From QCD Axion Stars to Boson Stars: Cosmology and Astrophysics

(arXiv: 1710.04729; 1804.07255; 1805.00430; 1906.02094)

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43rd Johns Hopkins Workshop
KAVLI IPMU 2019

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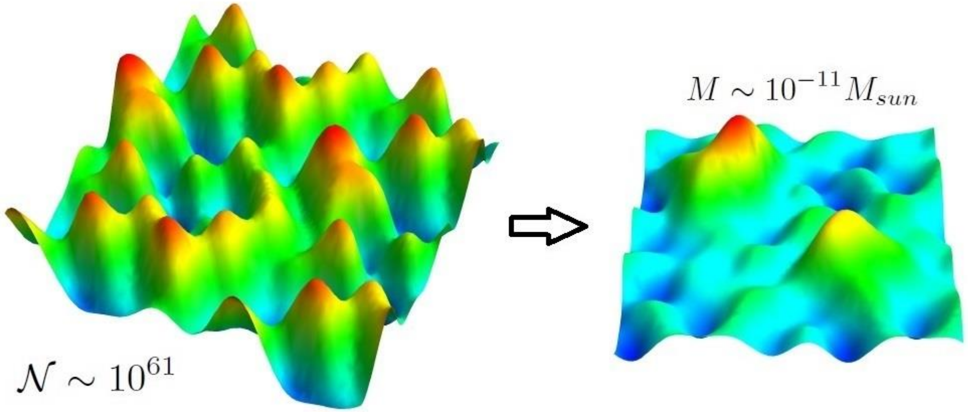
DARK MATTER AXION CLUMPS

- A popular dark matter candidate is the QCD axion: PG-boson associated with SSB of $U(1)_{PQ}$, which was introduced as a possible solution of the strong CP problem (Preskill et al., 1983; Peccei and Quin, 1977; Weinberg, 1978)
- We are interested in small scale axion substructure: (Sikivie and Yang, 2009)



On small scales axions can gravitationally thermalize leading to a type of BEC

(Guth, Hertzberg, and Prescod-Weinstein, 2015)

Condensate of short range order driven by attractive interactions : gravity + self interactions $\lambda\phi^4$

- Axions are described in field theory by a real scalar field $\phi(x,t)$ with a small potential $V(\phi)$ coming from nonperturbative QCD effects.
- Expanding around the CP preserving vacuum: $V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{\lambda}{4!}\phi^4 + \cdots$
- For the standard QCD axion $m_\phi^2 = m_\phi^2(m_{u,d,\pi}, f_\pi, f_a)$ and $\lambda = -\gamma \frac{m_\phi^2}{f_a^2} < 0$

$\gamma = 1$: $V(\phi) = m_\phi^2 f_a^2[1 - \cos(\phi/f_a)]$

$\gamma = 1 - 3m_u m_d/(m_u + m_d)^2 \approx 0.3$ (Grilli et al., 2016)

- In the non-relativistic regime we can rewrite the real axion field in terms of a complex Schrodinger field ψ

$$\phi(\boldsymbol{x},t) = \frac{1}{\sqrt{2m_\phi}}\left[e^{-im_\phi t}\psi(\boldsymbol{x},t) + e^{im_\phi t}\psi^*(\boldsymbol{x},t)\right]$$

- The dynamics of ψ is given by the standard non-relativistic Hamiltonian:

$$H_{nr} = H_{kin} + H_{int} + H_{grav}$$

(Guth, Hertzberg, and Prescod-Weinstein, 2015; Schiappacasse and Hertzberg, 2017)

$$H_{kin} = \frac{1}{2m_\phi}\int d^3x\, \nabla\psi^* \cdot \nabla\psi\,, \quad H_{int} = \frac{\lambda}{16m_\phi^2}\int d^3x \psi^{*2}\psi^2,$$

$$H_{grav} = -\frac{Gm_\phi^2}{2}\int d^3x \int d^3x' \frac{\psi^*(\boldsymbol{x})\psi^*(\boldsymbol{x}')\psi(\boldsymbol{x})\psi(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

- H_{nr} carries a global $\mathrm{U}(1)$ symmetry $\psi \rightarrow \psi e^{i\theta}$ associated with a conserved particle number

$$N = \int d^3x\, \psi^*(\boldsymbol{x})\psi(\boldsymbol{x})$$

True BEC : State of minimum energy at fixed N (spherically symmetry)
Other type of BEC: State of minimum energy at fixed N and angular momentum **L** (non-spherical configuration)

SPHERICALLY SYMMETRIC CLUMP CONDENSATES

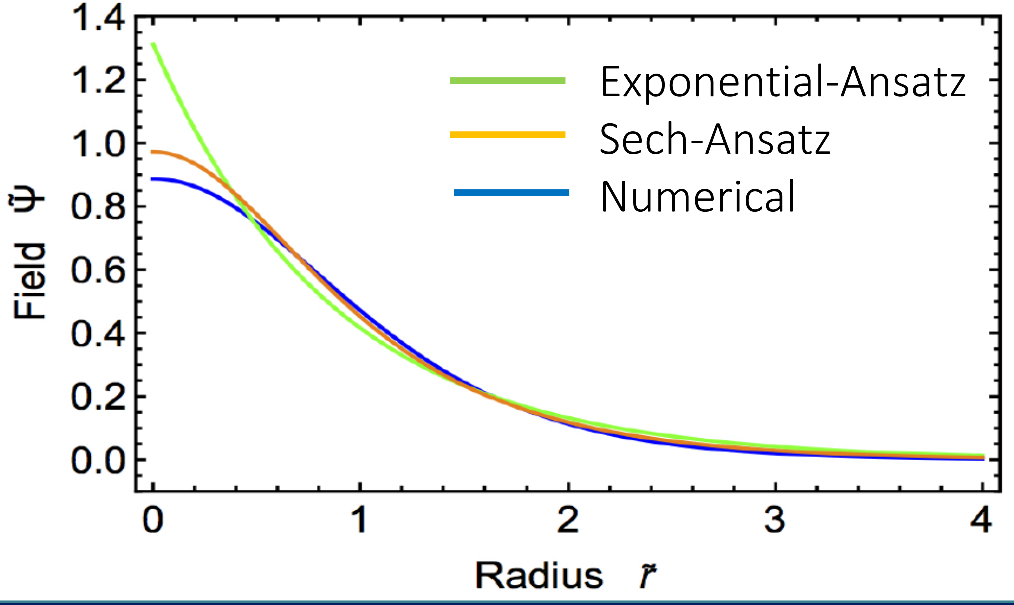
$\Psi(r)$ describes the radial profile
μ describes the correction to the frequency

- Gravity will inevitably cause a homogeneous condensate to fragment into an inhomogeneous field configuration: locally this leads to the formation of BEC clumps.
- The true BEC ground state is guaranteed to be spherically symmetric: $\psi(r,t) = \Psi(r)e^{-i\mu t}$
- The time independent field equation for a spherically symmetric eigenstate is

$$V_{nr} = -\frac{\Psi^4}{16 f_a{}^2}$$

$$\mu\Psi = -\frac{1}{2m}\left(\Psi'' + \frac{2}{r}\Psi'\right) - 4\pi G m^2 \Psi \int_0^\infty dr' r'^2 \, \frac{\Psi(r')^2}{r_>} + \frac{1}{2}\frac{\partial V_{nr}}{\partial \Psi}$$

(Schiappacasse and Hertzberg, **1710.04729**)



Far field region ($r \to \infty$) : Identical to the structure of the time independent Schrödinger equation for the hydrogen atom under replacement Gm^2N→ e^2

Near field region ($r \to 0$) : Corrections from self-interactions become important. There are no know full analytic solutions.

SPHERICALLY SYMMETRIC CLUMP CONDENSATES

SECH ANSATZ

$$\Psi(r) = \Psi_0 \,\mathrm{sech}(r/R) \text{ with } \Psi_0 = \sqrt{\frac{3N}{\pi^3 R^3}}$$

$$N_{max} \approx \frac{10.12}{|\lambda|\sqrt{\delta}}\,;$$

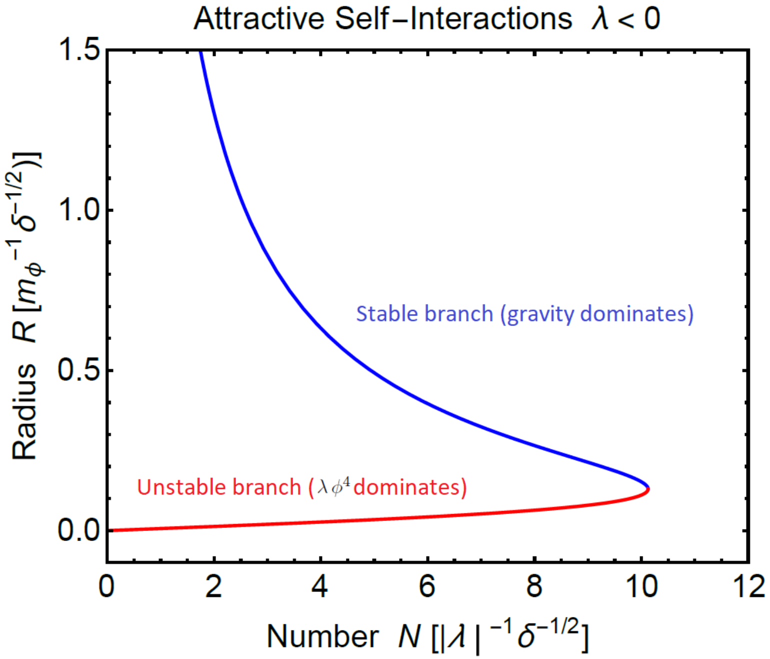
$$R_{min} \approx \frac{0.13}{m_\phi\sqrt{\delta}} \text{ with } \delta = \frac{G m_\phi^2}{|\lambda|} = \frac{G f_a^2}{\gamma}$$

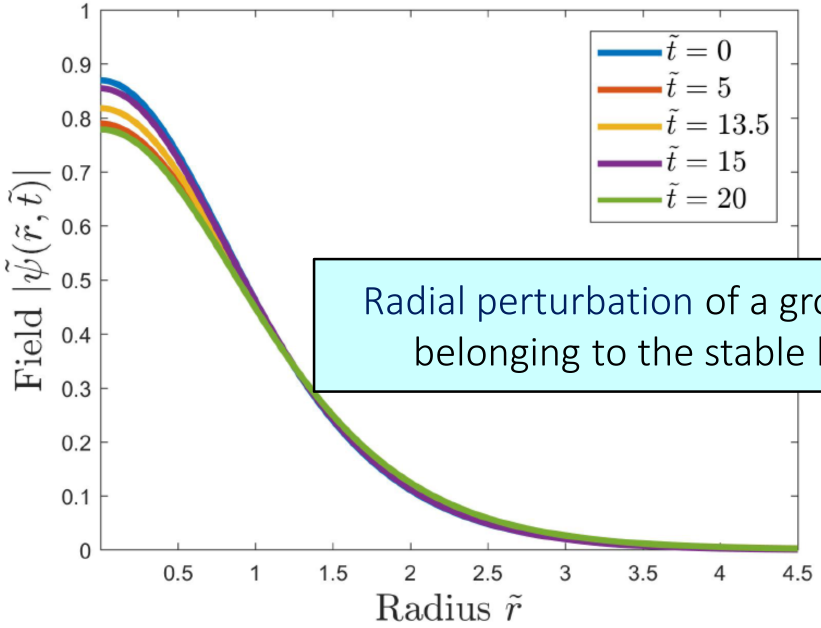
$$\mathrm{H}(R) = a\frac{N}{m_\phi R^2} - b\frac{G m_\phi^2 N^2}{R} + \mathrm{c}\frac{\lambda N^2}{m_\phi^2 R^3} \Leftarrow$$

Any localized anstaz of a single parameter R

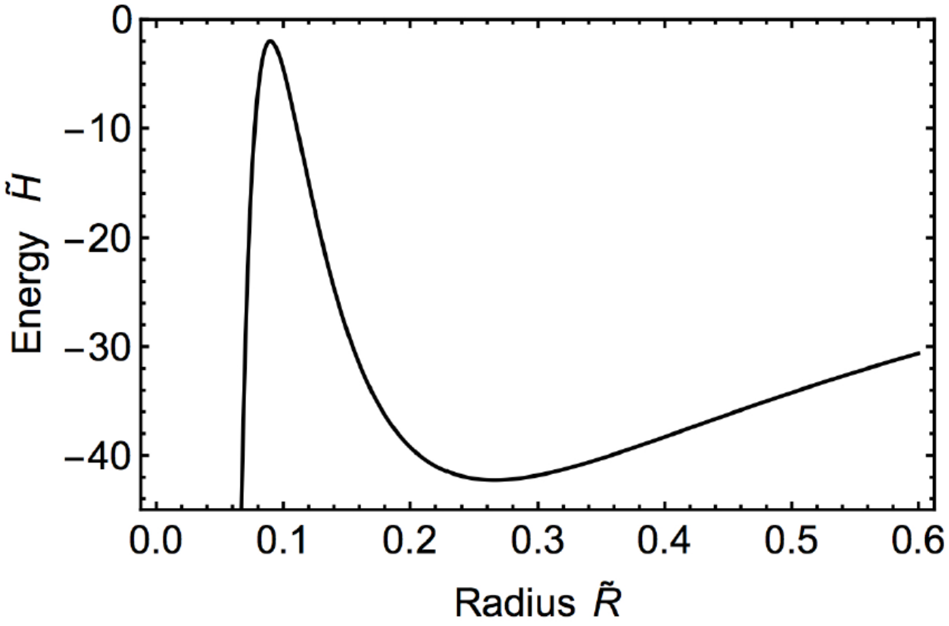
R is the effective radius of the solution (variational parameter)

$$\mathrm{N} = 4\pi \int_0^\infty |\psi(r,t)|^2 \, r^2 dr$$





Radial perturbation of a ground state belonging to the stable branch.



(Schiappacasse and Hertzberg, 1710.04729)

CLUMP CONDENSATES WITH ANGULAR MOMENTUM

- Clump condensates with non-zero angular momentum have a larger N_{max} (and field amplitude).

The angular momentum is $\mathbf{L} = (0,0,N\mathrm{m})$ with $N = 4\pi \int_0^\infty dr\, r^2 |\Psi(r)^2|$

- We take the field profile to be $\psi(x,t) = \sqrt{4\pi}\Psi(r) Y_{\mathrm{lm}}(\theta,\varphi) e^{-i\mu t}$
- We look for states which minimize the energy at fixed particle number and fixed angular momentum
- As usual we make an ansatz for the radial profile $\Psi(r)$: For non-zero l, the structure for small r behavior drastically changes in comparison to the l=0 case

$$\mu_{eff}\Psi \approx -\frac{1}{2m_\phi}\left(\Psi'' + \frac{2}{r}\Psi'\right) + \frac{\mathrm{l}(\mathrm{l}+1)}{2m_\phi r^2}\Psi \quad \text{(near region)}$$

It includes the gravitational term

These terms blow up when $r \longrightarrow 0$

We need $\Psi(r) = \Psi_\alpha r^{\mathrm{l}} - \frac{1}{2}\Psi_\beta r^{\mathrm{l}+2} + \cdots$ (near region)

MODIFIED GAUSSIAN ANSATZ

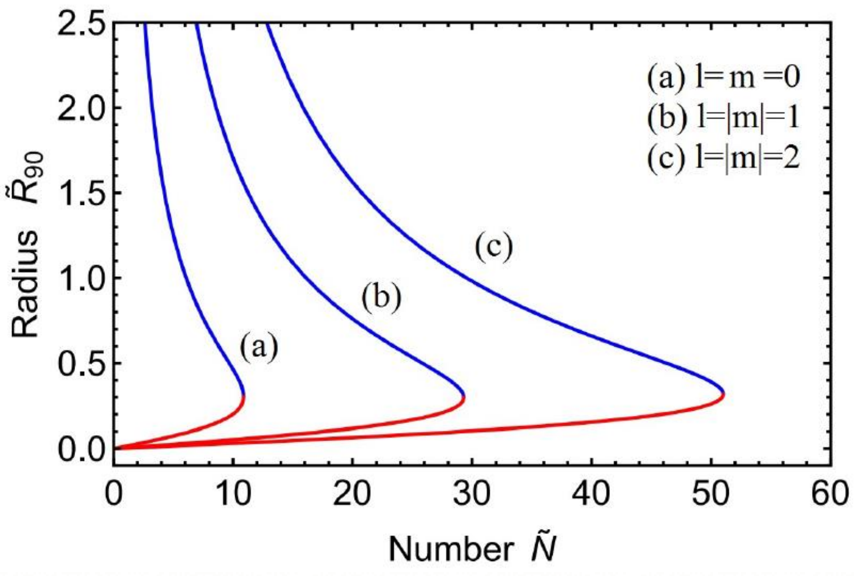
$$\Psi(r) = \sqrt{\frac{N}{2\pi\left(\mathrm{l}+\frac{1}{2}\right)! R^3}} \left(\frac{r}{R}\right)^{\mathrm{l}} e^{-r^2/(2R^2)}$$

The Hamiltonian is a generalization of the previous one for the $l = 0$ case: constant coefficients (a,b,c) become $\{\mathrm{l},\mathrm{m}\}$-dependent

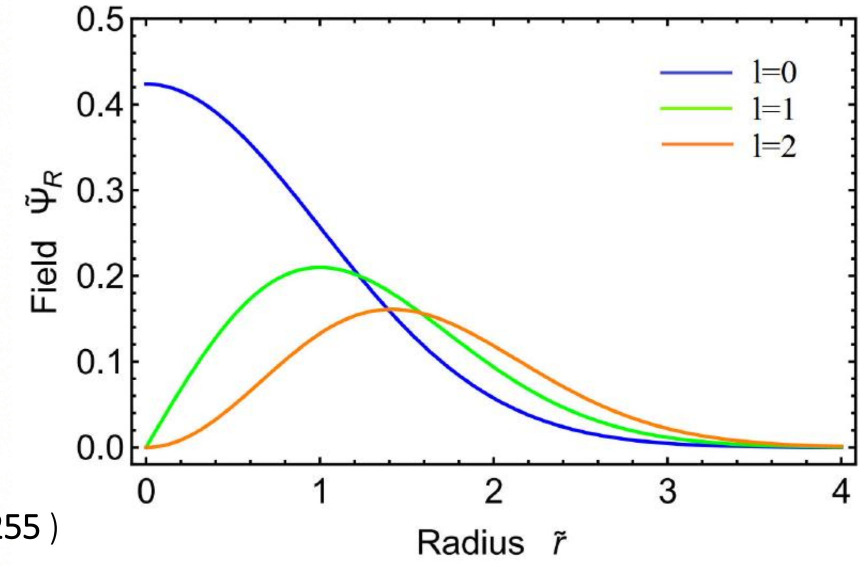
$$\mathrm{H}(R) = a_{\mathrm{lm}}\frac{N}{m_\phi R^2} - b_{\mathrm{lm}}\frac{G m_\phi^2 N^2}{R} + c_{\mathrm{lm}}\frac{\lambda N^2}{m_\phi^2 R^3}$$

CLUMP CONDENSATES WITH ANGULAR MOMENTUM

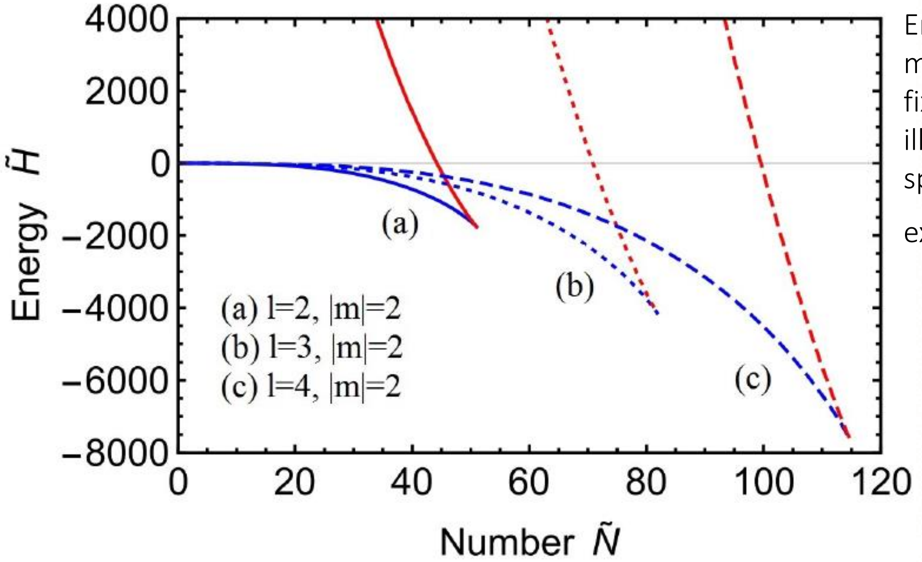
Field $\tilde{\Psi}_R = \Psi_R\sqrt{R^3/N}$ versus radius $\tilde{r} = r/R$ in the modified Gaussian ansatz for different values of spherical harmonic number l.



We plot R_{90} $(0.9N = 4\pi \int_0^{R_{90}} dr' r'^2 \Psi(r')^2)$, where $\tilde{R}_{90} = m_\phi \delta^{1/2} R_{90}$ and $\tilde{N} = |\lambda|\delta^{1/2}N$.



(Hertzberg and Schiappacasse, 1804.07255)



Energy of clump solution versus number with non-zero angular momentum parameter $|m| = 2$ for different values of l. At a fixed number N and angular momentum $L_z = Nm$, this illustrates that the configurations that minimizes the energy has spherical harmonic number $l = |m|$ (whenever the solution exists). Here $\tilde{H} = |\lambda|^{\frac{3}{2}}/(m_\phi^2\sqrt{G})H$ and $\tilde{N} = m_\phi\sqrt{G\,|\lambda|}$.

At high $l = |m|$,

$$N_{max} \approx \frac{10.52}{|\lambda|\sqrt{\delta}}\frac{(l^{3/2})}{(\ln l)^{1/4}} \; ; \; R_{min} \approx \frac{0.141}{m_\phi\sqrt{\delta}}\frac{(l^{1/2})}{(\ln l)^{1/4}}$$

PHYSICAL PARAMETERS FOR AXION STARS

- We compute the maximum number of particles, the maximum mass, and the minimum clump size for axion clumps as follows:

$$N_{max} \sim 1.5 \times 10^{60} \frac{\hat{a}_l}{\sqrt{\hat{b}_{lm}\hat{c}_{lm}}} \left(\hat{m}_\phi^{-2} \hat{f}_a \hat{\gamma}^{-1/2}\right)$$

$\hat{a}_l \sim l/2, \hat{b}_{lm} \sim \sqrt{(\ln l)/l}, \hat{c}_{lm} \sim 1/\sqrt{2l}$ for high $l = |m|$

$$M_{max} \sim 2.5 \times 10^{19}\,\mathrm{kg} \frac{\hat{a}_l}{\sqrt{\hat{b}_{lm}\hat{c}_{lm}}} \left(\hat{m}_\phi^{-1} \hat{f}_a \hat{\gamma}^{-1/2}\right)$$

Here $\hat{f}_a \equiv \frac{f_a}{6 \times 10^{11}\mathrm{GeV}}, \hat{m}_\phi \equiv \frac{m_\phi}{10^{-5}\mathrm{eV}}, \hat{\gamma} \equiv \gamma/0.3,$ and the coefficients $\hat{a}_l, \hat{b}_{lm}, \hat{c}_{lm}$ are normalized to their zero angular momentum value.

$$R_{90,min} \sim 70\,\mathrm{km} \left(\frac{\tilde{R}_{90}}{R_{90}}\right) \sqrt{\frac{\hat{c}_{lm}}{\hat{b}_{lm}}} \left(\hat{m}_\phi^{-1} \hat{f}_a^{\,-1} \hat{\gamma}^{1/2}\right)$$

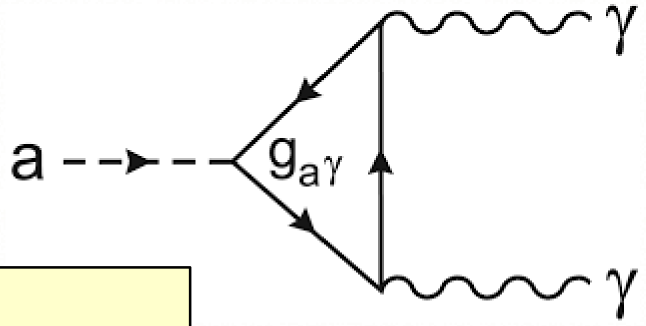
- Note that the number of axions within a correlation length in the scenario in which the PQ symmetry is broken after inflation is given by

(Guth, Hertzberg, and Prescod-Weinstein, 2015)

$$N_\xi \sim \frac{T_{eq} M_{pl}^3}{T_{QCD}^3 m_\phi} \sim 10^{61} \hat{m}_\phi^{-1}$$

For $l = |m| \gtrsim 5$, we have $N_{max} \gtrsim 10^{61}$

AXIONS AND PHOTONS



- The axion-photon decay channel runs through the chiral anomaly

$$L_{EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g_{a\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$g_{a\gamma} = \frac{\beta}{f_a}$ } KSVZ/DFSZ/Hidden sector
($\beta \sim 10^{-2}$ in QCD conventional axion models)

Kim (1979); Zhitnitsky (1980)

We send $g_{a\gamma} \to |g_{a\gamma}|$ for ease notation
(Only its magnitude is of significance here)

- Using $\hat{A}^\mu = (\hat{A}_0, \hat{\boldsymbol{A}})$ in the Coulomb Gauge, we obtain for the 2 degrees of freedom :

$$\ddot{\hat{\boldsymbol{A}}} - \nabla^2\hat{\boldsymbol{A}} + g_{a\gamma}\nabla \times \left[(\partial_t\phi)\hat{\boldsymbol{A}}\right] = 0 \longrightarrow \ddot{\hat{\boldsymbol{A}}}_{\boldsymbol{k}} + k^2\hat{\boldsymbol{A}}_{\boldsymbol{k}} + g_{a\gamma} i\boldsymbol{k} \times \int \frac{d^3k'}{(2\pi)^3}\partial_t\phi_{\boldsymbol{k}-\boldsymbol{k}'}\hat{\boldsymbol{A}}_{\boldsymbol{k}'} = 0$$

$|\nabla\phi| \ll |\partial_t\phi|$ in the non-relativistic limit for axions

HOMOGENEOUS CONDENSATES

- We begin by treating the axion field as homogeneous since this case is the simplest possible

The axion field is treat as a classical oscillatory background: $\phi(t) = \phi_0 \cos(\omega_0 t)$, where $\omega_0 \approx m_\phi$

In general such configuration is unstable to collapse from gravity and attractive self-interactions: Homogeneous Condensate → Clump Condensate

(Guth, Hertzberg, and Prescod-Weinstein, 2015)

- Clearly, the equation of motion for $\widehat{\boldsymbol{A}}_{\boldsymbol{k}}$ decouple in k-space. Then, expressing $\widehat{\boldsymbol{A}}_{\boldsymbol{k}}$ in function of vectors for circular polarizations and modes functions $s_{\boldsymbol{k}}(t)$ we have

$$\ddot{\widehat{\boldsymbol{A}}}_{\boldsymbol{k}} + k^2 \widehat{\boldsymbol{A}}_{\boldsymbol{k}} - g_{a\gamma}\omega_0\phi_0 i\boldsymbol{k} \times \widehat{\boldsymbol{A}}_{\boldsymbol{k}} = 0 \quad \longrightarrow \quad \ddot{s}_{\boldsymbol{k}} + \underbrace{[k^2 - g_{a\gamma}\omega_0\phi_0 \sin(\omega_0 t)]}_{\omega_k^2(t) = \omega_k^2(t + 2\pi/\omega_0)} s_{\boldsymbol{k}} = 0$$

- In the parameter space of Mathieu equation (ME) there is a band structure of unstable (resonant) and stable regions: (McLachlan, 1947)

$$s_{\mathbf{k}}(t) = P_{\mathbf{k}}(t)e^{\mu_k t} + P_{\mathbf{k}}(-t)e^{-\mu_k t}$$

GENERAL SOLUTION

If the Floquet exponent, μ_k, has a real part, the resonance occurs

- At small amplitude of weak coupling limit of $(k/\omega_0) \gg (g_{a\gamma}\phi_0/2)$, we have a spectrum of narrow bands equally spaced at $(k/\omega_0)^2 \approx (n/2)^2$ for n a positive integer.

The band width and Floquet exponent magnitude decreases as n increases

- Expand the solution for each circularized polarization as:

$$s_{\mathbf{k}}(t) = \sum_{\omega} e^{i\omega t} f_{\omega}(t)$$

Slowing varying

Plug into ME and focus on lowest frequencies $(\omega = \pm\omega_0/2)$

$$\frac{d}{dt}\begin{bmatrix} f_{\omega_0/2} \\ f_{-\omega_0/2} \end{bmatrix} = \frac{i}{\omega_0}\begin{bmatrix} k^2 - \frac{\omega_0^2}{4} & -i\frac{B}{2} \\ -i\frac{B}{2} & -k^2 + \frac{\omega_0^2}{4} \end{bmatrix}\begin{bmatrix} f_{\omega_0/2} \\ f_{-\omega_0/2} \end{bmatrix}$$

HOMOGENEOUS CONDENSATES
Small Amplitude Analysis

Positive eigenvalues previous matrix

- The growth rate is: $\mu_k = \sqrt{\dfrac{g_{a\gamma}^2 k^2 \phi_0^2}{4} - \dfrac{\left(k^2 - \frac{\omega_0^2}{4}\right)^2}{\omega_0^2}}$
- For $\Re(\mu_k) > 0$: Exponentially growing solutions

  (First instability band)
- For $\Re(\mu_k) = 0$: Instability band edges

$$k_{l/r_{edge}} = \sqrt{\frac{\omega_0^4}{4} + \frac{g_{a\gamma}^2 \omega_0^2 \phi_0^2}{16}} \mp \frac{g_{a\gamma}\omega_0\phi_0}{4}$$

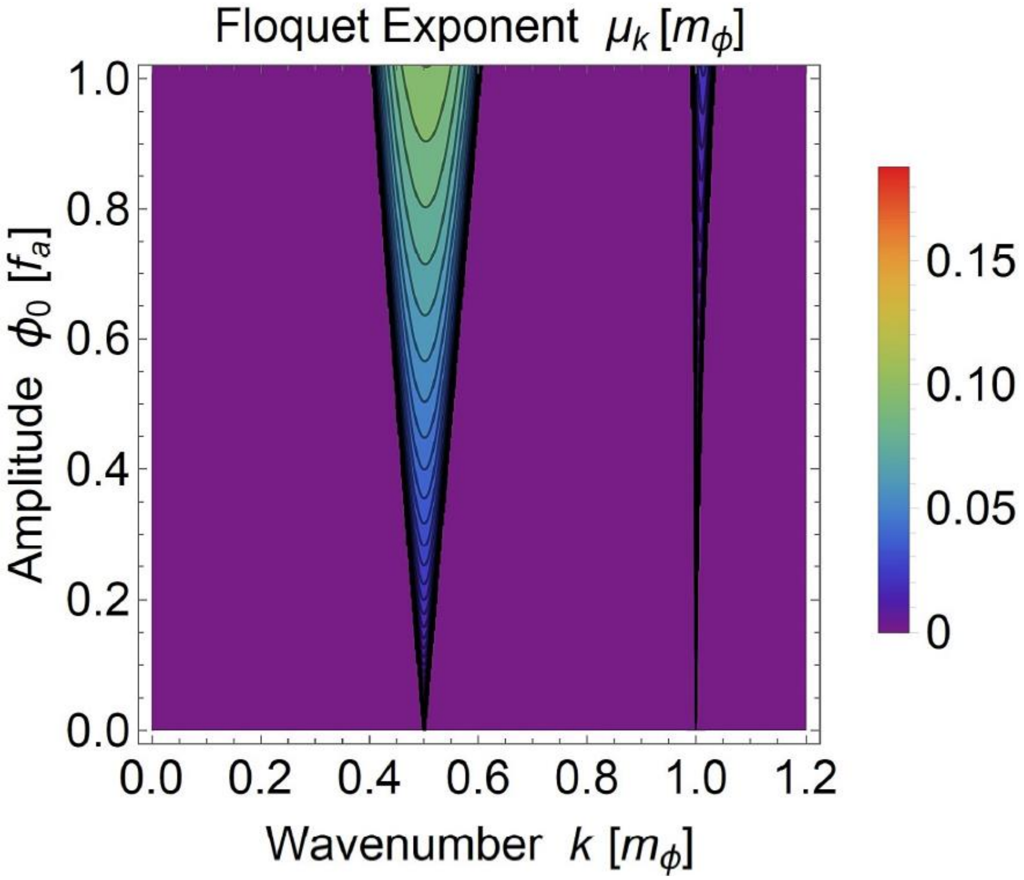
$$\boxed{k^* = (\omega_0/2)\sqrt{1 + g_{a\gamma}^2 \phi_0^2/2} \approx m_\phi/2}$$

Center of the band

Maximum Floquet Exponent

$$\boxed{\mu_H^* \approx \frac{g_{a\gamma} m_\phi \phi_0}{4}}$$

Numerical Analysis
(Standard Floquet Method)



Contour plot of the real part of Floquet exponent μ_k, describing parametric resonance of photons from a homogeneous condensate, as a function of wavenumber k and physical amplitude ϕ_0. We plot ϕ_0 in units of f_a and k & μ_k in units of m_ϕ. We have set $g_{a\gamma} = 0.4/f_a$ to illustrate the behavior, although in conventional QCD axions $g_{a\gamma} = \mathcal{O}(10^{-2})/f_a$ which would give narrower resonance bands.

(Hertzberg and Schiappacasse, 1805.00430)

- Since $\phi = \phi(r,t)$, the usual 3-dimensional Fourier transform of the equation of motion for the vector potential is not the best way to proceed.
- We prefer performing a vector spherical harmonic decomposition of $\hat{\boldsymbol{A}}$:

$$\hat{\boldsymbol{A}}(\boldsymbol{x},t) = \int \frac{d^3k}{(2\pi)^3} \sum_{lm} \left[\hat{a}(k) v_{lm}(k,t) \boldsymbol{M}_{lm}(k,\boldsymbol{x}) - \hat{b}(k) w_{lm}(k,t) \boldsymbol{N}_{lm}(k,\boldsymbol{x})\right]$$

Here $\boldsymbol{M}_{lm}$, $\boldsymbol{N}_{lm}$ are vector spherical harmonics, where $\boldsymbol{M}_{lm} = \frac{i j_l(kr)}{\sqrt{l(l+1)}} \left[\frac{im}{\sin\theta} Y_{lm} \hat{\theta} - \frac{\partial Y_{lm}}{\partial\theta} \hat{\varphi}\right]$ and $\nabla \times \boldsymbol{M}_{lm} = -ik\boldsymbol{N}_{lm}$, $\nabla \times \boldsymbol{N}_{lm} = ik\boldsymbol{M}_{lm}$

- Again neglecting gradients of the axion field,

$$\ddot{\hat{\boldsymbol{A}}} - \nabla^2 \hat{\boldsymbol{A}} + g_{a\gamma} \nabla \times \left[(\partial_t \phi) \hat{\boldsymbol{A}}\right] = 0$$

$$\Downarrow$$

$$\int \frac{d^3k}{(2\pi)^3} \sum_{lm} \left[\left(\ddot{v}_{lm} + k^2 v_{lm} - ik g_{a\gamma} \partial_t \phi w_{lm}\right) \boldsymbol{M}_{lm}(k,\boldsymbol{x}) - \left(\ddot{w}_{lm} + k^2 w_{lm} + ik g_{a\gamma} \partial_t \phi v_{lm}\right) \boldsymbol{N}_{lm}(k,\boldsymbol{x})\right] = 0$$

It can be solved numerically, but for any arbitrary sum over $\{l,m\}$ is quite complicated

- We focus on the $\{l = 1, m = 0\}$ channel for simplicity and leave a complete analysis for future work.
- We write out the individual vector components $(\hat{r}, \hat{\theta}, \hat{\varphi})$. Since the Coulomb Gauge reduces the system to only two independents equations $(\nabla \cdot \hat{A} = 0)$, we focus only on $(\hat{r}, \hat{\varphi})$ components.
- For the radial component, use $\phi = \Phi(r)\cos(\omega_0 t)$ and orthogonality properties of Spherical Bessel functions to obtain

$$\ddot{w}_{10}(k',t) + k'^2 w_{10}(k',t) - \frac{2i}{\pi} g_{a\gamma}\omega_0 k' \sin(\omega_0 t) \int dk\, k^2 v_{10}(k,t) \int dr\, r^2 \Phi(r) j_1(kr) j_1(k'r) = 0$$

The spherically symmetry of the axion field means that we can represent $\Phi(r)$ by a 1-d (real) Fourier transform $\tilde{\Phi}_{1d}(k)$

$$\boxed{\Phi(r) = \int \frac{d\tilde{k}}{2\pi} \cos(\tilde{k}r) \tilde{\Phi}_{1d}(\tilde{k})}$$

$$\boxed{\approx \frac{k^2 + k'^2}{8k^2 k'^2} \left[\tilde{\Phi}_{1d}(k - k') - \tilde{\Phi}_{1d}(k + k')\right]}$$

The 1d Fourier transform is dominated by $\tilde{k} \approx 0$. For example for sech ansatz $\tilde{\Phi}_{1d}(\tilde{k}) = \sqrt{\frac{3N}{\pi R}} \operatorname{sech}\left(\frac{\pi \tilde{k} R}{2}\right)$, we have $\tilde{k} \sim 1/R \ll m_\phi$

The resonance occurs when $k \approx k' \approx \frac{m_\phi}{2} \rightarrow$ the second term is exponential suppressed for $R \gg 1/m_\phi$

- A similar procedure can be applied for the other angular component equation, to obtain

$$\ddot{w}_{10}(k,t) + k^2 w_{10}(k,t) - i g_{a\gamma}\omega_0 k \sin(\omega_0 t) \int \frac{dk'}{2\pi} v_{10}(k',t)\widetilde{\Phi}_{1d}(k-k')=0$$

$$\ddot{v}_{10}(k,t) + k^2 v_{10}(k,t) + i g_{a\gamma}\omega_0 k \sin(\omega_0 t) \int \frac{dk'}{2\pi} w_{10}(k',t)\widetilde{\Phi}_{1d}(k-k')=0$$

- A self-consistent resonant solution is just given by $w_{10}(k,t) = \pm i v_{10}(k,t)$, which reduces the system to a single scalar differential equation
- We compute the resonance structure numerically using Floquet theory :

In the homogeneous case, there always exists a non-zero maximum Floquet exponent $\mu_H^* \approx g_{a\gamma} m_\phi \phi_0/4$

(1) We determine the maximum Floquet exponent μ^*
(2) We use various choices of $g_{a\gamma}$ and parameters of axion clump (R and N)
(3) We operate in the sech approximation on the stable branch:

$\phi(r,t) = \Phi(r)\cos(\omega_0 t) = \sqrt{\frac{2}{m_\phi}}\Psi(r)\cos(\omega_0 t)$ with $\omega_0 = m_\phi + \mu \approx m_\phi$

SPHERICALLY SYMMETRIC CLUMP CONDENSATES
Numerical Results

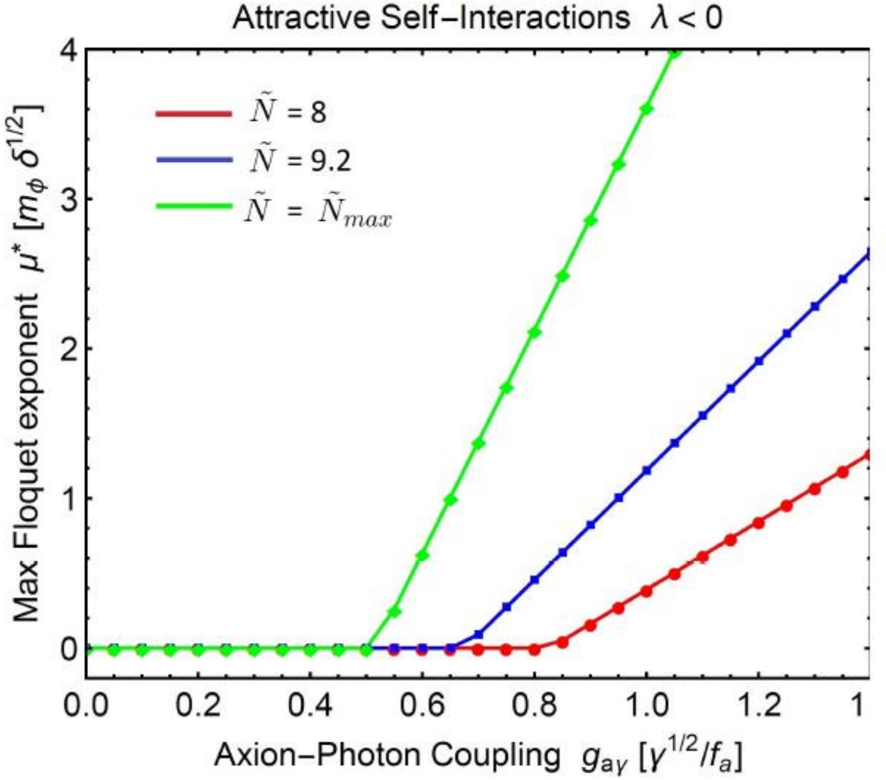
(Hertzberg and Schiappacasse, 1805.00430)

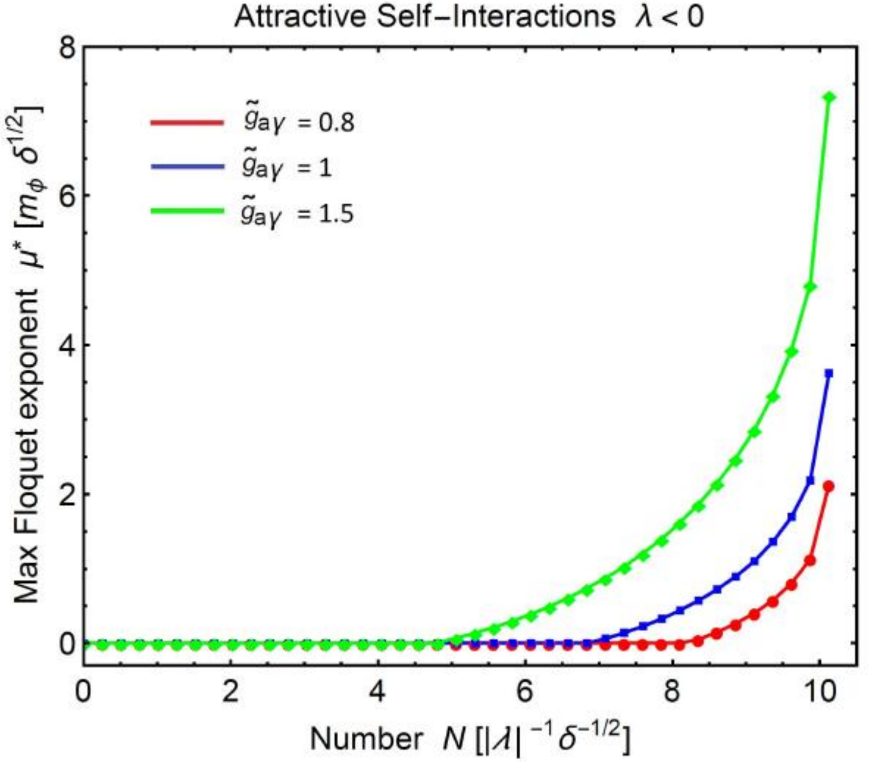
The real part of μ^* becomes zero below a critical $g_{a\gamma}$ or critical N.

For the QCD axions, when $N = N_{max}$ and $\gamma = 0.3$, parametric resonance happens for $g_{a\gamma} > g_{a\gamma,min} \approx \frac{\beta_c}{f_a}$ with $\beta_c \approx 0.3$

In conventional QCD axion models, $g_{a\gamma} = \frac{O(10^{-2})}{f_a}$. So, the resonance could be possible in unconventional axions models or for couplings to hidden sector photons

(Daido, Takahashi and N. Yokozaki 2018)





The maximum real part of Floquet exponent μ^*, describing parametric resonance of photons from a spherically symmetric clump condensate, as a function of axion-photon coupling $g_{a\gamma}$. (Left) We plot μ^* in units of $m_\phi\sqrt{\delta}$, $g_{a\gamma}$ in units of $\frac{\sqrt{\gamma}}{f_a}$, and $\tilde{N} = N/(|\lambda|\sqrt{\delta})$. (Right) We plot μ^* in units of $m_\phi\sqrt{\delta}$ and N in units of $|\lambda|^{-1}\delta^{-1/2}$. Here $\tilde{g}_{a\gamma} = g_{a\gamma}f_a/\sqrt{\gamma}$.

Resonance Condition : $\mu_H^* \approx \frac{g_{a\gamma} m_\phi \phi_0}{4} > \mu_{esc} \approx \frac{1}{2R}$

Hertberg (2010); Kawasaki and Yamada (2014)

Furthermore, excellent approximation to the growth rate from a localized clump: $\mu^* \approx \begin{cases} \mu_H^* - \mu_{esc}, & \mu_H^* > \mu_{esc} \\ 0, & \mu_H^* < \mu_{esc} \end{cases}$

CLUMP CONDENSATES WITH ANGULAR MOMENTUM
Approximate Treatment of Clump Resonance

- For an approximate treatment of clump resonance, we use the condition for resonance $\mu_H^* > \mu_{esc}$, which was established for spherically symmetric clumps.

- Since $\mu_H^* \approx g_{a\gamma} m_\phi \phi_0/4$, we need to determine the maximum field amplitude:

$$\phi_0 = \sqrt{\frac{2}{m_\phi}}\,\Psi_0\,\sqrt{4\pi}|Y_{lm}|_0$$

$$\sqrt{4\pi}|Y_{lm}|_0 \approx \frac{\sqrt{2}l^{1/4}}{\pi^{1/4}} \text{ for high } |m| = l$$

$$\Psi_0 = \sqrt{\frac{N_{max}}{R_{min}{}^3}}\,f_l \text{ with } f_l \approx \frac{1}{(2\pi)^{3/4}\sqrt{l}} \text{ for high } |m| = l$$

$$\boxed{\mu_H^* \approx 5.8\sqrt{l}(\ln l)^{1/4}\tilde{g}_{a\gamma} m_\phi \sqrt{\delta}}$$

$$\boxed{\mu_{esc} \approx 3.5(\ln l)^{\frac{1}{4}} m_\phi \sqrt{\delta}}$$

(for high $|m| = l$ and $N = N_{max}$ and $\tilde{g}_{a\gamma}$=$g_{a\gamma} f_a/\sqrt{\gamma}$)

- The radial profile is now peaked around $R_p = \sqrt{l}R$, but the full width in the radial direction is still $\omega_r \sim 2R$:

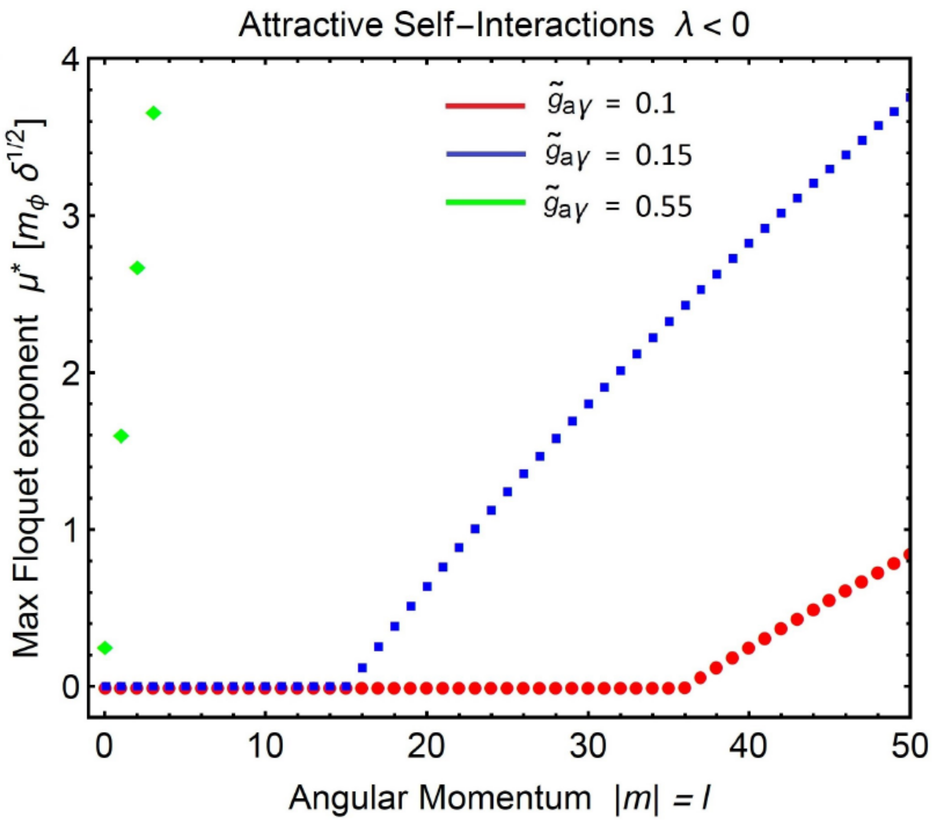
$$\Psi(r) \approx \sqrt{\frac{N}{(2\pi)^{3/2} l R^3}}\, e^{-(r-R_p)^2/R^2} \text{ for high } l$$

- The angular dependence is non-trivial. For $l = |m|$, the real field ϕ is

$$\phi(r,\theta,\varphi,t) = \sqrt{\frac{2}{m_\phi}}\sqrt{4\pi}|Y_{lm}|_0 \Psi(r)(-\sin\theta)^l \cos(\omega_0 t \pm l\varphi),$$

and the full width in the polar direction is $\omega_\theta \sim \Delta\theta R_p/\pi \sim 2R$ where $\Delta\theta = 2\cos^{-1}(e^{-\frac{1}{l}\ln 2})$. Then, we take as $\mu_{esc} \sim 1/2R$

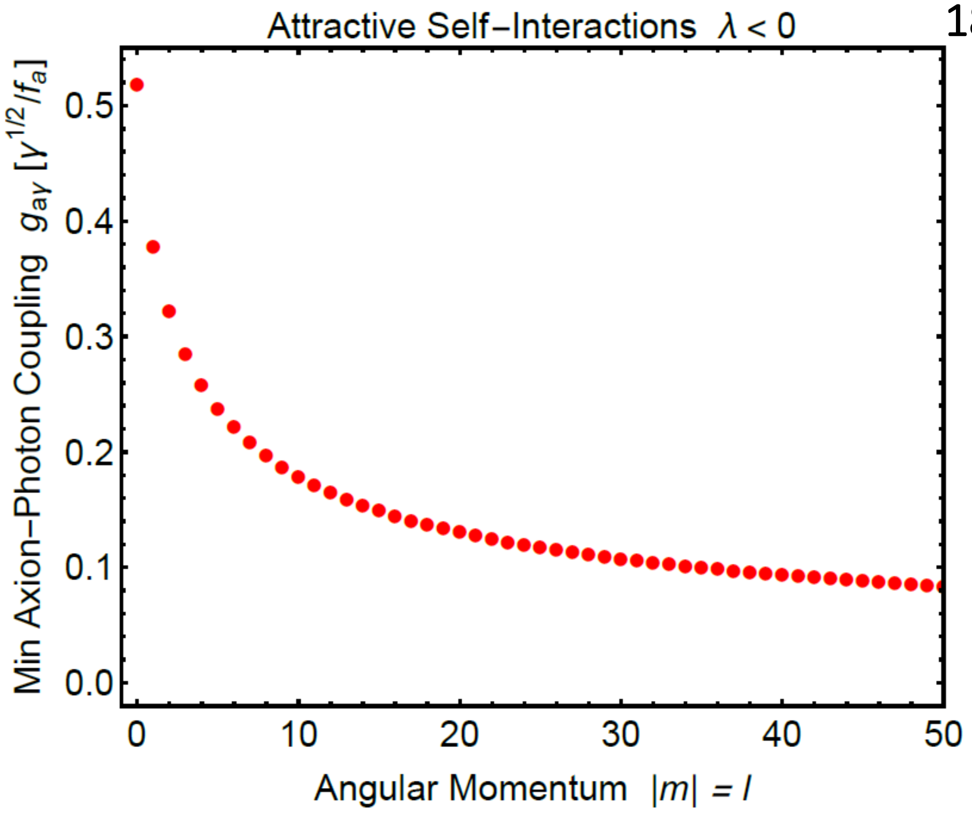
CLUMP CONDENSATES WITH ANGULAR MOMENTUM
Numerical Results



The maximum real part of Floquet exponent μ^*, describing parametric resonance of photons from a clump condensate as a function of its angular momentum $|m| = l$. We plot μ^* in units of $m_\phi\sqrt{\delta}$. This is for attractive self-interactions with $N = N_{max}$. Here $\tilde{g}_{a\gamma} = \frac{g_{a\gamma} f_a}{\sqrt{\gamma}}$.

$$\mu^* \approx Max(\mu_H^* - \mu_{esc}, 0)$$

The minimum axion-photon coupling $g_{a\gamma}$ that is necessary in order to have resonance from a clump condensate as a function of its angular momentum $|m| = l$. We plot in units of $\frac{\sqrt{\gamma}}{f_a}$. We take $N = N_{max}$.



(Hertzberg and Schiappacasse, 1805.00430)

By taking $\gamma = 0.3$ and $N = N_{max}$, for high angular momentum the minimum axion-photon coupling is just $g_{a\gamma} > g_{a\gamma,min} \approx \frac{\beta_c}{f_a\sqrt{l}}$ with $\beta_c = 0.3$

For the QCD axion $g_{a\gamma} = O(10^{-2})/f_a$, we need rather large angular momentum of $|m| = l \gtrsim O(10^3)$

GENERAL RELATIVISTIC BOSON STARS

- Here we analyze ground state configuration of boson stars with generic self-interaction with a full general relativistic treatment.
- Stability properties of boson stars have been studied extensively in the literature.

(Lee and Pang, 1989; Seidel and Suen, 1990 ; Schunk and Torres, 2000; Lai, 2004; Croon, Fan, and Sun, 2018)

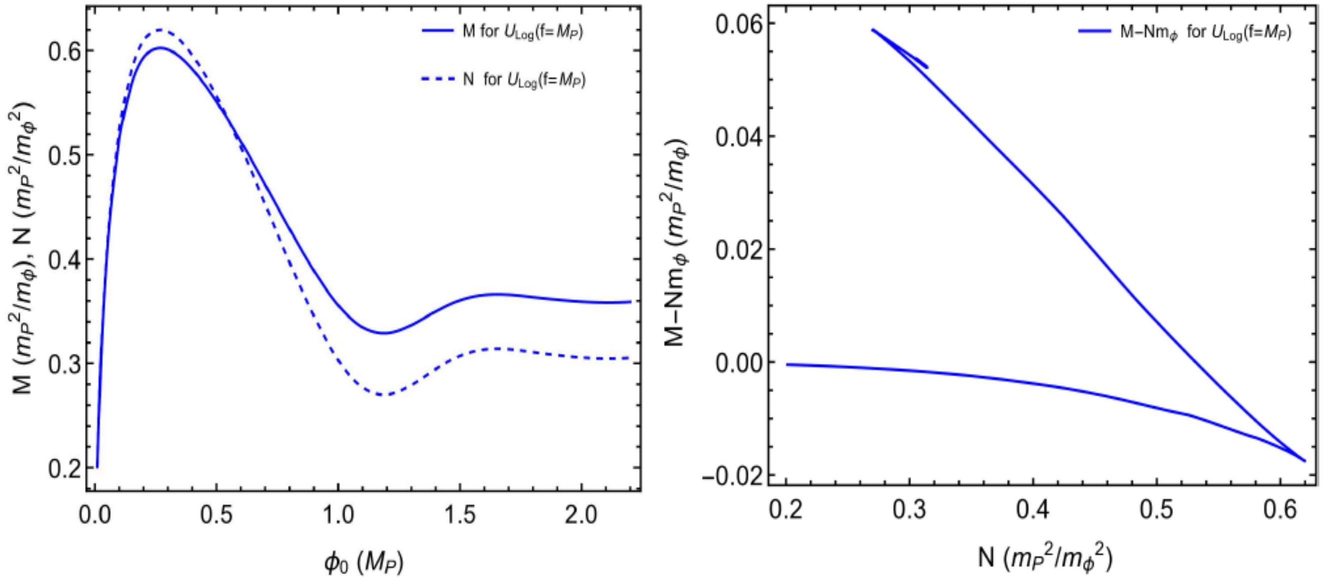
Mini-Boson Stars: $M_{max} = 0.633\, {m_P}^2/m_\Phi,\ C_{max} \cong 0.08$

Quartic Self-interaction: $M_{max} = 0.22\, \Lambda^{1/2} {m_P}^2/m_\Phi,\ C_{max}(\Lambda \to \infty) \cong 0.16$

- We are mainly interesting in two potentials with generic self-interactions ($\Phi(\boldsymbol{r}, t) = \phi(r) e^{i\omega t}$):

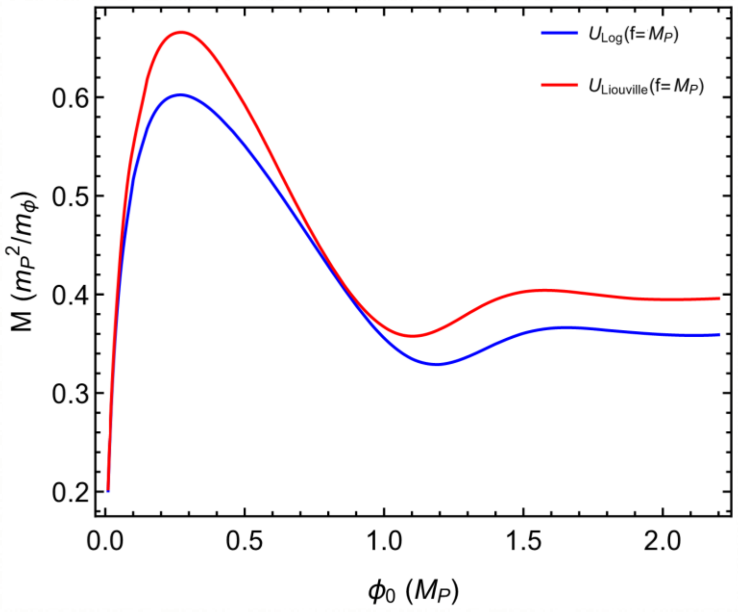
$$U_{Liouville} = f^2 {m_\Phi}^2 \left(e^{|\Phi|^2/f^2} - 1\right) \to U_{Liouville} = {m_\Phi}^2 \phi^2 + \frac{{m_\Phi}^2 \phi^4}{2f^2} + \frac{{m_\Phi}^2 \phi^6}{6f^4} + \frac{{m_\Phi}^2 \phi^8}{24 f^6} + \cdots$$

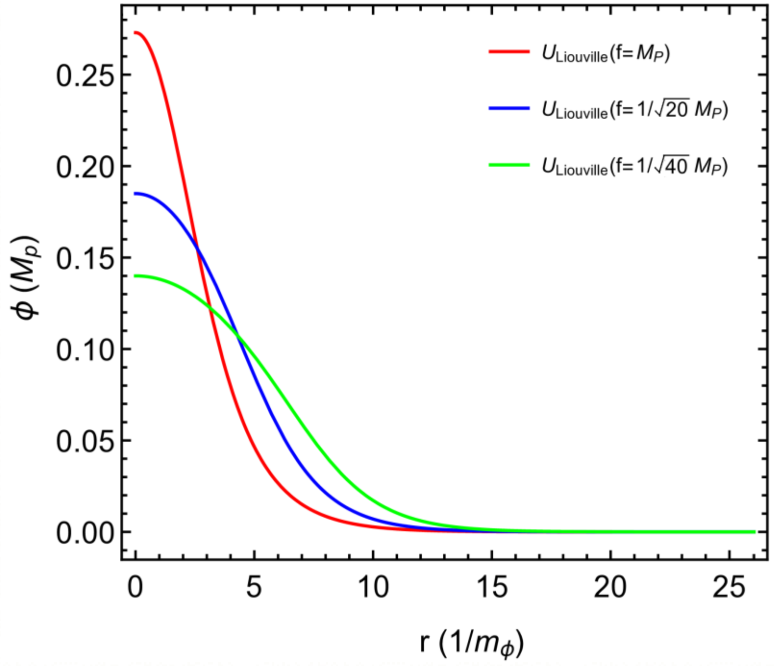
$$U_{Log} = f^2 {m_\Phi}^2 log\left(|\Phi|^2/f^2 + 1\right) \to U_{Log} = {m_\Phi}^2 \phi^2 - \frac{{m_\Phi}^2 \phi^4}{2f^2} + \frac{{m_\Phi}^2 \phi^6}{3f^4} - \frac{{m_\Phi}^2 \phi^8}{4 f^6} + \cdots$$



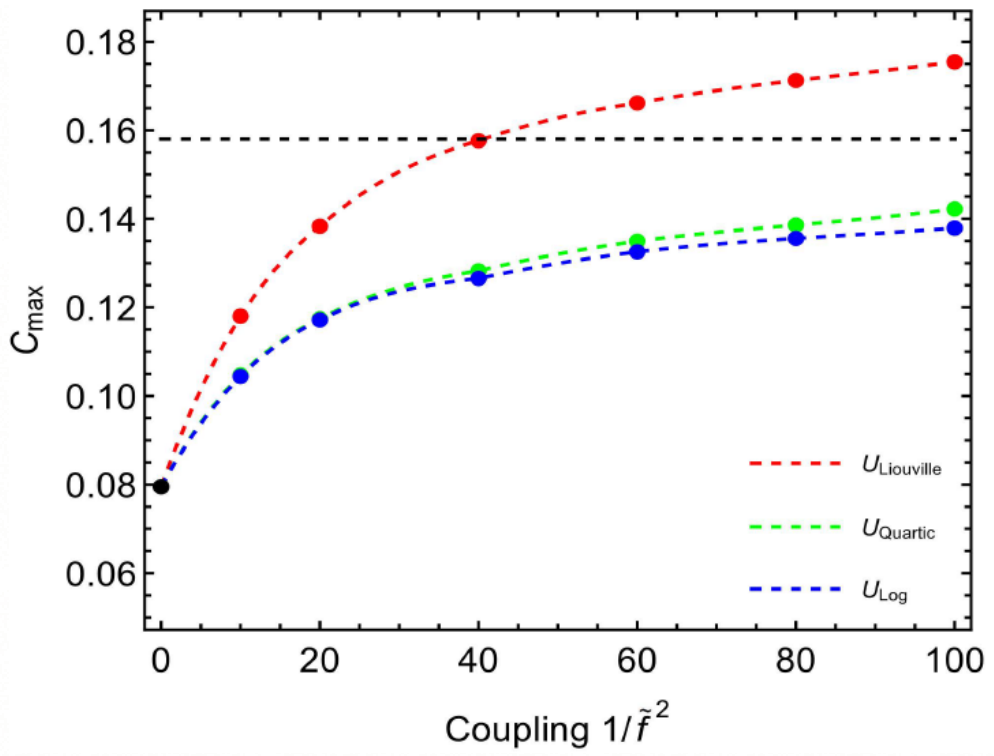
Analysis of the ground state configurations for boson stars with a logarithmic potential $U_{\rm Log}$, where $f = M_{\rm P}$. The mass M and number of particles N of the boson star are shown as functions of the central value of the scalar field ϕ_0 (left panel) and the corresponding bifurcation diagram (right panel). M, N, and ϕ_0 are shown in units of $m_{\rm p}^2/m_\Phi$, $m_{\rm p}^2/m_\Phi^2$, and $M_{\rm p}$, respectively.

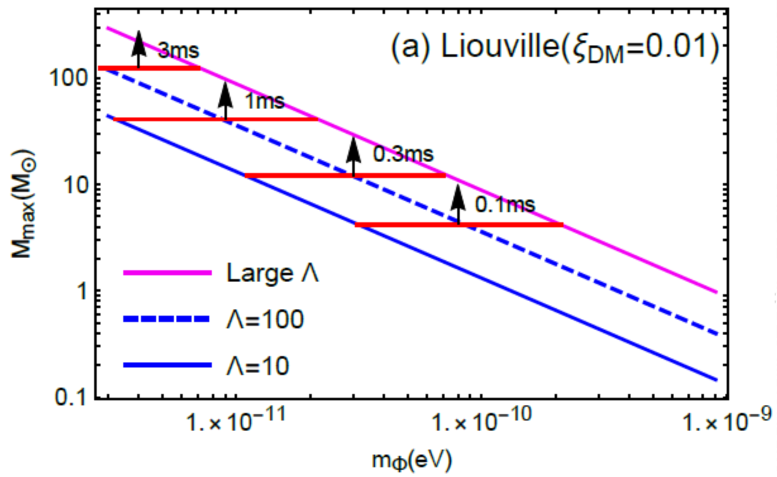
(Choi, He and Schiappacasse, **1906.02094**)

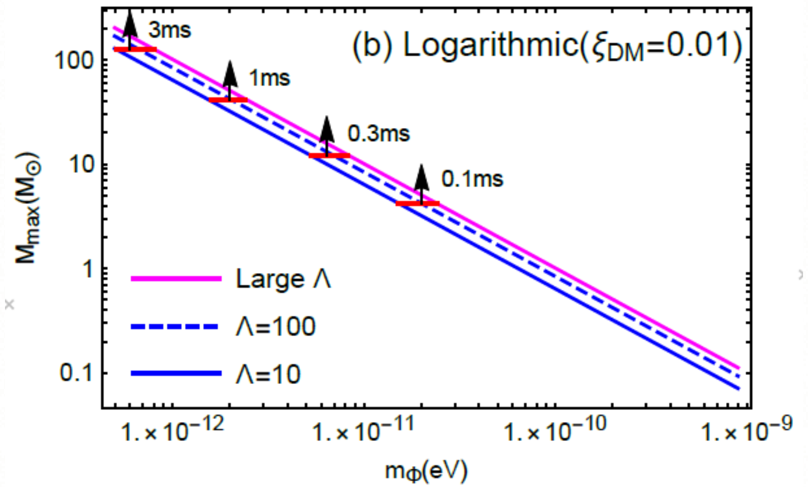




Possible Dectection: Chime is a novel transit radio telescope operating across the 400-800-MHz band. Wide bandwidth, high sensitivity, and a powerful correlator makes CHIME an excellent instrument for the detection of Fast Radio Bursts (FRBs). [CHIME/FRB Collaboration (1803.11235)]







Projected parameter space of the boson star models which can be probed by detecting the lensed fast radio burst (FRBs). Here ξ_{DM} is the dark matter fraction coming from boson stars. We assume ξ_{DM}=0.01. In each plot, the upper part of the red horizontal lines can be probed by the FRBs with the corresponding time delays $\overline{\Delta t}$. For each given coupling strength parameter Λ, the blue and magenta line show the relation between the scalar particle mass and the maximum mass of the boson star.

Effective maximum compactness $C_{max} = \frac{0.99\, M\, G_N}{R_{99}}$ as a function of $^1/_{f^2}$ (in units of $1/{M_P}^2$) for Liouville and Logarithmic potential. We have also added the usual case for a repulsive quartic self-interaction (green dashed curve). The black dashed curve corresponds to the asymptotic value of the maximum compactness when the coupling strength goes to infinity for the U_{Quartic}.

(Choi, He and Schiappacasse, **1906.02094**)

Scalar particle mass range derived from the mass constraint on the MACHO and the cold dark matter isocurvature modes constraint from the CMB.

$\boldsymbol{O(10^{-10})\ \mathrm{eV} < m_\Phi < O(10^{-3})\ \mathrm{eV}}$ for ξ_{DM}=0.01.

OUTLOOK

- In this work we have explored a possible novel consequence of the axion model, in which gravitationally axion bound clumps can form and undergo parametric resonance into electromagnetic radiation.
- For conventional values of axion-photon coupling, BEC (ground state) of axion dark matter can not undergo parametric resonance.
- For BEC axion dark matter with sufficiently large angular momentum, atypically large axion-photon coupling $g_{a\gamma} \gtrsim 1/f_a$, and for couplings to hidden sector, parametric resonance can occur.
- It would be interesting to further explore possible theoretical realizations of these more general possibilities as well as to explore possible hints of the idea of a clump mass pile-up.
- We have also explored generic scalar dark matter. In particular, we studied general relativistic boson stars with generic self-interactions: Liouville and Logarithmic scalar potential.
- Boson stars with both non-trivial scalar potentials are able to have stable ground state configurations.
- Since the Liouville potential can be seen as an infinite series of repulsive self-interacting terms after its Taylor expansion, boson stars under this scalar potential show a greater compactness in comparison to that of boson stars with either a Logarithmic potential or a repulsive quartic self-interacting term.
- Lensing of FRBs can be a plausible method of detection of these boson stars constraining their parameter space.

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Effective Photon Mass

- In the not-quite-empty space of the interstellar medium, photons acquire an effective mass equal to the plasma frequency as

$$\omega_p^2 = \frac{4\pi\alpha n_e}{m_e} = \frac{n_e}{0.03\text{cm}^{-3}}(6.4\times 10^{-12}\text{eV})^2$$

- Considering the spatial distribution of n_e and the fact that axion clump condensates are moving in the galactic halo: $\omega_p(t) \approx \omega_p f(t)$, where $f(t)$ is a non-periodic time dependent function of orden 1.
- The modified equation for the mode function of the vector potential for the homogeneous case is

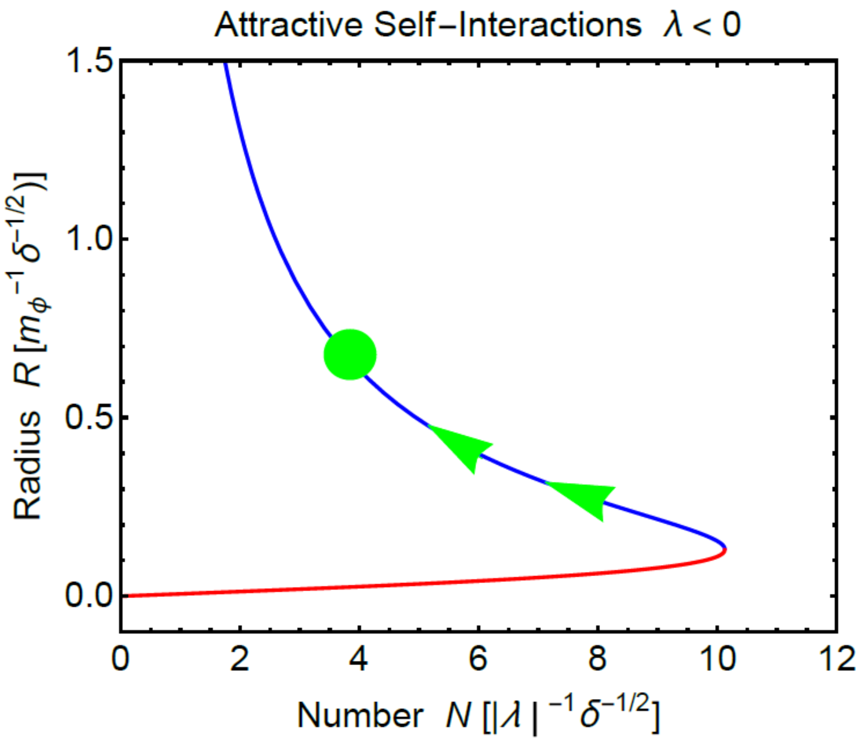
$$\ddot{s}_k + [k^2 + \omega_p^2(t) - g_{a\gamma}\omega_0 k\phi_0 \sin(\omega_0 t)]s_{\boldsymbol{k}} = 0$$

- Taking $k \approx (m_\phi/2)$ and using as reference the amplitude ϕ_0 evaluated for the case of a sech ansatz, we have

$$\frac{\omega_p^2}{(g_{a\gamma}\omega_0 k\phi_0)} \sim \frac{10^{-23}}{\left(\frac{\beta}{10^{-2}}\right)10^{-19}} \sim 10^{-4}$$

So, we expect that the effect of the effective photon mass in the resonance should be negligible

Astrophysical Consequences



Clump radius R as a function of clump number N for spherically symmetric clumps with attractive self-interactions. We have taken the axion-photon coupling to be $\tilde{g}_{a\gamma} = \frac{g_{a\gamma} f_a}{\sqrt{\gamma}} = 2$ here. For any clumps on the stable branch with number $N > N_c$ they will resonantly produce photons, lose mass, and pile-up at the critical value $N_c \approx 3.7/(|\lambda|\sqrt{\delta})$.

(Hertzberg and Schiappacasse, **1805.00430**)

- Suppose $g_{a\gamma} > g_{a\gamma,min}$. The photon occupancy number will increase from 0 to a large value, then the final output is a essentially classical electromagnetic waves.

- After clumps formation, clumps with sufficiently large mass will undergo parametric resonance into photons.

- If $N > N_c$, the clump will radiate into photons, losing mass until $M \to M_c = N_c m_\phi$: PILE-UP AT A UNIQUE VALUE OF CLUMPS.

Electromagnetic Emission in the Sky

- We can imagine a scenario in which the process of resonance is still occurring. Consider a pair of clump condensates, each with number N_1 and N_2 ($N_1 < N_c$ and $N_2 < N_c$).

- Suppose that these clumps merge together in the late universe. If $N_{Total} = N_1 + N_2 > N_c$, then the resonance will suddenly begin to occur, driving the total towards $N_{Total} \to N_c$.

- So, we expect a sudden emission of electromagnetic radiation in the galaxy:

$$\lambda_{EM} \approx {}^{2\pi}/_{k^*} \approx {}^{4\pi}/_{m_\phi}$$

$$\boxed{\lambda_{EM} \sim 10^{-1}\mathrm{m}}$$

- The typical mass of a clump is the order of $\sim 10^{-11} M_\odot$, which is comparable to the moon's mass. If the merger took place, an amount of energy comparable to Mc^2 equivalent to Moon's mass would be emitted to the galaxy.

Estimate Growth time-scale

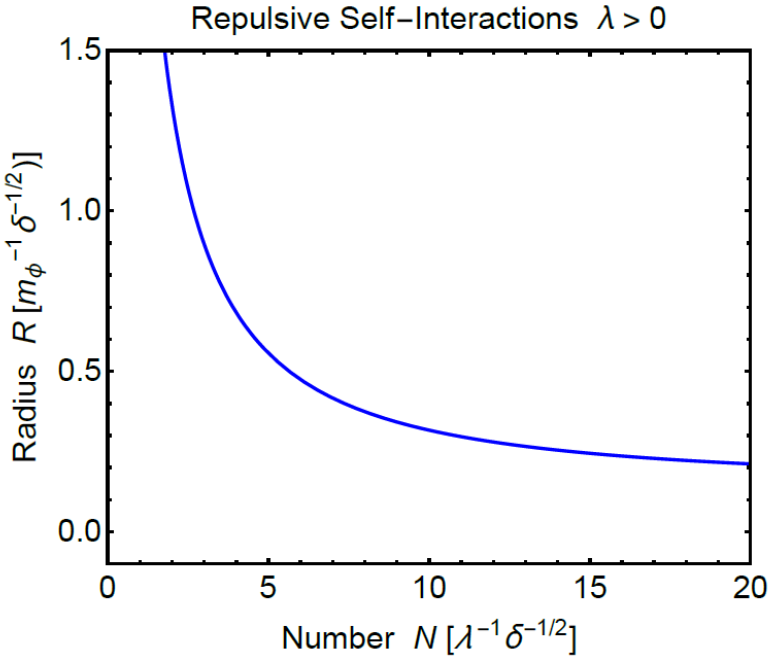
- Suppose $g_{a\gamma} > g_{a\gamma,min}$. This will lead to an exponential growth in the electromagnetic field.
- The photon occupancy number will increase from 0 to a very large value, then the final output is essentially classical electromagnetic waves.

- We can estimate a lower bound on the time-scale for this growth by taking $\mu^* \sim \mu_H^*$. Let us consider the true BEC ground states. Since the condition for resonance is given by $g_{a\gamma} f_a > 0.3$, we have

$$\mu^* \sim 15 g_{a\gamma} f_a m_\phi \sqrt{\delta} \gtrsim 5 m_\phi \sqrt{\delta}$$

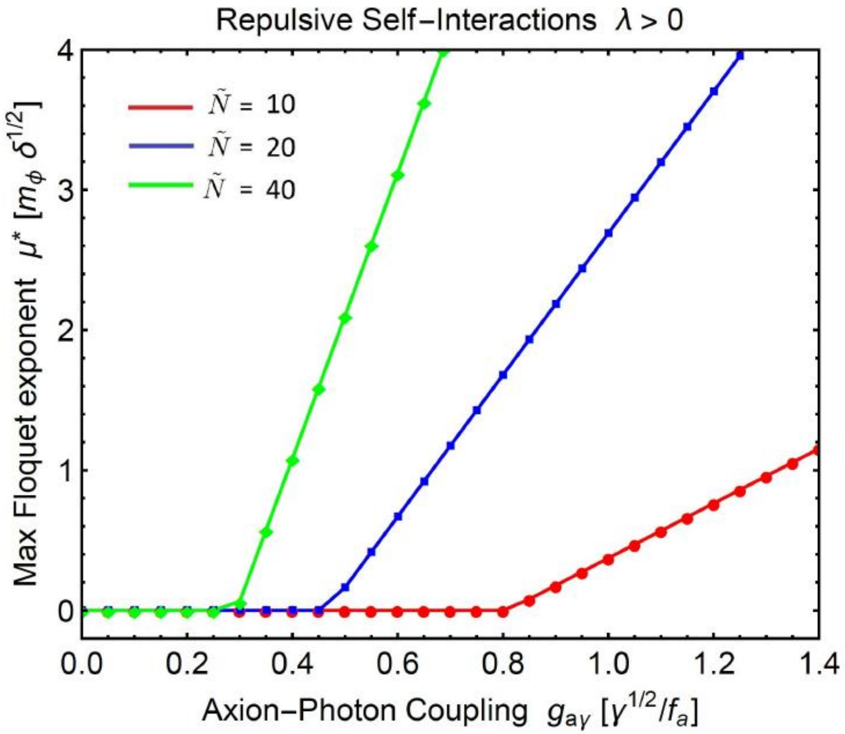
For typical values of QCD axion, we have $\tau = 1/\mu^* \lesssim O(10^{-4}\mathrm{sec})$

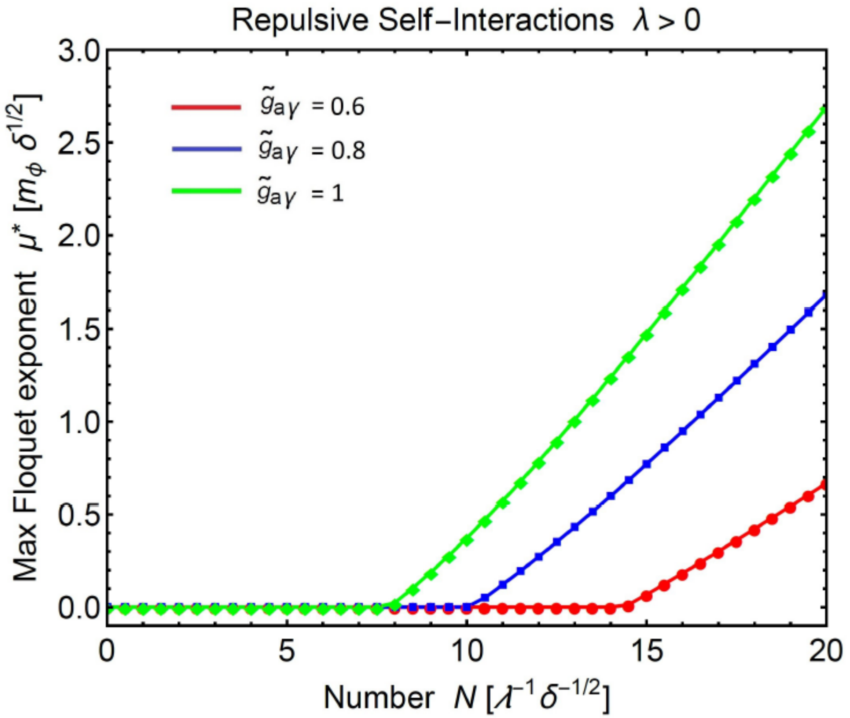
Repulsive Self-Interactions (Non-relativistic Regime)



- For repulsive interactions we have only $\lambda > 0$ there is only 1 branch of solution. This branch is stable and there is no a maximum number of particles.

(Hertzberg and Schiappacasse, **1805.00430**)





The maximum real part of Floquet exponent μ^*, describing parametric resonance of photons from a spherically symmetric clump condensate, as a function of axion-photon coupling $g_{a\gamma}$. (Left) We plot μ^* in units of $m_\phi\sqrt{\delta}$, $g_{a\gamma}$ in units of $\frac{\sqrt{\gamma}}{f_a}$, and $\tilde{N} = N/(|\lambda|\sqrt{\delta})$. (Right) We plot μ^* in units of $m_\phi\sqrt{\delta}$ and N in units of $|\lambda|^{-1}\delta^{-1/2}$. Here $\tilde{g}_{a\gamma} = g_{a\gamma}f_a/\sqrt{\gamma}$.