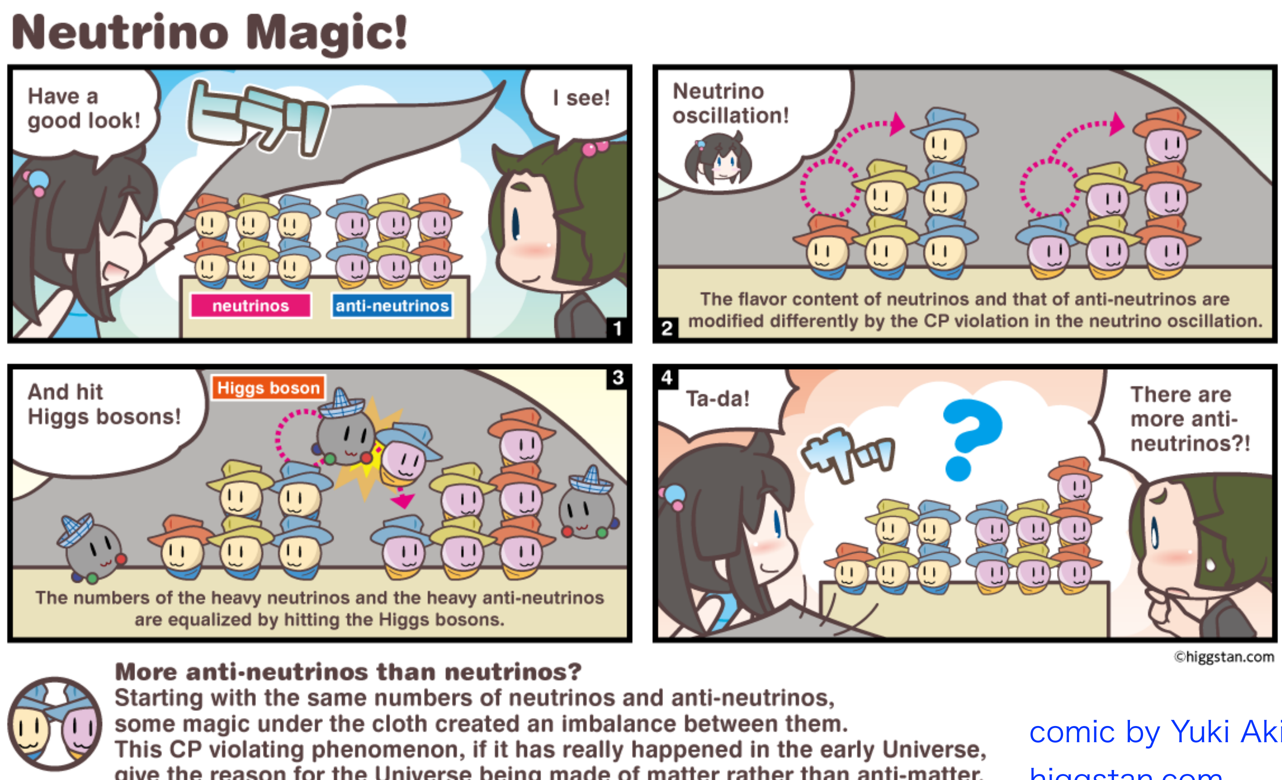
1807.06582 with Ryuichiro Kitano, and Yuta Hamada

Leptogenesis via active neutrino oscillation

Wen Yin, KAIST in Korea



comic by Yuki Akimoto, higgstan.com

Prospects of Neutrino Physics at IPMU, 11th May 2019

1. Introduction

How to generate the baryon asymmetry?

Sakharov's conditions

**Baryon/Lepton number violation*

**C and CP violation*

**Out of thermal equilibrium*

Unfortunately, the SM does not sufficiently satisfy…

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A clear New Physics! Neutirno mass

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$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \mathrm{h.c.}$$

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**Baryon/Lepton number violation* ✓

**C and CP violation* ✓

Neutrino oscillation can provide CP violation. It is favored to happen.

**Out of thermal equilibrium*

T2K Collaboration, 1701.00432

See Lisi's talk slide.

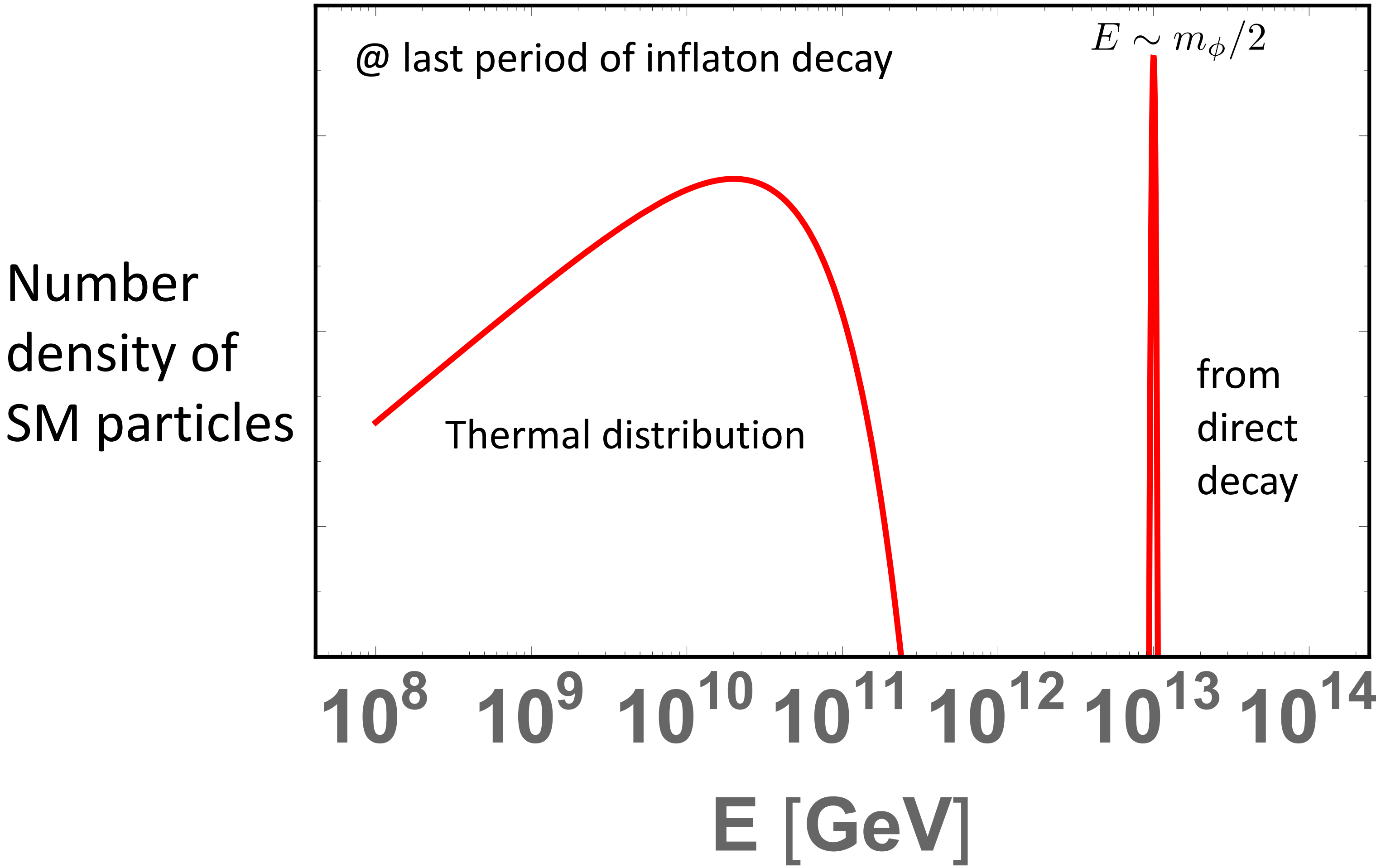
A clear New Physics! Neutirno mass

The SM (probably) is an effective theory with Majorana neutrino mass term

$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \text{h.c.}$$

Big bang is a one way process

Inflaton decay: $\phi \rightarrow \text{SM particles}$



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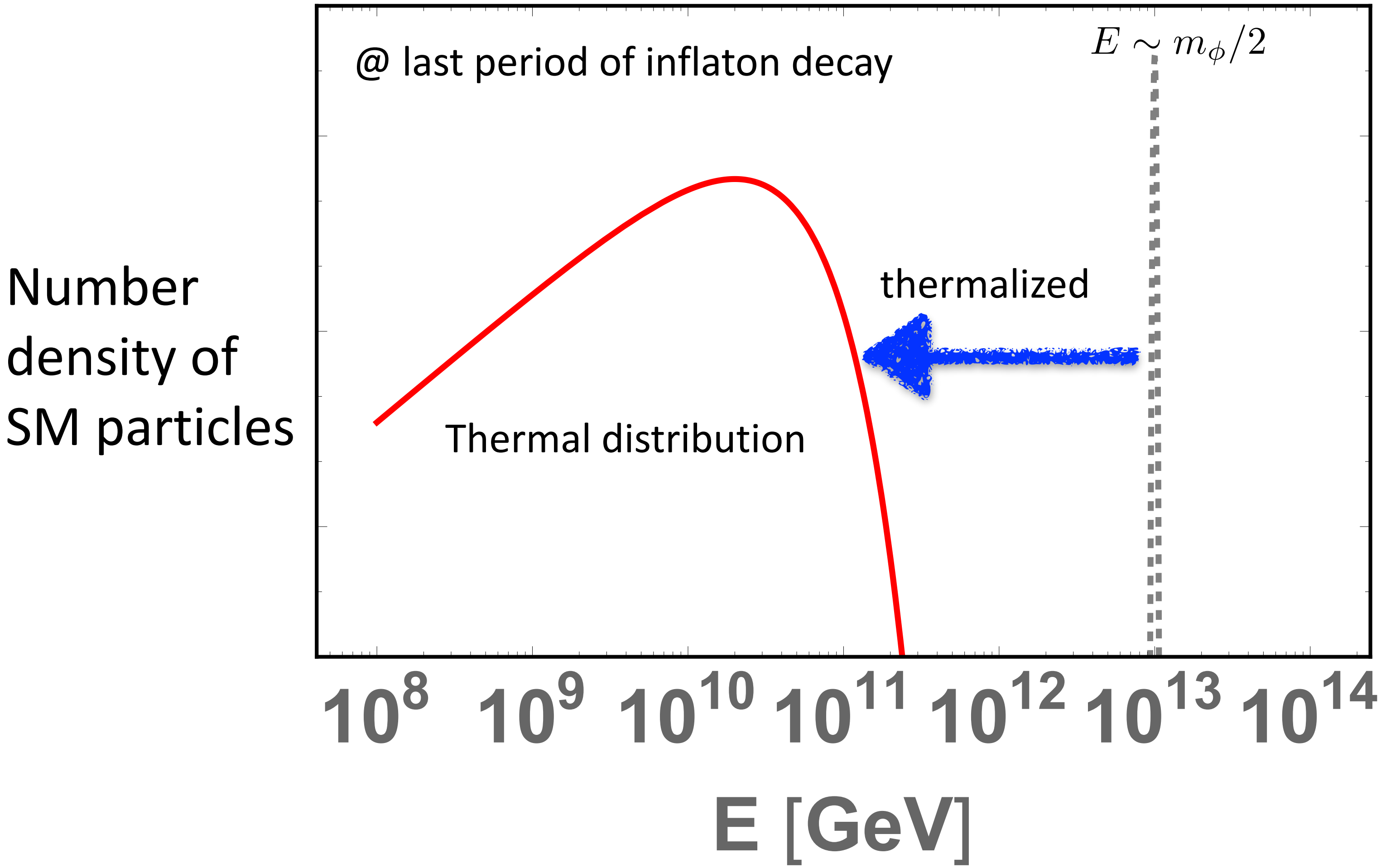


What I will show

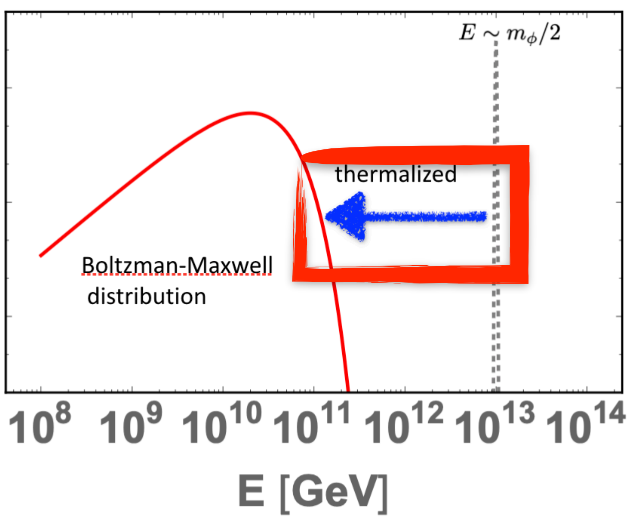
Kitano, Hamada, WY 1807.06582

Setup:

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \text{h.c.}$$

Baryogengesis due to active neutrino oscillation during the reheating/thermalization.

Number density of SM particles



Leptogenesis from active neutrino oscillation with two higher dimensional terms.
Kitano, Hamada 1609.05028.

c.f. Leptogenesis with light enough right-handed neutrinos.
Fukugida, Yanagida, 86; Pilaftsis, 97;
Akhmedov et al, 98;
Asaka, Shaposhnikov, 0505013;
Asaka, Yoshida, 1812.11323;
See Asaka, Turner, and Yoshida 's talks.

2. Neutrino Oscillation at big bang

$$\phi \to \nu_{\rm ini} + X, \bar{\nu}_{\rm ini} + \bar{X}$$

$\langle \nu_{\rm ini} |$

$$P_{{\rm ini}\to I} \simeq | \sum_{\alpha} \langle \nu_{\rm ini} | \nu_\alpha \rangle e^{i t_{\rm MFP} (m_{\nu_\alpha}^{\rm th})^2 / k} \langle \nu_\alpha | \nu_I \rangle |^2$$

c.f. $P_{e\to\mu} \simeq | \sum_{\alpha} \langle \nu_e | \nu_\alpha \rangle e^{i t m_{\nu_\alpha}^2 / k} \langle \nu_\alpha | \nu_\mu \rangle |^2$ @ vacuum

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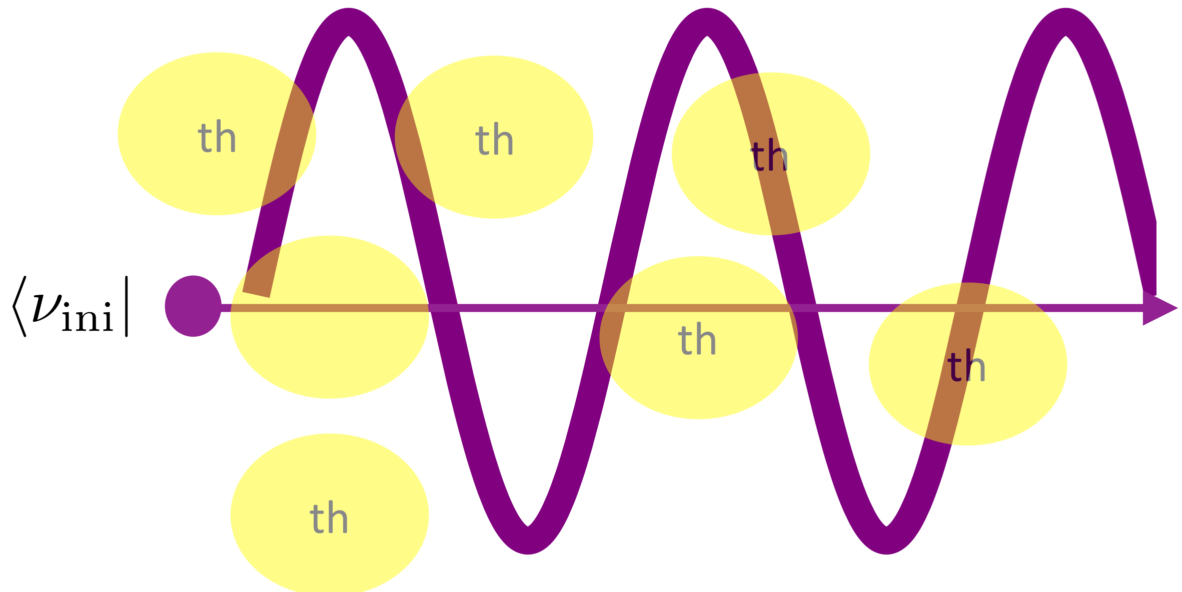

$$P_{{\rm ini}\to I} \simeq | \sum_\alpha \langle \nu_{\rm ini} | \nu_\alpha \rangle e^{i t_{\rm MFP} (m^{\rm th}_{\nu_\alpha})^2/k} \langle \nu_\alpha | \nu_I \rangle |^2$$

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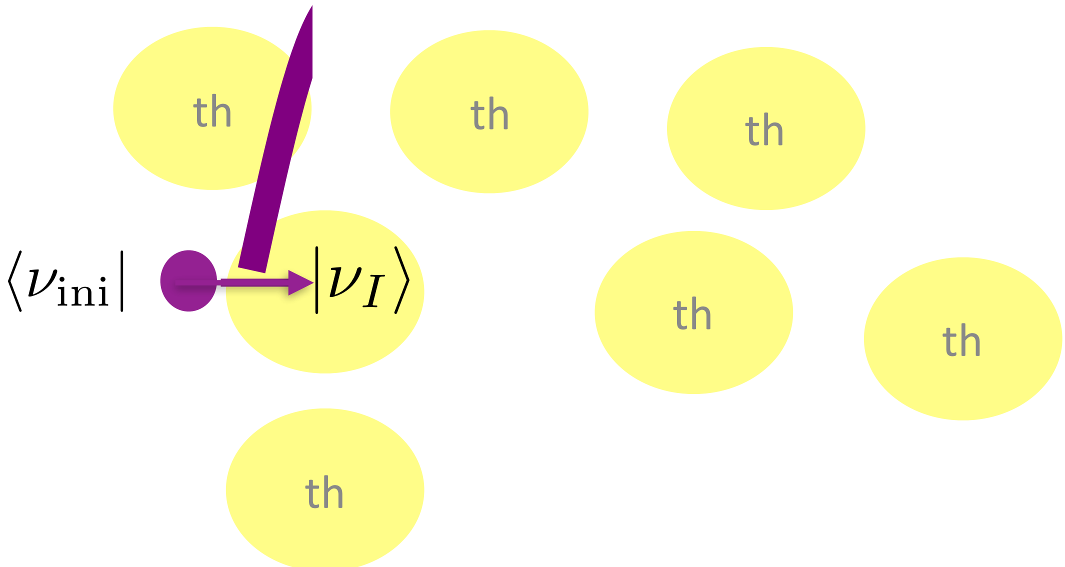

$$P_{{\rm ini}\to I} \simeq |\sum_\alpha \langle \nu_{\rm ini}|\nu_\alpha\rangle e^{it_{\rm MFP}(m^{\rm th}_{\nu_\alpha})^2/k}\langle \nu_\alpha|\nu_I\rangle|^2$$

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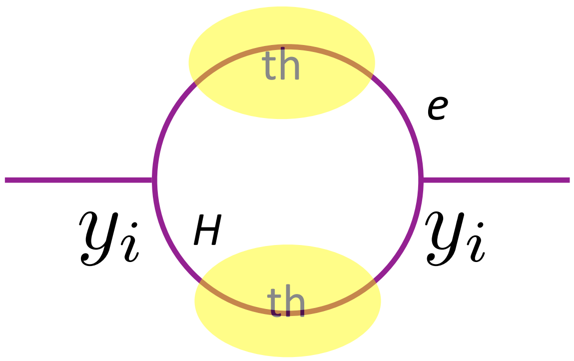
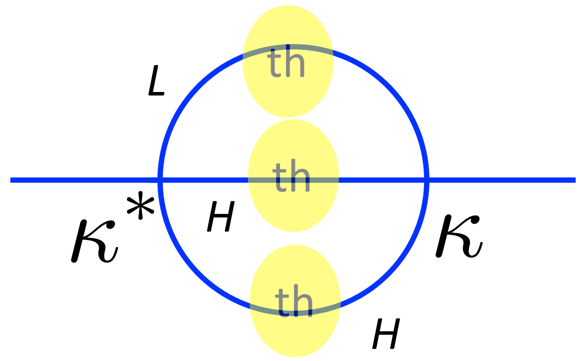
Oscillating frequency during reheating.

$$P_{\mathrm{ini}\to I} \simeq |\sum_{\alpha} \langle \nu_{\mathrm{ini}}|\nu_\alpha\rangle e^{it_{\mathrm{MFP}}(m^{\mathrm{th}}_{\nu_\alpha})^2/k}\langle\nu_\alpha|\nu_I\rangle|^2$$

$$(m^{\mathrm{th}}_{\nu,\alpha})^2 = \mathrm{eigen}[\frac{y_i^2T^2}{16}\delta_{ij} + 0.046(\kappa^*\kappa)_{ij}T^4] + C\delta_{ij}$$



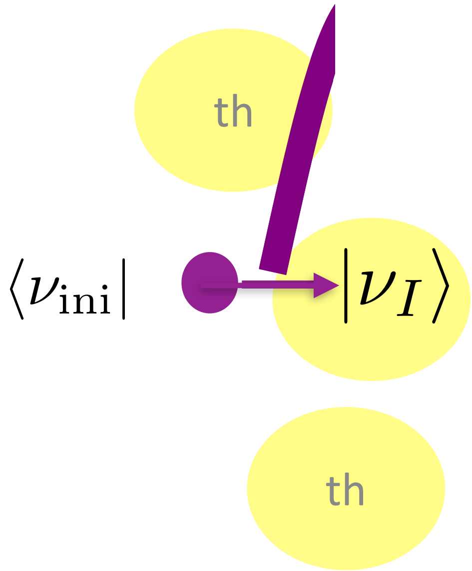


$|\nu_\alpha\rangle$ is the mass eigen state.

The mean free path of the neutrino.

$$P_{\mathrm{ini}\to I} \simeq |\sum_{\alpha} \langle \nu_{\mathrm{ini}} | \nu_\alpha \rangle e^{i t_{\mathrm{MFP}} (m^{\mathrm{th}}_{\nu_\alpha})^2 / k} \langle \nu_\alpha | \nu_I \rangle |^2$$

$$t_{MFP} \sim \frac{1}{\alpha_2^2 T}\sqrt{\frac{k}{T}} \qquad (m^{\mathrm{th}}_{\nu,\alpha})^2 = \mathrm{eigen}[\frac{y_i^2 T^2}{16}\delta_{ij} + 0.046(\kappa^* \kappa)_{ij} T^4] + C\delta_{ij}$$



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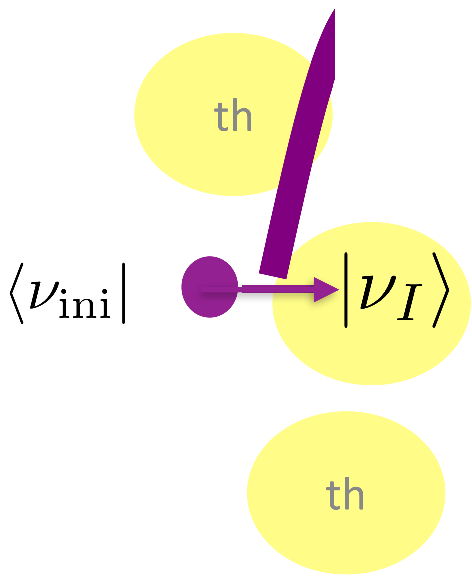


Neutrino Oscillation provides CP violation

$$P_{\text{ini}\to I} \simeq \Big| \sum_{\alpha} \langle \nu_{\text{ini}} | \nu_\alpha \rangle e^{i t_{\text{MFP}} (m^{\text{th}}_{\nu_\alpha})^2 / k} \langle \nu_\alpha | \nu_I \rangle \Big|^2$$

$$t_{MFP} \sim \frac{1}{\alpha_2^2 T} \sqrt{\frac{k}{T}}$$

$$(m^{\text{th}}_{\nu,\alpha})^2 = \text{eigen}[\frac{y_i^2 T^2}{16} \delta_{ij} + 0.046 (\kappa^* \kappa)_{ij} T^4] + C \delta_{ij}$$

$$P_{\text{ini}\to I} - P_{\overline{\text{ini}}\to \bar{I}} \propto \frac{\Delta (m^{\text{th}}_\nu)^2}{k} t_{MFP} \sim 0.01 \sqrt{T/k}$$

c.f. $P_{e\to\mu} - P_{\bar{e}\to\bar{\mu}} \propto \sin[t \Delta m_\nu^2 / k]$ @vacuum

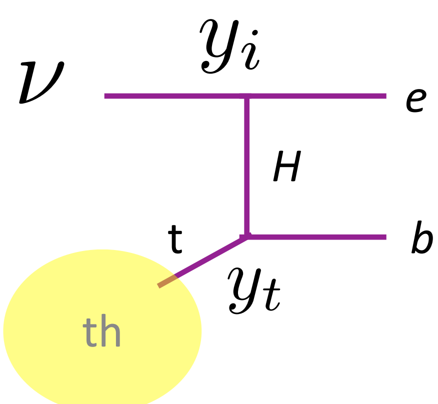
Oscillation phase is not too small at the reheating era.

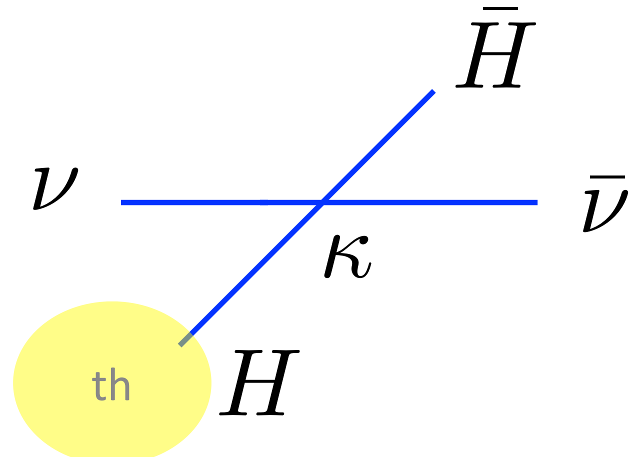
How to observe the "flavor"?

$$P_{\mathrm{ini}\to I} \simeq \left| \sum_{\alpha} \langle \nu_{\mathrm{ini}} | \nu_\alpha \rangle e^{i t_{\mathrm{MFP}} (m^{\mathrm{th}}_{\nu_\alpha})^2 / k} \langle \nu_\alpha | \nu_I \rangle \right|^2$$

Only flavor dependent process can identify the flavor.

"Observation" is made due to the following interaction process.





$|\nu_I\rangle$ is the state defined by the interaction.
The formalism will be given later.

Lepton number violation happens through "observation".

$$P_{\mathrm{ini}\to I} \simeq |\sum_\alpha \langle \nu_{\mathrm{ini}}|\nu_\alpha\rangle e^{it_{\mathrm{MFP}}(m_{\nu_\alpha}^{\mathrm{th}})^2/k}\langle\nu_\alpha|\nu_I\rangle|^2$$

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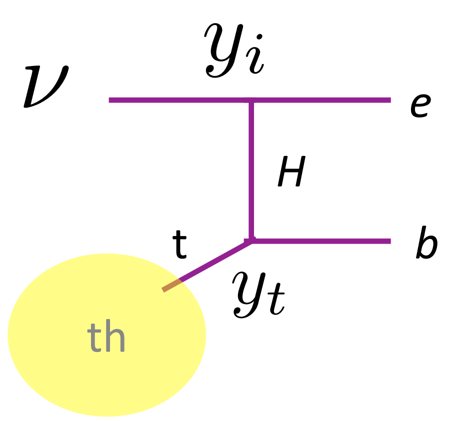
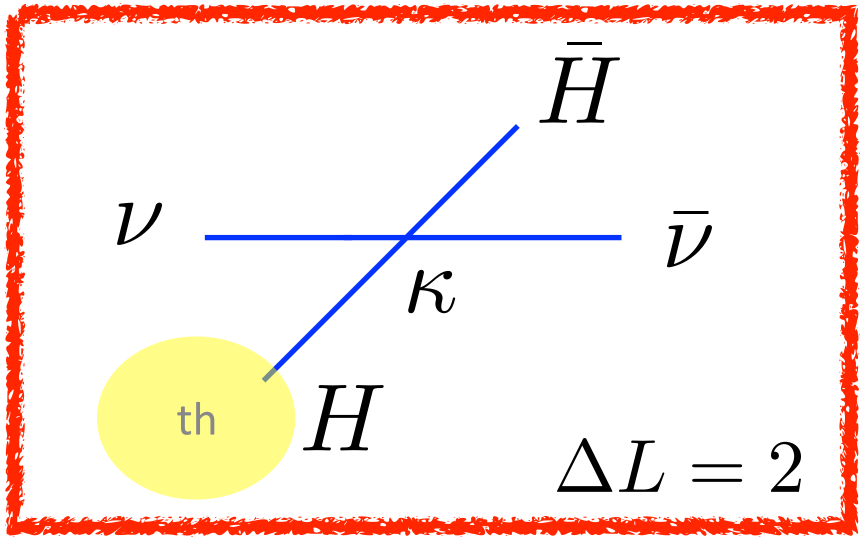
Thus, "observation" is made due to the following interaction process.





Lepton asymmetry can be made!

The (rough) estimation of lepton asymmetry

$m_\phi \sim T \ll 10^{12} GeV$ with certain CP phase

$$\frac{\Delta n_L}{s} \propto Br_{\phi \to \nu_{\rm ini} + X / \overline{\nu_{\rm ini}} + \overline{X}} \times t_{MFP} \frac{\Delta m_\nu^2}{T} \times \frac{\sigma^{\rm th}_{llHH}}{\sigma^{\rm th}_{\rm yukawa}}$$

CP violation (blue underline: $t_{MFP} \frac{\Delta m_\nu^2}{T}$) — lepton # violation (red underline: $\frac{\sigma^{\rm th}_{llHH}}{\sigma^{\rm th}_{\rm yukawa}}$)

Blue line: Flavor dependent asymmetry of order $\frac{\Delta m^2_{\rm th}}{T}\frac{1}{\Gamma_{\rm th}} \sim 0.01$

Red line: How frequently the flavor is observed by the llHH interaction.

$$\frac{\sigma^{th}_{llHH}}{\sigma^{th}_{\rm yukawa}} \sim \frac{\Delta m_\nu^2 / v^4 T^2}{y_\tau^2 y_t^2}$$

The (rough) estimation of lepton asymmetry

$m_\phi \sim T \ll 10^{12} GeV$

$$\frac{\Delta n_L}{s} \propto Br_{\phi \to \nu_{\rm ini} + X / \bar{\nu}_{\rm ini} + \bar{X}} \times 10^{-9} \left(\frac{T_R}{10^9 GeV} \right)^2$$

c.f. required asymmetry $$|\Delta n_L / s| \sim 10^{-10}$$

Enough asymmetry can be made for sufficiently high reheat temperature.

However, for a precise prediction or more complicated processes, we need a systematic approach.

3. Kinetic Equation (Extended Boltzmann Eqs)

density matrix for left-handed leptons $\rho(\mathbf{p}) \equiv \rho_{ij}(\mathbf{p})$ $i, j = e, \mu, \tau$

Sigl, Raffelt, 1993

$$i\frac{d\rho(\mathbf{p})}{dt} = [\Omega(\mathbf{p}), \rho(\mathbf{p})] - \frac{i}{2}\{\Gamma^d_{\mathbf{p}}, \rho(\mathbf{p})\} + \frac{i}{2}\{\Gamma^p_{\mathbf{p}}, 1 - \rho(\mathbf{p})\},$$

Oscillation term Interaction terms (with CP phase)

$$\left(i\frac{d\overline{\rho}(\mathbf{p})}{dt} = - [\Omega(\mathbf{p}), \bar{\rho}(\mathbf{p})] - \frac{i}{2}\{\Gamma^d_{\mathbf{p}}, \overline{\rho}(\mathbf{p})\} + \frac{i}{2}\{\Gamma^p_{\mathbf{p}}, 1 - \overline{\rho}(\mathbf{p})\}, \right)^*$$

Hamiltonian: $$\Omega_{ij}(\mathbf{p}) \simeq \frac{y_i^2 T^2}{16|\mathbf{p}|}\delta_{ij} + 0.046(\kappa^*\kappa)_{ij}\frac{T^4}{|\mathbf{p}|}, \quad \text{for } |\mathbf{p}| \gtrsim T.$$

This is absent in ordinary Boltzmann eqs.

Equations to be solved

$$i\frac{d\rho_{\mathbf{k}}}{dt} = [\Omega_{\mathbf{k}}, \rho_{\mathbf{k}}] - \frac{i}{2}\{\Gamma^d_{\mathbf{k}}, \rho_{\mathbf{k}}\},$$

$$i\frac{d\delta\rho_T}{dt} = [\Omega_T, \delta\rho_T] - \frac{i}{2}\{\Gamma^d_T, \delta\rho_T\} + i\delta\Gamma^p_T,$$

+ eqs of right-handed/anti leptons

$$\Omega_k = \frac{y_i^2 T^2}{16 m_\phi}\delta_{ij} + 0.046(\kappa^*\kappa)_{ij}\frac{T^4}{m_\phi} \qquad \Omega_T = \frac{y_i^2 T}{16}\delta_{ij} + 0.046(\kappa^*\kappa)_{ij} T^3$$

$$\left(\Gamma^d_{\mathbf{k}}\right)_{ij} \simeq C\alpha_2^2 T\sqrt{\frac{T}{|\mathbf{k}|}}\delta_{ij} + \frac{9y_t^2}{64\pi^3|\mathbf{k}|}T^2(\delta_{i\tau}\delta_{\tau j}y_\tau^2 + \delta_{i\mu}\delta_{\mu j}y_\mu^2) + \frac{21\zeta(3)}{32\pi^3}(\kappa^*\cdot\kappa)_{ij}T^3,$$

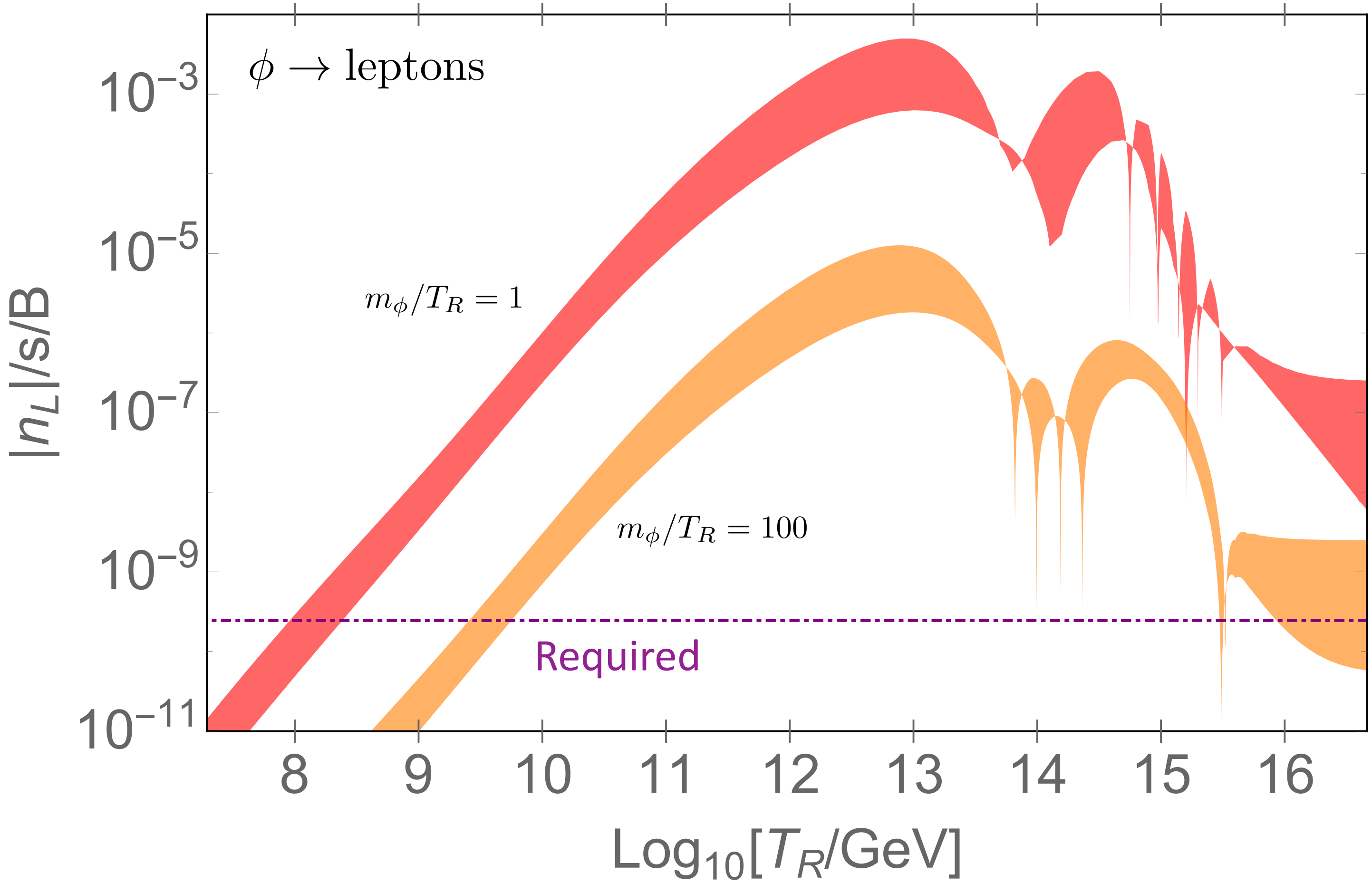
$$\left(\Gamma^d_T\right)_{ij} \simeq C'\alpha_2^2 T\delta_{ij} + \frac{9y_t^2}{64\pi^3}T(\delta_{i\tau}\delta_{\tau j}y_\tau^2 + \delta_{i\mu}\delta_{\mu j}y_\mu^2) + \frac{21\zeta(3)}{32\pi^3}(\kappa^*\cdot\kappa)_{ij}T^3,$$

$$(\delta\Gamma^p_T)_{ij} \simeq C\alpha_2^2 T\sqrt{\frac{T}{|\mathbf{k}|}}\,(\rho_{\mathbf{k}})_{ij} - C'\alpha_2^2 T\,(\delta\bar{\rho}_T)_{ij}$$
$$+ \frac{3\zeta(3)}{8\pi^3}(\kappa^*\cdot(\bar{\rho}_{\mathbf{k}} - 3/4\rho_{\mathbf{k}})^t\cdot\kappa)_{ij}T^3 + \frac{3\zeta(3)}{8\pi^3}(\kappa^*\cdot(\delta\bar{\rho}_T - 3/4\delta\rho_T)^t\cdot\kappa)_{ij}T^3.$$

Some formula can be also found in Akhmedov,et al. 9803255; Abada, et al. 0601083.; Asaka, et al. 1112.5565.
See Refs. [Landau and Pomeranchuk 1953; Migdal 1956] for LPM effect.

Solution (Normal Hierarchy)

By solving kinetic equations for leptons, we get



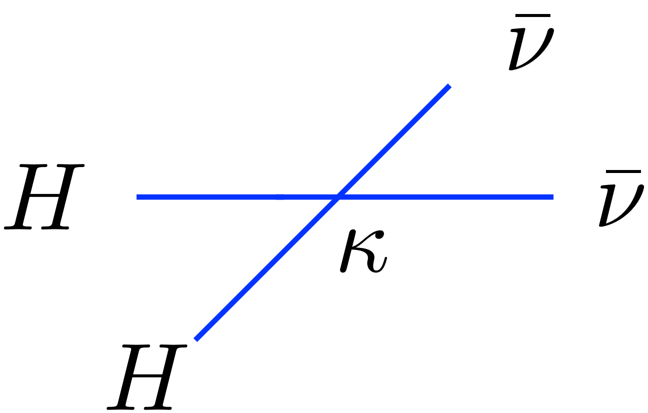
Dirac phase=-pi/2, Majorana phase=0.3 pi, V∝{1,1,1}

This scenario works for $T_R \gtrsim 10^8\, GeV$

CASE B: Inflaton dominantly to Higgs

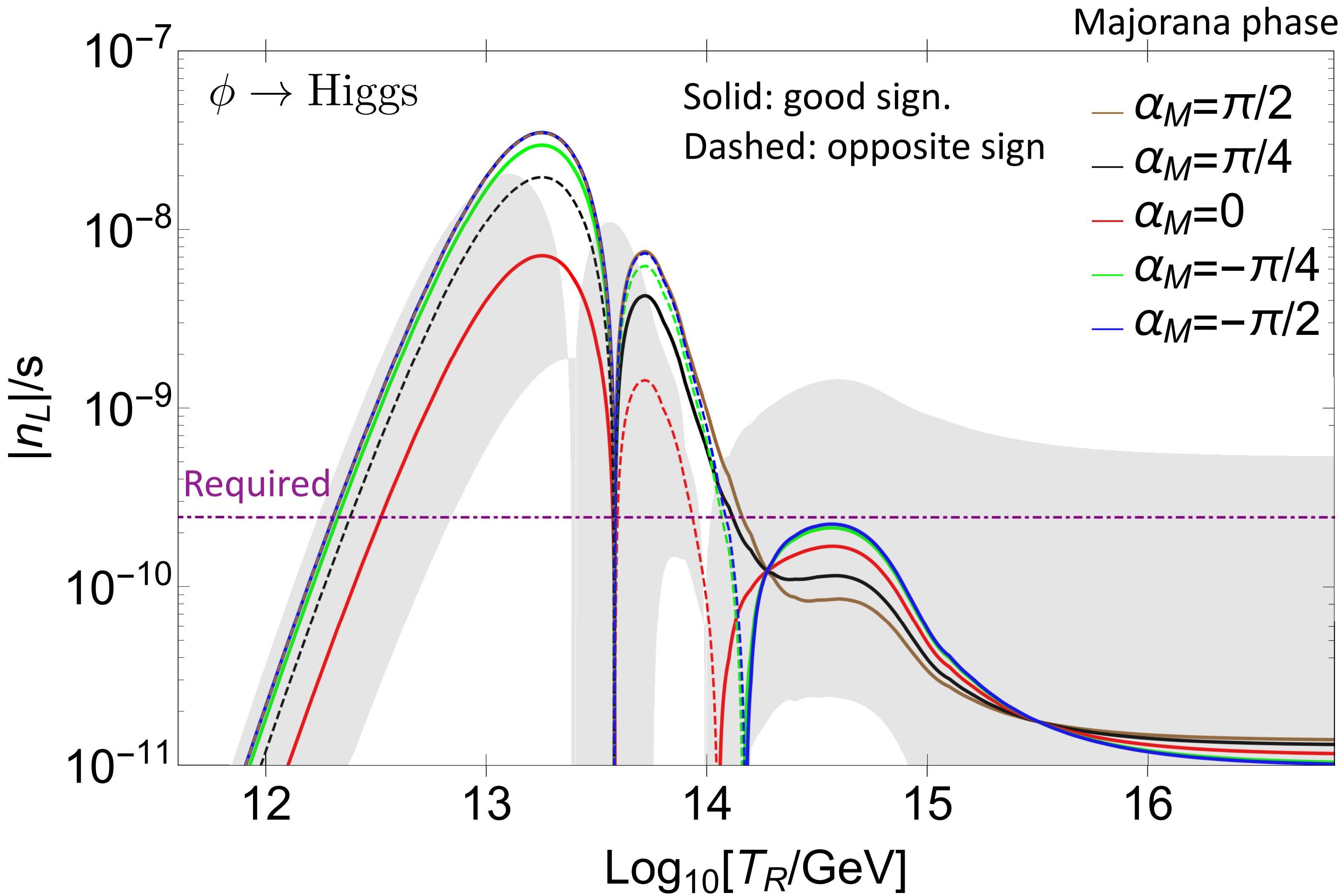
$$\mathcal{L} \supset A\phi|H|^2$$

$$\phi \to HH^*$$



$\rightarrow$ Flavor oscillation

Solution (Normal Hierarchy)

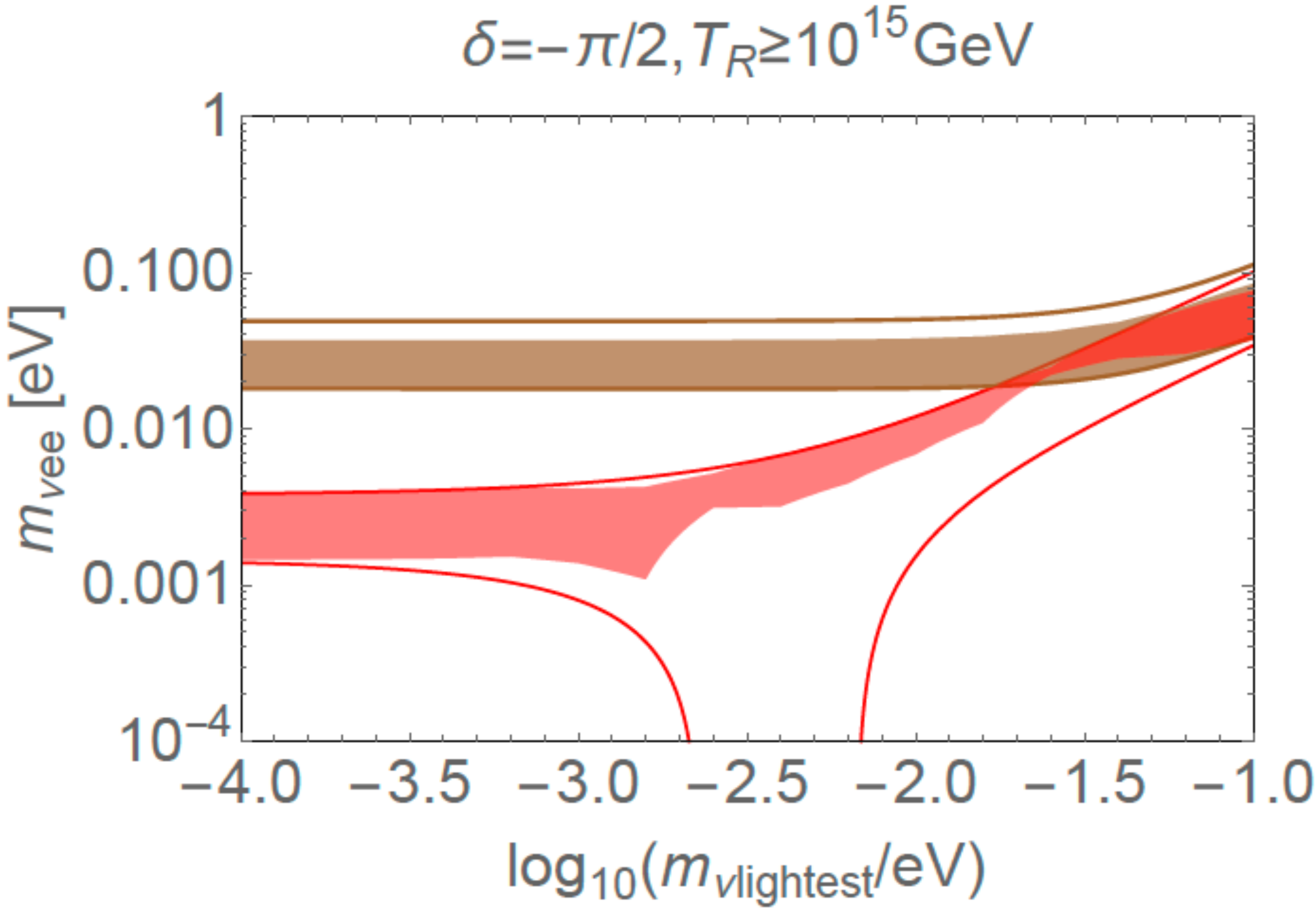


Inflaton mass =100 T_R

Two massive neutrinos. Dirac Phase=-Pi/2

Neutrinoless double beta decay

The CP phase and neutrino mass can be tested from neutrino exps.



Summary

Baryon asymmetry explained within SM with

$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \text{h.c.}$$

- The observed asymmetry can be explained at reheat temp > $10^8\ GeV$

- The scenario can be tested in future neutrino exps, especially for inflaton dominantly decays to Higgs.

Backups

CASE A

Inflaton $\longrightarrow$ $$V^e|\,l_e\rangle + V^\mu|\,l_\mu\rangle + V^\tau|\,l_\tau\rangle\,,$$ $$(V^e)^*|\,\bar{l}_e\rangle + (V^\mu)^*|\,\bar{l}_\mu\rangle + (V^\tau)^*|\,\bar{l}_\tau\rangle$$

Initial condition

$$\rho_{\mathbf{k}}|_{t=t_R} = \bar{\rho}_{\mathbf{k}}|_{t=t_R} = \mathcal{N} V_i V_j^*, \quad \delta\rho_T|_{t=t_R} = \delta\bar{\rho}_T|_{t=t_R} = 0.$$

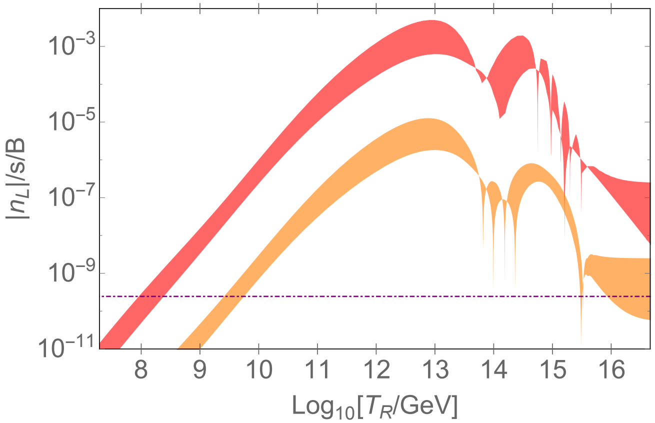
$$\mathcal{N} = \frac{3}{4}\frac{T_R}{m_\phi}B,$$

Parameters

$$T_R, m_\phi, B, V_i$$

and those for neutrino oscillation.

Analytic Formula T_R<10^13 GeV



This scenario works for

$$T_R \gtrsim 10^8\, GeV$$

$$\frac{n_L}{s} \sim -2 \times 10^{-6} \cdot \xi_{CP} \cdot B \cdot \left(\frac{T_R}{10^{11}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right)^{-1}, \qquad (T_R \gtrsim 10^{11}\ \mathrm{GeV}),$$

and

$$\frac{n_L}{s} \sim -2 \times 10^{-6} \cdot \xi_{CP} \cdot B \cdot \left(\frac{T_R}{10^{11}\ \mathrm{GeV}}\right)^3 \left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right)^{-1}, \qquad (T_R \lesssim 10^{11}\ \mathrm{GeV}),$$

Baryon asymmetry could be explained by ***any model***, with

$$\mathcal{L} = \mathcal{L}_{SM} + \kappa llHH + \ldots \quad @\ \mu_{RG} > 10^8\mathrm{GeV}$$

CASE B

$$\mathcal{L} \supset A\phi|H|^2$$

Inflaton $\longrightarrow$ Higgs, Higgs* $\longrightarrow$ $l, \bar{l}$

Initial conditions

$$(\rho_{\mathbf{k}})_{ij} = (\bar{\rho}_{\mathbf{k}})_{ij} = \mathcal{N} \cdot \frac{\frac{21\zeta(3)}{32\pi^3}(\kappa^*\kappa)_{ij}T_R^3}{C\alpha_2^2 T_R\sqrt{\frac{T_R}{m_\phi}}} \sim 7\times 10^{-2}\cdot\mathcal{N}\left(\frac{m_\phi}{10^{15}\ \mathrm{GeV}}\right)^{1/2}\left(\frac{T_R}{10^{13}\ \mathrm{GeV}}\right)^{3/2}.$$

$\rho_T\big|_{t=t_R} = \bar{\rho}_T\big|_{t=t_R} = 0$ (Leptons are not thermalized initially)

Parameters

$$T_R, m_\phi$$

and those for neutrino oscillation.

Not complete washout

with degenerate case

$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \text{h.c.}$$

$$\kappa \simeq 1/v^2 \text{diag}[m_\nu, m_\nu, m_\nu]$$

$SO(3)$ sym.

Conserved current for lepton flavor sym.
Certain flavor dependent asymmetry remains.

Even with very high Temp, conserved current protects lepton asymmetry!

Oscillation phase

$$t_{osc}^{-1} \sim \frac{y_\tau^2}{16} T \sim 10^7 \left(\frac{T}{10^{12} GeV} \right) GeV$$

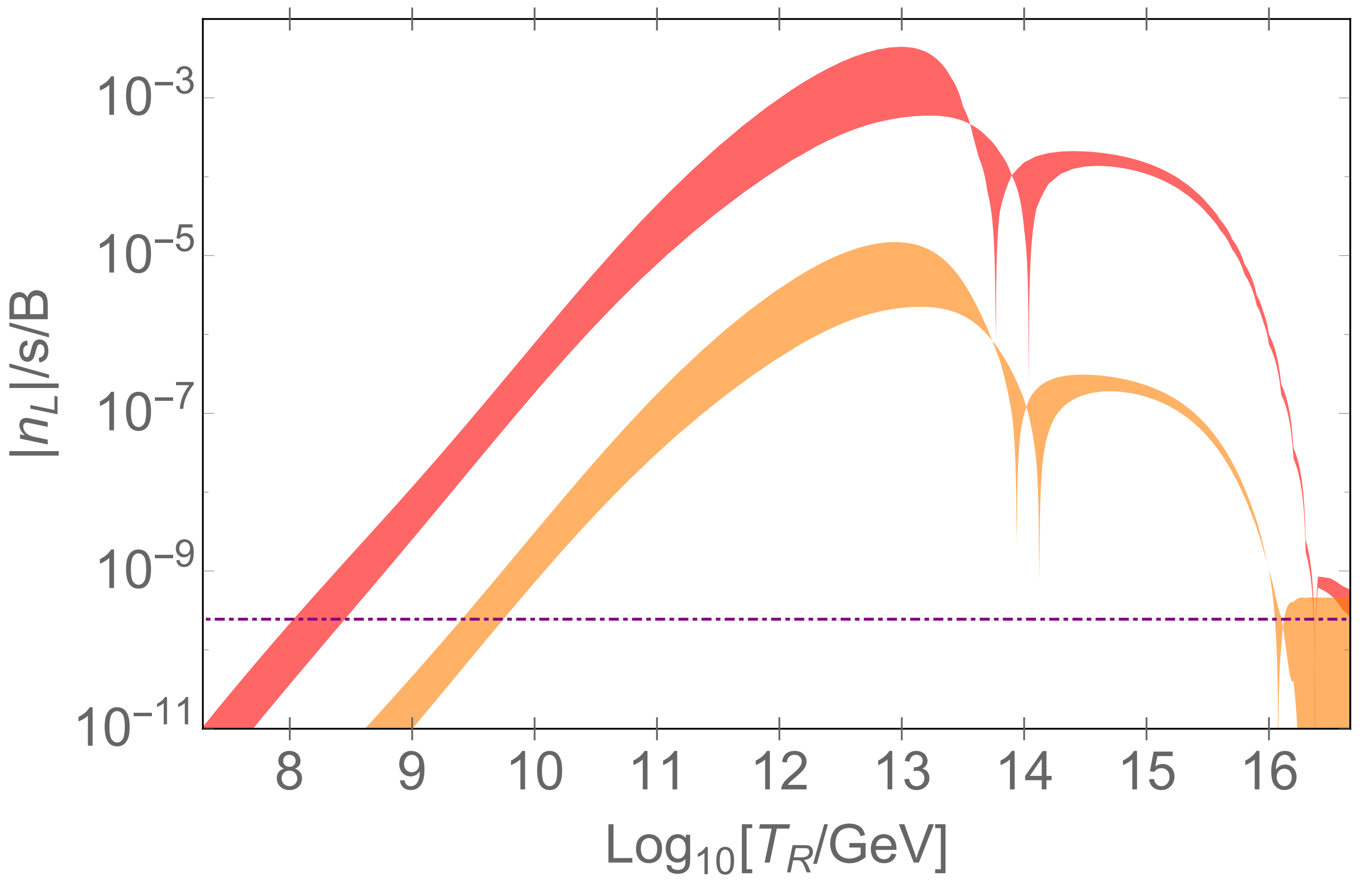
$$t_{scat}^{-1} \sim \alpha_2^2 T \sim 10^9 \left(\frac{T}{10^{12} GeV} \right) GeV$$

$$\mathrm{Phase} \sim t_{osc}^{-1} t_{scat} \ll 1$$

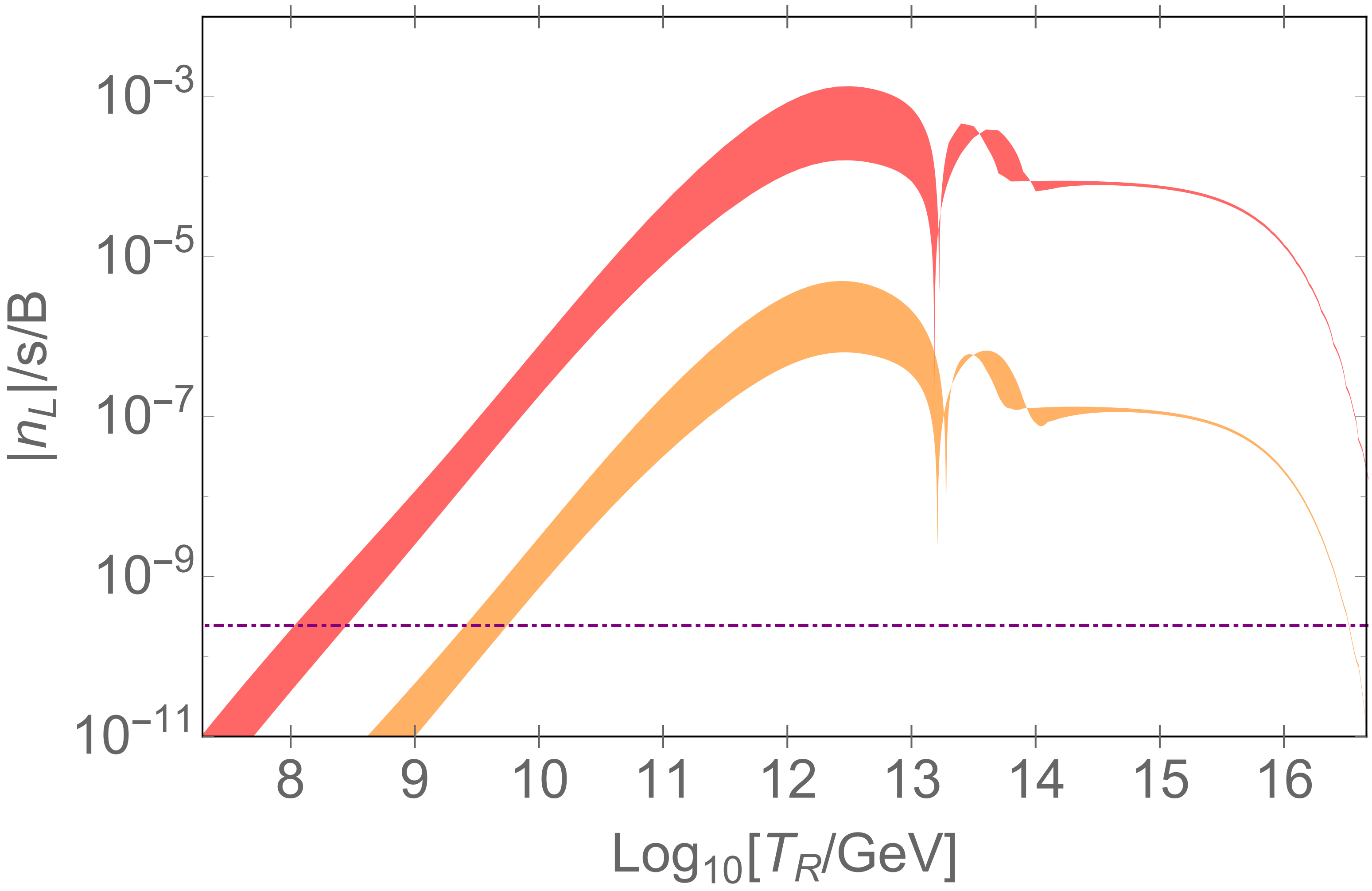
Coherently scattering (LPM effect) becomes important and it is included in the analysis.

FIGURES FOR INFLATON->LEPTON

Inverted hierarchy one massless neutrino.
Other parameters are same as the main part one.



normal order degenerate mass. mnulightest=0.07eV
Other parameters are same as the main part one.



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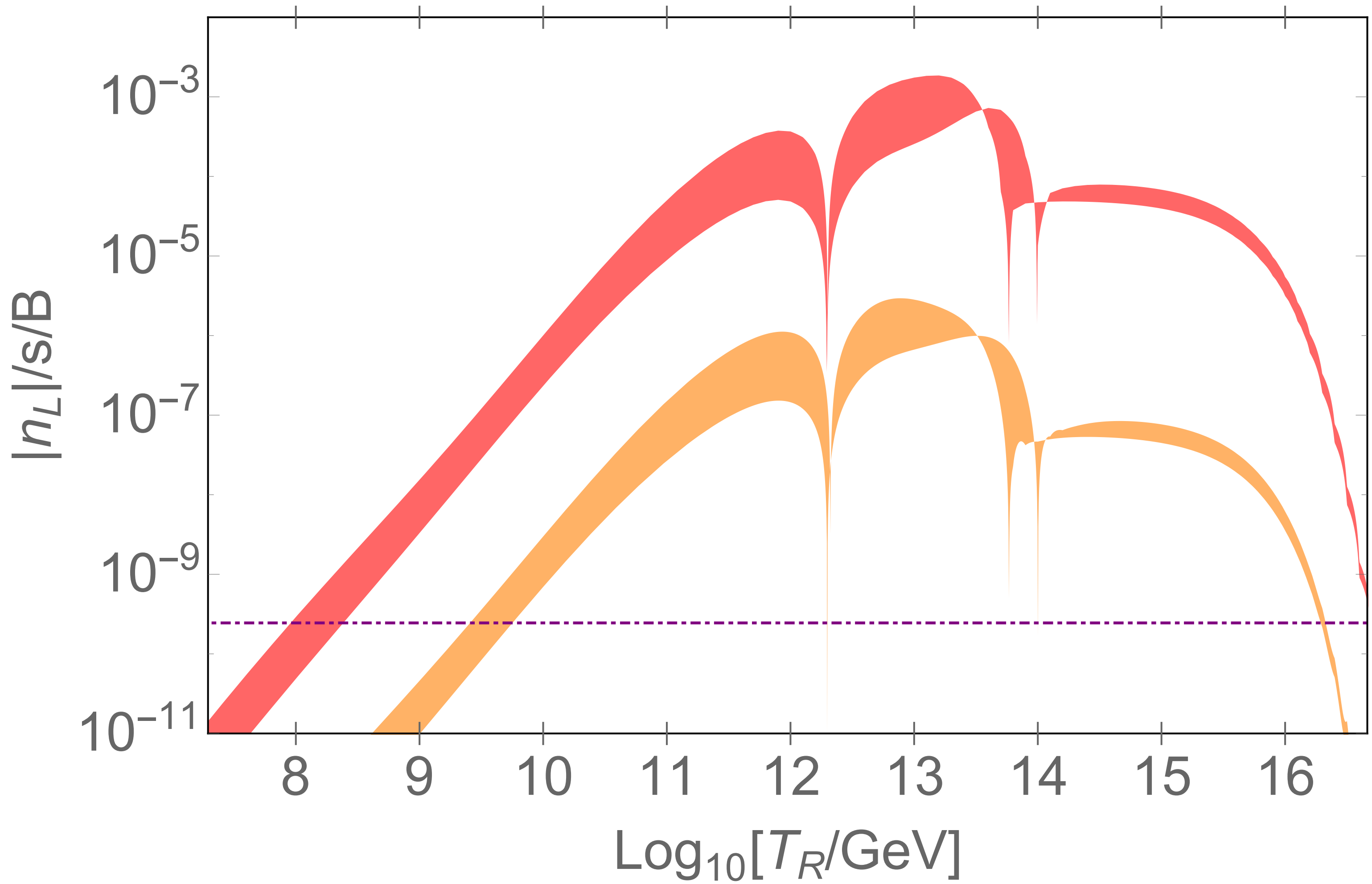
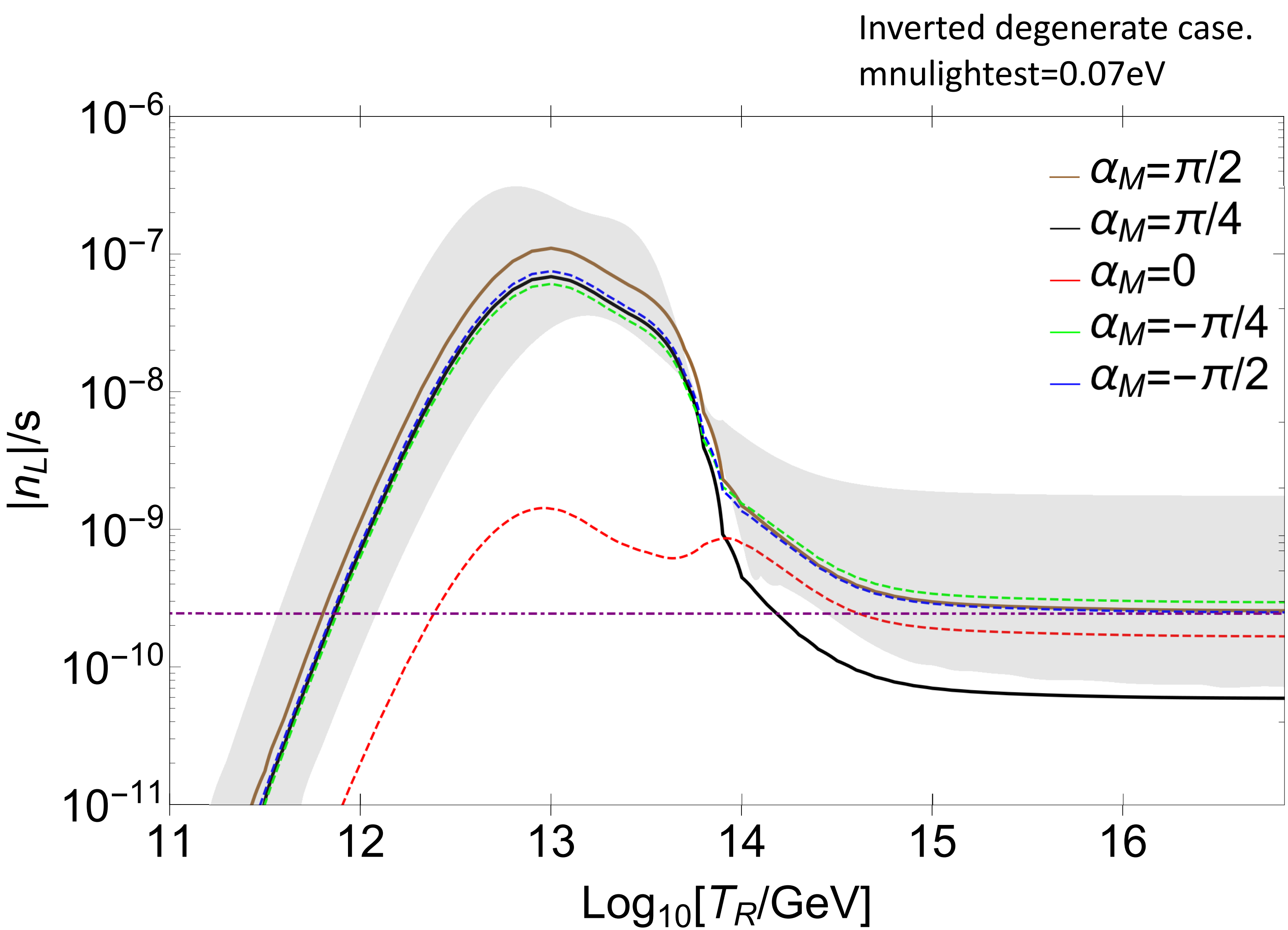
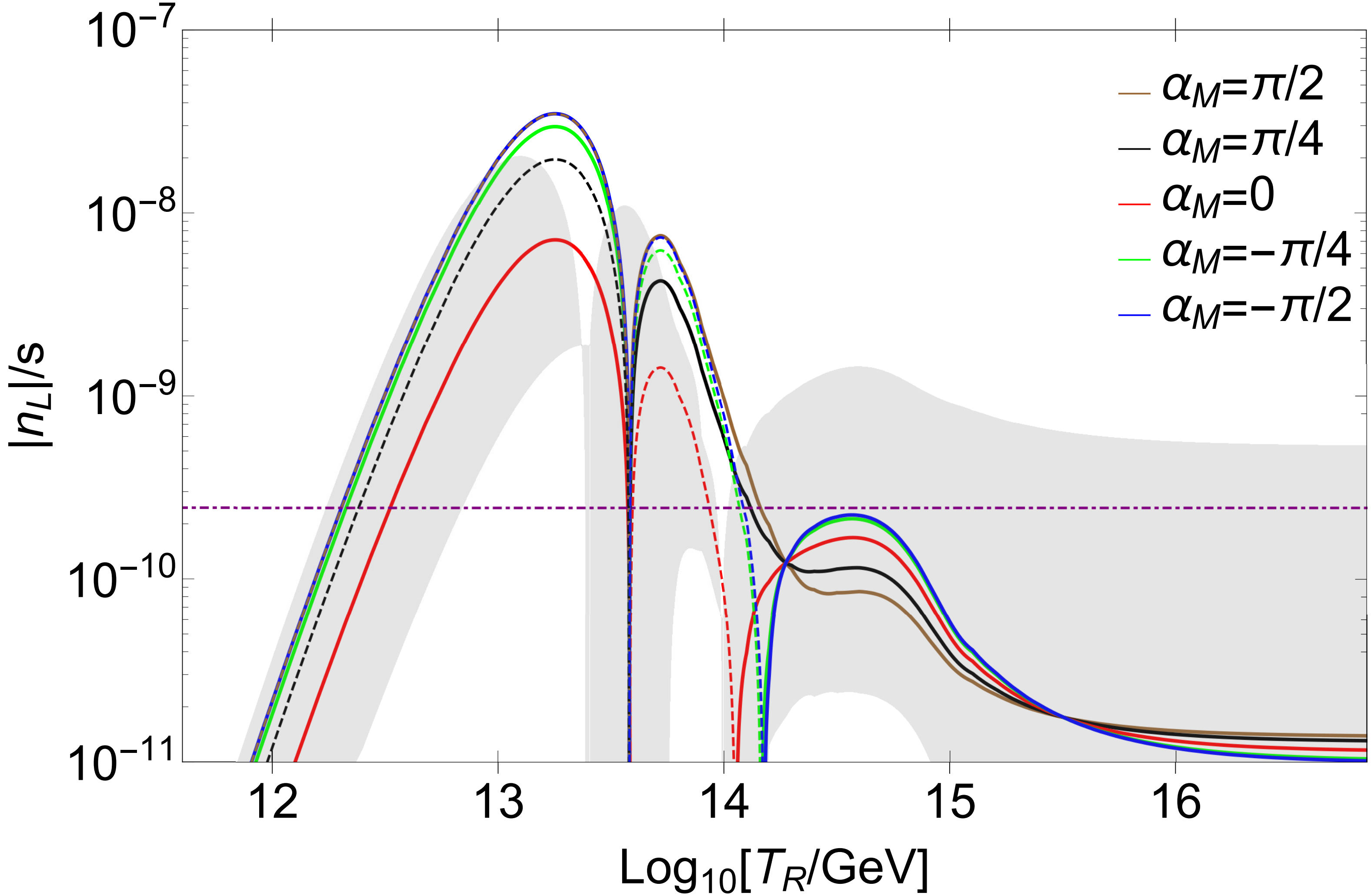


FIGURES FOR INFLATON->HIGGS



Inverted degenerate case.
mnulightest=0.07eV
$\alpha_M=\pi/2$
$\alpha_M=\pi/4$
$\alpha_M=0$
$\alpha_M=-\pi/4$
$\alpha_M=-\pi/2$
$|n_L|/s$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
11
12
13
14
15
16
$\mathrm{Log}_{10}[T_R/\mathrm{GeV}]$

Normal hierarchy one massless neutrino.



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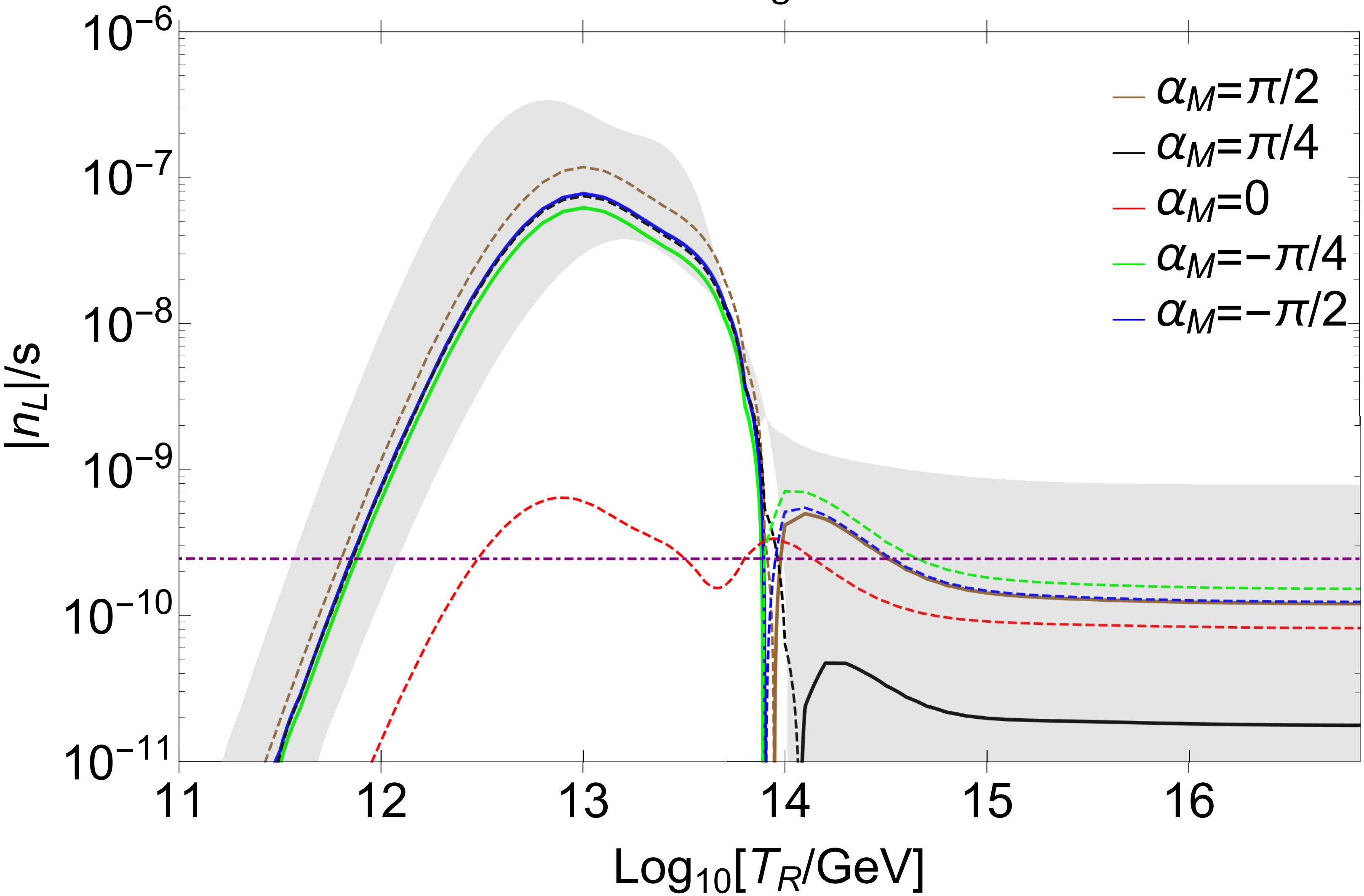
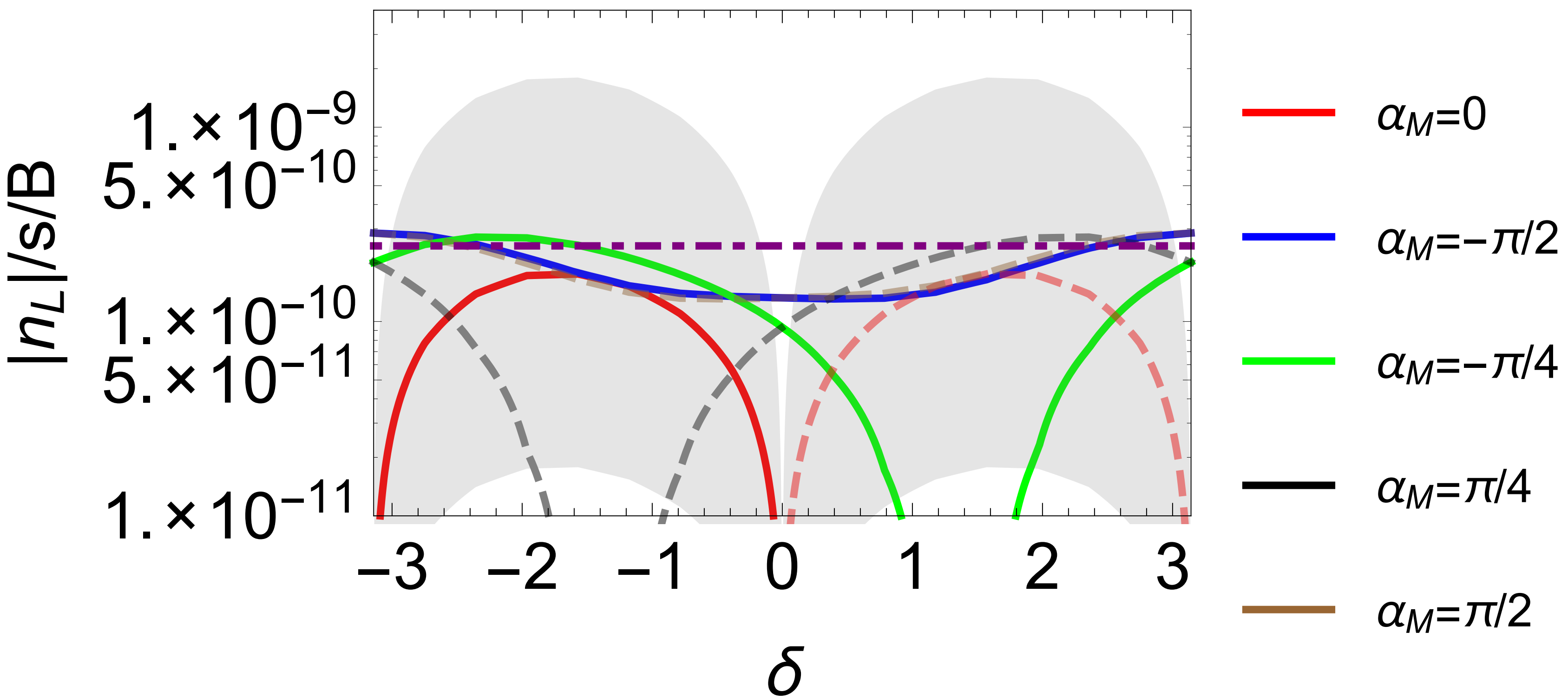
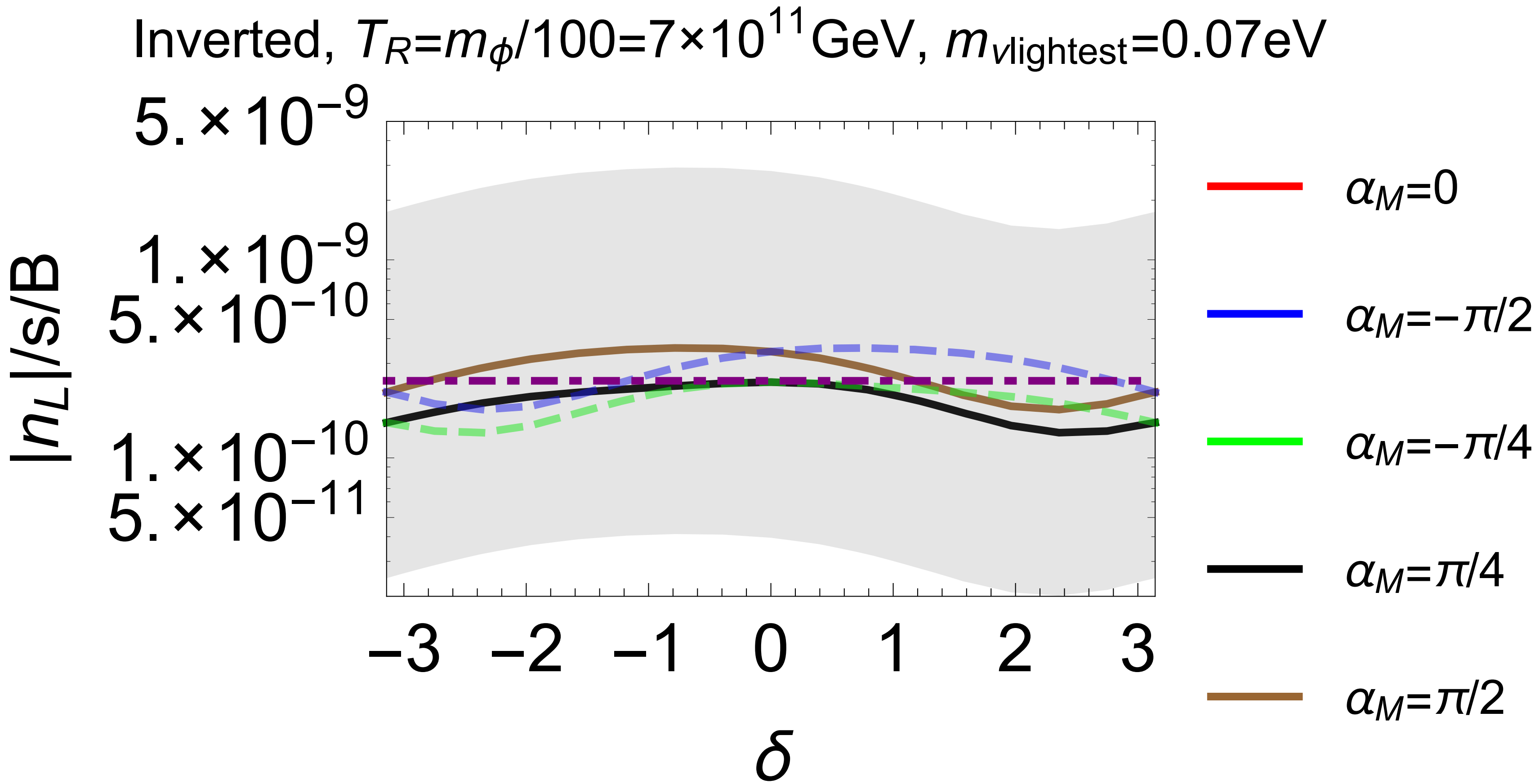
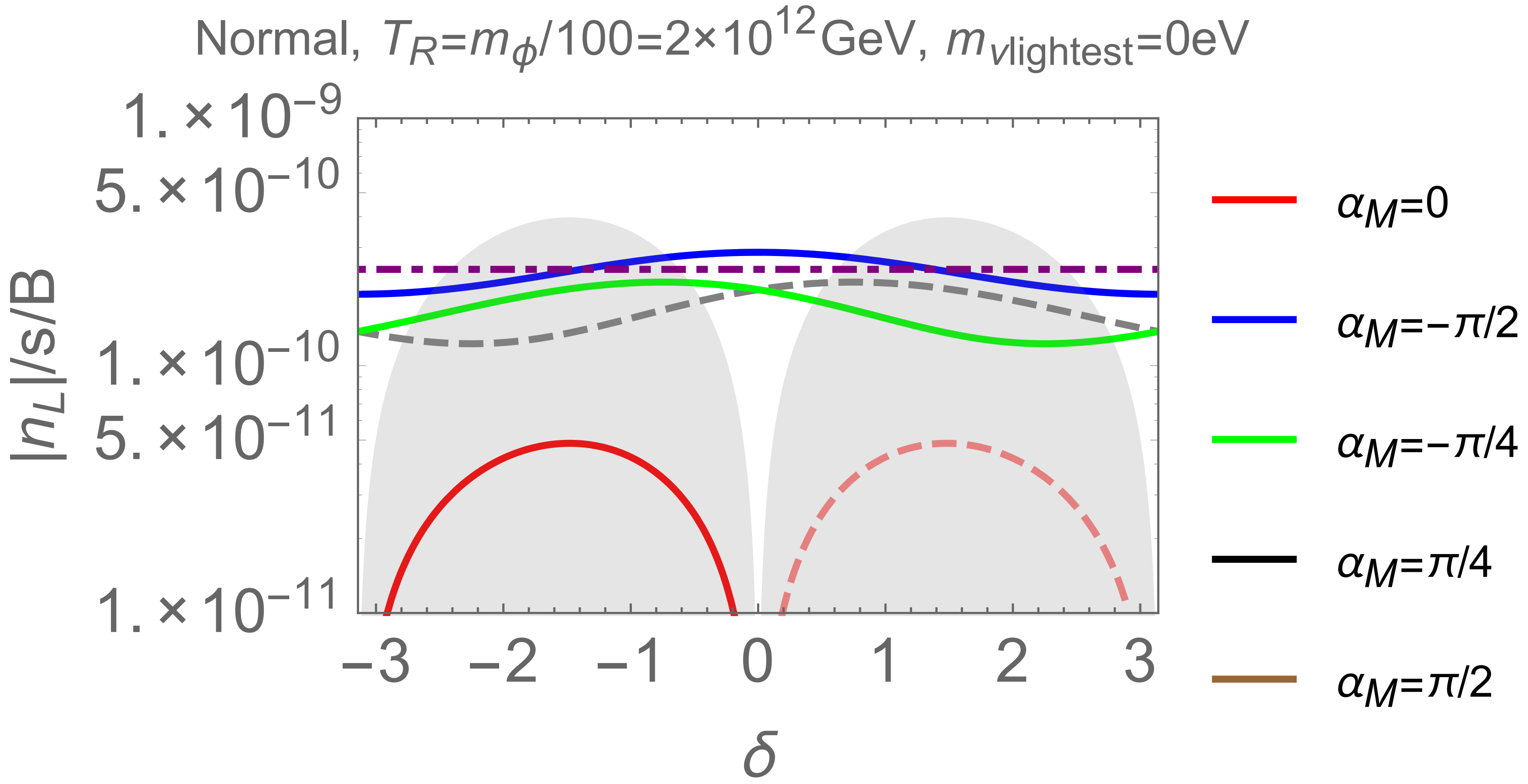


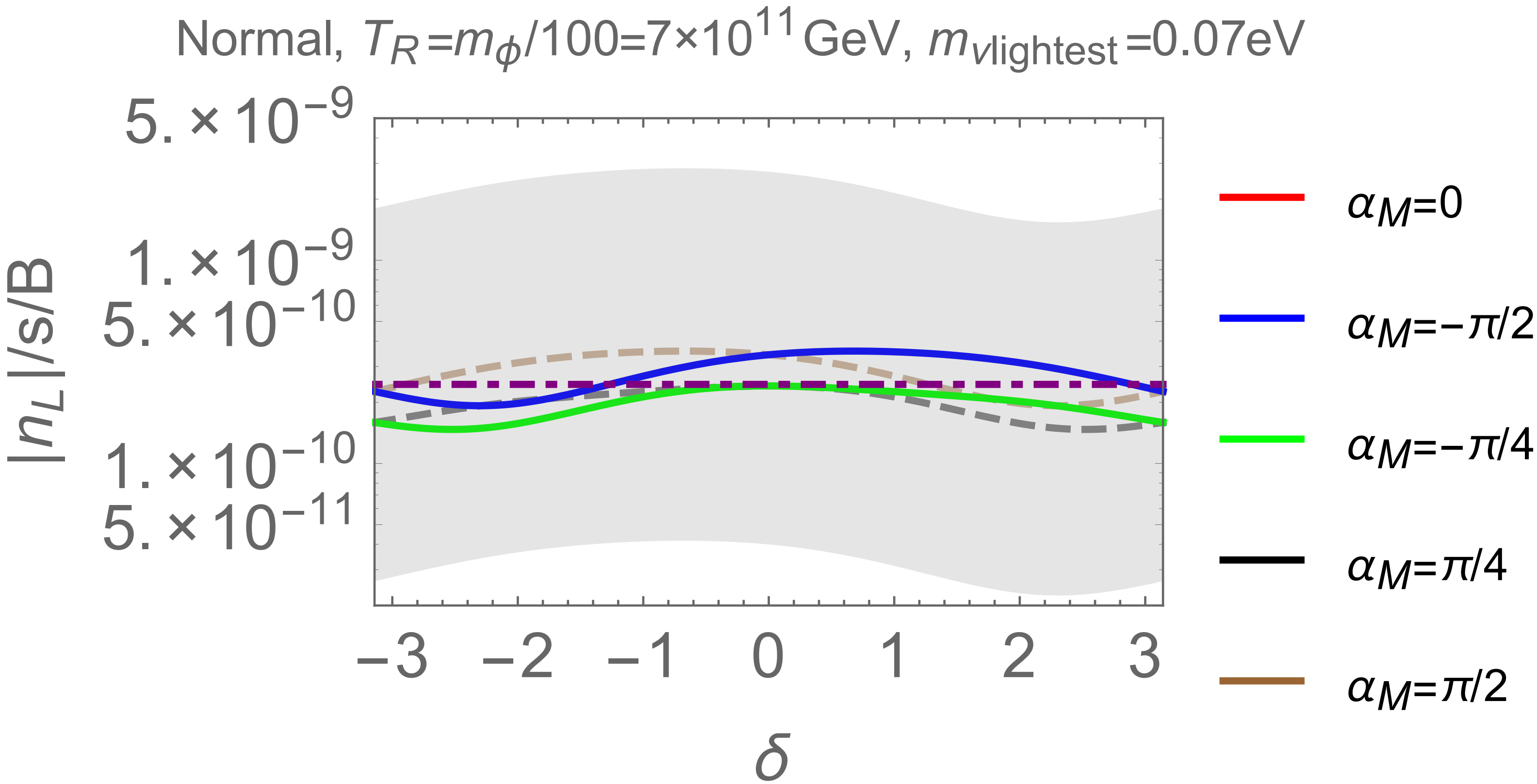
Inverted, $T_R = m_\phi/100 = 4\times10^{12}$GeV, $m_{\nu\text{lightest}}$=0eV



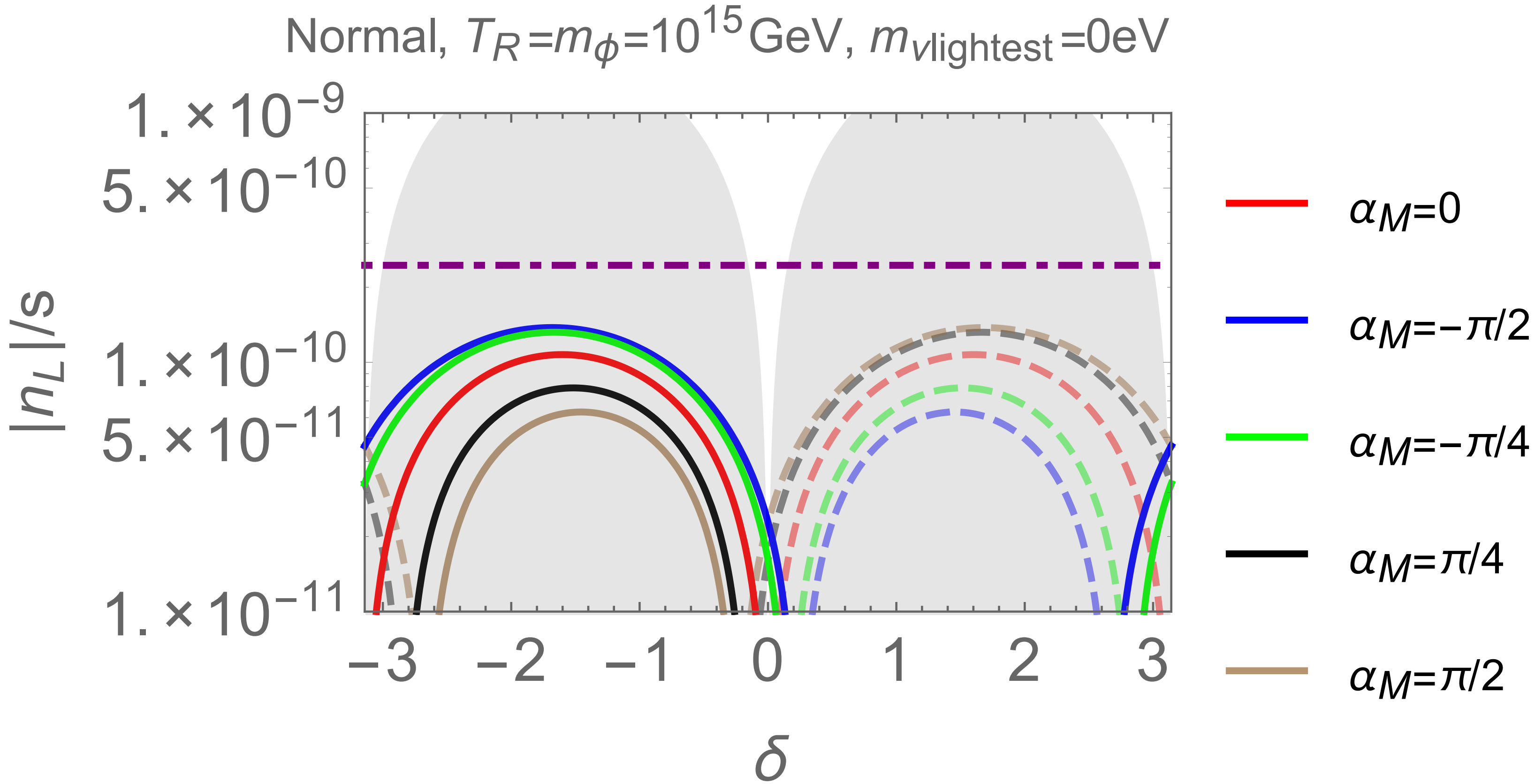

Inverted, $T_R = m_\phi/100 = 7\times10^{11}$GeV, $m_{\nu\mathrm{lightest}} = 0.07$eV
$5.\times10^{-9}$
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$|n_L|/s/B$
−3
−2
−1
0
1
2
3
δ
$\alpha_M=0$
$\alpha_M=-\pi/2$
$\alpha_M=-\pi/4$
$\alpha_M=\pi/4$
$\alpha_M=\pi/2$



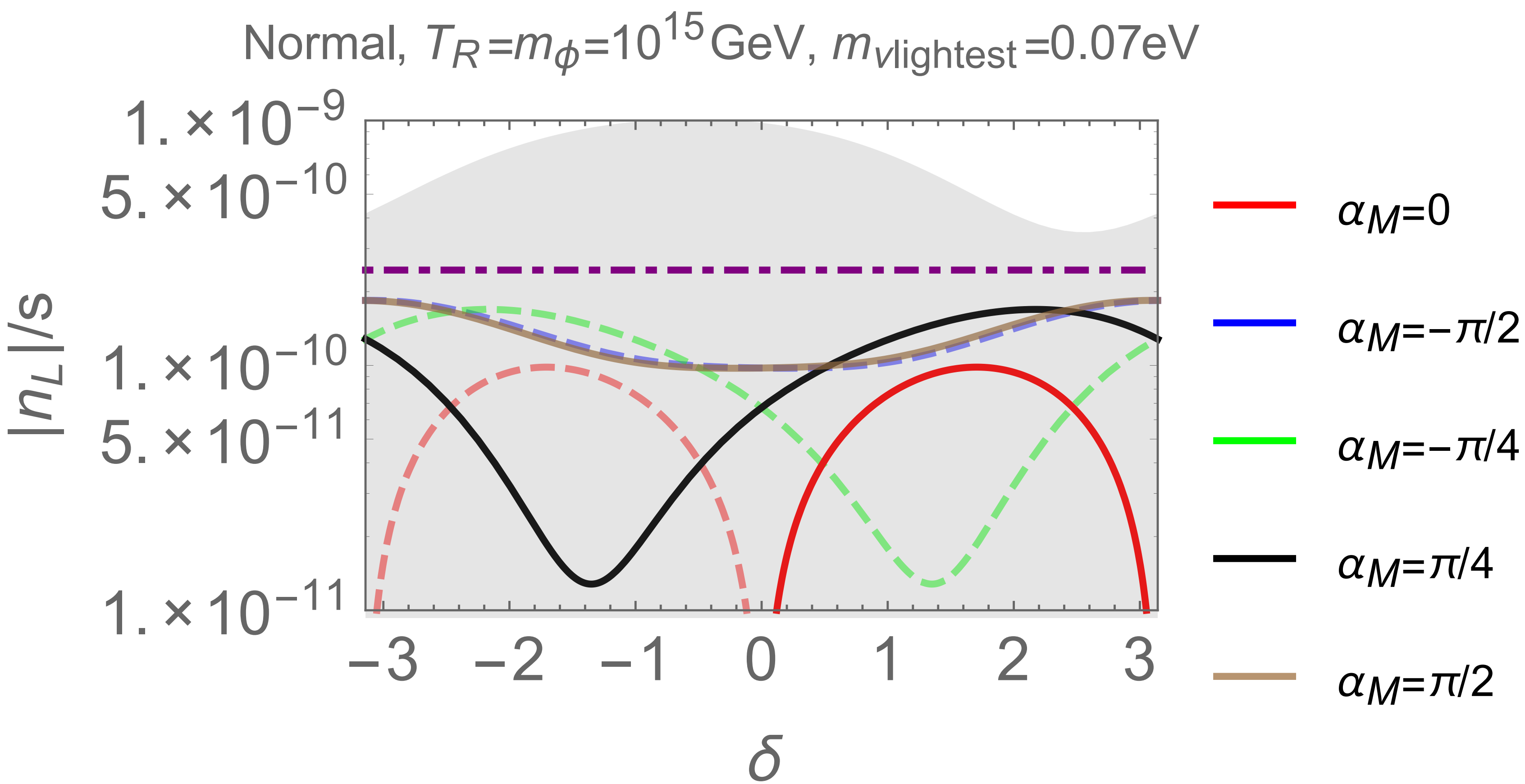
Normal, $T_R = m_\phi/100 = 2\times10^{12}$GeV, $m_{\nu\mathrm{lightest}}$=0eV
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$1.\times10^{-11}$
$|n_L|/s/B$
−3
−2
−1
0
1
2
3
δ
α_M=0
α_M=−π/2
α_M=−π/4
α_M=π/4
α_M=π/2



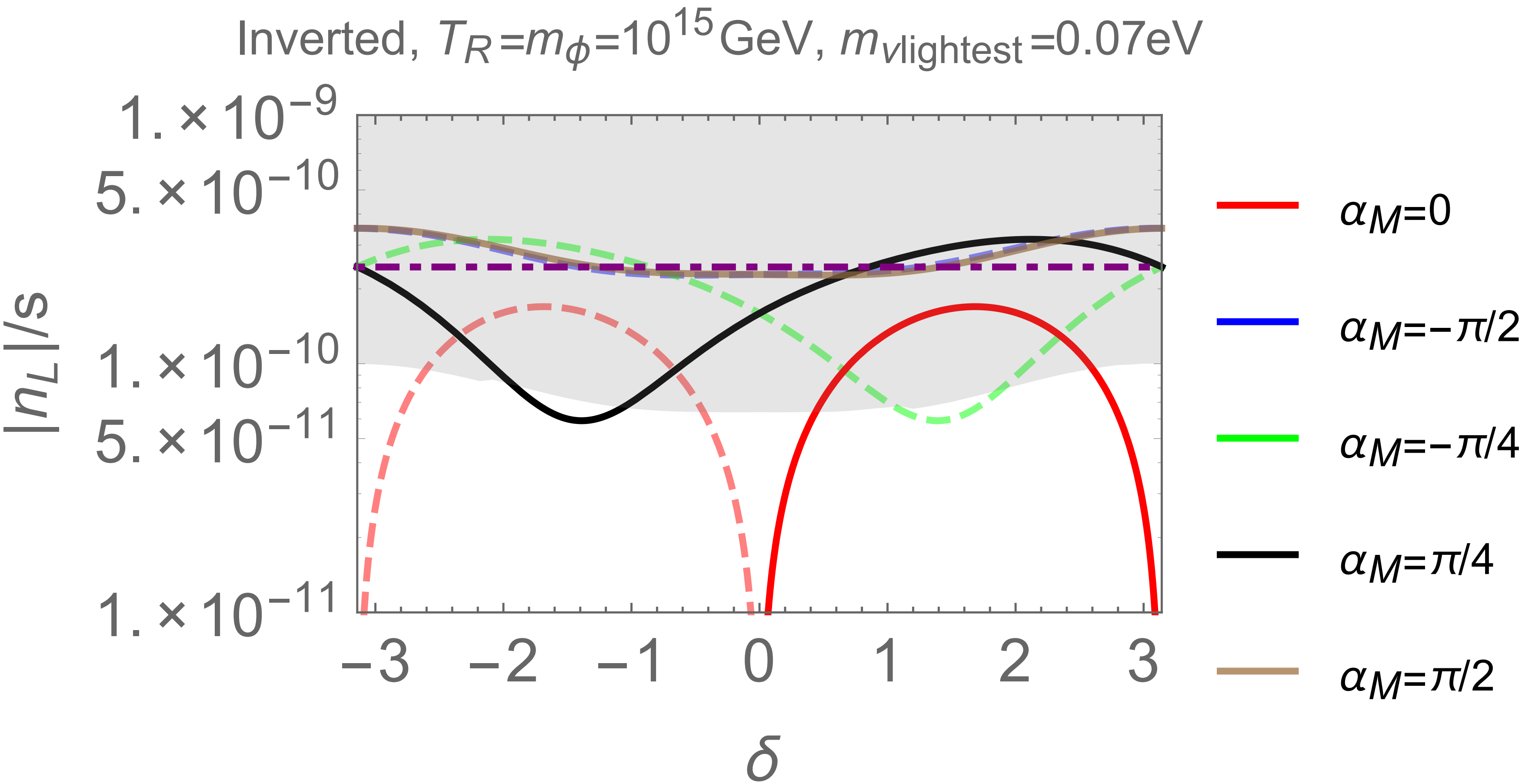
Normal, $T_R = m_\phi/100 = 7\times10^{11}$ GeV, $m_{\nu\text{lightest}} = 0.07$eV
$5.\times10^{-9}$
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$|n_L|/s/B$
−3
−2
−1
0
1
2
3
δ
$\alpha_M=0$
$\alpha_M=-\pi/2$
$\alpha_M=-\pi/4$
$\alpha_M=\pi/4$
$\alpha_M=\pi/2$



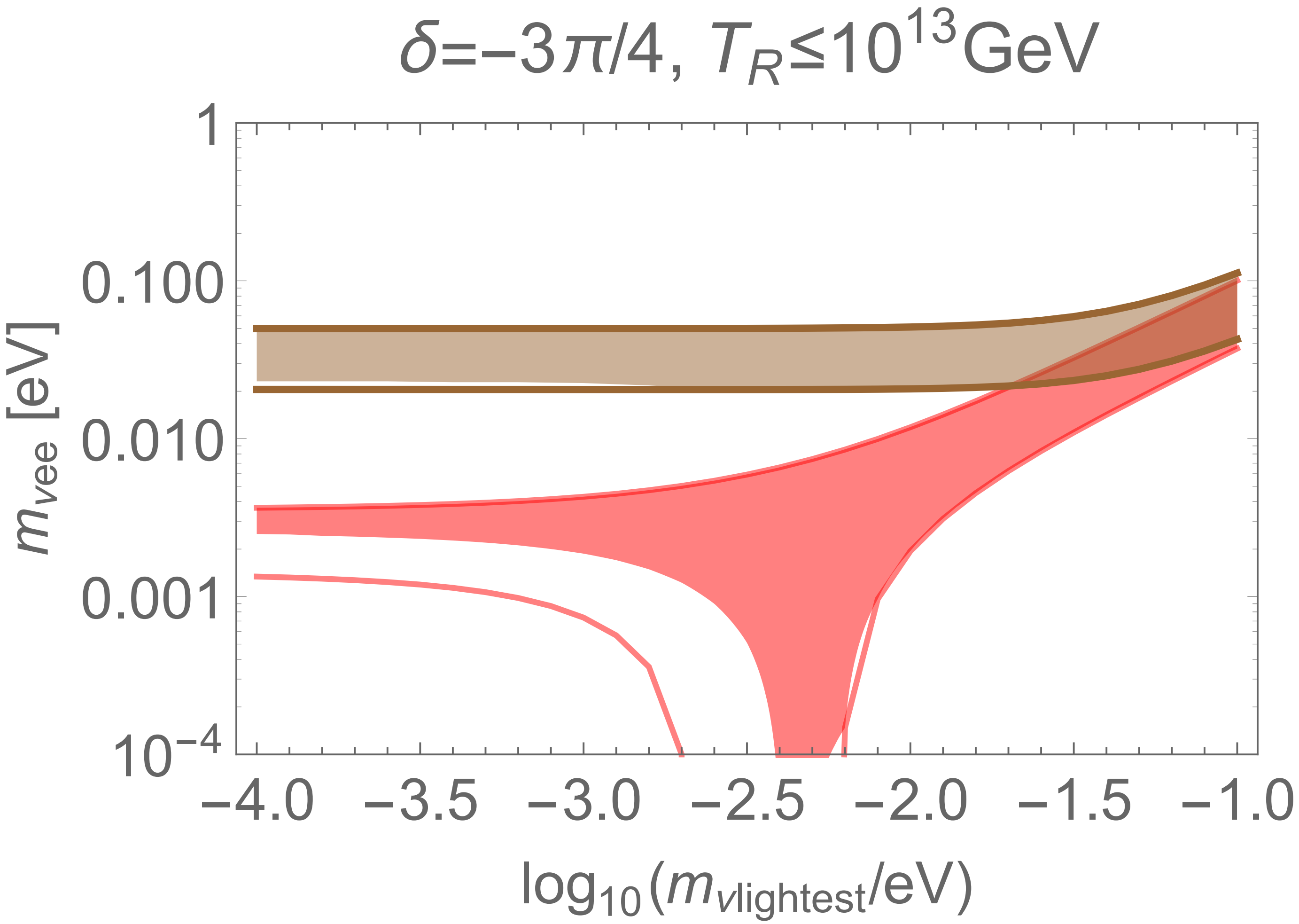
Normal, $T_R=m_\phi=10^{15}$GeV, $m_{\nu\text{lightest}}$=0eV
1. × 10−9
5. × 10−10
1. × 10−10
5. × 10−11
1. × 10−11
$|n_L|/s$
−3
−2
−1
0
1
2
3
δ
α_M=0
α_M=−π/2
α_M=−π/4
α_M=π/4
α_M=π/2



Normal, $T_R = m_\phi = 10^{15}$ GeV, $m_{\nu\text{lightest}} = 0.07$eV
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$1.\times10^{-11}$
$|n_L|/s$
−3
−2
−1
0
1
2
3
δ
α_M=0
α_M=−π/2
α_M=−π/4
α_M=π/4
α_M=π/2



Inverted, $T_R = m_\phi = 10^{15}$ GeV, $m_{\nu\text{lightest}} = 0.07$eV
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$1.\times10^{-11}$
$|n_L|/s$
−3
−2
−1
0
1
2
3
δ
α_M=0
α_M=−π/2
α_M=−π/4
α_M=π/4
α_M=π/2



δ=−3π/4, T_R≤10^{13}GeV
1
0.100
0.010
0.001
10^{-4}
$m_{νee}$ [eV]
−4.0
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
$\log_{10}(m_{νlightest}/eV)$