Neutrino Masses and Mixing: Theoretical Aspects

S. T. Petcov

SISSA/INFN, Trieste, Italy, and
Kavli IPMU, University of Tokyo, Japan

Johns Hopkins Workshop
Kavli IPMU, Kashiwa (Tokyo), Japan
June 5, 2019

Understanding the origin of the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years is one of the most challenging problems in neutrino physics. It is part of the more general fundamental problem in particle physics of understanding the origins of flavour in the quark and lepton sectors, i.e., of the patterns of quark masses and mixing, and of the charged lepton and neutrino masses and of neutrino mixing.

"Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesnt have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses."

From Model Physicist, CERN Courier, 13 October 2017.

Of fundamental importance are also

- the determination of the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos (which is one of the most challenging and pressing problems in present day elementary particle physics) (GERDA, CUORE, KamLAND-Zen, EXO, LEGEND, nEXO,...);

- determining the status of CP symmetry in the lepton sector (T2K, NOνA; T2HK, DUNE);

- determination of the type of spectrum neutrino masses possess, or the "neutrino mass ordering" (T2K + NOνA; JUNO; PINGU, ORCA; T2HKK, DUNE);

- determination of the absolute neutrino mass scale, or $\min(m_j)$ (KATRIN, new ideas; cosmology).

The program of research extends beyond 2030.

There have been remarkable discoveries in neutrino physics in the last ~ 21 years.

Experimental Proofs for $\nu-$Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of** $\mu-$**like events**

Dominant $\nu_{\mu} \rightarrow \nu_{\tau}$ K2K, MINOS, T2K; CNGS (OPERA)

$-\nu_{\odot}$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ **BOREXINO**

$-\bar{\nu}_e$ (from reactors): **Daya Bay, RENO, Double Chooz**

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS, NOν**A (**ν_μ **from accelerators):** $\nu_{\mu} \rightarrow \nu_e$

T2K, NOν**A (**$\bar{\nu}_\mu$ **from accelerators):** $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$

Compelling Evidences for $\nu-$Oscillations: ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U^*_{lj}\, |\nu_j>\,, \quad \nu_j:\; m_j \neq 0\,; \quad l = e, \mu, \tau\,; \quad n \geq 3\,;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\, \nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x):\; m_j \neq 0\,; \quad l = e, \mu, \tau\,.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 0.5$ eV.

These data imply that

$$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q,\ q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim$ 1 eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^{2}$

These discoveries suggest the existence of New Physics beyond that of the ST.

The New Physics can manifest itself (can have a variety of different "flavours"):

- In the existence of more than 3 massive neutrinos: $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...).
- In the observed pattern of neutrino mixing and in the values of the CPV phases in the PMNS matrix.
- In the Majorana nature of massive neutrinos ($L \neq$ const.).
- In the existence of new particles, e.g., at the TeV scale: heavy Majorana Neutrinos N_j, doubly charged scalars,...

- In the existence of new (FChNC, FCFNSNC) neutrino interactions ($U(1)_X$, $M_X \lesssim 50$ MeV).
- In the existence of LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu - e$ conversion, etc., which proceed with rates close to the existing upper limits.
- In the existence of "unknown unknowns"...

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...) if, e.g., sterile ν_R, $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,...} \sim 1$ eV;
in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analises of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly"), data of radioactive source callibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly"); tests (DANSS, NEOS, PROSPECT, STEREO, ICARUS (at Fermilab), ...).

ii) $M_{4,5,...} \sim (10 - 10^3)$ GeV, low scale seesaw models;
$M_{4,5,...} \sim (10^9 - 10^{13})$ GeV, "classical" seesaw models.

Reference Model: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \to \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., May, 1980
J. Schechter, J.W.F. Valle, June, 1980
M. Doi, T. Kotani, E. Takasugi, July, 1980

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,

- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;

- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky et al., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.37 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.297$, $\cos 2\theta_{12} \gtrsim 0.29$ (3σ),

- $|\Delta m^2_{31(32)}| \cong 2.53$ (2.43) [2.56 (2.54)] $\times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437$ (0.569) [0.425 (0.589)], NO (IO) ,

- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0214$ (0.0218) [0.0215 (0.0216)], NO (IO).

F. Capozzi et al. (Bari Group), arXiv:1601.07777 [arXiv:1703.04471].

• $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31(32)})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering (IO)}$$

Convention: $m_1 < m_2 < m_3$ - NO, $\quad m_3 < m_1 < m_2$ - IO

$$\Delta m^2_{31}(NO) = -\,\Delta m^2_{32}(IO)$$

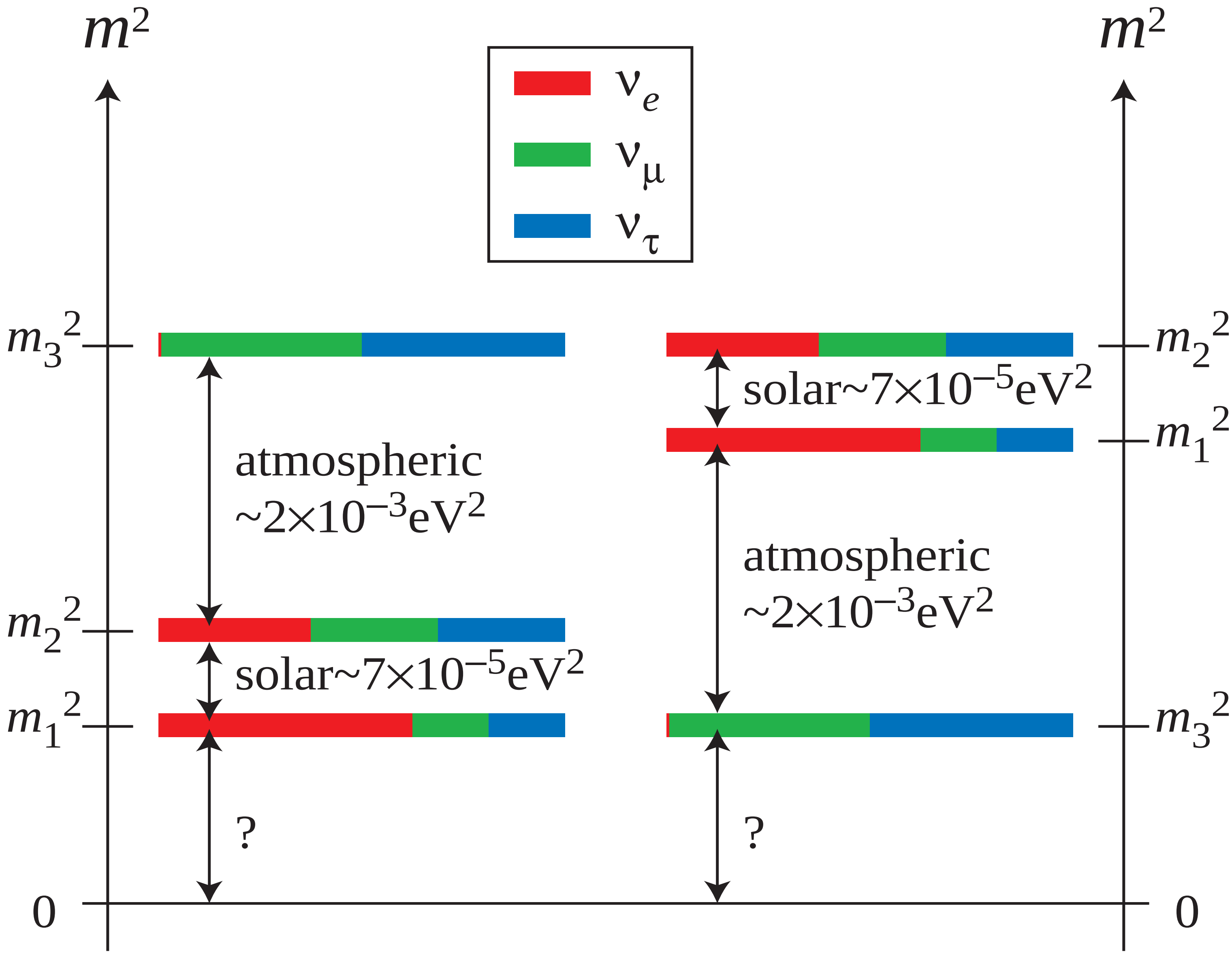
$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> |\Delta m^2_{31(32)}|,\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

• $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}},\ \ m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$ - NO;

• $m_1 = \sqrt{m_3^2 + \Delta m^2_{23} - \Delta m^2_{21}},\ \ m_2 = \sqrt{m_3^2 + \Delta m^2_{23}}$ - IO;



m^2
m^2
ν_e
ν_μ
ν_τ
$m_3{}^2$
atmospheric
$\sim 2\times 10^{-3}\mathrm{eV}^2$
$m_2{}^2$
solar$\sim 7\times 10^{-5}\mathrm{eV}^2$
$m_1{}^2$
?
0
$m_2{}^2$
solar$\sim 7\times 10^{-5}\mathrm{eV}^2$
$m_1{}^2$
atmospheric
$\sim 2\times 10^{-3}\mathrm{eV}^2$
$m_3{}^2$
?
0

Table 3: Best fit values and allowed ranges at $N\sigma = 1, 2, 3$ for the 3ν oscillation parameters, in either NO or IO. The latter column shows the formal "1σ accuracy" for each parameter, defined as 1/6 of the 3σ range divided by the best-fit value (in percent).

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
$\Delta m^2_{\odot}/10^{-5}$ eV^2	NO	7.34	7.20 – 7.51	7.05 – 7.69	6.92 – 7.91	2.2
	IO	7.34	7.20 – 7.51	7.05 – 7.69	6.92 – 7.91	2.2
$\lvert\Delta m^2_{\rm A}\rvert/10^{-3}$ eV^2	NO	2.49	2.46 – 2.53	2.43 – 2.56	2.39 – 2.59	1.4
	IO	2.48	2.44 – 2.51	2.41 – 2.54	2.38 – 2.58	1.4
$\sin^2\theta_{12}$	NO	3.04	2.91 – 3.18	2.78 – 3.32	2.65 – 3.46	4.4
	IO	3.03	2.90 – 3.17	2.77 – 3.31	2.64 – 3.45	4.4
$\sin^2\theta_{13}/10^{-2}$	NO	2.14	2.07 – 2.23	1.98 – 2.31	1.90 – 2.39	3.8
	IO	2.18	2.11 – 2.26	2.02 – 2.35	1.95 – 2.43	3.7
$\sin^2\theta_{23}/10^{-1}$	NO	5.51	4.81 – 5.70	4.48 – 5.88	4.30 – 6.02	5.2
	IO	5.57	5.33 – 5.74	4.86 – 5.89	4.44 – 6.03	4.8
δ/π	NO	1.32	1.14 – 1.55	0.98 – 1.79	0.83 – 1.99	14.6
	IO	1.52	1.37 – 1.66	1.22 – 1.79	1.07 – 1.92	9.3

$$\Delta m^2_{\odot} \equiv \Delta m^2_{21};\quad \Delta m^2_{\rm A} \equiv \Delta m^2_{31(32)},\ \text{NO (IO)}.$$

F. Capozzi et al. (Bari Group), arXiv:1804.09678.

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\mathrm{CP}} \propto J_{\mathrm{CP}} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

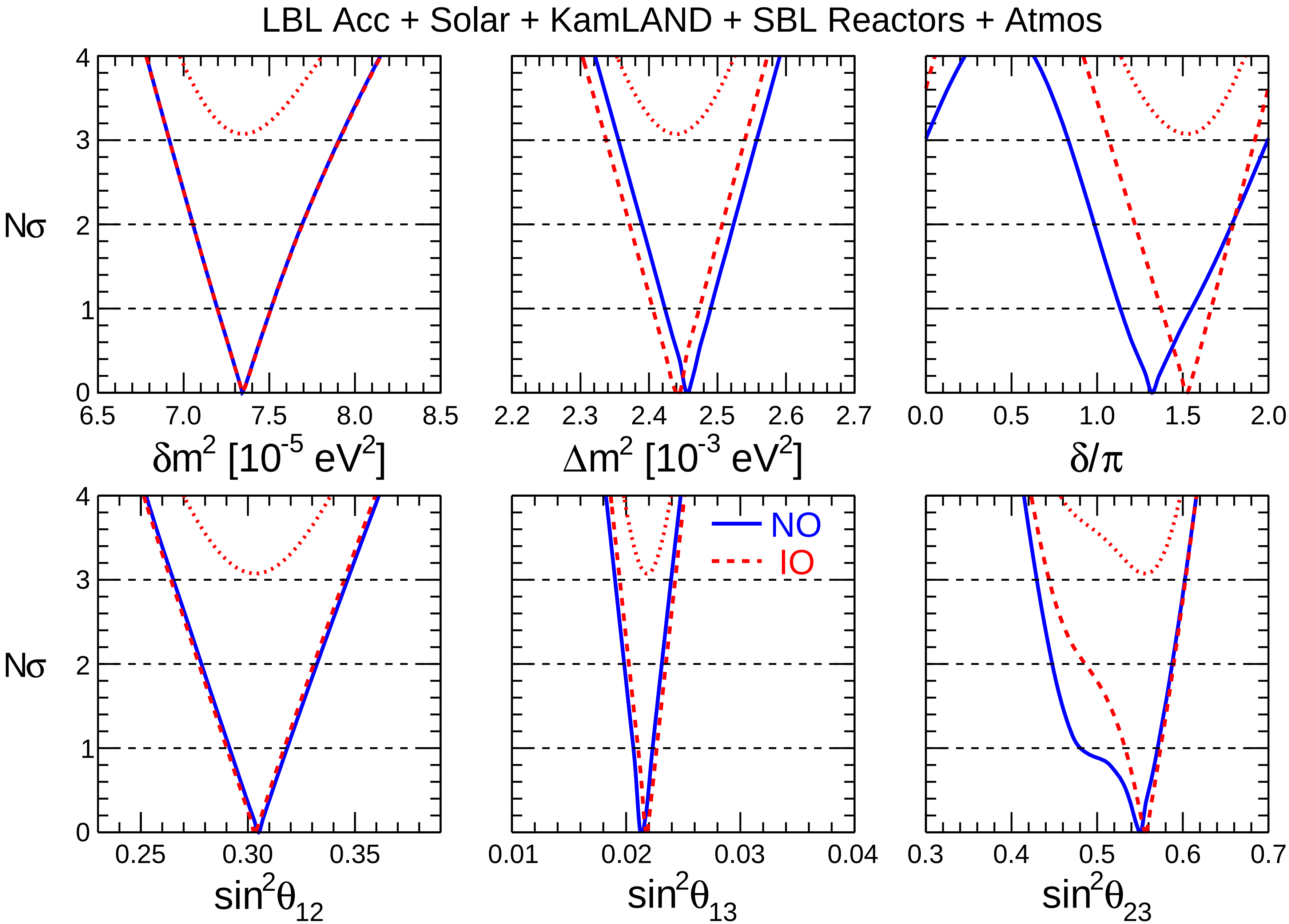
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky et al., 1980;
P. Langacker et al., 1987

  - $\left|<m>\right|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

- **Best fit value:** $\delta = 1.32\,(1.52)\pi$ **[**$1.30\,(1.54)\pi$**];**
- $\delta = 0$ **or** 2π **are disfavored at** $3.0\,(3.6)\sigma$ **[**$2.6\,(3.0)\sigma$**];**
- $\delta = \pi$ **is disfavored at** $1.8\,(3.6)\sigma$ **[**$1.7\,(3.3)\sigma$**];**
- $\delta = \pi/2$ **is strongly disfavored at** $4.4\,(5.2)\sigma$ **[**$4.3\,(5.0)\sigma$**].**
- **At** 3σ**:** δ/π **is found to lie in 0.83-1.99 (1.07-1.92) [1.07-1.97 (0.80-2.08)].**

F. Capozzi, E. Lisi *et al.*, arXiv:1804.09678 [E. Esteban *et al.*, NuFit 3.2 (Jan., 2018)]



F. Capozzi et al. (Bari Group), arXiv:1804.09678.

Latest global analysis: data favors NO

IO disfavored at 3.1σ.

F. Capozzi et al., 1804.09678.

The Simplest Model of $m_\nu \neq 0$, $U \neq 1$

ST + 3 $\nu_R(x)$ - RH singlet ν fields + L=const.

$$\mathcal{L}_{\rm Y}(x) \;=\; Y^{\nu}_{l'l}\,\overline{\nu_{l'R}}(x)\,\Phi^T(x)\,(i\tau_2)\,\psi_{lL}(x) + \textbf{h.c.}\,,$$

$$M_D \;=\; \frac{v}{\sqrt{2}} Y^{\nu}\,, \quad v = 246 \text{ GeV}\,.$$

ν_j, m_j - **Dirac particles; no explanation why** $m(\nu_j) <<< m_l, m_q$.

No DM candidate.

No mechanism for generation of the observed BAU.

The LFV processes $\mu^+ \rightarrow e^+ + \gamma$ **decay,** $\mu^- \rightarrow e^- + e^+ + e^-$ **decay,** $\tau^- \rightarrow e^- + \gamma$ **decay, etc. are allowed.**
However, they are predicted to proceed with unobservable rates:

$$BR(\mu \rightarrow e + \gamma) = \frac{3\alpha}{32\pi}\left|U_{ej}\,U^*_{\mu j}\,\frac{m_j^2}{M_W^2}\right|^2 \cong (2.5 - 3.9)\times 10^{-55}\,,$$

$$M_W \cong 80 \text{ GeV, the } \mathrm{W}^{\pm} - \text{mass}$$

S.T.P., 1976

"New Physics": $m_j \neq 0$, $\nu_l \rightarrow \nu_{l'}$, $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$ **oscillations.**

Problem: the assumption L=const.

“In modern understanding of particle physics global symmetries are approximate.” Global $U(1)$ symmetry leading to L=const. is expected to be broken by quantum gravity effects.

See E. Witten, 1710.01791; S. Weinberg, CERN Courier, 13 October 2017

Qualitative understanding of $m_{\nu_j} <<< m_{e,\mu,\tau}, m_q$

- **Seesaw mechanisms of neutrino mass generation**

P. Minkowski, 1977; T. Yanagida, 1979; M. Gell-Mann, P. Ramond, R. Slansky, 1979; R. Mohapatra, G. Senjanovic, 1980 (type I).

W. Konetschny, W. Kummer 1977; M. Magg, C. Watterich, 1980; T.P. Cheng, L.-F. Li, 1980 (type II).

R. Foot *et al.*, 1989 (type III).

Explains the smallness of $\nu-$masses (naturalness); connection to grand unification.

Through leptogenesis theory link the $\nu-$mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

- **Radiative generation of ν masses and mixing**

A. Zee, 1980; K. Babu, 1985;...; recent review: Y.Cai *et al.*, 1706.08524

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism: ν_{lR} - RH νs' (heavy).

Type II seesaw mechanism: $\mathbf{H(x)}$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

W. Konetschny, W. Kummer 1977; M. Magg, C. Watterich, 1980; T.P. Cheng, L.-F. Li, 1980; J. Schecter, J.F.W. Valle, 1980 G. Lazarides, Q. Shafi, C. Watterich, 1981.

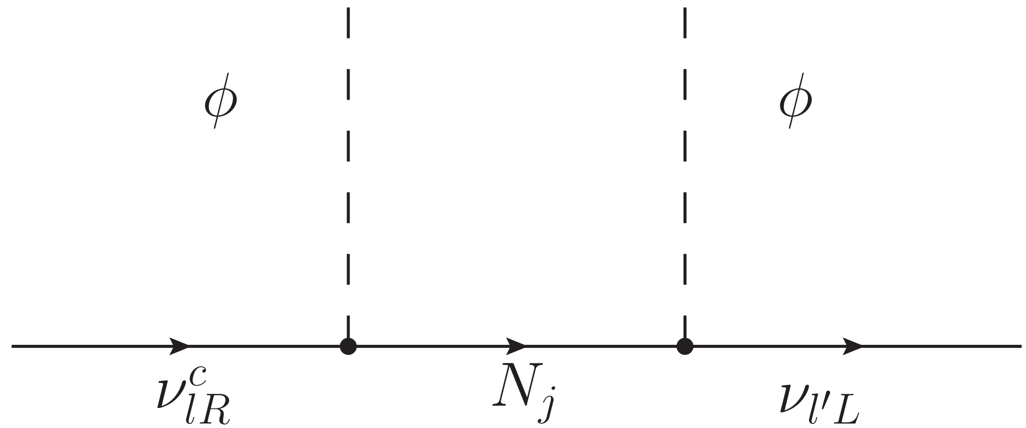
Type III seesaw mechanism: $\mathbf{T(x)}$ - a triplet of fermion fields.

R. Foot *et al.*, 1989

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)_{0\nu}$-decay, LFV processes, etc.) and New Physics at LHC.



• $\nu_{l'R}(x)$: **Majorana mass term at "high scale" ($\sim$TeV; or $(10^9 - 10^{13})$ GeV in $SO(10)$ GUT)**

$$\mathcal{L}^{\nu}_{M}(x) = + \frac{1}{2}\,\nu^{\mathsf{T}}_{l'R}(x)\;C^{-1}\;(M^{RR})^{\dagger}_{l'l}\;\nu_{lR}(x) + h.c. = -\frac{1}{2}\sum_j \bar{N}_j\,M_j\,N_j\;,$$

• **Yukawa type coupling of $\nu_{lL}(x)$ and $\nu_{l'R}(x)$ involving $\Phi(x)$:**

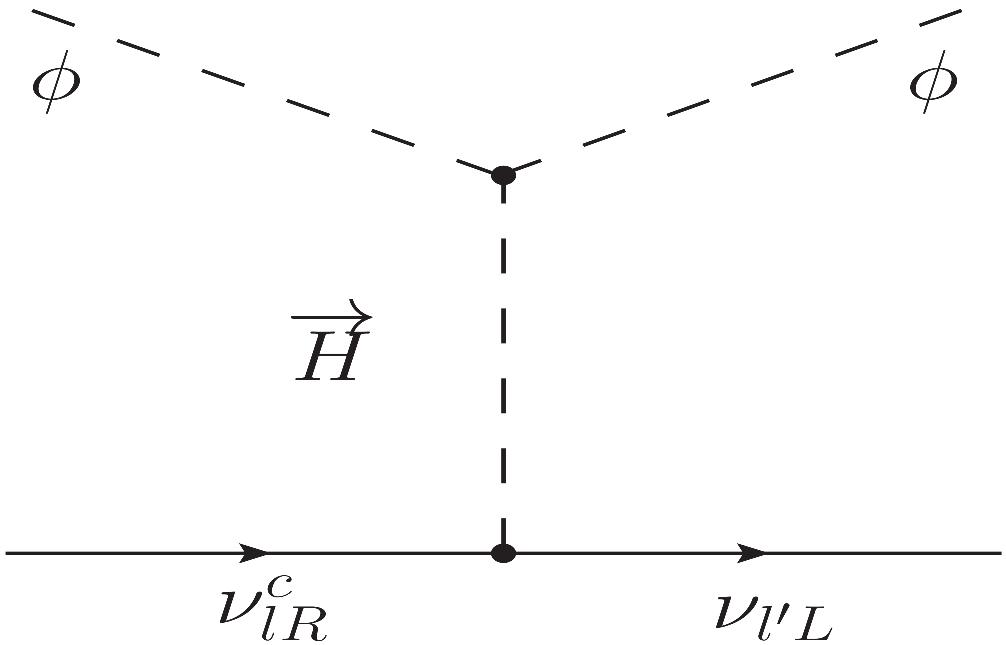
$$\begin{aligned}
\mathcal{L}_{\mathsf{Y}}(x) &= \bar{Y}^{\nu}_{l'l}\,\overline{\nu_{l'R}}(x)\,\Phi^T(x)\,(i\tau_2)\,\psi_{lL}(x) + \textbf{h.c.}\,,\\
&= Y^{\nu}_{jl}\,\overline{N_{jR}}(x)\,\Phi^T(x)\,(i\tau_2)\,\psi_{lL}(x) + \textbf{h.c.}\,,\\
M_D &= \frac{v}{\sqrt{2}}\,Y^{\nu}\,,\quad v = 246\ \text{GeV}\,.
\end{aligned}$$

For sufficiently large M_j, Majorana mass term for $\nu_{lL}(x)$:

$$M_{\nu} \cong v_u^2\;(Y^{\nu})^T\,M_j^{-1}\,Y^{\nu} = U^{*}_{\text{PMNS}}\;m^{\text{diag}}_{\nu}\;U^{\dagger}_{\text{PMNS}}\;.$$

$v_u\,Y^{\nu} = M_D$, $M_D \sim 1$ **GeV**, $M_j = 10^{10}$ **GeV:** $M_{\nu} \sim 0.1$ **eV.**

Type II Seesaw Mechanism



$$M_\nu \cong h\, v^2\, \mu_H M_H^{-2} = U^*_{\text{PMNS}}\; m_\nu^{\text{diag}}\; U^\dagger_{\text{PMNS}}\ .$$

$h = 10^{-2}$, $\mu_H \sim M_H$, $v = 246$ GeV, $M_H \sim 6 \times 10^{12}$ GeV:
$M_\nu \sim 0.1$ eV.

The Higgs Triplet Model (HTM)

The TeV scale version of the type II seesaw model - H^{--}**,** H^{-}**,** H^{0}**, have masses at the TeV scale ("Higgs Triplet Model (HTM)"),**

$$\frac{\tau}{2}\mathbf{H}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} H^{-}/\sqrt{2} & H^{0} \\ H^{--} & -H^{-}/\sqrt{2} \end{pmatrix}.$$

$$\mathcal{L}_{\rm II}(x) = \left(h_{l'l}\overline{\psi_{l'L}}(x)\frac{\tau}{2}\,i\tau_2\,\psi^{C}_{lR}(x)\,\mathbf{H}(x) + h.c\right) - V(\mathbf{H},\Phi)$$

$V(\mathrm{H},\Phi)$ **rather "elaborate"; leads to** $< H^0 > /\sqrt{2} \equiv v_T \neq 0$**.**

The flavour neutrino Majorana mass matrix generated by $< H^0 >$**:**

$$(M_\nu)_{\ell\ell'} \cong 2\,h_{\ell\ell'}\,v_T\,.$$

An upper limit on v_T – from its effect $\rho = M_W^2/M_Z^2\cos^2\theta_W = 1.0003 \pm 0.00023$ (in the SM, $\rho = 1$ at tree-level):

$$\rho \equiv 1 + \delta\rho = \frac{1+2x^2}{1+4x^2}\,,\;\; x \equiv \sqrt{2}v_T/v \;: \qquad v_T \lesssim 2.43\ \text{GeV}\,.$$

A lower limit on v_T - from the magnitude of $|(M_\nu)_{\ell\ell'}|$ and the requirement of perturbative values of $h_{\ell\ell'}$, $|h_{\ell\ell'}|^2 \leq 4\pi$:

for $|(M_\nu)_{\ell\ell'}| = 0.1$ eV, one finds $v_T \gtrsim 10^{-2}$ eV.

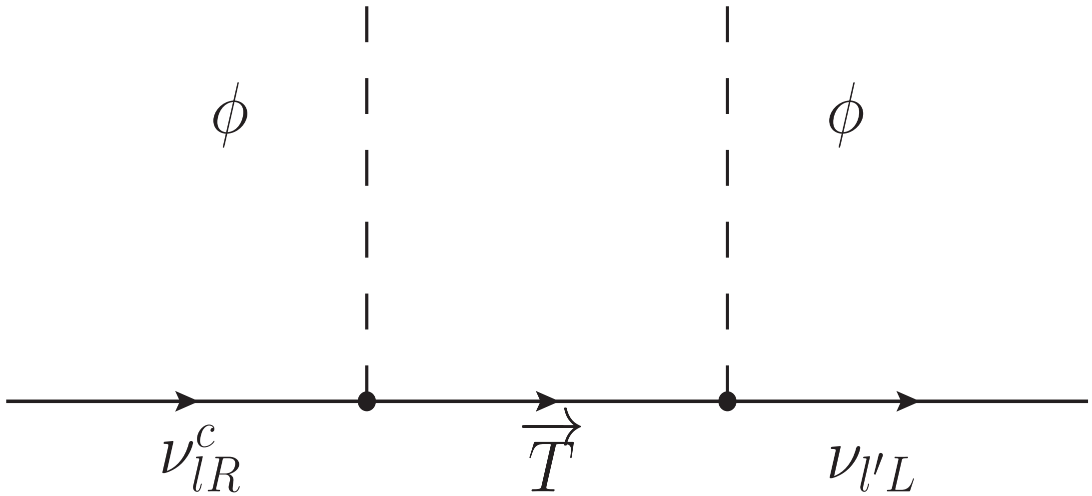
For $M_H \sim (100-1000)$ GeV, the model predicts a plethora of beyond the SM physics phenomena most of which can be probed at the LHC and in the experiments on charged lepton flavour violation, if $v_H \sim (1-100)$ eV, so that the couplings $h_{ll'}$ are sufficiently large in magnitude.

E. Ma et al., 2001; E.J. Chun et al., 2003; J. Garayoa, T. Schwetz, 2008; F. del Aguila, J.A. Aguilar-Saavedra, 2009; A.G. Akeroyd et al., 2009, 2010 and 2011; H. Sugiyama, 2009; D.N. Dinh et al. 2012 and 2013; S. Antusch et al., 2018

Assuming $M_{H^-} > M_{H^{--}}$, for $v_T < 10^4$ eV the decays of $H^{--} \to l^- + l'^-$, $l, l' = e, \mu, \tau$ are dominant.
At LHC, the pair $H^{--} + H^{++}$ is produced via virtual γ and Z^0.
The search for pairs of same sign charged leptons $\mu^\pm + \mu^\pm$, $e^\pm + e^\pm$ and $e^\pm + \mu^\pm$ at LHC leads for $v_T \sim (10-10^4)$ eV to $M_{H^{--}} > 620$ GeV (95% C.L.).

S. Antusch et al., 2018

Type III Seesaw Mechanism



$$M_\nu \cong v^2\;(Y_T)^T\,M_T^{-1}\,Y_T = U^*_{\text{PMNS}}\;m_\nu^{\text{diag}}\;U^\dagger_{\text{PMNS}}\;.$$

$v\,Y_T \sim 1$ GeV, $M_T \sim 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.

In EO formalism M_ν in all 3 mechanisms - due to the Weinberg dim 5 operator:

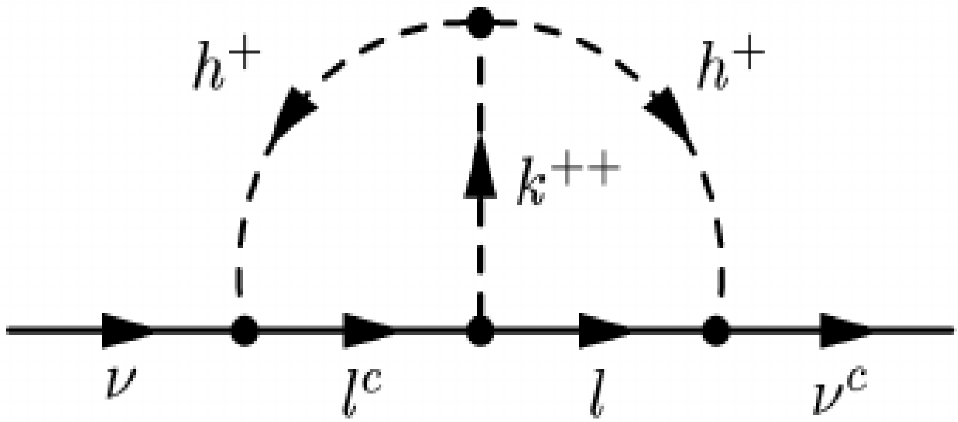
$$\frac{\lambda_{ll'}}{\Lambda} L_l \, H \, L_{l'} \, H$$

Radiative generation of ν masses and mixing

A. Zee, 1980; K.S. Babu, 1985;...; Y. Farzan *et al.*, 1208.2732; recent review: Y.Cai et al., 1706.08524

Generic features

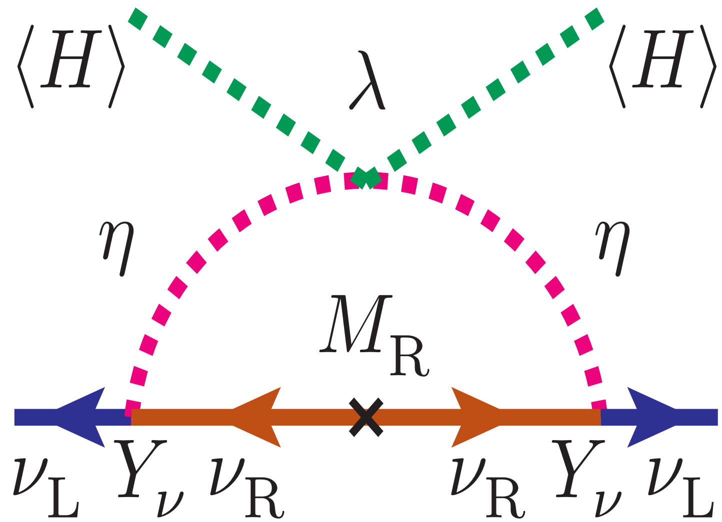
- **Loop suppression helps explaining the smallness of $\nu-$masses.**
- **New particles need not be super heavy - can be at the TeV scale.**
- **Models at the TeV scale - testable.**
- **No need to introduced ν_R.**
- **Typically includes extended scalar sector.**



K.S. Babu, 1988

$Y_W(k^{++}) = 4$, $L(k^{++}) = -2$; $Y_W(h^+) = 2$, $L(h^+) = -2$; no ν_R;
ν_j- Majorana fermions.
$M(k^{++}) \sim$ TeV, $M(h^+) \sim$ TeV - possible.

$k^{++} \rightarrow l^+ + l'^+$, $l, l' = e, \mu, \tau$;
$\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, $\mu^- + (A,Z) \rightarrow e^- + (A,Z)$;
$(\beta\beta)_{0\nu}-$ decay.



E. Ma, hep-ph/0601225 (figure from T. Ohlsson, Sh. Zhou, 1311.3846)

$SU(2)_L \times U(1)_{Y_W} \times Z_2$;
ν_R - present, M_R- Majorana mass of ν_R;
η - additional $SU(2)_L$ doublet; $< \eta^0 >= 0$ (scalar potential);
$Z_2(\eta) = Z_2(\nu_R) = -1$; $Z_2(H) = Z_2(L_l) = Z_2(l^c_L) = +1$.
ν_j- Majorana fermions;
DM candidate available: either N_1 ($N_{1,2,3}$, $M_{1,2,3}$, min$(M_j) = M_1$), or $\sqrt{2}$Reη^0.
$M(\eta^\pm) \sim$ TeV, $M_{1,2,3} \sim$ TeV - possible;
$M(\eta^\pm) > M_{1,2,3}$: $\eta^\pm \rightarrow l^\pm N_{1,2,3}$, $N_2 \rightarrow l^\pm l^\mp N_1$, $N_3 \rightarrow l^\pm l^\mp N_{1,2}$
$M(\eta^\pm) < M_{1,2,3}$: $N_{1,2,3} \rightarrow l^\pm \eta^\mp$
$\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, $\mu^- + (A,Z) \rightarrow e^- + (A,Z)$;
$(\beta\beta)_{0\nu}-$ decay.

More cases of M_ν generated at 1-loop, 2-loop, 3-loop,..., level are discussed in Y. Farzan *et al.*, 1208.2732.

Effective operators which generate M_ν at 1-loop or 2-loop level are discussed in the recent review: Y.Cai et al., 1706.08524.

In particular, M_ν can be generated at loop level by dim 7 operators:

$$O_3 = L\,L\,Q\,\overline{d}\,H, \quad O_8 = L\,\overline{d}\,\overline{e}^\dagger\,\overline{u}^\dagger\,H$$

UV completion: LQ with (3,1,-2/3) and (3,3,-2/3) - well known candidates to explain the flavour anomalies (the indications for breaking of lepton universality) B-meson decays.

R. Volkas, talk at ν'2018

Understanding the Pattern of Neutrino Mixing

ANARCHY:

A. De Gouvea, H. Murayama, hep-ph/0301050; PLB, 2015.

L. Hall, H. Murayama, N. Weiner, hep-ph/9911341.

$U_{\rm PMNS}$ **from random draw of unbiased distribution of** 3×3 **unitary matrices.**

θ_{ij} **random qunatities, no correlations whatsoever between the values of** θ_{12} **and/or** θ_{13} **and/or** θ_{23}**.**
Predicts distributions (not values) of θ_{ij}**;**
values of $\theta_{ij} \sim \pi/4$ **most probable.**
Three large mixing angles - most natural for the approach.
However, $\theta_{13} \cong 0.15$**...**
Values of m_j**,** Δm^2_{ij} **- not predicted.**

Family Symmetries (Continuous)

Spontaneouesly broken $U(1)_{\mathsf{FN}}$ family symmetry at scale M (can be generalised to $U(2)$, $SU(3)$...):

Frogatt, Nielsen, 1979

$$Y^{\ell}_{ll'}\,\overline{\psi_{lL}}(x)\Phi(x)\,l'_R(x) \rightarrow A_{ll'}\left(\frac{\varphi}{M}\right)^{n_{ll'}}\overline{\psi_{lL}}(x)\Phi(x)\,l'_R(x),$$

φ - flavon field: $\dfrac{<\varphi>}{M} \sim \lambda = \sin\theta_C = 0.22$.

$n_{ll'} = n(\overline{\psi_{lL}}) + n(\Phi) + n(l'_R)$, $n(\varphi) = -1$; $A_{ll'} \sim 1$.

$m_e/m_\mu \sim \lambda^4$, $m_\mu/m_\tau \sim \lambda^2$; $m_d/m_s \sim \lambda^2$, $m_d/m_b \sim \lambda^3$, $m_u/m_c \sim \lambda^3$, $m_u/m_t \sim \lambda^5$.

Similar generalisation for the two terms in the see-saw Lagrangian:

$$Y^{\nu}_{l'l}\overline{\nu_{l'R}}(x)\Phi^T(x)\,(i\tau_2)\psi_{lL}(x),\ \frac{1}{2}\,\nu^{\mathsf{T}}_{l'R}(x)\,C^{-1}\,(M_{RR})^{\dagger}_{l'l}\,\nu_{lR}(x).$$

Provides an understanding of the hierarchical structure of charged lepton and quark masses. Mixing angles typically related to fermion mass ratios.

Understanding the Pattern of Neutrino Mixing

With the observed pattern of neutrino mixing Nature is sending us a message. The message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. We do not know at present what is the content of Nature's message. However, I believe that it reads

SYMMETRY

The Quest for Nature's Message

Towards Quantitative Understanding of $U_{\rm PMNS}$, m_j

The observed pattern of 3-ν mixing, two large and one small mixing angles,
$\theta_{12} \cong 33^\circ$, $\theta_{23} \cong 45^\circ \pm 6^\circ$ **and** $\theta_{13} \cong 8.4^\circ$,
can most naturally be explained by extending the Standard Model (SM) with a flavour symmetry corresponding to a non-Abelian discrete (finite) group G_f**.**

$G_f = A_4$, T', S_4, A_5, D_{10}, D_{12},...

Vast literature; reviews: G. Altarelli, F. Feruglio, 1002.0211; H. Ishimori et al., 1003.3552; M. Tanimoto, AIP Conf.Proc. 1666 (2015) 120002; S. King and Ch. Luhn, 1301.1340; D. Meloni, 1709.02662; STP, 1711.10806

Examples of Predictions and Correlations.

- $\sin^2\theta_{23} = \frac{1}{2}$.
- $\sin^2\theta_{23} \cong \frac{1}{2}(1 \mp \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong \frac{1}{2}(1 \mp 0.022)$.
- $\sin^2\theta_{23} = 0.455;\ 0.463;\ 0.537;\ 0.545;\ 0.604$ **(small uncert.)**.
- $\sin^2\theta_{12} \cong \frac{1}{3}(1 + \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.340$.
- $\sin^2\theta_{12} \cong \frac{1}{3}(1 - 2\sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.319$.
- **and/or** $\cos\delta = \cos\delta(\theta_{12},\theta_{23},\theta_{13};\theta^{\nu}_{12},...)$,

$J_{CP} = J_{CP}(\theta_{12},\theta_{23},\theta_{13},\delta) = J_{CP}(\theta_{12},\theta_{23},\theta_{13};\theta^{\nu}_{12},...)$,

θ^{ν}_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Prospective (useful/requested) precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE; T2K+NOνA(?)).**

$\delta(\delta) = 10^\circ$ **at** $\delta = 3\pi/2$ **(T2HK+THKK?)**

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\rm atm} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\rm PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \sin^{-1}\frac{1}{\sqrt{3}} - 0.020$; $\theta_{12} \cong \pi/4 - 0.20$, $\theta_{13} \cong 0 + \pi/20$, $\theta_{23} \cong \pi/4 \mp 0.10$.
- $U_{\rm PMNS}$ **due to new approximate symmetry?**

A Natural Possibility (vast literature):

$$U = U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})\; Q(\psi,\omega) U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1, \xi_2)\,,$$

with

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})$ - from diagonalization of the l^- mass matrix;
- $U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1, \xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi,\omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\rm LC}$, $U_{\rm GRAM}$, $U_{\rm GRBM}$, $U_{\rm HGM}$:

$$U_{\rm LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c^{\nu}_{23}}{\sqrt{2}} & \frac{c^{\nu}_{23}}{\sqrt{2}} & s^{\nu}_{23} \\ \frac{s^{\nu}_{23}}{\sqrt{2}} & -\frac{s^{\nu}_{23}}{\sqrt{2}} & c^{\nu}_{23} \end{pmatrix}; \quad \mu-\tau \ \text{symmetry}: \quad \theta^{\nu}_{23} = \mp\pi/4\,;$$

$$U_{\rm GR} = \begin{pmatrix} c^{\nu}_{12} & s^{\nu}_{12} & 0 \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \theta^{\nu}_{12} = \pi/6\,.$$

$U_{\rm GRAM}$: $\sin^2\theta^{\nu}_{12} = (2+r)^{-1} \cong 0.276$, $r = (1+\sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$)

$U_{\rm GRBM}$: $\sin^2\theta^{\nu}_{12} = (3-r)/4 \cong 0.345$.

GRB and HG mixing: W. Rodejohann et al., 2009.

$U_{\text{TBM(BM)}}$: Groups A_4, T', S_4 (S_4),... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

• U_{GRA}: Group A_5,...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.276$ and $s^2_{23} = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057;...

• U_{LC}: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

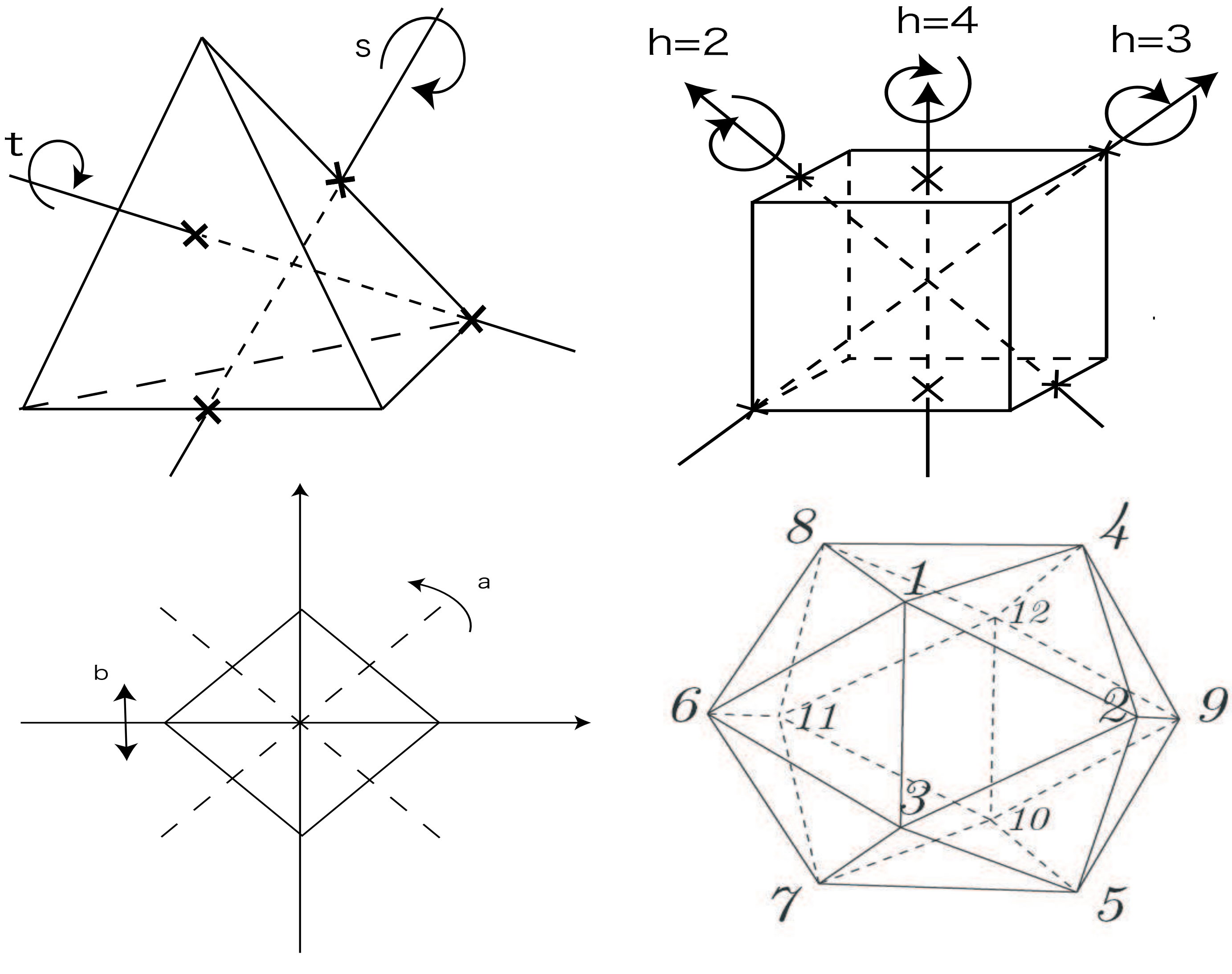
S.T.P., 1982

• U_{LC}: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^ν_{23} - free parameter; $s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

• $U_{\rm GRB}$: Group D_{10},...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.345$ and $s^2_{23} = 1/2$ must be corrected.

• $U_{\rm HG}$: Group D_{12},...; $s^2_{13} = 0$, $s^2_{12} = 0.25$ and possibly $s^2_{23} = 1/2$ must be corrected.

For all symmetry forms considered we have: $\theta^{\nu}_{13} = 0$, $\theta^{\nu}_{23} = \mp\pi/4$.
They differ by the value of θ^{ν}_{12}:
TBM, BM, GRA, GRB and HG forms correspond to $\sin^2\theta^{\nu}_{12} = 1/3;\ 0.5;\ 0.276;\ 0.345;\ 0.25$.



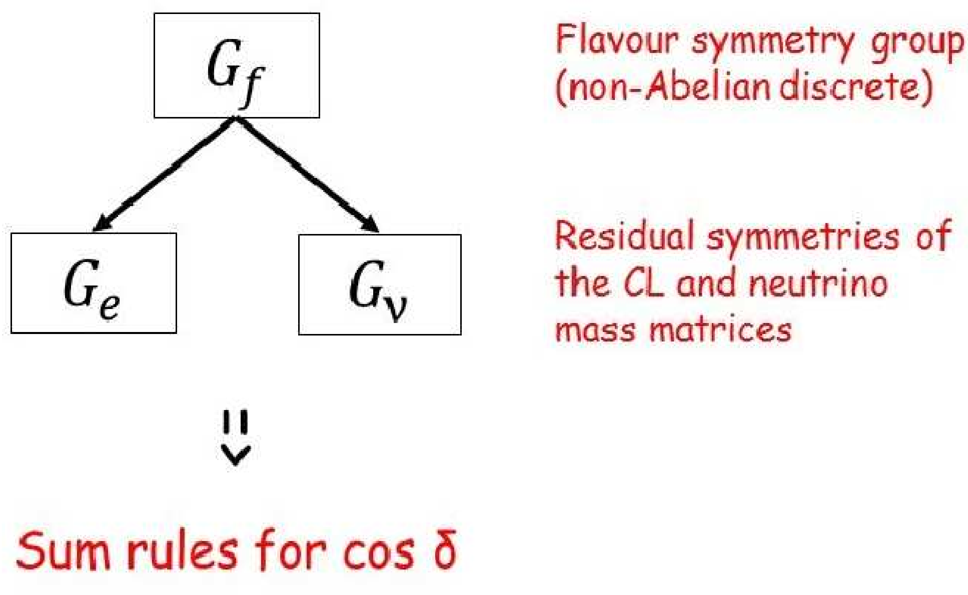
Examples of symmetries: A_4, S_4, D_4, A_5

From M. Tanimoto et al., arXiv:1003.3552

Group	Number of elements	Generators	Irreducible representations
S_4	24	$S,\ T\ (U)$	$\mathbf{1},\ \mathbf{1}',\ \mathbf{2},\ \mathbf{3},\ \mathbf{3}'$
A_4	12	$S,\ T$	$\mathbf{1},\ \mathbf{1}',\ \mathbf{1}'',\ \mathbf{3}$
T'	24	$S,\ T\ (R)$	$\mathbf{1},\ \mathbf{1}',\ \mathbf{1}'',\ \mathbf{2},\ \mathbf{2}',\ \mathbf{2}'',\ \mathbf{3}$
A_5	60	$\tilde{S},\ \tilde{T}$	$\mathbf{1},\ \mathbf{3},\ \mathbf{3}',\ \mathbf{4},\ \mathbf{5}$
D_{10}	20	$A,\ B$	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4$
D_{12}	24	$\tilde{A},\ \tilde{B}$	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4,\ \mathbf{2}_5$

Number of elements, generators and irreducible representations of some discrete groups.

How does it Work.



G_f: non-Abelian discrete (finite) flavour symmetry.

Typically (but not uniquely) $l_L(x)$, $\nu_{lL}(x)$, $l = e, \mu, \tau$ **- triplets of** G_f.

$G_\nu -$ **symmetry of the neutrino mass term;**

$G_e -$ **symmetry of the charged lepton mass term.**

Majorana Mass Term for $\nu_{lL}(x)$, $l = e, \mu, \tau$ **:**

$$\mathcal{L}^{\nu}_{M}(x) = \frac{1}{2}\, \nu^{\mathsf{T}}_{l'L}(x)\, C^{-1}\, M_{l'l}\, \nu_{lL}(x) + h.c.\,,\; C^{-1}\, \gamma_\alpha\, C = -\,\gamma^{\mathsf{T}}_{\alpha}$$

Charged lepton mass term:

$$\mathcal{L}_\ell(x) = -\, \overline{l'_L}(x)\;\, M_{el'l}\;\, l_R(x) + h.c.$$

U_e: $U_e^\dagger M_e M_e^\dagger U_e = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$.
G_e - **residual symmetry group of** $M_e M_e^\dagger$

U_ν: $U_\nu^T M_\nu U_\nu = \text{diag}(m_1, m_2, m_3)$.
G_ν - **residual symmetry group of** M_ν

$U_{\text{PMNS}} = U_e^\dagger U_\nu$

$G_e = Z_2$; Z_n, $n > 2$; $Z_n \times Z_m$, $n, m \geq 2$
(max. $G_e = U(1) \times U(1) \times U(1)$ **+** G_e **subgroup of** $SU(3)$**: max.** $G_e = U(1) \times U(1)$**)**

ν_j, **Majorana mass term,** $m_j \neq m_k$, $j \neq k = 1, 2, 3$: $G_\nu = Z_2 \times Z_2$, Z_2
(max. $G_\nu = Z_2 \times Z_2 \times Z_2$ **+** G_ν **subgroup of** $SU(3)$**: max.** $G_\nu = Z_2 \times Z_2$**)**

U_e **and** U_ν **constrained or determined (**U_ν **up to Majorana phases):**

$U_{\mathrm{PMNS}} = U_e^\dagger U_\nu$ **constrained or determined (up to the Majorana phases)**

The constraints depend on: G_f, $\rho(g_f)$, G_e **and** G_ν.

$G_f = A_4,\ S_4,\ T',\ A_5$.

A_4**: 3** Z_2**, 4** Z_3**, 1** $Z_2 \times Z_2$ **subgroups (total 8).**

T'**: similar to** A_4.

S_4**: 9** Z_2**, 4** Z_3**, 3** Z_4**, 4** $Z_2 \times Z_2$ **subgroups (total 20).**

A_5**: has 15** Z_2**, 10** Z_3**, 6** Z_5**, 5** $Z_2 \times Z_2$ **subgroups (36).**

Predictions and Correlations I

$U_\nu = U_{\rm TBM,BM,GRA,GRB,HG}\,\bar{P}(\xi_1,\xi_2)$; θ^ν_{12};

$U_\ell^\dagger = R_{12}(\theta^\ell_{12})\,Q$, $Q = {\rm diag}(e^{i\varphi},1,1)$
(the "minimal" = simplest case ($SU(5)\times T'$,...)

$U_\ell^\dagger = R_{12}(\theta^\ell_{12})\,R_{23}(\theta^\ell_{23})Q$, $Q = {\rm diag}(1,e^{-i\psi},e^{-i\omega})$
(next-to-minimal case)

$\cos\delta = \cos\delta(\theta_{12},\theta_{23},\theta_{13};\theta^\nu_{12},...)$,

$J_{CP} = J_{CP}(\theta_{12},\theta_{23},\theta_{13},\delta) = J_{CP}(\theta_{12},\theta_{23},\theta_{13};\theta^\nu_{12},...)$,

θ^ν_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

For arbitrary fixed θ^{ν}_{12} and any θ_{23} ("minimal" and "next-to-minimal" cases):

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12}\right.$$

$$\left. + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right]\,.$$

S.T.P., arXiv:1405.6006

This results is exact.

"Minimal" case: $\sin^2\theta_{23} = \frac{1}{2}\frac{1-2\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$.

In all cases TBM, BM (LC), GRA, GRB, HG:

- **New sum rules relating θ_{12},θ_{13}, θ_{23} and δ;**
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.

• $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.

• TBM case: $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij}:
$\delta \cong 263.5^\circ$ or 96.5°, $\cos\delta = -0.114$, $J_{CP} \cong \mp 0.034$.

• GRAM case, b.f.v. of θ_{ij}: $\delta \cong 286.8^\circ$ or 73.2°;
$\cos\delta = 0.289$, $J_{CP} \cong \mp 0.0327$.

• GRBM case, b.f.v. of θ_{ij}: $\delta \cong 258.5^\circ$ or 101.5°;
$\cos\delta = -0.200$, $J_{CP} \mp 0.0333$.

• HGM case, b.f.v. of θ_{ij}: $\delta \cong 298.4^\circ$ or 61.6°;
$\cos\delta = 0.476$, $J_{CP} \cong \mp 0.0299$.

• BM, LC cases: $\delta \cong \pi$, $\cos\delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij}: F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos\delta$ or δ and using high precision data on θ_{12}, θ_{23} and θ_{13}, one can distiguish between different symmetry forms of $\tilde{U}_{\nu}$!

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, (DUNE, T2HK, T2HKK) see, e.g., R. Acciarri *et al.* [DUNE Collab.], arXiv:1512.06148, 1601.05471 and 1601.02984; K. Abe *et al.* [T2HK Proto-Collab.], arXiv:1502.05199 (PTEP 2015 (2015) 053C02), and arXiv:1611.06118 (PTEP 2018 (2018) 1).

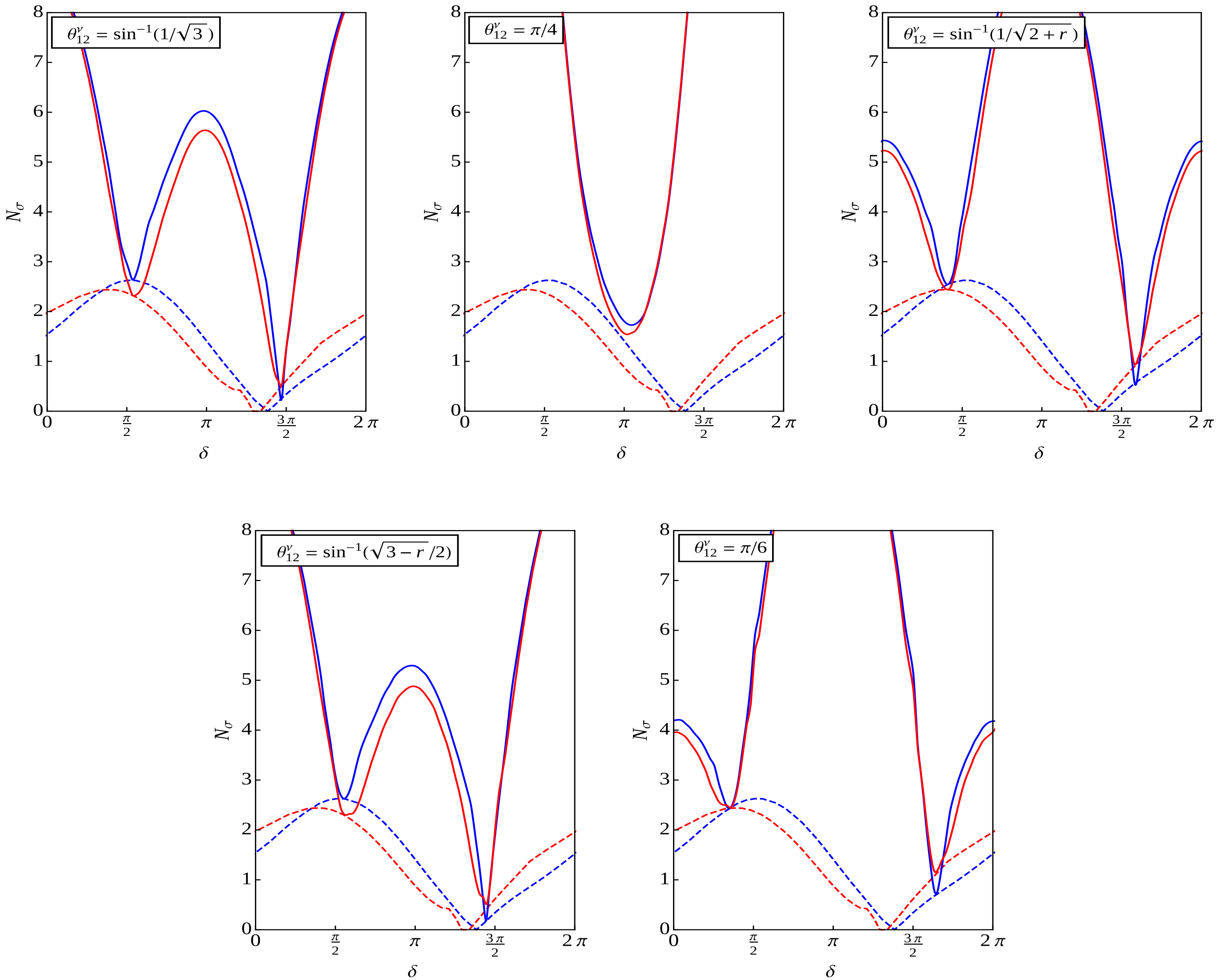
Statistical analysis, likelihood method; input "data": $\sin^2\theta_{13}$, $\sin^2\theta_{12}$, $\sin^2\theta_{12}$, δ from F. Capozzi et al., arXiv:1312.2878v2 (May 5, 2014).

$$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

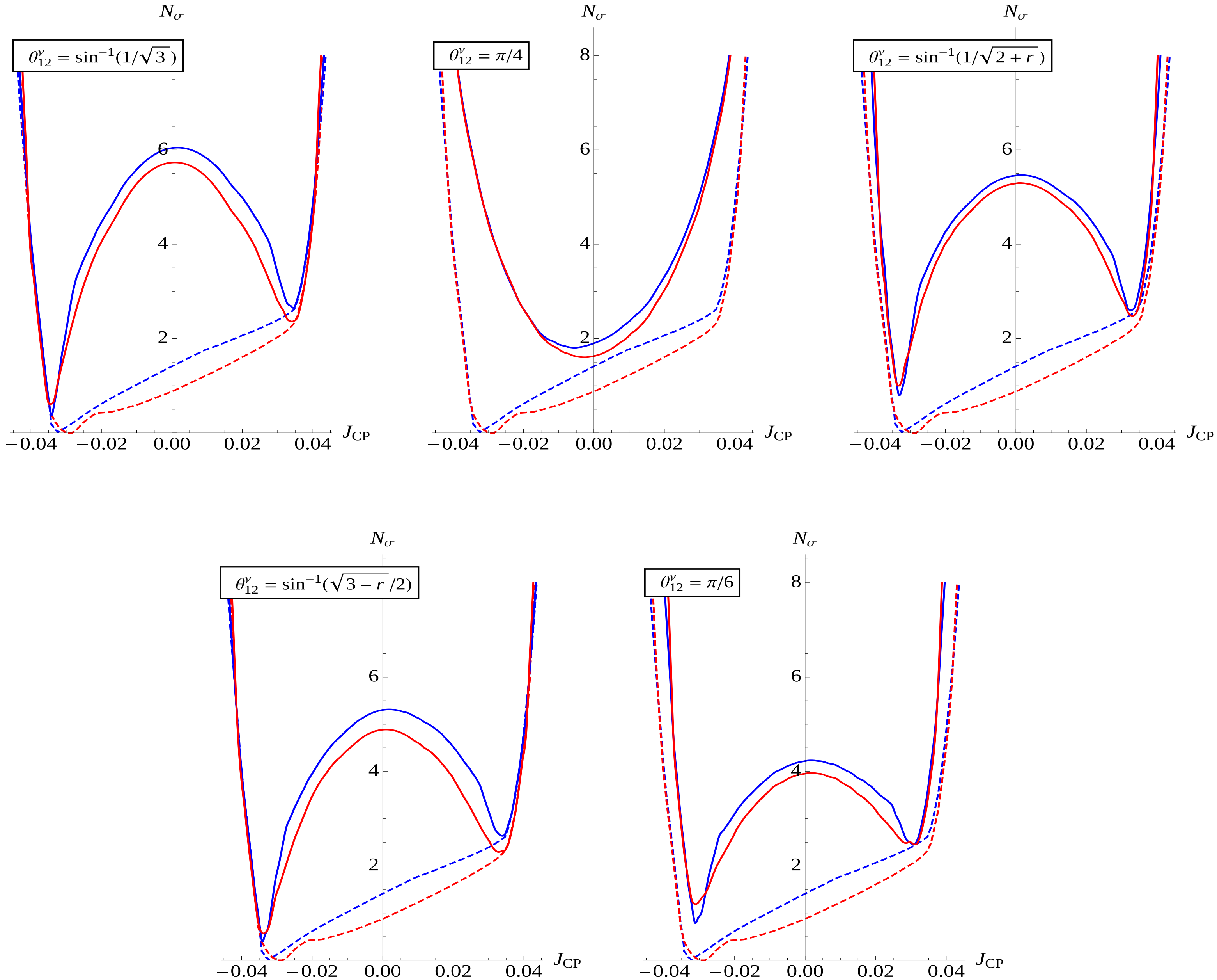
nσ confidence level interval of values of $\cos\delta$:

$$L(\cos\delta) \geq L(\chi^2_{\rm min}) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

TBM, GRA, GRB, HG: $J_{\rm CP} = 0$ **excluded at** 5σ, 4σ, 4σ, 3σ **confidence level.**

At 3σ: $0.020 \leq |J_{\rm CP}| \leq 0.039$.

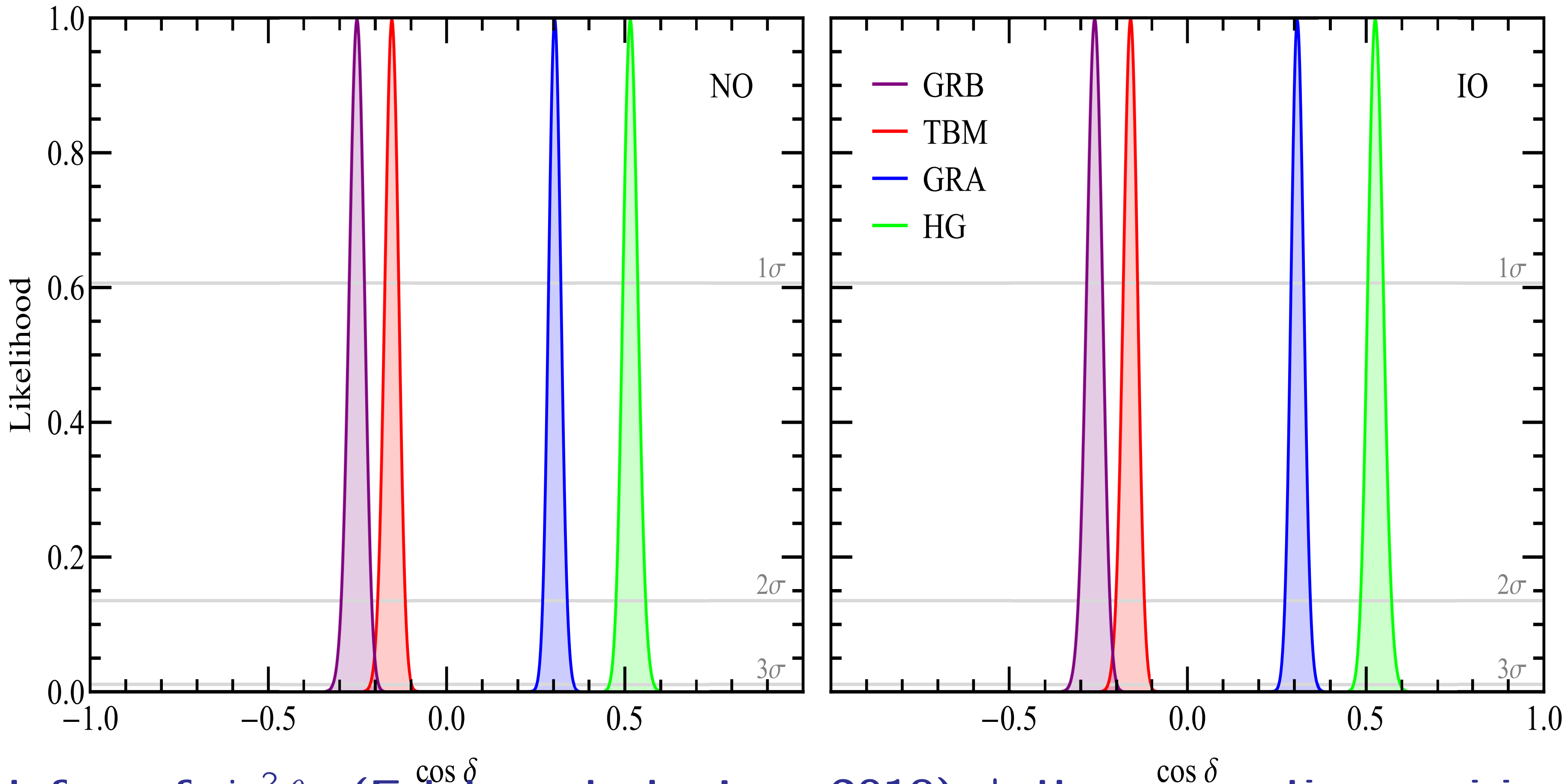
BM (LC), b.f.v.: $J_{\rm CP} = 0$;
at 3σ: -0.026 $(-0.025) \leq J_{\rm CP} \leq 0.021$ (0.023) **for NO (IO) neutrino mass spectrum.**

Prospective precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

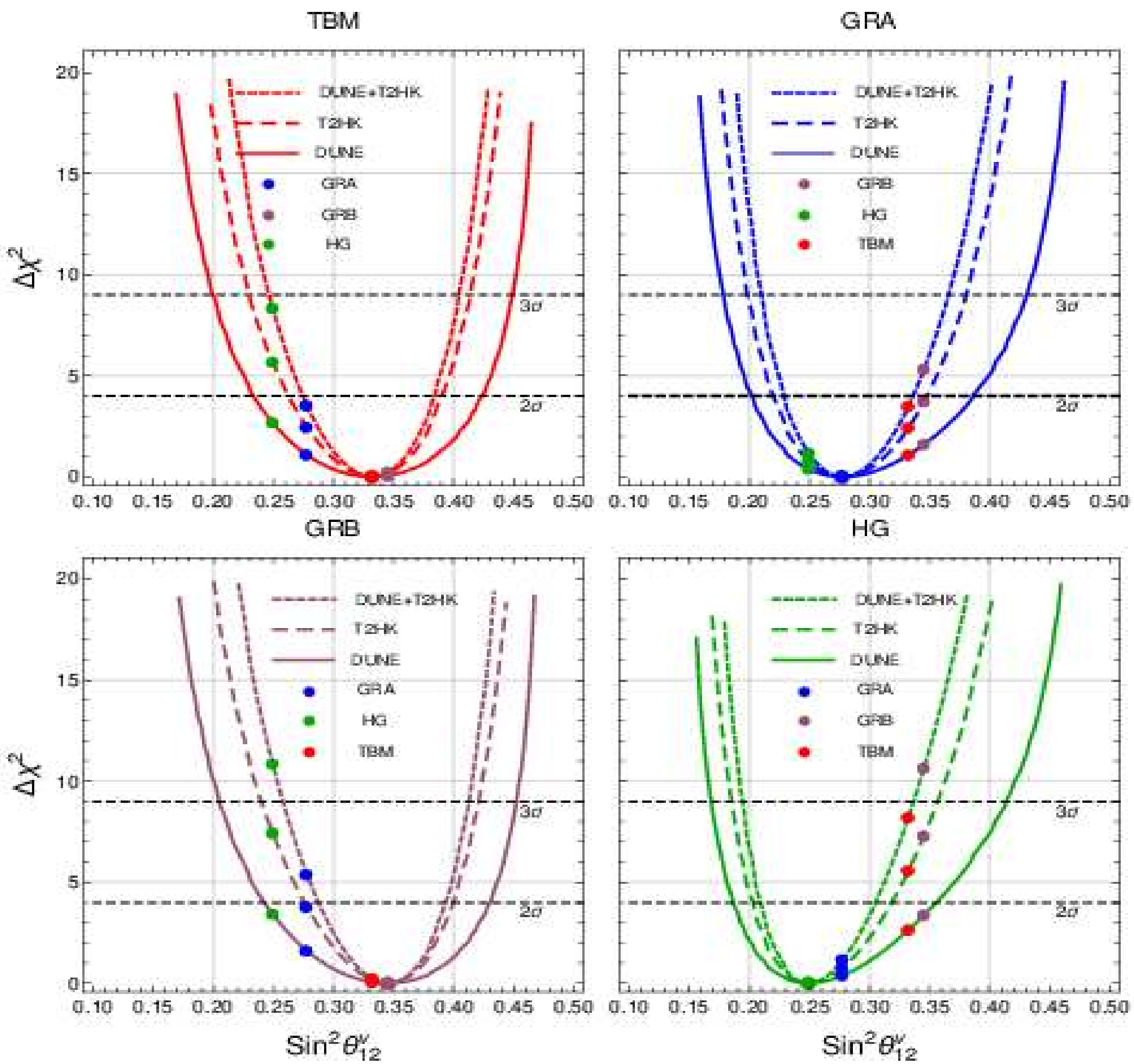
$\delta(\sin^2\theta_{23}) = 5\%$ **(T2K, NOνA combined).**



b.f.v. of $\sin^2\theta_{ij}$ (Esteban et al., Jan., 2018) + the prospective precision used.

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right] .$$

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE).**



Agarwalla, Chatterjee, STP, Titov, arXiv:1711.02107

GRB - HG $> 3\sigma$; GRA - GRB $\geq 2\sigma$; TMB - HG $\cong 3\sigma$; TMB - GRA $\cong 2\sigma$. With T2HKK data - better sensitivity.

The Power of Data

Systematic analysis (I. Girardi *et al.*, 2016): **all possible combinations of residual symmetries G_e and G_ν of the lepton flavour symmetry groups $G_f = S_4$, A_4, T' and A_5, leading to correlations between some of the three neutrino mixing angles and/or between the neutrino mixing angles and the Dirac CPV phase δ, were considered.**

(A) $G_e = Z_2$ **and** $G_\nu = Z_k$, $k > 2$ **or** $Z_m \times Z_n$, $m, n \geq 2$;

(B) $G_e = Z_k$, $k > 2$ **or** $Z_m \times Z_n$, $m, n \geq 2$ **and** $G_\nu = Z_2$;

(C) $G_e = Z_2$ **and** $G_\nu = Z_2$.

In these cases $U_e^\dagger$ **and/or** U_ν **of** $U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$, **are partially (or fully) determined by residual discrete symmetries of** $G_f = S_4$, A_4, T' **and** A_5.

More specifically:

A. $G_e = Z_2$, $G_\nu = Z_n$, $n > 2$ **or** $Z_n \times Z_m$, $n, m \geq 2$;
U_ν **fixed; A1, A2 (A3):** θ_{23}, $\cos\delta$ **(**θ_{12}, θ_{13}**) predicted.**

B. $G_e = Z_n$, $n > 2$ **or** $G_e = Z_n \times Z_m$, $n, m \geq 2$, $G_\nu = Z_2$;
U_e **fixed; B1,B2 (B3):** θ_{12}, $\cos\delta$ **(**θ_{23}, θ_{13}**) predicted.**

C. $G_e = Z_2$ **and** $G_\nu = Z_2$: θ_{12} **or** θ_{23} **or** $\cos\delta$ **predicted.**

$G_f = A_4$, S_4, T', A_5.

A_4**:** **3** Z_2**, 4** Z_3**, 1** $Z_2 \times Z_2$ **subgroups (total 8).**

T'**: similar to** A_4**.**

S_4**:** **9** Z_2**, 4** Z_3**, 3** Z_4**, 4** $Z_2 \times Z_2$ **subgroups (total 20).**

A_5**: has 15** Z_2**, 10** Z_3**, 6** Z_5**, 5** $Z_2 \times Z_2$ **subgroups (36).**

In the case of A_4 (T') symmetry only there are 64 models (up to permutation of rows and columns).

A_4:
$(G_e, G_\nu) = (Z_2, Z_3)$, **A1 - A3;**
$(G_e, G_\nu) = (Z_2, Z_2)$, **A1 - A3;**
$(G_e, G_\nu) = (Z_3, Z_2)$, **B1 - B3;**
$(G_e, G_\nu) = (Z_2 \times Z_2, Z_2)$, **B1 - B3;**
$(G_e, G_\nu) = (Z_2, Z_2)$, **C1 - C9.**

For A_4, S_4 and A_5 the total number of models to be analysed is extremely large. However, a total of only 14 models survive the 3σ constraints on $\sin^2\theta_{ij}$ from the current data and the requirement $|\cos\delta| \leq 1$.

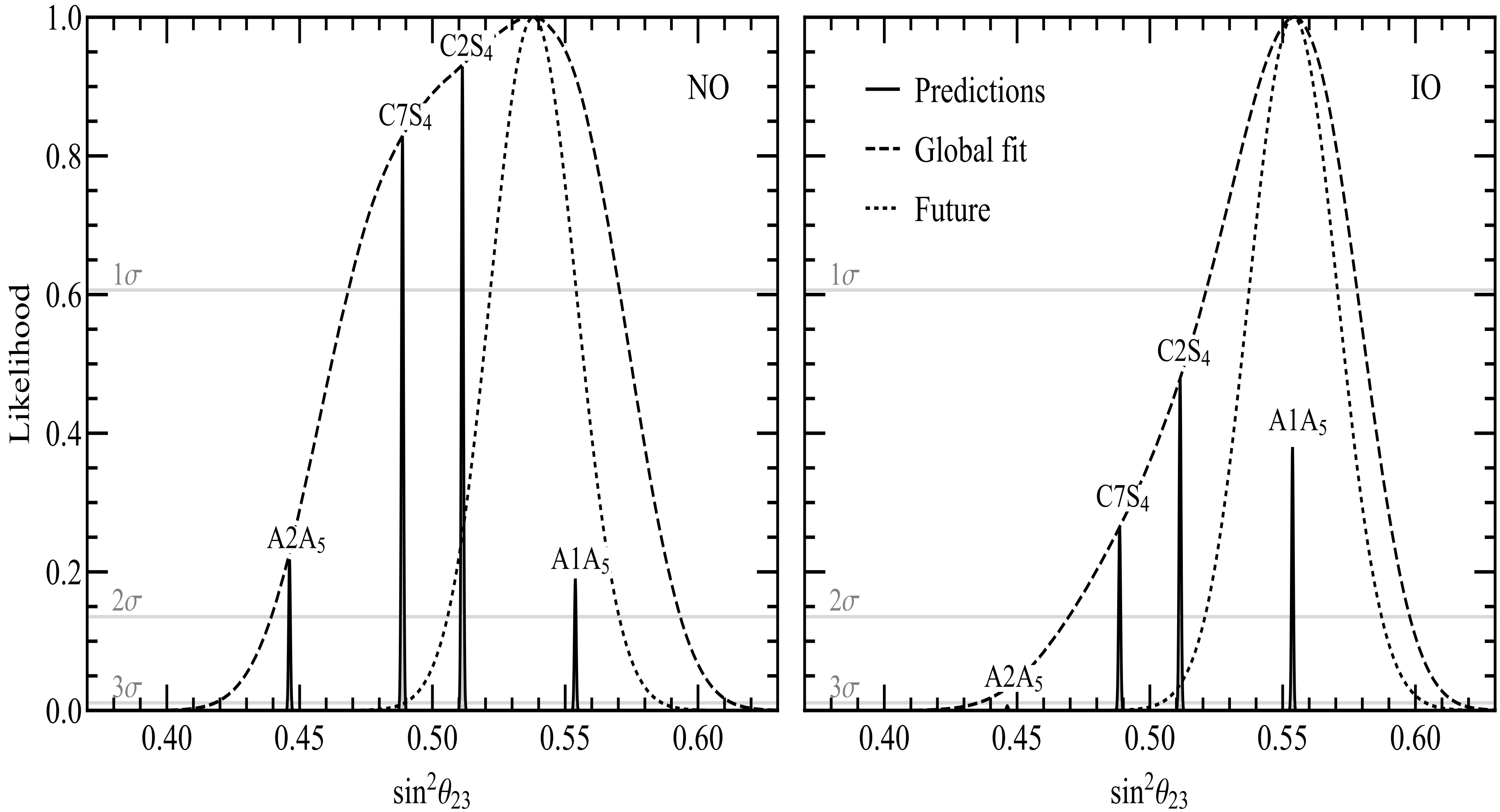
Phenomenologically Viable Predictions

A1 (A2), A_5 **(**$G_e = Z_2$**,** $G_\nu = Z_3$ **(Dirac** ν_j**)):**
$\sin^2\theta_{23} \cong 0.553$ (0.447); $\cos\delta \cong 0.716$ (-0.716).

A1, S_4: $\sin^2\theta_{23} \cong 0.5(1 - \sin^2\theta_{13}) \cong 0.489$;
$\cos\delta \cong -1$ **requires** $\sin^2\theta_{12} \cong 0.348$ **(!)**

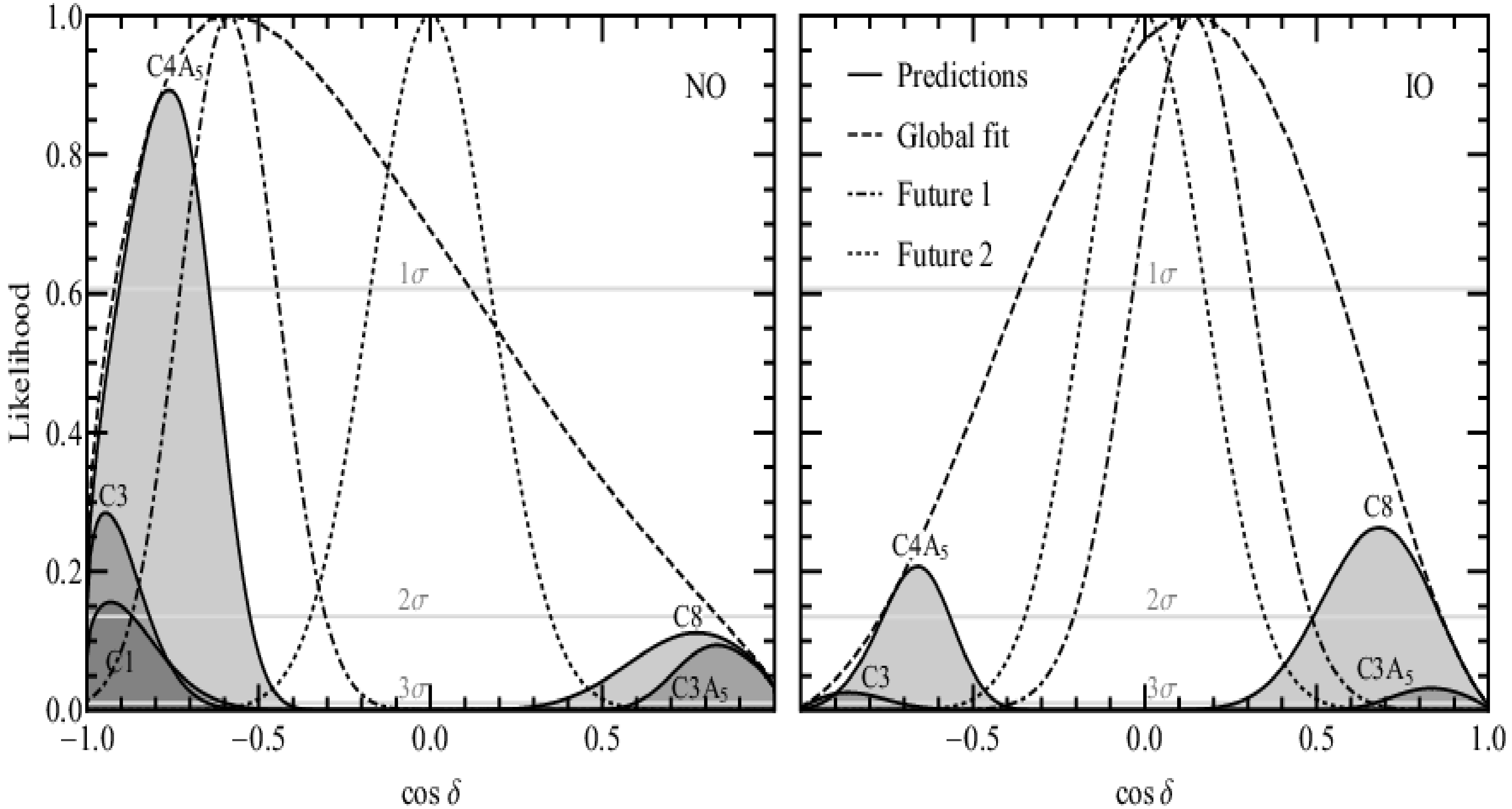
B1, A_4 **(**T'**,** S_4**,** A_5**)** **(**$G_e = Z_3^T$**,** $G_\nu = Z_2^S$**):**
$U_{\rm PMNS} = U_{\rm TBM} U_{13}(\theta^\nu_{13}, \delta_{13}) Q_0$;
$\sin^2\theta_{12} = 1/(3\cos^2\theta_{13}) \cong 0.340$; $\cos\delta \cong 0.570$.

B2, S_4 **(**$G_e = Z_3^T$**,** $G_\nu = Z_2^{SU}$**):**
$\sin^2\theta_{12} \cong (1 - 2\sin^2\theta_{13})/3 = 0.319$; $\cos\delta \cong -0.269$.



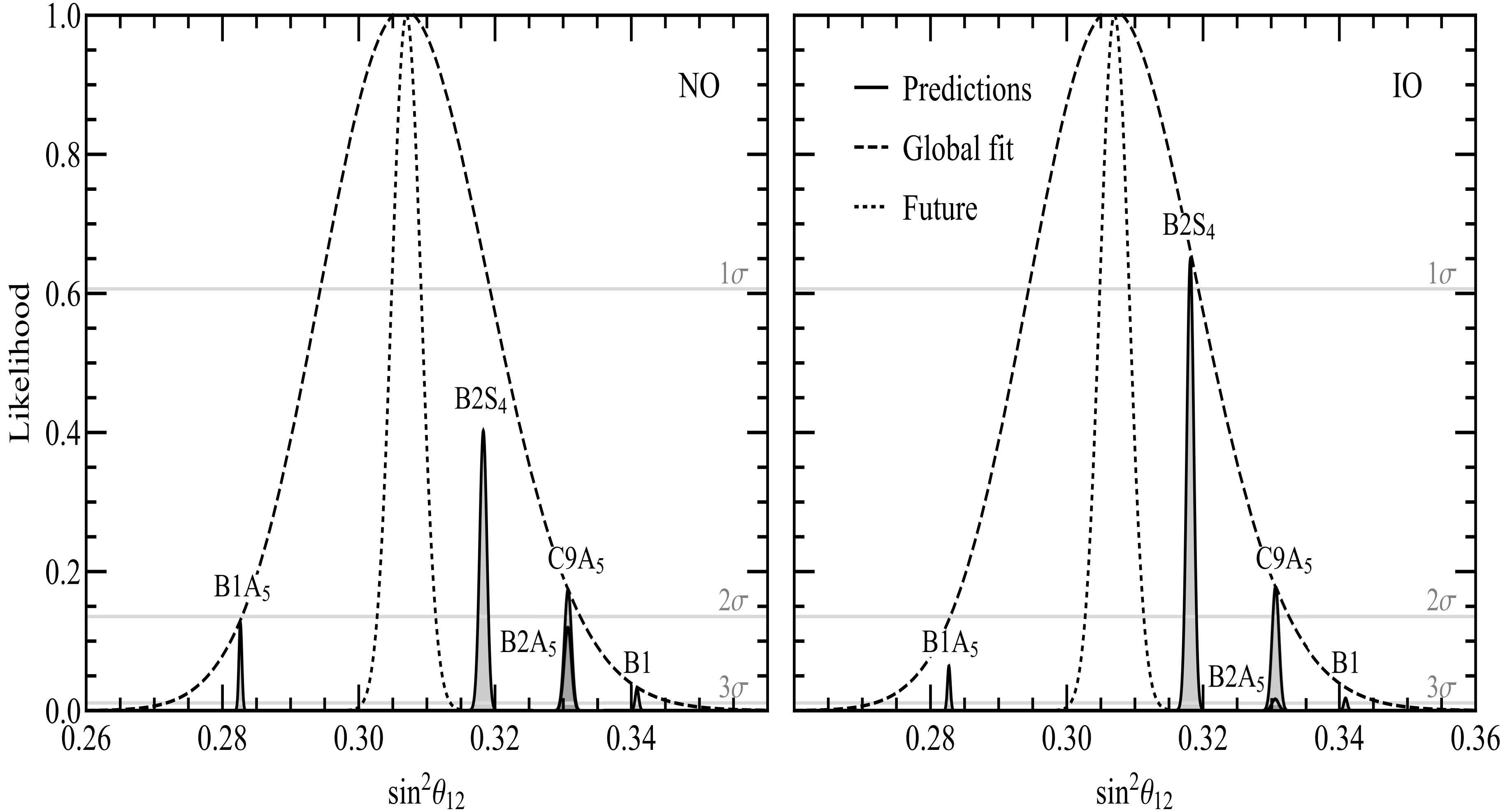
S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE).**



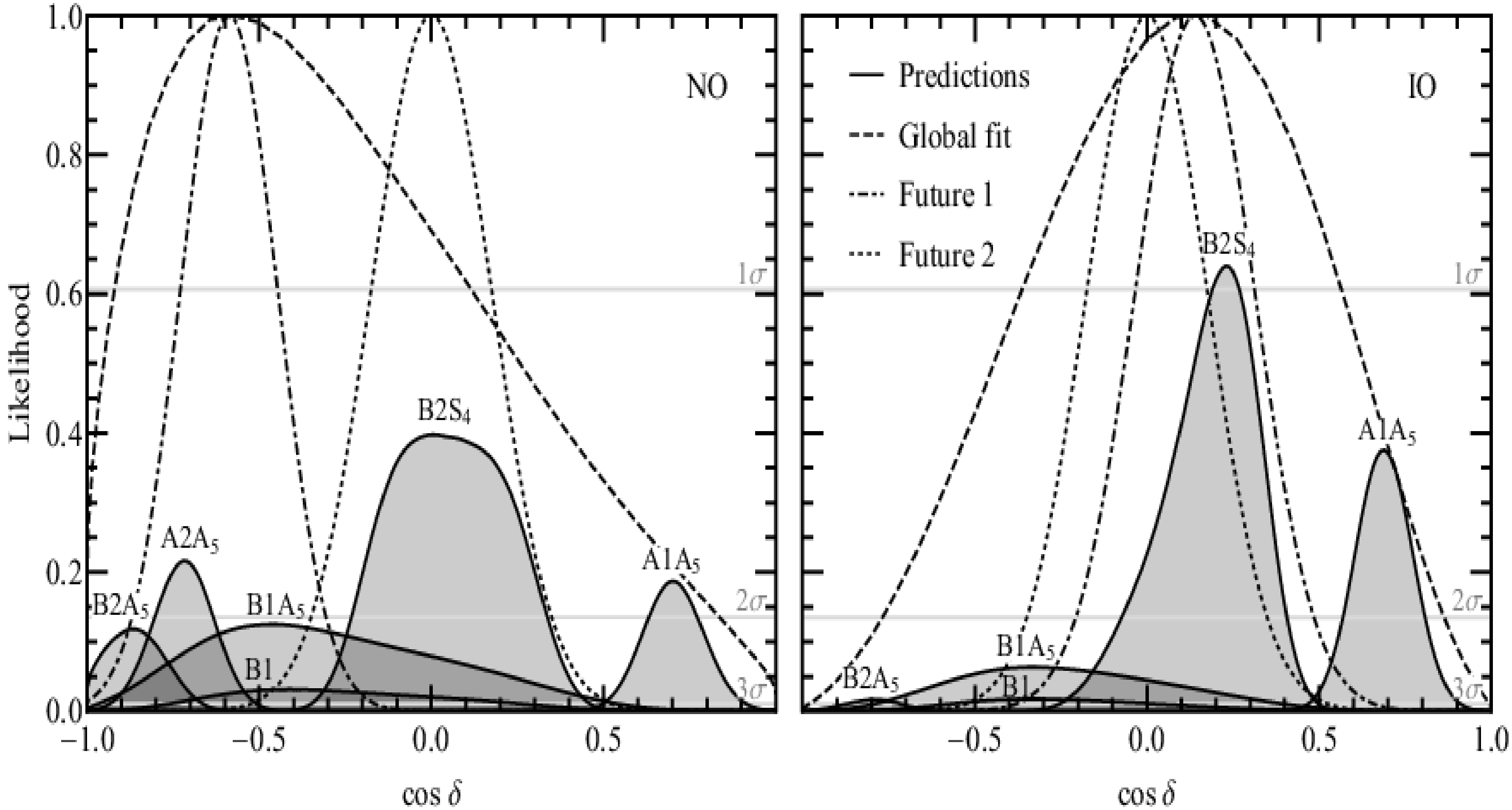
S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\delta) = 10^{\circ}$.



S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO).**



S.T.P., A. Titov, arXiv:1804.00182

A total of 6 models would survive out of the currently viable 14 (of the extremely large number) considered if $\delta(\sin^2\theta_{23}) = 3\%$, $\delta(\sin^2\theta_{12}) = 0.7\%$ and the current b.f.v. would not change:

A1A_5, C2S_4, C3, C3A_5, C4A_5, C8.

Will be constrained further by the data on δ.

The predictions obtained for $\cos\delta$ are valid in a large class of theoretical models of (lepton) flavour based on discrete symmetries.

(See, e.g., the reviews D. Meloni, 1709.02662 and STP, 1711.10806 for extensive lists of references.)

Conclusions.

- Understanding the origin of the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years is one of the most challenging problems in neutrino physics.

- The observed pattern of neutrino mixing can be due to a new basic (approximate non-Abelian discrete) symmetry of particle interactions leading to an approximate symmetry form of the PMNS matrix.

- The most important testable consequence of the symmetry approach to understanding the pattern of neutrino mixing is the correlation between the values of some of the neutrino mixing angles and/or the value of $\cos\delta$ and the values of the neutrino mixing angles: $\delta = \delta(\theta_{12}, \theta_{13}, \theta_{23}; \theta^{\nu}_{12})$. The second correlation depends on the underlying approximate symmetry form of the $U_{\rm PMNS}$.

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Conclusions (contd.)

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

Dirac and Majorana CPV may have the same source.

The see-saw mechanism provides a link between the ν-mass generation and the baryon asymmetry of the Universe (BAU).

Any of the CPV phases in $U_{\rm PMNS}$ can be the leptogenesis CPV parameters.

Low energy leptonic CPV can be directly related to the existence of BAU.

These results underline further the importance of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.