Gravitational Particle Production and Dark Matter

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arXiv:1502.02475, 1504.07119, 1604.08898, 1804.10378, 1903.10973, 1905.09143

Johns Hopkins Workshop @ Kavli IPMU (2019/6/8)

Contents

- Particle production in cosmology
- Gravitational particle production
- Purely gravitational dark matter
- Gravitational particle production in extended gravity

Particle Production

Particle Production

- Important concept in Cosmology
  - Reheating after inflation
  - Dark matter production
  - Baryogenesis
  - Dangerous relics (moduli, gravitino, etc.)
  - Primordial density perturbation
  - Primordial gravitational waves

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Gravitational production of longwave modes

Particle Production

- Important concept in Cosmology
  - Reheating after inflation
  - Dark matter production
  - Baryogenesis
  - Dangerous relics (moduli, gravitino, etc.)

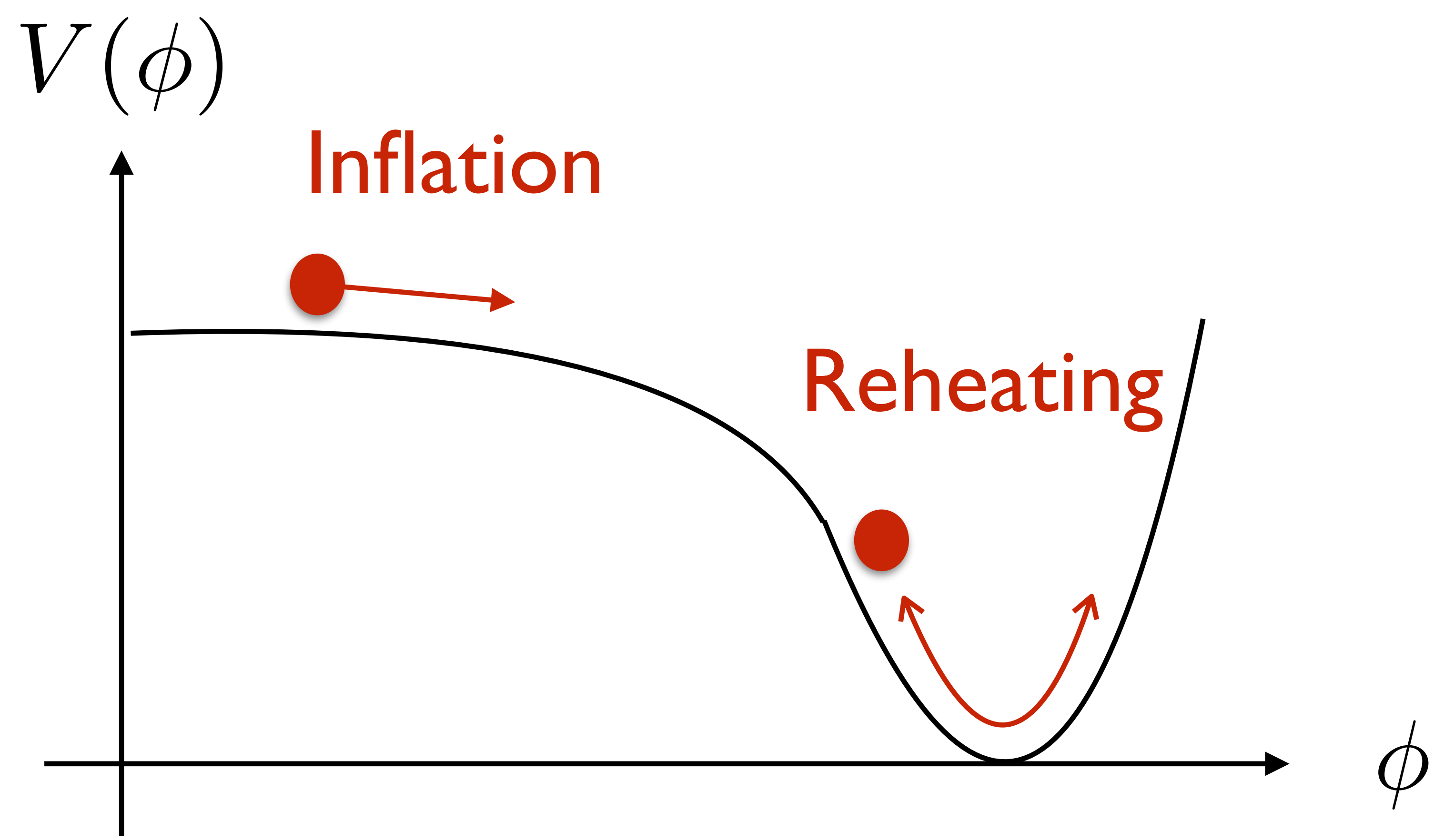
"Gravitational particle production" may play crucial role!

  - Primordial density perturbation
  - Primordial gravitational waves

Gravitational production of longwave modes

Reheating after inflation

- After inflation, inflaton cohenrent oscillation begins
- Inflaton oscillation induces particle production
- Eventually inflaton energy is converted into radiation



Example of particle production

$$\boxed{V = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + g\phi^2\chi^2 + \mu\phi\chi^2}$$

- Inflaton coherent oscillation $\phi = \widetilde{\phi}(t)\cos(m_\phi t)$
- Effective mass of χ rapidly oscillates

$$-\partial^2\chi + m_\chi^{(\mathrm{eff})2}\chi = 0 \qquad m_\chi^{(\mathrm{eff})2} = m_\chi^2 + 2g\phi^2 + 2\mu\phi$$

- It excites zero-point fluctuation of χ (c.f. parametric resonance)

$\longrightarrow$ Particle production of χ

Dolgov, Kirilova (1990), Traschen, Brandenberger (1990), Kofman, Linde, Starobinsky (1994)

QFT treatment of particle production

- Define "in vacuum" at $t = t_{\rm in}$: $a_{\vec{k}} \left|0\right\rangle = 0$

$$\chi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3} \left[\chi_k(t) a_{\vec{k}} + \chi_k^*(t) a^\dagger_{-\vec{k}}\right] e^{i\vec{k}\cdot\vec{x}} \qquad \omega_k(t) = \omega_k^{(\rm in)}$$

- Measure quanta at $t = t_{\rm out}$

$$\chi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3} \left[\psi_k(t) b_{\vec{k}} + \psi_k^*(t) b^\dagger_{-\vec{k}}\right] e^{i\vec{k}\cdot\vec{x}} \qquad \omega_k(t) = \omega_k^{(\rm out)}$$

Bogoliubov transformation : $b_k \sim \sum_{k'} (\alpha_{kk'} a_{k'} + \beta_{kk'} a^\dagger_{k'})$

$$\boxed{n_k = \left\langle 0\right| b_k^\dagger b_k \left|0\right\rangle \sim \sum_{k'} |\beta_{kk'}|^2}$$

A Method for Calculation

- Useful parametrization

$$\chi_k(\tau) = \alpha_k(\tau) v_k(\tau) + \beta_k(\tau) v_k^*(\tau) \qquad v_k(\tau) \equiv \frac{1}{\sqrt{2\omega_k}} e^{-i\int \omega_k d\tau}$$

- Equaion of motion:

$$\chi_k'' + \omega_k^2 \chi_k = 0, \qquad \omega_k^2 \equiv k^2 + m_\chi^{(\text{eff})2}.$$

$$\longrightarrow \quad \alpha_k' v_k = \frac{\omega_k'}{2\omega_k} v_k^* \beta_k, \qquad \beta_k' v_k^* = \frac{\omega_k'}{2\omega_k} v_k \alpha_k$$

- Energy density:

$$\rho_\chi = \int \frac{d^3 k}{(2\pi)^3} \omega_k \left(\frac{1}{2} + |\beta_k|^2 \right)$$

$$f_\chi(k) \equiv |\beta_k|^2$$

Vacuum conrtribution
(subtracted by cosmological constant)

Phase space density of
produced particles

- For $m_\chi^{(\mathrm{eff})2} = m_\chi^2 + 2\mu\phi$

$$n_\chi(t) = \int \frac{d^3k}{(2\pi)^3} f_\chi(k,t) \simeq \frac{\mu^2 \widetilde{\phi}^2}{8\pi} t$$

$\longrightarrow$ Inflaton perturbative "decay" rate $\Gamma(\phi \to \chi\chi) = \dfrac{\mu^2}{8\pi m_\phi}$

- For $m_\chi^{(\mathrm{eff})2} = m_\chi^2 + 2g\phi^2$

$$n_\chi(t) = \int \frac{d^3k}{(2\pi)^3} f_\chi(k,t) \simeq \frac{g^2 \widetilde{\phi}^4}{8\pi} t$$

$\longrightarrow$ Inflaton "annihilation" rate $\Gamma(\phi\phi \to \chi\chi) \sim \dfrac{g^2 \widetilde{\phi}^2}{8\pi m_\phi}$

Time dependence of background determines production rate

$m_\chi^{(\mathrm{eff})2} = A + B\cos(m_\phi t)$ $\longrightarrow$ "Decay"

$m_\chi^{(\mathrm{eff})2} = A + B\cos^2(m_\phi t)$ $\longrightarrow$ "Annihilation"

Gravitational Particle Production

Gravitational Particle Production

Parker (1969), Ford (1986)

- Real scalar field interacting only through gravity

$$\boxed{S=\int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2R-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{2}m_\chi^2\chi^2\right)}$$

FRW metric: $ds^2=-dt^2+a^2(t)d\vec{x}^2=a^2(\tau)(-d\tau^2+d\vec{x}^2)$

$$\rightarrow \boxed{S=\int d\tau d^3x\frac{a^2(\tau)}{2}\left[\chi'^2-(\nabla\chi)^2-a^2m_\chi^2\chi^2\right]}$$

- χ does not have direct interaction with inflaton. However, it feels inflaton dynamics through gravity.

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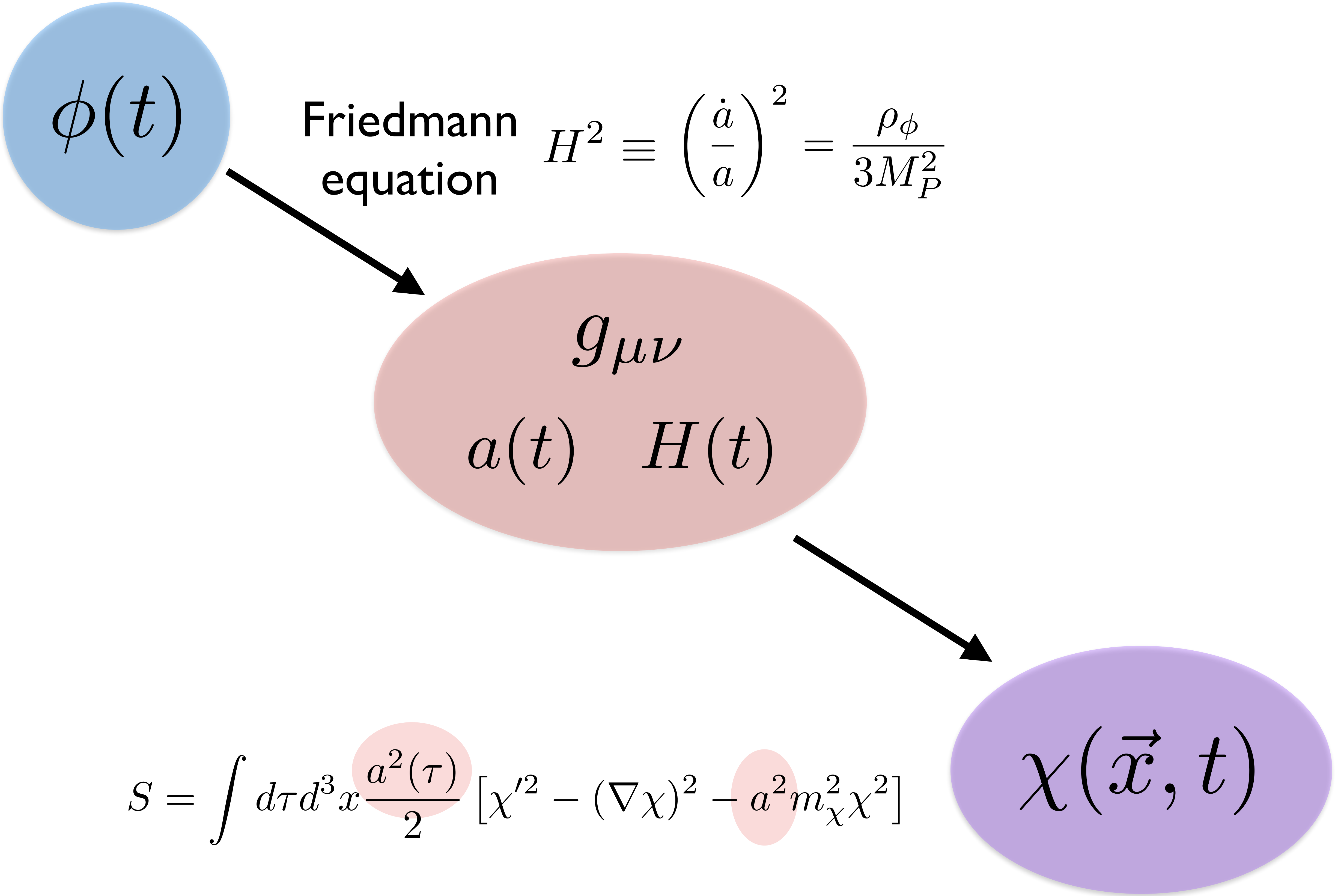


$\phi(t)$
Friedmann equation
$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_\phi}{3M_P^2}$
$g_{\mu\nu}$
$a(t) \quad H(t)$
$S = \int d\tau d^3x \frac{a^2(\tau)}{2}\left[\chi'^2 - (\nabla\chi)^2 - a^2 m_\chi^2 \chi^2\right]$
$\chi(\vec{x}, t)$

Background evolution

- Friedmann equation $3M_P^2 H^2 = \rho_\phi$
- Inflaton EoM $\ddot{\phi} + 3H\dot{\phi} + \dfrac{\partial V}{\partial \phi} = 0$

Two time scales: m_ϕ , H

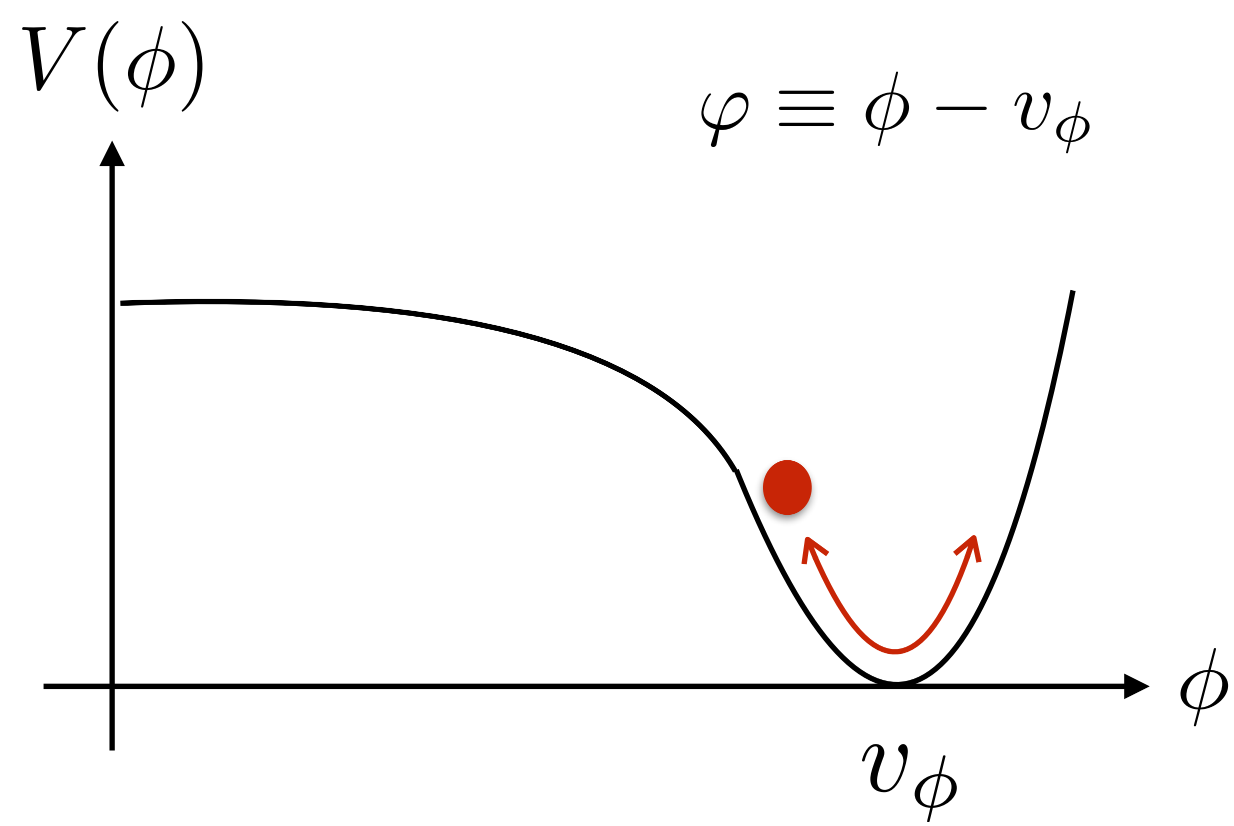
H : expansion rate "Slow"

m_ϕ : oscillation rate "Fast"

Background quantities have both "slow" and "fast" part

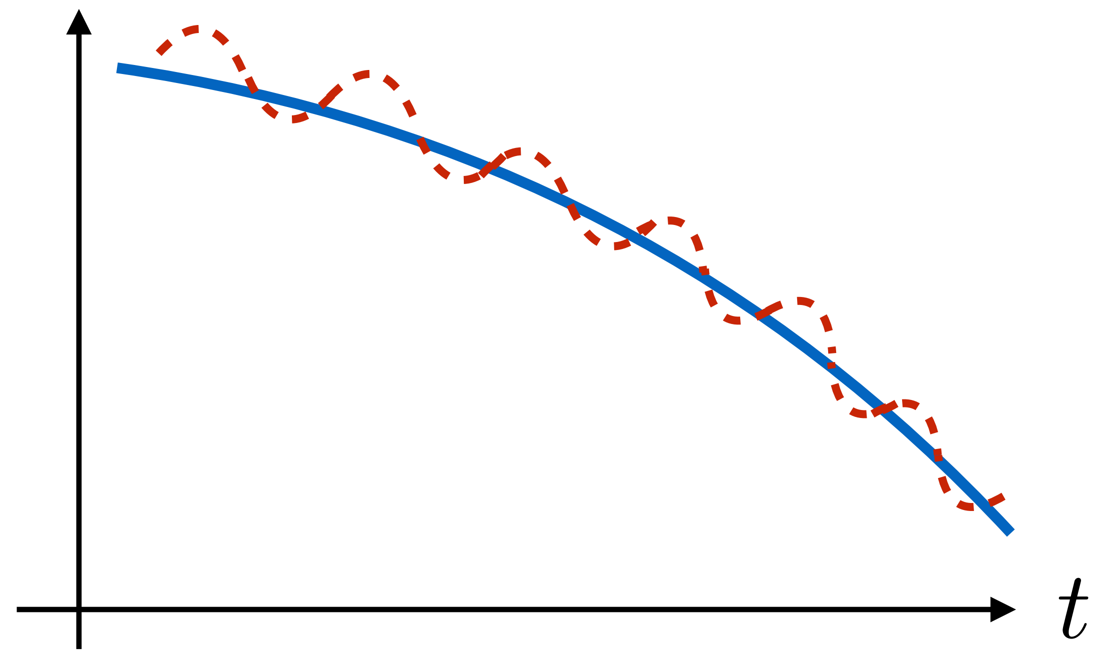
$$V(\phi) \simeq \frac{1}{2} m_\phi^2 \varphi^2$$

$$\varphi \equiv \phi - v_\phi$$



- Slow-Fast decomposition

$$H = \langle H \rangle + \delta H$$
$$\rho_\phi = \langle \rho_\phi \rangle + \delta \rho_\phi$$



"Slow" part $\dot{X} \sim \mathcal{O}(HX)$

"Fast" oscillating part $\dot{X} \sim \mathcal{O}(m_\phi X)$

- **Slow part**

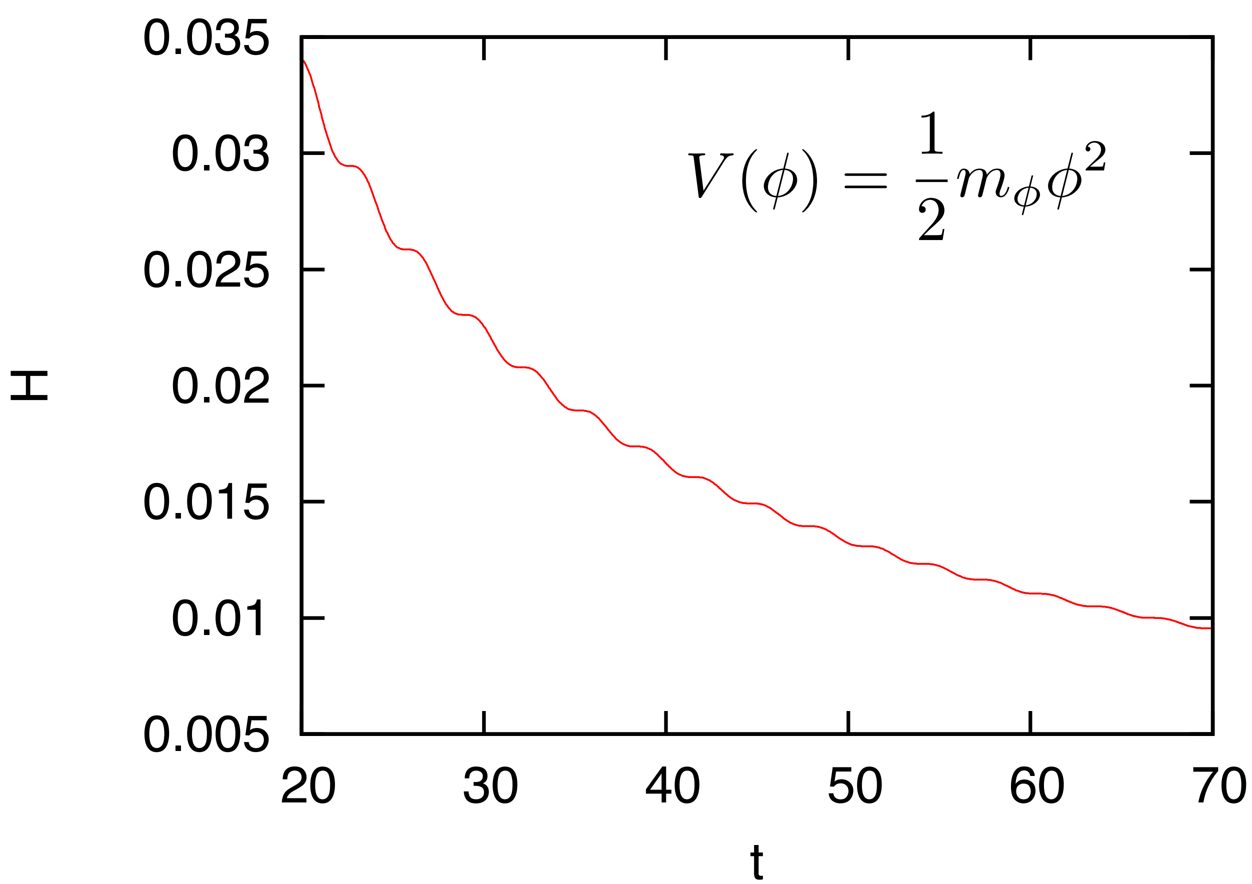
$$\boxed{\langle H \rangle = \frac{2}{3t} \qquad \langle a(t) \rangle \propto t^{2/3}}$$

- **Fast part**

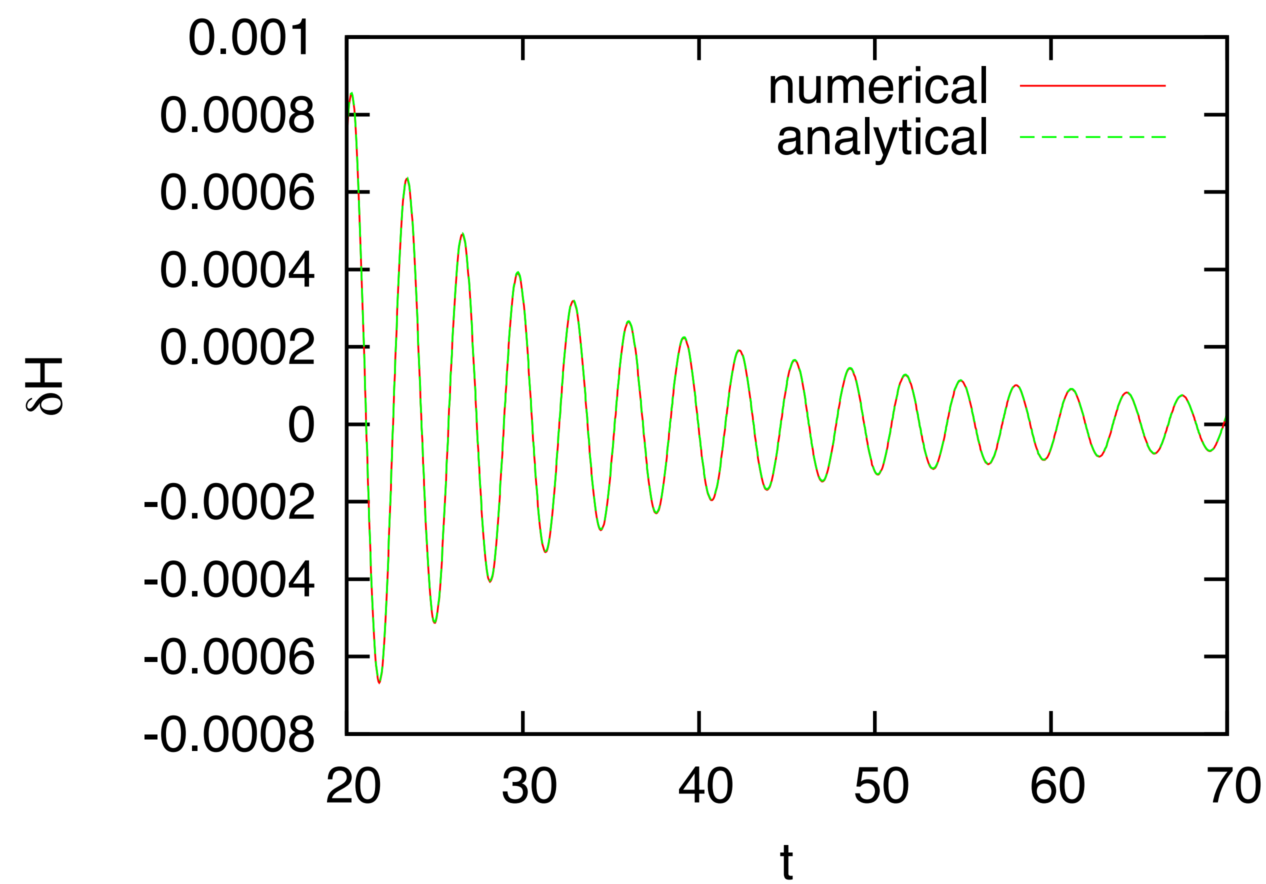
$$\boxed{\delta H \simeq -\frac{1}{4}\frac{\varphi\dot{\varphi}}{M_P^2} \qquad a(t) \simeq \langle a(t) \rangle \left(1 - \frac{\varphi^2 - \langle \varphi^2 \rangle}{8 M_P^2}\right)}$$

Numerical calculation of Hubble parameter

H



δH



Hubble parameter & scale factor contain rapidly oscillating part

$$\frac{\delta \rho_\phi}{\rho_\phi} \sim \frac{\delta H}{H} \sim \mathcal{O}\left(\frac{\varphi}{M_P}\right) \qquad \frac{\delta a}{a} \sim \mathcal{O}\left(\frac{\varphi^2}{M_P^2}\right)$$

Ema, Jinno, Mukaida, KN (2015)

Scalar particle production

- Action of minimal scalar

$$\boxed{S = \int d\tau d^3x \frac{a^2(\tau)}{2}\left[\chi'^2 - (\nabla\chi)^2 - a^2 m_\chi^2 \chi^2\right]}$$

- Canonical field $\widetilde{\chi} \equiv a\chi$

$$\boxed{S = \int d\tau d^3x \frac{1}{2}\left[\widetilde{\chi}'^2 - (\partial_i\widetilde{\chi})^2 - m_\chi^{(\mathrm{eff})2}\widetilde{\chi}^2\right]}$$

Effective mass: $m_\chi^{(\mathrm{eff})2} = a^2 m_\chi^2 - \frac{a''}{a} = a^2\left(m_\chi^2 - 2H^2 + \frac{\dot{\phi}^2}{2M_P^2}\right)$

Time dependent mass just from gravitational effect

- Abundance of scalar particle

$$m_\chi^{(\mathrm{eff})2} = \langle a^2 \rangle \left[m_\chi^2 - 2\langle H \rangle^2 - \left(m_\chi^2 - 2\langle H \rangle^2 \right) \frac{\varphi^2}{4M_P^2} + \langle H \rangle \frac{\varphi\dot{\varphi}}{M_P^2} + \frac{\dot{\varphi}^2}{2M_P^2} \right]$$

- "Fast" contribution from inflaton "annihilation"

$$\boxed{n_\chi^{(\mathrm{fast})}(t) \sim \mathcal{C} H_{\mathrm{inf}}^3 \left(\frac{a(t_{\mathrm{end}})}{a(t)} \right)^3}$$

$$\Gamma(\phi\phi \to \chi\chi) \sim \frac{\mathcal{C}}{16\pi} \frac{\Phi^2}{M_P^2} \frac{m_\phi^3}{M_P^2}$$

Φ : oscillation amplitude

t_{end} : end of inflaton

Ema, Jinno, Mukaida, KN (2015); Ema, KN, Tang (2018); Chung, Kolb, Long (2018)

- "Slow" contribution: produced by Hubble expansion

$$\boxed{n_\chi^{(\mathrm{slow})}(t) \sim H_{\mathrm{inf}}^3 \, e^{-m/H_{\mathrm{inf}}} \left(\frac{a(t_{\mathrm{end}})}{a(t)} \right)^3}$$

Ford (1986)

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Ford (1986)

Comments

- Generation of superhorizon fluctuation of light scalar during inflation is understood in the same way.

$$m_\chi^{(\mathrm{eff})2} \simeq a^2(m_\chi^2 - 2H_{\mathrm{inf}}^2)$$

Superhorizon modes $k/a \lesssim H_{\mathrm{inf}}$ are enhanced.

- "Gravitational particle production" often refers to "slow" contribution at transition from dS to MD/RD

It is suppressed as $e^{-m_\chi/H_{\mathrm{inf}}}$ for $m_\chi \gg H_{\mathrm{inf}}$

Ford (1986), Chung, Kolb, Riotto (1999)

- Gravitational production from inflaton oscillation is efficient even for $m_\chi \gg H_{\mathrm{inf}}$ (as far as $m_\chi < m_\phi$)

Graviton production

- Graviton action is the same as minimal massless scalar

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j \qquad \partial_i h_{ij} = h_{ii} = 0$$

$$\boxed{S = \int d\tau d^3x\, a^2(t) \frac{M_P^2}{8} \left[\left(\frac{\partial h_{ij}}{\partial \tau} \right)^2 - (\partial_k h_{ij})^2 \right]}$$

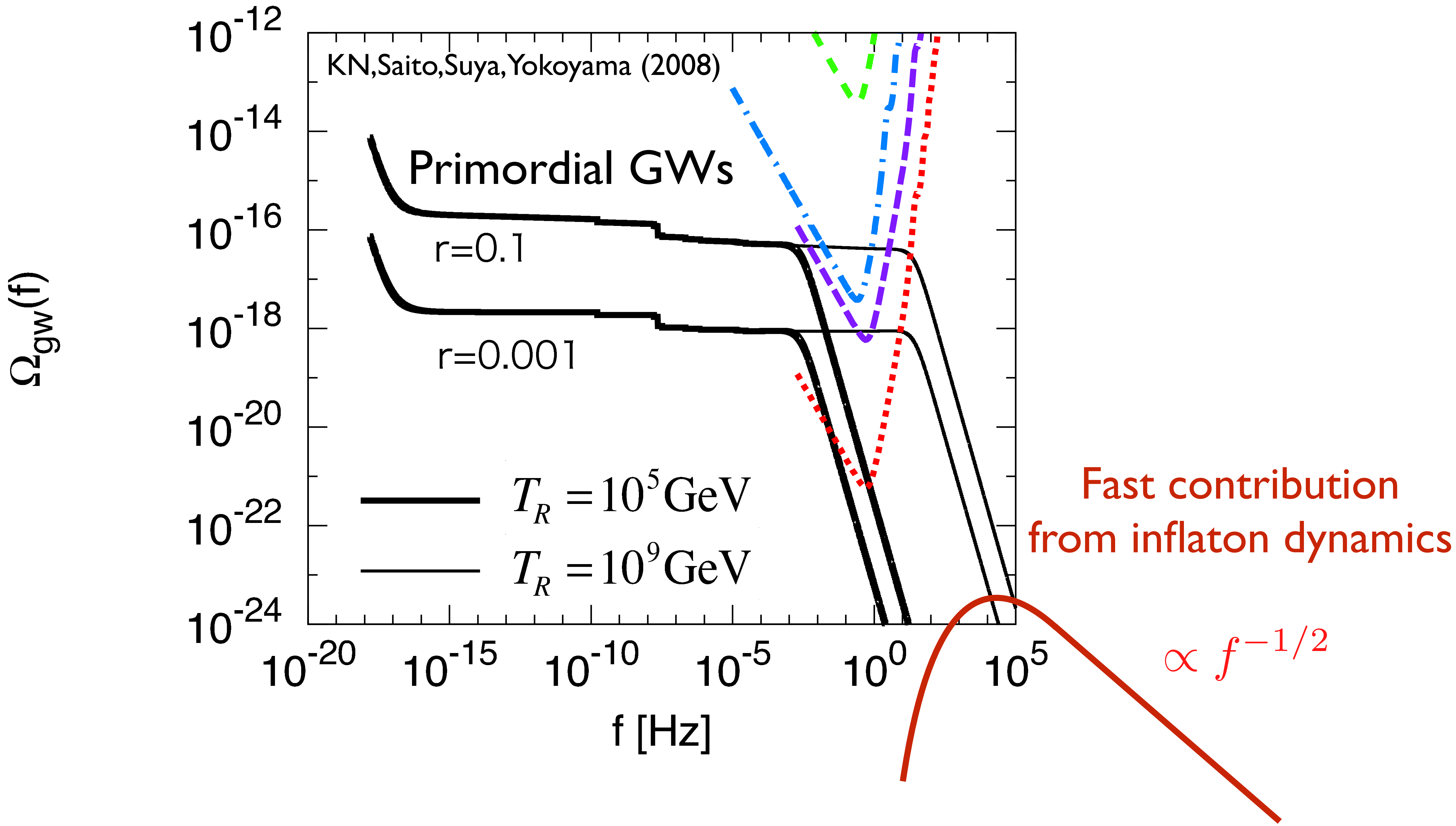
- Production rate of graviton is similar to scalar

$$\widetilde{h}_{ij} \equiv a h_{ij} \qquad m^{(\mathrm{eff})2} = -\frac{a''}{a} = a^2 \left(-2H^2 + \frac{\dot{\phi}^2}{2M_P^2} \right)$$

"Slow" Superhorizon mode ("primordial GW")

"Fast" High frequency GWs induced by inflaton oscillation

Stochastic GW spectrum



$$f_{\rm GW} \simeq \frac{m_\phi a(H_{\rm inf})}{2\pi} \simeq 2\times 10^5\,{\rm Hz}\left(\frac{m_\phi}{10^{13}\,{\rm GeV}}\right)\left(\frac{T_{\rm R}}{10^{10}\,{\rm GeV}}\right)^{1/3}\left(\frac{H_{\rm inf}}{10^{14}\,{\rm GeV}}\right)^{-2/3}$$

Ema, Jinno, Mukaida, KN (2015)

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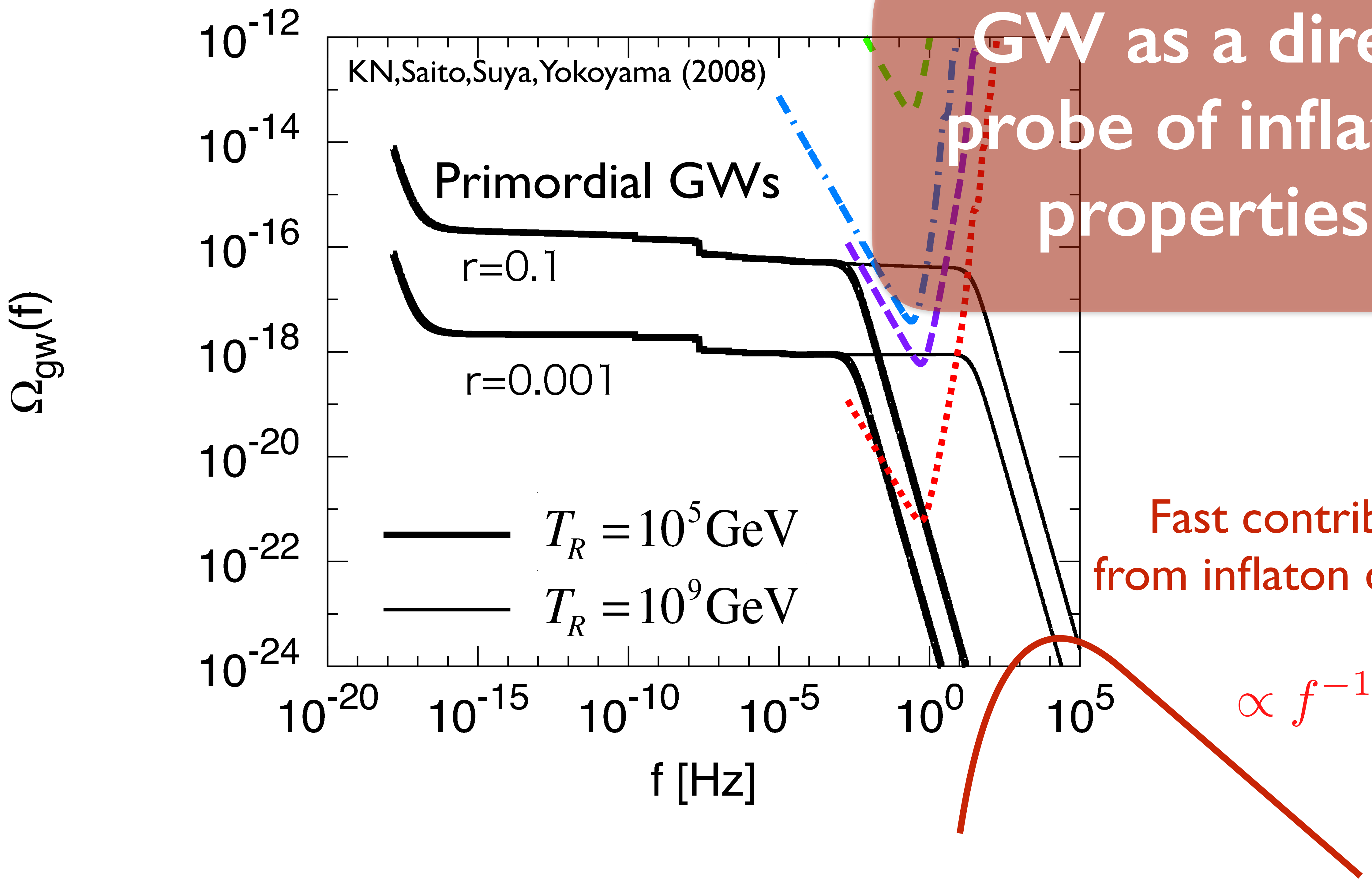

GW as a direct probe of inflaton properties!

Fast contribution from inflaton dynamics

$\propto f^{-1/2}$

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Purely Gravitational Dark Matter

Models of dark matter

- WIMP DM

SUSY neutralino	Weak SU(2), sfermion exchange
Z2 scalar	Higgs-portal coupling

- Light particle

Sterlile neutrino	Mixing with active neutrino
Axion	Anomalous interaction suppressed by PQ scale
Hidden photon	Kinetic mixing with photon

- FIMP, SIMP, …

- Purely Gravitational DM (PGDM) Only gravitational interaction

1. Scalar PGDM

2. Fermion PGDM

3. Vector PGDM

1. Scalar PGDM

- Real scalar field interacting only through gravity

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}m_\chi^2\chi^2\right)$$

- Several production mechanisms of PGDM
  - Thermal scattering of SM particle with graviton exchange

    Garny, Sandora, Sloth (2015); Tang, Wu (2016)

    $$\mathrm{SM} + \mathrm{SM} \to \mathrm{graviton} \to \chi\chi$$

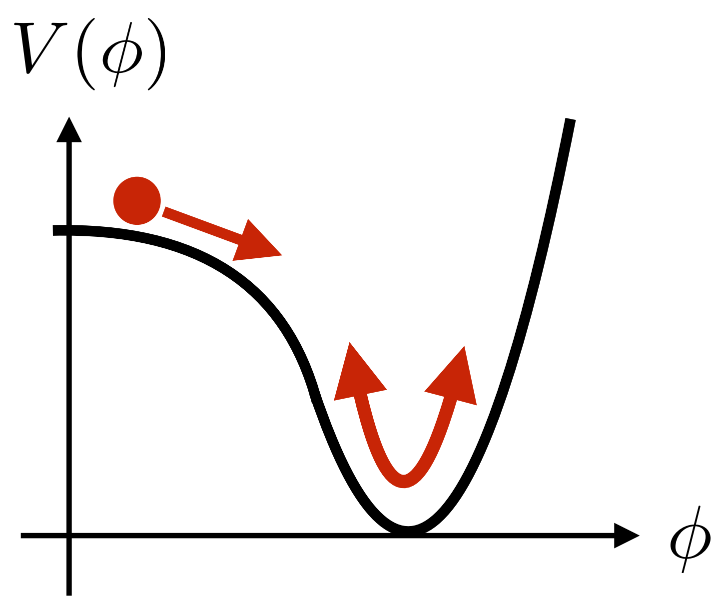
  - Gravitational particle production   Ema, KN, Tang (2018)

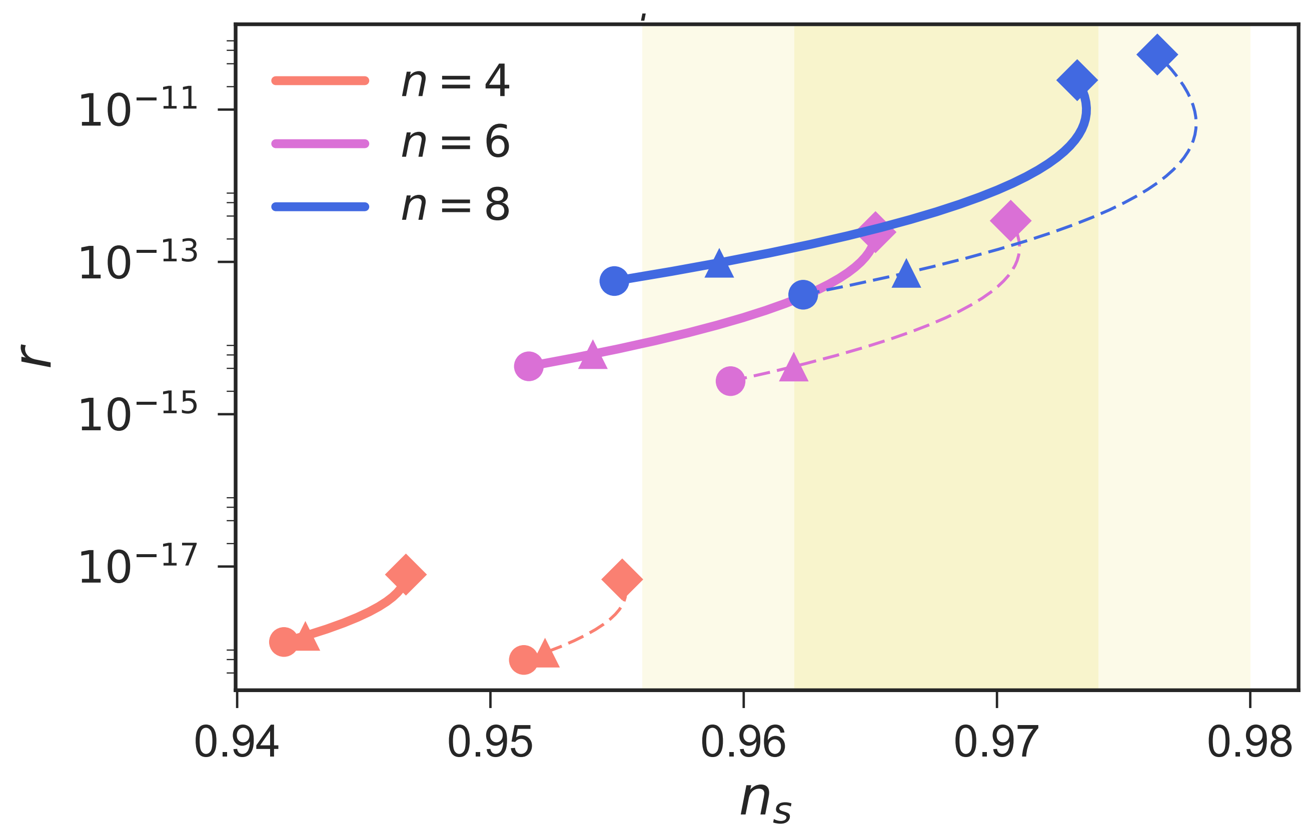
    In most cases, this is larger than thermal production.

Concrete Calculation

- **New inflation**

$$V(\phi) = M^4\left[1 - \left(\frac{\phi}{v_\phi}\right)^n\right]^2$$



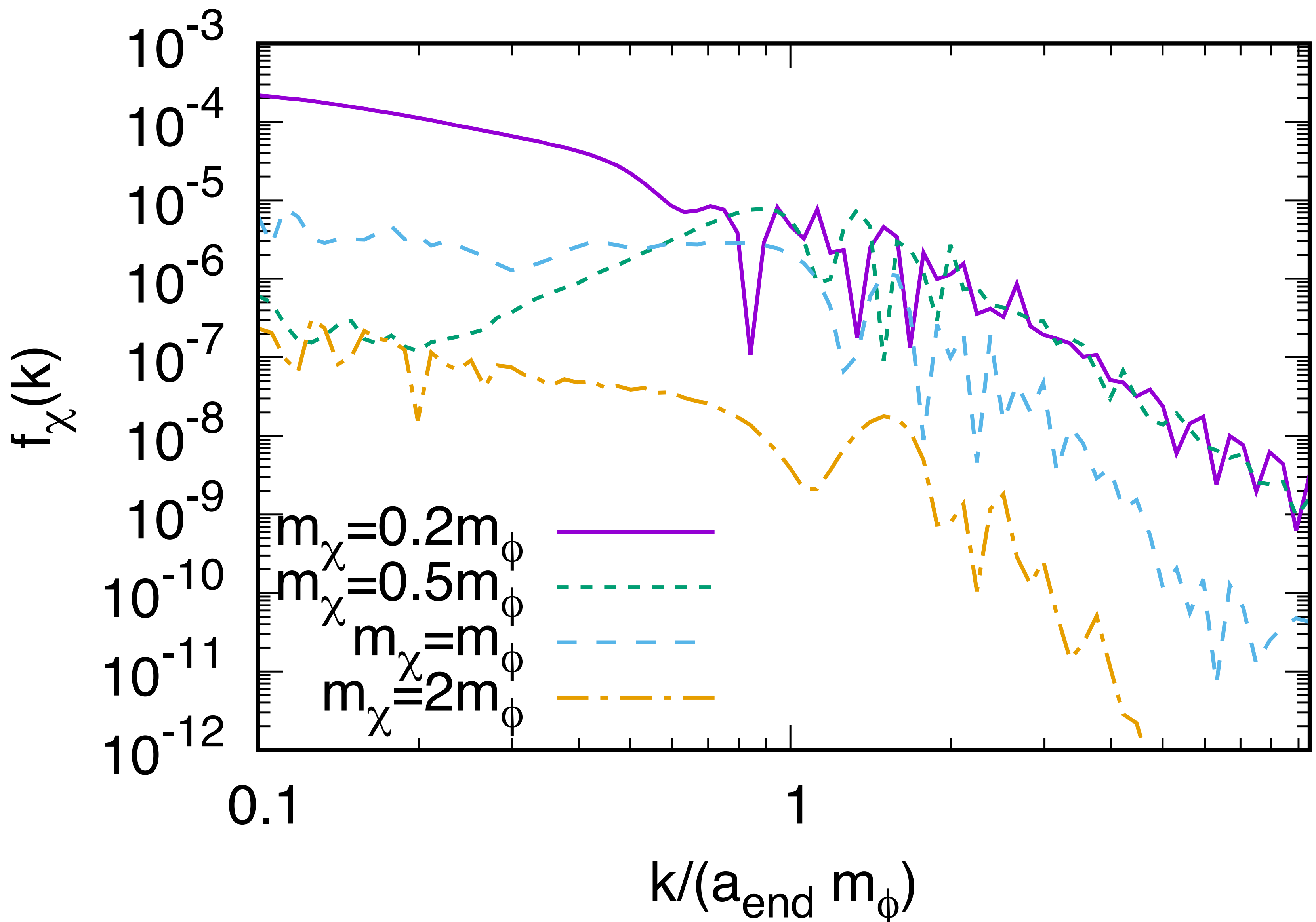


- $\chi_k(\tau) = \alpha_k(\tau)v_k(\tau) + \beta_k(\tau)v_k^*(\tau),\qquad v_k(\tau) \equiv \frac{1}{\sqrt{2\omega_k}}\exp\left(-i\int \omega_k d\tau\right)$

- EoM: $\alpha_k' v_k = \frac{\omega_k'}{2\omega_k} v_k^* \beta_k,\qquad \beta_k' v_k^* = \frac{\omega_k'}{2\omega_k} v_k \alpha_k$

- We can numerically evaluate $f_\chi(k) = |\beta_k|^2$ given inflation model

- Phase space distribution well after inflation



Y.Ema, KN, Y.Tang (2018)

$$v_\phi = 0.5 M_P$$

$$\left(\frac{m_\phi}{H_{\rm inf}} \sim 30\right)$$

$a_{\rm end}$: scale factor at the end of inflation

Energy density is peaked around $k \sim a_{\rm end} m_\phi$

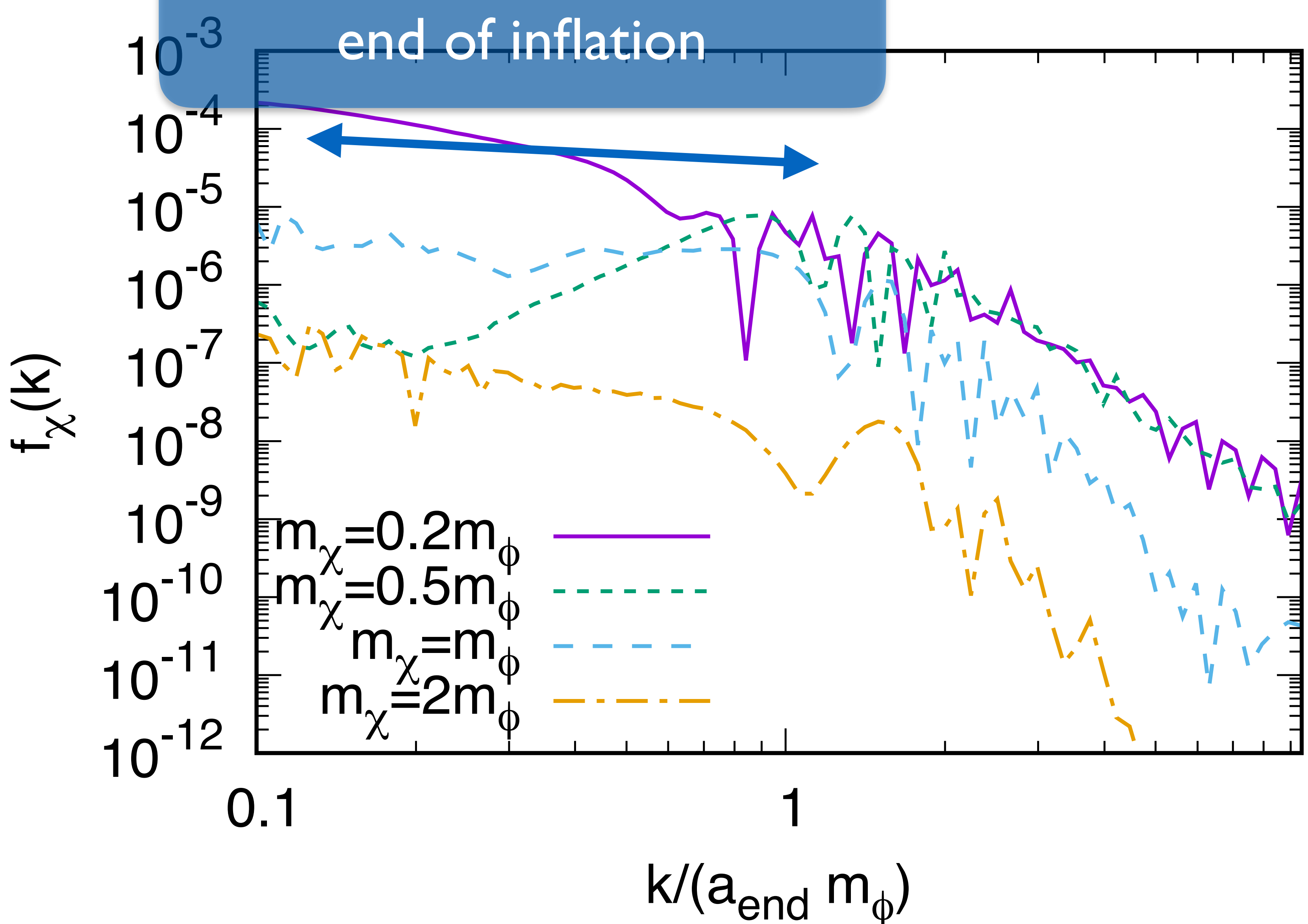
$$a^4 \rho_\chi = \int \frac{d^3k}{(2\pi)^3} \omega_k f_\chi(k)$$

$$\frac{\rho_\chi^{\rm (GP)}}{s} \sim \frac{\mathcal{C}}{4} \frac{m_\chi H_{\rm inf} T_{\rm R}}{M_P^2} \simeq 3 \times 10^{-10}\,{\rm GeV}\,\mathcal{C} \left(\frac{m_\chi}{10^9\,{\rm GeV}}\right) \left(\frac{H_{\rm inf}}{10^9\,{\rm GeV}}\right) \left(\frac{T_{\rm R}}{10^{10}\,{\rm GeV}}\right)$$

- Phase space distribution well after inflation

Production around the end of inflation

Y.Ema, KN, Y.Tang (2018)



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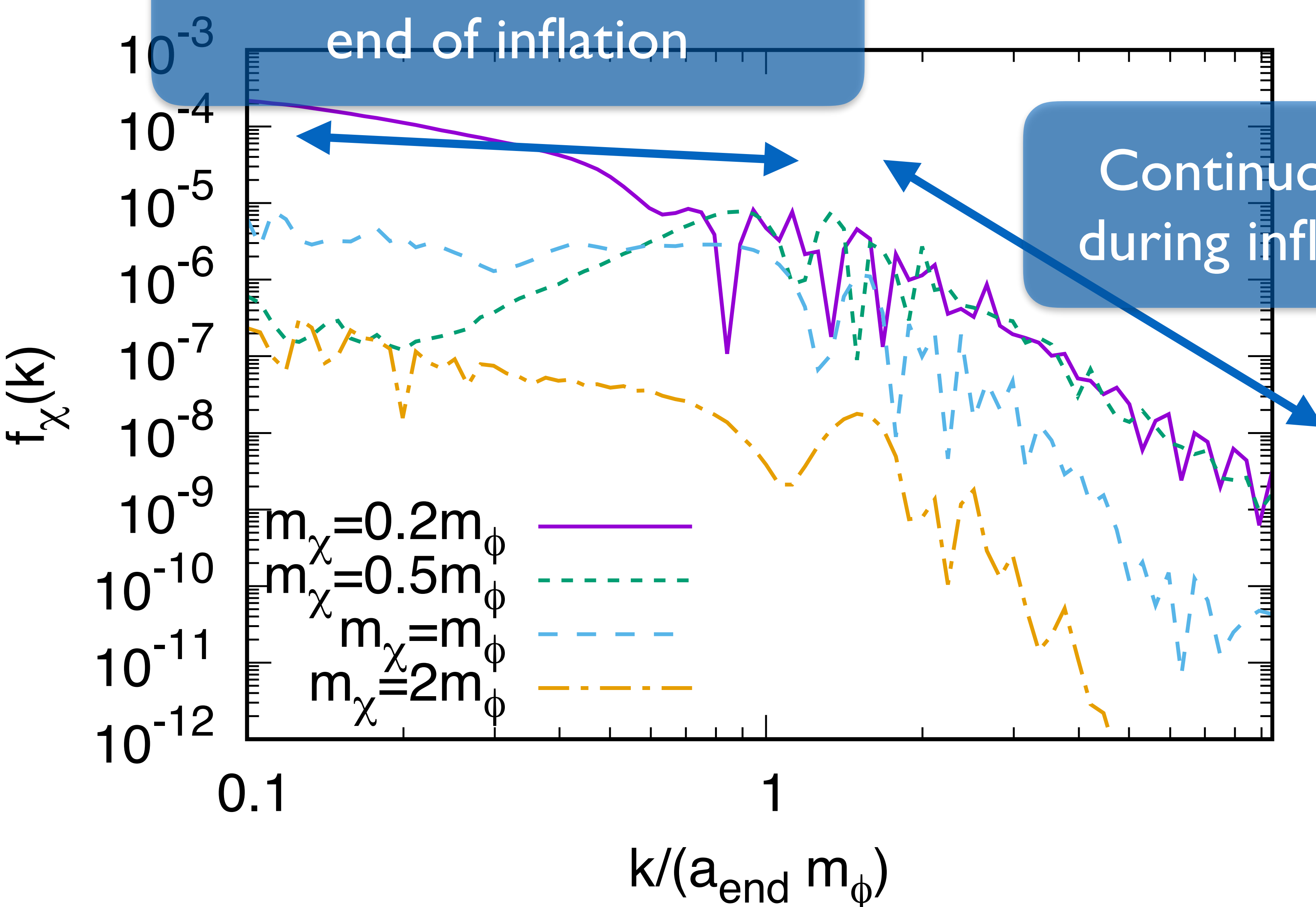

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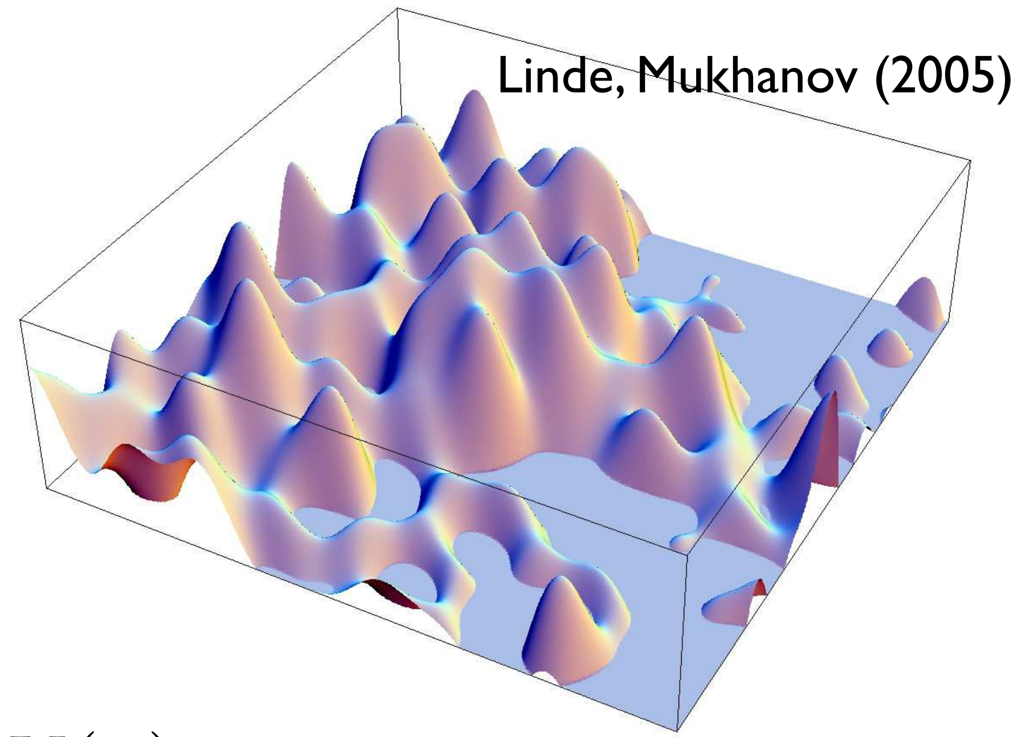
PGDM from coherent oscillation



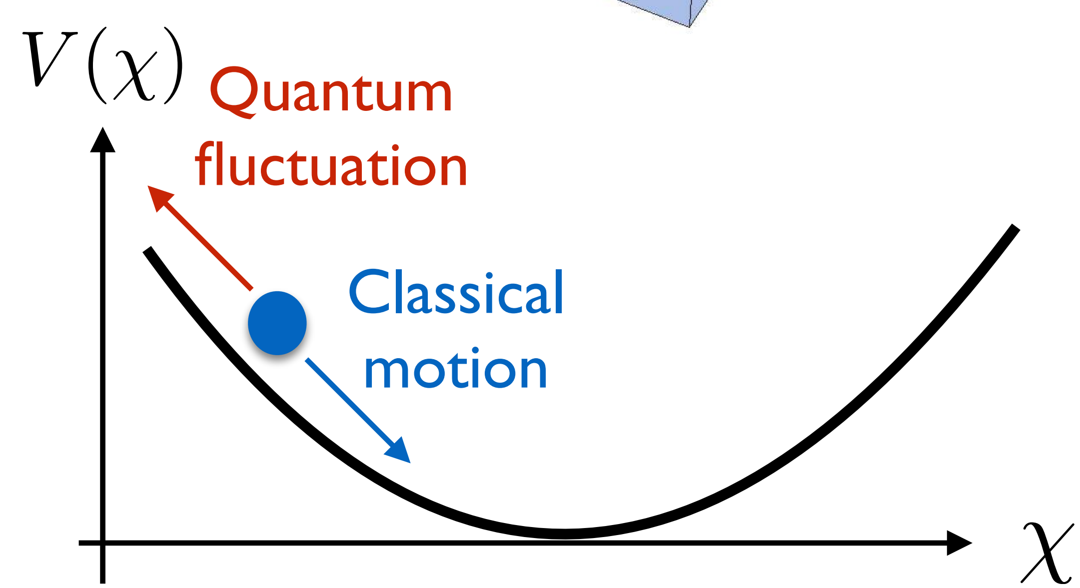
- $\omega_k^2 \simeq k^2 + a^2(m_\chi^2 - 2H_{\rm inf}^2)$

  Superhorizon modes are enhanced during inflation if $m_\chi \lesssim H_{\rm inf}$

- Field value is saturated at

$$\langle \chi^2 \rangle \simeq \frac{3H_{\rm inf}^4}{8\pi^2 m_\chi^2}$$

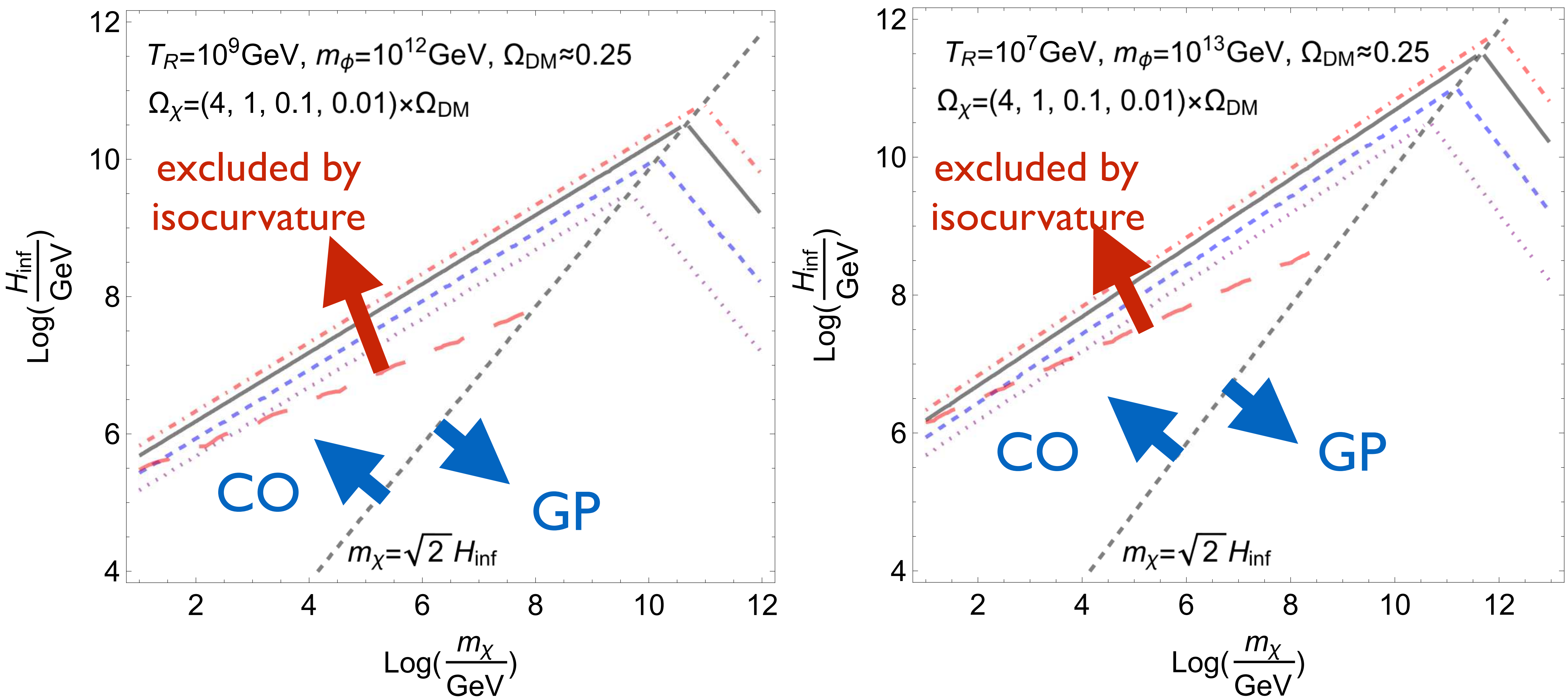
- Coherent oscillation after inflation



$$\boxed{\frac{\rho_\chi^{(\rm CO)}}{s} \simeq \frac{T_{\rm R}}{8}\frac{\langle\chi^2\rangle}{M_P^2} \simeq 8\times 10^{-12}\,{\rm GeV}\left(\frac{H_{\rm inf}}{10^9\,{\rm GeV}}\right)^4\left(\frac{10^9\,{\rm GeV}}{m_\chi}\right)^2\left(\frac{T_{\rm R}}{10^{10}\,{\rm GeV}}\right)}$$

NOTE: isocurvature constraint is severe

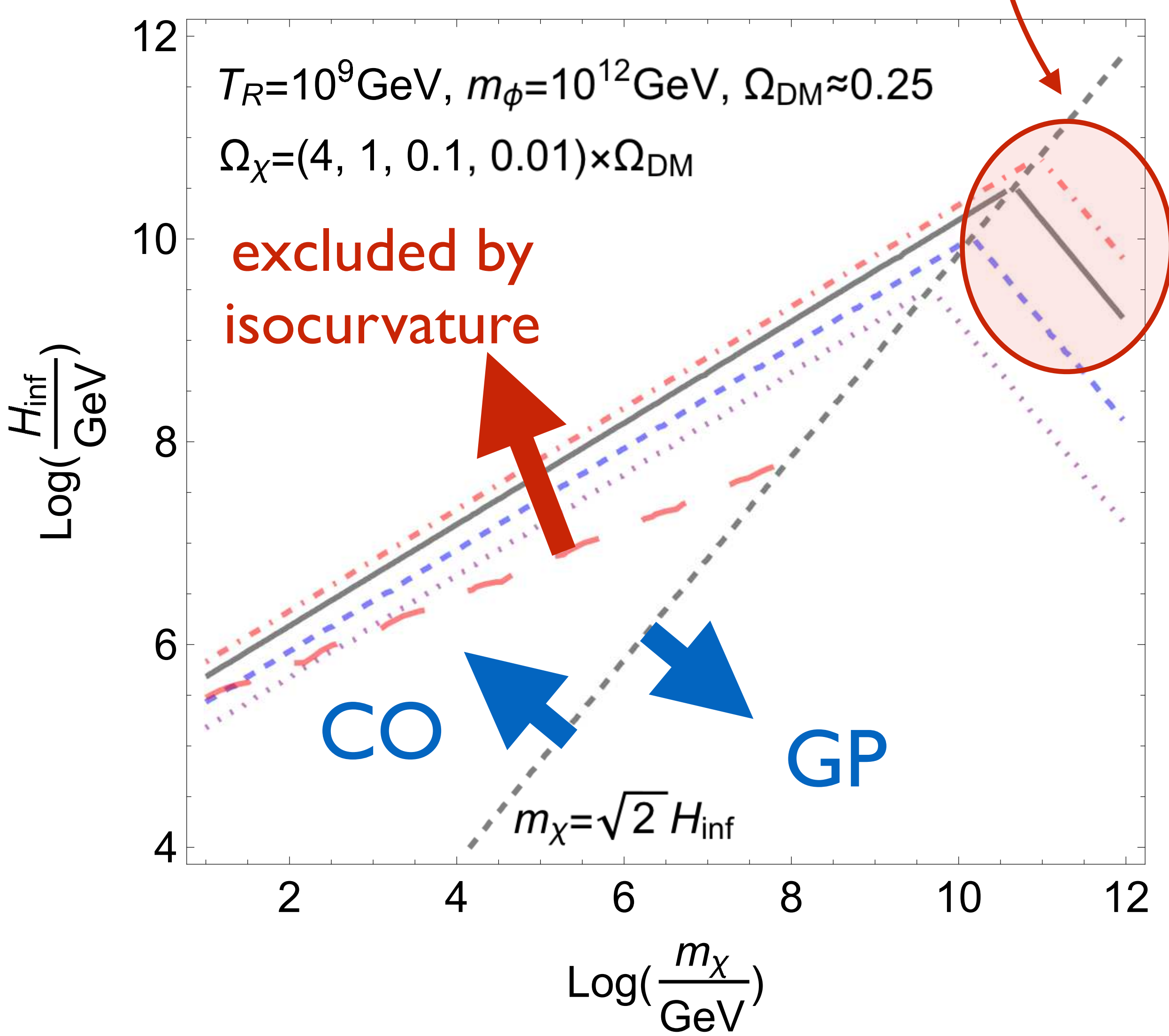
Contour of scalar PGDM abundandce

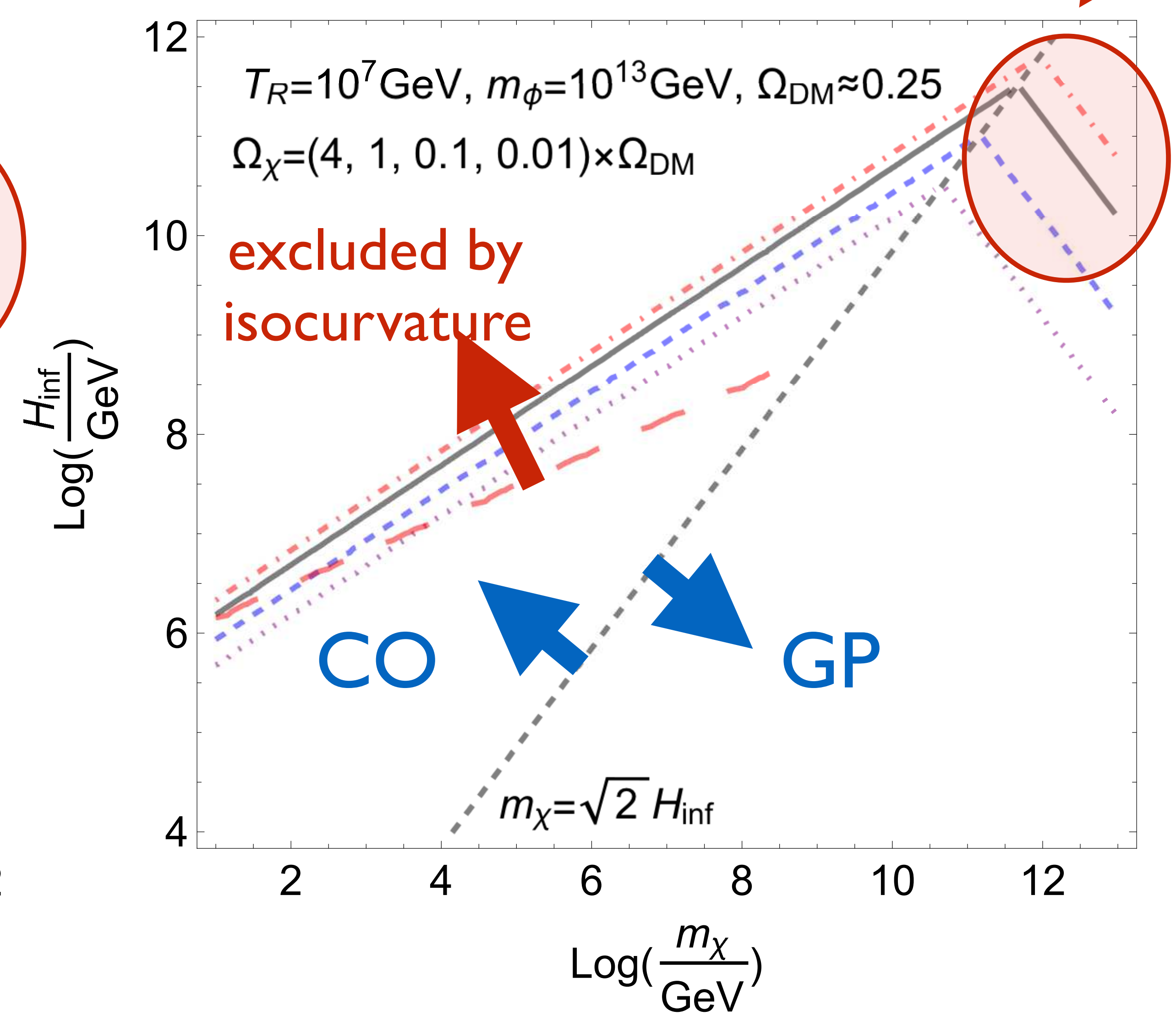


Y.Ema, KN, Y.Tang (2018)

Contour of scalar PGDM abundandce

Region of consistent PGDM abundance





Y.Ema, KN, Y.Tang (2018)

2. Fermion PGDM

- Fermion may be more natural candidate of PGDM

E.g.) Renormalizable interaction with SM is naturally forbidden,
Stability is ensured by Z2(B-L) for B-L singlet fermion.

- Free fermion minimally coupled to gravity

$$S = \int d^4x\, e \left[-\frac{1}{2} \overline{\psi} \left(e^\mu_a \gamma^a D_\mu - m \right) \psi \right]$$

- Canonical fermion $\widetilde{\psi} \equiv a^{3/2} \psi$

$$S = \int d\tau d^3x \left[-\frac{1}{2} \overline{\widetilde{\psi}} \left(\delta^\mu_a \gamma^a \partial_\mu - am \right) \widetilde{\psi} \right]$$

It does not "feel" gravity in the massless limit $m \to 0$
(a fermion is conformal in massless limit)

Formalism to calculate fermion production

- Decomposition: $\psi_{\vec{k}}(\tau) = \sum_{h=\pm} \left[u_{\vec{k},h}(\tau) b_{\vec{k},h} + v_{\vec{k},h}(\tau) b^{\dagger}_{-\vec{k},h} \right]$ $\quad \left\{ b_{\vec{k},h}, b^{\dagger}_{\vec{k}',h'} \right\} = (2\pi)^3 \, \delta(\vec{k} - \vec{k}') \delta_{hh'}$

- EoM: $\partial_\tau^2 u^{\pm}_{\vec{k},h}(\tau) + \left[\omega_k^2(\tau) \pm i(am)' \right] u^{\pm}_{\vec{k},h}(\tau) = 0, \qquad \omega_k^2(\tau) \equiv k^2 + a^2 m^2$

- Useful parametrization:

$$u^{+}_{\vec{k},h}(\tau) = A_{k,h}(\tau) g_+ e^{-i \int^\tau \omega_k(\tau') d\tau'} + B_{k,h}(\tau) g_- e^{i \int^\tau \omega_k(\tau') d\tau'} \qquad g_{\pm} \equiv \sqrt{(\omega_k \pm am)/(2\omega_k)}$$

- EoM becomes:

$$A'_{k,h}(\tau) = -\frac{g'_-}{g_+} e^{2i \int^\tau \omega_k(\tau') d\tau'} B_{k,h}(\tau), \qquad B'_{k,h}(\tau) = -\frac{g'_+}{g_-} e^{-2i \int^\tau \omega_k(\tau') d\tau'} A_{k,h}(\tau)$$

- Phase space density:

$$f_\psi(\vec{k}, \tau) = |B_k(\tau)|^2 \qquad a^3(\tau) n_\psi(\tau) = 2 \int \frac{d^3k}{(2\pi)^3} f_\psi(k, \tau)$$

- Fermion abundance

$$m_\psi^{(\mathrm{eff})} = am \simeq \langle a(t) \rangle\, m \left(1 - \frac{\varphi^2}{8M_P^2}\right)$$

- "Fast" contribution from inflaton annihilation

$$\Gamma(\phi\phi \to \psi\psi) \sim \frac{\mathcal{C}}{16\pi} \frac{\Phi^2}{M_P^2} \frac{m_\phi^3}{M_P^2} \left(\frac{m}{m_\phi}\right)^2$$

$$\boxed{n_\psi^{(\mathrm{fast})}(t) \sim \mathcal{C} H_{\mathrm{inf}}^3 \left(\frac{m}{m_\phi}\right)^2 \left(\frac{a(t_{\mathrm{end}})}{a(t)}\right)^3}$$

Y.Ema, KN, Y.Tang (2019)

- "Slow" contribution from Hubble expansion

$$\boxed{n_\psi^{(\mathrm{slow})}(t) \sim m H_{\mathrm{inf}}^2\, e^{-m/H_{\mathrm{inf}}} \left(\frac{a(t_{\mathrm{end}})}{a(t)}\right)^3}$$

Chung, Everett, Yoo, Zhou (2011)

- Fermion abundance

$$m_\psi^{(\mathrm{eff})} = am \simeq \langle a(t) \rangle\, m \left(1 - \frac{\varphi^2}{8M_P^2}\right)$$

"Slow" part

- "Fast" contribution from inflaton annihilation

$$\Gamma(\phi\phi \to \psi\psi) \sim \frac{\mathcal{C}}{16\pi} \frac{\Phi^2}{M_P^2} \frac{m_\phi^3}{M_P^2} \left(\frac{m}{m_\phi}\right)^2$$

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Y. Ema, KN, Y. Tang (2019)

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$$m_\psi^{(\mathrm{eff})} = am \simeq \langle a(t) \rangle m \left(1 - \frac{\varphi^2}{8M_P^2}\right)$$

"Slow" part "Fast" part

- "Fast" contribution from inflaton annihilation

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Y. Ema, KN, Y. Tang (2019)

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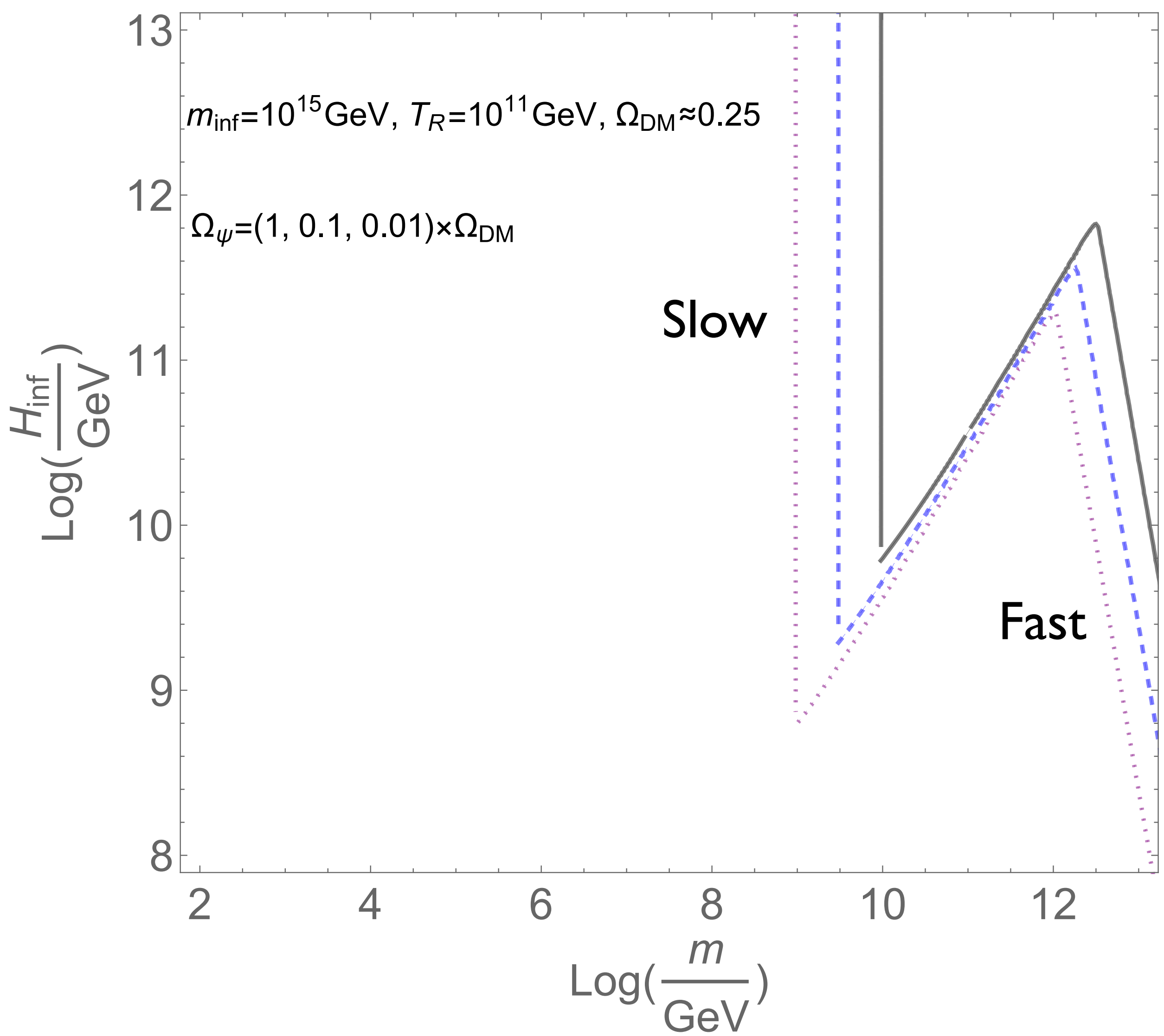
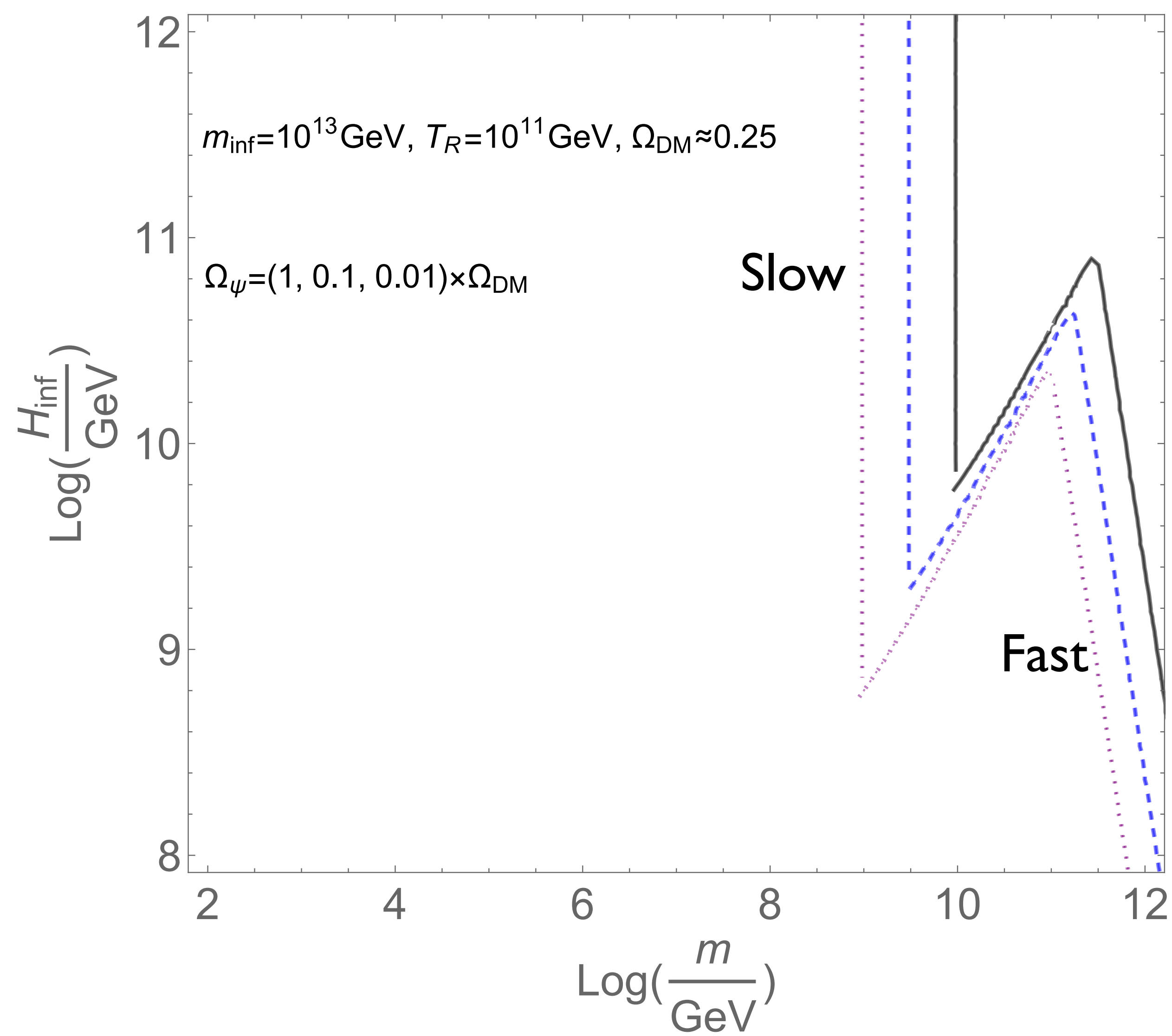
Chung, Everett, Yoo, Zhou (2011)

Contour of fermion abundance





There is lower bound on fermion mass
in order for it to become DM

Y.Ema, KN, Y.Tang (2019)

3. Vector PGDM

- Hidden vector boson is also a candidate of DM if kinetic mixing with photon is small enough.
- Massive vector boson minimally coupled to gravity

$$\boxed{S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right]}$$

- Transverse-longitudinal decomposition

$$\vec{A} = \vec{A}_T + \hat{k} A_L \qquad \vec{k} \cdot \vec{A}_T = 0$$

Graham, Mardon, Rajendran (2015)

$$S = S_T + S_L,$$

$$S_T = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \vec{A}_T|^2 - (k^2 + a^2 m^2) |\vec{A}_T|^2 \right),$$

$$S_L = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(\frac{a^2 m^2}{k^2 + a^2 m^2} |\partial_\tau A_L|^2 - a^2 m^2 |A_L|^2 \right)$$

Transverse mode

$$S_T = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \vec{A}_T|^2 - (k^2 + a^2 m^2)|\vec{A}_T|^2 \right)$$

- Conformal in the massless limit
  (similar to massive conformal scalar)

- Inflaton "annihilation" rate: $\Gamma(\phi\phi \to A_T A_T) \sim \frac{\mathcal{C}}{16\pi} \frac{\Phi^2}{M_P^2} \frac{m_\phi^3}{M_P^2} \left(\frac{m}{m_\phi} \right)^4$

- Produced number density:

$$n_{A_T}(t) \simeq H_{\rm inf}^3 \left[\mathcal{C}_T \frac{m^4}{m_{\rm inf}^4} + \eta \frac{m}{H_{\rm inf}} \right] \left(\frac{a(t_{\rm end})}{a(t)} \right)^3$$

Subdominant compared with longitudinal mode

Longitudinal mode

$$S_L = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \widetilde{A_L}|^2 - \omega_L^2 |\widetilde{A_L}|^2 \right) \qquad \omega_L^2 \equiv k^2 + m_L^2$$

$$m_L^2 = a^2 m^2 - \frac{k^2}{k^2 + a^2 m^2} \left(\frac{a''}{a} - \frac{a'^2}{a^2} \frac{3a^2 m^2}{k^2 + a^2 m^2} \right)$$

- High momentum limit: $m_L^2 \simeq a^2 m^2 - \frac{a''}{a}$

  Same as minimal scalar (note the Goldstone picture)

$$\Gamma(\phi\phi \to A_L A_L) \sim \frac{\mathcal{C}}{16\pi} \frac{\Phi^2}{M_P^2} \frac{m_\phi^3}{M_P^2}$$

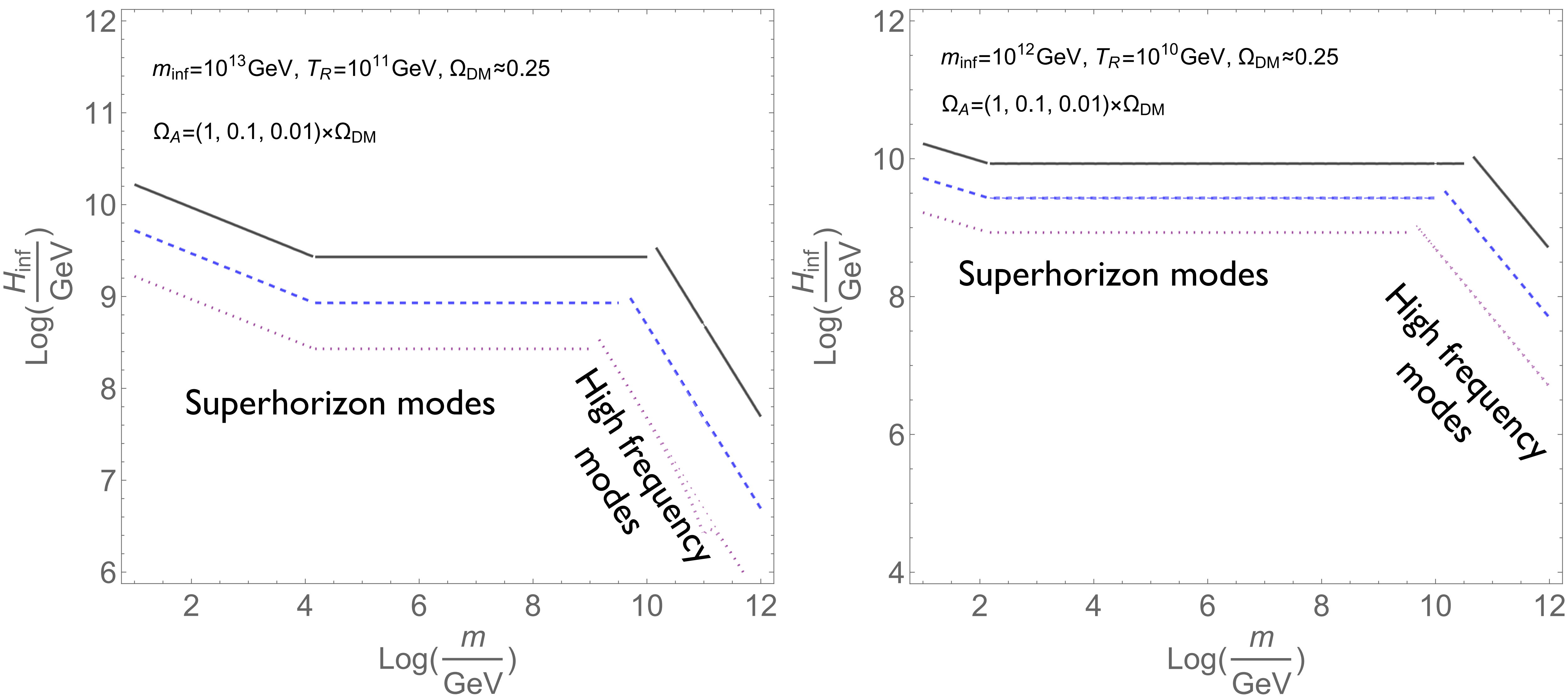
- Superhorizon modes are also produced if $m \lesssim H_{\rm inf}$

  Modes with $m \lesssim k/a \lesssim H_{\rm inf}$ are enahnced.

  $\longrightarrow$ Isocurvature constraint can be easily avoided

Graham, Mardon, Rajendran (2015)

Contour of vector boson abundance



Vector PGDM is possible for
wide range of mass & inflation scale

Y.Ema, KN, Y.Tang (2019)

Gravitational particle production in extended gravity

Extended gravity

- Einstein gravity needs not be exactly true
  - Starobinsky inflation: $$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \xi R^2\right)$$ Starobinsky (1980)
  - Higgs inflation: $$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \xi |H|^2 R\right)$$ Bezrukov, Shaposhnikov (2009)
  - New Higgs inflation: $$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{G^{\mu\nu}}{M^2}D_\mu H^\dagger D_\nu H\right)$$ Germani, Kehagias (2010)
- Expansion law is modified

  $\longrightarrow$ Gravitational production rate is modified
- Particle production can be a probe of gravity theory

Background evolution in f(Φ)R gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

- Friedmann equation:

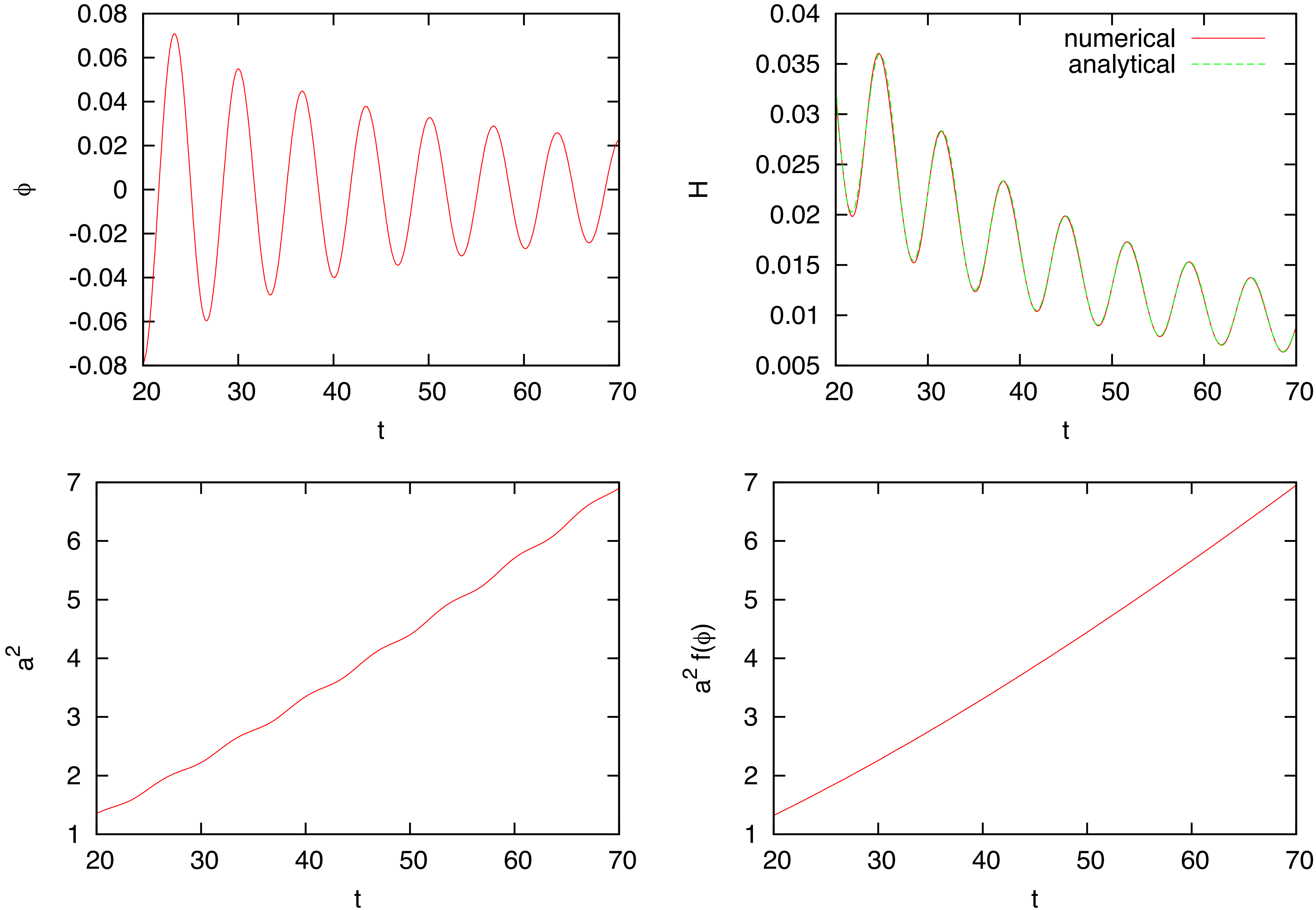
$$3fH^2 = \rho_\phi - 3H\dot{f}$$

$$(3H^2 + 2\dot{H})f + \ddot{f} + 2H\dot{f} = -\frac{1}{2}\dot{\phi}^2 + V.$$

- Assume: $f(\phi) = M_P^2\left(1 + c_1\frac{\phi}{M_P} + \cdots\right)$

$$H = H_0 + H_1 \qquad \phi = \phi_0 + \phi_1$$ (Einstein + small deviation)

$$\longrightarrow \quad \boxed{H_1 \simeq -\frac{c_1}{2M_P}\dot{\phi}_0 \qquad a(t) \simeq a_0(t)\left(1 - \frac{c_1}{2}\frac{\phi}{M_P}\right)}$$



Ema, Jinno, Mukaida, KN (2015)

- ## Scalar paticle production

$$S = \int d\tau d^3x \frac{a^2(\tau)}{2}\left[\chi'^2 - (\nabla\chi)^2 - a^2 m_\chi^2 \chi^2\right] \qquad a(t) \simeq a_0(t)\left(1 - \frac{c_1}{2}\frac{\phi}{M_P}\right)$$

Inflaton decay rate: $\Gamma(\phi \to \chi\chi) = \dfrac{c_1^2 m_\phi^3}{128\pi M_P^2}$

(cf). Inflaton "annihilation" rate in Einstein gravity

$$\Gamma(\phi\phi \to \chi\chi) \simeq \frac{\mathcal{C}}{32\pi}\frac{\Phi^2}{M_P^2}\frac{m_\phi^3}{M_P^2} \qquad a(t) \simeq \langle a(t)\rangle\left(1 - \frac{\varphi^2 - \langle\varphi^2\rangle}{8M_P^2}\right)$$

)

- ## Graviton paticle production

$$S = \int d\tau d^3x \, a(t)^2 f(\phi) \frac{1}{8}\left[h_{ij}'^2 - (\partial_l h_{ij})^2\right] \qquad a^2(t) f(\phi) \simeq a_0^2(t)$$

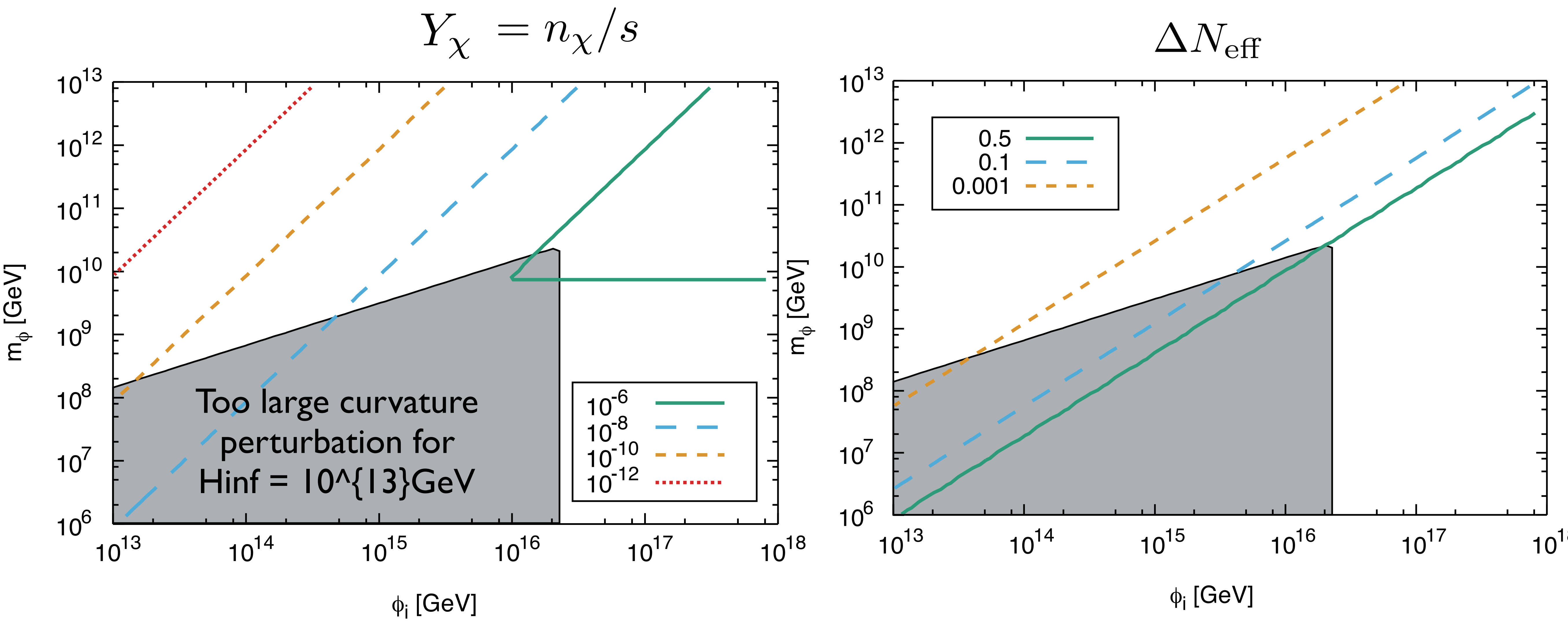
Production rate is same as Einstein gravity

(Note that f(Φ)R is conformally equivalent to Einstein gravity)

Scalar abundance in f(Φ) theory

Here we assume that Φ is subdominant component of the Universe.

Light particle abundnace is determined just by the branching ratio.



Ema, Jinno, Mukaida, KN (2016)

Background evolution in derivative coupling

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2}\right)\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

- Friedmann equation and EoM:

$$H^2 = \frac{\rho_\phi}{3M_P^2} \qquad \rho_\phi \equiv \left(1 + \frac{9H^2}{M^2}\right)\frac{\dot\phi^2}{2} + V,$$

$$3H^2 + 2\dot H = -\frac{p_\phi}{M_P^2} \qquad p_\phi \equiv \left(1 - \frac{3H^2}{M^2}\right)\frac{\dot\phi^2}{2} - V - \frac{1}{M^2}\frac{d}{dt}(H\dot\phi^2)$$

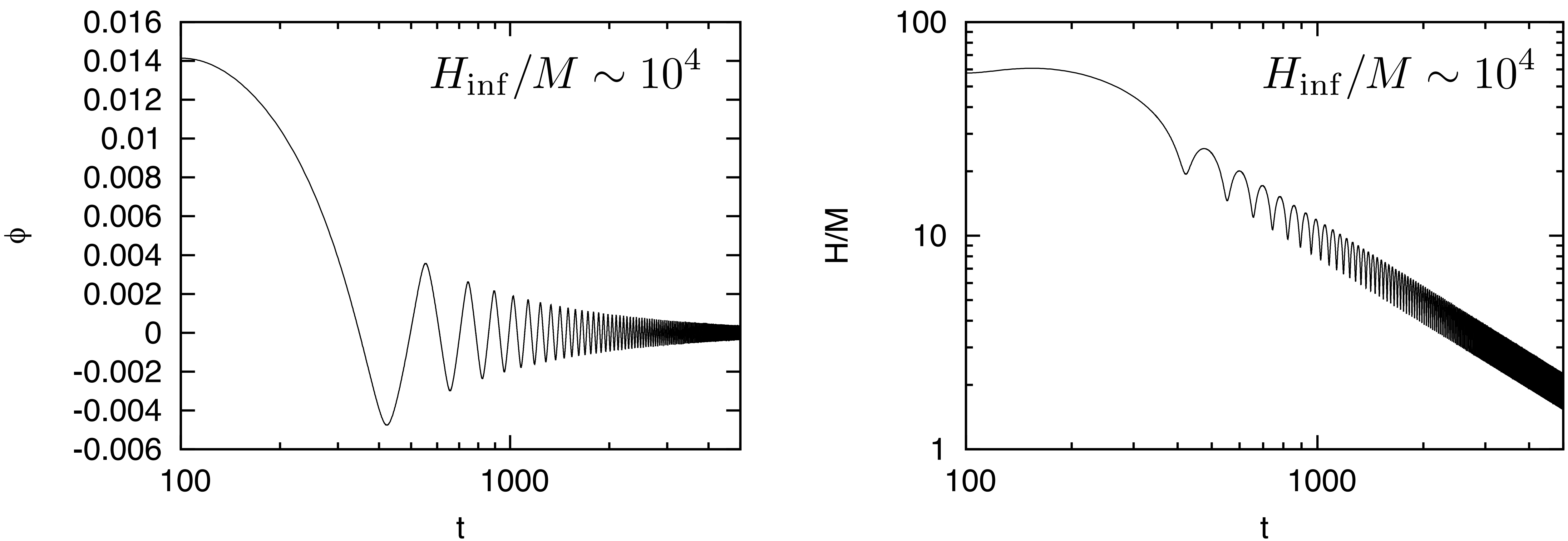
$$\left(1 + \frac{3H^2}{M^2}\right)\ddot\phi + 3H\left(1 + \frac{3H^2}{M^2} + \frac{2\dot H}{M^2}\right)\dot\phi + \frac{\partial V}{\partial\phi} = 0.$$

- Case 1: H << M Similar to Einstein gravity

  Case 2: H >> M $|p_\phi| \sim \mathcal{O}(m_\phi\rho_\phi/H) \gg \rho_\phi \longrightarrow |\dot H| \sim m_\phi H (\gg H^2)$

H is violently oscillating! Jinno, Mukaida, KN (2013)

Numerical results



- Particle production (even the graviton production) can be signifucantly enhanced for H>>M

- However, there is gradient instability in this regime.
  Inflaton fluctuation exponentially develops in shortest time range.

$$|p_\phi| \gg \rho_\phi \longrightarrow c_s^2 \ll 0$$

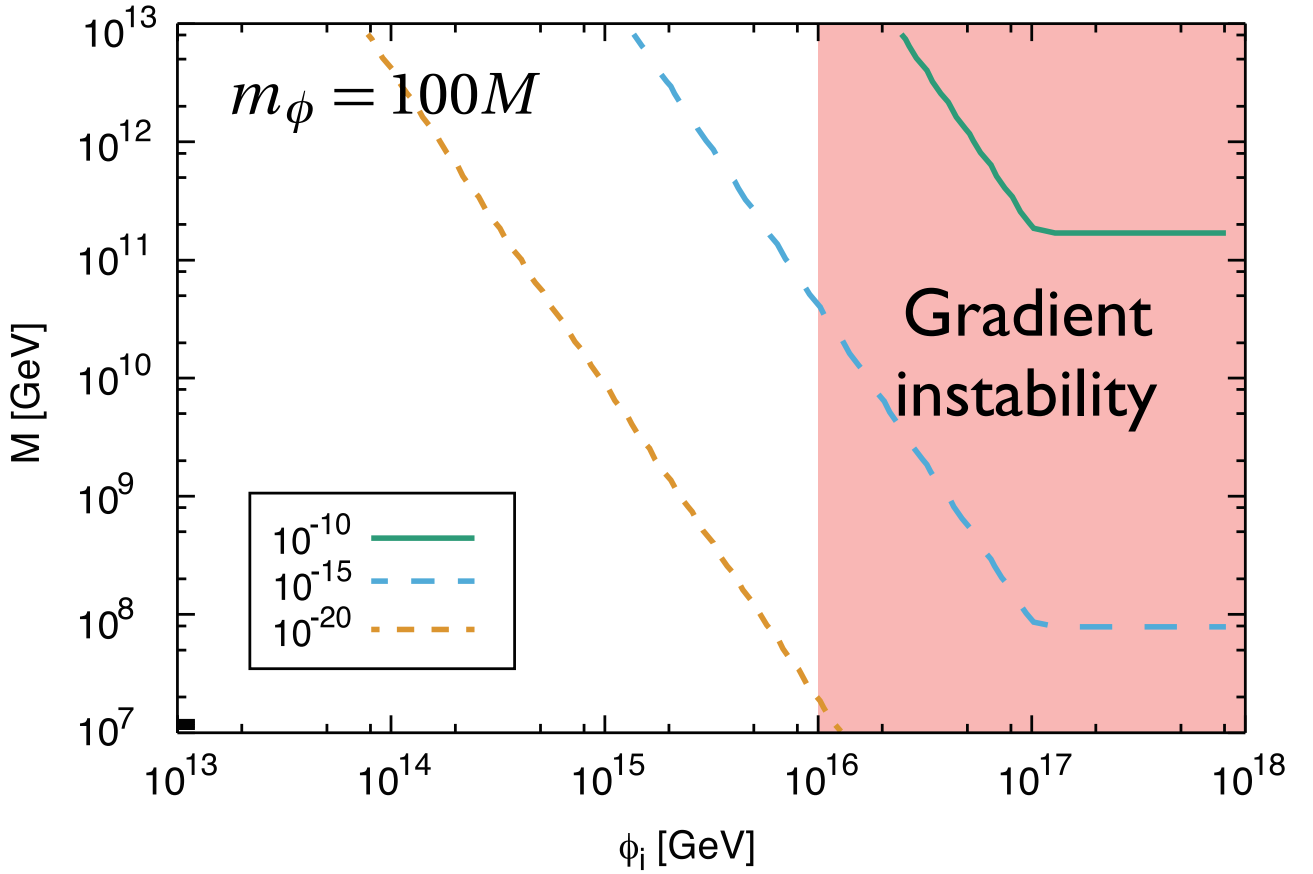
Ema, Jinno, Mukaida, KN (2015)

- Analysis assuming homogeneous inflaton breaks down.

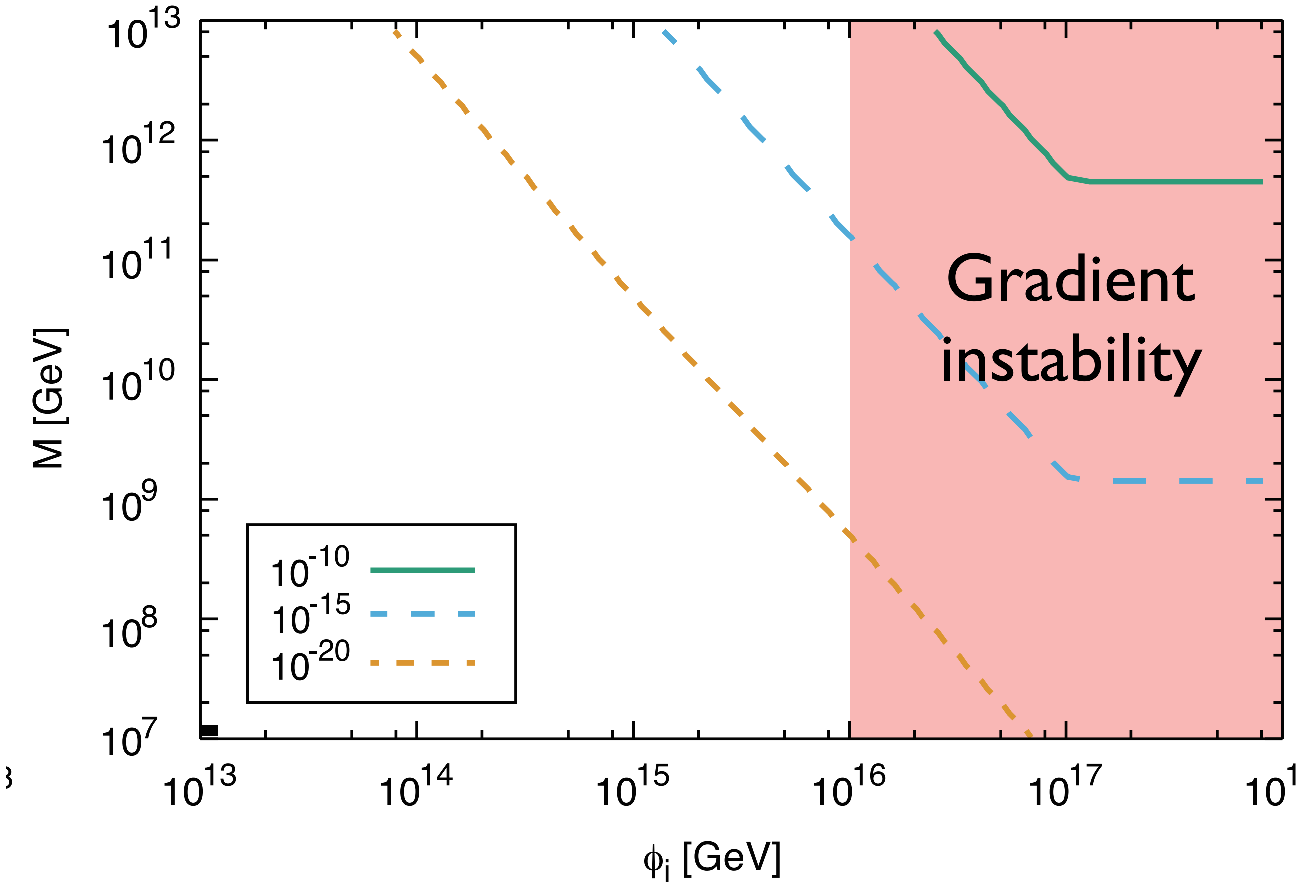
Scalar abundance in derivative coupling model

Here we assume that Φ is subdominant component of the Universe

$$Y_\chi = n_\chi / s$$



$$\Delta N_{\rm eff}$$



Ema, Jinno, Mukaida, KN (2016)

Summary

- Gravitational particle production is ubiquitos phenomena that happen in the early Universe.
- It is efficient in the reheating era where inflaton oscillates rapidly.
- Gravitational production rate is sensitive to the extension of gravity theory.
- Particles with only gravitational interaction can be produced to become dominant dark matter.
- Implications: moduli problem, baryogenesis, etc…

Appendix

Non-minimal coupling

- PGDM nonminimal coupling to gravity

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{2}(M_P^2-\xi\chi^2)R-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{2}m_\chi^2\chi^2\right)$$

- Canonical action in FRW $\qquad g_{\mu\nu}dx^\mu dx^\nu=a^2(\tau)(-d\tau^2+d\vec{x}^2)$

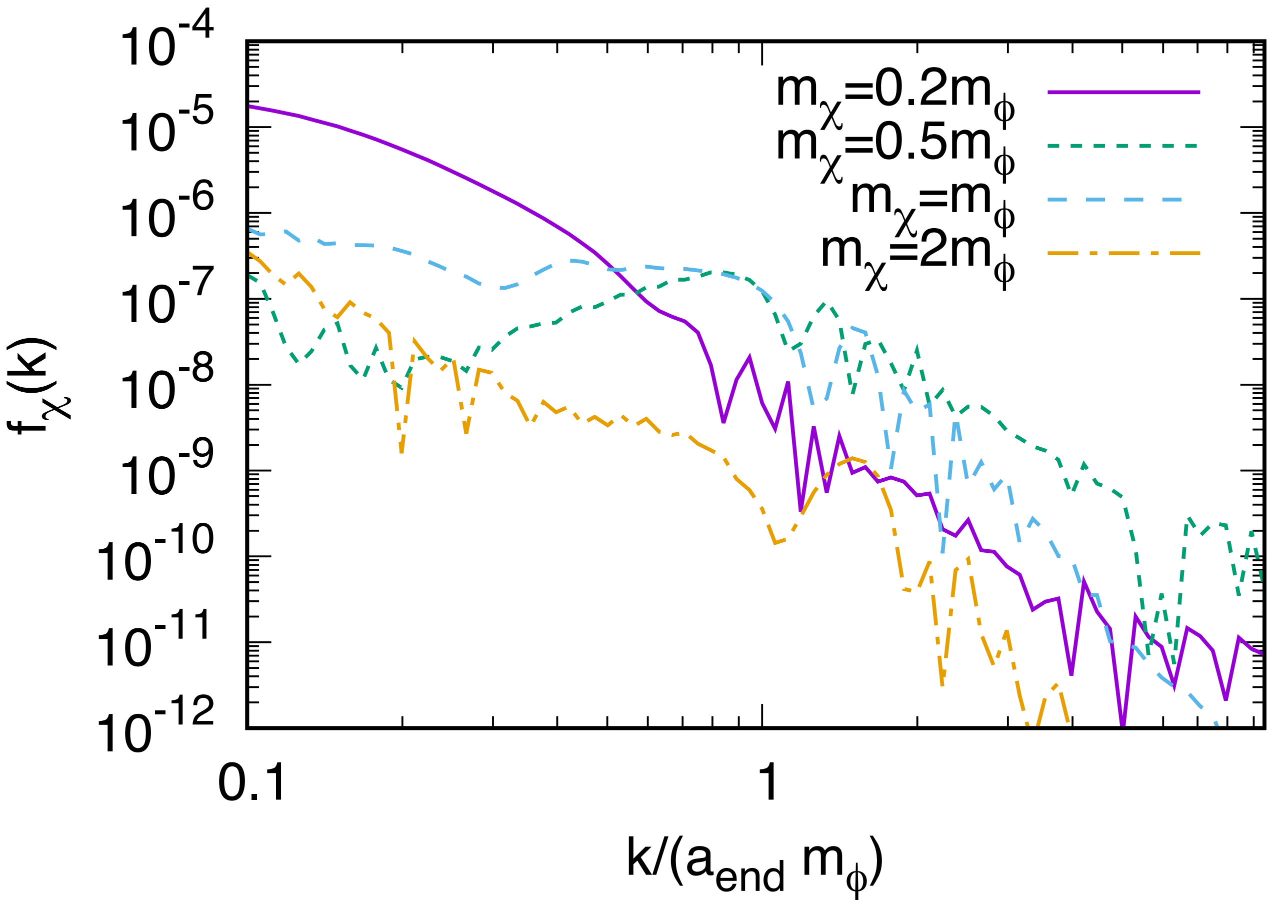
$$S=\int d\tau d^3x\frac{1}{2}\left[\widetilde{\chi}'^2-(\partial_i\widetilde{\chi})^2-m_\chi^{(\mathrm{eff})2}\widetilde{\chi}^2\right], \qquad \widetilde{\chi}\equiv a\chi$$

$$m_\chi^{(\mathrm{eff})2}\equiv a^2m_\chi^2-(1-6\xi)\frac{a''}{a}$$

- A special case: $\xi=\dfrac{1}{6}$ (conformal coupling)
- $\xi\gtrsim\dfrac{m_\phi^2}{H_{\mathrm{inf}}^2}$: tachyonic instability

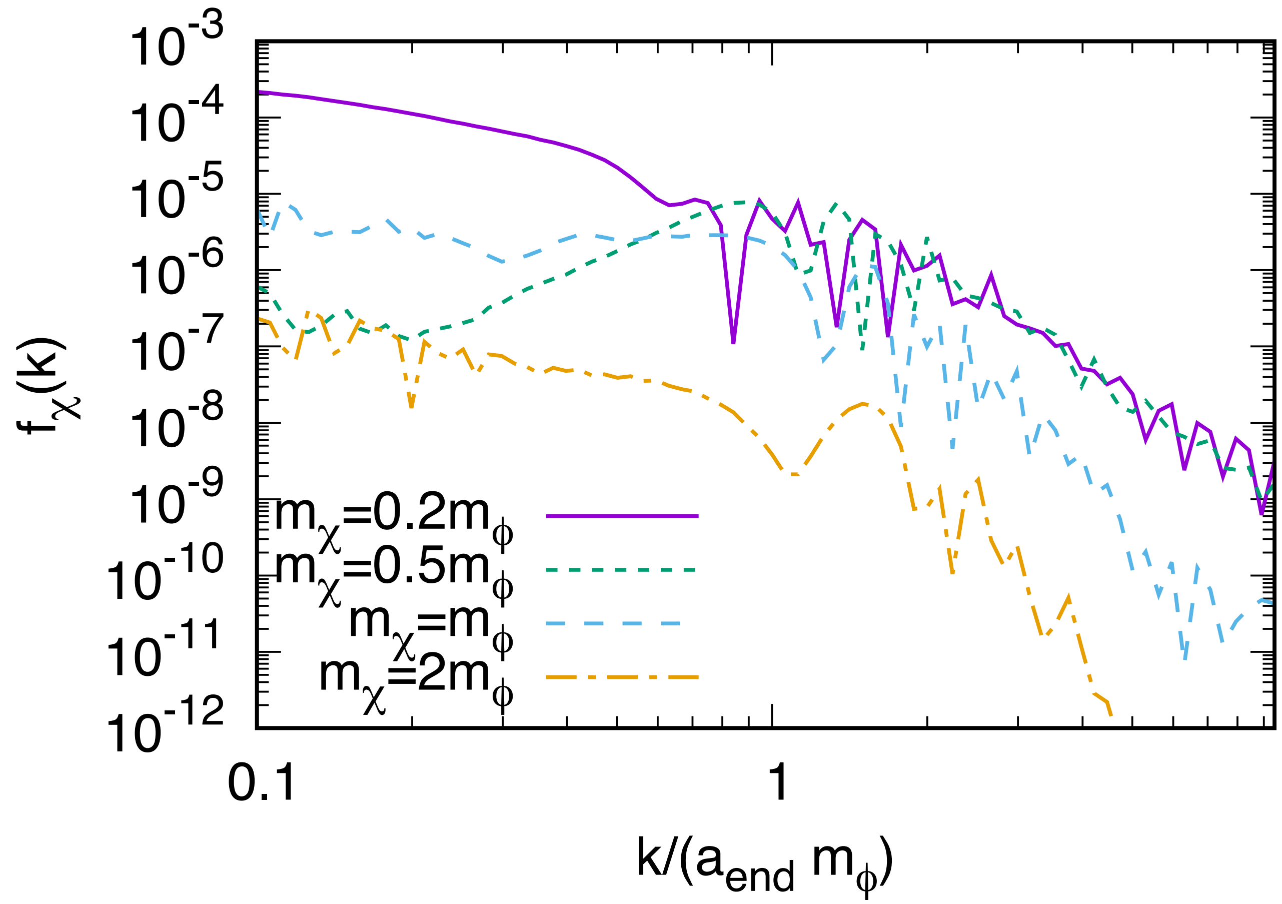
Bassett, Liberati (1998),
Tsujikawa,Maeda,Torii (1999)

Conformal coupling



$$n_\chi \sim H_{\rm inf}^3 \left(\frac{m_\chi}{m_\phi}\right)^4$$

Minimal coupling



$$n_\chi \sim H_{\rm inf}^3$$