!

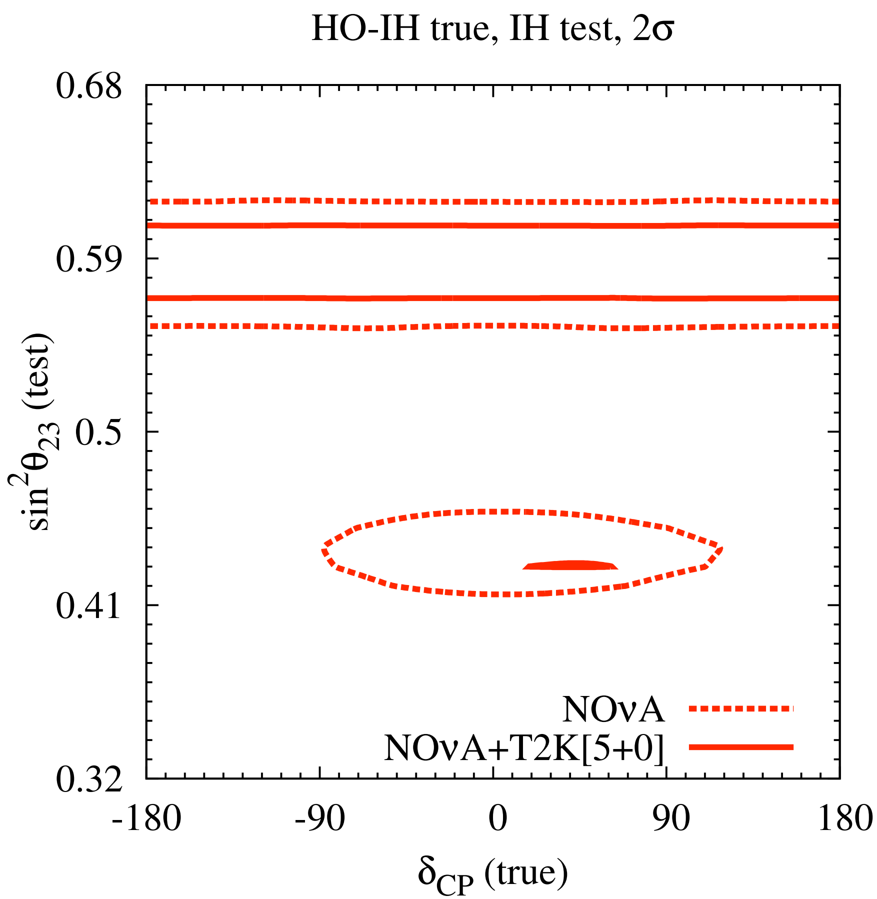
Allowed regions in test $sin^2\theta_{23}$ - true δ_{CP} plane

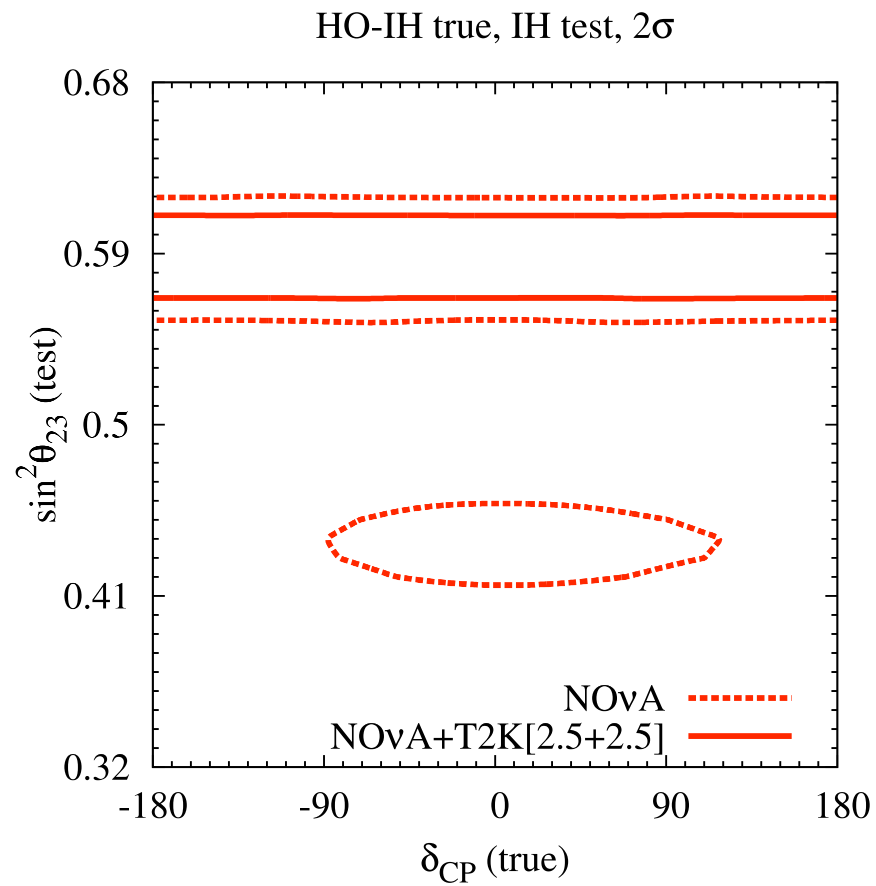


Agarwalla, Prakash, Sankar, arXiv:1301.2574 [hep-ph]

Balanced neutrino & anti-neutrino runs from T2K are mandatory if HO turns out to be the right octant!

Allowed regions in test $\sin^2\theta_{23}$ - true δ_{CP} plane

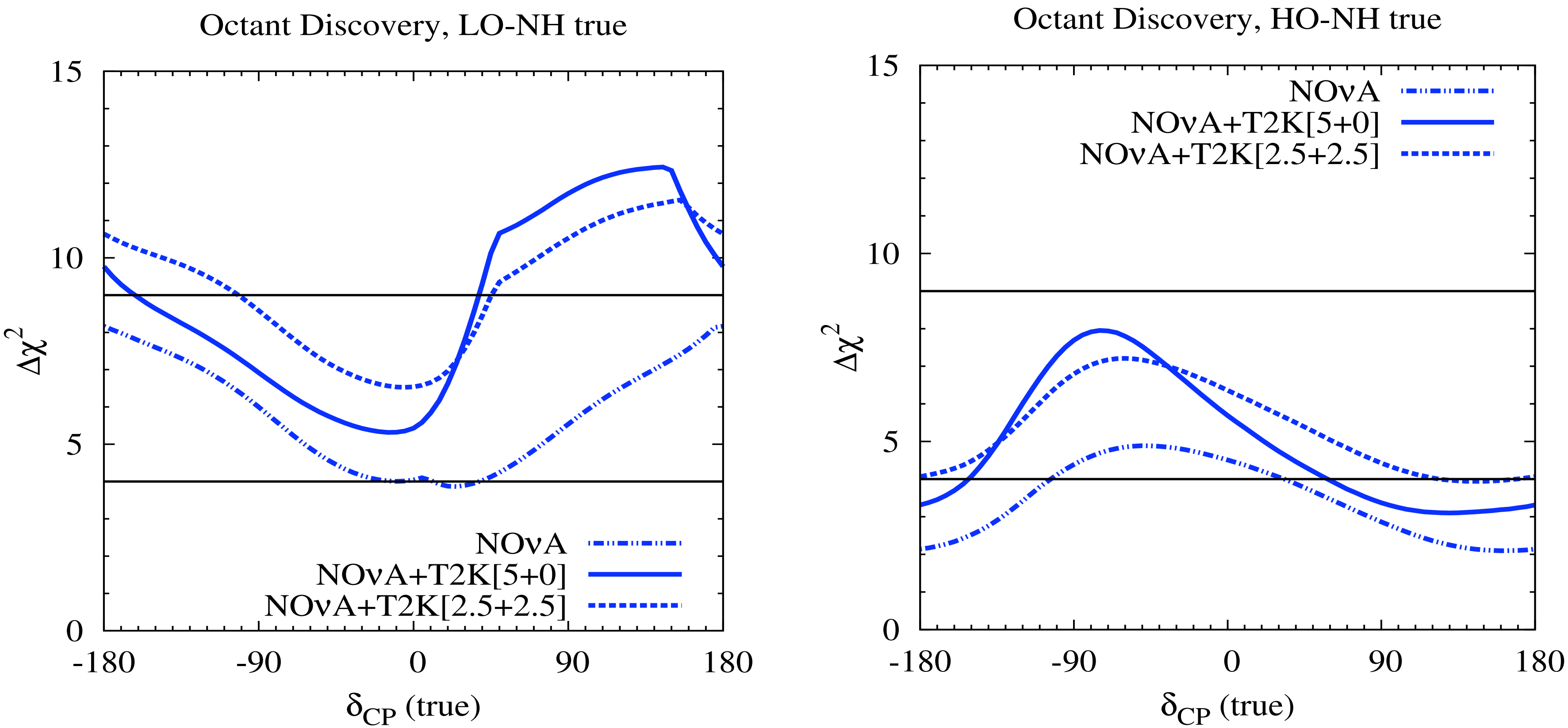




Agarwalla, Prakash, Sankar, arXiv:1301.2574 [hep-ph]

Balanced neutrino & anti-neutrino runs from T2K are mandatory if HO turns out to be the right octant!

Resolving Octant of θ_{23} with T2K and NOvA

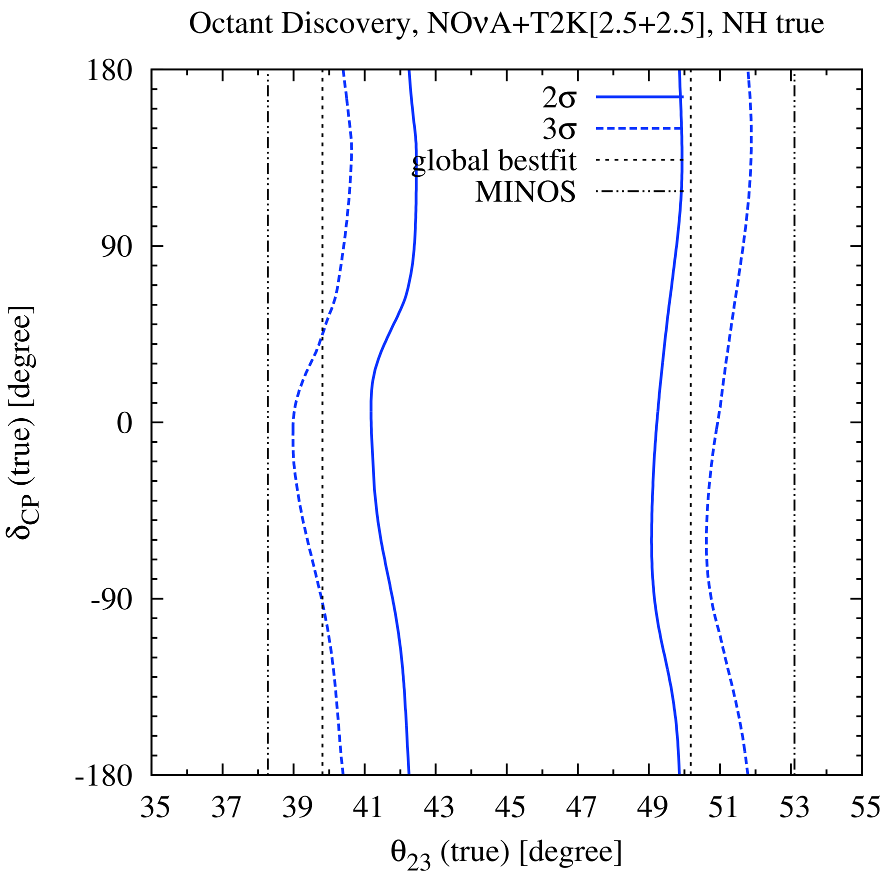


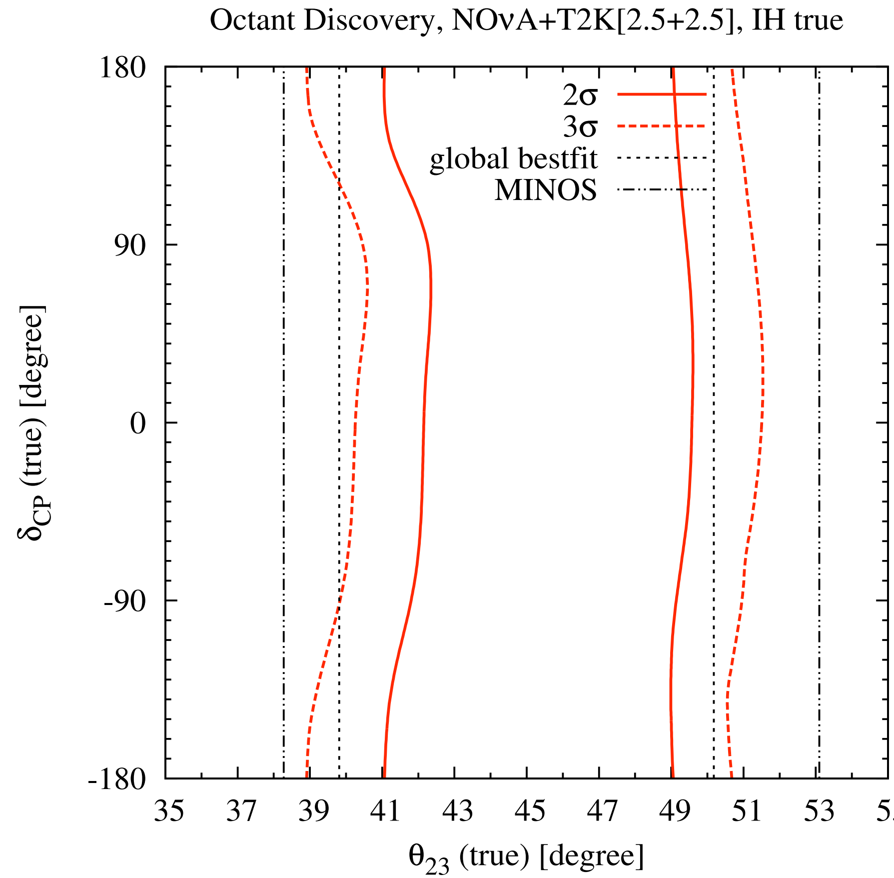
Agarwalla, Prakash, Sankar, arXiv:1301.2574 [hep-ph]

A 2σ resolution of the octant, for all combinations of neutrino parameters, becomes possible if we add the balanced neutrino and anti-neutrino runs from T2K (2.5 years ν + 2.5 years anti-ν) and NOνA (3 years ν + 3 years of anti-ν)

Important message: T2K must run in anti-neutrino mode in future!

Octant discovery in θ_{23} (true) – δ_{CP} (true) plane with T2K & NOvA





Agarwalla, Prakash, Sankar, arXiv:1301.2574 [hep-ph]

With Normal Hierarchy

If $\theta_{23} < 41°$ or $\theta_{23} > 50°$, we can resolve the octant issue at 2σ irrespective δ_{CP}

If $\theta_{23} < 39°$ or $\theta_{23} > 52°$, we can resolve the octant issue at 3σ irrespective δ_{CP}