New Inflation in the Landscape and the Observed Cosmic Perturbation

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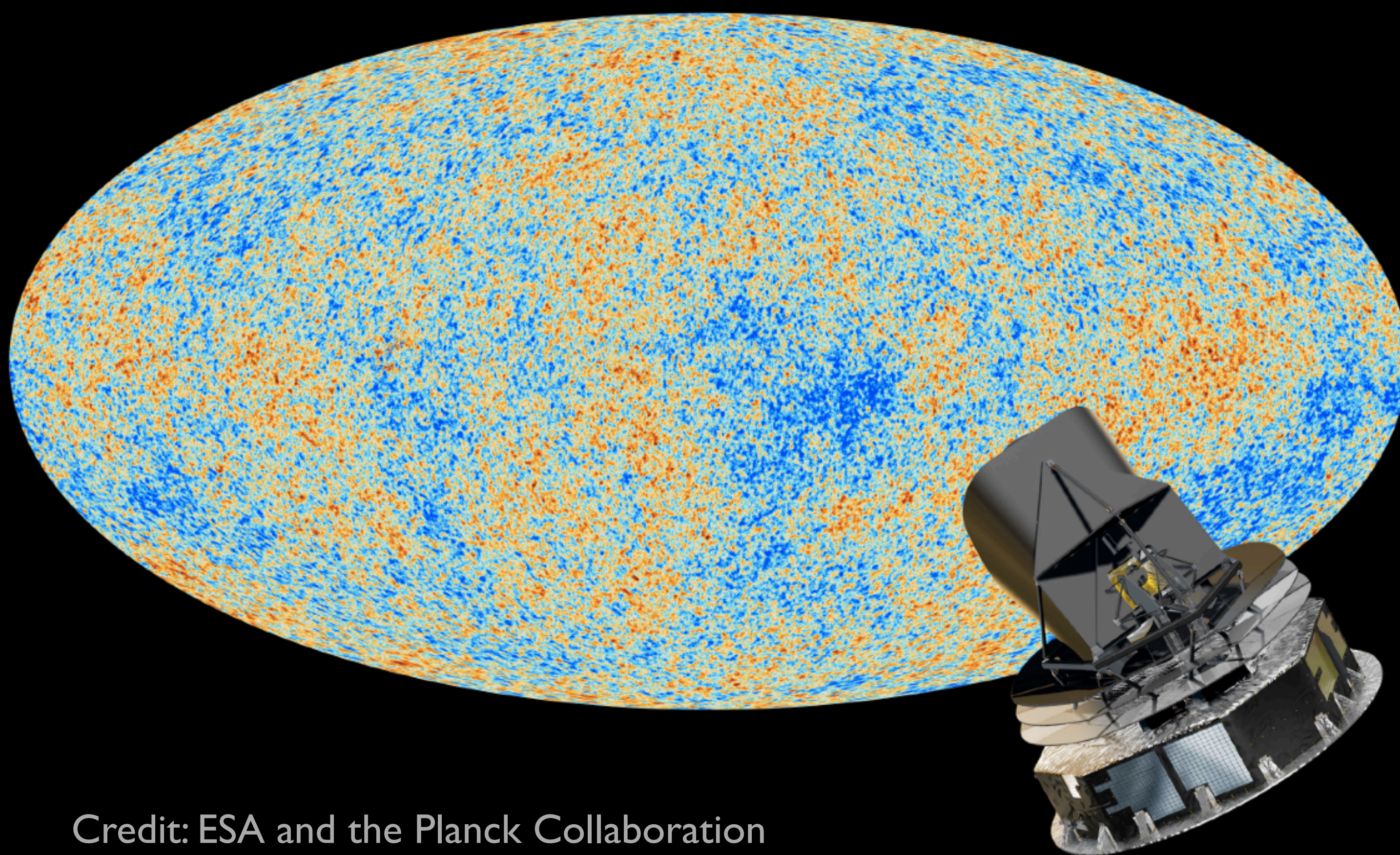
based on the work done with Keisuke Harigaya, **arXiv: 1811.01994**

Era of Precision Cosmology



Credit: ESA and the Planck Collaboration

Amplitude of dimensionless scalar power spectrum

$$P_\zeta = (2.101^{+0.031}_{-0.034}) \times 10^{-9}$$

Spectral index of the scalar spectrum

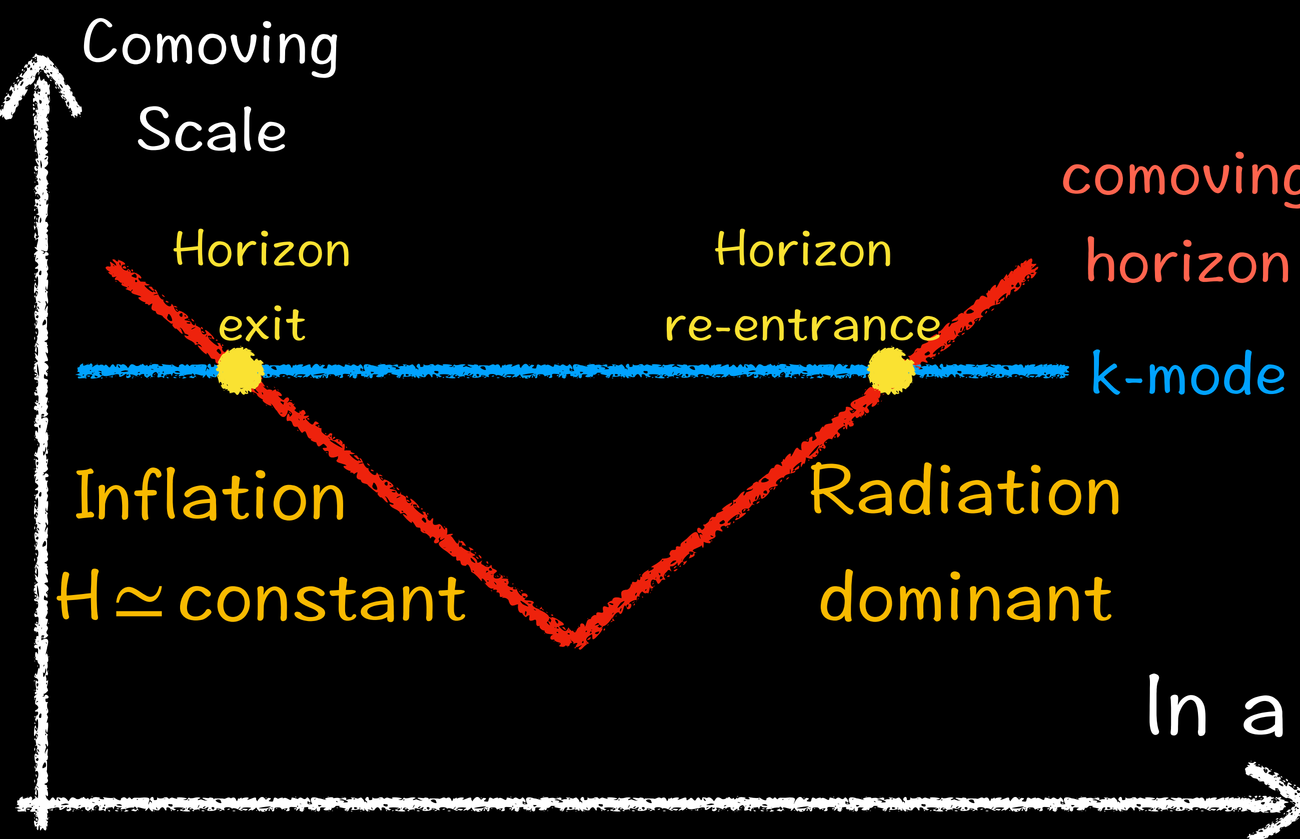
$$n_s = 0.965 \pm 0.004$$

(Planck 2018)

Why these numbers??

Inflation generically predicts nearly scale-invariant power spectrum

But $n_s \simeq 0.965$ seems fine-tuned



Single field slow-roll: $n_s = 1 + 2\eta_V - 6\epsilon_V$

$$\epsilon_V \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V \equiv \frac{V''}{V}$$

$n_s \simeq 0.965$ implies fine-tuning in potential parameters unless protected by symmetry

η-problem

Large Field Inflation
Approximate or Discrete Shift Symmetry

Chaotic Inflation: $V(\phi) = \mu^{4-p}\phi^p$

$$n_s = 1 - \frac{(2+p)}{2\mathcal{N}_e^*}$$

(Linde 1983)

Natural Inflation: $V(\phi) = \frac{V_0}{2}\left[1 - \cos\left(\frac{\phi}{f}\right)\right]$

$$n_s \underset{(f \gg M_{\rm Pl})}{\rightarrow} 1 - \frac{2}{\mathcal{N}_e^*}$$

(Freese et.al 1990)

Small Field Inflation
Fine-tuning parameters

Hilltop (New) Inflation: $V(\phi) = V_0 \left[1 + \frac{1}{2}\eta_0 \frac{\phi^2}{M_{\rm Pl}^2} + \cdots \right]$ $\boxed{\eta_0 < 0}$

(Linde 1982)

$$n_s = 1 + 2\eta_0$$

(Albrecht & Steinhart 1982)

Inflection Point Inflation:

$$V(\phi) = V_0 \left[1 + \lambda_0 \frac{\phi}{M_{\rm Pl}} + \frac{1}{2}\eta_0 \frac{\phi^2}{M_{\rm Pl}^2} + \frac{1}{3!}\mu_0 \frac{\phi^3}{M_{\rm Pl}^3} + \cdots \right]$$

$$n_s = 1 - 4\sqrt{\frac{\lambda_0 \mu_0}{2}} \cot\left(\mathcal{N}_e^* \sqrt{\frac{\lambda_0 \mu_0}{2}} \right)$$

(Baumann et.al 2007)

(Bueno Sanchez et.al 2007)

- It's not clear if large field inflation models are consistent with quantum gravity.
  e.g. distance conjecture and weak gravity conjecture.

- Small field inflation models require fine-tuning potential parameters.

- For most cases, perturbation amplitude (energy scale of inflation) is an input parameter to match observation instead of being dynamically determined.

How to determine the parameters?

- String Landscape and Anthropic Principle

Credit: Quanta Magazine

Inflationary solution/dS vacua may lie in the Swampland?

See Jacob's talk on the phenomenological implications of dS Swampland conjecture

- "Spectrum" of determining parameters

In this work we adopt a moderate view

Dynamics ⟷ Anthropic Principle

Anthropic Consideration on Perturbation Amplitude

- Not strong enough to yield strict bounds on P_ζ

- The probability distribution of P_ζ has the form

$$\mathcal{P}_{P_\zeta} = \mathcal{P}_{\rm post}\mathcal{P}_{\rm inflation}$$

- The anthropic consideration on P_ζ is affected by cosmological constant ρ_Λ. If ρ_Λ is too large, structures cannot form. Assuming ρ_Λ has a uniform probability distribution function, this leads to

$$\int_0^{\rho_\Lambda^{max}} d\rho_\Lambda = \rho_\Lambda^{max} \propto P_\zeta^{3/2}$$

Anthropic Consideration on Perturbation Amplitude

- If P_ζ is too large, galaxy would be too dense such that the time scale of orbital disruption and close encounter with nearby planets is too short.

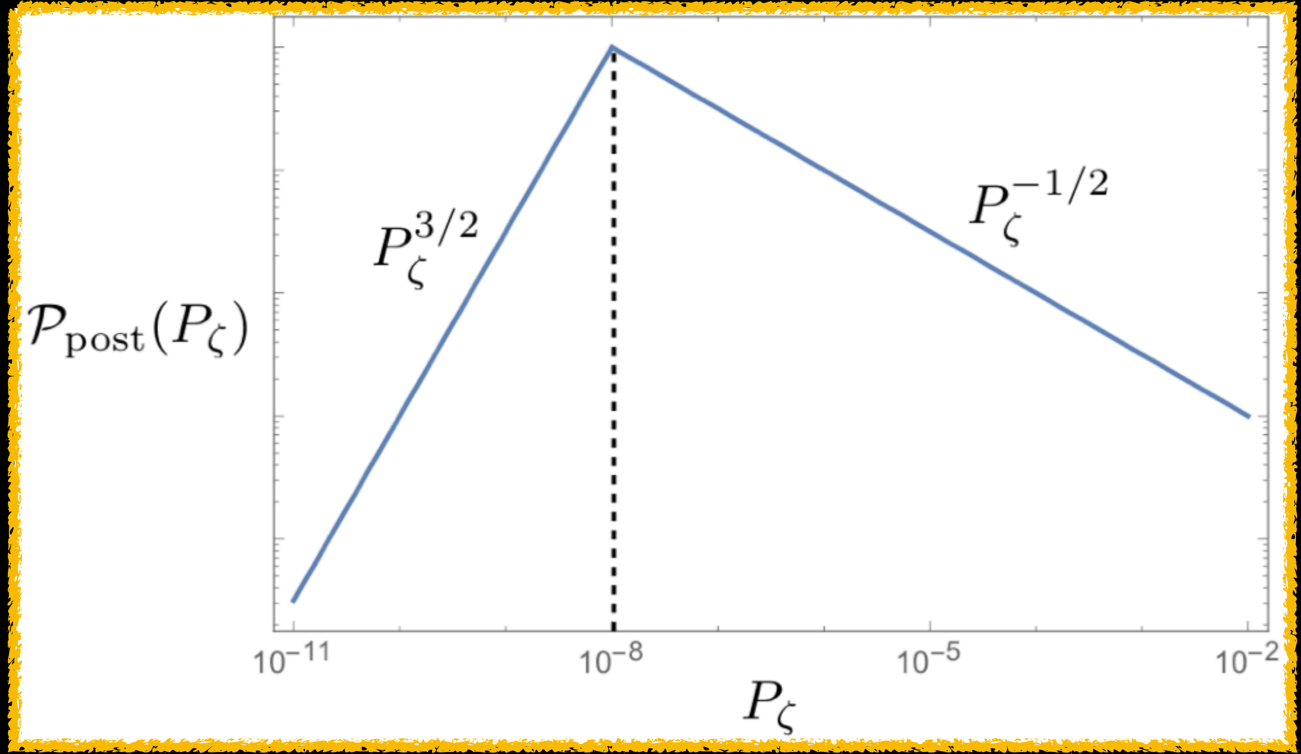
We take this critical value to be $P_\zeta \sim 10^{-8}$ (Tegmark et.al 1998, 2006)

- Even with $P_\zeta > 10^{-8}$, we may live in a region of the universe with smaller density contrast. Assume the distribution to populate at a region with $\delta \equiv \delta\rho/\rho$ is Gaussian, we have

$$\int_0^{10^{-4}} d\delta \, \frac{1}{\sqrt{2\pi P_\zeta}} e^{-\frac{\delta^2}{2P_\zeta}} \simeq \int_0^{10^{-4}} d\delta \, \frac{1}{\sqrt{2\pi P_\zeta}} \propto P_\zeta^{-1/2}$$

Anthropic Consideration on Perturbation Amplitude

- Rudimentary discussion yields:



- $\mathcal{P}_{P_\zeta} = \mathcal{P}_{\rm post}\mathcal{P}_{\rm inflation}$ Cannot bias toward large P_ζ too much

- Note that $P_\zeta = \frac{V}{24\pi^2\epsilon}$. If the inflation energy scale is simply given by a mass parameter, then it would naturally biased toward the fundamental scale linearly. The observed $P_\zeta \sim 10^{-9}$ then requires fine-tuning.

- A very rigorous discussion on $\mathcal{P}_{\rm post}$ seems rather difficult. However, the requirement $\mathcal{P}_{\rm inflation}$ should not bias toward large P_ζ seems to be necessary.

Can

$n_s = 0.965 \pm 0.004$ **&** $P_\zeta = (2.101^{+0.031}_{-0.034}) \times 10^{-9}$

be natural?

R-Symmetry Breaking Inflation

(Kumekawa, Moroi, & Yanagida 1994)

(Izawa & Yanagida 1996)

superpotential: $W = v^2\Phi - \frac{g}{N+1}\Phi^{N+1}$ $\quad (Z_{2N}\ R\text{-symmetry})$

Kahler Potential: $K = \Phi^\dagger\Phi + \frac{1}{4}k(\Phi^\dagger\Phi)^2 \cdots$

$D_iW = \partial_i W + W\partial_i K$

potential:
$$V = e^K\left(D_iW K^{i\bar{j}} D_{\bar{j}}W^* - 3|W|^2\right)$$
$$= |v^2 - g\varphi^N|^2 - kv^4|\varphi|^2 + \cdots$$
$$= v^4 - kv^4|\varphi|^2 - \left(gv^2\varphi^N + \text{ h.c.}\right) + \cdots$$
$$= v^4 - \frac{1}{2}kv^4\phi^2 - \frac{g}{2^{\frac{N-2}{2}}}v^2\phi^N\cos(N\theta) + \cdots$$

$\varphi = \frac{\phi}{\sqrt{2}}e^{i\theta}$

Interested in the region $\phi \ll 1$

with the initial condition

$\theta = 0 \mod \frac{2\pi}{N}$

(Well-motivated in Landscape Scenario)

R-Symmetry Breaking Inflation

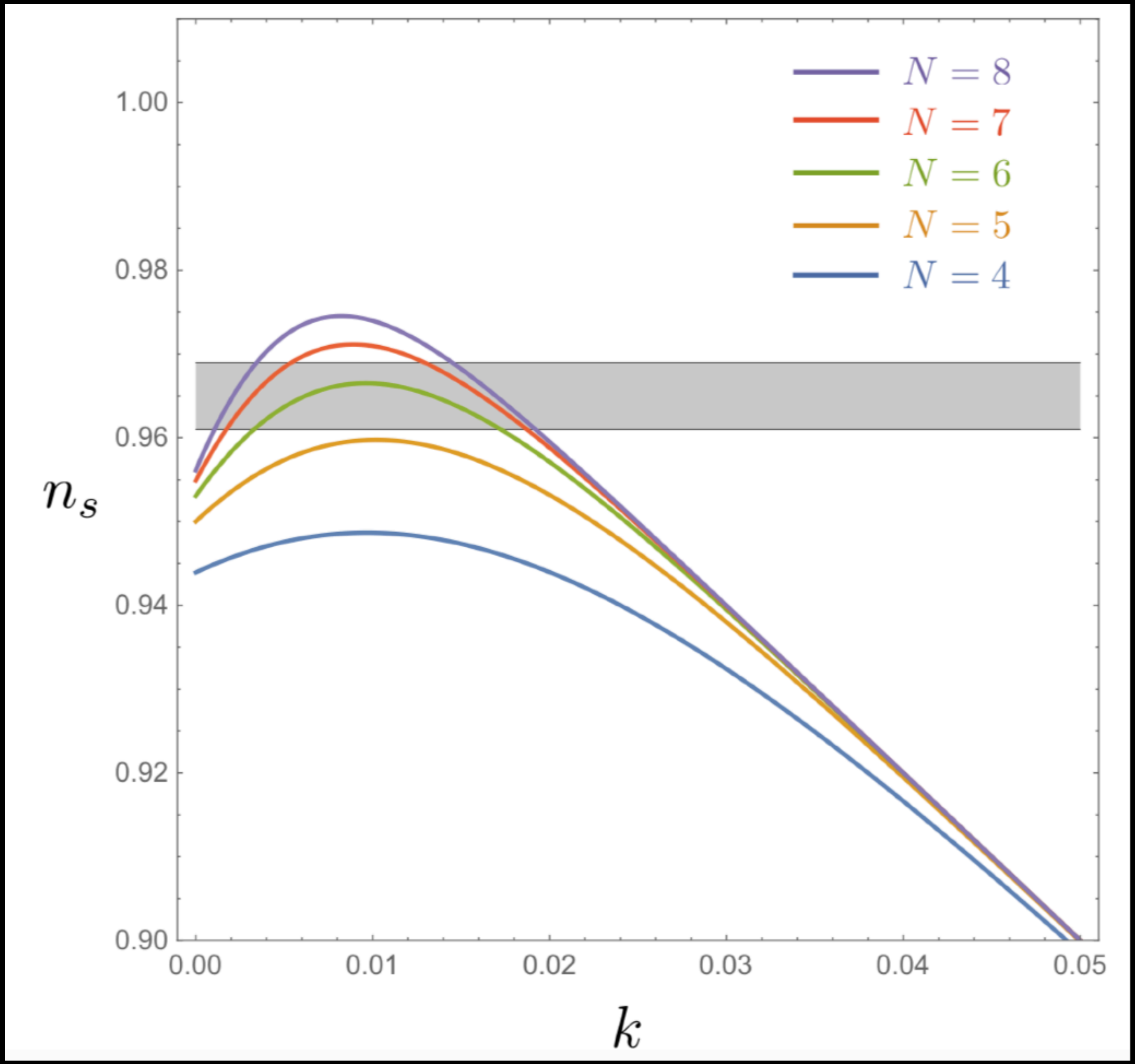
$$n_s \simeq 1 - 2k - 2N(N-1)kf_N^*$$

$$k \simeq 0.01 \overset{N=6}{\Longleftrightarrow} n_s = 0.965$$

For **fixed** P_ζ , if v is biased toward large value, then $k \simeq 0.01$ is natural.

(See later slides)

(Harigaya, Ibe & Yanagida 2013)



$$f_N \equiv f_N(k, \mathcal{N}_e) = \frac{1}{N}\frac{1}{\left([1+(N-1)k]\, e^{(N-2)k\mathcal{N}_e} - 1\right)}$$

R-Symmetry Breaking Inflation

- To obtain the probability distribution of the observables, need to assume probability distribution of Lagrangian parameters.

- Parametrize the probability distribution as

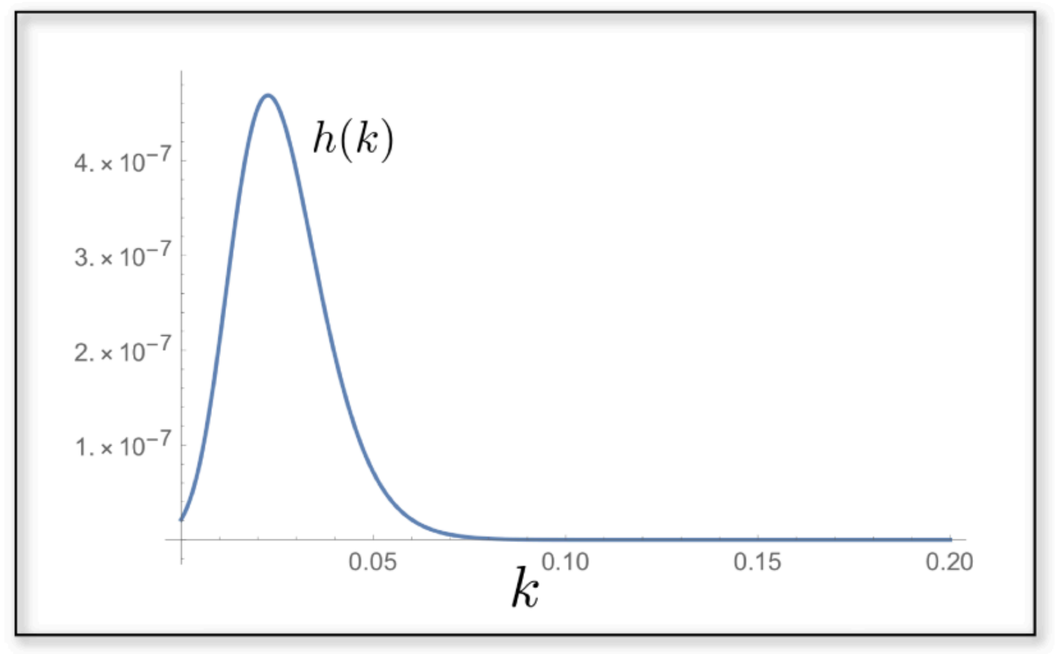
$$\int dk\, dg\; dv^2\;\; g^q\, v^{2p}$$

- The distribution of g does not affect the distribution of n_s and P_ζ much so we won't discuss it in this talk.

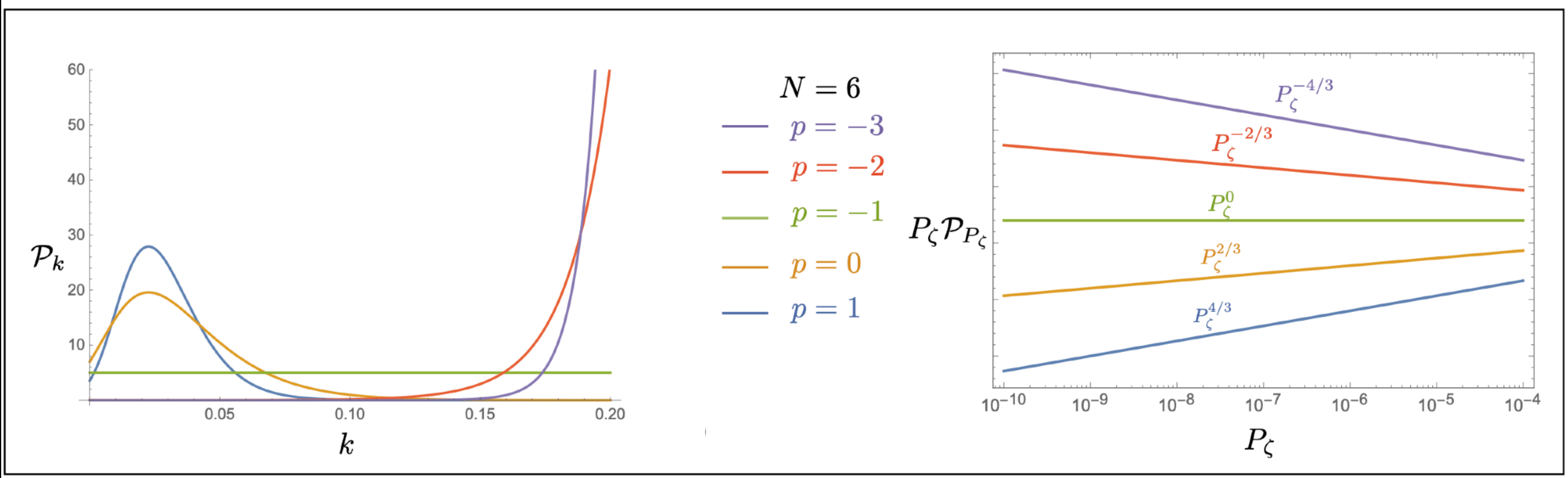
- With a change of variable one can obtain the distribution for observables.

$$\mathcal{P}_k \propto h^{\frac{(p+1)}{(N-3)}}$$

$$P_\zeta \mathcal{P}_{P_\zeta} \propto P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}$$



$$h(k) \equiv k^{N-1} f_N^* \left[24\pi^2(1+N f_N^*)^2\right]^{\frac{N-2}{2}}$$



- For fixed P_ζ ,when the inflation energy scale is biased toward larger value ($p = 0$and $p = 1$ in above plots), k is biased toward small value.

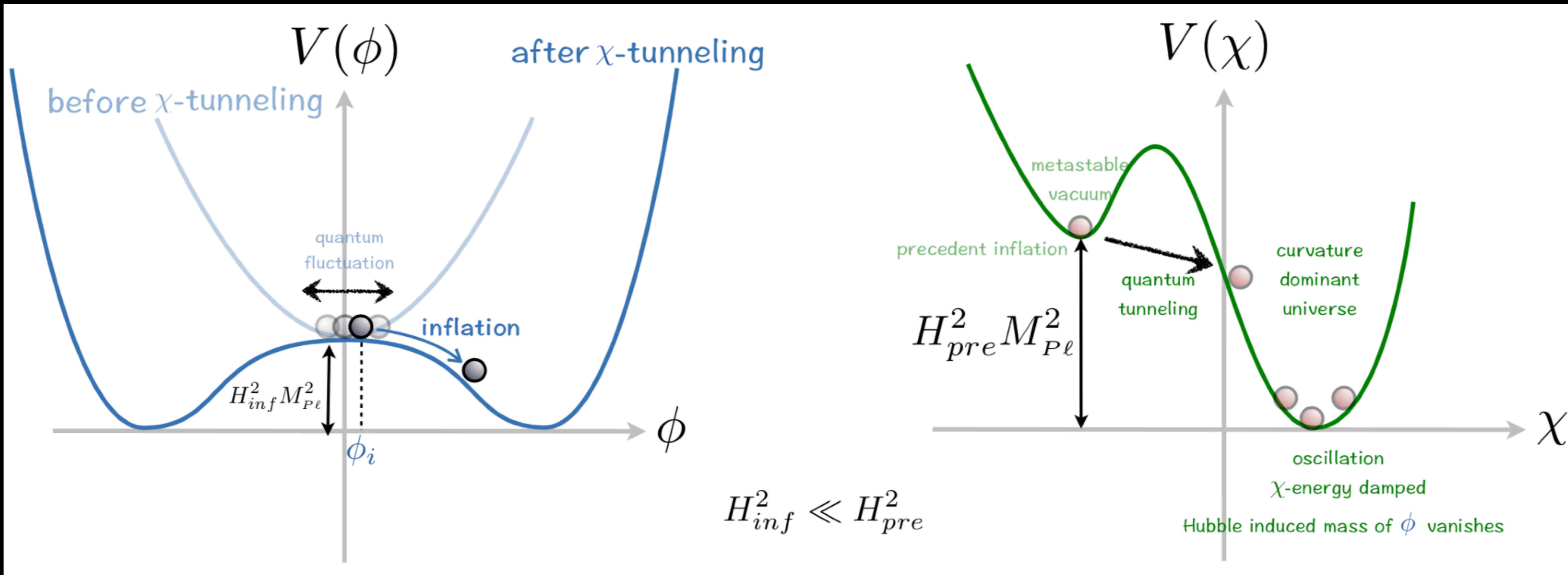
η-problem solved

- HOWEVER, in such cases, P_ζ is biased toward large value. The observed value becomes atypical.

- If the energy scale v is originated from dimensional transmutation, we expect a distribution with $p = -1$. We get no further bias toward large P_ζ , but k is no longer suppressed at large $\mathcal{P}_k$.

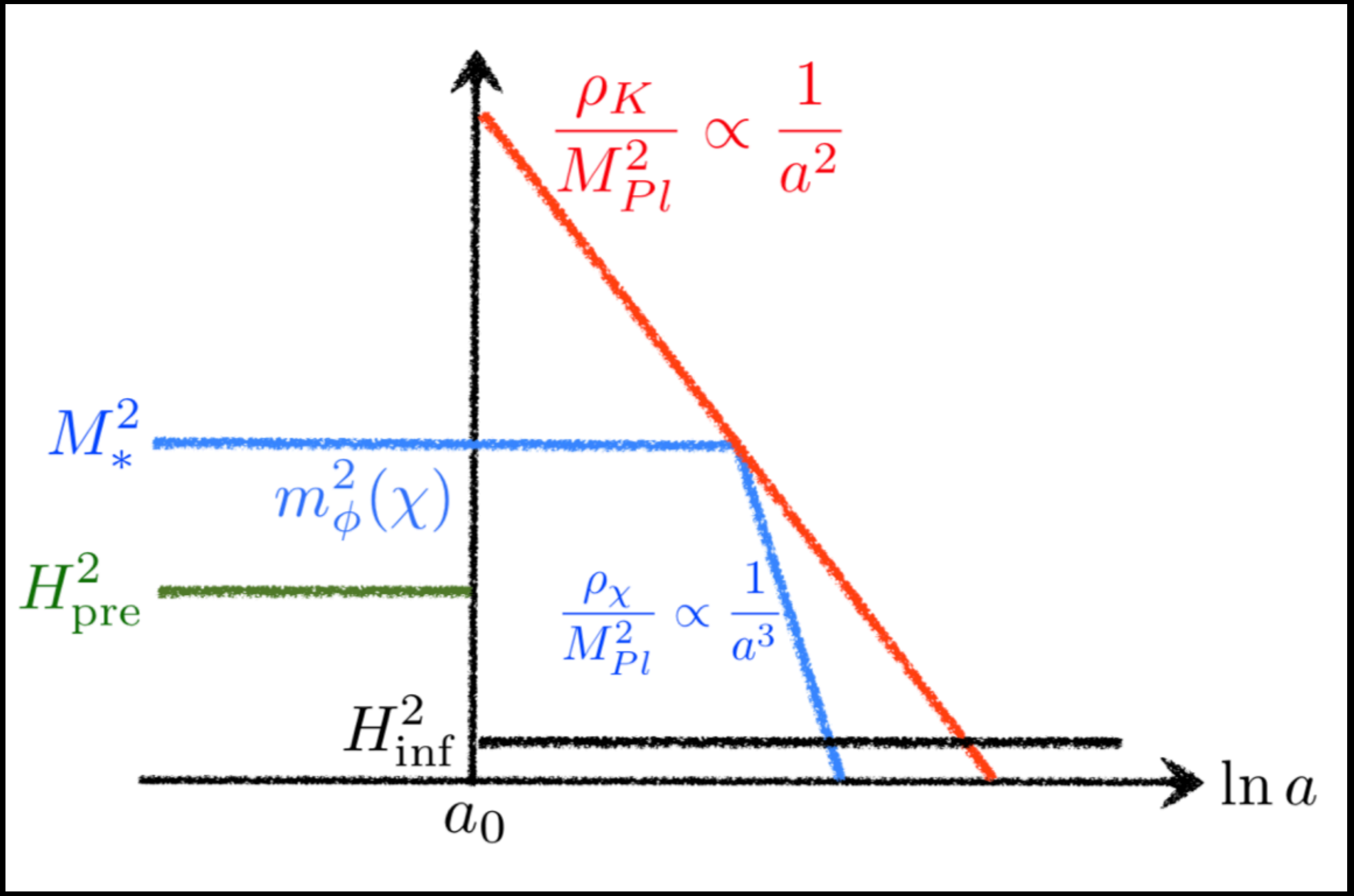
Need more ingredients to solve BOTH fine-tunings

New Inflation in the Landscape



- The universe was first trapped in the meta-stable vacua of χ, undergoing a *precedent* inflation. The inflaton trapped at the origin by Hubble induced mass.
- After quantum tunneling of χ, the universe became open FRW. After thawing from Hubble damping, χ rolled down to minimum.
- Eventually the potential energy of the inflaton took over and inflation began.

Quantum fluctuation of the inflaton during precedent inflation was suppressed by Hubble induced mass, but the long wavelength mode that exited the horizon right before the quantum tunneling of χ survives.



The inflaton can have a nonzero initial value ϕ_i due to this zero mode

The zero mode has a Gaussian distribution, which for small ϕ_i can be approximated as uniform distribution.

$$\int \mathcal{P}_{\phi_i}\, d\phi_i = \int d\phi_i$$

R-Symmetry Breaking Inflation in the Landscape

- Probability distribution of ϕ_i should be included.

$$\int d\phi_i \, dk \, dg \; dv^2 \;\; g^q \, v^{2p}$$

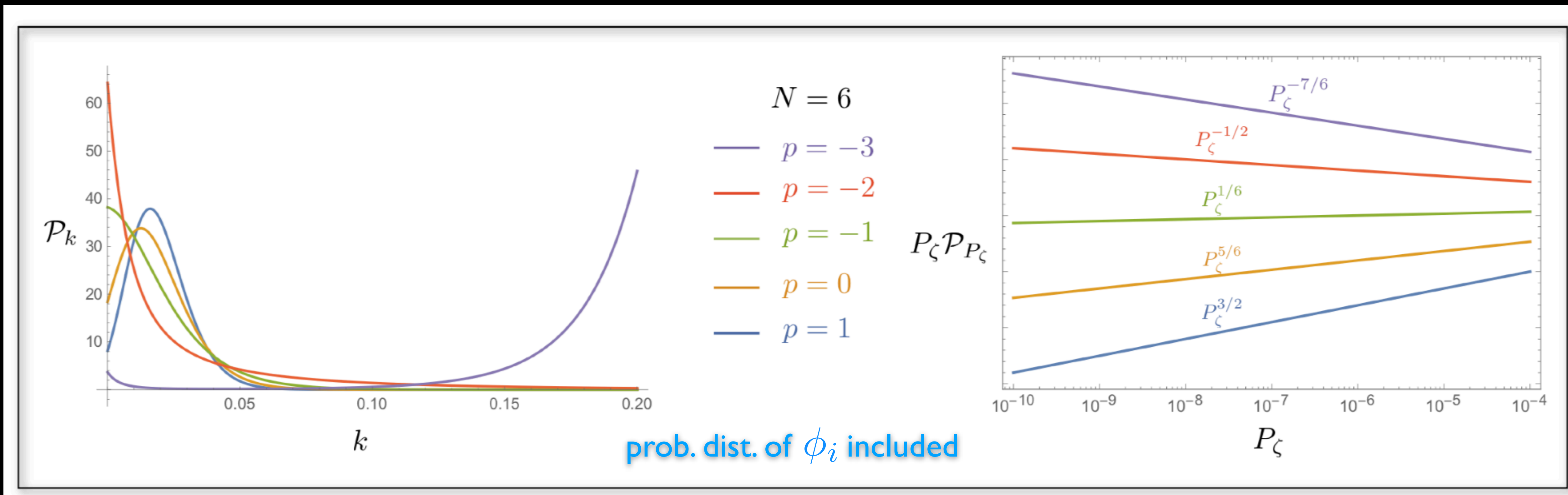
- Inflaton can start off from a point away from the origin.

- Inflation needs to last long enough to wipe out curvature energy density and to have structure on the galaxy scale.

- This imposes anthropic bound $\phi_{\rm ant}$ on ϕ_i , $\phi_i < \phi_{\rm ant}$

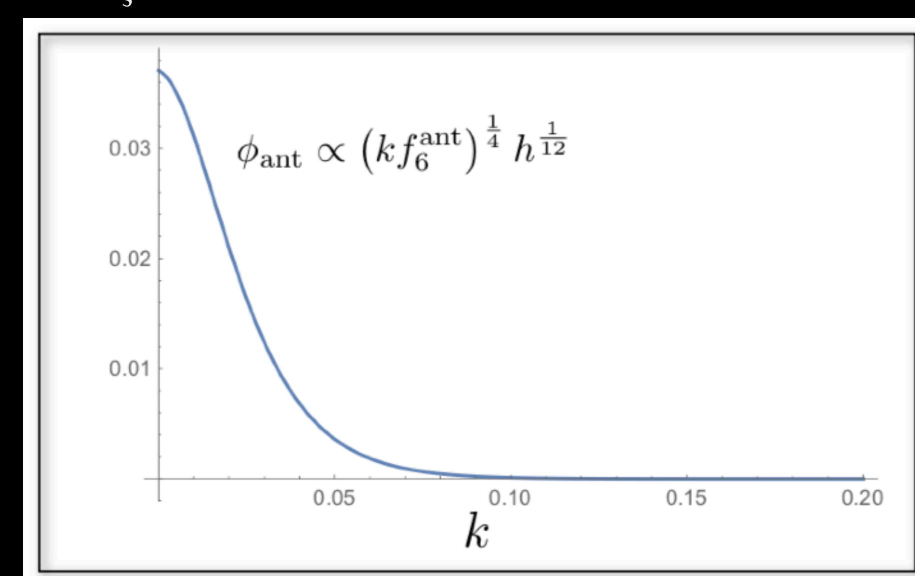
- An additional bias toward smaller k, as large enough e-folds of inflation requires flatter potential.

R-Symmetry Breaking Inflation in the Landscape



$$\mathcal{P}_k \propto h^{\frac{(p+1)}{(N-3)}}$$
$$P_\zeta \mathcal{P}_{P_\zeta} \propto P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}$$

$$\mathcal{P}_k \propto h^{\frac{(p+1)}{(N-3)}} \left[\left(k f_N^{60}\right)^{\frac{1}{N-2}} h^{\frac{1}{(N-3)(N-2)}} \right]$$
$$P_\zeta \mathcal{P}_{P_\zeta} \propto P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}} P_\zeta^{\frac{1}{2(N-3)}}$$



Note $p=-2$ can *simultaneously* give bias toward small k and small P_ζ

R-Symmetry Breaking Inflation in the Landscape

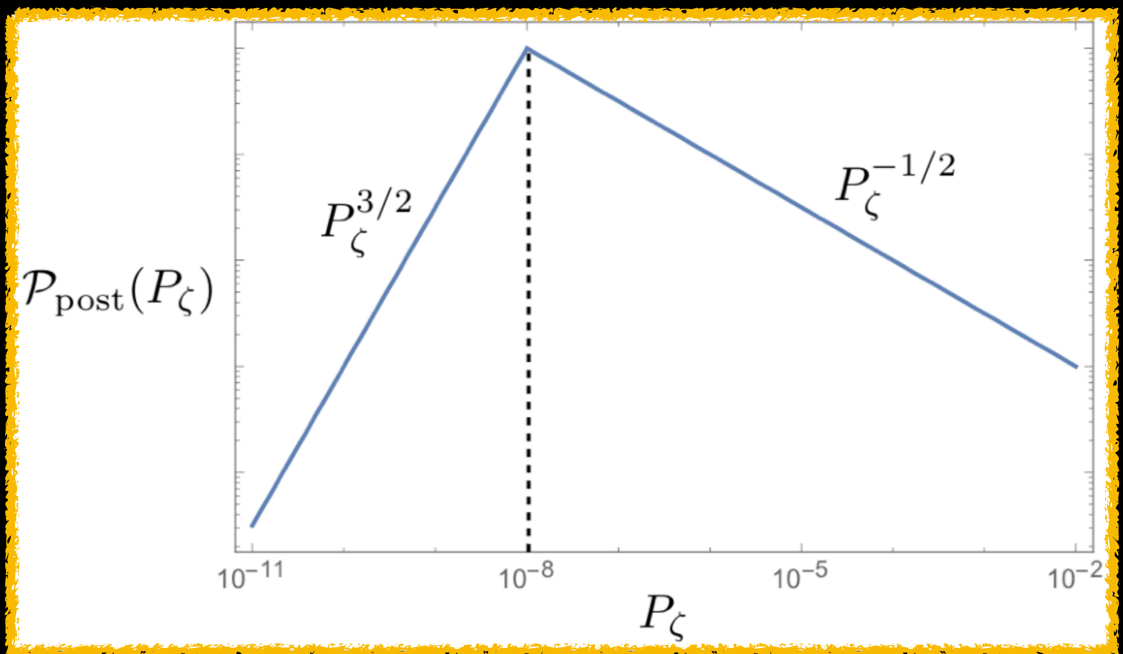
- The probability distribution $\mathcal{P}_k$ at large k

$$\mathcal{P}_k \propto k^{\frac{(N-2)(p+1)+1}{N-3}} e^{-\frac{(N-2)(p+2)}{N-3} k \mathcal{N}_e}$$

→ Need $p \geq -2$ to suppress $\mathcal{P}_k$ at large k

- For prob. dist. of P_ζ, recall that $\mathcal{P}_{P_\zeta} = \mathcal{P}_{\text{post}} \mathcal{P}_{\text{inflation}}$

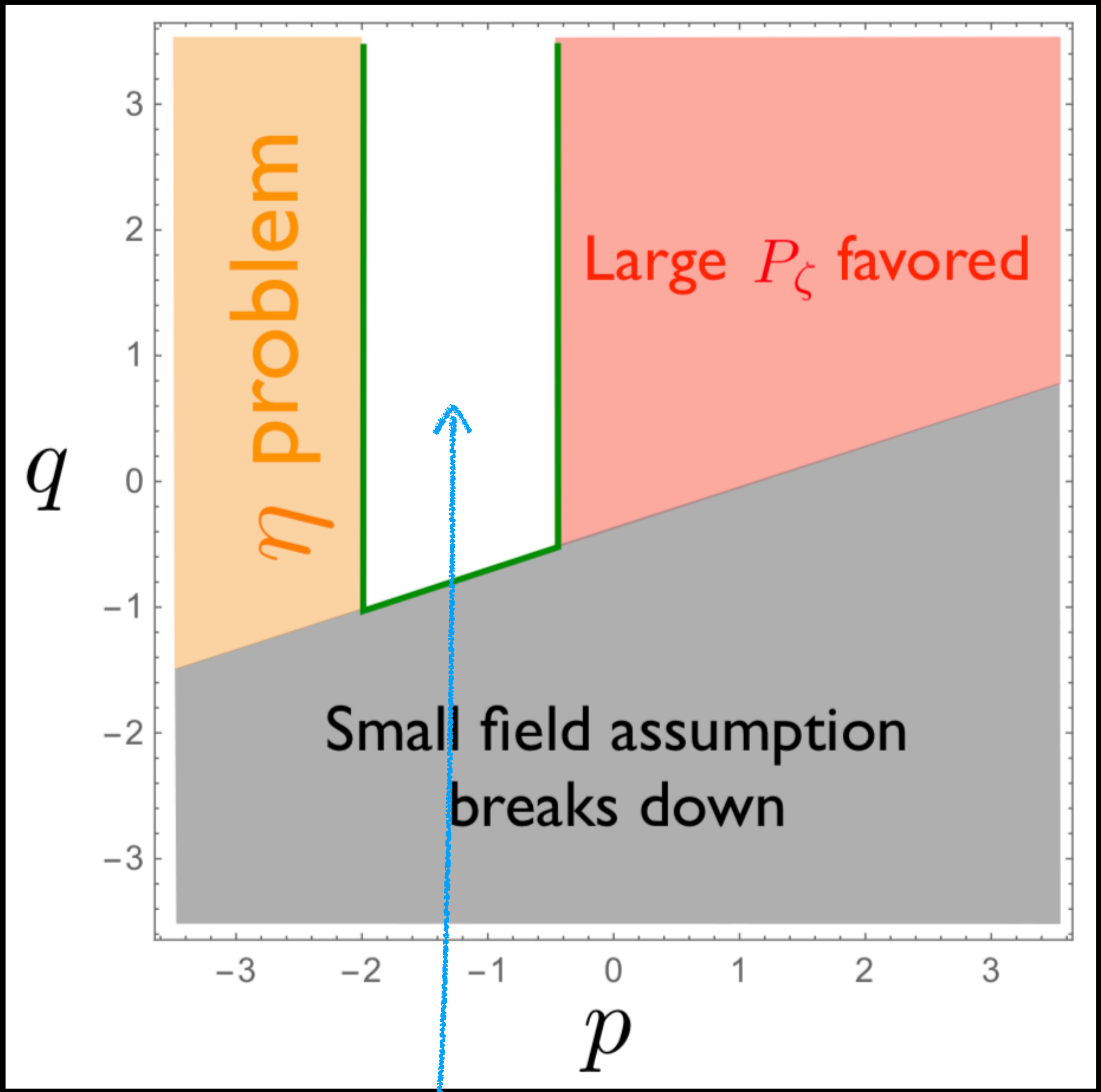
$$P_\zeta \mathcal{P}_{P_\zeta} \propto P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}} P_\zeta^{\frac{1}{2(N-3)}}$$

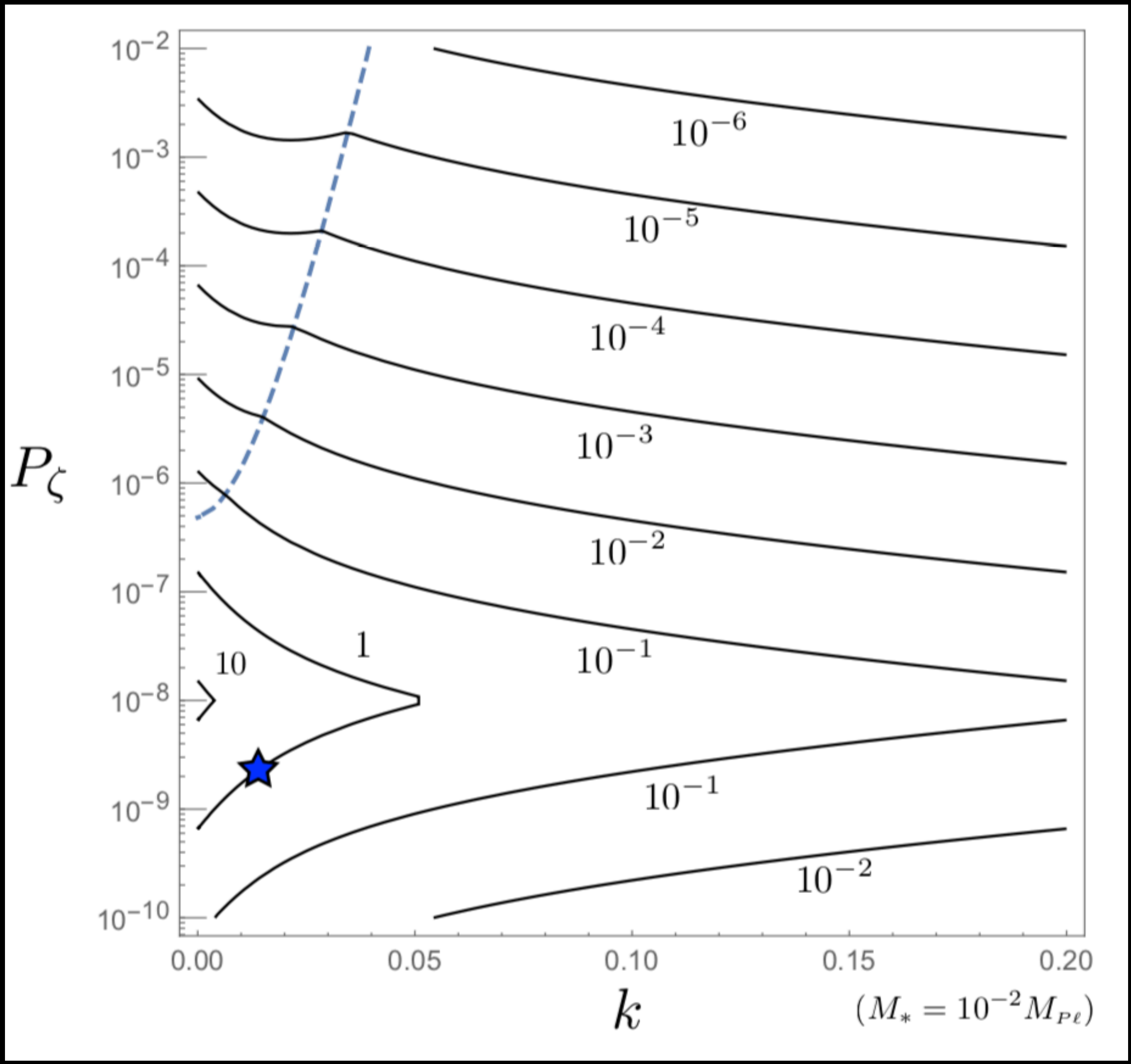


→ Need $p \leq \frac{-2}{N-2}$ so that P_ζ is not biased toward large value

R-Symmetry Breaking Inflation in the Landscape

$(N = 6)$





This open window wouldn't exist if $\int \mathcal{P}_{\phi_i}\, d\phi_i$ were not included

The contour plot of $\frac{P_\zeta \mathcal{P}_{\rm net}(k, P_\zeta)}{P_\zeta \mathcal{P}_{\rm net}\big|_{k_{obs}, P_\zeta^{obs}}}$ for $N = 6$, $p = -2$

Summary and Discussion

- We now have very precise measurement of P_ζ and n_s, but their values require explanations.

- Anthropic principle alone seems impossible to achieve this goal. Particularly, it's hard to imagine anthropic argument can strongly constrain n_s.

- By dynamics alone, P_ζ is mostly used as input parameter.

- String landscape provides a stage to naturally combine the two.

Thank you for your attention!