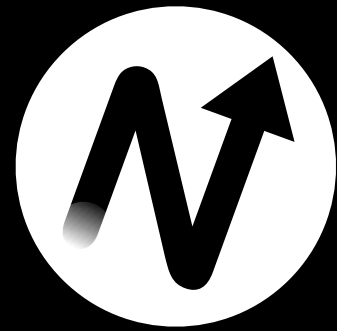


Bouncing Universe from Nothing

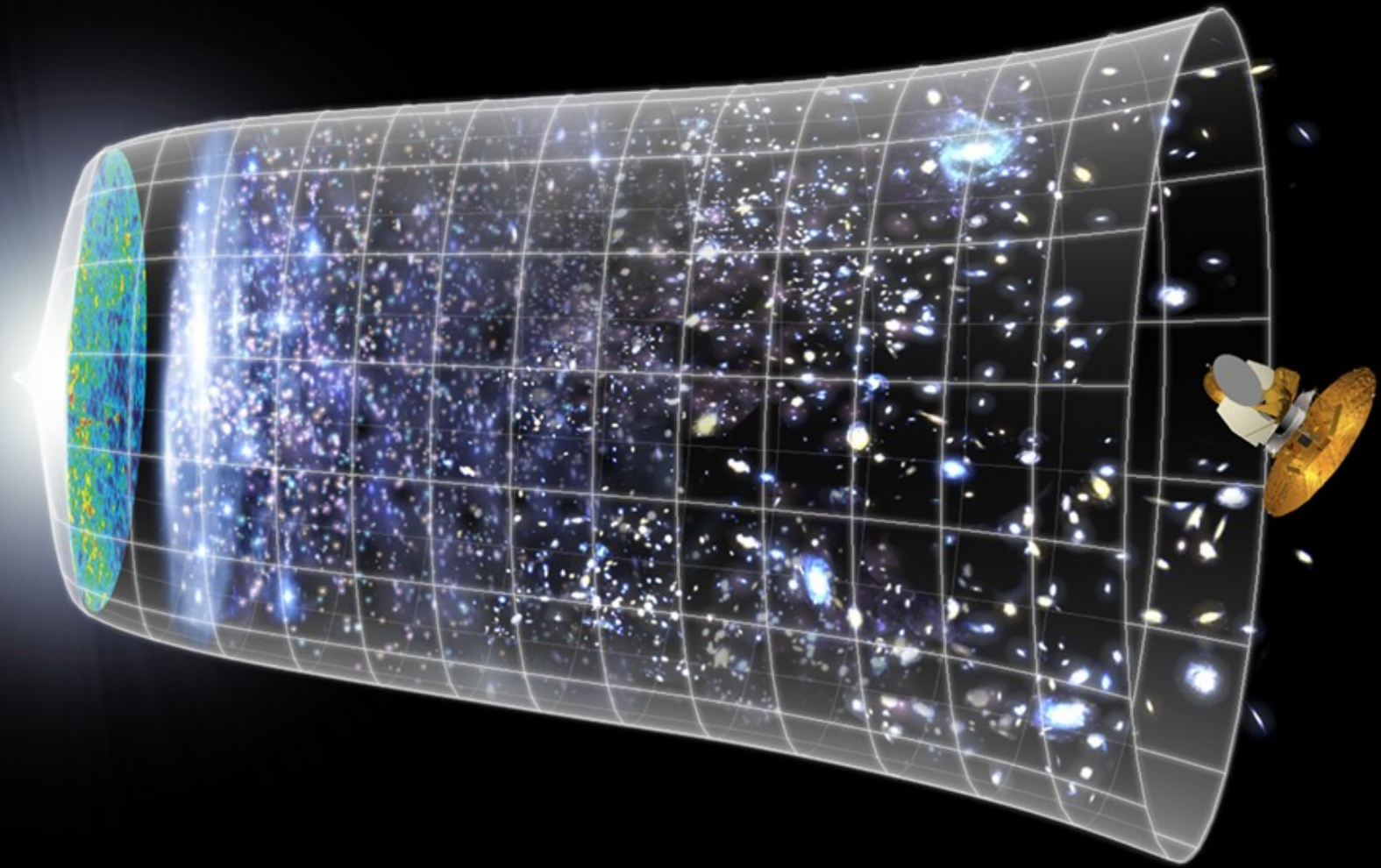
Takahiro Terada
(KEK, JSPS fellow)

Hiroki Matsui, Fuminobu Takahashi, TT, arXiv:1904.12312 [gr-qc]

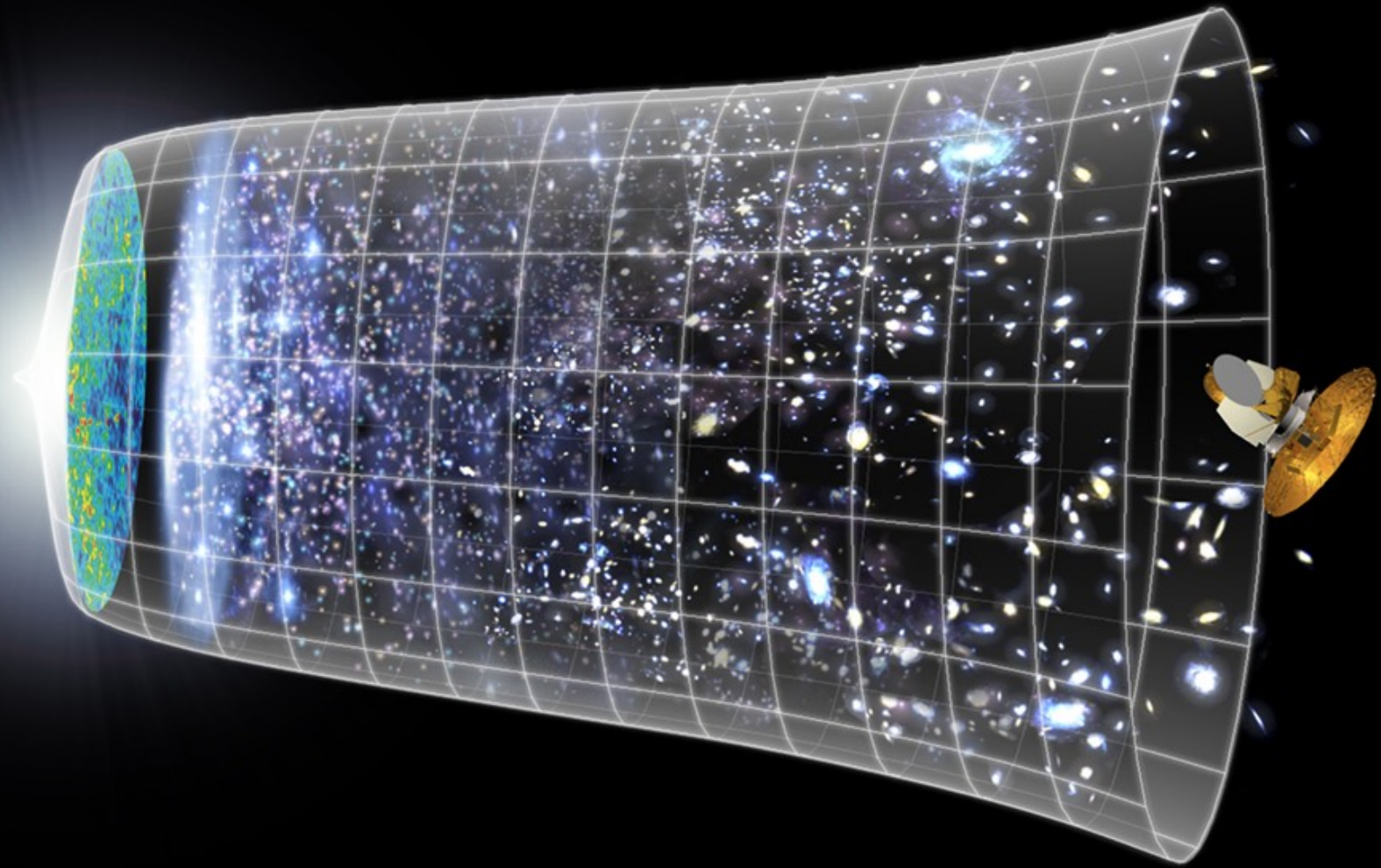


Introduction

Cosmological History



Cosmological History



Always Expanding?

Usually, yes.

For a flat universe,

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Friedmann eq.

Hubble parameter

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

$$H^2 = \frac{\rho}{3}$$

Energy density

(reduced Planck unit: $8\pi G = 1$)

As long as $\rho > 0$, it keeps expanding.

Universe can contract

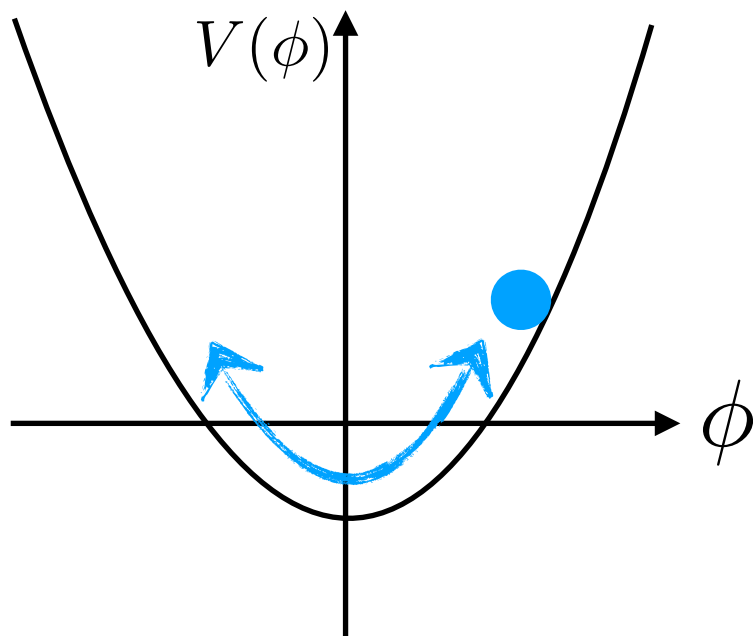
If $\rho = 0$ (i.e., $H = 0$) is realized, the universe starts to contract.

$$\dot{H} = -\frac{1}{2}(\rho + P)$$

Null Energy Condition (NEC): $\rho + P \geq 0$

For example,

it is possible for the scalar field with a **negative potential**.

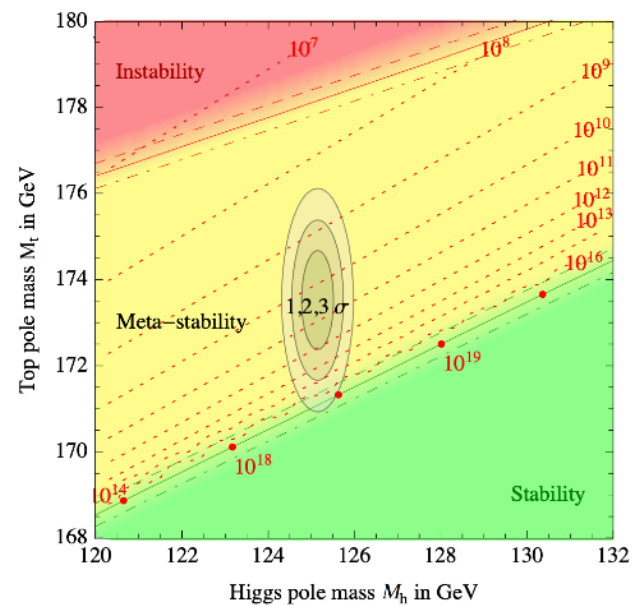
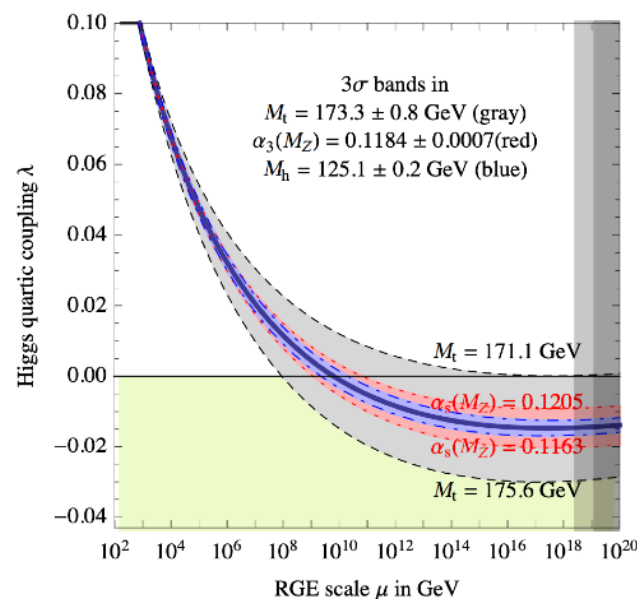


However, it **keeps contracting**,
eventually leading to a **big crunch**.

[Linde, hep-th/0110195] [Felder et al., hep-th/0202017]

Negative Potential

Standard Model



[Buttazzo et al., 1307.3536]

Supergravity

R-symmetry breaking,
 negative semi-definite

$$V = e^K \left(g^{\bar{j}i} D_i W \bar{D}_{\bar{j}} \bar{W} - 3|W|^2 \right)$$

SUSY breaking,
 positive semi-definite

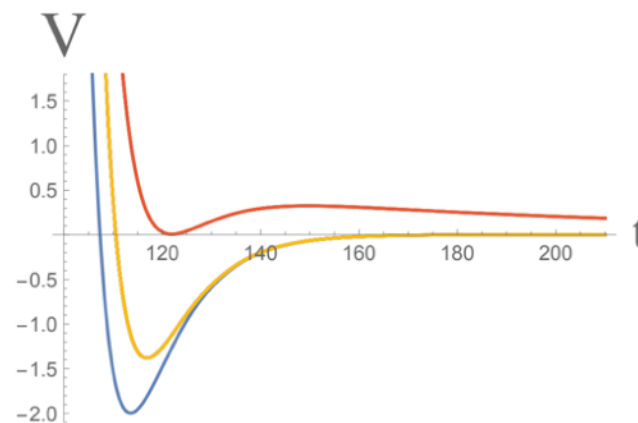
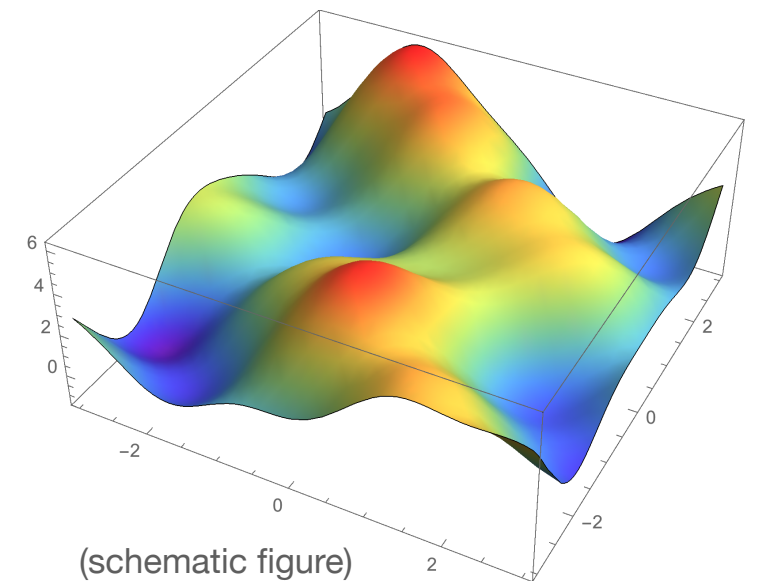


figure from [Kallosh et al., 1808.09428]

String Landscape



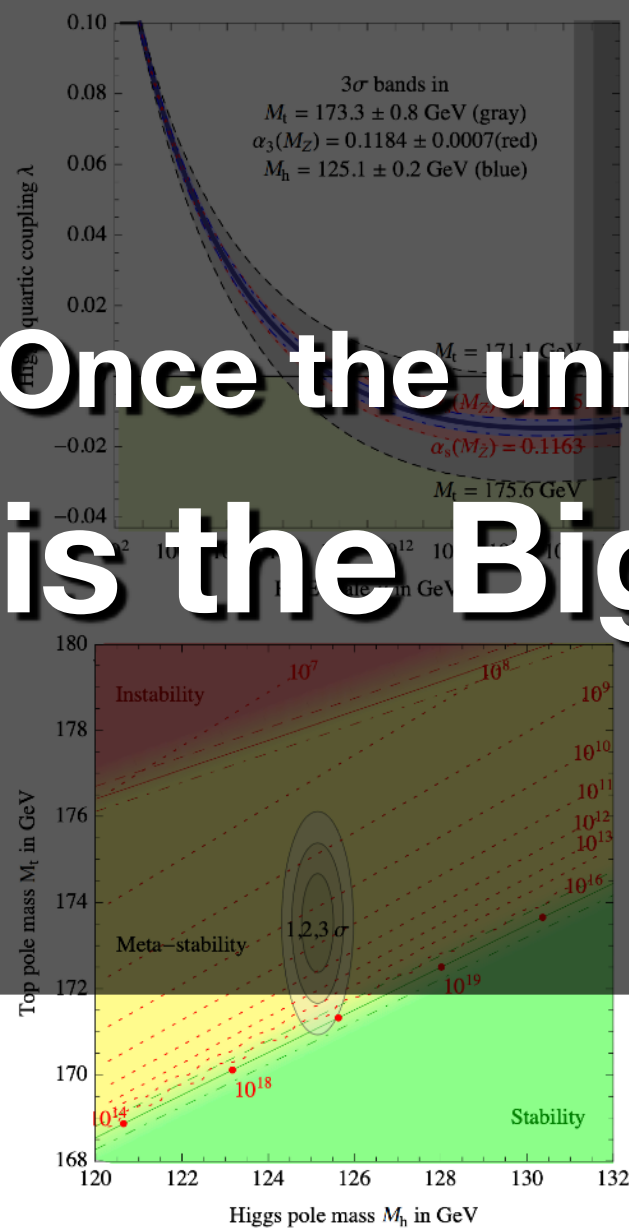
De Sitter vacuum
 construction is nontrivial
 in superstring theory.

(cf. Swampland de Sitter conjecture)

[Obied et al., 1806.08362]

Negative Potential

Standard Model



[Buttazzo et al., 1307.3536]

Supergravity

R-symmetry breaking,
 negative semi-definite

positive semi-definite

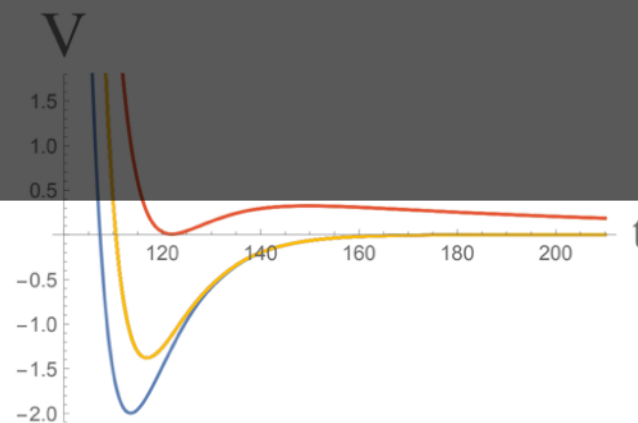
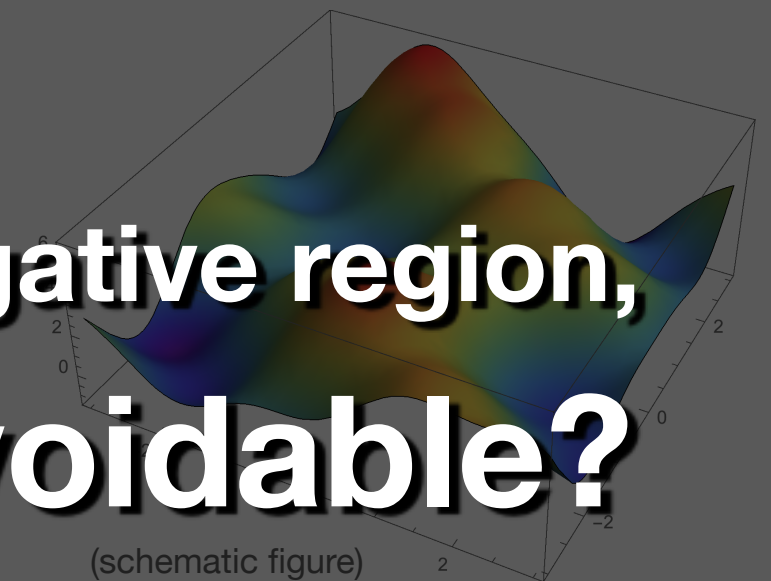


figure from [Kallosh et al., 1808.09428]

String Landscape



De Sitter vacuum construction is nontrivial in superstring theory.

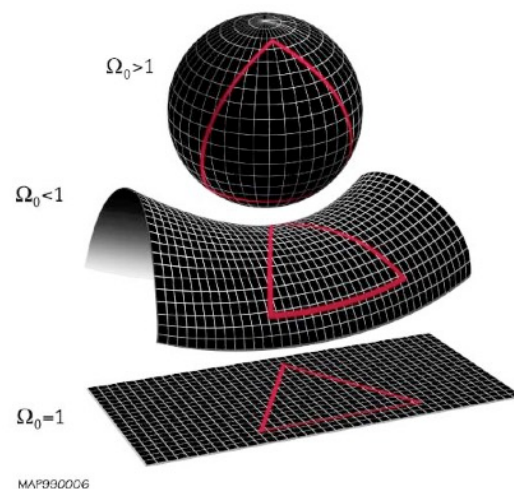
(cf. Swampland de Sitter conjecture)
 [Obied et al., 1806.08362]

Once the universe enters the negative region,
 is the Big Crunch unavoidable?

Spatial Curvature

Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - \mathcal{K}r^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right]$$



spatial curvature

positive (closed), zero (flat), or negative (open)

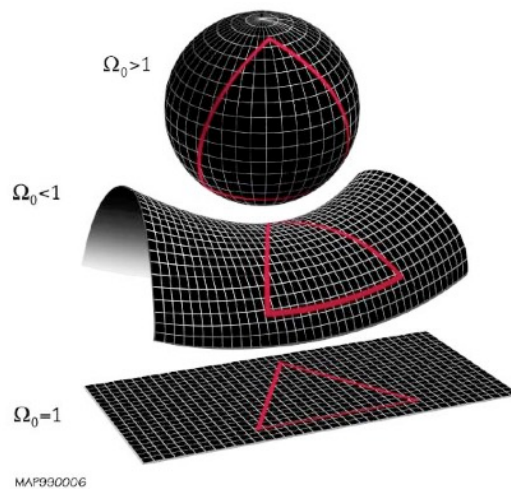
$$H^2 = \frac{\rho}{3} - \frac{\mathcal{K}}{a^2}$$

$$\dot{H} = -\frac{1}{2}(\rho + P) + \frac{\mathcal{K}}{a^2}$$

Spatial Curvature

Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - \mathcal{K}r^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right]$$



spatial curvature

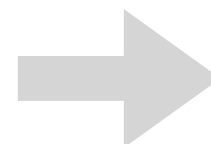
positive (closed), zero (flat), or negative (open)

Contraction → Expansion is possible.

simple example of a “bounce”

$$\rho = -P = \Lambda \quad (\text{cosmological constant})$$

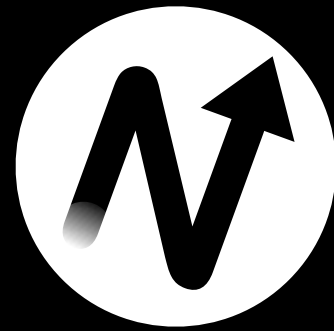
$$\mathcal{K} > 0$$



$$a(t) = \sqrt{\frac{3\mathcal{K}}{\Lambda}} \cosh \left(\sqrt{\frac{\Lambda}{3}} t \right)$$

$$H^2 = \frac{\rho}{3} - \frac{\mathcal{K}}{a^2}$$

$$\dot{H} = -\frac{1}{2}(\rho + P) + \frac{\mathcal{K}}{a^2}$$



Expansion, Contraction, and Expansion Again

Einstein Gravity & a Scalar Field

Action

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

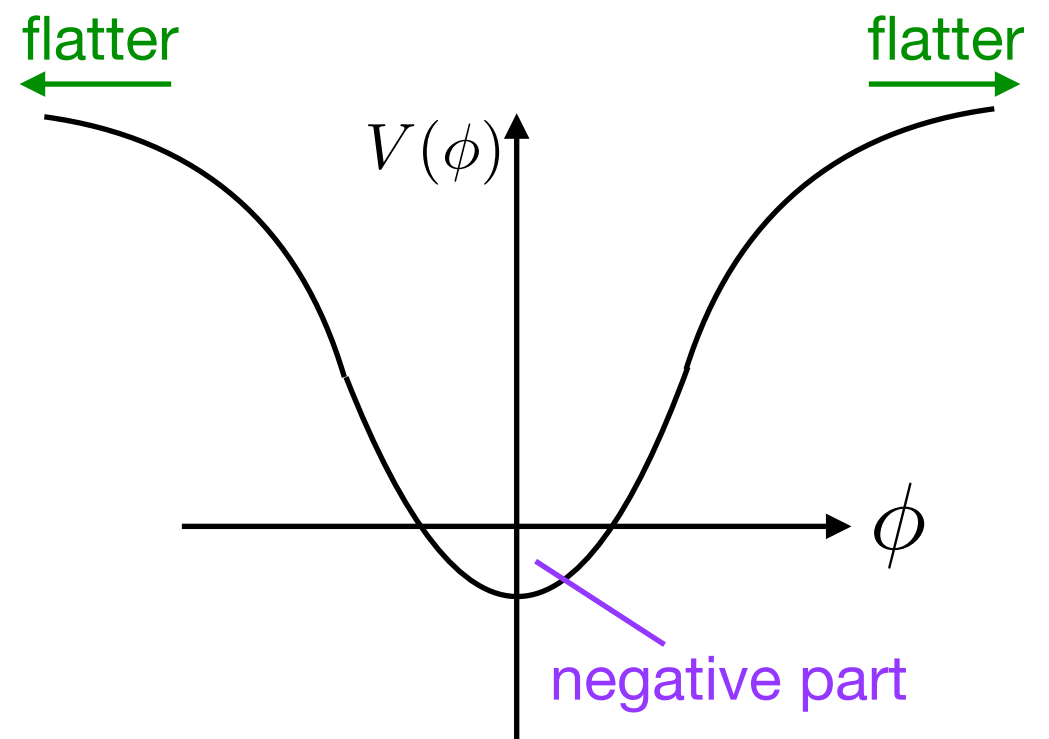
Eqs. of motion

$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{\mathcal{K}}{a^2},$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 + \frac{\mathcal{K}}{a^2},$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

Scalar Potential



1. Initial Expansion Phase

Eqs. of motion

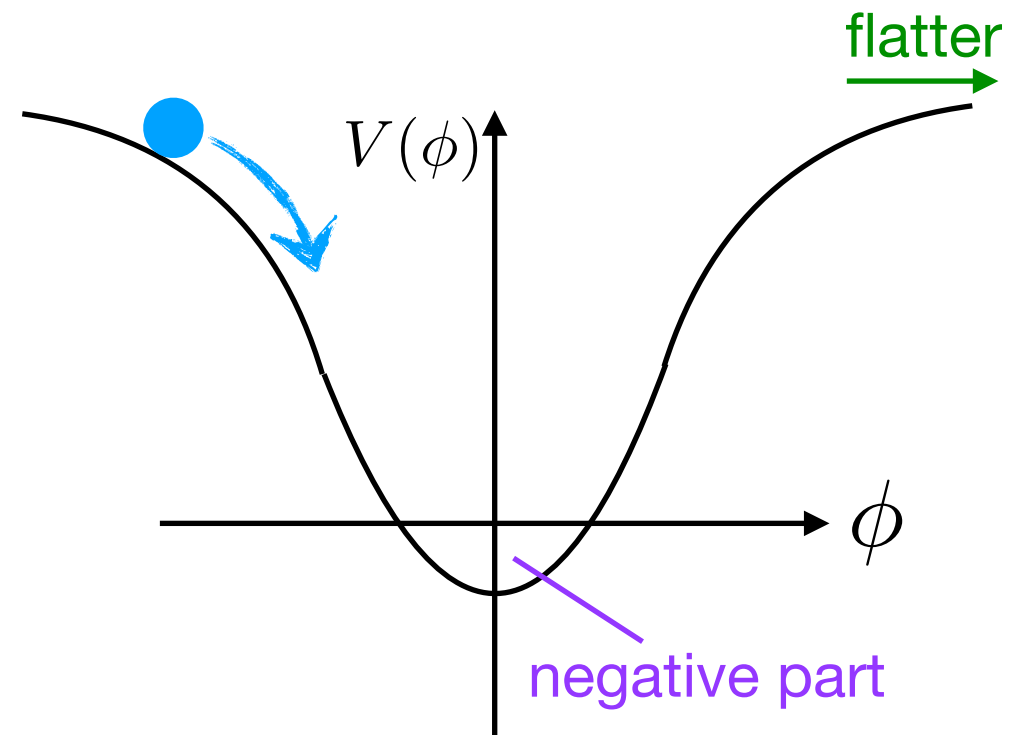
$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{\mathcal{K}}{a^2},$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 + \frac{\mathcal{K}}{a^2},$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

Suppose $\rho \gg \frac{\mathcal{K}}{a^2} > 0$ initially.

Scalar Potential



1. Initial Expansion Phase

Eqs. of motion

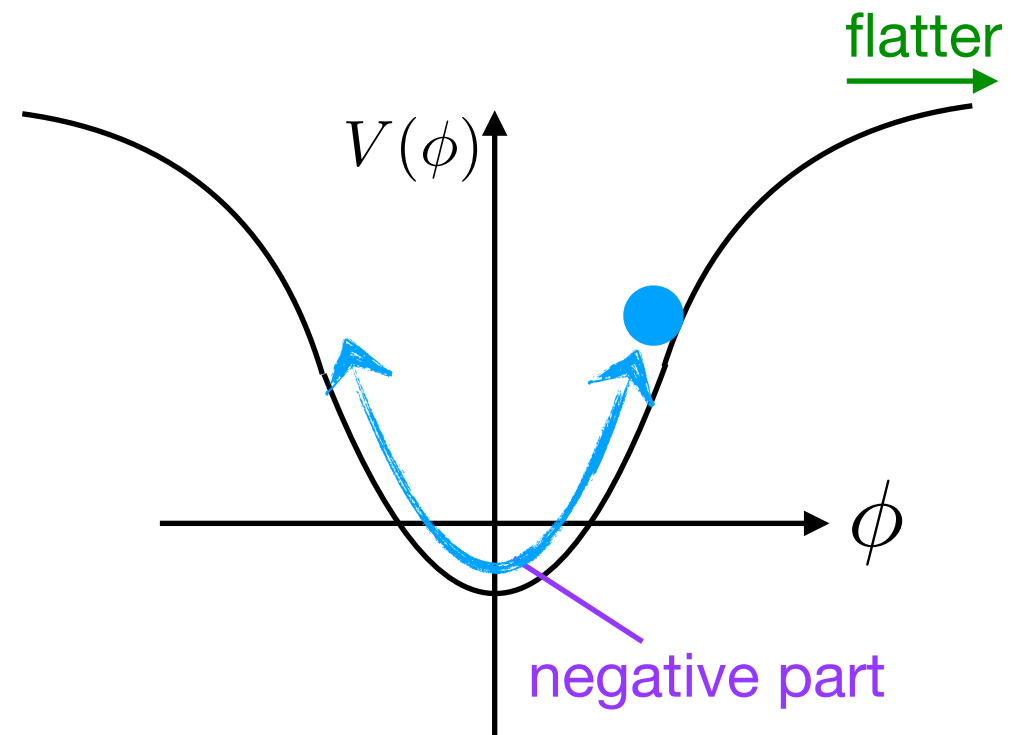
$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{\mathcal{K}}{a^2},$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 + \frac{\mathcal{K}}{a^2},$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

Suppose $\rho \gg \frac{\mathcal{K}}{a^2} > 0$ initially.

Scalar Potential



1. Initial Expansion Phase

Eqs. of motion

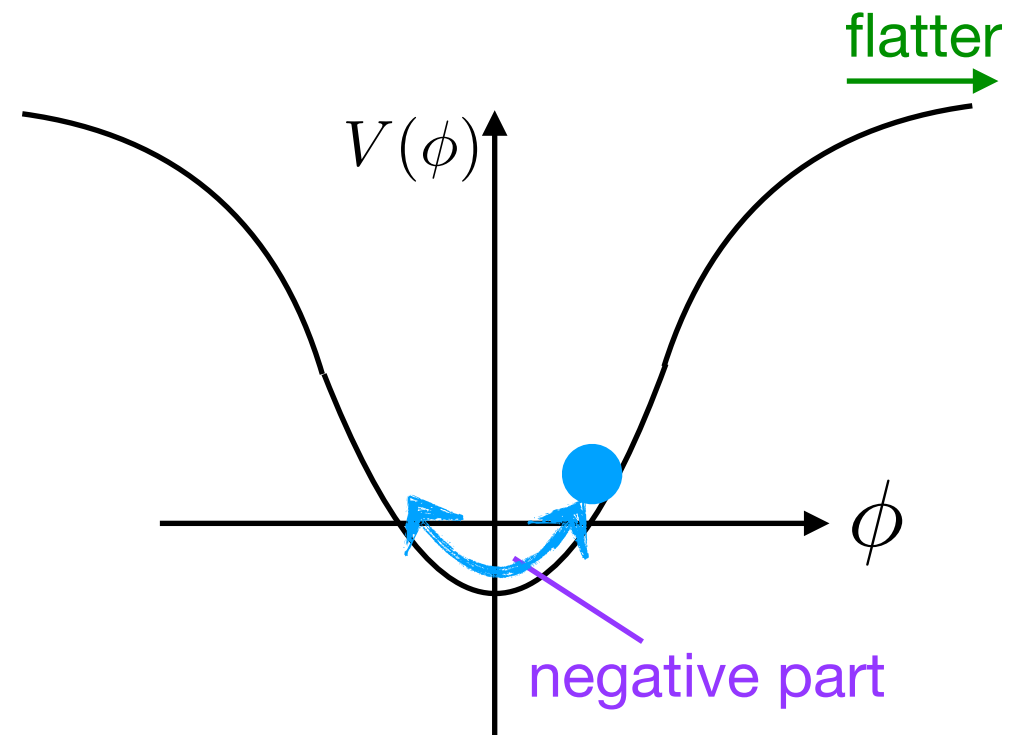
$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{\mathcal{K}}{a^2},$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 + \frac{\mathcal{K}}{a^2},$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

Suppose $\rho \gg \frac{\mathcal{K}}{a^2} > 0$ initially.

Scalar Potential



2. Contraction Phase

Eqs. of motion

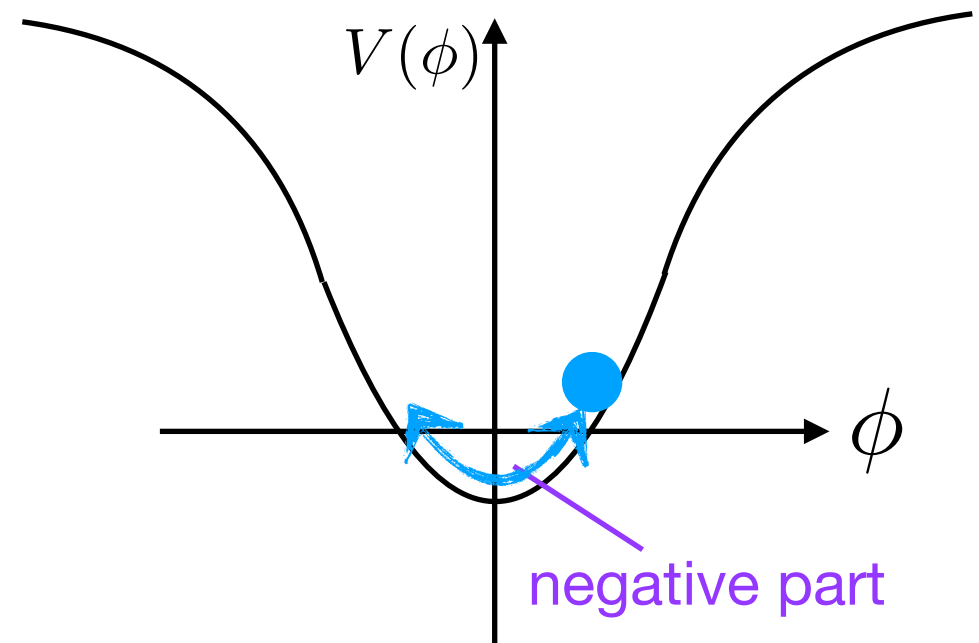
$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{\mathcal{K}}{a^2},$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 + \frac{\mathcal{K}}{a^2},$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

During contraction, $H < 0$ works as ***anti-friction***.

Scalar Potential



2. Contraction Phase

Eqs. of motion

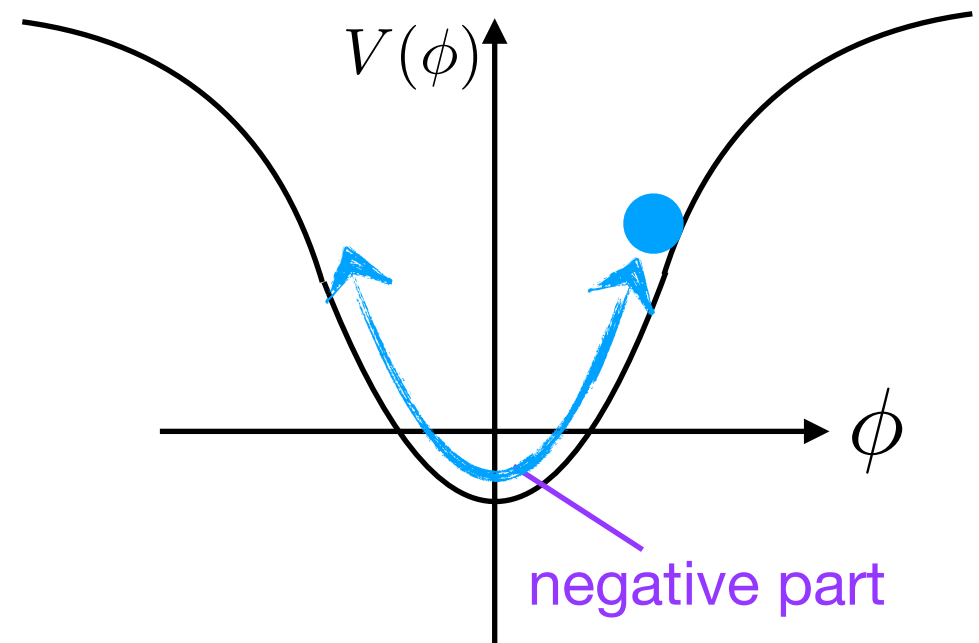
$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{\mathcal{K}}{a^2},$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 + \frac{\mathcal{K}}{a^2},$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

During contraction, $H < 0$ works as ***anti-friction***.

Scalar Potential



2. Contraction Phase

Eqs. of motion

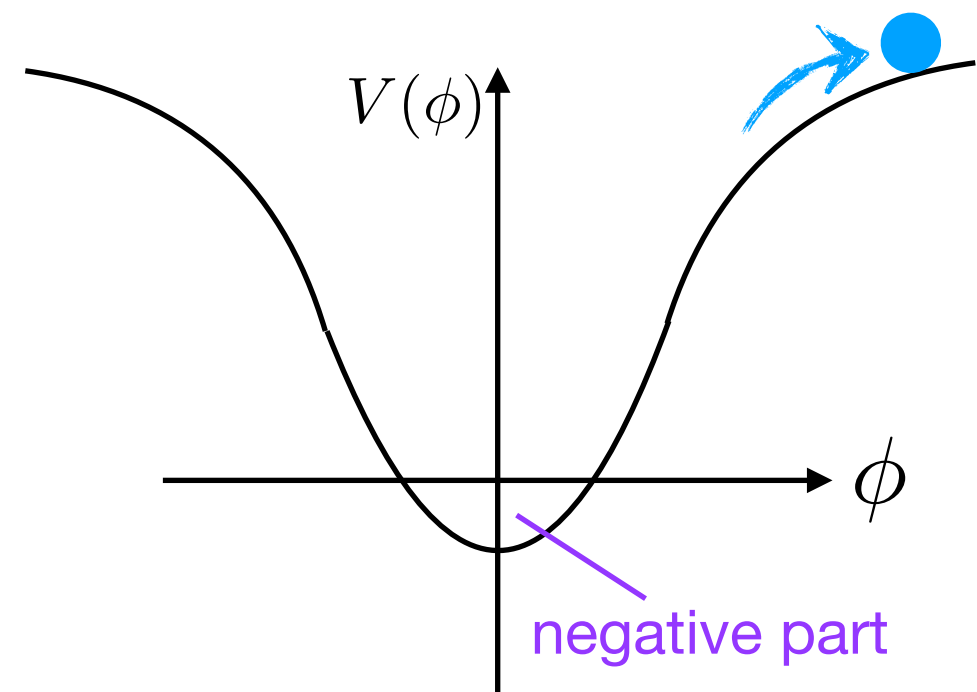
$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{\mathcal{K}}{a^2},$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 + \frac{\mathcal{K}}{a^2},$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

During contraction, $H < 0$ works as ***anti-friction***.

Scalar Potential



If the kinetic energy is sufficiently suppressed on the plateau,
the positive curvature can make the universe expand again.

3. Second Expansion Phase

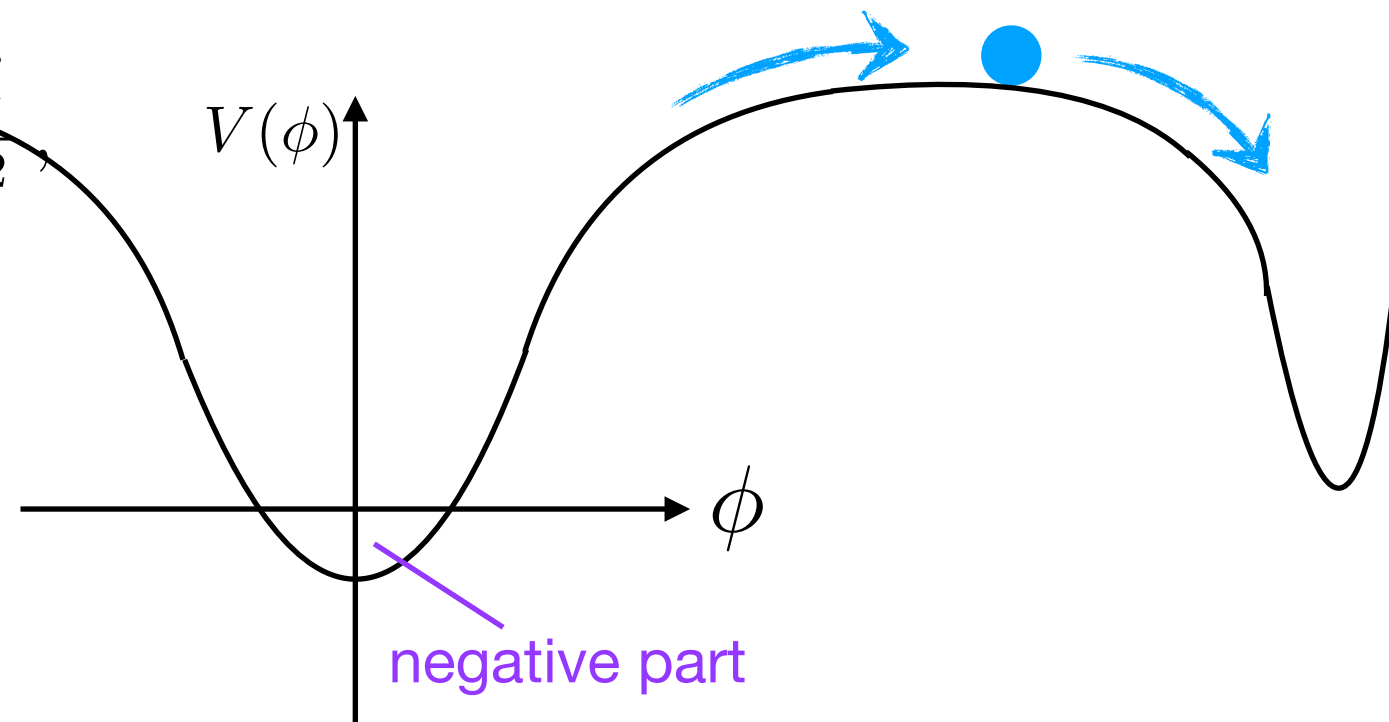
Eqs. of motion

$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{\mathcal{K}}{a^2},$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 + \frac{\mathcal{K}}{a^2},$$

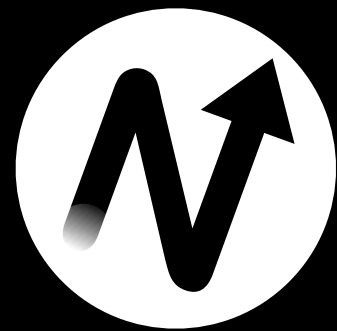
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

Scalar Potential



The flat part of the potential, *built in for the bounce purpose*, lets the universe naturally enter the **slow-roll inflationary** regime.

The resultant cosmology can be **consistent with observation**.



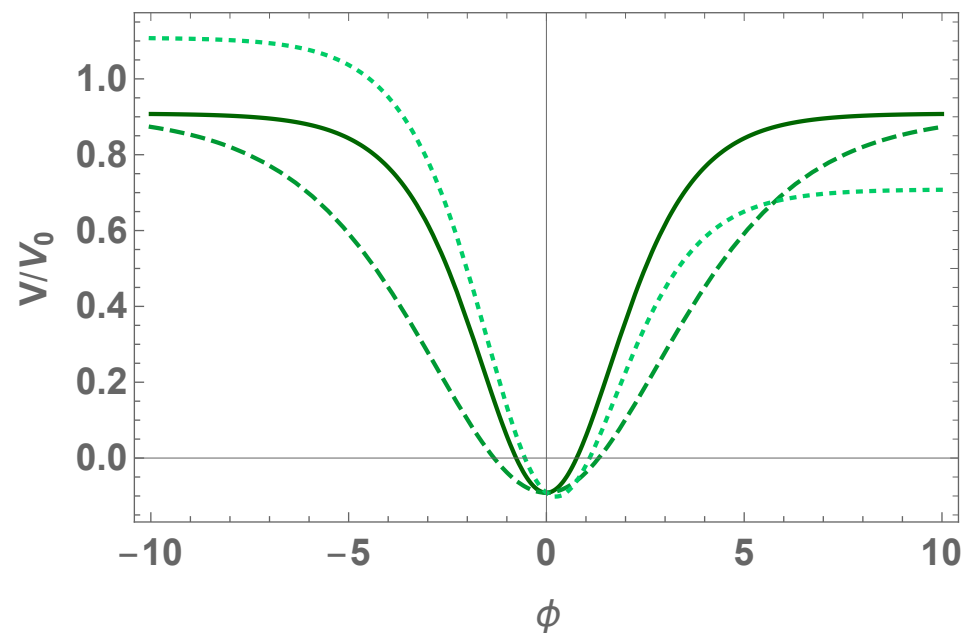
Example Model

Example Model

Scalar potential

$$V(\phi) = V_0 \left(\tanh^2 \left[\frac{\phi}{\sqrt{6\alpha}} \right] + \beta \tanh \left[\frac{\phi}{\sqrt{6\alpha}} \right] + \gamma \right)$$

$$\alpha > 0, \quad -1 < \beta < 1, \quad -1 < \gamma \leq 0$$

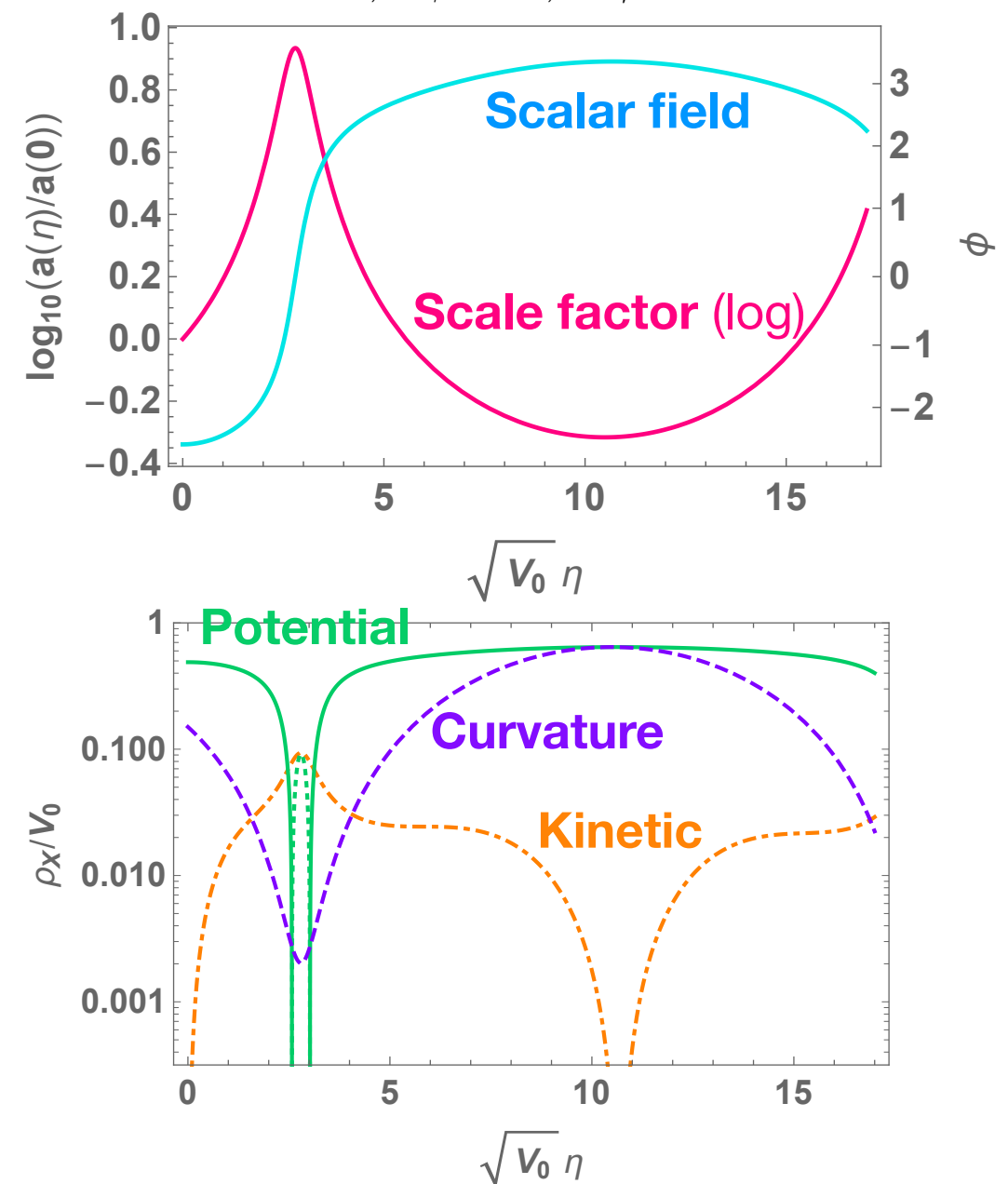


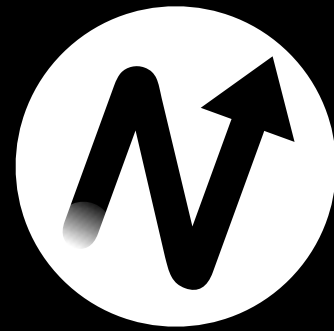
Initial conditions for numerical calculation

$$\phi(0) = -\sqrt{6\alpha}, \quad \dot{\phi}(0) = 0, \quad \text{and} \quad \frac{\mathcal{K}}{a(0)^2} = 0.05\sqrt{V_0}.$$

Numerical results

$$\alpha = 1, \quad \beta = 0, \quad \gamma = -0.09143$$





Possible origin of the positive curvature

Birth of Closed Universe

Mini-superspace approximation

Wave function of the universe: $\Psi[g_{\mu\nu}(t, x), \phi(t, x)] = \Psi[a(t), \phi(t)]$

Wheeler-De Wit eq.

$$\mathcal{H}(a, \phi) \Psi(a, \phi) = 0$$

where

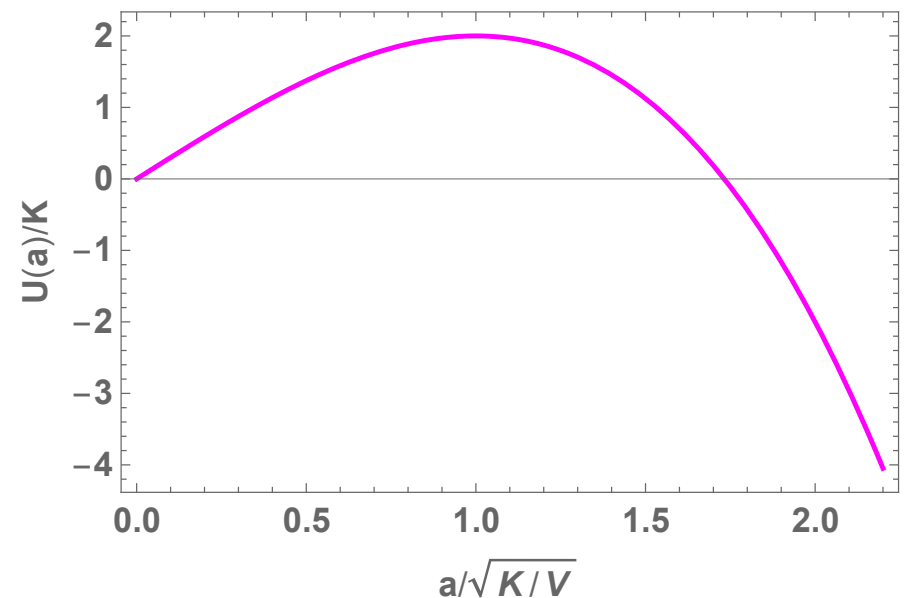
$$\mathcal{H}(a, \phi) = \frac{1}{12a^2} \frac{\partial}{\partial a} \left(a \frac{\partial}{\partial a} \right) - \frac{1}{2a^3} \frac{\partial^2}{\partial \phi^2} - U(a, \phi) \quad [\text{Kiefer, Sandhoefer, 0804.0672}]$$

$$U(a, \phi) = a^3 \left(\frac{3\mathcal{K}}{a^2} - V(\phi) \right)$$

Initial conditions [Vilenkin, PRD37, 888 (1988)]

$$a(0) = \sqrt{\frac{3\mathcal{K}}{V(\phi)}} \quad \dot{a}(0) = 0$$

$$\phi(0) = \text{const.} \quad \dot{\phi}(0) = 0$$



Birth of Closed Universe

Mini-superspace approximation

Wave function of the universe: $\Psi[g_{\mu\nu}(t, x), \phi(t, x)] = \Psi[a(t), \phi(t)]$

Wheeler-De Wit eq.

$$\mathcal{H}(a, \phi) \Psi(a, \phi) = 0$$

where

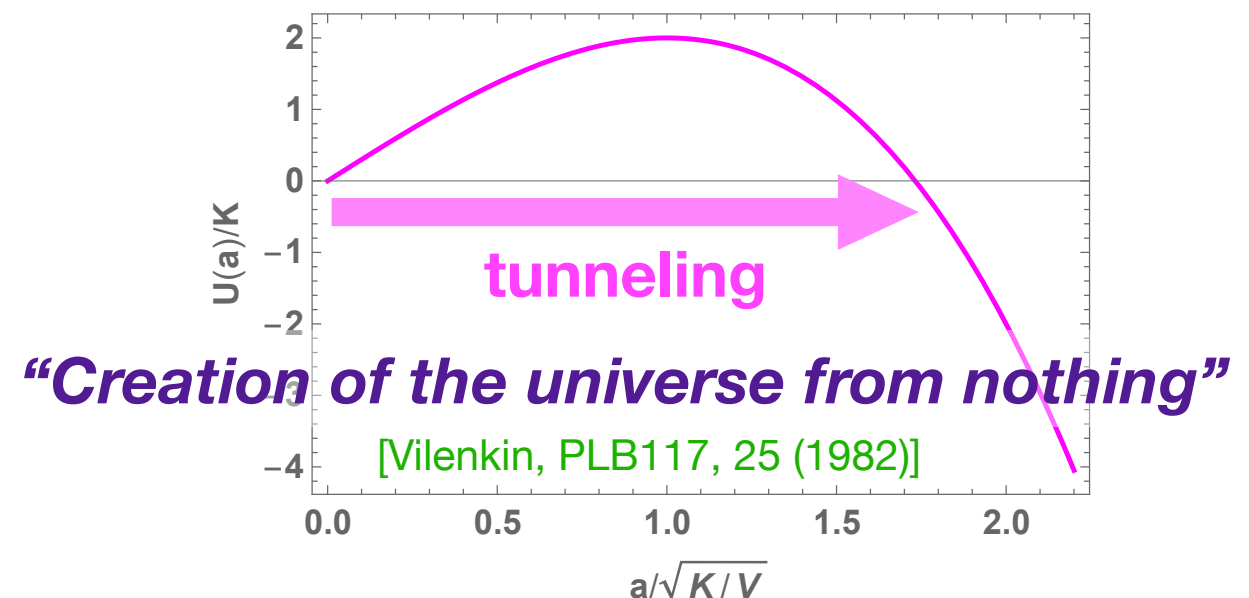
$$\mathcal{H}(a, \phi) = \frac{1}{12a^2} \frac{\partial}{\partial a} \left(a \frac{\partial}{\partial a} \right) - \frac{1}{2a^3} \frac{\partial^2}{\partial \phi^2} - U(a, \phi) \quad [\text{Kiefer, Sandhoefer, 0804.0672}]$$

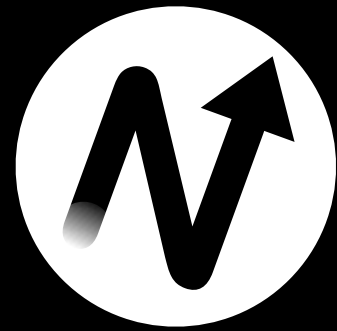
$$U(a, \phi) = a^3 \left(\frac{3\mathcal{K}}{a^2} - V(\phi) \right)$$

Initial conditions [Vilenkin, PRD37, 888 (1988)]

$$a(0) = \sqrt{\frac{3\mathcal{K}}{V(\phi)}} \quad \dot{a}(0) = 0$$

$$\phi(0) = \text{const.} \quad \dot{\phi}(0) = 0$$





Summary and conclusions

Summary & Conclusion

Summary & Conclusion

- We find new nontrivial cosmological solutions.
 - (Creation from Nothing $\rightarrow$) Expansion $\rightarrow$ Contraction $\rightarrow$ Inflationary Expansion
 - (Creation from Nothing $\rightarrow$) Cyclic

Summary & Conclusion

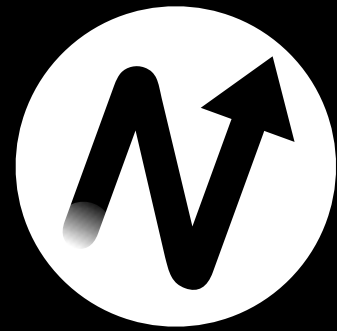
- We find new nontrivial cosmological solutions.
 - (Creation from Nothing $\rightarrow$) Expansion $\rightarrow$ Contraction $\rightarrow$ Inflationary Expansion
 - (Creation from Nothing $\rightarrow$) Cyclic
- 4dim, Einsteing gravity, a real canonical scalar.
No violation of Null Energy Condition. No singularity.

Summary & Conclusion

- We find new nontrivial cosmological solutions.
 - (Creation from Nothing $\rightarrow$) Expansion $\rightarrow$ Contraction $\rightarrow$ Inflationary Expansion
 - (Creation from Nothing $\rightarrow$) Cyclic
- 4dim, Einsteing gravity, a real canonical scalar.
No violation of Null Energy Condition. No singularity.
- Our solutions open interesting possibilities
for General Relativity and Our Universe (past and future).

Summary & Conclusion

- We find new nontrivial cosmological solutions.
 - (Creation from Nothing $\rightarrow$) Expansion $\rightarrow$ Contraction $\rightarrow$ Inflationary Expansion
 - (Creation from Nothing $\rightarrow$) Cyclic
- 4dim, Einsteing gravity, a real canonical scalar.
No violation of Null Energy Condition. No singularity.
- Our solutions open interesting possibilities for General Relativity and Our Universe (past and future).
- Many things to be explored.



Appendix

Dynamics of Scalar Field

approximation	regime	relevance
$\ddot{\phi} + \cancel{3H\dot{\phi}} + \frac{\partial V(\phi)}{\partial \phi} = 0$	“No friction” e.g.) oscillation	relevant around H=0
$\cancel{\ddot{\phi}} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	“Slow-Roll regime” Potential energy dominate.	attractor solution during expansion
$\ddot{\phi} + 3H\dot{\phi} + \cancel{\frac{\partial V(\phi)}{\partial \phi}} = 0$	“Ultra-Slow-Roll regime” Kinetic energy is important.	attractor solution during contraction
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	“Scaling solution” (special situation)	

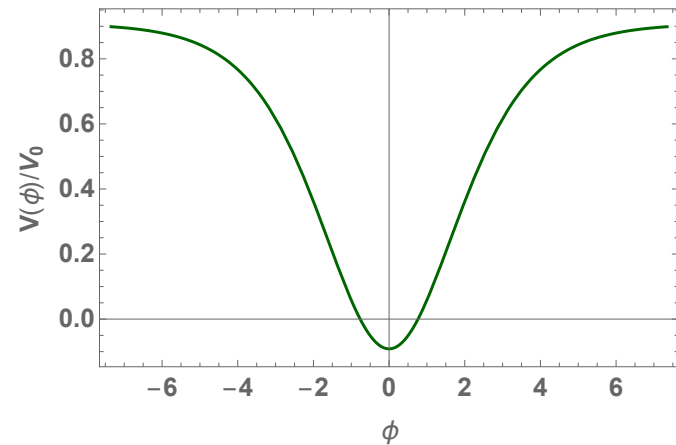
Dynamics of Scalar Field

approximation	regime	relevance
$\ddot{\phi} + \cancel{3H\dot{\phi}} + \frac{\partial V(\phi)}{\partial \phi} = 0$	“No friction” e.g.) oscillation	relevant around $H=0$
$\cancel{\ddot{\phi}} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	“Slow-Roll regime” Potential energy dominate.	attractor solution during expansion
$\ddot{\phi} + 3H\dot{\phi} + \cancel{\frac{\partial V(\phi)}{\partial \phi}} = 0$	“Ultra-Slow-Roll regime” Kinetic energy is important.	attractor solution during contraction
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	“Scaling solution” (special situation)	

**How to suppress the kinetic energy
is the key for a successful bounce.**

Tuning of the offset

$$\alpha = 1, \quad \beta = 0, \quad \gamma = -0.09143$$

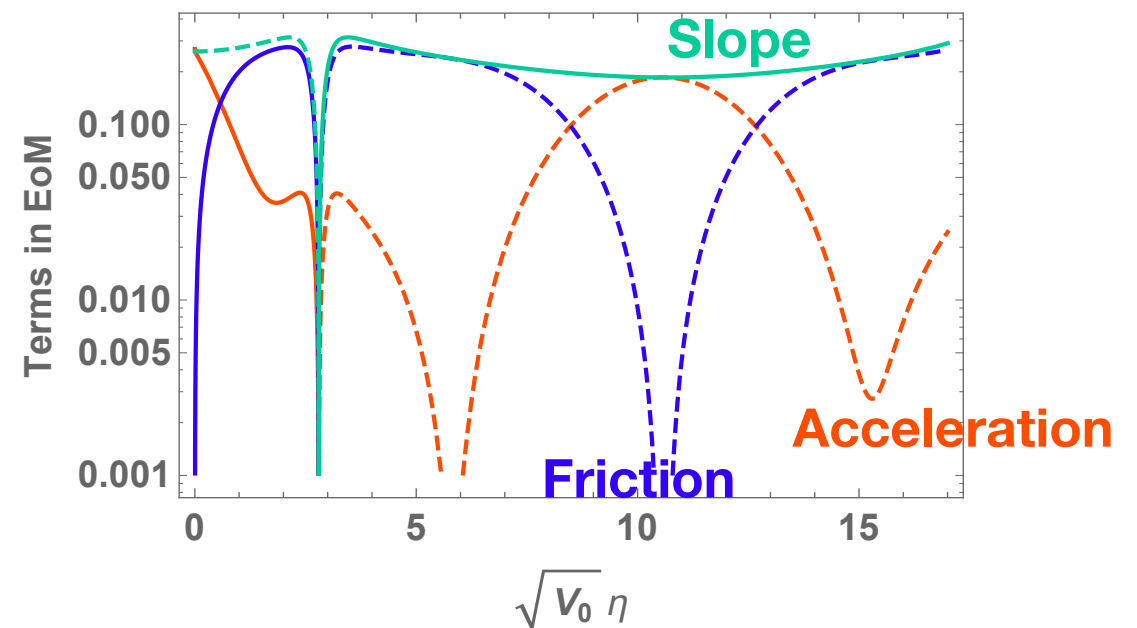
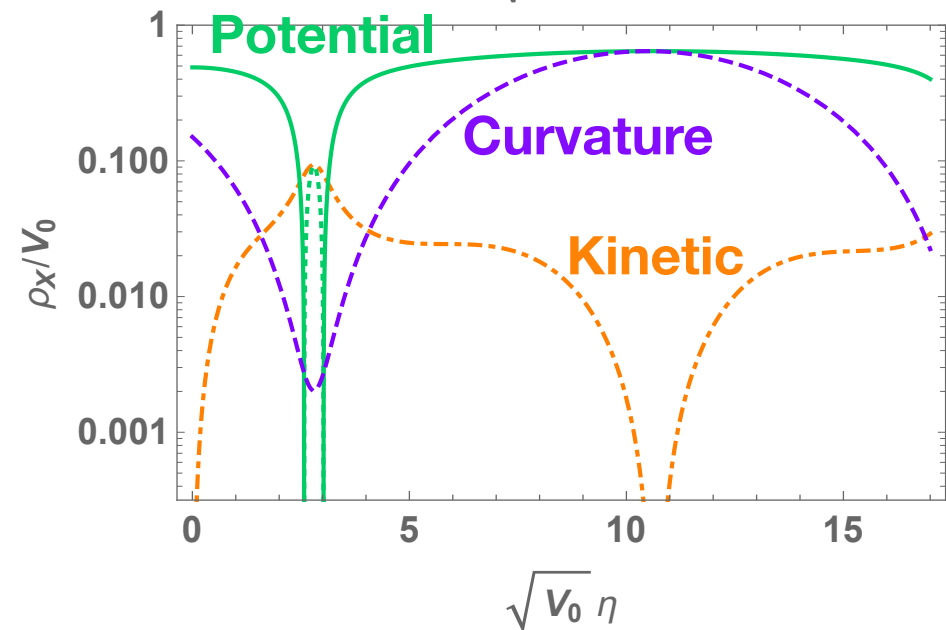
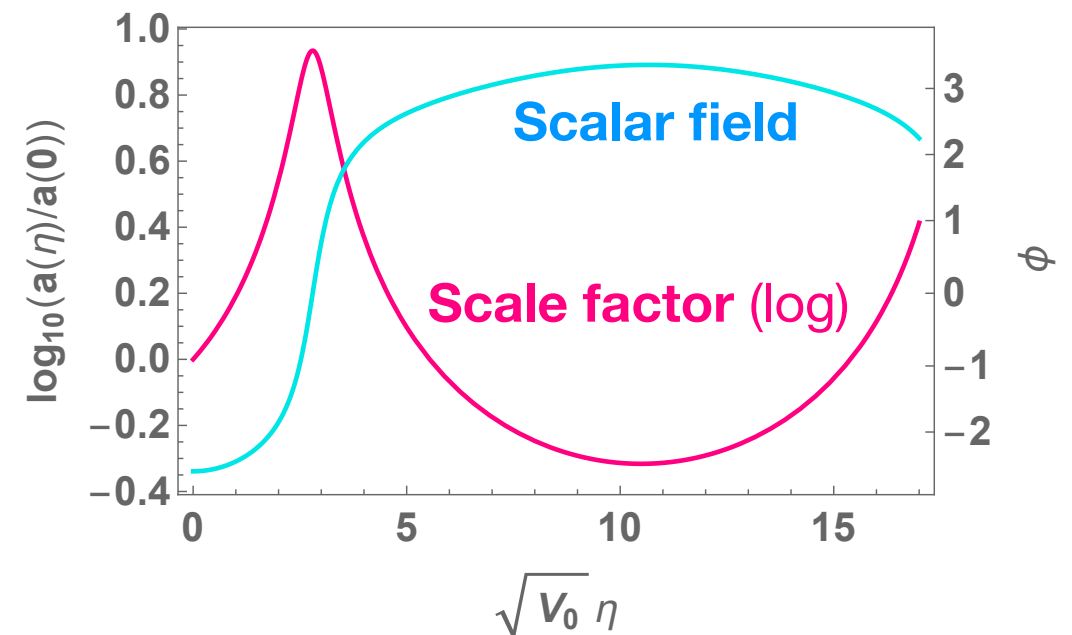


$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V - \frac{3\mathcal{K}}{a^2}$$

Potential energy
Kinetic energy
“Curvature energy”

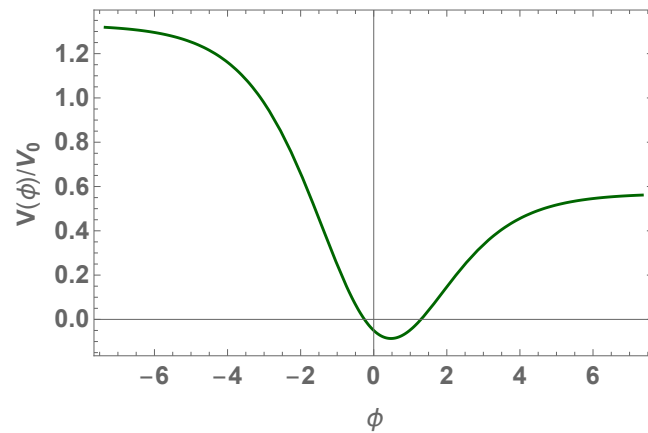
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

Acceleration
Friction
Slope



Tuning of the left-right asymmetry

$$\alpha = 1, \quad \beta = -0.3805885, \quad \gamma = -0.05$$

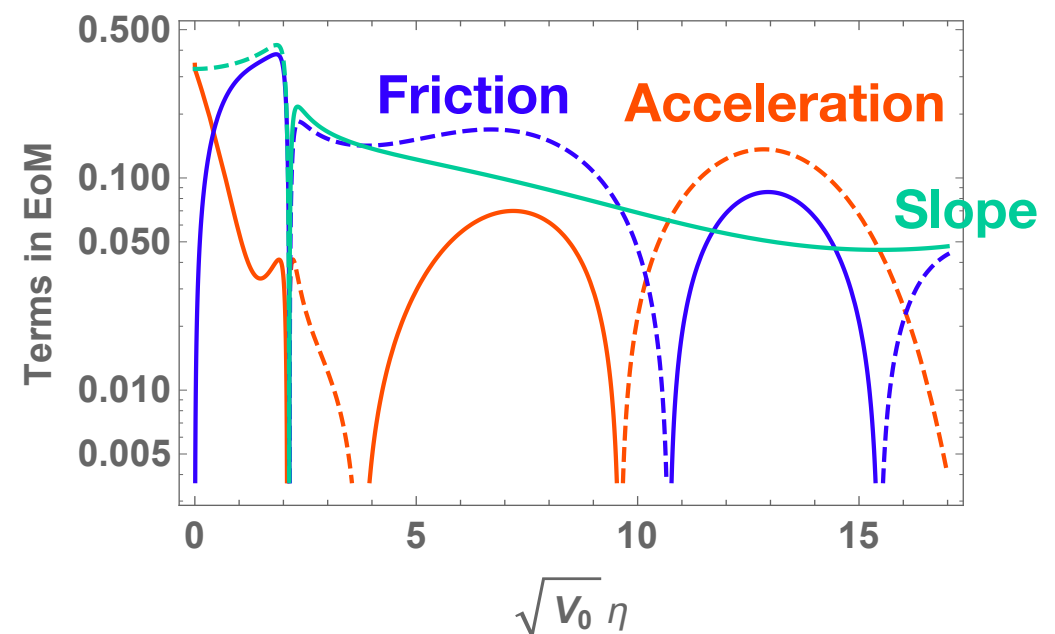
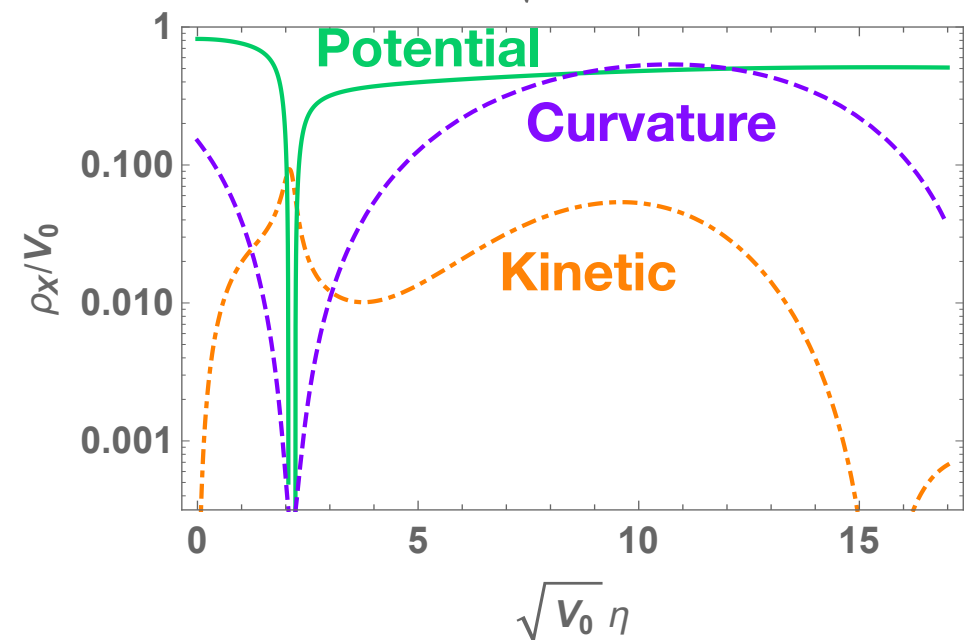
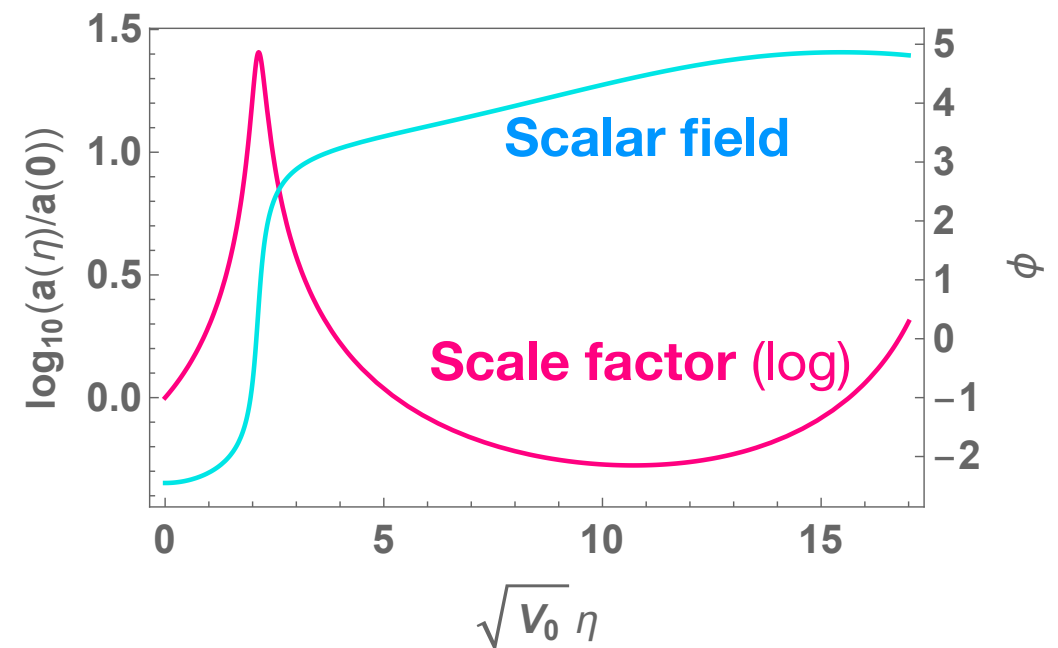


$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V - \frac{3\mathcal{K}}{a^2}$$

Potential energy
Kinetic energy
“Curvature energy”

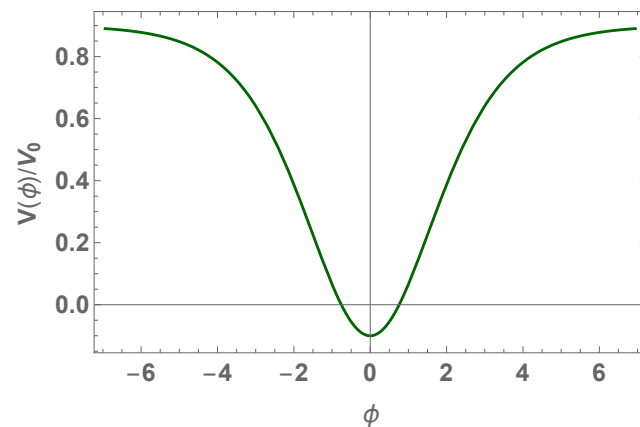
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

Acceleration
Friction
Slope



Tuning of the width

$$\alpha = 0.8924, \quad \beta = 0, \quad \gamma = -0.1$$

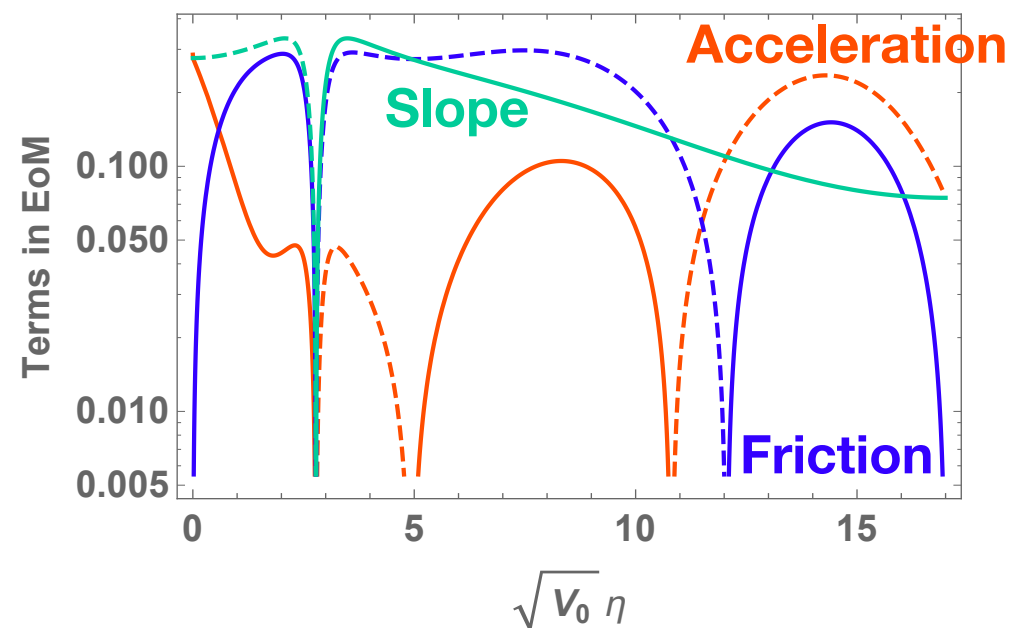
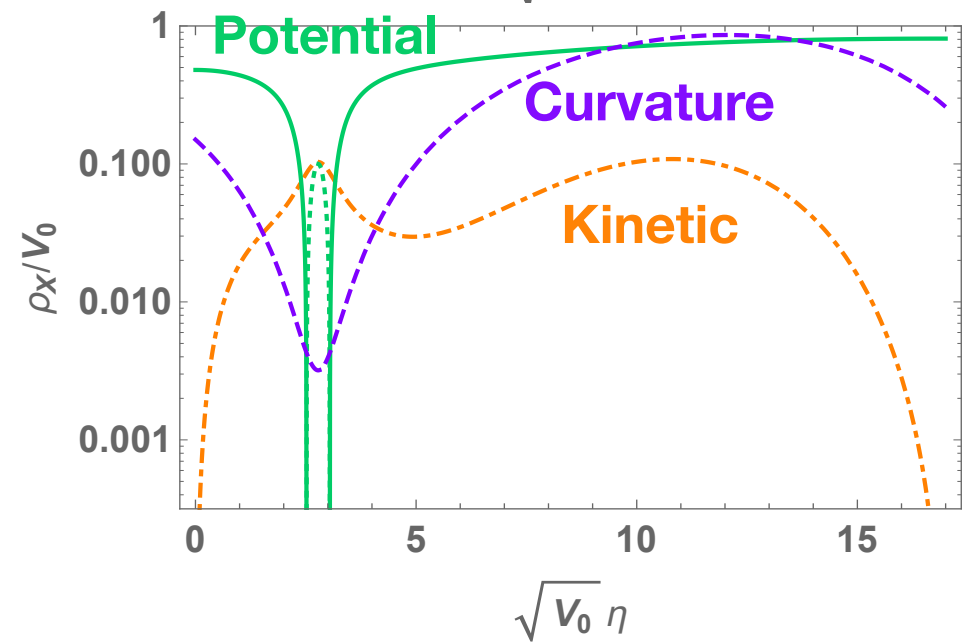
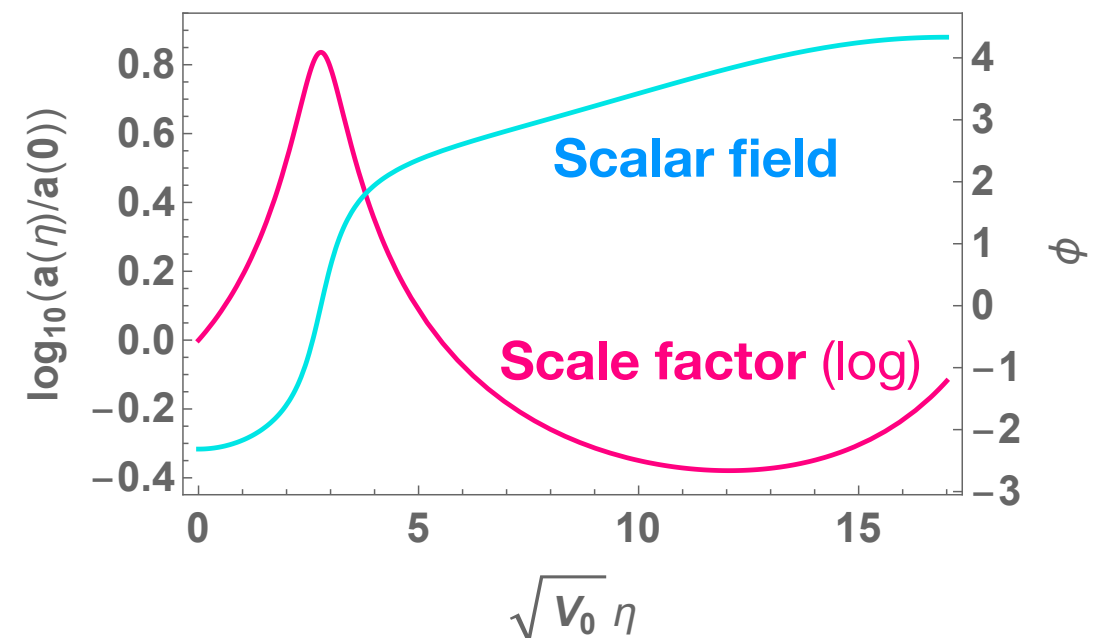


$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V - \frac{3\mathcal{K}}{a^2}$$

Potential energy
Kinetic energy
“Curvature energy”

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

Acceleration
Slope
Friction



Birth of Closed Universe

Initial conditions [Vilenkin, PRD37, 888 (1988)]

$$a(0) = \sqrt{\frac{3\mathcal{K}}{V(\phi)}} \quad \dot{a}(0) = 0$$

$$\phi(0) = \text{const.} \quad \dot{\phi}(0) = 0$$



How this is determined is controversial.

Two proposals [Vilenkin, PRD37, 888 (1988)]
for nucleation probability

$$\mathcal{P}(a, \phi) \propto \exp\left(\mp \frac{24\pi^2 M_{\text{P}}^4}{V(\phi)}\right)$$

[Vilenkin, PRD30, 509 (1984)]

[Hartle, Hawking, PRD28, 2960 (1983)]
“no-boundary proposal”

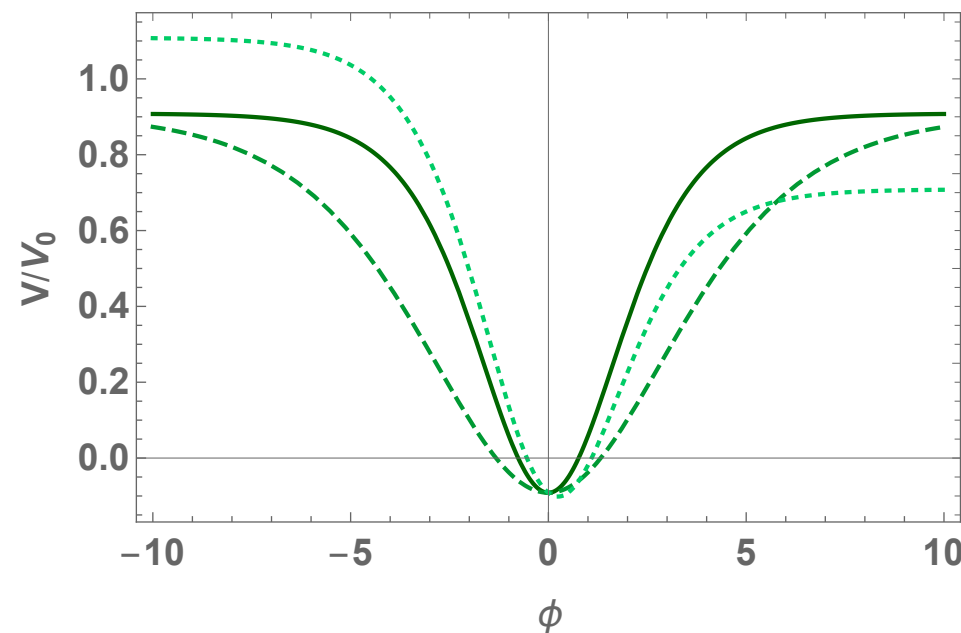
Even if the universe is born with tiny energy density by the Hartle-Hawking process, the energy density can increase in the contraction phase, which makes the scenario viable.

Cyclic solution

Scalar potential

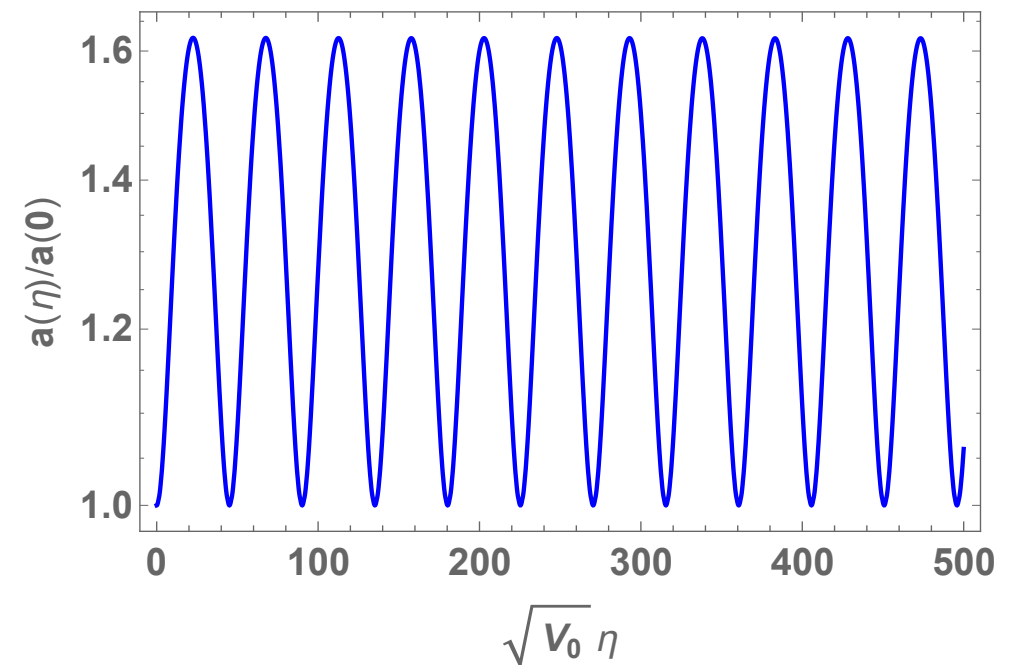
$$V(\phi) = V_0 \left(\tanh^2 \left[\frac{\phi}{\sqrt{6\alpha}} \right] + \beta \tanh \left[\frac{\phi}{\sqrt{6\alpha}} \right] + \gamma \right)$$

$$\alpha > 0, \quad -1 < \beta < 1, \quad -1 < \gamma \leq 0$$



Numerical results

$$\alpha = 5 \times 10^{-5}, \quad \beta = \gamma = 0, \quad \mathcal{K} = 0.05V_0$$



Initial conditions for numerical calculation

$$a(0) = \sqrt{\frac{3\mathcal{K}}{V(\phi)}} \quad \dot{a}(0) = 0 \quad \phi(0) = -3\sqrt{6\alpha} \quad \dot{\phi}(0) = 0$$

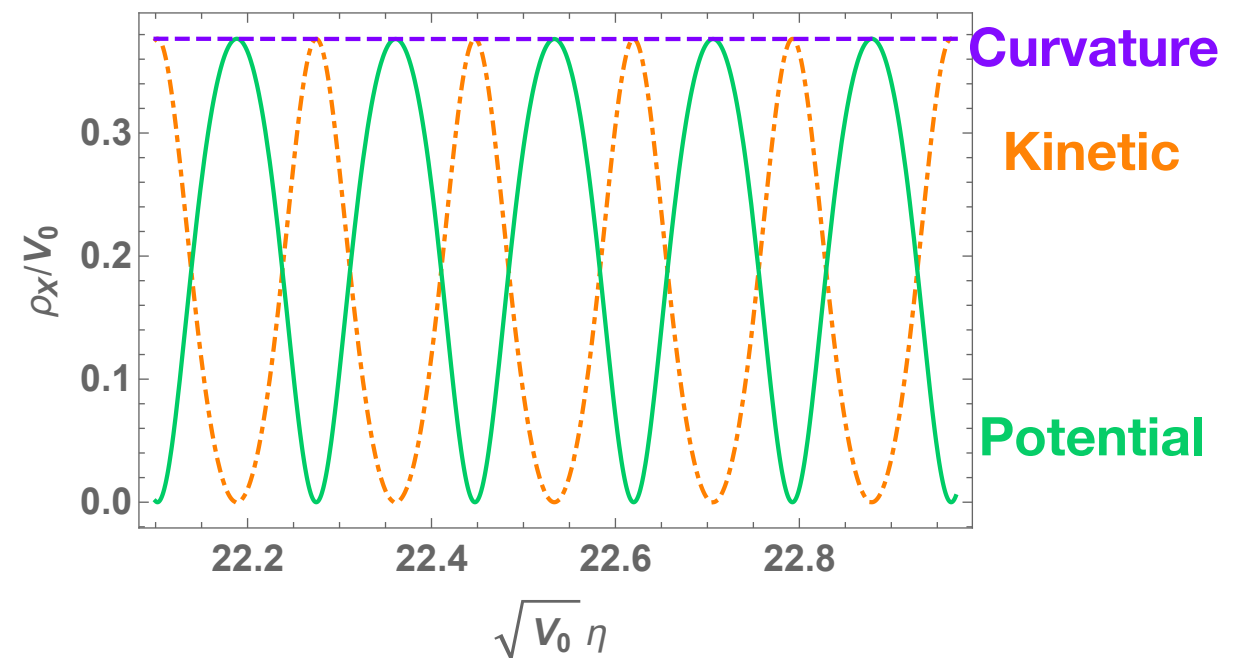
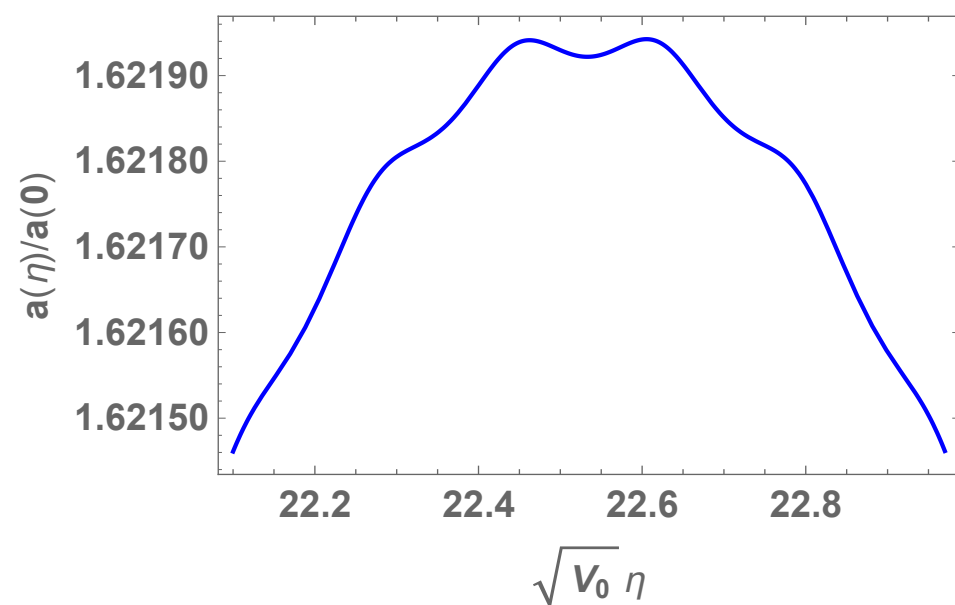
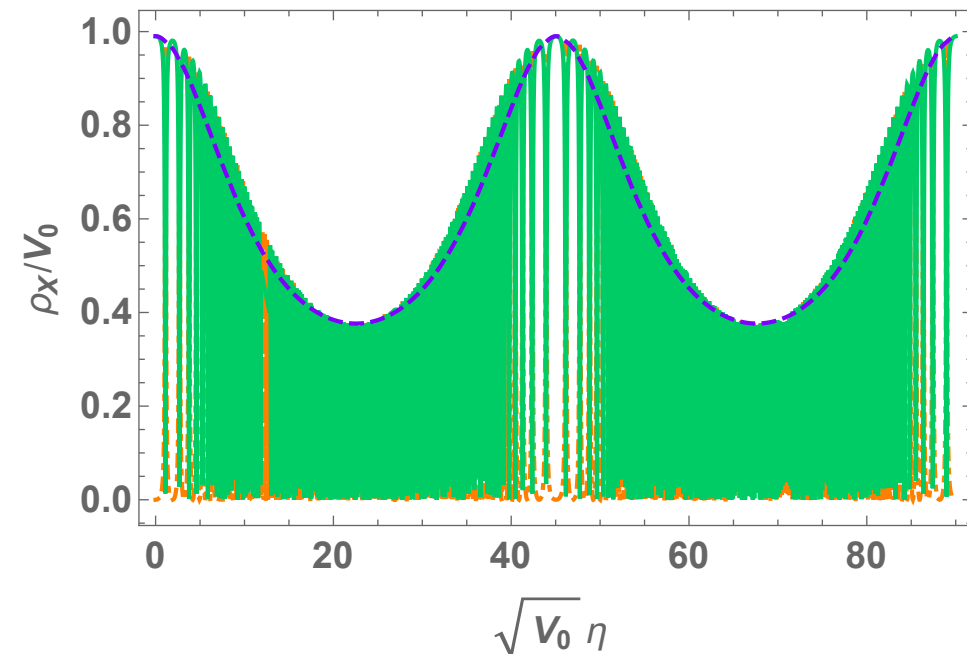
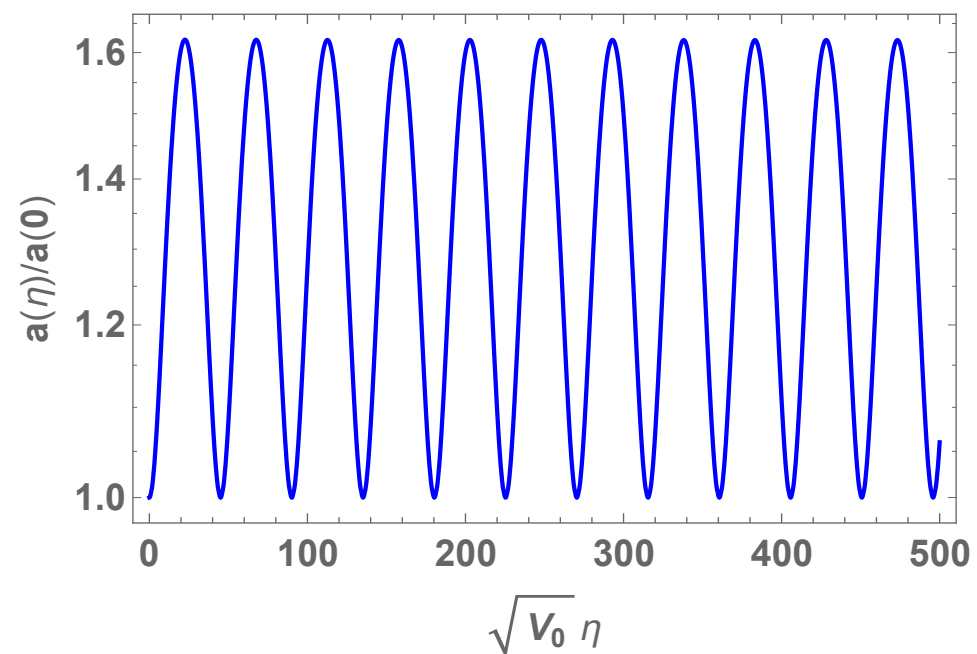
(Details in the next slide)

Cyclic solution

$$\alpha = 5 \times 10^{-5}, \quad \beta = \gamma = 0, \quad \mathcal{K} = 0.05V_0$$

Scale factor

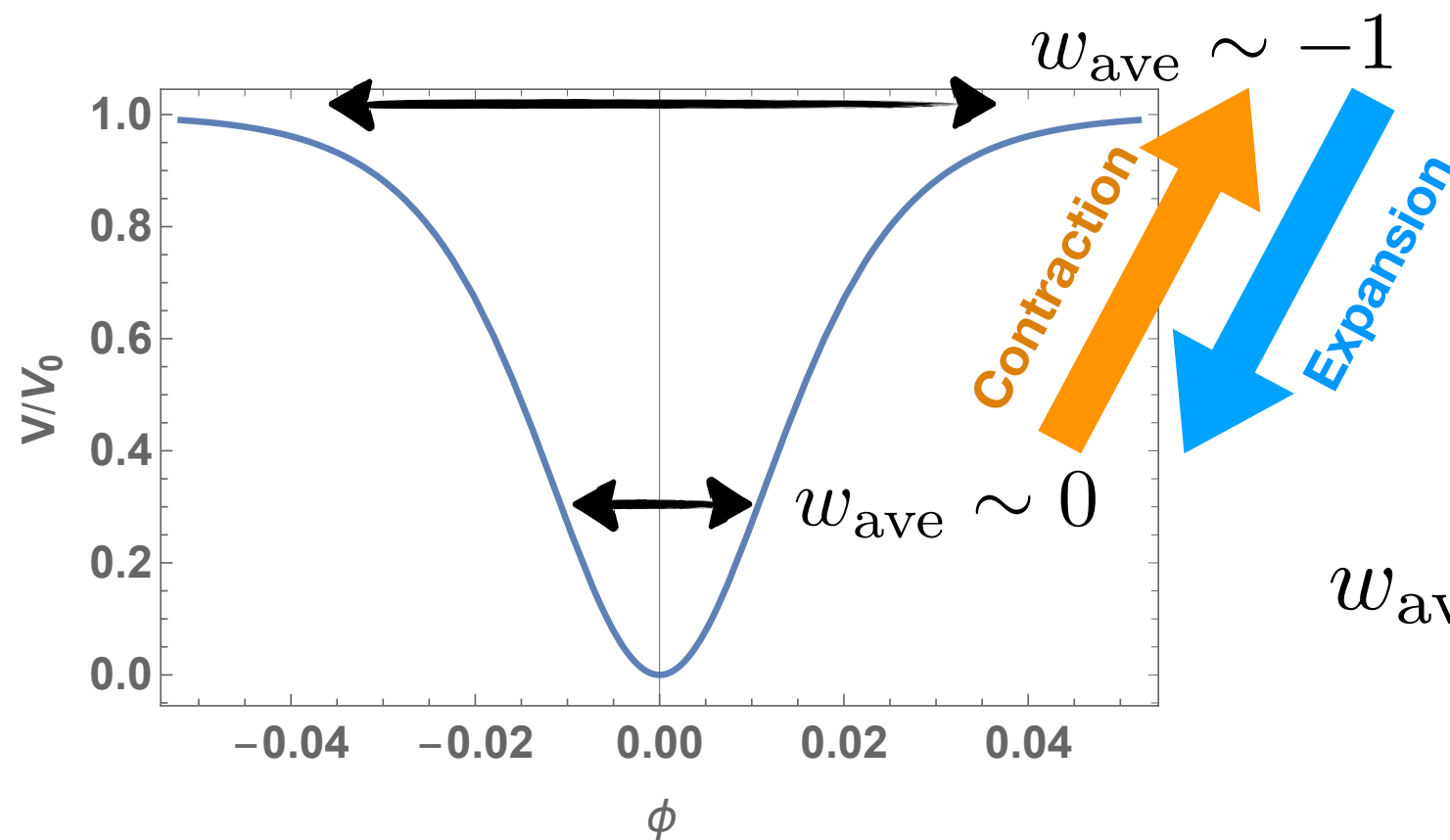
Energy densities



Mechanism of Cycles

Coarse graining oscillations of the scalar field

$$w_{\text{ave}} = \langle w \rangle_{\text{osc}}$$



w_{ave} oscillates around $-1/3$.

Discussion

Discussion

- Not restricted to our **past**. It may occur in **future**.

Discussion

- Not restricted to our **past**. It may occur in **future**.
- **Tuning is necessary.** Anthropic justification?

Discussion

- Not restricted to our **past**. It may occur in **future**.
- **Tuning is necessary**. Anthropic justification?
- **Eternal inflation** or “**just enough**” **inflation**. Observable effects?

Discussion

- Not restricted to our **past**. It may occur in **future**.
- **Tuning is necessary**. Anthropic justification?
- **Eternal inflation** or “**just enough**” **inflation**. Observable effects?
- **CMB data** slightly favor positive curvature. It can relax the **H₀ tension**.

Discussion

- Not restricted to our **past**. It may occur in **future**.
- **Tuning is necessary**. Anthropic justification?
- **Eternal inflation** or “**just enough**” **inflation**. Observable effects?
- **CMB data** slightly favor positive curvature. It can relax the **H₀ tension**.
- In contrast with bubble universes in the **string landscape**.

Discussion

- Not restricted to our **past**. It may occur in **future**.
- **Tuning is necessary**. Anthropic justification?
- **Eternal inflation** or “**just enough**” **inflation**. Observable effects?
- **CMB data** slightly favor positive curvature. It can relax the **H₀ tension**.
- In contrast with bubble universes in the **string landscape**.
- **Vacuum energy can change** during the contraction phase.

Discussion

- Not restricted to our **past**. It may occur in **future**.
- **Tuning is necessary**. Anthropic justification?
- **Eternal inflation** or “**just enough**” **inflation**. Observable effects?
- **CMB data** slightly favor positive curvature. It can relax the **H₀ tension**.
- In contrast with bubble universes in the **string landscape**.
- **Vacuum energy can change** during the contraction phase.
- **Bounce at an arbitrarily higher energy scale** may be possible.

Constraints on Curvature

$$\Omega_K = -0.056^{+0.028}_{-0.018}$$

(68%, Planck TT+lowE)

$$\Omega_K = -0.044^{+0.018}_{-0.015}$$

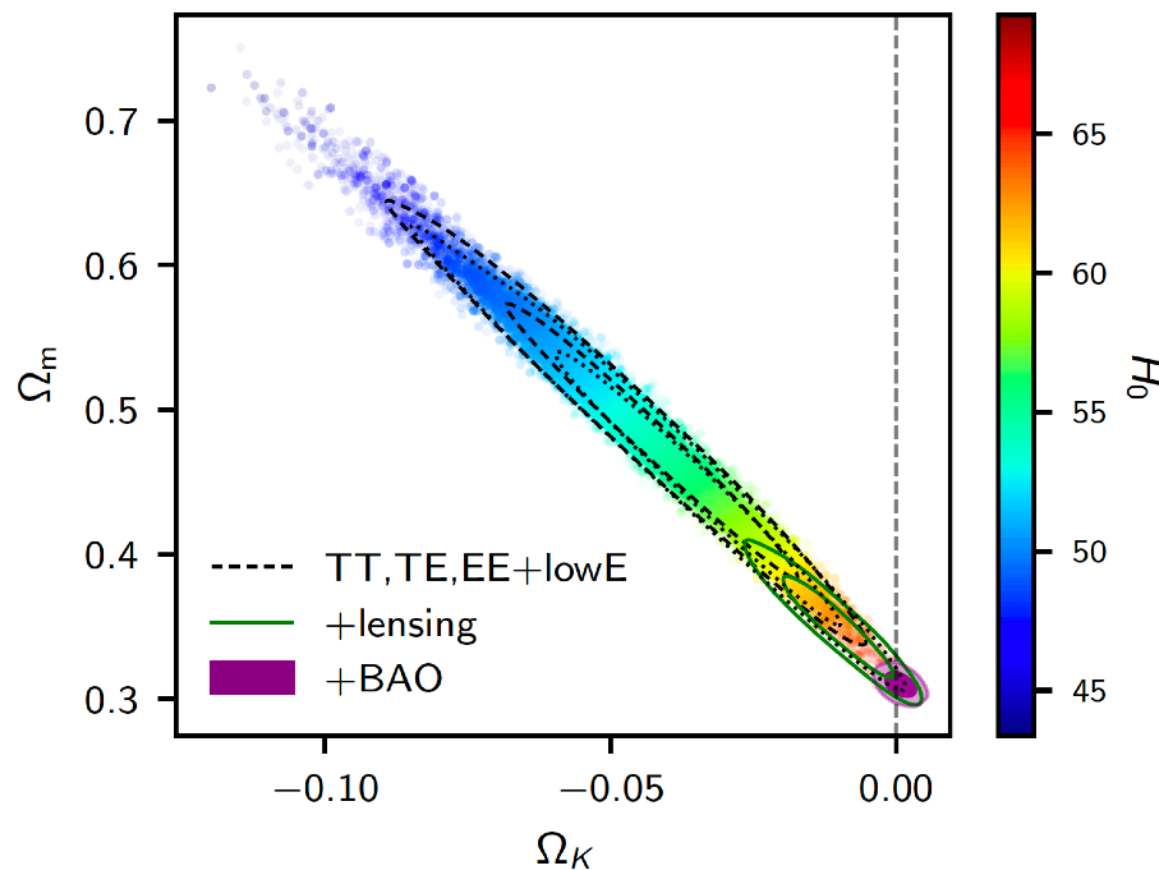
(68%, Planck TT,TE,EE+lowE)

$$\Omega_K = -0.0106 \pm 0.0065$$

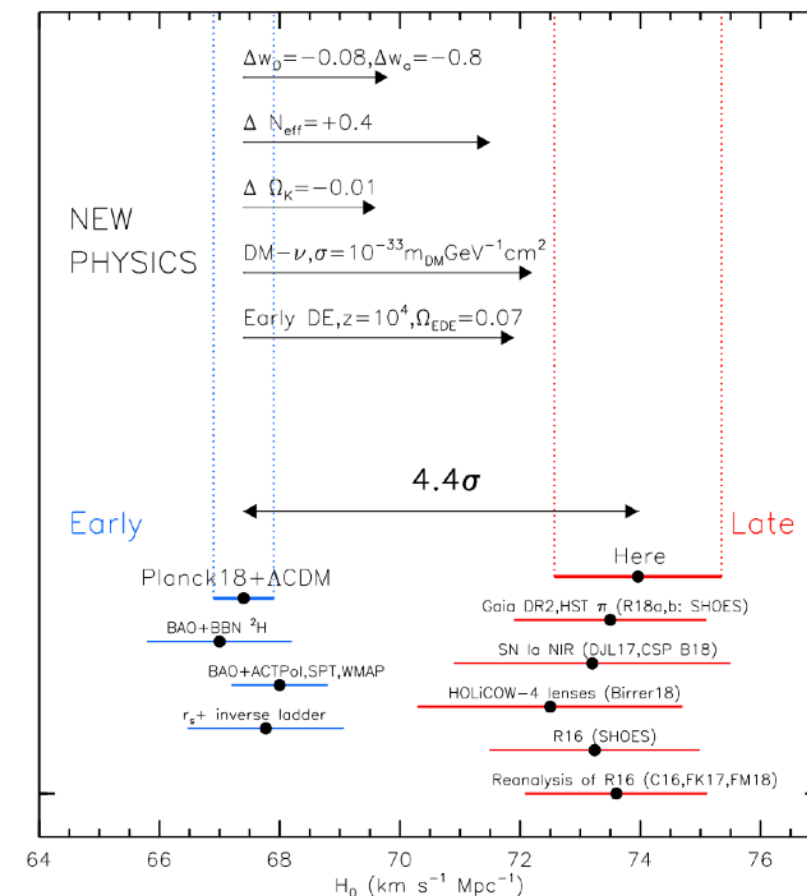
(68%, Planck TT,TE,EE+lowE+lensing)

$$\Omega_K = 0.0007 \pm 0.0019$$

(68%, Planck TT,TE,EE+lowE+lensing+BAO)

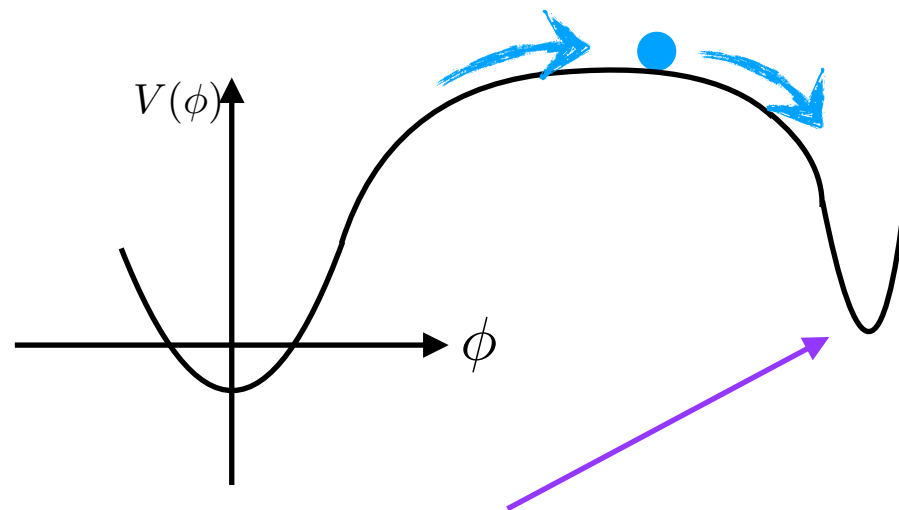


[Planck collaboration 2018: Cosmological parameters]



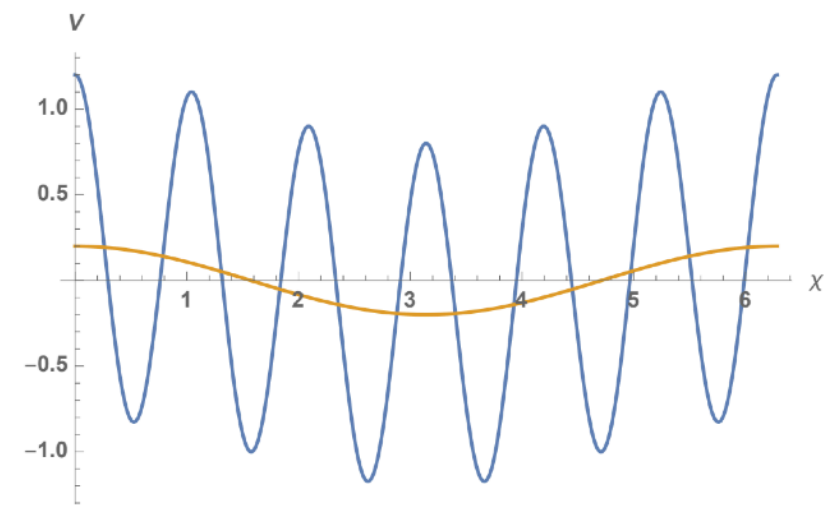
[Riess, Casertano, Yuan, Macri, Scolnic, 1903.07603]

Uplifting Vacuum Energy



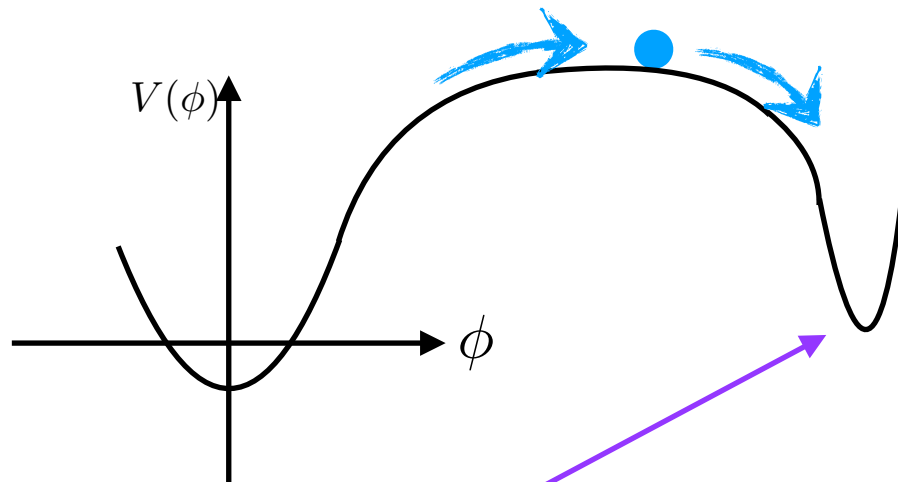
Another minimum for ϕ
may not be necessary.

Potential of light (axionic) field

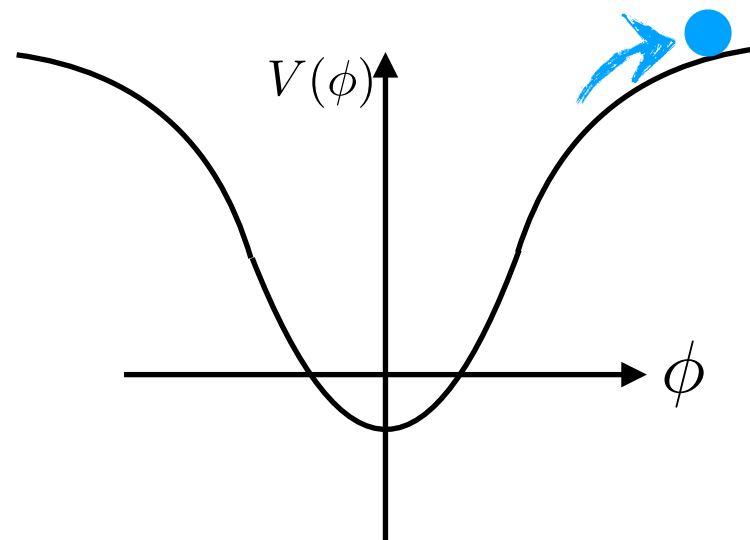


[Graham, Kaplan, Rajendran, 1902.06793]

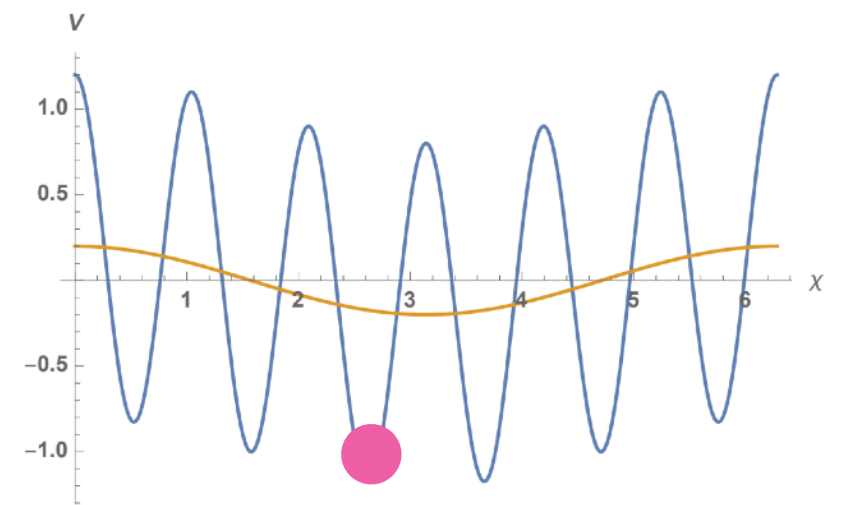
Uplifting Vacuum Energy



Another minimum for ϕ
may not be necessary.

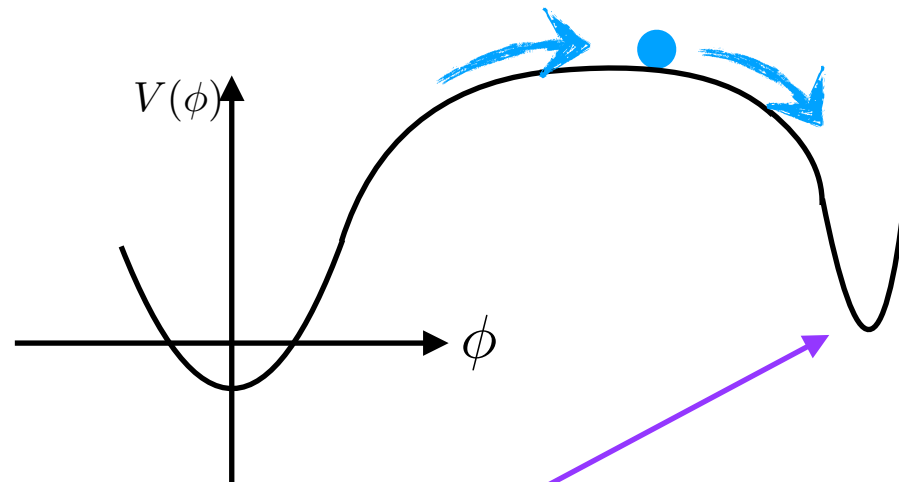


Potential of light (axionic) field

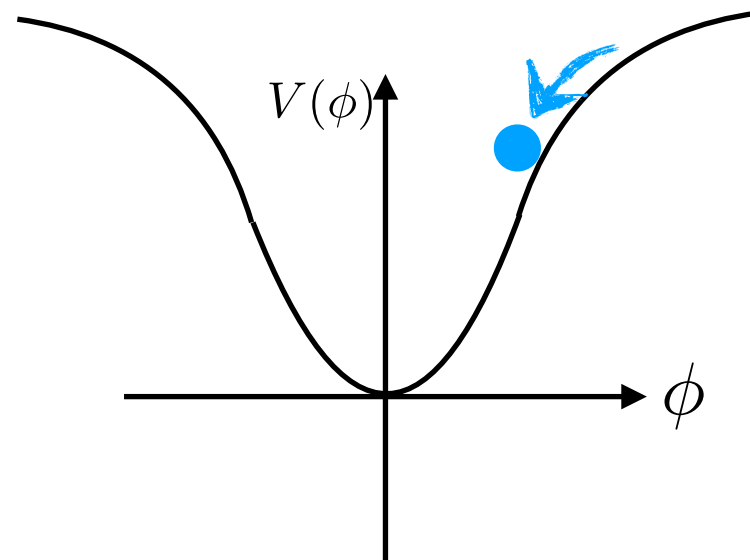


[Graham, Kaplan, Rajendran, 1902.06793]

Uplifting Vacuum Energy

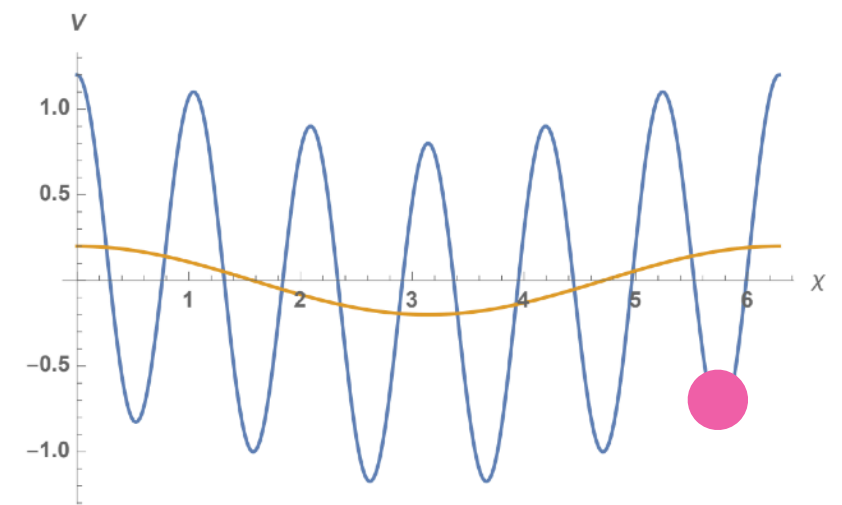


Another minimum for ϕ
may not be necessary.



Uplift of the cosmological constant is possible.

Potential of light (axionic) field



[Graham, Kaplan, Rajendran, 1902.06793]

Increase Energy before Bounce

Dynamics of Scalar Field

approximation	regime	relevance
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"No friction" e.g.) oscillation	relevant around $H=0$
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Slow-Roll regime" Potential energy dominate.	attractor solution during expansion
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Ultra-Slow-Roll regime" Kinetic energy is important.	attractor solution during contraction
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Scaling solution" (special situation)	

How to suppress the kinetic energy
is the key for a successful bounce.

Kinetic-potential scaling solution

$$\frac{1}{2}\dot{\phi}^2(t) \propto V(\phi(t))$$

$$V(\phi) \sim \exp\left(-\sqrt{3(1+w)}\phi\right)$$

Increase Energy before Bounce

Dynamics of Scalar Field

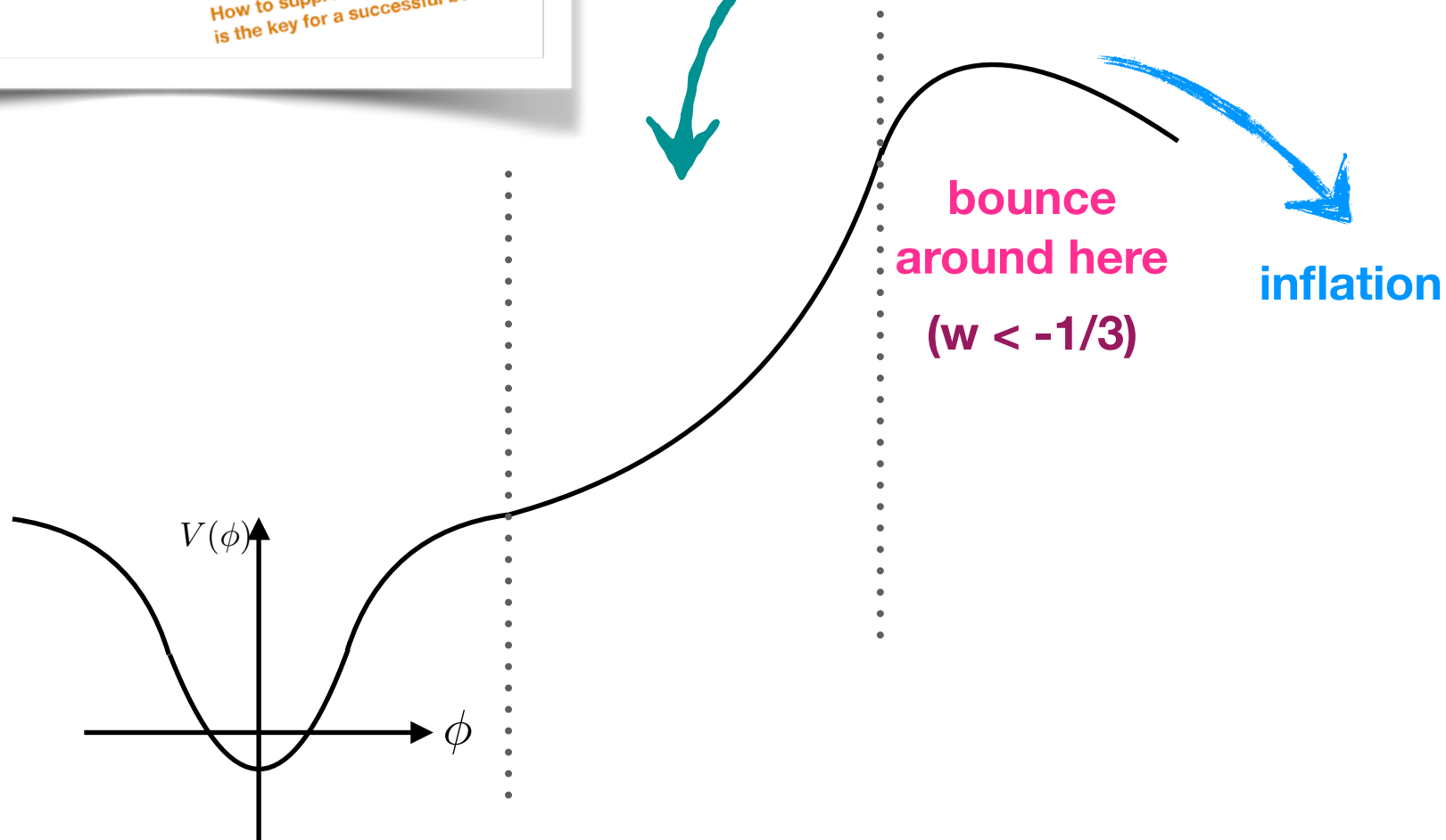
approximation	regime	relevance
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"No friction" e.g.) oscillation	relevant around $H=0$
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Slow-Roll regime" Potential energy dominate.	attractor solution during expansion
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Ultra-Slow-Roll regime" Kinetic energy is important.	attractor solution during contraction
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Scaling solution" (special situation)	

How to suppress the kinetic energy
is the key for a successful bounce.

Kinetic-potential scaling solution

$$\frac{1}{2}\dot{\phi}^2(t) \propto V(\phi(t))$$

$$V(\phi) \sim \exp\left(-\sqrt{3(1+w)}\phi\right)$$



Increase Energy before Bounce

Dynamics of Scalar Field

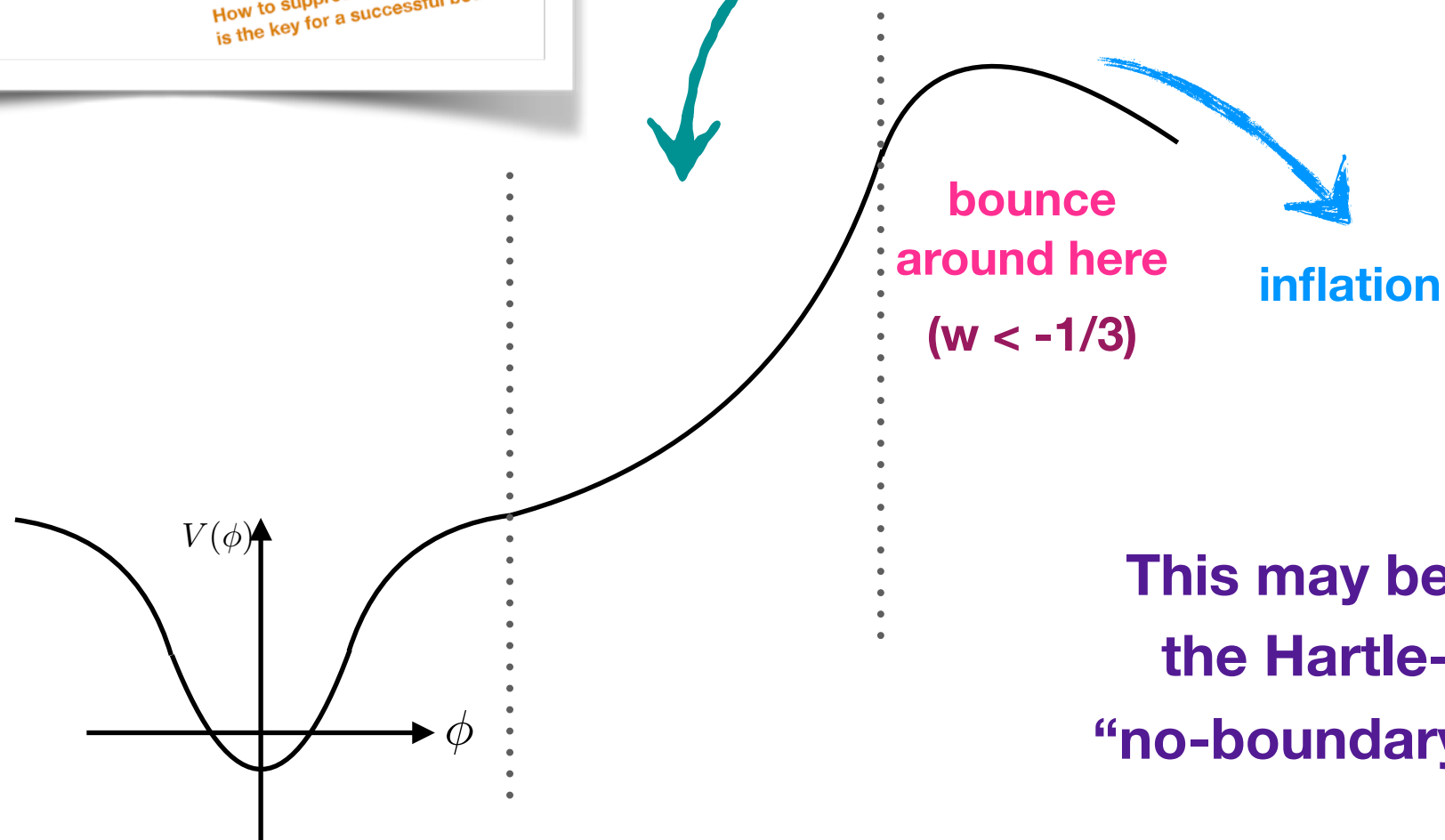
approximation	regime	relevance
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"No friction" e.g.) oscillation	relevant around $H=0$
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Slow-Roll regime" Potential energy dominate.	attractor solution during expansion
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Ultra-Slow-Roll regime" Kinetic energy is important.	attractor solution during contraction
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Scaling solution" (special situation)	

How to suppress the kinetic energy
is the key for a successful bounce.

Kinetic-potential scaling solution

$$\frac{1}{2}\dot{\phi}^2(t) \propto V(\phi(t))$$

$$V(\phi) \sim \exp\left(-\sqrt{3(1+w)}\phi\right)$$



Related literature

Our scenario *as a whole* gives us a new interesting possibility,
but *each part* has been studied well in the literature.

Note: We do NOT violate the null energy condition.

We consider only NON-singular bounce.

Contraction by a negative potential

[Linde, hep-th/0110195]

[Felder et al., hep-th/0202017]

Cyclic universe

[Kardashev, MNRAS 243, 252 (1990)]

[Dabrowski, gr-qc/9503017]

[Graham, Horn, Kachru, Rajendran, Torroba, 1109.0282]

[Graham, Horn, Rajendran, Torroba, 1405.0282]

See also

[Biswas, 0801.1315]

[Biswas, Alexander, 0812.3182]

[Barrow, Ganguly, 1703.05969]

[Ganguly, Barrow, 1710.00747]

Bounce with positive curvature

[Martin, Peter, hep-th/0307077]

[Gordon, Turok, hep-th/0206138]

[Falciano, Lilley, Peter, 0802.1196]

[Haro, 1511.05048]

[Parker, Fulling, PRD7, 2357 (1973)]

[Starobinsky, SAL4, 82 (1978)]

[Barrow, Matzner, PRD21, 336 (1980)]

[Hawking, Les Houches 1983]

[Page, CQG 1, 417 (1984)]

[Schmidt, gr-qc/0108087]

[Cornish, Shellard, gr-qc/9708046]