Effects of an early matter dominated era on gravitational waves induced by scalar perturbations

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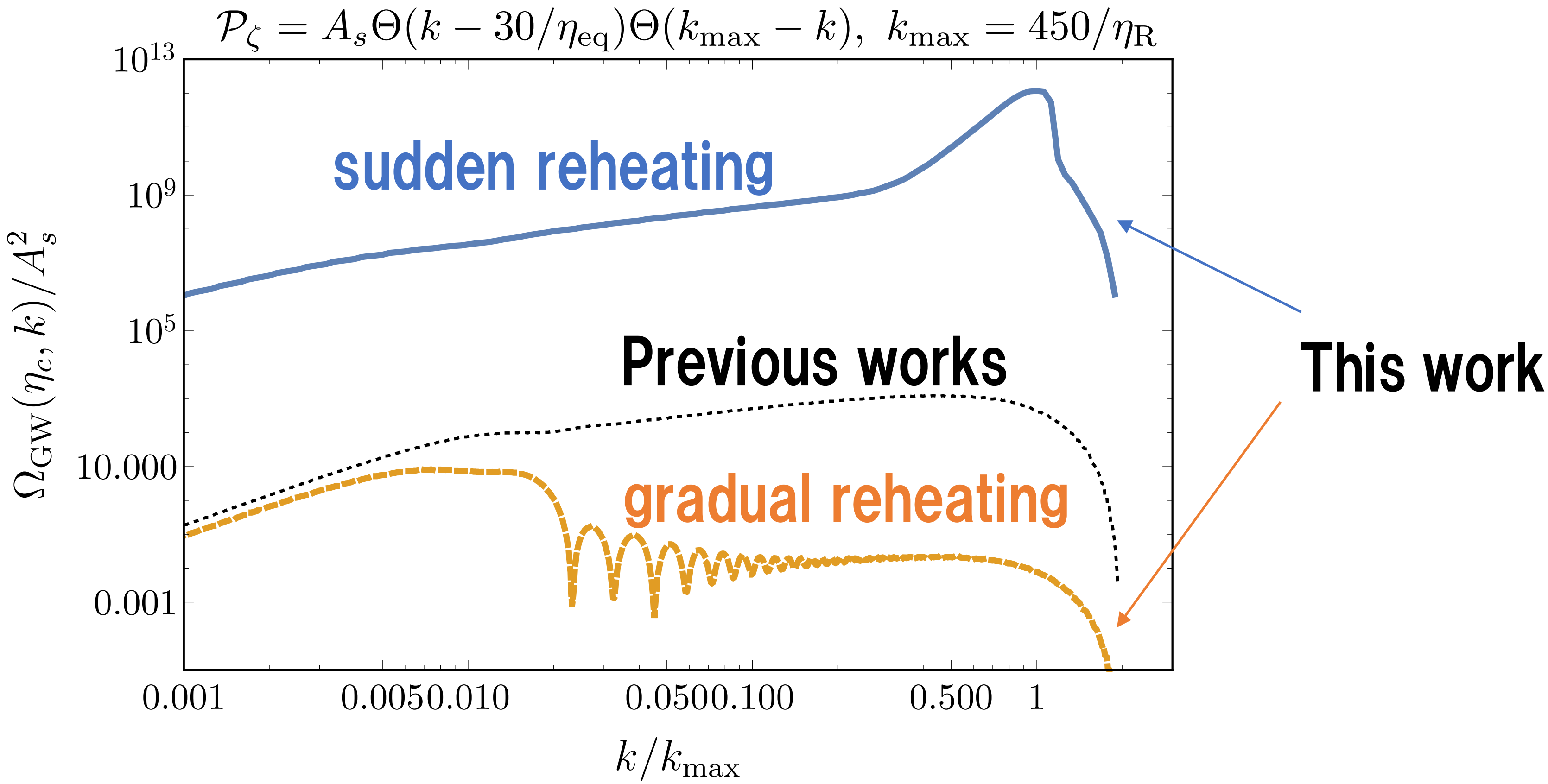
Overview



We revisit the effect of an early matter dominated era on the gravitational waves induced by scalar perturbations.

As a result, we find that the induced GWs strongly depend on how quickly the reheating occurs.

Outline

- Introduction
- eMD effects in gradual reheating scenario
- eMD effects in sudden reheating scenario
- Summary

GWs induced by curvature perturbations

At linear order, scalar perturbations do not induce GWs.

However,

at second order, scalar perturbations can induce GWs.

(Ananda et al 2006, Baumann et al. 2007)

Metric perturbations:

Scalar perturbations (related to curvature perturbations)

$$\mathrm{d}s^2 = a^2\left[-(1+2\Phi)\mathrm{d}\eta^2 + \left((1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x_i \mathrm{d}x_j\right]$$

Tensor perturbations (GWs)

E.o.M for GWs:

$$h_k^{\lambda''} + 2\mathcal{H}h_k^{\lambda'} + k^2 h_k^{\lambda} = 4\mathcal{S}^{\lambda}(k,\eta)$$

Source term from second order scalar perturbations

$$\mathcal{S}^{\lambda}(k,\eta) = \int \frac{\mathrm{d}^3k'}{(2\pi)^3}\mathrm{e}^{\lambda lm}(\hat{k})k'_l k'_m \left[2\Phi_{k'}(\eta)\Phi_{k-k'}(\eta) + \frac{4}{3(1+w)}\left(\Phi_{k'}(\eta) + \frac{\Phi'_{k'}(\eta)}{\mathcal{H}}\right)\left(\Phi_{k-k'}(\eta) + \frac{\Phi'_{k-k'}(\eta)}{\mathcal{H}}\right)\right]$$

Motivations for the induced GWs

The frequencies of the induced GWs correspond to the scales of the scalar perturbations inducing GWs.

Since the wide range of GW frequencies could be observed in the future, we could investigate small-scale (scalar) perturbations using the induced GWs!
$(\lambda \lesssim 1\,\mathrm{Mpc})$

Small-scale perturbations enter the horizon early, the induced GW could be a good probe of the early Universe.

Main question of this work

What effects does the early matter dominated era make on the induced GWs?

The history of the Universe

Reheating

Inflation era

We focus on this!

(Early matter dominated (eMD) era)

Radiation dominated (RD) era

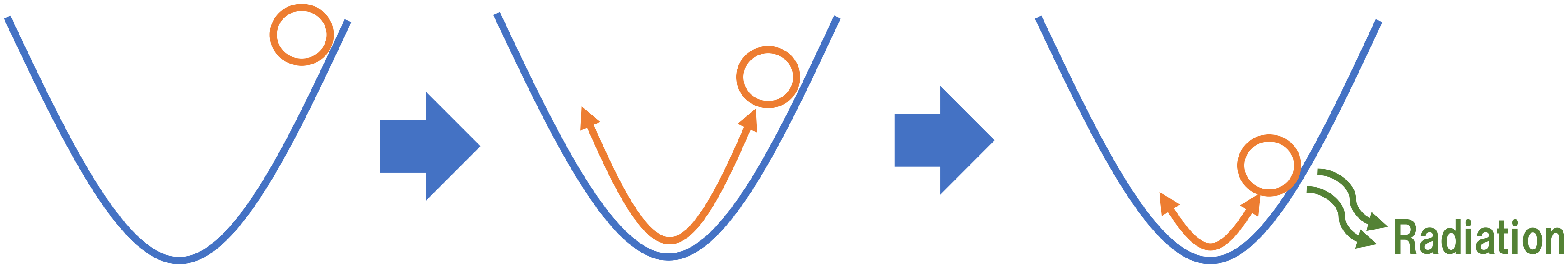
(Late) matter dominated era (from z~3400)

Dark energy dominated era (from z~0.5)

What causes eMD era ?

One scenario: oscillation of inflaton

Chaotic inflation



Inflation occurs the vacuum energy of the inflaton.

During the oscillation of an inflaton, the Universe behaves as MD era.

The oscillation amplitude decays due to the expansion and the decay to radiation.

During the oscillation

Time average

$$\langle \rho \rangle = \left\langle \frac{1}{2}\dot{\phi}^2 + \frac{m^2}{2}\phi^2 \right\rangle = m^2 \left\langle \phi^2 \right\rangle$$

$$\langle P \rangle = \left\langle \frac{1}{2}\dot{\phi}^2 - \frac{m^2}{2}\phi^2 \right\rangle = 0$$

MD era !
$(w = P/\rho = 0)$

RD era starts.

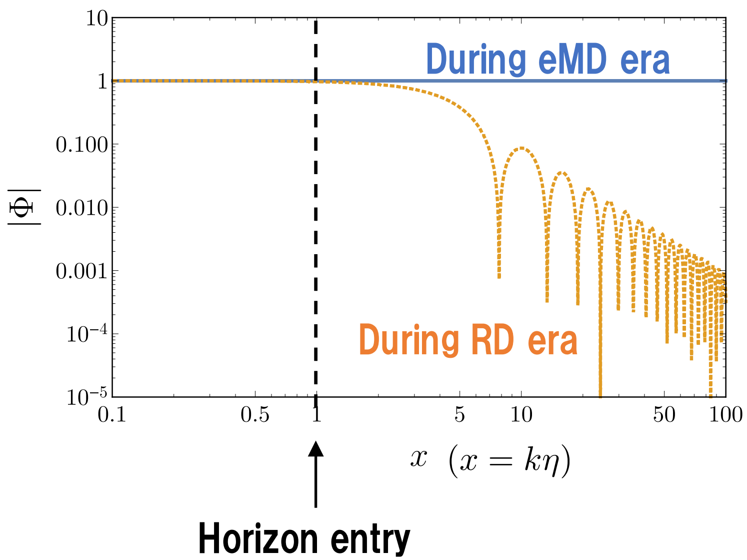
(Other scenario: oscillation of curvaton)

Results in previous works

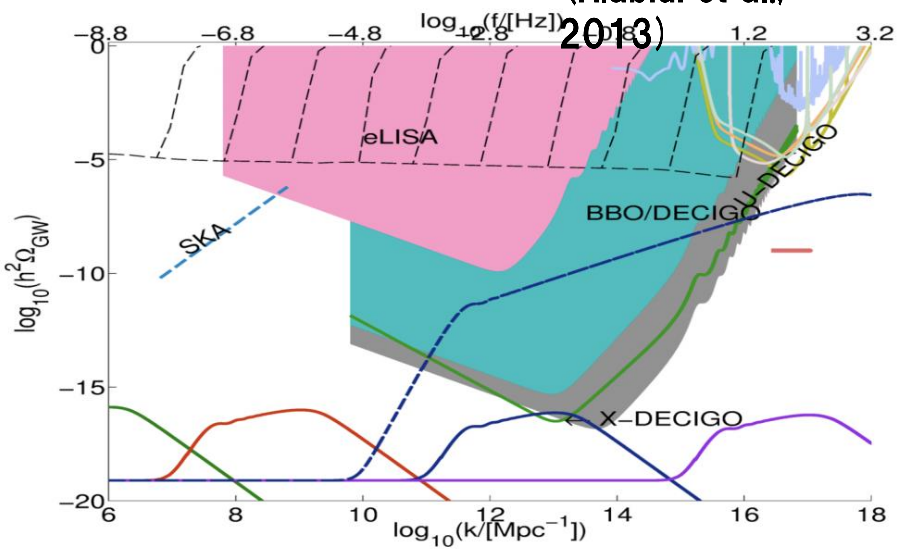
There are works discussing the effects of an eMD era on the induced GWs.

(Assadullahi, Wands, 2009, Alabidi, Kohri, Sasaki, Sendaouda, 2013, Kohri, Terada, 2018)

During an eMD era, the transfer function of the gravitational potential (Φ), the source of GWs, does not decay even on subhorizon because of the growing matter perturbations.



(Alabidi et al., 2013)



Induced GWs are enhanced?

Implicit assumptions in previous works

In previous works, the authors put the following (implicit) assumptions.
(Assadullahi, Wands, 2009, Alabidi, Kohri, Sasaki, Sendaouda, 2013, Kohri, Terada, 2018)

1. The transfer function of the gravitational potential (Φ), the source of the induced GWs, is unity ($\Phi=1$) until the beginning of the RD era.

2. The perturbations that enter the horizon during an eMD era does not induce GWs during the RD era.

What we do in this work

We remove the above assumptions and revisit the effects of an eMD era on the induced GWs.

We consider two cases:

1. Gradual reheating, whose time scale is comparable to the Hubble time scale at that time.
2. Sudden reheating, whose time scale is much shorter than the Hubble time scale at that time.

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Gradual reheating

We consider the case where the field dominating the Universe during an eMD era (inflaton or curvaton) decays to radiation with a constant decay rate Γ.

$$(a^3 \rho_m \propto \mathrm{e}^{-\Gamma t})$$

In this gradual reheating scenario, the situation is similar to the decaying dark matter scenario.

E.o.M in decaying DM scenario (Poulin, Serpico, Lesgourgues, 2018)

$$\rho_m' = -(3\mathcal{H} + a\Gamma)\rho_m,$$

$$\rho_r' = -4\mathcal{H}\rho_r + a\Gamma\rho_m$$

ρ_m : Energy density of matter

ρ_r : Energy density of radiation

δ : Energy density perturbation

$\theta\ (\equiv i k_j v^j)$: Velocity divergence

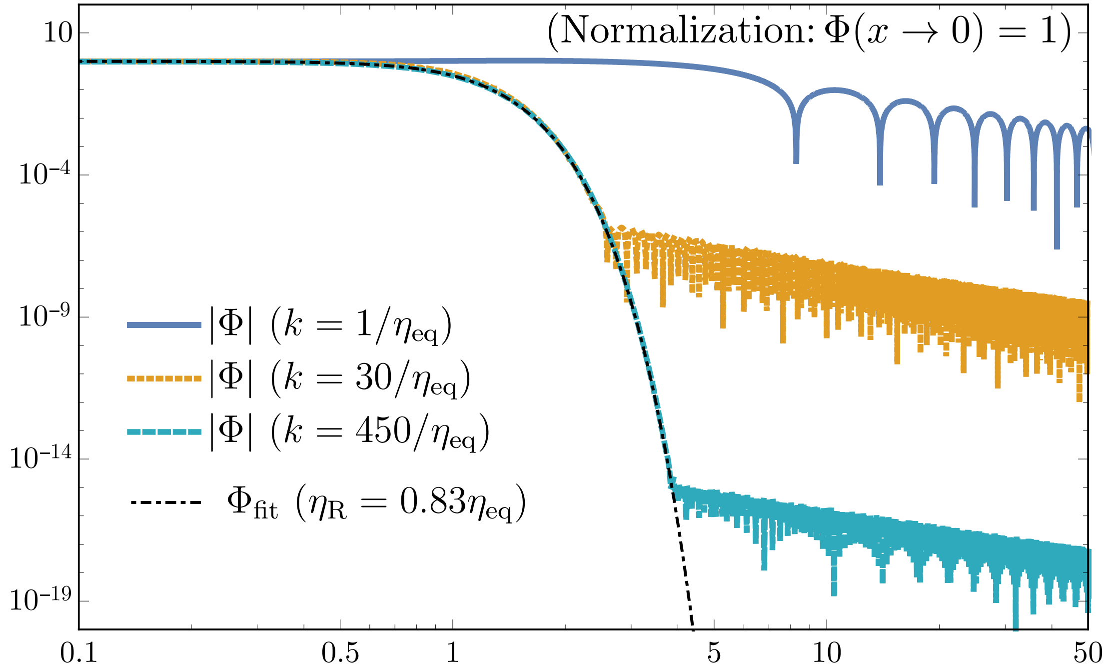
$$\delta_m' = -\theta_m + 3\Phi' - a\Gamma\Phi,$$

$$\theta_m' = -\mathcal{H}\theta_m + k^2\Phi,$$

$$\delta_r' = -\frac{4}{3}(\theta_r - 3\Phi') + a\Gamma\frac{\rho_m}{\rho_r}(\delta_m - \delta_r + \Phi),$$

$$\theta_r' = \frac{k^2}{4}\delta_r + k^2\Phi - a\Gamma\frac{3\rho_m}{4\rho_r}\left(\frac{4}{3}\theta_r - \theta_m\right)$$

Perturbations in gradual reheating



$\rho_m = \rho_r$ at $\eta = \eta_{\rm eq}$

On subhorizon scale,

$$k^2\Phi \simeq \frac{3}{2}\mathcal{H}^2\left(\frac{\rho_m}{\rho_{\rm tot}}\delta_m + \frac{\rho_r}{\rho_{\rm tot}}\delta_r\right)$$

During eMD era, δ_m ($\propto a$) grows on subhorizon unlike δ_r.

Φ is mainly determined by $\rho_m\delta_m$ for a while even after $\eta_{\rm eq}$.

We naively expect

$$\Phi \propto \rho_m \propto \exp\left(\int^\eta {\rm d}\bar{\eta}\, a\Gamma\right)$$

If we take $\eta_{\rm R} = 0.83\,\eta_{\rm eq}$, the fitting formula fits the numerical result during the exponential decay very well!

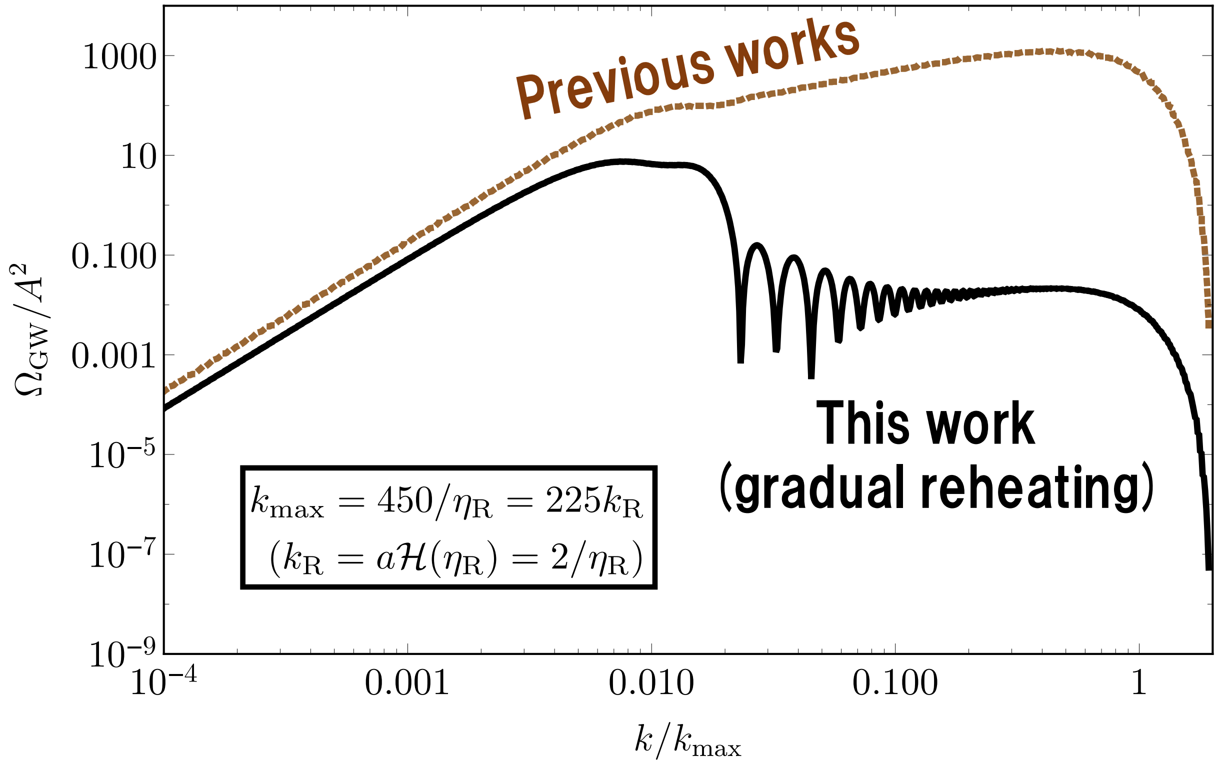
Fitting formula

$$\Phi_{\rm fit} \equiv \exp\left(\int^\eta {\rm d}\bar{\eta} a(\bar{\eta})\Gamma\right) \qquad (a(\eta_{\rm R})\Gamma = \mathcal{H}(\eta_{\rm R}) = 2/\eta_{\rm R})$$

$$= \begin{cases} \exp\left(-\frac{2}{3}\left(\frac{\eta}{\eta_{\rm R}}\right)^3\right) & (\eta < \eta_{\rm R}) \\ \exp\left(-2\left(\left(\frac{\eta}{\eta_{\rm R}}\right)^2 - \frac{\eta}{\eta_{\rm R}} + \frac{1}{3}\right)\right) & (\eta \geq \eta_{\rm R}) \end{cases}$$

Results

GWs induced by curvature perturbations with $\mathcal{P}_\zeta = A\,\Theta(k - 30/\eta_{\rm eq})\Theta(k_{\rm max} - k)$



The induced GWs are suppressed compared to the previous results.

Main reason

A damping of tensor perturbation occurs during the gradual transition.

Why does the damping occur?

E.o.M for tensor perturbations:

$$h_{\boldsymbol{k}}^{\lambda''} + 2\mathcal{H}h_{\boldsymbol{k}}^{\lambda'} + k^2 h_{\boldsymbol{k}}^{\lambda} = 4\mathcal{S}_{\boldsymbol{k}}^{\lambda}$$

For $\eta \ll \eta_R$, the source is almost constant and the tensor perturbations on subhorizon scale become

$$h_{\boldsymbol{k}}^{\lambda} \simeq \frac{4\mathcal{S}_{\boldsymbol{k}}^{\lambda}}{k^2}$$

For $\eta_R \lesssim \eta \lesssim 2\eta_R$, the source decays gradually and the tensor perturbations follows the decay for a while. During this phase, the damping occurs.

$$h_{\boldsymbol{k}}^{\lambda} \simeq \frac{4\mathcal{S}_{\boldsymbol{k}}^{\lambda}}{k^2}$$

(The source and h are damping.)

For $2\eta_R \lesssim \eta$, the source decays rapidly and the time derivative of the tensor perturbations dominates the E.o.M. Then, tensor perturbations decouple from the source.

$$h_{\boldsymbol{k}}^{\lambda} \simeq A_{\mathrm{dec}} \frac{a_{\mathrm{dec}}}{a} e^{ik\eta}$$

(Freely propagating GWs)

Amplitude at the decoupling

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- <u>eMD effects in sudden reheating scenario</u>
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Toy model for sudden reheating

Here, we consider the sudden reheating scenario, where the transition time scale is much shorter than the Hubble time scale at that time.

Toy model:

$$\mathcal{L} = -\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2}\partial^\mu \chi \partial_\mu \chi - \frac{1}{2}\partial^\mu \tau \partial_\mu \tau - V$$

$$V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\tau^2 + \frac{\lambda}{4}\tau^2\chi^2 + \frac{c}{2}M\phi\chi^2$$

ϕ : Dominant field (such as inflaton or curvaton)

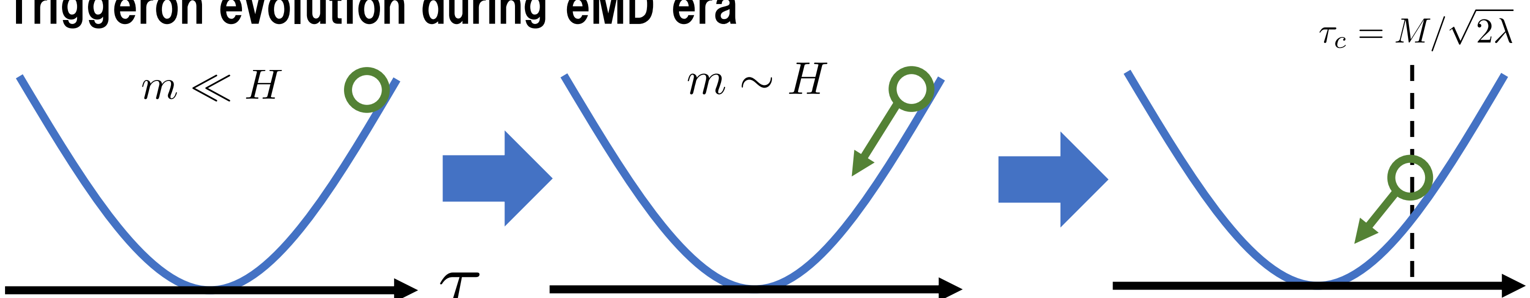
τ : Trigger field ("triggeron")

χ : Field denoting the daughter particles of ϕ

$$\Gamma = \frac{c^2 M}{32\pi}\sqrt{1-\frac{m^2_{\chi,\text{eff}}}{(M/2)^2}}\,\Theta\left(M^2 - 4m^2_{\chi,\text{eff}}\right)$$ (Decay rate of $\phi \to 2\chi$) $\quad m^2_{\chi,\text{eff}} = \langle \lambda\tau^2/2 \rangle$

We take $M \gg m$ and initial condition of τ as $\tau_0 \gg M$.

Triggeron evolution during eMD era

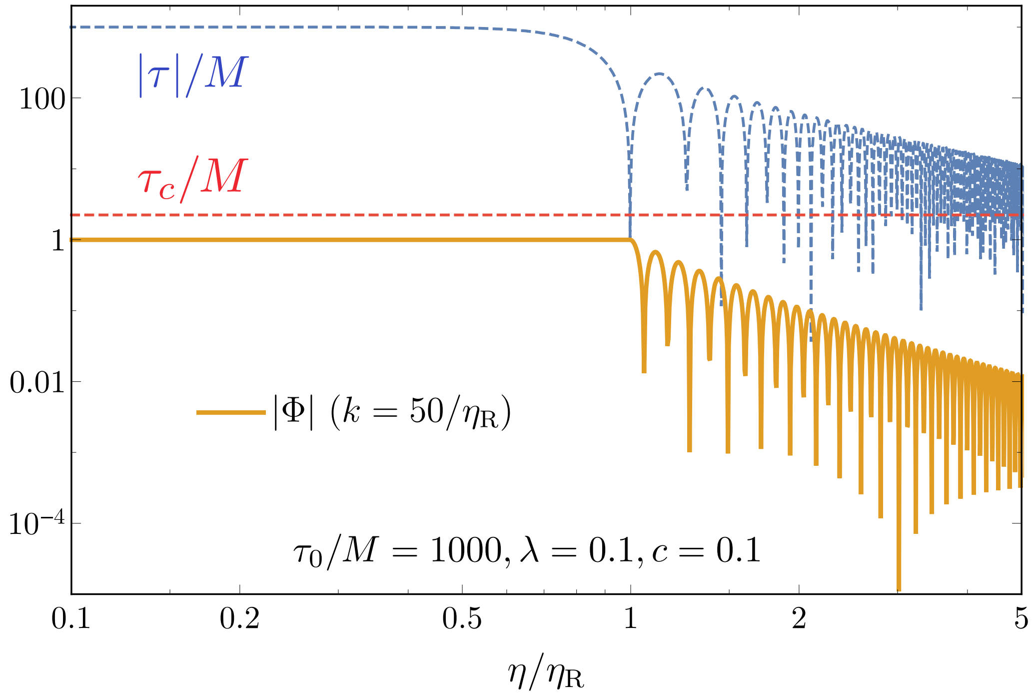


At first, decay of ϕ is kinematically blocked.

τ starts to roll at m~H.

When τ passes by τ_c, the decay channel of ϕ opens and reheating occurs instantaneously.

Evolutions in sudden reheating



There is no damping in Φ during the transition in the sudden reheating scenario.
After the transition, perturbations that enters the horizon during eMD era oscillates with the time scale shorter than the Hubble time at that time (kη≫ 1).

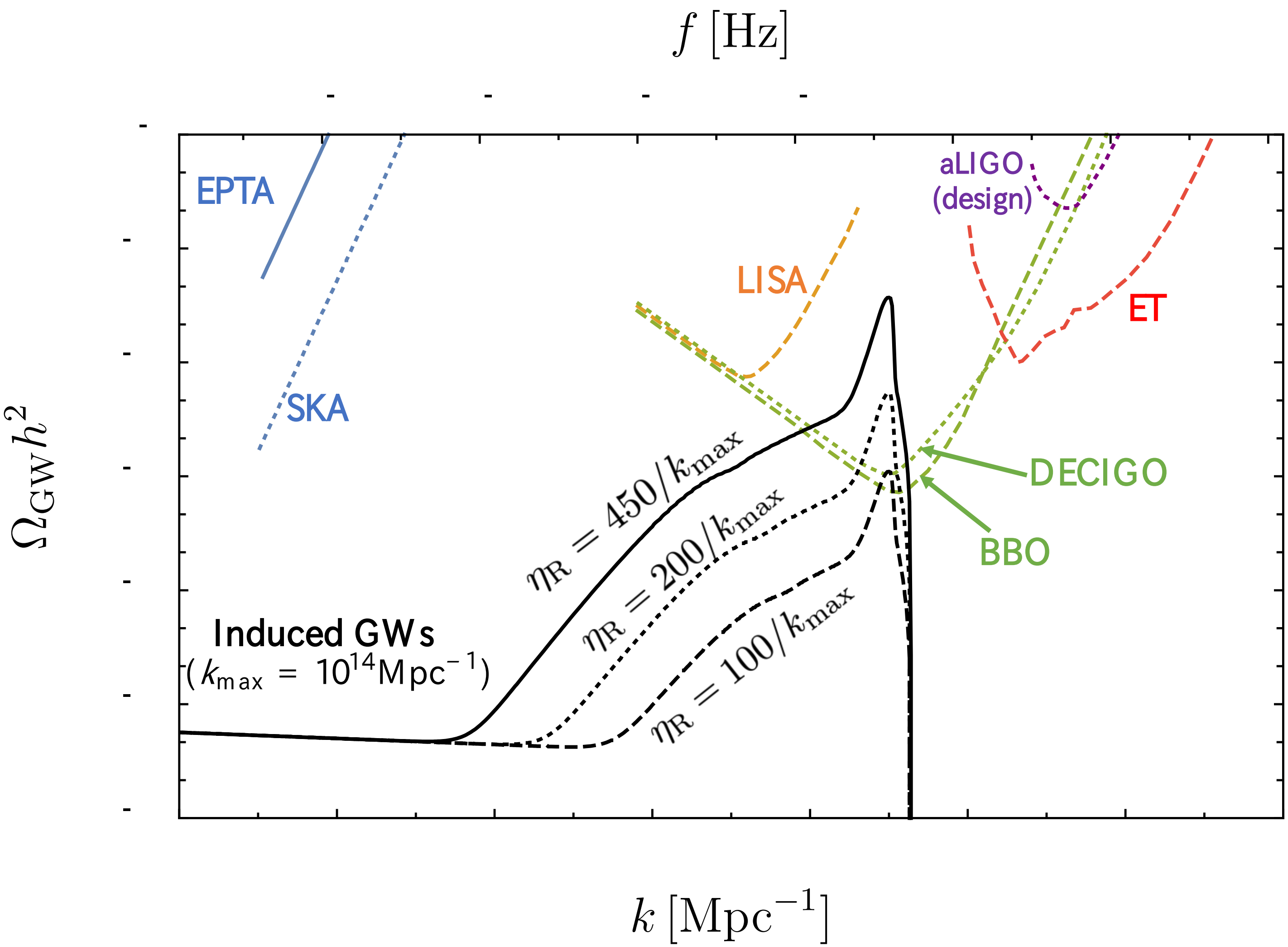
Enhance GWs !

Source term

Dominant

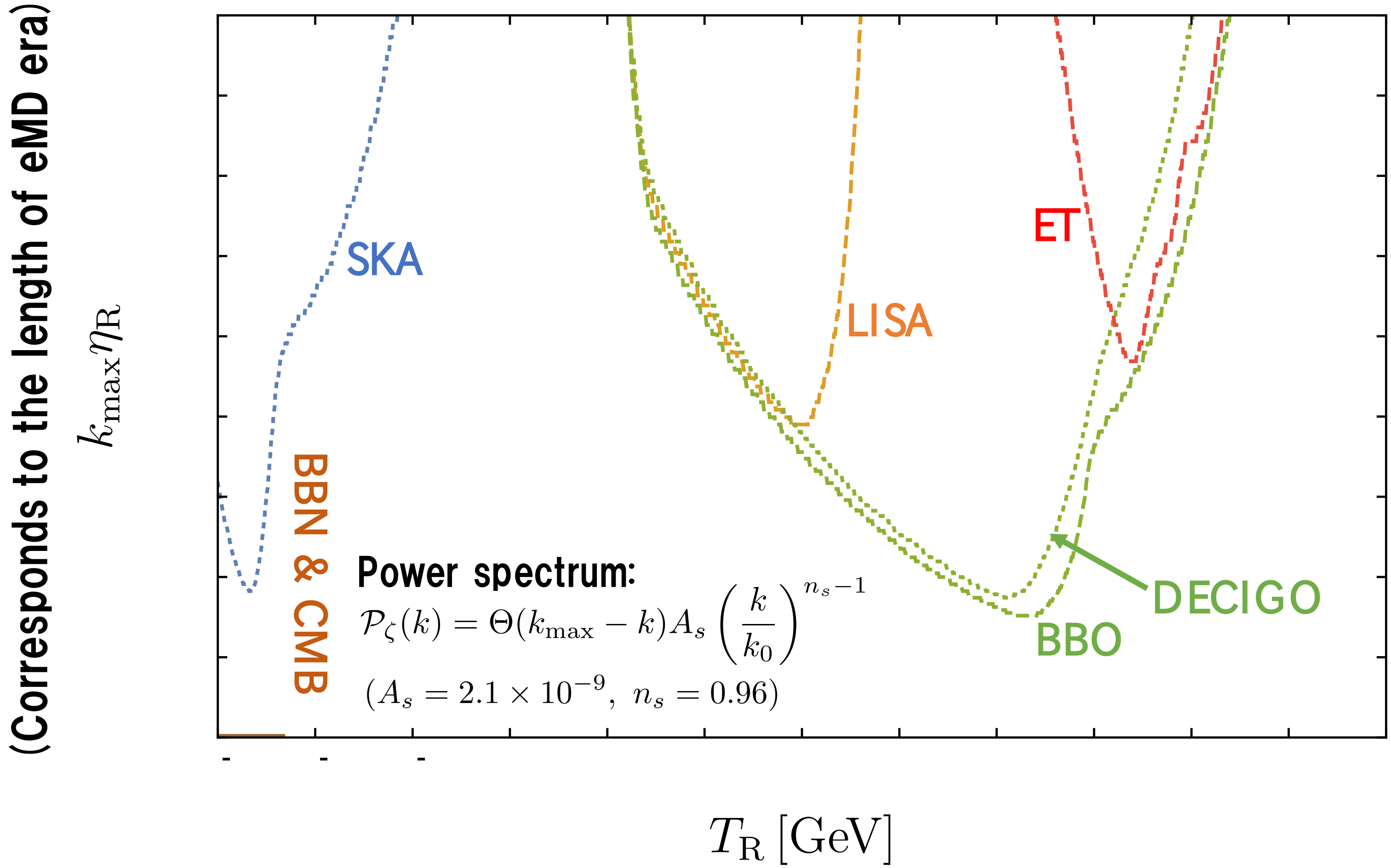
$$\mathcal{S}^\lambda(k,\eta) = \int \frac{\mathrm{d}^3k'}{(2\pi)^3} \mathrm{e}^{\lambda lm}(\hat{k}) k'_l k'_m \left[2\Phi_{k'}(\eta)\Phi_{k-k'}(\eta) + \frac{4}{3(1+w)} \left(\Phi_{k'}(\eta) + \frac{\Phi'_{k'}(\eta)}{\mathcal{H}}\right) \left(\Phi_{k-k'}(\eta) + \frac{\Phi'_{k-k'}(\eta)}{\mathcal{H}}\right)\right]$$

Induced GWs



$$\mathcal{P}_\zeta(k) = \Theta(k_{max} - k) A_s \left(\frac{k}{k_0}\right)^{n_s - 1} \quad A_s = 2.1 \times 10^{-9}, \quad n_s = 0.96$$

Future constraints on reheating temperature



The future observations could constrain the wide range of reheating temperature in the sudden reheating scenario.

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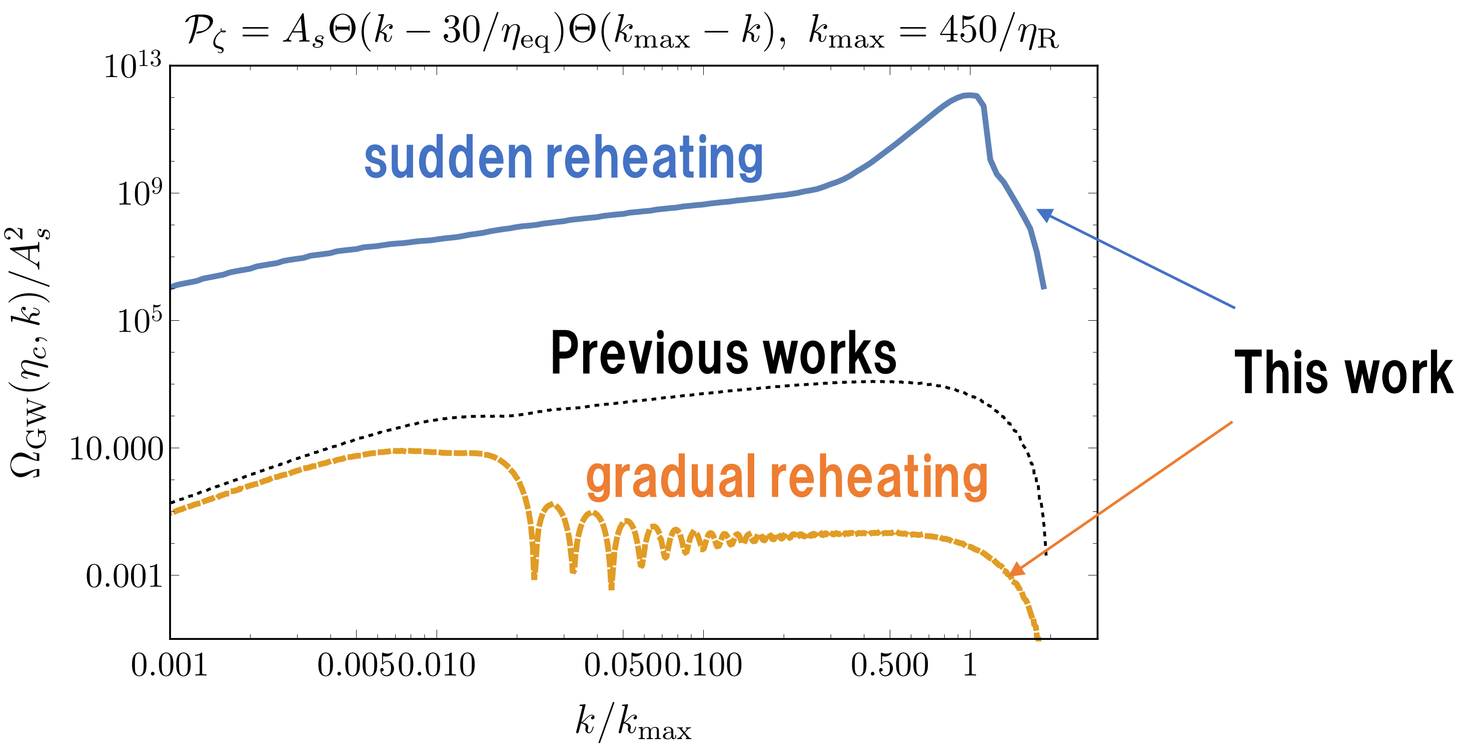
Summary of this talk



We have revisited the effect of the early matter dominated era on the gravitational waves induced by curvature perturbations, taking into account the perturbations throughout the transition.

As a result, we have found that the induced GWs strongly depend on how quickly the reheating occurs.

Backup

Foregrounds

In future, stochastic GWs of astrophysical origin or cosmological origins that are different from the induced GWs, such as those from phase transition, may be detected.
In that case, the constraints would become weak.

Foreground example:

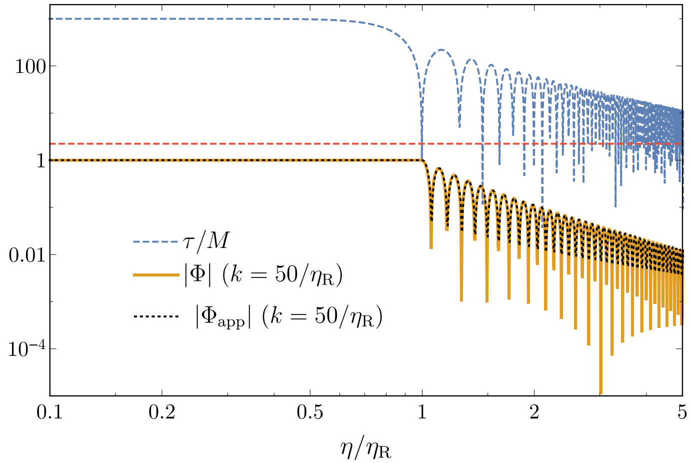
Around PTA target frequency
GWs from supermassive-black-hole binaries

Around LISA target frequency
GWs from galactic white-dwarf binaries

Around BBO target frequency
GWs from neutron star binaries

We may use non-Gaussianity of the induced GWs to discriminate them from other sources. (Bartolo et al., 2018)

Evolutions in sudden reheating



$$\Phi_{\rm app}(x, x_{\rm R}) = \begin{cases} 1 & (\text{for } \eta \leq \eta_{\rm R}) \\ A(x_{\rm R})\mathcal{J}(x) + B(x_{\rm R})\mathcal{Y}(x) & (\text{for } \eta \geq \eta_{\rm R}) \end{cases}$$

J and Y are the independent solutions of the following equation for Φ during RD era.

$$\Phi'' + 3(1+w)\mathcal{H}\Phi' + wk^2\Phi = 0 \quad (w = 1/3)$$

$$\mathcal{J}(x) = \frac{3\sqrt{3}\, j_1\left(\frac{x - x_{\rm R}/2}{\sqrt{3}}\right)}{x - x_{\rm R}/2} \qquad j_1(x) = \frac{\sin x - x\cos x}{x^2}$$

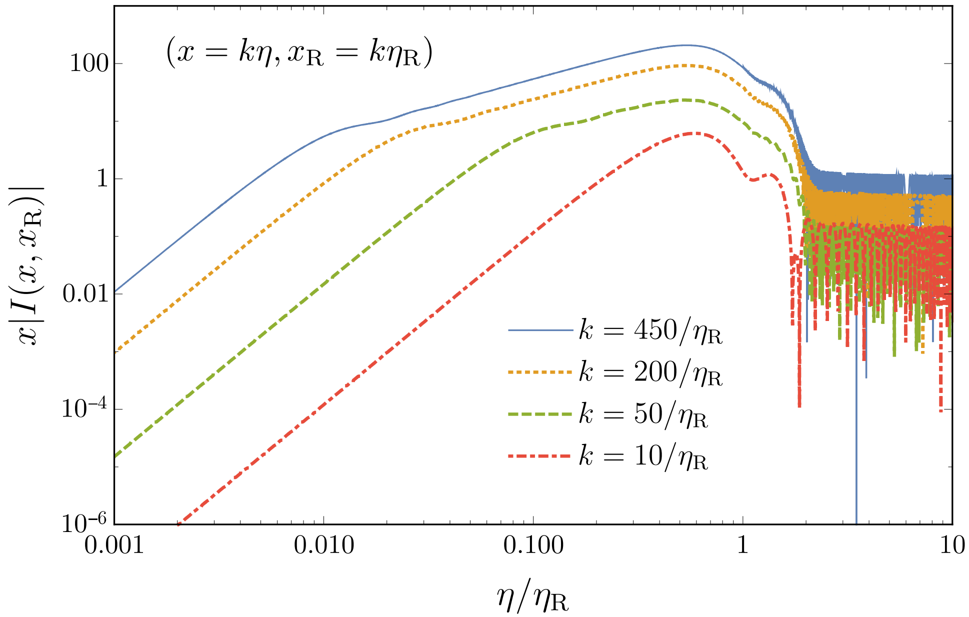
$$\mathcal{Y}(x) = \frac{3\sqrt{3}\, y_1\left(\frac{x - x_{\rm R}/2}{\sqrt{3}}\right)}{x - x_{\rm R}/2} \qquad y_1(x) = -\frac{\cos x + x\sin x}{x^2}$$

A and B are determined so that Φ and Φ′ are continuous at $\eta_{\rm R}$.

$$A(x_{\rm R}) = \frac{1}{\mathcal{J}(x_{\rm R}) - \frac{\mathcal{Y}(x_{\rm R})}{\mathcal{Y}'(x_{\rm R})}\mathcal{J}'(x_{\rm R})}$$

$$B(x_{\rm R}) = -\frac{\mathcal{J}'(x_{\rm R})}{\mathcal{Y}'(x_{\rm R})}A(x_{\rm R})$$

Why does the damping occur?



$$\overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \times \overline{I^2(k\eta,k\eta_{\rm R})}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)} \propto x^2\overline{I^2(x,x_{\rm R})}$$

$$x_{\rm R}|I(x_{\rm R},x_{\rm R})| \sim x|I_{\rm eMD}(x,x_R)|$$

$$x|I(x,x_{\rm R})| = x|I_{\rm eMD}(x,x_{\rm R})| + x|I_{\rm RD}(x,x_{\rm R})| \quad (\eta > \eta_{\rm R})$$

$I_{\rm eMD}$: Describing the GWs induced during eMD era

$I_{\rm RD}$: Describing the GWs induced during RD era

E.o.M for tensor perturbations:

$$h_{\boldsymbol{k}}^{\lambda\,\prime\prime} + 2\mathcal{H}h_{\boldsymbol{k}}^{\lambda\,\prime} + k^2 h_{\boldsymbol{k}}^\lambda = 4\mathcal{S}_{\boldsymbol{k}}^\lambda$$

$$\Rightarrow$$

$$h_{\boldsymbol{k}}^\lambda \simeq \frac{4\mathcal{S}_{\boldsymbol{k}}^\lambda}{k^2} \quad (\eta \lesssim 2\eta_{\rm R})$$

$$h_{\boldsymbol{k}}^\lambda \simeq \frac{e^{ik\eta}}{a} \quad (\eta \gtrsim 2\eta_{\rm R})$$

From this result, we can see the main reason of the cancellation between the components comes from the behaviors of Φ around the transition.

Stochastic gravitational waves

LIGO and Virgo have detected gravitational waves (GWs) induced by the mergers of BHs and NSs.
The direct detection of GWs has opened the door to a new era of astronomy and cosmology!

In this work, we discuss the early Universe in terms of GWs.

Image of GWs from mergers

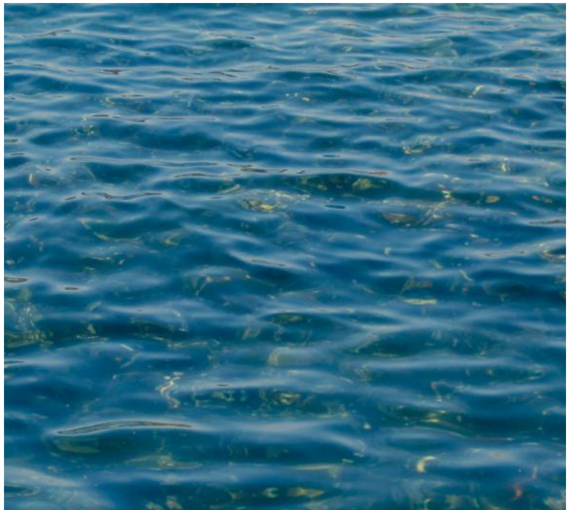


Image of Stochastic GWs

In particular, we focus on stochastic GWs.

Stochastic GWs:
GWs created by superposition of a large number of independent sources.

Astrophysical source

Mergers of compact objects, such as BHs, NSs, and WDs

Cosmological source

Vacuum fluctuation during inflation
Cosmic strings
Second order scalar perturbations

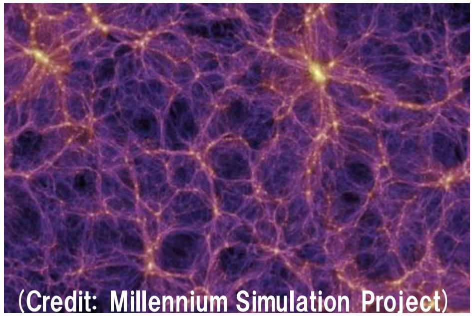
We focus on this !

Scalar perturbation in Cosmology

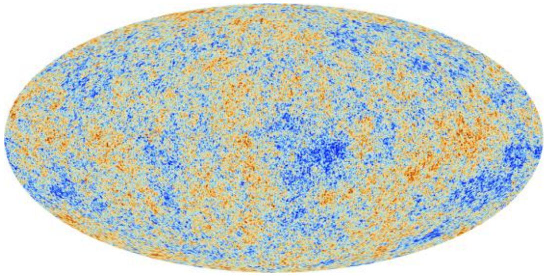
Scalar perturbations induce the perturbations of energy density ($\delta\rho/\rho$).

Scalar perturbations are origins of many things!

Examples



Large Scale Structure



(Credit: ESA and the Planck Collaboration)

CMB perturbations

Galaxies

From the observations, we already know the amplitude of the scalar perturbations.

$$\mathcal{P}_\zeta = 2.1 \times 10^{-9}$$
$$(\delta\rho/\rho \sim 10^{-5})$$

(Planck 2018)

$$\mathcal{P}_\zeta = \frac{9}{4}\mathcal{P}_\Phi$$

curvature perturbation scalar perturbation

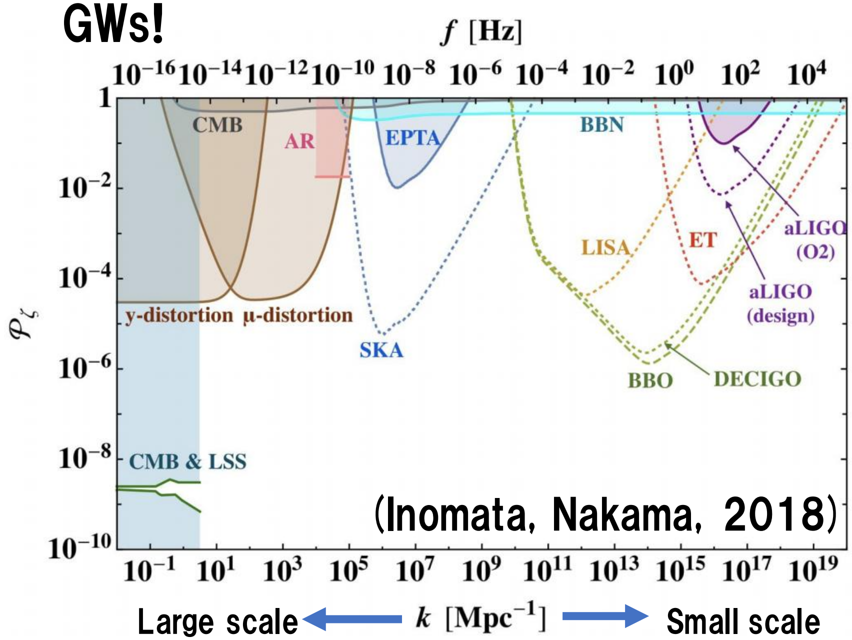
Scalar perturbations originate from vacuum fluctuations of an inflaton during the inflation era.

Motivations for the induced GWs

The frequencies of the induced GWs correspond to the scales of the scalar perturbations inducing GWs.

$(\lambda \lesssim 1\,\mathrm{Mpc})$

We can investigate small-scale (scalar) perturbations using the induced GWs!



The future and current GW projects can investigate the power spectrum of curvature perturbations.

(The left figure is just an advertisement of our previous work, arXiv: 1812.00674.)

Since small-scale perturbations enter the horizon early, the induced GW could be a good probe of the early Universe.

Main question of this work

What kind of effects does the early matter dominated era make on the induced GWs?

GWs induced by scalar perturbations

Metric perturbations: (Newtonian gauge, neglecting anisotropic stress)

Scalar perturbations (related to curvature perturbations)

$$ds^2 = a^2\left[-(1+2\Phi)d\eta^2 + 2V_i d\eta dx^i + \left((1-2\Phi)\delta_{ij} + \frac{1}{2}h_{ij}\right)dx^i dx^j\right]$$

Vector perturbations (neglect in the following)

Tensor perturbations (GWs) $h^i_i = 0$, $\partial_i h^i_j = 0$

Substituting this metric into the Einstein tensor, we derive the following expression up to second order,

$$G^i_j = a^{-2}\left[\frac{1}{4}(h^{i\prime\prime}_j + 2\mathcal{H}h^{i\prime}_j - \Delta h^i_j) + 4\Phi\partial^i\partial_j\Phi + 2\partial^i\Phi\partial_j\Phi + A^i_j\right]$$

Energy momentum tensor is written as

$$T^i_j = (\rho + P)\delta u^{,i}\delta u_{,j} + (P+\delta P)\delta^i_j$$

Einstein eq.

$$G_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}$$

$(c=1, M_{\rm Pl} = 1/\sqrt{8\pi G})$

Terms irrelevant to tensor perturbations $(\hat{\mathcal{T}}_{ij}{}^{lm}A_{lm} = 0)$

$$\delta u_{,i} = -\frac{2M_{\rm Pl}^2}{a(\rho+P)}\partial_i(\Phi' + \mathcal{H}\Phi)$$ (from linear perturbation theory)

From Einstein eq., we derive the following equation for tensor perturbations

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \Delta h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm}$$

$\hat{\mathcal{T}}_{ij}{}^{lm}$: Projection operator onto the transverse and traceless tensor

$$S_{ij} \equiv 4\Phi\partial_i\partial_j\Phi + 2\partial_i\Phi\partial_j\Phi - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Phi' + \mathcal{H}\Phi)\partial_j(\Phi' + \mathcal{H}\Phi)$$ $(w = P/\rho)$

Equations for tensor perturbations

E.o.M for tensor perturbation in real space:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \Delta h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm}$$

We perform the Fourier transform.

$$h_{ij}(\eta, \boldsymbol{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \left(e^{+}_{ij}(\boldsymbol{k}) h^{+}_{\boldsymbol{k}} + e^{\times}_{ij}(\boldsymbol{k}) h^{\times}_{\boldsymbol{k}}\right) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

Then we derive the following equation:

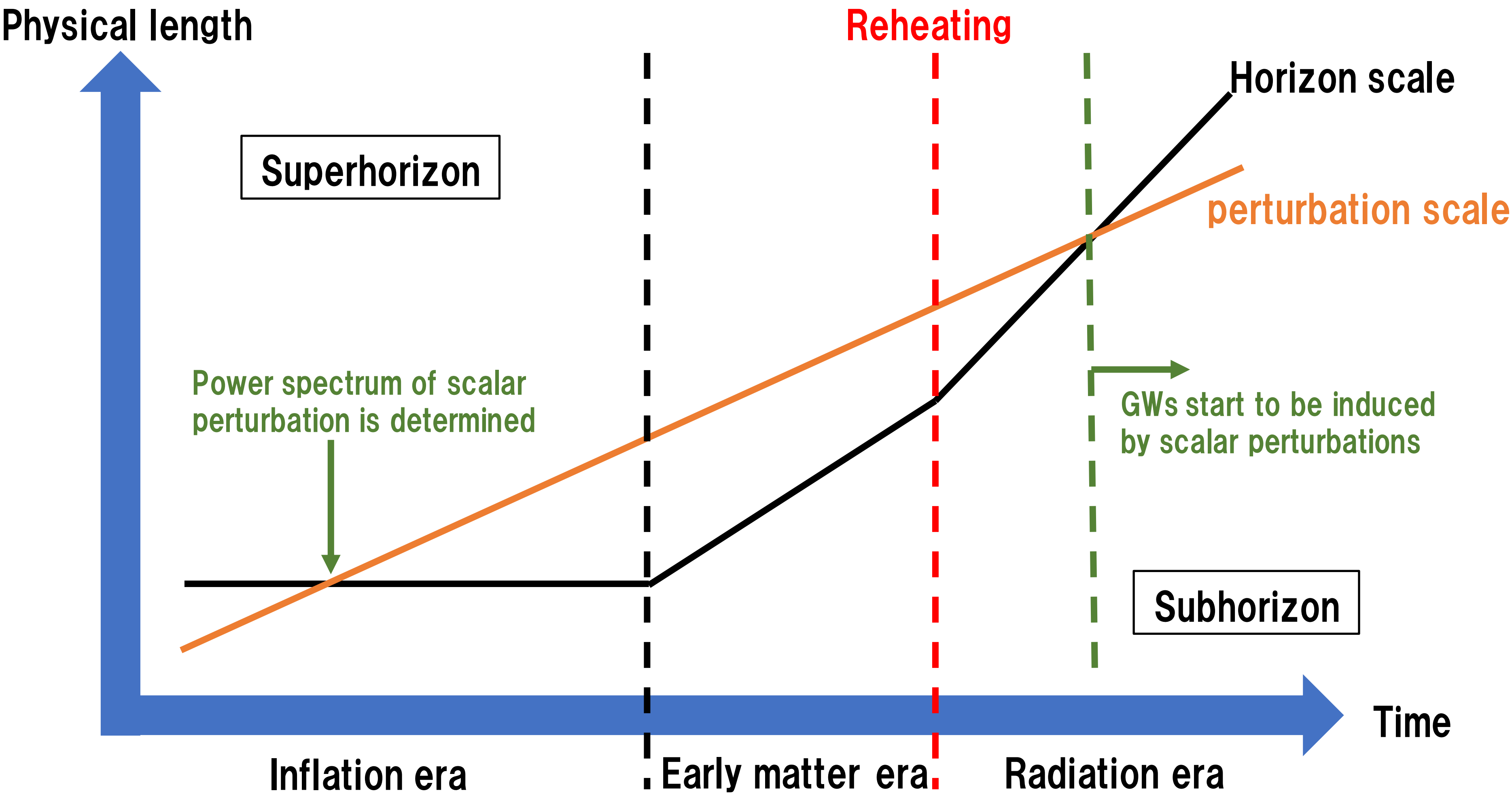
$$h^{\lambda\,\prime\prime}_{\boldsymbol{k}} + 2\mathcal{H}h^{\lambda\,\prime}_{\boldsymbol{k}} + k^2 h^{\lambda}_{\boldsymbol{k}} = 4\mathcal{S}^{\lambda}(\boldsymbol{k}, \eta) \qquad (\lambda : +, \times)$$

Source term from second order scalar perturbations

$$\mathcal{S}^{\lambda}(\boldsymbol{k}, \eta) = \int \frac{\mathrm{d}^3 k'}{(2\pi)^3} \mathrm{e}^{\lambda\, lm}(\hat{\boldsymbol{k}}) k'_l k'_m \left[2\Phi_{\boldsymbol{k}'}(\eta)\Phi_{\boldsymbol{k}-\boldsymbol{k}'}(\eta) + \frac{4}{3(1+w)} \left(\Phi_{\boldsymbol{k}'}(\eta) + \frac{\Phi'_{\boldsymbol{k}'}(\eta)}{\mathcal{H}}\right)\left(\Phi_{\boldsymbol{k}-\boldsymbol{k}'}(\eta) + \frac{\Phi'_{\boldsymbol{k}-\boldsymbol{k}'}(\eta)}{\mathcal{H}}\right)\right]$$

Point At linear order, scalar perturbations do not induce GWs.
However,
at second order, scalar perturbations can induce GWs.

Evolutions of perturbations

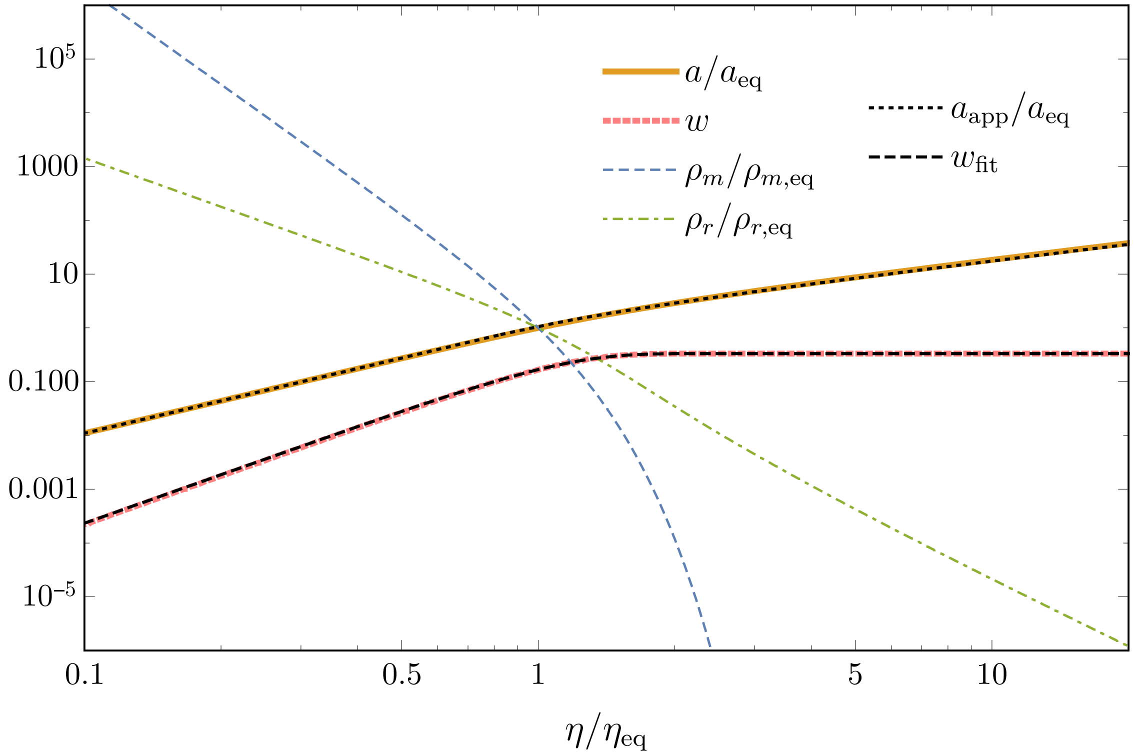


Outline

- General Introduction
- Introduction of this work
- eMD effects in gradual reheating scenario
- eMD effects in sudden reheating scenario
- Summary

Background in gradual reheating

The evolutions of background quantities.



$$w \equiv P/\rho$$

$$\rho_m = \rho_r \text{ at } \eta = \eta_{\rm eq}$$

The fitting formula for a:

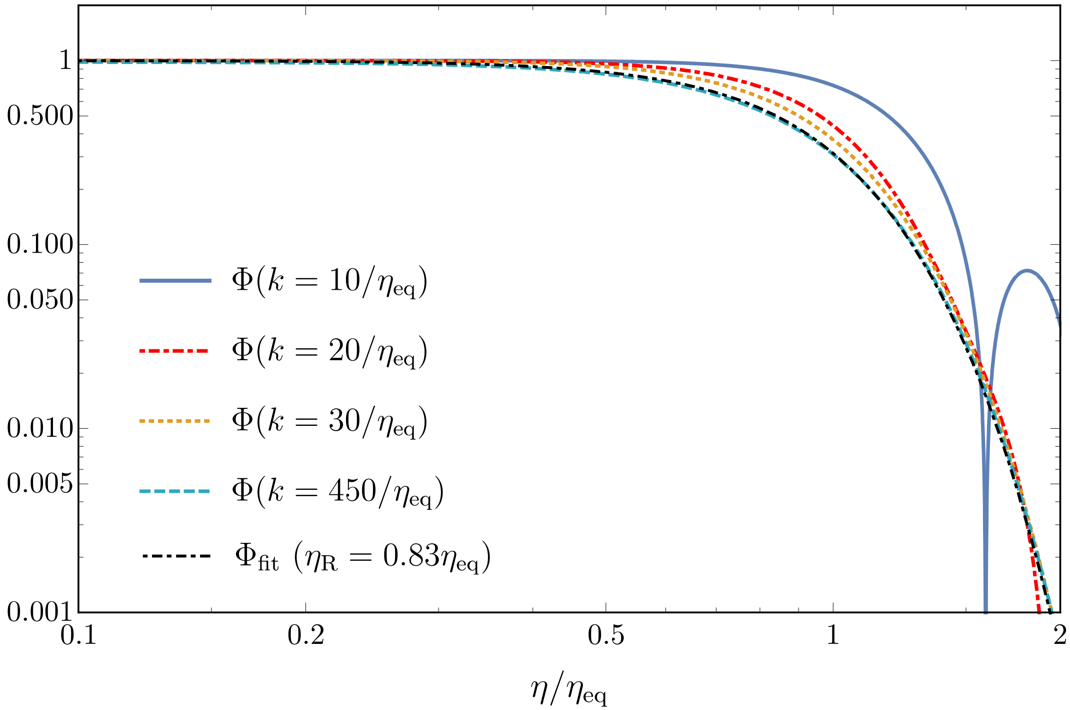
$$\frac{a_{\rm app}(\eta)}{a_{\rm app}(\eta_{\rm R})} = \begin{cases} \left(\frac{\eta}{\eta_{\rm R}}\right)^2 & (\eta < \eta_{\rm R}) \\ 2\frac{\eta}{\eta_{\rm R}} - 1 & (\eta \geq \eta_{\rm R}) \end{cases}$$

The fitting formula for w:

$$w_{\rm fit} = \frac{1}{3}\left(1 - \exp\left(-0.7\left(\frac{\eta}{\eta_{\rm eq}}\right)^3\right)\right)$$

In the above figure, we take $\eta_{\rm R} = 0.83\,\eta_{\rm eq}$.

Limits of applications of the formulas



If we consider the scale entering the horizon around the transition ($k<30/\eta_{\rm eq}$), the fitting formula does not fit the numerical result very well.

To avoid high computational costs, we neglect the contribution from the perturbations ($k<30/\eta_{\rm eq}$) in the following. (The analysis for $1/\eta_{\rm eq}<k<$ $k<30/\eta_{\rm eq}$ is future work.)

In addition, the perturbations entering much before the transition can be non-linear during eMD, to which our analysis cannot be applied.

Nonlinear scale: $$k_{\rm NL} \sim \sqrt{\frac{5}{2}} \mathcal{P}_\zeta^{-1/4} \mathcal{H}(\eta_{\rm R}) \sim 470/\eta_{\rm R}.$$

As a concrete example, we consider

$$\mathcal{P}_\zeta = A\,\Theta(k-30/\eta_{\rm eq})\Theta(450/\eta_{\rm R}-k) \qquad (\eta_{\rm R}=0.83\eta_{\rm eq})$$

Formulas we use to calculate induced GWs

Summary of the formulas we use: (Kohri, Terada, 2018)

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)}$$

Energy density of the induced GWs

$$\overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \times \overline{I^2(v,u,k\eta)}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

Power spectrum of the induced GWs

Here, we assume that the scale factor is given as, (because this fits the numerical result well)

$$\frac{a_{\rm app}(\eta)}{a_{\rm app}(\eta_{\rm R})} = \begin{cases} \left(\frac{\eta}{\eta_{\rm R}}\right)^2 & (\eta < \eta_{\rm R}) \\ 2\frac{\eta}{\eta_{\rm R}} - 1 & (\eta \geq \eta_{\rm R}) \end{cases}$$

Then we can express I as $(x = k\eta, x_{\rm R} = k\eta_{\rm R})$

$$\begin{aligned} I(u,v,x,x_{\rm R}) =& \int_0^{x_{\rm R}} {\rm d}\bar{x} \left(\frac{1}{2(x/x_{\rm R})-1}\right)\left(\frac{\bar{x}}{x_{\rm R}}\right)^2 kG_k^{\rm eMD\to RD}(\eta,\bar{\eta}) f(u,v,\bar{x},x_{\rm R}) \\ &+ \int_{x_{\rm R}}^{x} {\rm d}\bar{x} \left(\frac{2(\bar{x}/x_{\rm R})-1}{2(x/x_{\rm R})-1}\right) kG_k^{\rm RD}(\eta,\bar{\eta}) f(u,v,\bar{x},x_{\rm R}) \\ \equiv& I_{\rm eMD}(u,v,x,x_{\rm R}) + I_{\rm RD}(u,v,x,x_{\rm R}) \end{aligned}$$

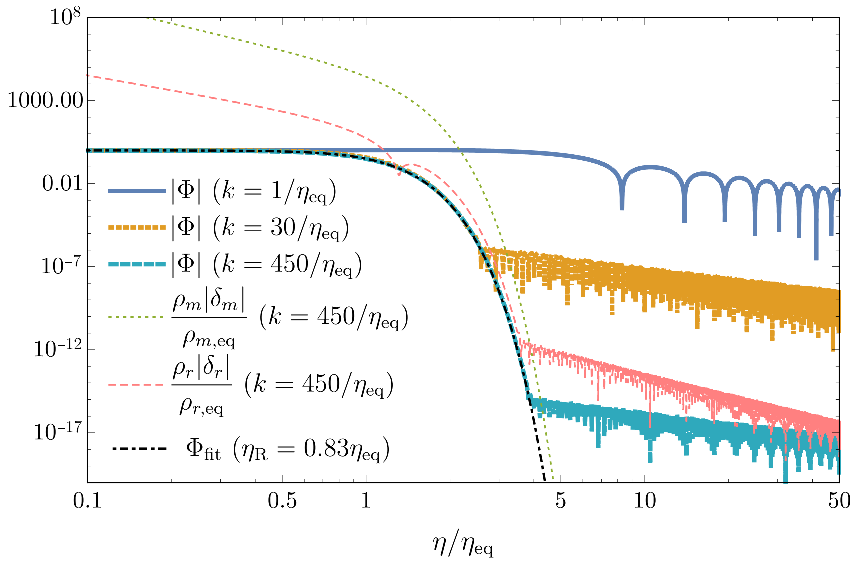
$(\Phi(x\to 0) = 1,\ \Phi'(x) = \partial\Phi(x)/\partial\eta)$

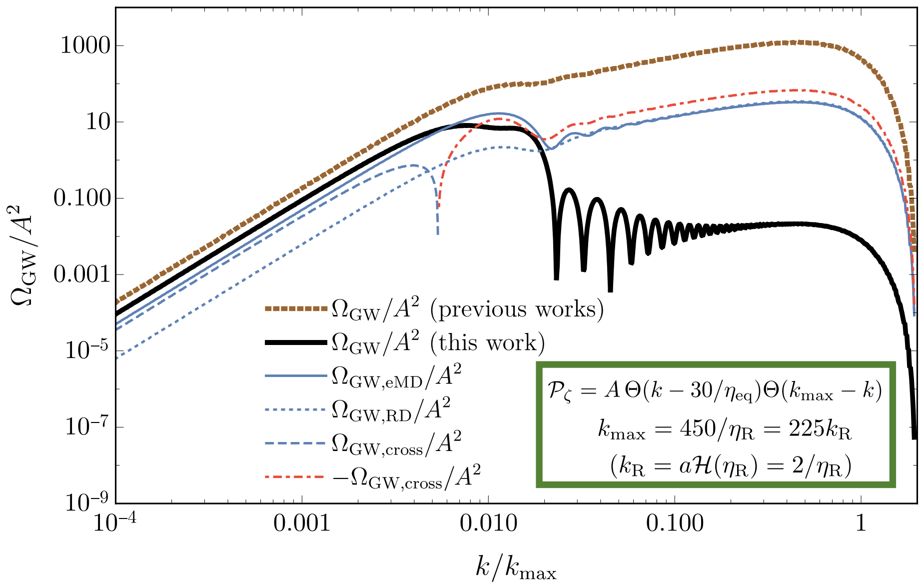
$$f(u,v,\bar{x},x_{\rm R}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}$$

Summary for gradual reheating

We have revisited the effects of eMD era on the induced GWs in a gradual reheating scenario, taking into account the evolution of Φ, the source of induced GWs, during the transition.

As a result, we have found that the induced GWs are suppressed, compared to the previous results.





Formulas we use

Summary of the formulas we use: (Kohri, Terada, 2018)

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)}$$

Energy density of the induced GWs

$$\overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \times \overline{I^2(v,u,k\eta)}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

Power spectrum of the induced GWs

$$\begin{aligned} I(u,v,x,x_{\rm R}) =& \int_0^{x_{\rm R}} {\rm d}\bar{x} \left(\frac{1}{2(x/x_{\rm R})-1}\right)\left(\frac{\bar{x}}{x_{\rm R}}\right)^2 kG_k^{\rm eMD\to RD}(\eta,\bar{\eta}) f(u,v,\bar{x},x_{\rm R}) \\ &+ \int_{x_{\rm R}}^{x} {\rm d}\bar{x}\left(\frac{2(\bar{x}/x_{\rm R})-1}{2(x/x_{\rm R})-1}\right) kG_k^{\rm RD}(\eta,\bar{\eta}) f(u,v,\bar{x},x_{\rm R}) \\ \equiv& I_{\rm eMD}(u,v,x,x_{\rm R}) + I_{\rm RD}(u,v,x,x_{\rm R}) \end{aligned}$$

$$(x = k\eta,\, x_{\rm R} = k\eta_{\rm R})$$

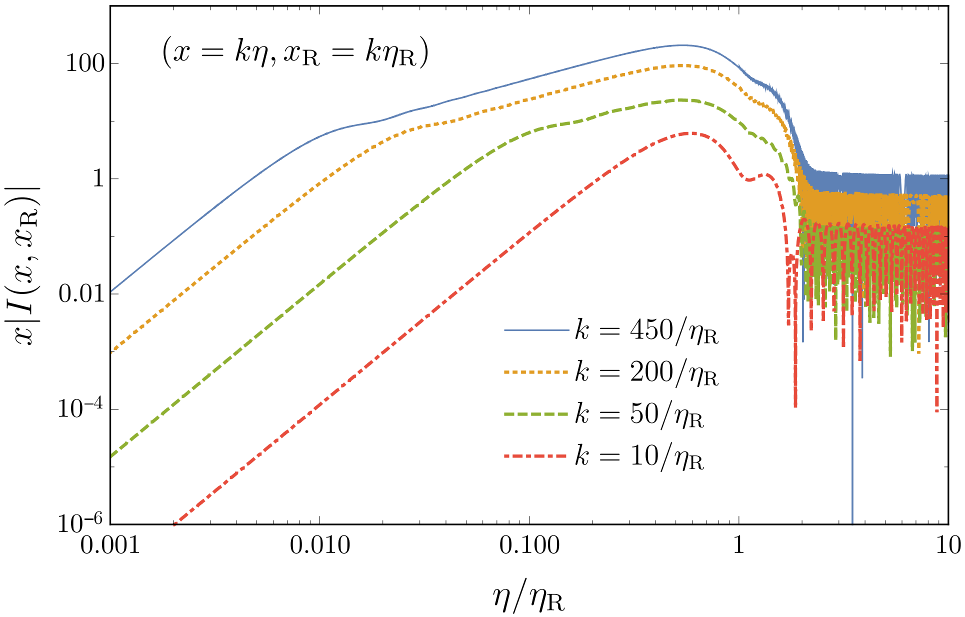
$$(\Phi(x\to 0) = 1,\ \Phi'(x) = \partial\Phi(x)/\partial\eta)$$

$$\boxed{f(u,v,\bar{x},x_{\rm R}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}}$$

Substitute

$$\Phi(x,x_{\rm R}) = \begin{cases} 1 & (\text{for } x \le x_{\rm R}) \\ A(x_{\rm R})\mathcal{J}(x) + B(x_{\rm R})\mathcal{Y}(x) & (\text{for } x \ge x_{\rm R}) \end{cases}$$

Why does the cancellation occur?



$$\overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \times \overline{I^2(k\eta,k\eta_\mathrm{R})}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

$$\Omega_\mathrm{GW}(\eta,k) = \frac{\rho_\mathrm{GW}(\eta,k)}{\rho_\mathrm{tot}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)} \propto x^2\overline{I^2(x,x_\mathrm{R})}$$

$$x_\mathrm{R}|I(x_\mathrm{R},x_\mathrm{R})| \sim x|I_\mathrm{eMD}(x,x_R)|$$

$$x|I(x,x_\mathrm{R})| = x|I_\mathrm{eMD}(x,x_\mathrm{R})| + x|I_\mathrm{RD}(x,x_\mathrm{R})| \quad (\eta > \eta_\mathrm{R})$$

I_eMD : **Describing the GWs induced during eMD era**

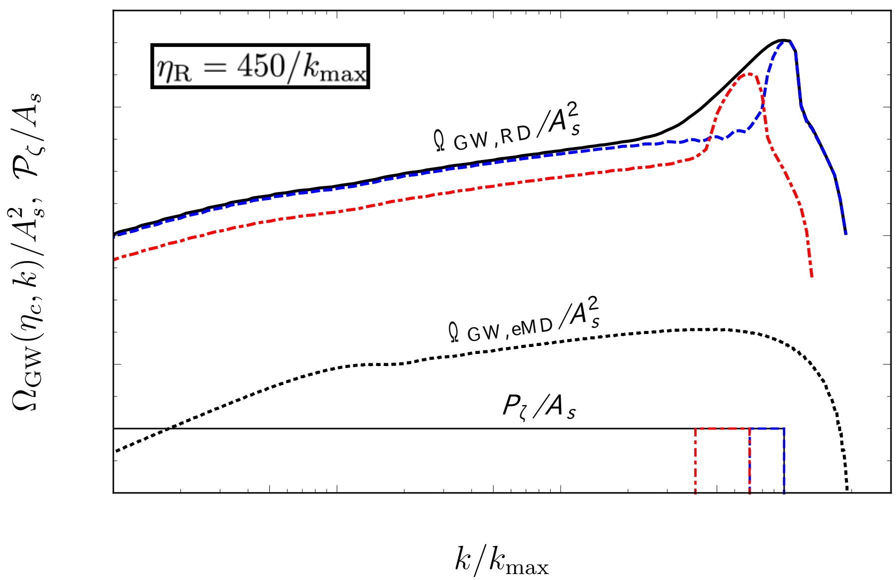
I_RD : **Describing the GWs induced during RD era**

$$I(x,x_\mathrm{R}) = \int_0^x \mathrm{d}\bar{x}\frac{a(\bar\eta)}{a(\eta)}kG_k(\eta,\bar\eta)f(\bar{x},x_\mathrm{R}) = \begin{cases} \int_0^{x_\mathrm{R}} \mathrm{d}\bar{x}\left(\frac{\bar{x}}{x}\right)^2 kG_k^\mathrm{eMD}(\eta,\bar\eta)f(\bar{x},x_\mathrm{R}) & (\eta \leq \eta_\mathrm{R}) \\ \int_0^{x_\mathrm{R}} \mathrm{d}\bar{x}\left(\frac{1}{2(x/x_\mathrm{R})-1}\right)\left(\frac{\bar{x}}{x_\mathrm{R}}\right)^2 kG_k^{\mathrm{eMD}\to\mathrm{RD}}(\eta,\bar\eta)f(\bar{x},x_\mathrm{R}) & \\ \quad + \int_{x_\mathrm{R}}^x \mathrm{d}\bar{x}\left(\frac{2(\bar{x}/x_\mathrm{R})-1}{2(x/x_\mathrm{R})-1}\right)kG_k^\mathrm{RD}(\eta,\bar\eta)f(\bar{x},x_\mathrm{R}) & (\eta > \eta_\mathrm{R}) \end{cases}$$

$$kG_k^\mathrm{RD}(\eta,\bar\eta) = \sin(x-\bar{x})$$
$$kG_k^{\mathrm{eMD}\to\mathrm{RD}}(\eta,\bar\eta) = C(x,x_\mathrm{R})\bar{x}j_1(\bar{x}) + D(x,x_\mathrm{R})\bar{x}y_1(\bar{x})$$
$$kG_k^\mathrm{eMD}(\eta,\bar\eta) = -x\bar{x}(j_1(x)y_1(\bar{x}) - y_1(x)j_1(\bar{x}))$$

$$C(x,x_\mathrm{R}) = \frac{\sin x - 2x_\mathrm{R}(\cos x + x_\mathrm{R}\sin x) + \sin(x-2x_\mathrm{R})}{2x_\mathrm{R}^2}$$
$$D(x,x_\mathrm{R}) = \frac{(2x_\mathrm{R}^2-1)\cos x - 2x_\mathrm{R}\sin x + \cos(x-2x_\mathrm{R})}{2x_\mathrm{R}^2}$$

Results of Induced GWs



$\Omega_{GW,RD}$: **GWs induced during RD era**

$\Omega_{GW,eMD}$: **GWs induced during eMD era**

$\mathcal{P}_\zeta(k) = A_s\Theta(k_{max} - k)$ **(Black)**

$\mathcal{P}_\zeta(k) = A_s\Theta(k - 0.7k_{max} - k)\Theta(k_{max} - k)$ **(Blue)**

$\mathcal{P}_\zeta(k) = A_s\Theta(k - 0.4k_{max} - k)\Theta(0.7k_{max} - k)$ **(Red)**

The GWs are mainly induced by the perturbations around the smallest scales during the RD era.

Source function: $(\Phi'(x) = \partial\Phi(x)/\partial\eta)$

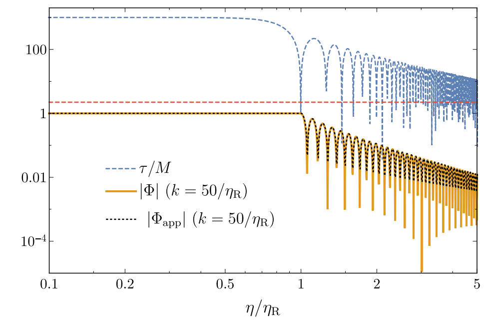
Dominant

$$f(u,v,\bar{x},x_R) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}$$

After the transition, the gravitational potential that enters the horizon during eMD era oscillates faster than the Hubble scale.

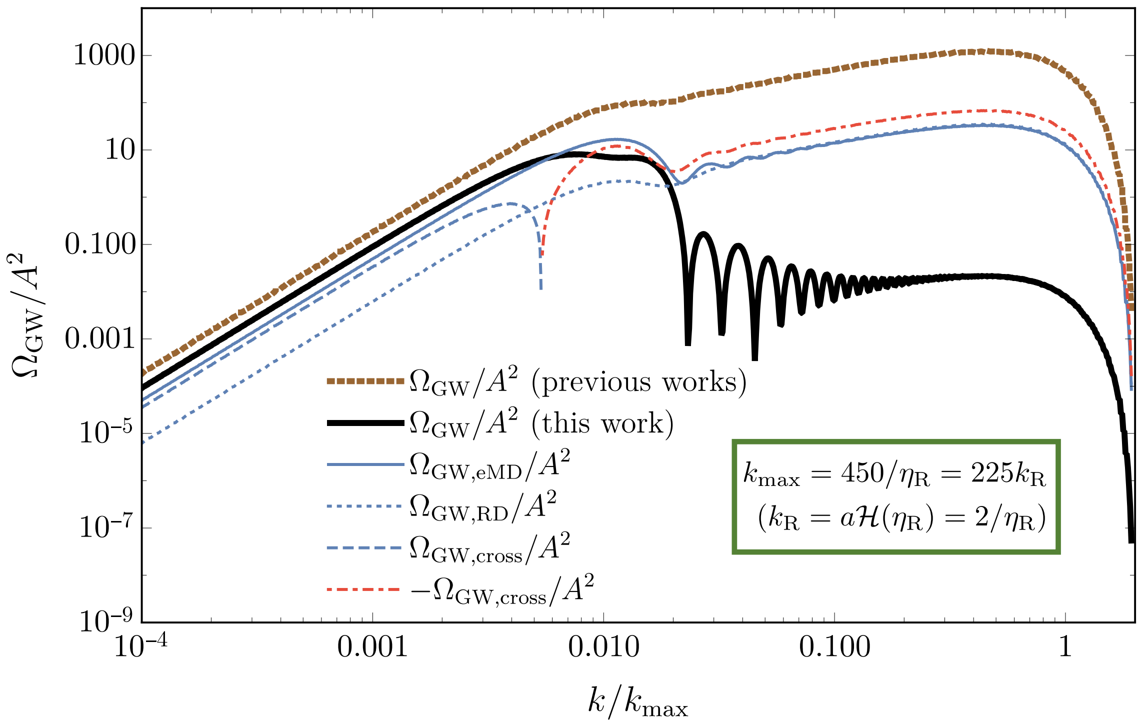
The smaller the scale is, the faster the oscillation frequency is.

The contribution from the smallest scale is dominant.



Results

GWs induced by curvature perturbations with $\mathcal{P}_\zeta = A\,\Theta(k - 30/\eta_{\rm eq})\Theta(k_{\rm max} - k)$



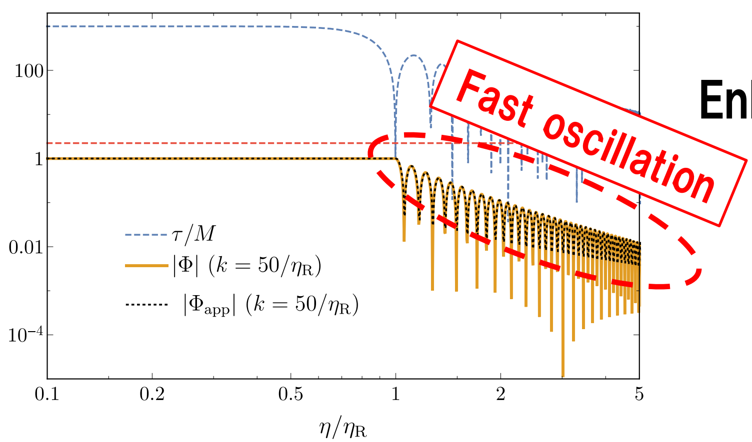
The induced GWs are suppressed compared to the previous results.
There are two reasons:

1. Each component of the induced GWs is suppressed because Φ is smaller than unity around the transition.
2. There is a damping of energy density after $\eta_{\rm R}$. (Note that $\Omega_{\rm GW,eMD}$ corresponds to the energy density at $\eta_{\rm R}$ up to some factor.)

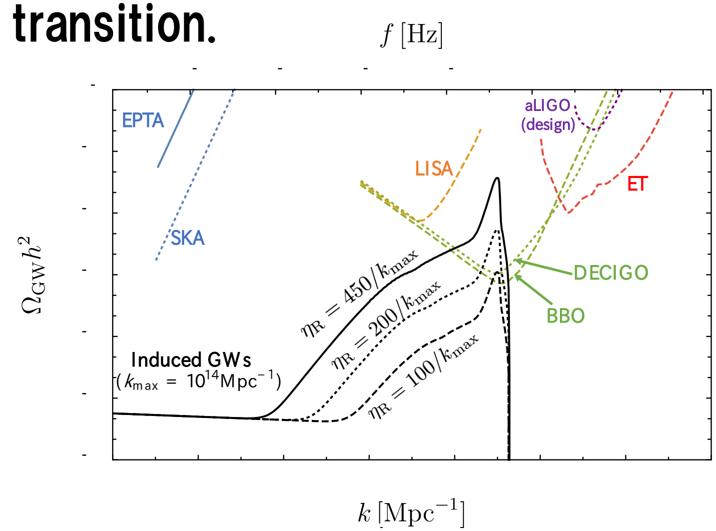
Summary for sudden reheating

We have considered the sudden scenario, in which the transition time scale from eMD era to RD era is much shorter than the Hubble time scale at that time.

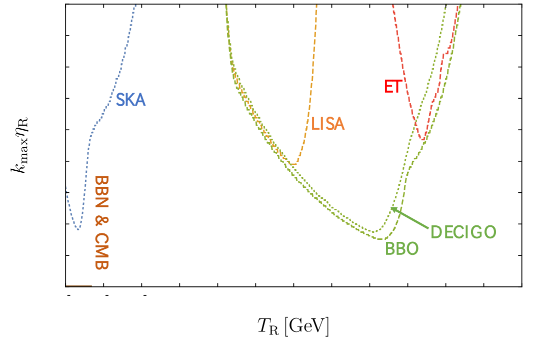
As a result, we have found that the induced GWs could be enhanced due to the fast oscillation of the perturbation after the transition.



Enhance GWs



In this scenario, the reheating temperature could be constrained by the future GW observations.

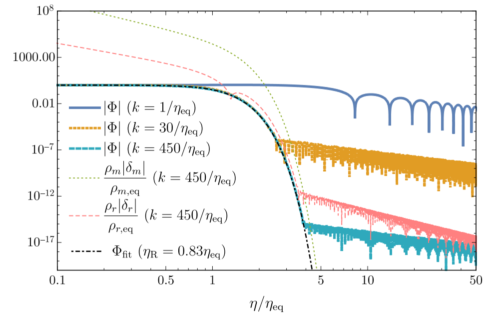


How to calculate GWs in gradual reheating

We substitute the fitting formulas of Φ and w into f, given as

$$f(u,v,\bar{x},x_{\mathrm{R}}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}$$

Note that we can neglect the contribution from the oscillation of Φ due to its small amplitude.
(Φ with $k=30/\eta_{\mathrm{eq}}$ starts to oscillate with $\Phi \sim O(10^{-6})$.)



Then we calculate the induced GWs with the following formulas.

$$\begin{aligned} I(u,v,x,x_{\mathrm{R}}) =& \int_0^{x_{\mathrm{R}}} \mathrm{d}\bar{x} \left(\frac{1}{2(x/x_{\mathrm{R}})-1}\right)\left(\frac{\bar{x}}{x_{\mathrm{R}}}\right)^2 k G_k^{\mathrm{eMD}\to\mathrm{RD}}(\eta,\bar{\eta}) f(u,v,\bar{x},x_{\mathrm{R}}) \\ &+ \int_{x_{\mathrm{R}}}^{x} \mathrm{d}\bar{x} \left(\frac{2(\bar{x}/x_{\mathrm{R}})-1}{2(x/x_{\mathrm{R}})-1}\right) k G_k^{\mathrm{RD}}(\eta,\bar{\eta}) f(u,v,\bar{x},x_{\mathrm{R}}) \\ \equiv& I_{\mathrm{eMD}}(u,v,x,x_{\mathrm{R}}) + I_{\mathrm{RD}}(u,v,x,x_{\mathrm{R}}) \end{aligned}$$

$$\begin{aligned} \overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \\ \times \overline{I^2(v,u,k\eta)} \mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku) \end{aligned}$$

$$\begin{aligned} \Omega_{\mathrm{GW}}(\eta,k) &= \frac{\rho_{\mathrm{GW}}(\eta,k)}{\rho_{\mathrm{tot}}(\eta)} \\ &= \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)} \end{aligned}$$

Specifically, I^2 can be expressed as

$$\overline{I^2} = \overline{I_{\mathrm{eMD}}^2} + \overline{I_{\mathrm{RD}}^2} + 2\overline{I_{\mathrm{eMD}} I_{\mathrm{RD}}}$$

For later convenience, we split the induced GWs into three parts as

$$\Omega_{\mathrm{GW}} = \Omega_{\mathrm{GW,eMD}} + \Omega_{\mathrm{GW,RD}} + \Omega_{\mathrm{GW,cross}}$$

These components are related to I^2_{eMD}, I^2_{RD}, and $2I_{\mathrm{eMD}} I_{\mathrm{RD}}$, respectively.

What we do in this work

We revisit the effects of an eMD era on induced GWs.

As we will see, the results are sensitive to the time scale of the reheating. Then, we consider two cases:

1. Gradual reheating, whose time scale is comparable to the Hubble time scale at that time.
2. Sudden reheating, whose time scale is much faster than the Hubble time scale at that time.

In the following, to spotlight the effects of an eMD era on induced GWs, we focus on the GWs induced by the perturbations entering the eMD era.

(The effects mainly come from the behaviors of the perturbations on subhorizon during eMD era, which are different from those during RD era.)

Gradual reheating

We consider the case where the field dominating the Universe during an eMD era (inflaton or curvaton) decays to radiation with a constant decay rate Γ.

To consider GWs induced during RD era by the perturbations entering the horizon during eMD era, we need to take into account the evolution of the gravitational potential around the transition correctly.

In this gradual reheating scenario, we can apply the formulas for the decaying dark matter scenario to follow the evolutions of background or perturbation quantities.

E.o.M in decaying DM scenario (Poulin, Serpico, Lesgourgues, 2018)

$$\rho_m' = -(3\mathcal{H} + a\Gamma)\rho_m,$$
$$\rho_r' = -4\mathcal{H}\rho_r + a\Gamma\rho_m$$

ρ_m : Energy density of matter

ρ_r : Energy density of radiation

δ : Energy density perturbation

$\theta \ (\equiv i k_j v^j)$: Velocity divergence

$$\delta_m' = -\theta_m + 3\Phi' - a\Gamma\Phi, \quad \theta_m' = -\mathcal{H}\theta_m + k^2\Phi,$$
$$\delta_r' = -\frac{4}{3}(\theta_r - 3\Phi') + a\Gamma\frac{\rho_m}{\rho_r}(\delta_m - \delta_r + \Phi),$$
$$\theta_r' = \frac{k^2}{4}\delta_r + k^2\Phi - a\Gamma\frac{3\rho_m}{4\rho_r}\left(\frac{4}{3}\theta_r - \theta_m\right)$$

What we do in this work

We revisit the effects of an eMD era on induced GWs.

As we will see, the results are sensitive to the time scale of the reheating. Then, we consider two cases:

1. Gradual reheating, whose time scale is comparable to the Hubble time scale at that time.
2. Sudden reheating, whose time scale is much shorter than the Hubble time scale at that time.

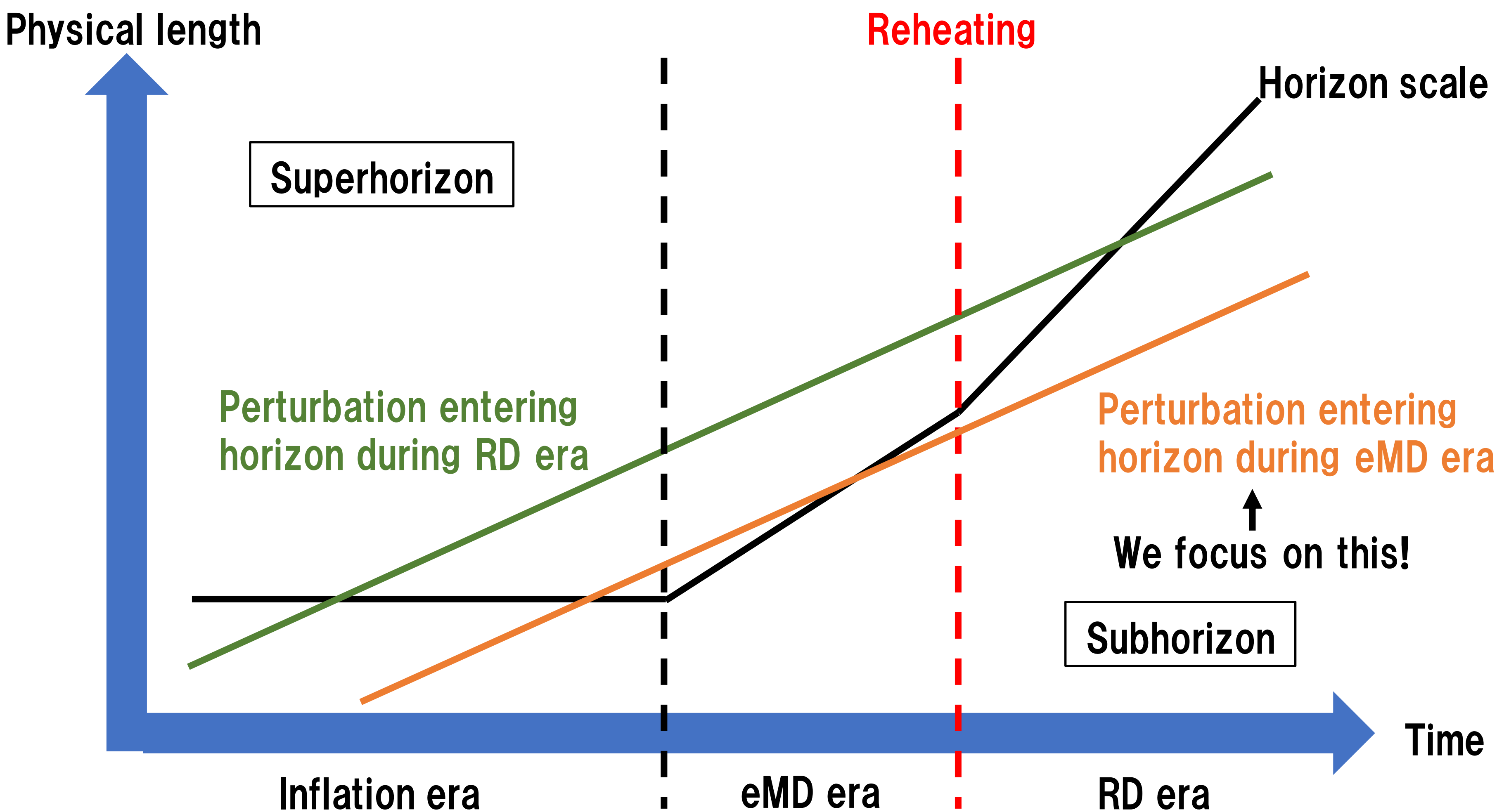
In the following, to spotlight the effects of an eMD era on induced GWs, we focus on the GWs induced by the perturbations entering the horizon during an eMD era.

When GWs are induced

We take into account this!

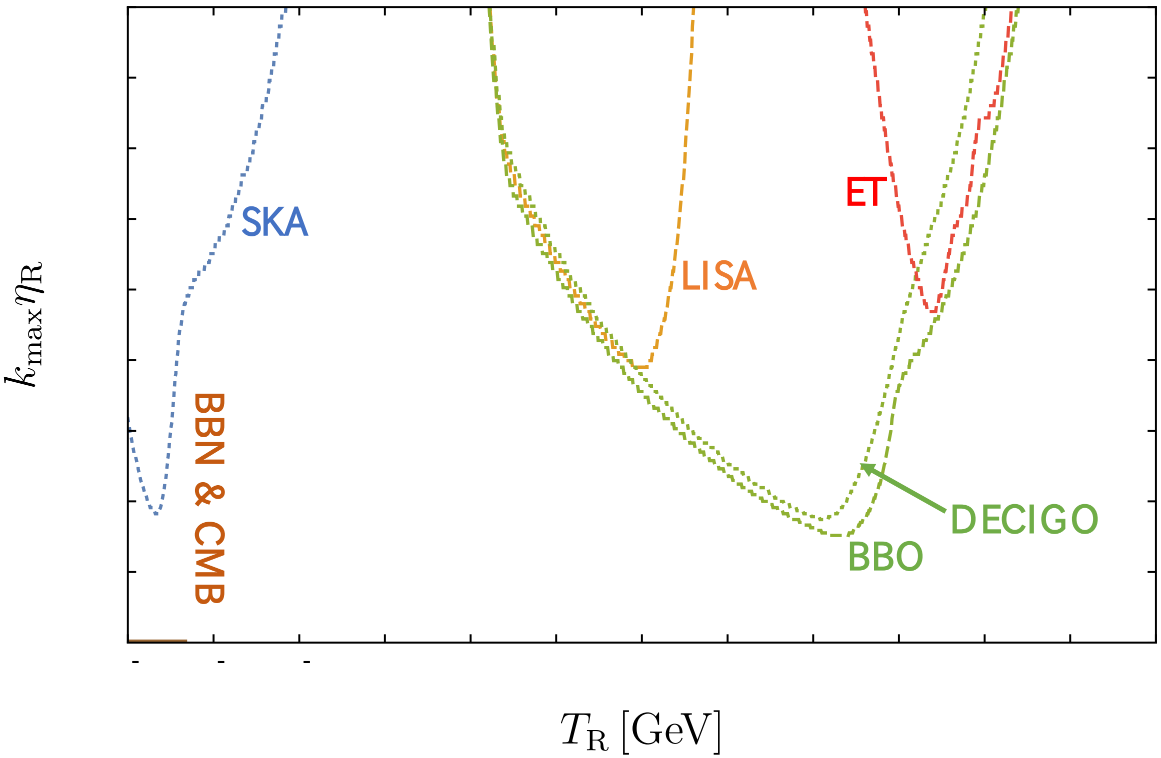
When the source perturbations enter the horizon		eMD	RD
	eMD	Discussed so far	Never discussed
	RD		No eMD effect

Evolutions of perturbations



Future constraints on reheating temperature

(We have neglected foregrounds)



Power spectrum:

$$\mathcal{P}_\zeta(k) = \Theta(k_{\max} - k) A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$(A_s = 2.1 \times 10^{-9},\ n_s = 0.96)$$

Signal to noise ratio:

(Allen & Romano, 1997)

Induced GWs

$$\rho = \sqrt{2T_{\rm obs}} \left[\int_{f_{\min}}^{f_{\max}} df \left(\frac{\Omega_{\rm GW}(f)}{\Omega_{\rm GW,eff}(f)}\right)^2\right]^{1/2}$$

Observation time

$T_{\rm obs} = 20{\rm yr}$ for SKA

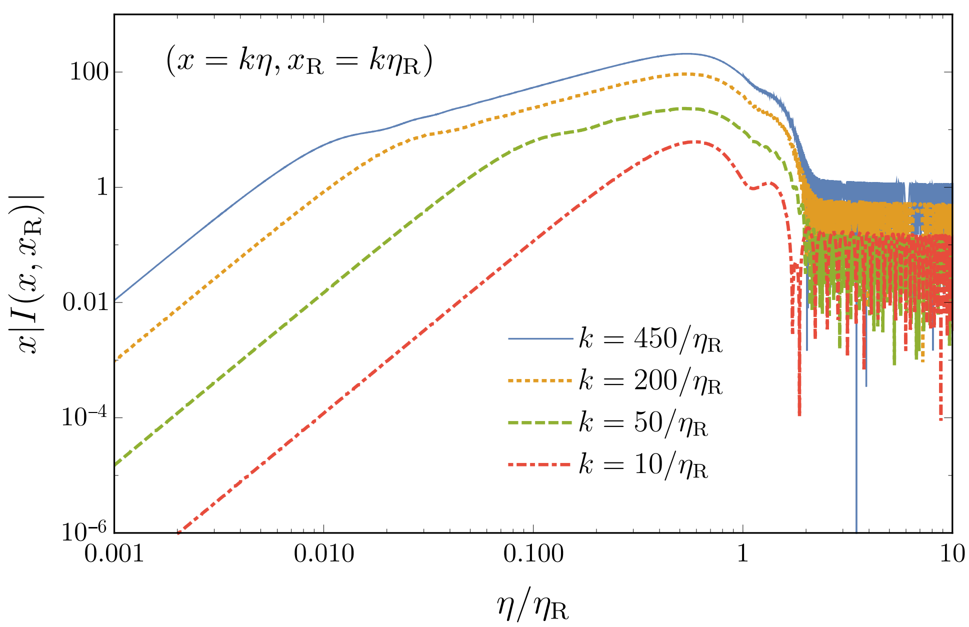
$T_{\rm obs} = 1{\rm yr}$ for otehrs

Effective noise spectrum of each project

ρ =1 is taken in the figure.

The future observations could constrain the wide range of reheating temperature in the sudden reheating scenario.

Why does the cancellation occur?



$$\overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \times \overline{I^2(k\eta,k\eta_{\rm R})}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)} \propto x^2\overline{I^2(x,x_{\rm R})}$$

$$x_{\rm R}|I(x_{\rm R},x_{\rm R})| \sim x|I_{\rm eMD}(x,x_R)|$$

$$x|I(x,x_{\rm R})| = x|I_{\rm eMD}(x,x_{\rm R})| + x|I_{\rm RD}(x,x_{\rm R})| \quad (\eta > \eta_{\rm R})$$

$I_{\rm eMD}$: **Describing the GWs induced during eMD era**

$I_{\rm RD}$: **Describing the GWs induced during RD era**

$x|I(x,x_R)|$ decays around $\eta = \eta_R$ due to the decay of the gravitational potential Φ, which means,

$$x|I_{\rm eMD}(x,x_{\rm R}) + I_{\rm RD}(x,x_{\rm R})| \ll x|I_{\rm eMD}(x,x_{\rm R})| \quad (\eta \gg \eta_{\rm R})$$

$$\rightarrow \quad \Omega_{\rm GW} \ll \Omega_{\rm GW,eMD}, \Omega_{\rm GW,RD}$$

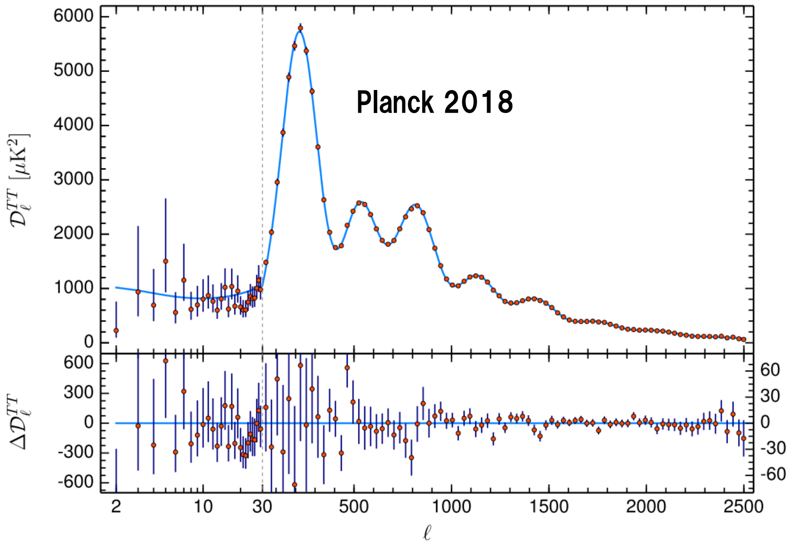
From this result, we can see the main reason of the cancellation between the components comes from the behaviors of Φ around the transition.

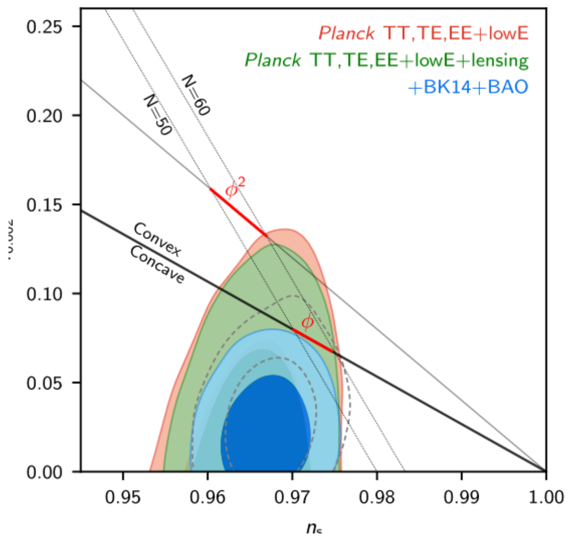
Perturbations and Cosmology

The inflation theory predicts power spectra of perturbations, including scalar (curvature), tensor (GWs) perturbations.

Therefore

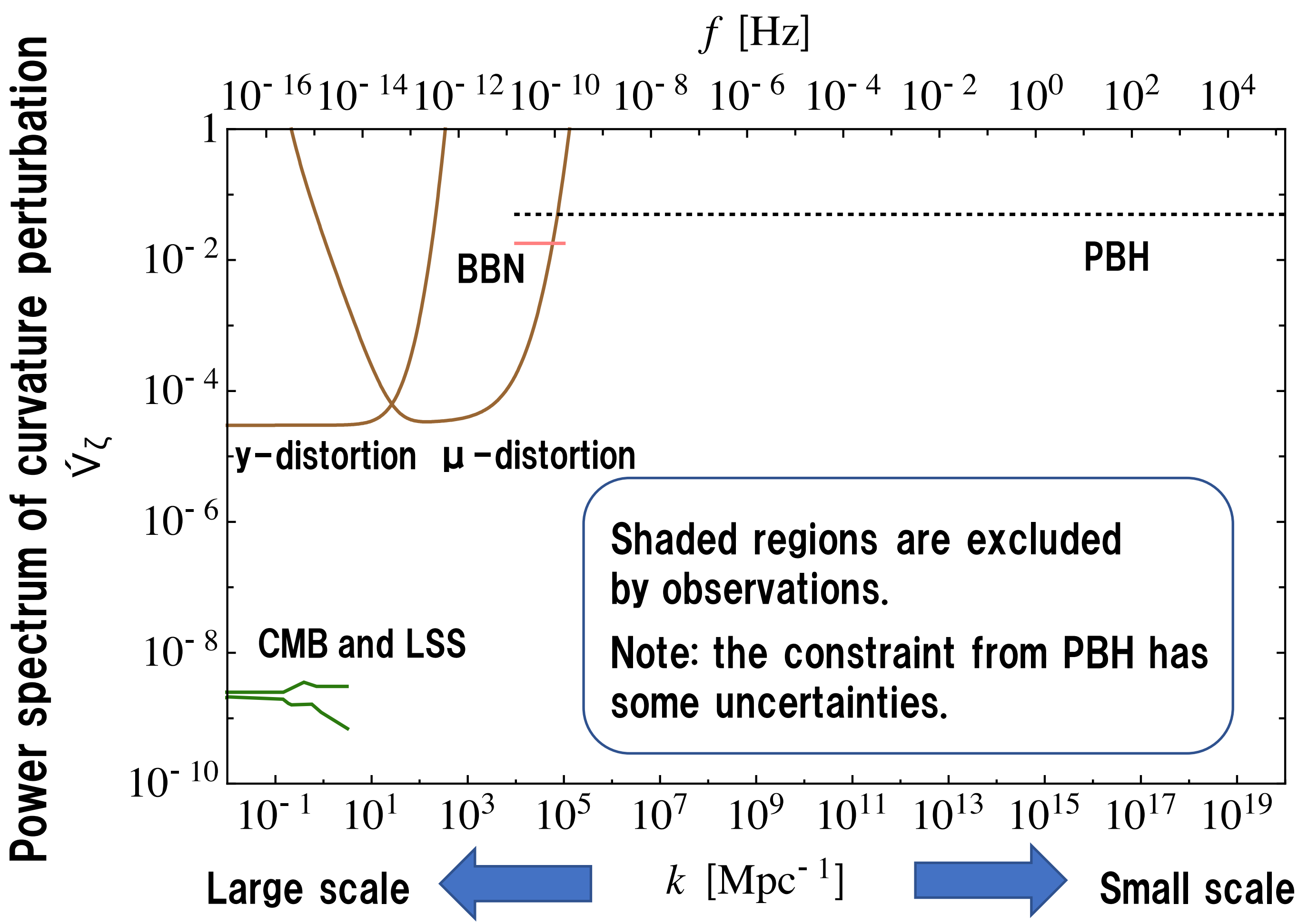
To understand the nature of the inflation theory, it is important to investigate the perturbations.





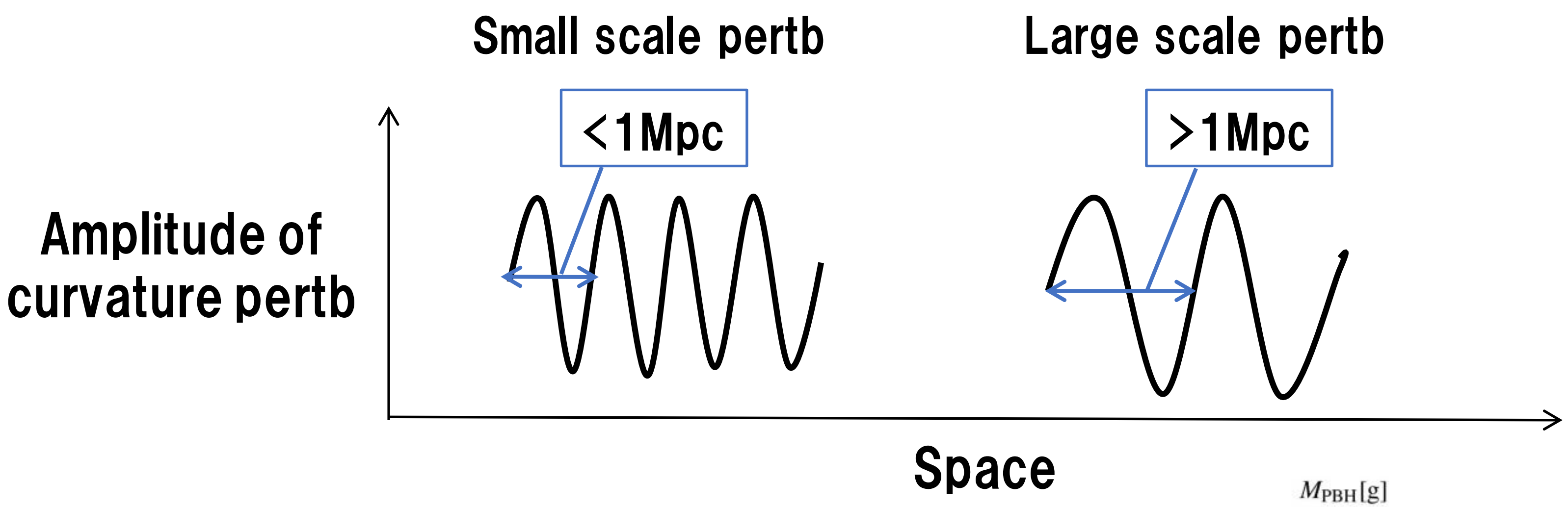
On large scale (>1Mpc), curvature perturbations are observed very well.

Current constraints on the small scale



It is difficult to observe small scale perturbations due to Silk damping or non-linear growth of large scale structure (LSS).

Small scale perturbation and PBH

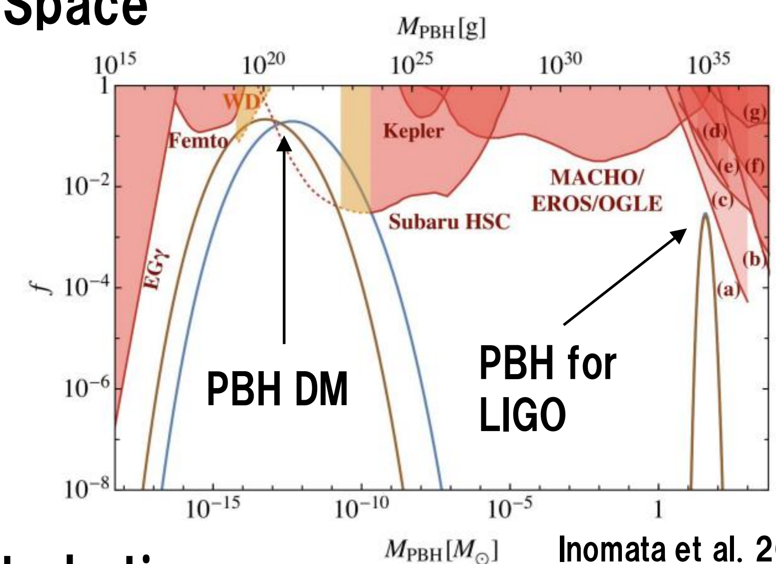


Some inflation models predict large perturbations on small scale and sizable amount of PBHs.

PBH is one of the candidate of DM and the heavy BHs detected by LIGO.

Therefore

It is important to discuss small scale perturbations.



Inomata et al. 2017

What we do in this work

GWs can be induced by scalar (curvature) perturbations at second order. (Terada-san's talk)

If there is a large perturbation on small scale, the induced GWs can be constrained by observations.

We can constrain the small scale perturbations using the induced GWs!

Related at second order

Scalar perturbations ⟷ GWs

Constraints on small scale perturbation from induced GWs ⟵ Constraints on GWs from observations

We do this!

Related work: Assadullahi & Wands, 2009

Brief review of the induced GWs

GWs can be induced by scalar (curvature) perturbations at second order. (Terada-san's talk)

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)},$$

Energy density of the induced GWs

$$\overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \times \overline{I^2(v,u,k\eta)}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

Power spectrum of the induced GWs

$$\overline{I^2(v,u,x\to\infty)} = \frac{1}{2}\left(\frac{3(u^2+v^2-3)}{4u^3v^3x}\right)^2 \left(\left(-4uv+(u^2+v^2-3)\log\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2 + \pi^2(u^2+v^2-3)^2\Theta(v+u-\sqrt{3})\right)$$

$$\Omega_{\rm GW}(\eta_0,k) = 0.83\left(\frac{g_c}{10.75}\right)^{-1/3}\Omega_{r,0}\,\underline{\Omega_{\rm GW}(\eta_c,k)} \qquad (1/k \ll \eta_c \ll \eta_{\rm eq})$$

What we calculate with above formula

Constraints on the induced GWs

When multiple detectors or pulsars are available, it is beneficial to use a cross-correlation between them to look for the correlated signal due to stochastic GWs.

Signal to noise ratio is defined as (Allen & Romano, 1997)

Induced GWs

$$\rho = \sqrt{2T}\left[\int_{f_{\rm min}}^{f_{\rm max}} df \left(\frac{\Omega_{\rm GW}(f)}{\Omega_{\rm GW,eff}(f)}\right)^2\right]^{1/2}$$

Observation time

Effective sensitivity

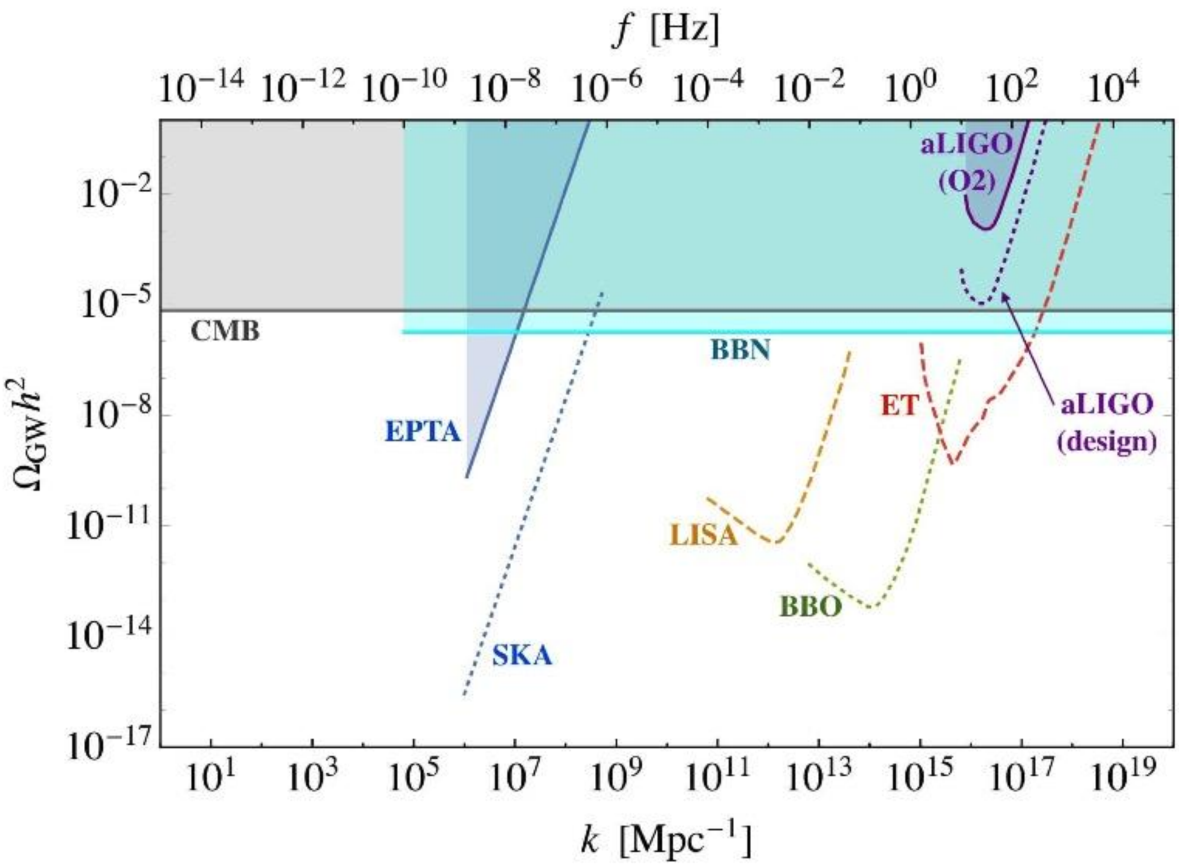
The effective sensitivity is given by

Overlap reduction function, coming from cross correlation

$$\Omega_{\rm GW,eff}(f) H_0^2 = \frac{2\pi^2}{3} f^3 \left[\sum_{I=1}^{M}\sum_{J>I}^{M} \frac{\Gamma_{IJ}^2(f)}{P_{nI}(f) P_{nJ}(f)}\right]^{-1/2}$$

Noise power spectrum

Summary of GW constraints



Shaded regions are excluded by current observations.

Outline

- Introduction
- Gravitational waves induced by scalar perturbations
- Constraints on the power spectrum on small scale
- Summary

How to derive the constraints

To be specific, we assume the peak-like profile for the power spectrum

$$\mathcal{P}_\zeta(k) = A \exp\left(-\frac{(\log k - \log k_{\rm p})^2}{2\sigma^2}\right).$$

1. Fix σ.
2. Fix T（observation time）for each project.

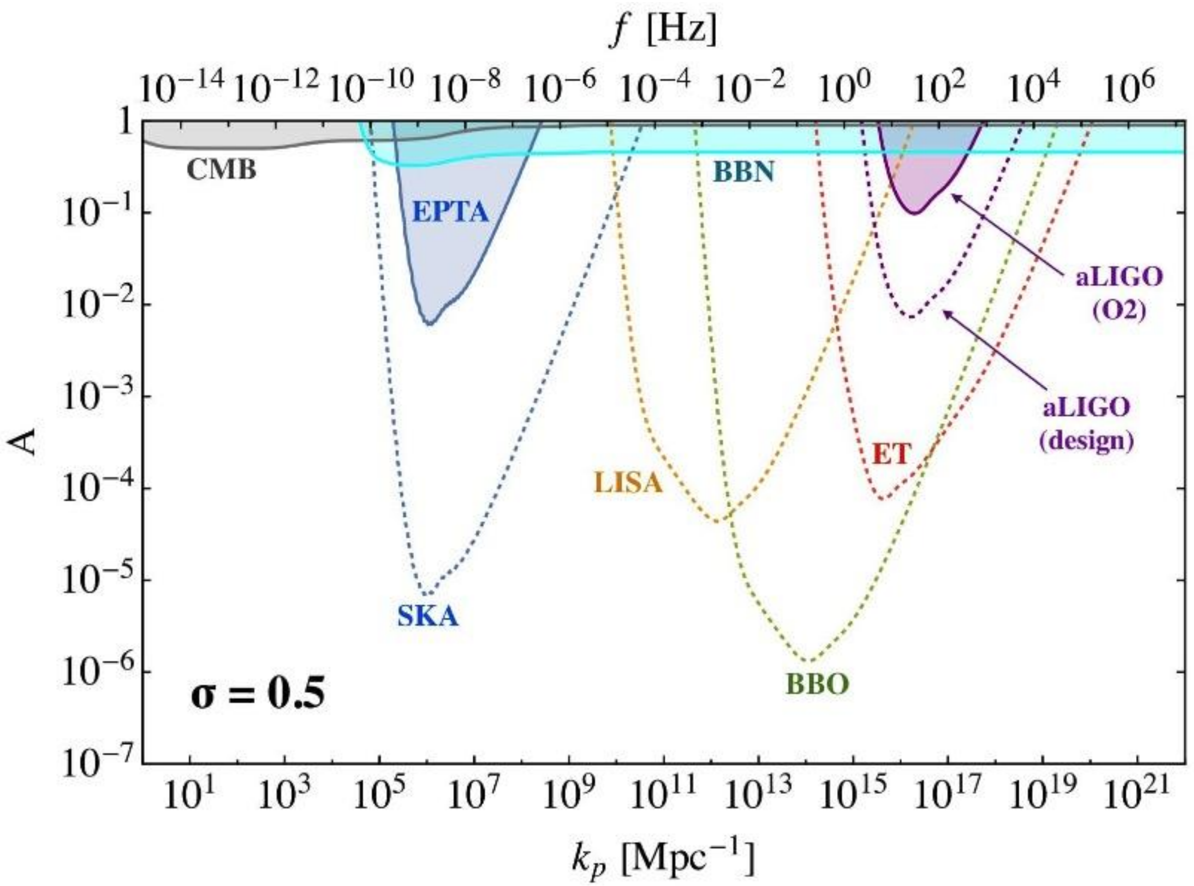
   EPTA:18 years, SKA: 20 years, aLIGO（O2）: 4 months, Others: 1 year.

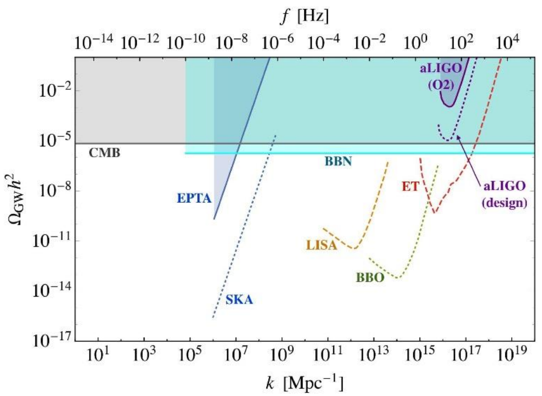
3. Fix k_p and calculate A that yields signal-to-noise ratio unity（ρ=1）for each project.

$$\rho = \sqrt{2T}\left[\int_{f_{\rm min}}^{f_{\rm max}} df \left(\frac{\Omega_{\rm GW}(f)}{\Omega_{\rm GW,eff}(f)}\right)^2\right]^{1/2}$$

4. Change k_p and calculate A repeatedly and get the constraint curves on A. $(A \propto \rho^{1/2} T^{-1/4})$

Constraints on small scale perturbations





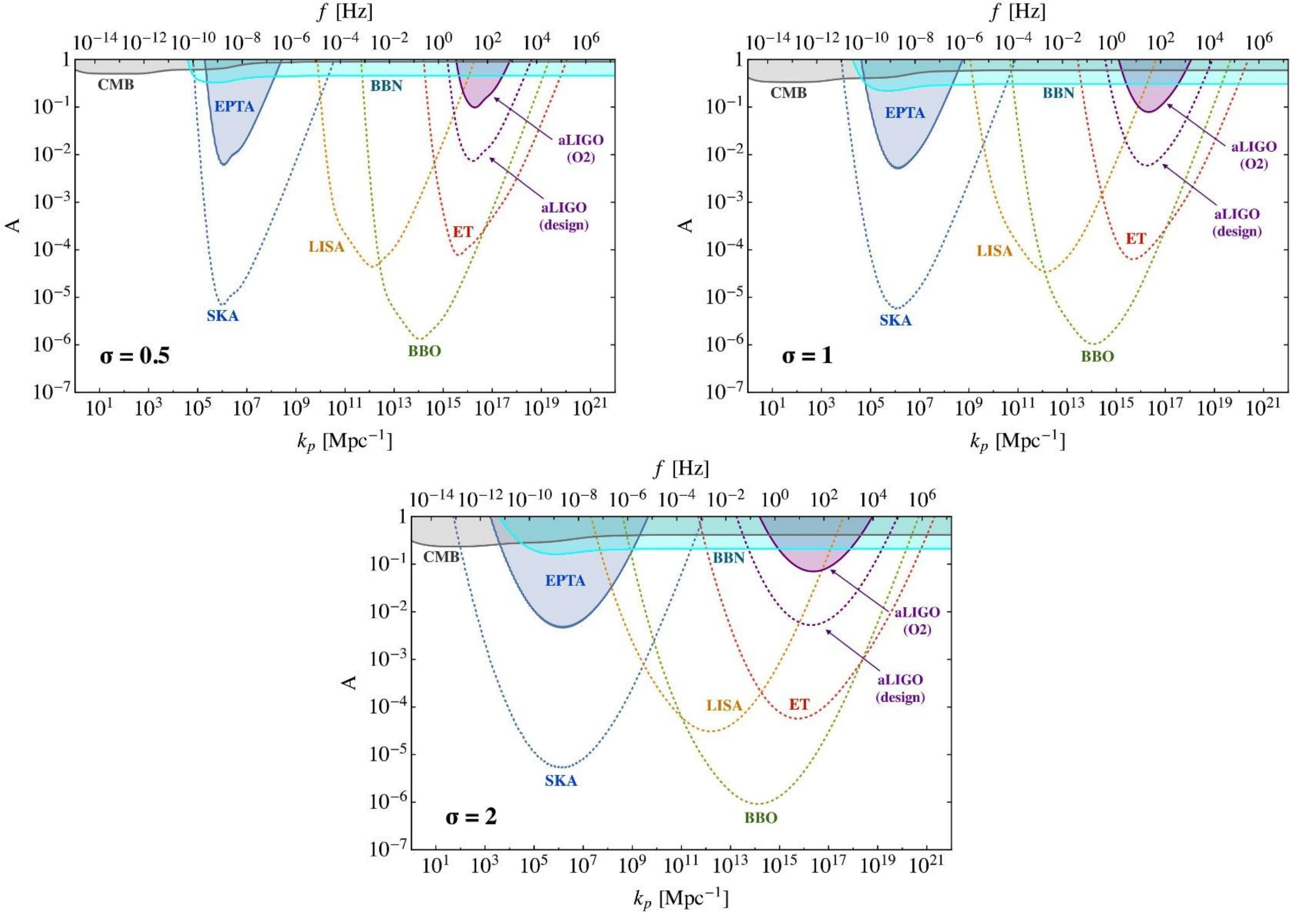
$$\mathcal{P}_\zeta(k) = A \exp\left(-\frac{(\log k - \log k_\mathrm{p})^2}{2\sigma^2}\right).$$

$$\rho = \sqrt{2T}\left[\int_{f_\mathrm{min}}^{f_\mathrm{max}} df \left(\frac{\Omega_\mathrm{GW}(f)}{\Omega_\mathrm{GW,eff}(f)}\right)^2\right]^{1/2}$$

The constraints on the smaller-scale are stronger compared to the effective sensitivities to GWs.

This is because of the frequency integration in the definition of signal-to-noise ratio.

Profile dependence



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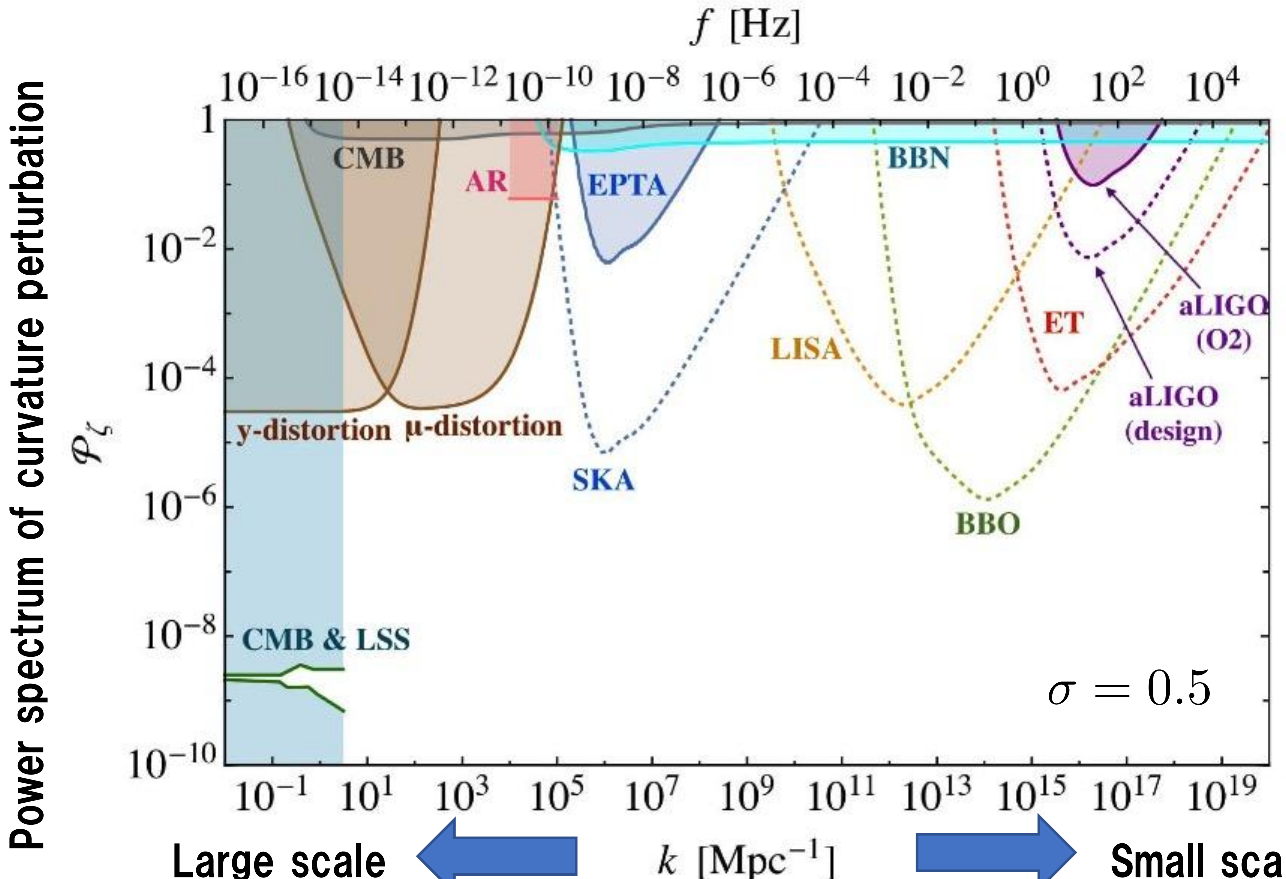
What we have done



Large scale ← → Small scale

We have derived the constraints on small scale perturbations using GWs induced by scalar perturbations at 2nd order.

Foregrounds

In future, stochastic GWs of astrophysical origin or cosmological origins that are different from the induced GWs, such as those from phase transition, may be detected.
In that case, the constraints would become weak.

Foreground example:

Around PTA target frequency
GWs from supermassive-black-hole binaries

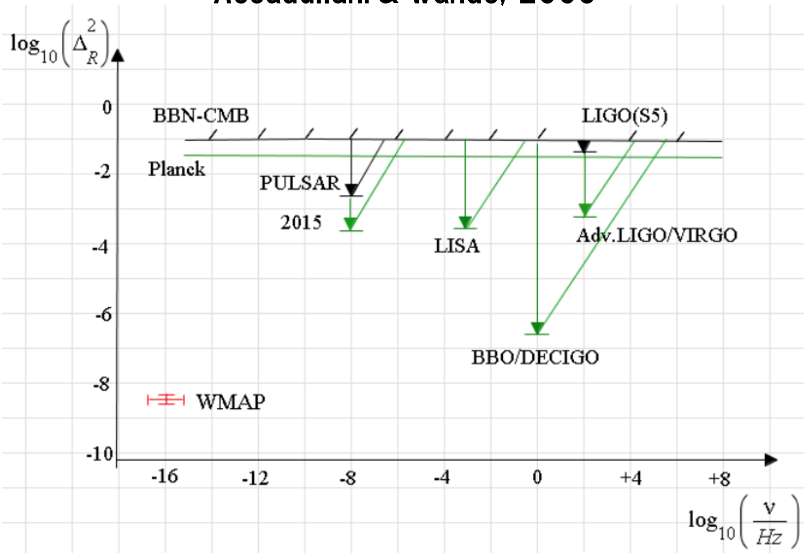
Around LISA target frequency
GWs from galactic white-dwarf binaries

Around BBO target frequency
GWs from neutron star binaries

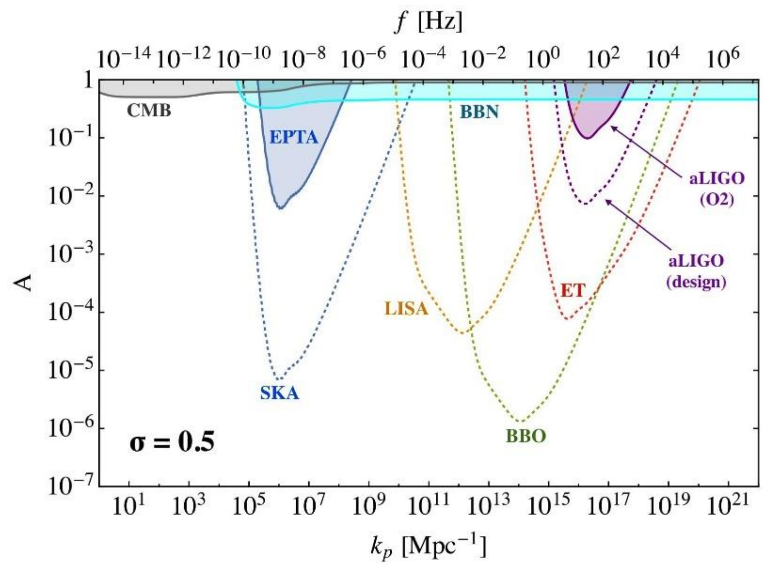
We may use non-Gaussianity of the induced GWs to discriminate them from other sources. (Bartolo et al., 2018)

Difference between our work and previous work

Assadullahi & Wands, 2009



Our work



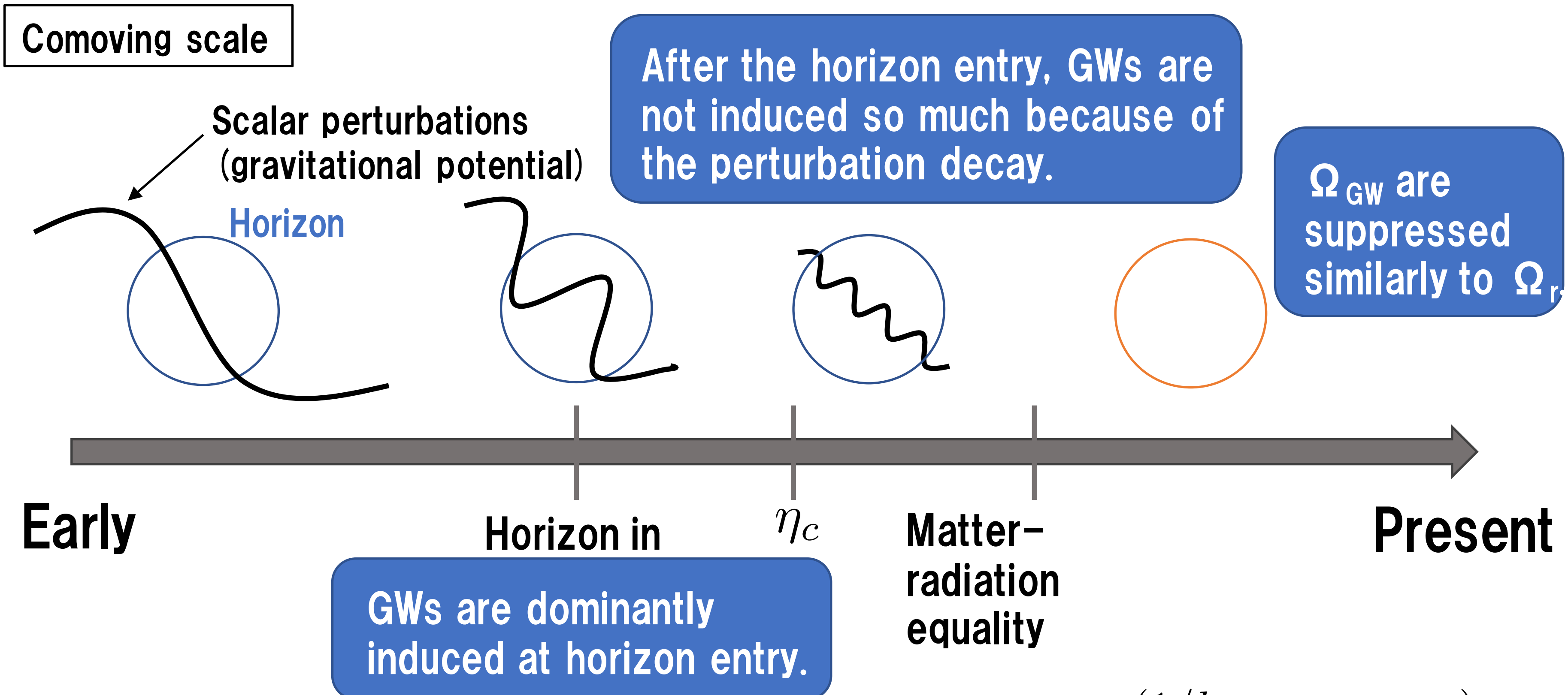
In previous work, they use the old formula for the induced GWs.

For scale invariant spectrum: ($P_\zeta = A$)

$$\Omega_{\rm GW}(\eta_c, k) \simeq \begin{cases} 33.3\, A^2 & \text{(for previous work)} \\ 0.8222\, A^2 & \text{(for Kohri\&Terada, 2018)} \end{cases}$$

In addition, they do not take into account frequency dependence of the sensitivity curves.

Evolution of the induced GWs

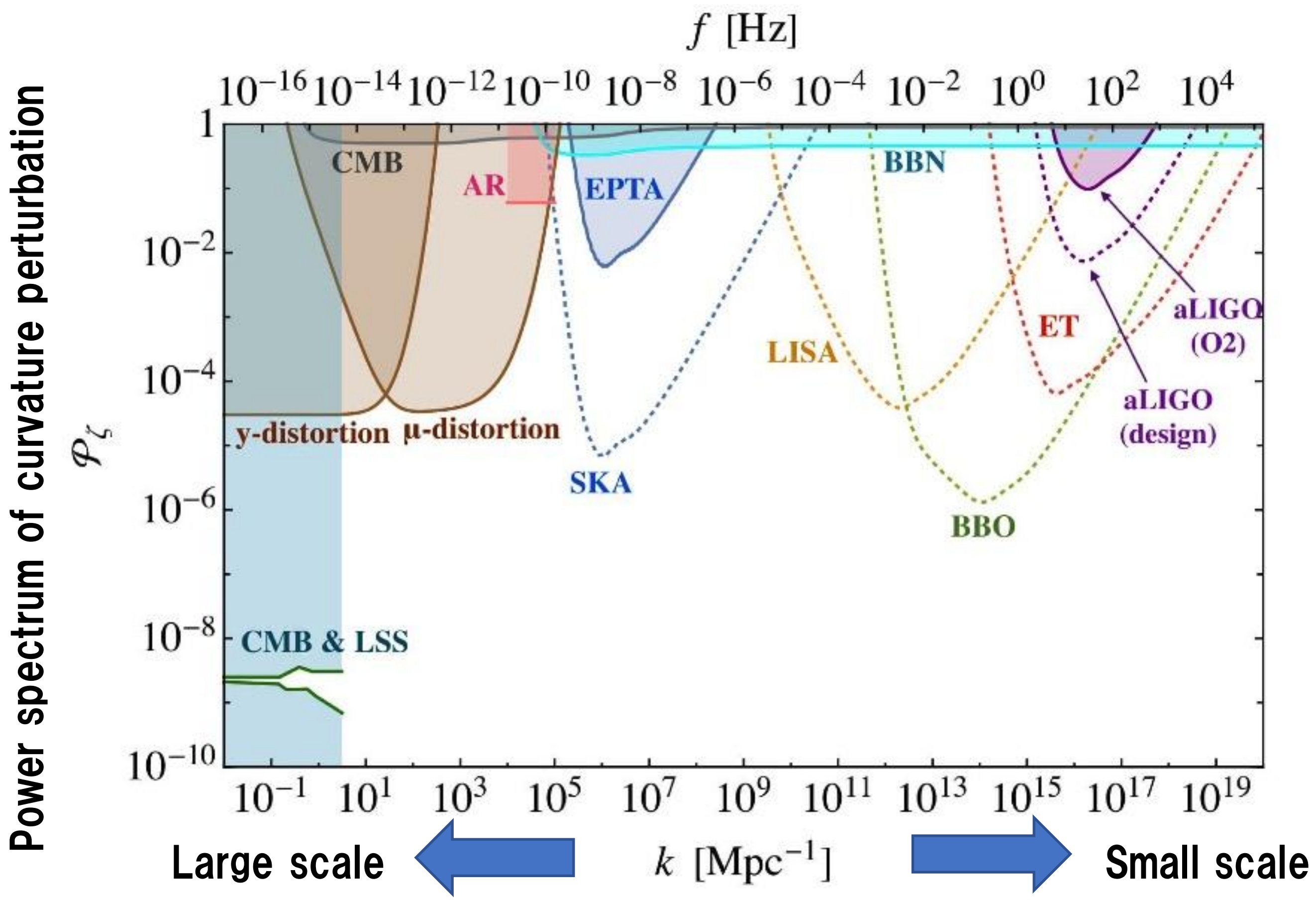


$$(1/k \ll \eta_c \ll \eta_{\rm eq})$$

$$\Omega_{\rm GW}(\eta_0, k) = 0.83 \left(\frac{g_c}{10.75}\right)^{-1/3} \Omega_{r,0}\, \underline{\Omega_{\rm GW}(\eta_c, k)}$$

What we calculate with the formula

What we do



We derive constraints on small scale curvature perturbations using GWs induced by scalar perturbations.