3/6/2019 Johns Hopkins workshop

Dark Matter Sommerfeld-enhanced annihilation and bound-state decay at finite temperature

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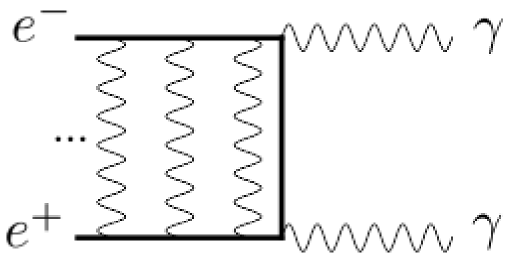
based on

[arXiv:1808.06472] Phys.Rev. **D98** (2018) no.11, 115023

in collaboration with

Laura Covi (Göttingen) and **Kyohei Mukaida** (DESY).

Non-relativistic Positronium annihilation



- Bound-state decay **[J. Wheeler 1946]**:

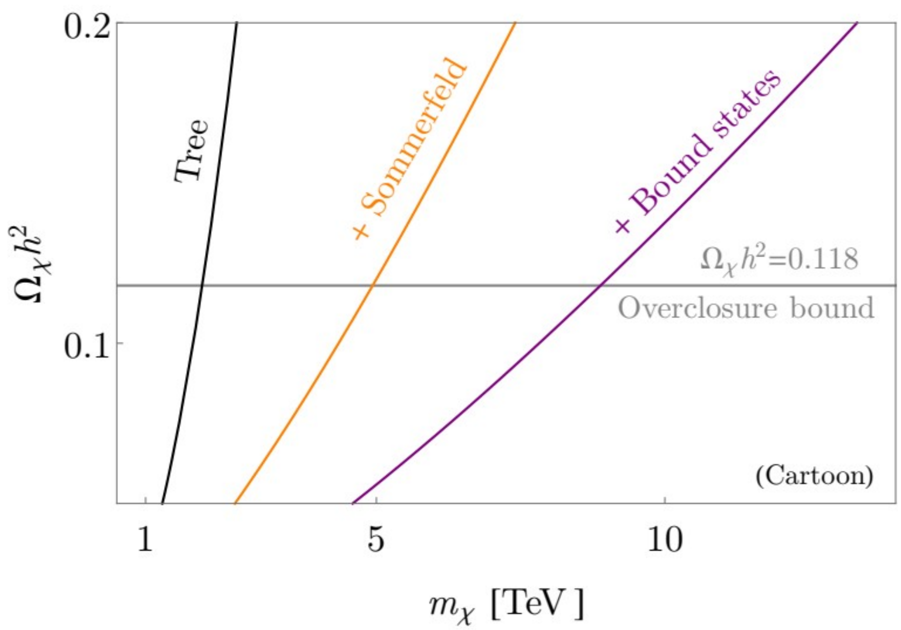
$$\Gamma_n = 4(\sigma v)_0 \times |\psi_n(r=0)|^2$$

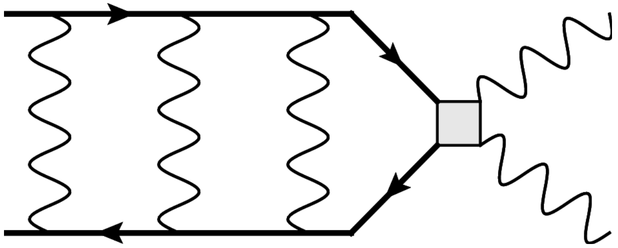
- Sommerfeld-enhanced annihilation **[A. Sakharov 1948]**:

$$(\sigma v) = (\sigma v)_0 \times |\psi(r=0)|^2$$
$$\propto (\sigma v)_0 \, (\alpha / v_{\rm rel}) \, , \ \text{for } v_{\rm rel} \lesssim \alpha$$

Motivation

- **How heavy can (neutralino) dark matter be?**
  (title of [H. Fukuda *et al.* '19])





Ex.: Wino, co-annihilation of colored charged particles, Higgs mediated bound states, hidden charged dark sectors, SIDM with light mediators, ...
[J. Hisano *et al.* '07, ...] [J. Feng *et al.* '09, Harling&Petraki '14, ..., Harz&Petraki '19]

⇒ Effects generically allow for **larger DM masses**.

⇒ **Evading detection** at, e.g., the LHC.

⇒ Eventually consulting construction of future colliders.

Literature and methods

Decompose problem into two questions:

1) **Effective in-medium potential?**

System similar to heavy quarkonium decay in QGP.

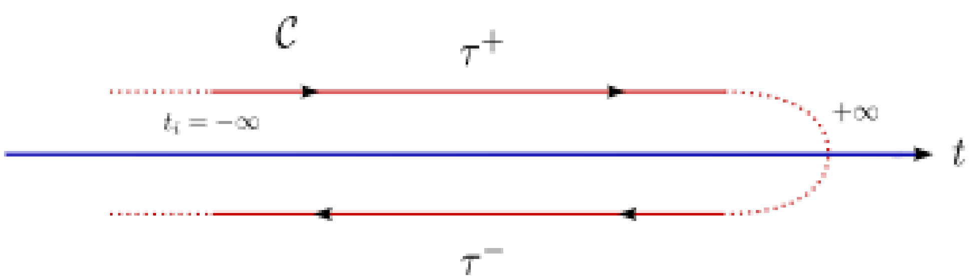
⇒ **Equilibrium 4-Polyakov loop (Wilson lines) method** **[M. Laine *et al.* '07]**

2) **DM number density equation (What is the "BE" to use?)**

Langevin equation: $\delta\dot{n} = -\Gamma_{\text{chem}}\delta n(t) + \xi(t)$ **[Bödeker&Laine '12]**

Our approach: **Non-equilibrium QFT ⇒ In-medium effects + full dynamics**

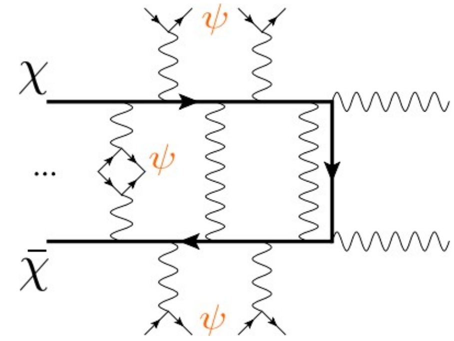
$$G_O(x,y) \equiv \langle T_{\mathcal{C}} O(x) O^\dagger(y) \rangle$$
$$\langle ... \rangle \equiv \mathrm{Tr}[\rho ...]$$



In some limits, previous literature results should be (are) recovered.

Scheme

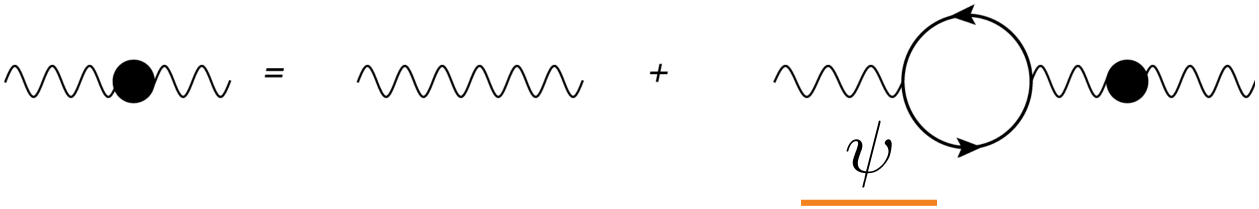
$$\boxed{\mathcal{L} \supset g_\chi \bar{\chi}\gamma^\mu \chi A_\mu + \underline{g_\psi \bar{\psi}\gamma^\mu \psi A_\mu}}$$



I Non-relativistic effective action on CTP-contour

$$S_{\rm NR}[\eta,\xi] = \int_{x\in\mathcal{C}} \eta^\dagger\left[i\partial_t + \frac{\Delta}{2M}\right]\eta + \xi^\dagger\left[i\partial_t - \frac{\Delta}{2M}\right]\xi + \int_{x,y\in\mathcal{C}} i\frac{g_\chi^2}{2}\underbrace{J(x)D(x,y)J(y)}_{\text{“potential scattering”}} + i\underbrace{O^\dagger(x)\Gamma(x,y)O(y)}_{\text{“annihilation”}},$$

- Separation of short- and long-range contributions, $J \equiv \eta^\dagger\eta + \xi^\dagger\xi, O \equiv \xi^\dagger\eta$
- Electric correlator: $D(x,y) \equiv \langle T_\mathcal{C} A_0(x) A_0(y)\rangle$



Scheme

II Number density equation from EoM of two-point function

$$\dot{n}_\eta + 3Hn_\eta = -2(\sigma v)_0 \left[G^{++--}_{\eta\xi}(x,x,x,x) - G^{++--}_{\eta\xi}(x,x,x,x)\Big|_{\rm eq} \right].$$

$$n_\eta(x) \equiv \langle \eta^\dagger(x)\eta(x)\rangle, \; G_{\eta\xi}(x,y,z,w) \equiv \langle T_{\mathcal{C}}\eta(x)\xi^\dagger(y)\xi(w)\eta^\dagger(z)\rangle$$

- Exact result! "Only" have to compute a 4-point function.
- Truncation of Martin-Schwinger hierarchy introduces approximations.
- To lowest order:

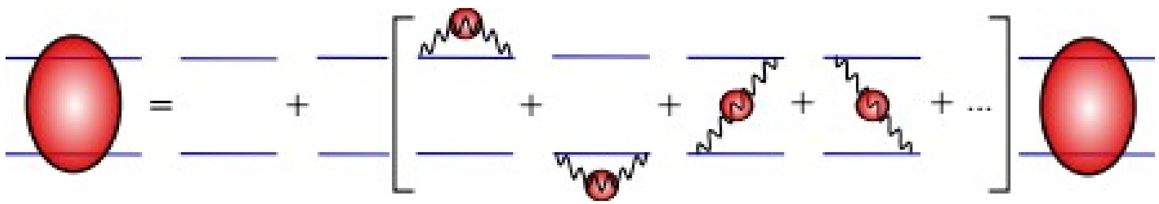
$$2G^{++--}_{\eta\xi}(x,x,x,x) \simeq n_\eta n_\xi$$

→ *Lee-Weinberg equation*

- For bound states to occur, need non-perturbative solution

Scheme

III Resummation and effective potential



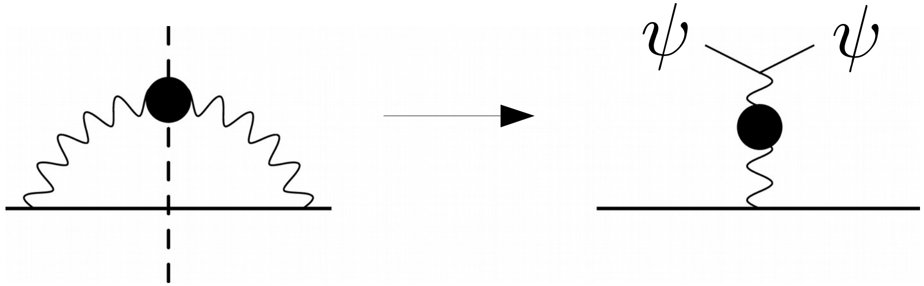
$$\left[\frac{\nabla_{\mathbf{r}}^2}{M} + E + i\epsilon - V_{\rm eff}(\mathbf{r}, T)\right] G^R_{\eta\xi}(\mathbf{r}, \mathbf{r}'; E) = 2i\delta(\mathbf{r} - \mathbf{r}')$$

In static limit and Hard-Thermal-Loop approximation:

$$\boxed{V_{\rm eff}(\mathbf{r}, T) \equiv -ig_\chi^2 \int \frac{d^3p}{(2\pi)^3}(1 - e^{i\mathbf{p}\mathbf{r}})D^{++}(0, \mathbf{p}) = -\alpha_\chi m_D - \frac{\alpha_\chi}{r}e^{-m_D r} \underline{- i\alpha_\chi T\phi(m_D r)}}$$

consistent with **[M. Laine *et al.* '07]**

Interpretation of thermal width:



Soft bath particle scattering
aka "Landau damping"

Scheme

IV Kubo-Martin-Schwinger relation and two-particle spectral function

$$\dot{n}_\eta + 3Hn_\eta = -2(\sigma v)_0 \left[G^{++--}_{\eta\xi}(x,x,x,x) - G^{++--}_{\eta\xi}(x,x,x,x)\Big|_{\mathrm{eq}} \right].$$

- Treat annihilation as perturbation
- Assume grand canonical state $\rho \propto e^{-\beta(\hat{H}-\mu_\eta \hat{N}_\eta - \mu_\xi \hat{N}_\xi)}$ to relate:

$$\boxed{G^{++--}_{\eta\xi}(x,x,x,x) = e^{-2\beta(M-\mu)} \int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).}$$

$$G^{\rho}_{\eta\xi} = 2\Im[iG^R]$$

- Positive and negative energy (bound state) solutions included.

Consistency check

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\rm eq} = e^{-2\beta M}\int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta\mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

$$V_{\rm eff}(r,T) = 0$$

Free spectral function:

$$G^{\rho}_{\eta\xi} \propto \theta(E) E^{1/2}$$

+

Ideal dilute gas:

$$n = n_s^{\rm eq} e^{\beta\mu}$$

$$\Rightarrow \beta\mu = \ln[n/n_s^{\rm eq}]$$

$$\dot{n} + 3Hn = -(\sigma v)_0\left[n^2 - (n_s^{\rm eq})^2\right]$$

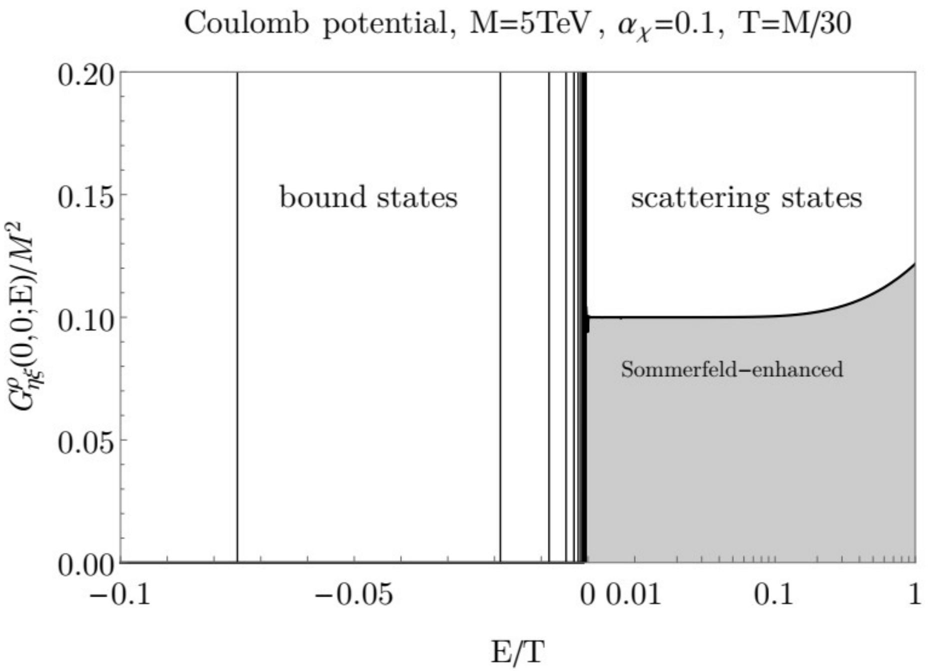
Lee-Weinberg equation ✓

Consistency check

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\text{eq}} \left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\text{eq}} = e^{-2\beta M} \int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

$$\lim_{T\to 0} V_{\text{eff}}(r,T)$$



Consistency check

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

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$$\lim_{T\to 0} V_{\rm eff}(r,T)$$

Spectral function:

$$(\sigma v)_0 G^{\rho}_{\eta\xi}(E)|_{E>0} \propto E^{1/2}(\sigma v)_0 |\psi(0)|^2$$

$$(\sigma v)_0 G^{\rho}_{\eta\xi}(E)|_{E<0} \propto \sum_n \delta(E - E_n)\Gamma_n$$

Chemical potential:

$$n = n_s^{\rm eq} e^{\beta\mu} + \sum_i n_i^{\rm eq} e^{2\beta\mu}$$

$$\Rightarrow \beta\mu = \ln[\alpha n / n_s^{\rm eq}]$$

$$\dot{n} + 3Hn = -\left(\langle(\sigma v)_0 S\rangle + \sum_i \Gamma_i \frac{n_i^{\rm eq}}{(n_s^{\rm eq})^2}\right)\left[(\alpha n)^2 - (n_s^{\rm eq})^2\right]$$

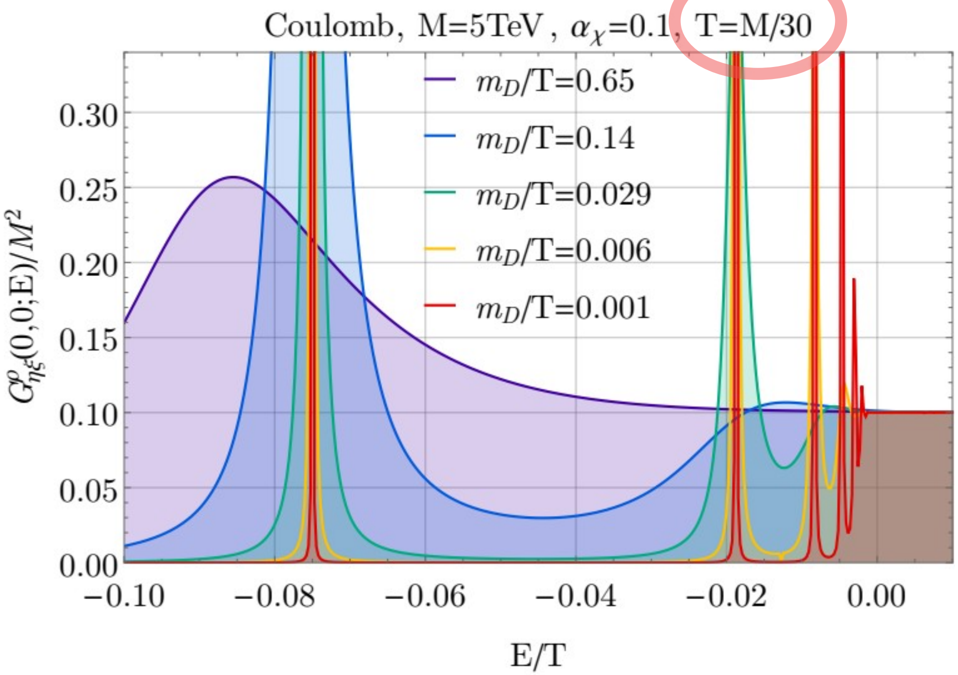
BEs in (Saha) ionization equilibrium ✓

Finite temperature corrections

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\rm eq} = e^{-2\beta M}\int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

full $V_{\rm eff}(r,T)$



⇒ Thermal width can exceed binding energy

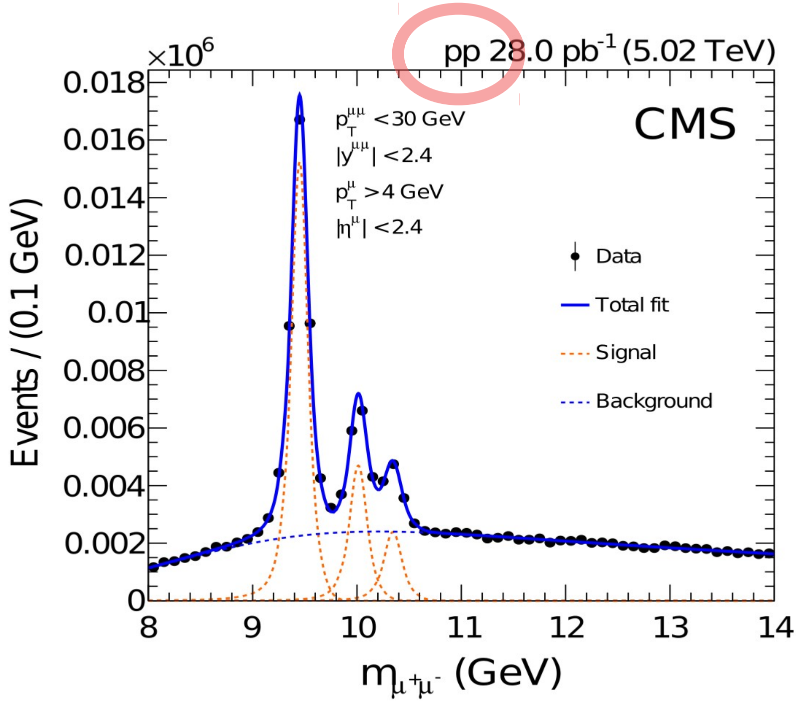
⇒ Spectrum is continuous (can not distinguish bound from scattering state)

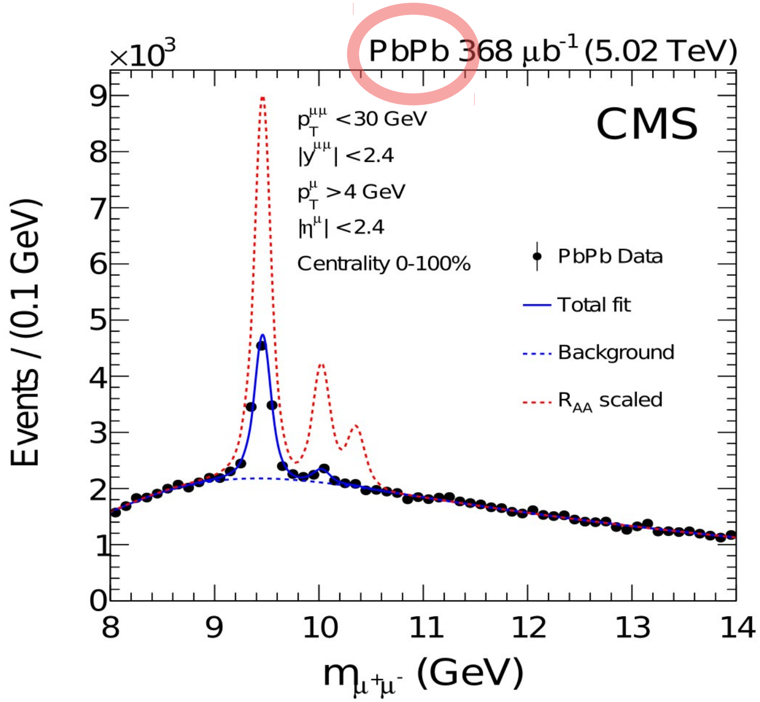
⇒ Melting of ground-state pole at the time of DM freeze-out possible!

⇒ Rate is exponentially sensitive to changes in spectral function.

Heavy quarkonium annihilation/decay in QGP

CMS collaboration, Phys.Lett. **B790** (2019) 270-293





- Sequential melting of [Υ(1S), Υ(2S), Υ(3S)] in QGP observed.

Summary

- Analytically derived new number density equation, accounting for finite temperature corrections to Sommerfeld-enhanced annihilation and bound-state decay.
- Contributed to set a theoretical basis for quantifying the impact of finite temperature corrections in a self-consistent approach.
- Finite temperature corrections arise in spectral function and chemical potential.
- Clear understanding of the limitation (e.g. ionization equilibrium).
- More theoretical developments required to obtain full picture.
- Non-equilibrium QFT promising approach.

Backup: BEs and ionization equilibrium

$$
\begin{aligned}
\dot{n}_s + 3Hn_s =& - \langle (\sigma v)_{\rm an} \rangle \left[n_s^2 - (n_s^{\rm eq})^2 \right] \\
& - \sum_i \langle (\sigma v)_i \rangle \left[n_s^2 - n_i K_i^{-1} \right] ,
\end{aligned}
$$

$$K_i \equiv n_i^{\rm eq}/(n_s^{\rm eq})^2$$

$$
\begin{aligned}
\dot{n}_i + 3Hn_i =& - \Gamma_i \left[n_i - n_i^{\rm eq} \right] \\
& + \langle (\sigma v)_i \rangle \left[n_s^2 - n_i K_i^{-1} \right] \\
& - \sum_j \Gamma_{i \to j} \left[n_i - n_j R_{ij} \right]
\end{aligned}
$$

$$R_{ij} \equiv n_i^{\rm eq}/n_j^{\rm eq}$$

Ionization equilibrium:

$$\boxed{\left(\frac{n_s}{n_s^{\rm eq}} \right)^2 = \frac{n_i}{n_i^{\rm eq}}, \ \forall i \ \Rightarrow 2\mu \equiv 2\mu_s = \mu_i \ \forall i}$$

Different approaches

- Dynamical formulation exists for the *linear regime close to chemical eq*.

[Bödeker&Laine '12]

$$\dot{\delta n} = -\Gamma_{\rm chem}\delta n(t) + \xi(t) \;\; (\text{Langevin equation})$$

$\Gamma_{\rm chem}$ be related to spectral function (previous slide)

matching $\Gamma_{\rm chem}$

$$\dot{n} + 3Hn = -\Gamma_{\rm chem}(n - n_{\rm eq}) + \mathcal{O}([n - n_{\rm eq}]^2) \;\; (\text{linearized BE})$$

$$\dot{n} + 3Hn = -\frac{\Gamma_{\rm chem}}{2n_{\rm eq}}(n^2 - n_{\rm eq}^2) \;\; (\text{quadratic BE})$$

- To check, we need a derivation from **non-equilibrium quantum field theory** (this work)

Summary consistency checks

$$\boxed{\dot{n} + 3Hn = -2(\sigma v)_0 G_4^{++--}\Big|_{\text{eq.}} \left[\left(\alpha n / n_{s,0}^{\text{eq}}\right)^2 - 1\right]}$$

Consistency check

$V_{\text{eff}}(r,T) = 0$ $\longrightarrow$

$$\dot{n} + 3Hn = -(\sigma v)_0 \left[n^2 - n_{\text{eq}}^2\right]$$

Lee-Weinberg equation ✓

$\lim\limits_{T\to 0} V_{\text{eff}}(r,T)$ $\longrightarrow$

$$\dot{n} + 3Hn = -\left(\langle(\sigma v)_0 S\rangle + \sum_i \Gamma_i R_i\right)\left[(\alpha n)^2 - (n_s^{\text{eq}})^2\right]$$

BEs in ionization equilibrium ✓

full $V_{\text{eff}}(r,T)$, $n \sim n_{\text{eq}}$ $\longrightarrow$

$$\dot{n} + 3Hn = -\Gamma_{\text{chem}}\left[n - n_{\text{eq}}\right]$$

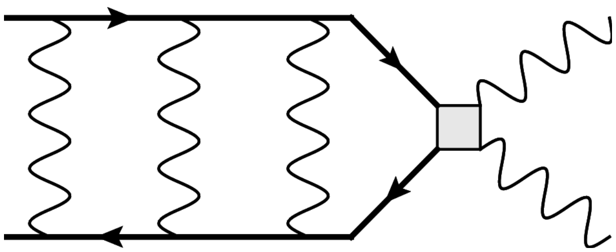
consistent with Langevin approach

in linear regime close to chem. equil. ✓

Motivation

- **How heavy can (neutralino) dark matter be?**
  (title of [H. Fukuda *et al.* '19])

- **Sommerfeld enhancement + Bound states**



Ex.: Wino, co-annihilation of colored charged particles, Higgs mediated bound states, hidden charged dark sectors, SIDM with light mediators, ...
[J. Hisano *et al.* '07, ...] [J. Feng *et al.* '09, Harling&Petraki '14, ..., Harz&Petraki '19]

Criteria for single attractive Yukawa potential: $\frac{1}{\alpha m_\chi} \lesssim \frac{1}{m_\phi}$

$$
\begin{aligned}
(\chi\bar{\chi})_{\rm sc} &\rightleftharpoons \text{SM SM}', && \textbf{annihilation}\\
(\chi\bar{\chi})_{\rm sc} &\rightleftharpoons \phi + (\chi\bar{\chi})_{B_i}, && \\
(\chi\bar{\chi})_{B_i} &\rightleftharpoons \phi + (\chi\bar{\chi})_{B_j}, && \text{radiative processes}\\
(\chi\bar{\chi})_{B_i} &\rightleftharpoons \text{SM SM}'. && \textbf{decay}
\end{aligned}
$$

⇒ Effects generically allow for **larger DM masses**.

⇒ **Evading detection** at, e.g., the LHC.

⇒ Eventually consulting construction of future colliders.