Gravitational waves induce by non-Gaussian scalar perturbations

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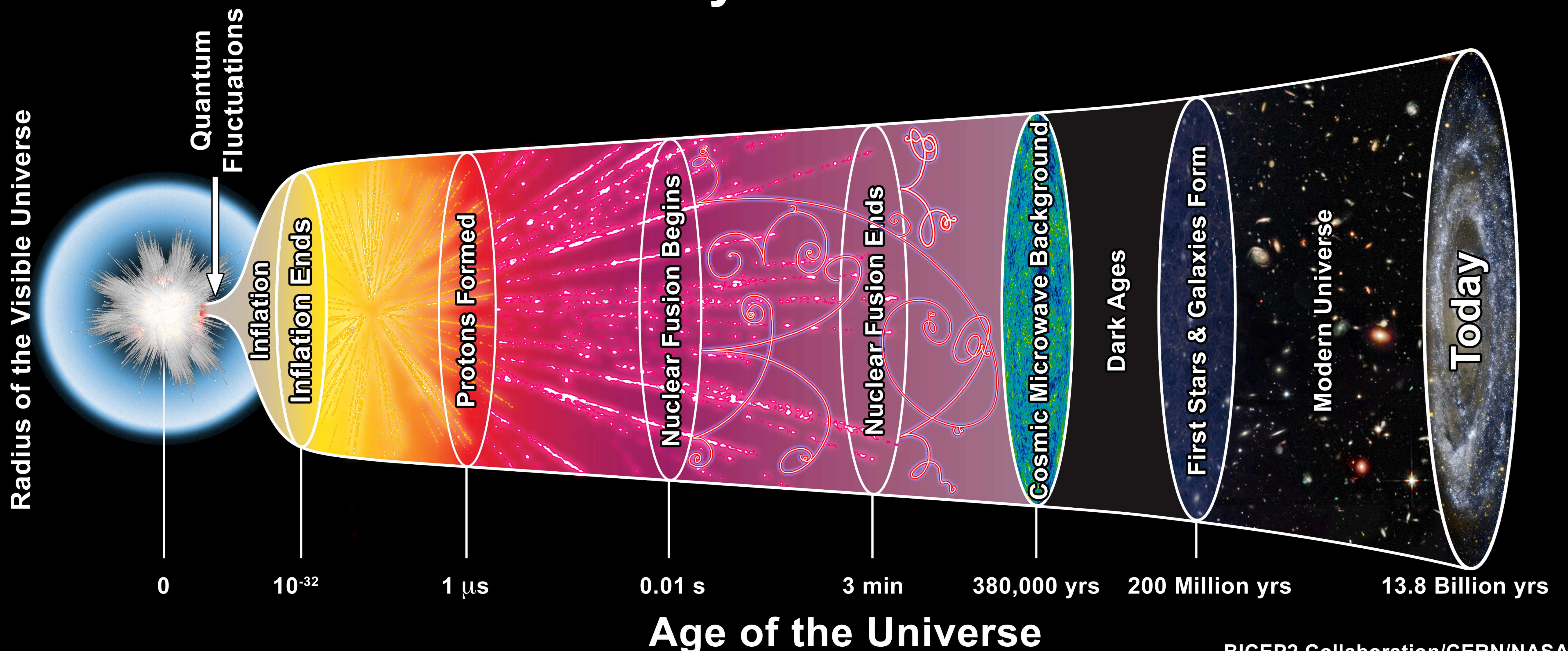
Based on: Rong-gen Cai, SP and Misao Sasaki,
arXiv:1810.11000, Phys.Rev.Lett.**122**.201101;
arXiv:1906.XXXXX, in preparation.

43rd Johns Hopkins Workshop, Kavli IPMU, June 6th, 2019

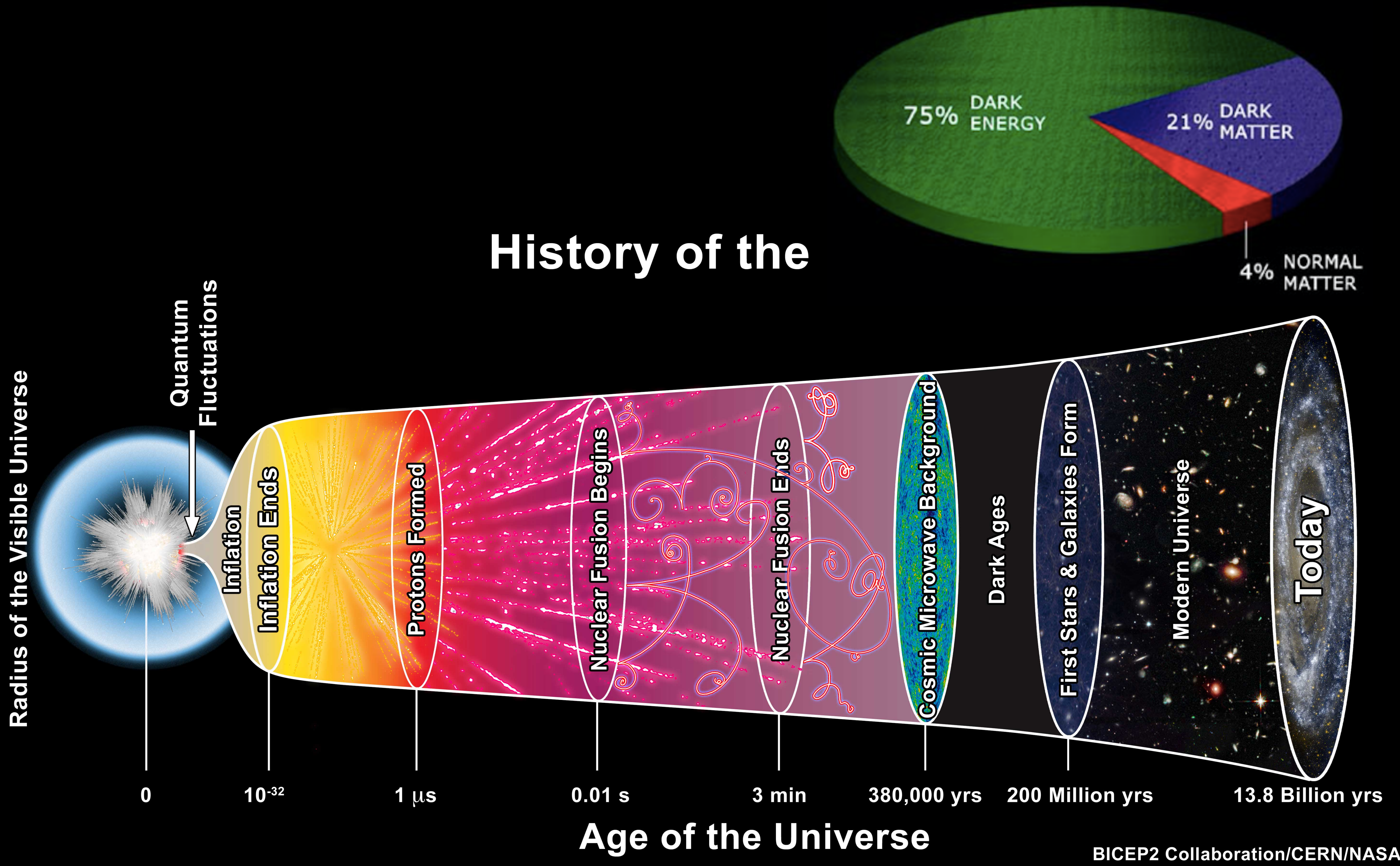
Content

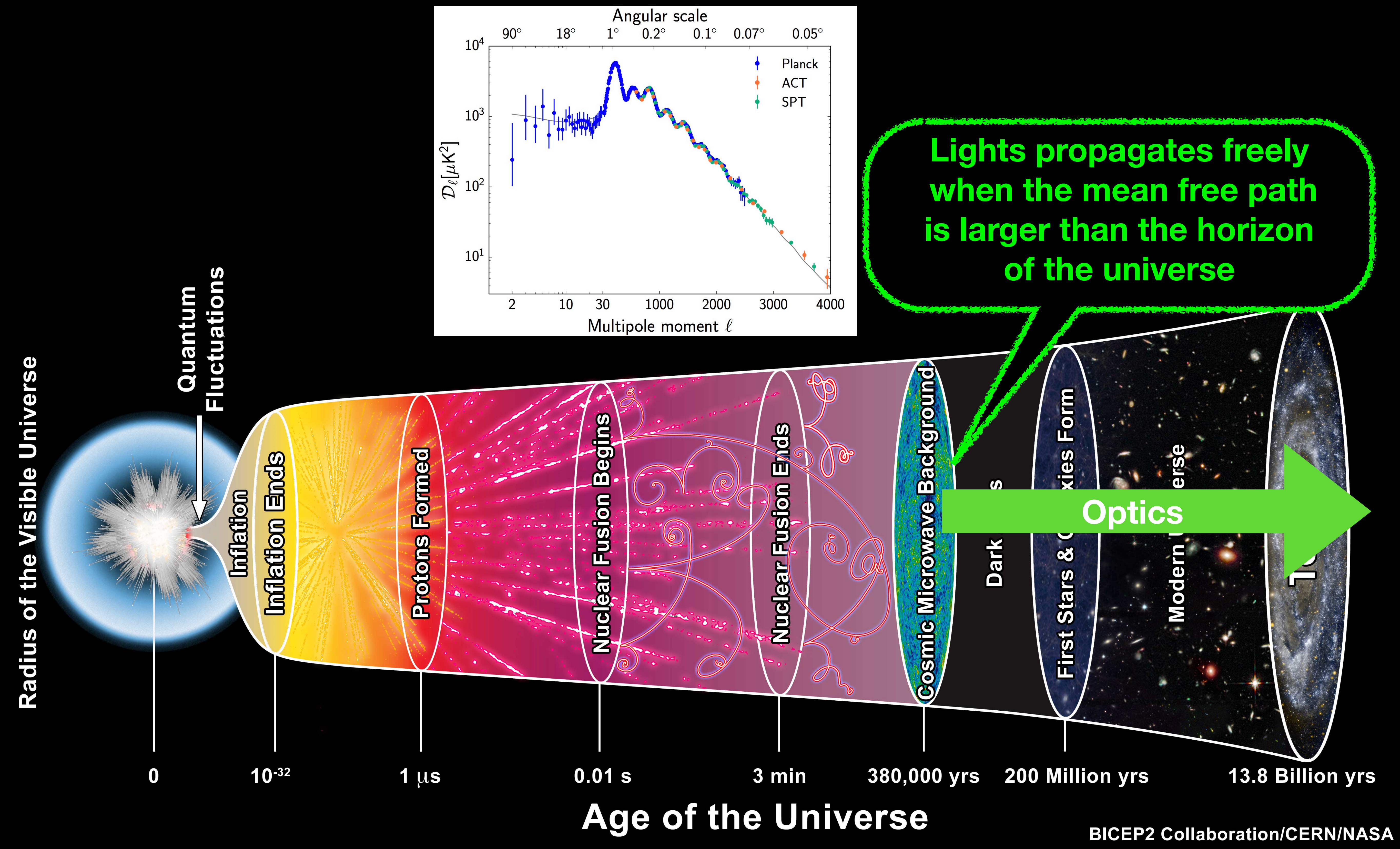
- Cosmic History and Stochastic GWs
- PBH formation
- Induced GWs
- Summary

History of the Universe

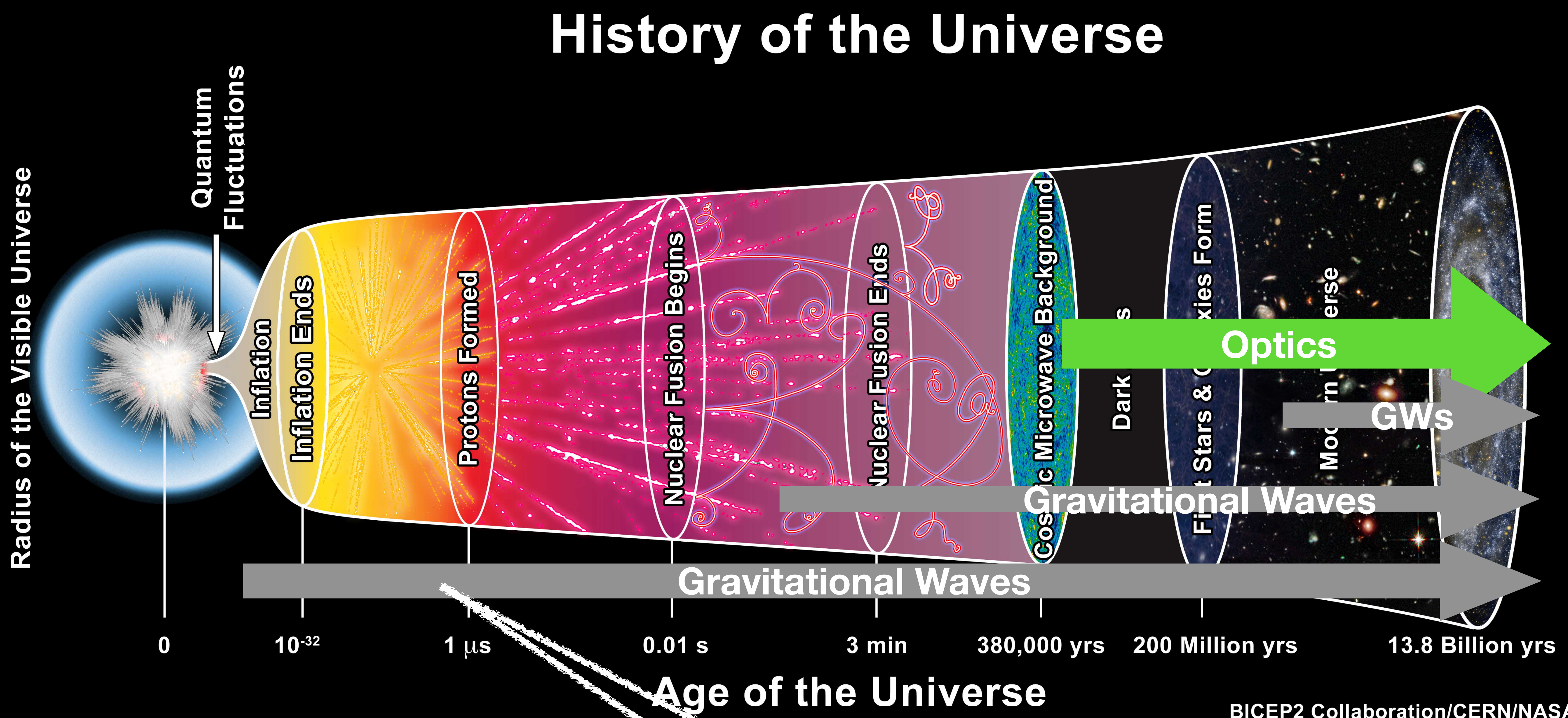


BICEP2 Collaboration/CERN/NASA

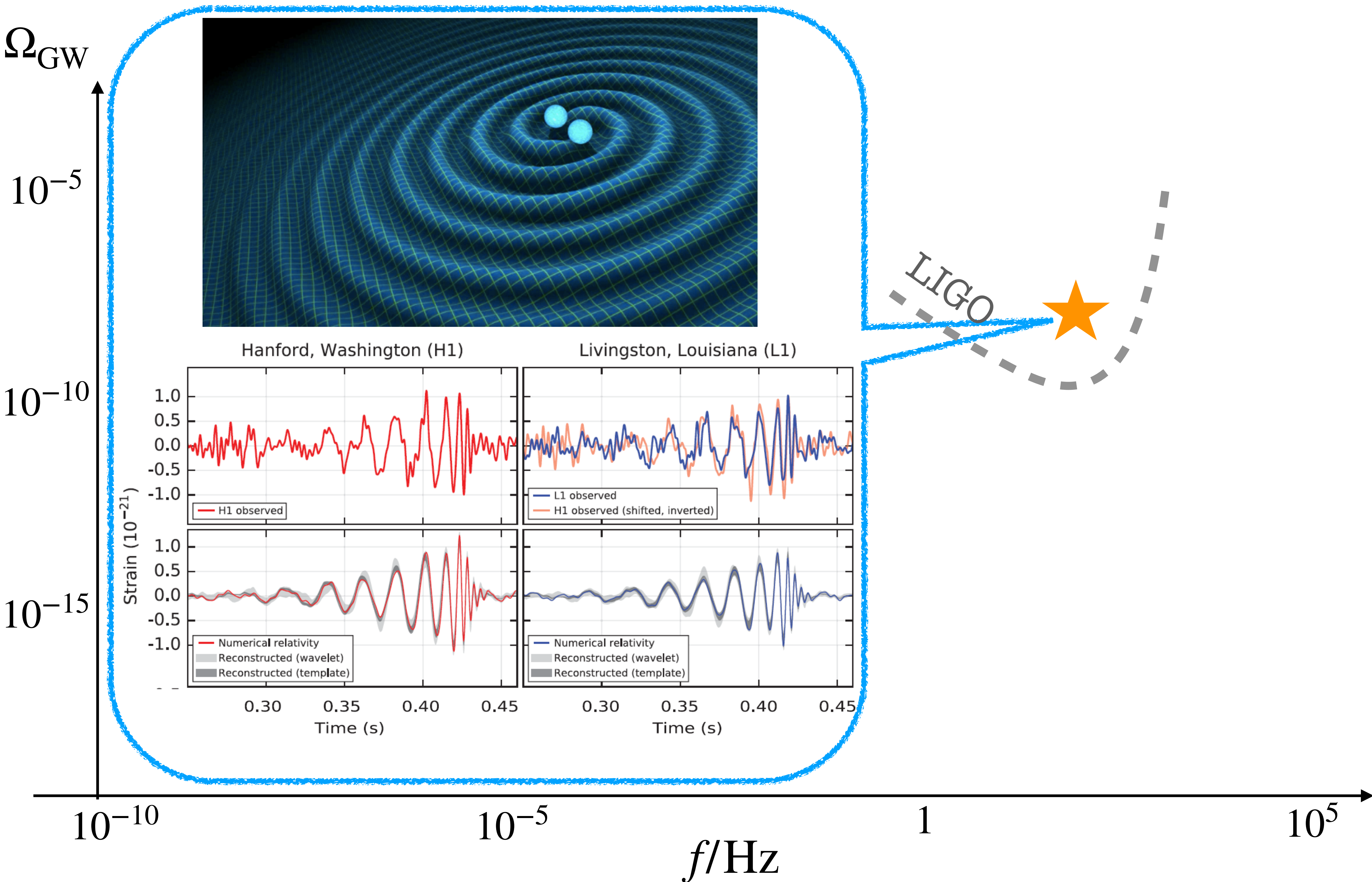

75% DARK ENERGY
21% DARK MATTER
4% NORMAL MATTER
History of the
Radius of the Visible Universe
Quantum Fluctuations
Inflation
Inflation Ends
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Dark Ages
First Stars & Galaxies Form
Modern Universe
Today
0
10-32
1 μs
0.01 s
3 min
380,000 yrs
200 Million yrs
13.8 Billion yrs
Age of the Universe
BICEP2 Collaboration/CERN/NASA



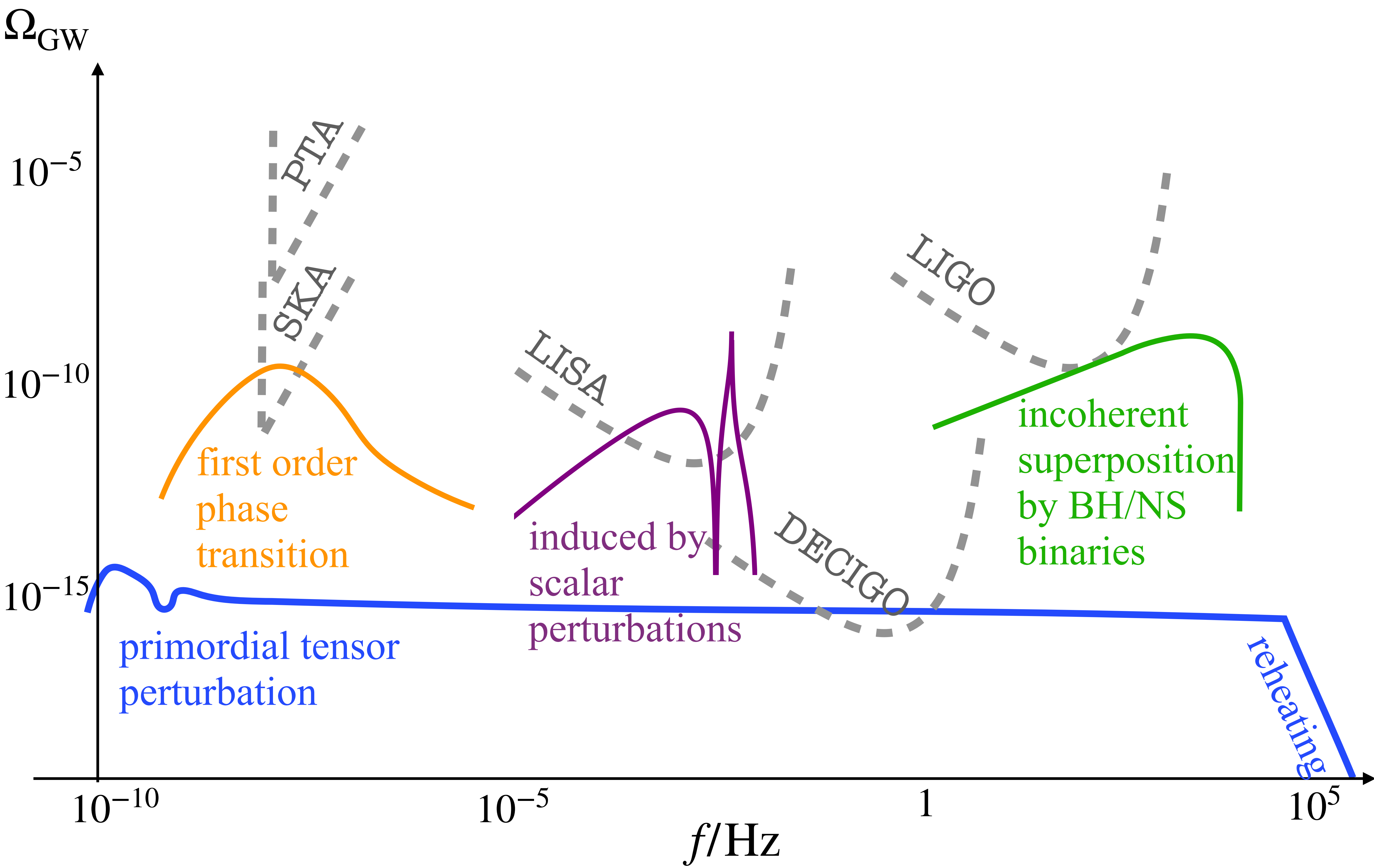
Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
0.05°
10^4
10^3
10^2
10^1
$\mathcal{D}_\ell [\mu K^2]$
Planck
ACT
SPT
2
10
30
1000
2000
3000
4000
Multipole moment ℓ
Lights propagates freely when the mean free path is larger than the horizon of the universe
Radius of the Visible Universe
Quantum Fluctuations
Inflation
Inflation Ends
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Dark
First Stars &
Modern
Optics
0
10^{-32}
1 μs
0.01 s
3 min
380,000 yrs
200 Million yrs
13.8 Billion yrs
Age of the Universe
BICEP2 Collaboration/CERN/NASA



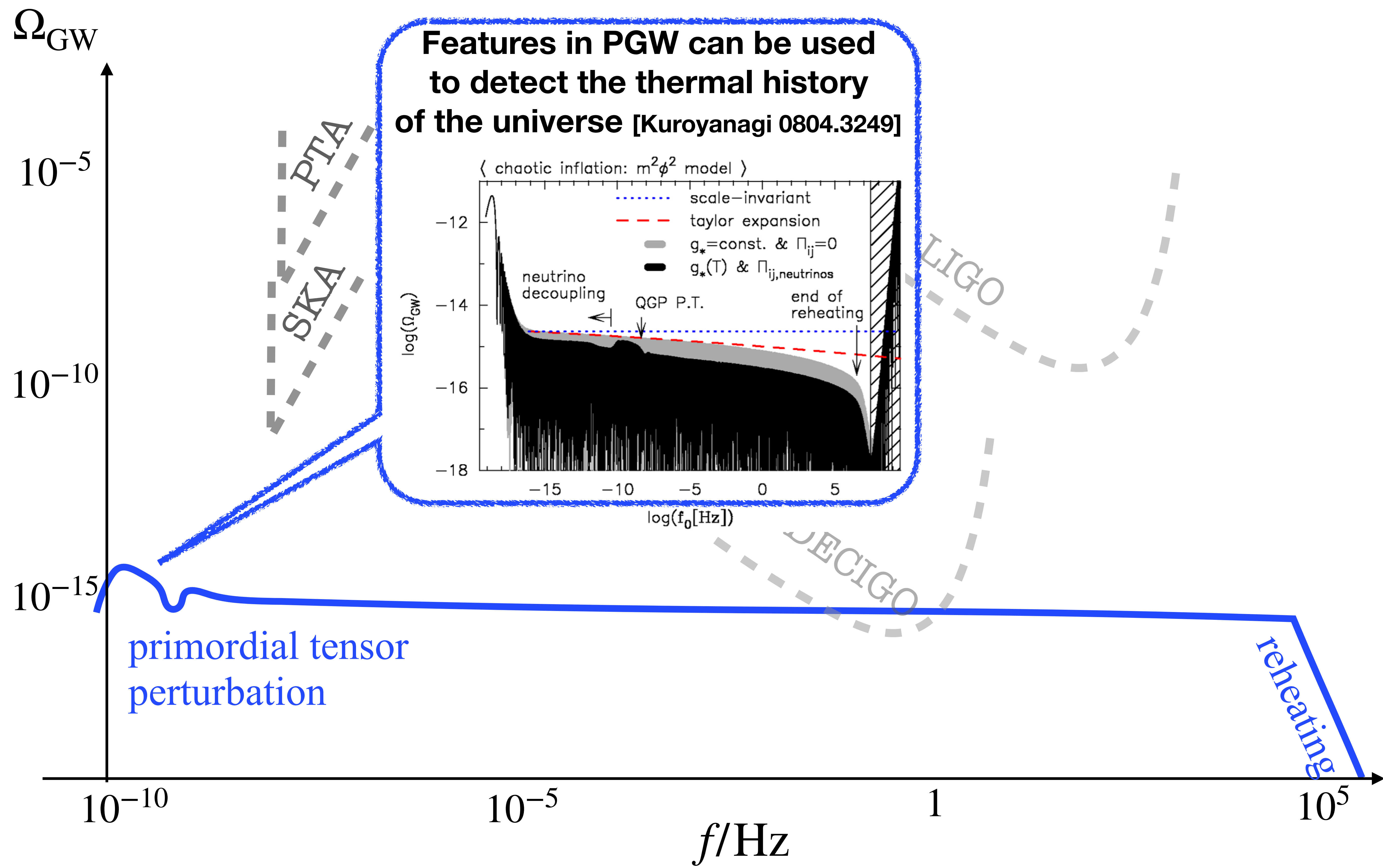
GWs propagates freely all the time!



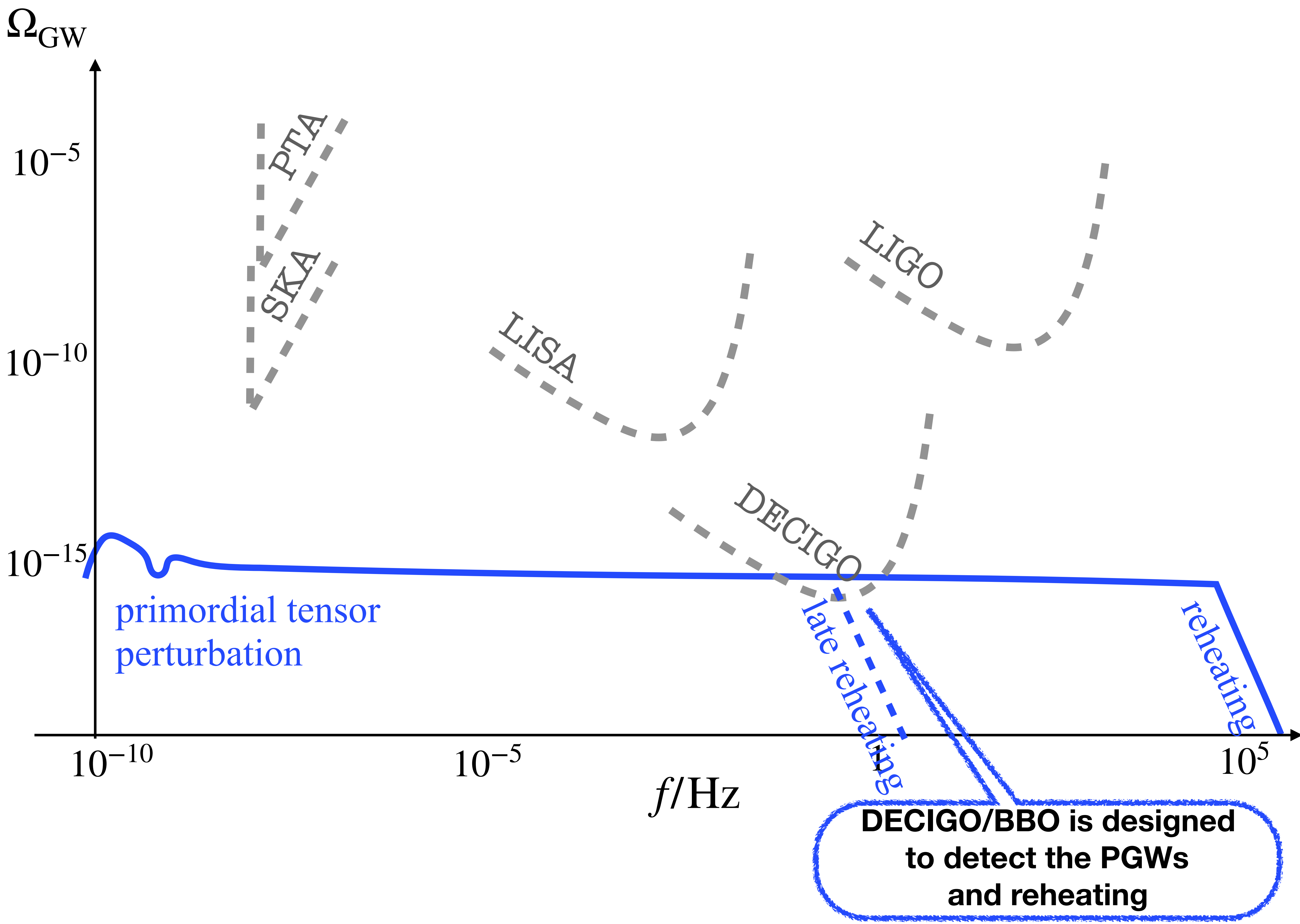
Fractional energy density: $\Omega_{GW} = \dfrac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$



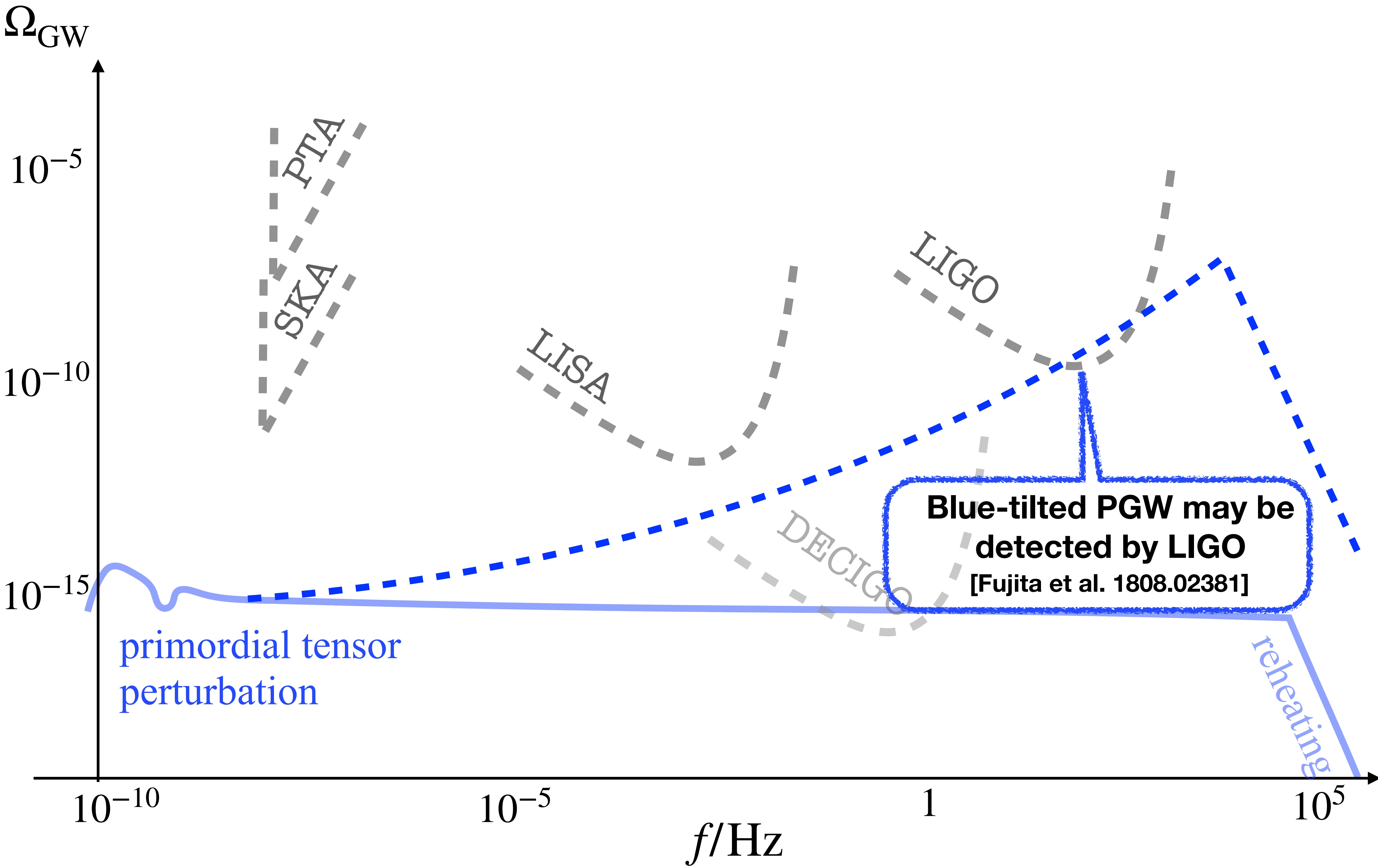
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
first order phase transition
induced by scalar perturbations
incoherent superposition by BH/NS binaries
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz



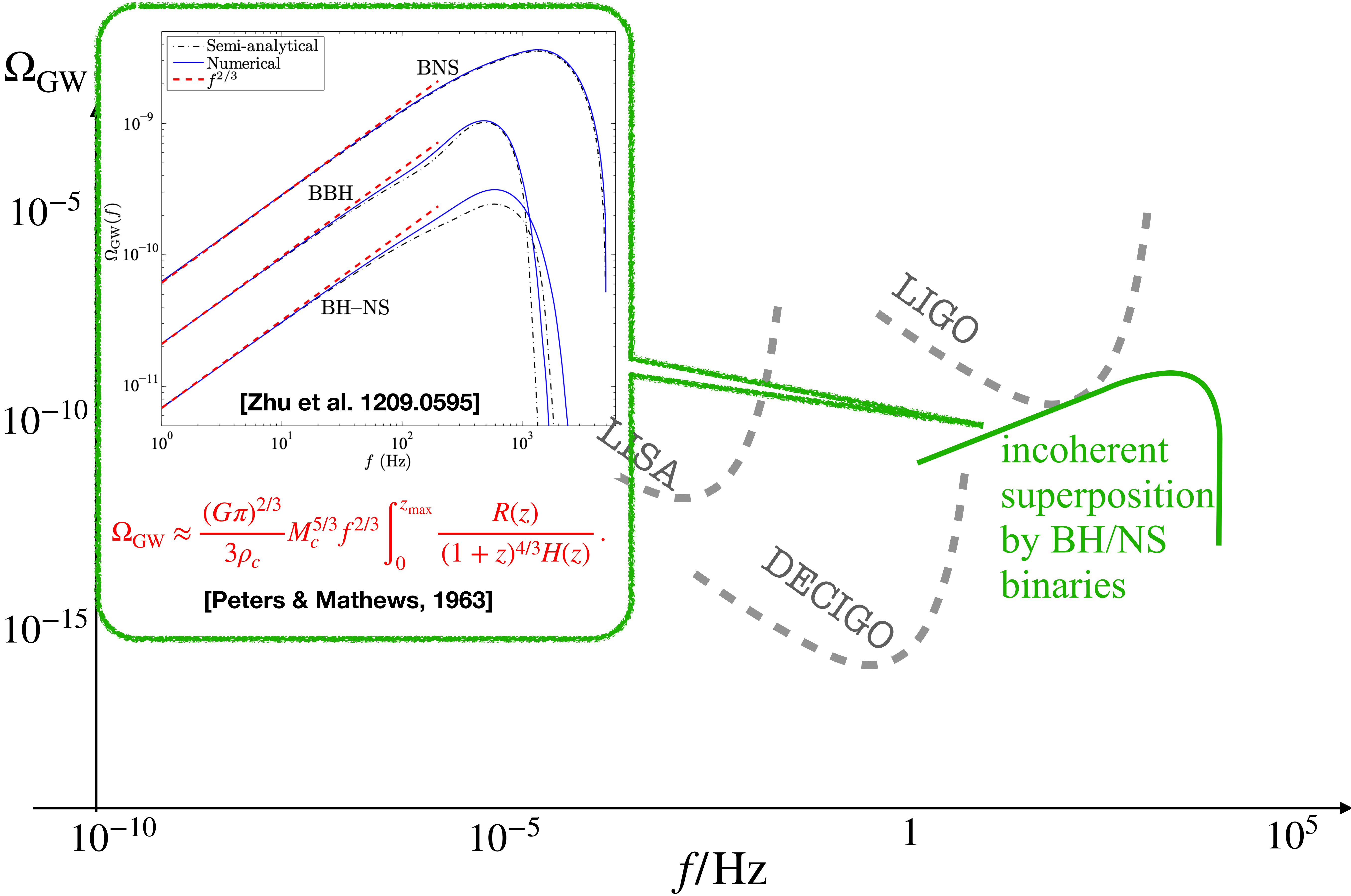
Features in PGW can be used to detect the thermal history of the universe [Kuroyanagi 0804.3249]
⟨ chaotic inflation: m²φ² model ⟩
scale−invariant
taylor expansion
g*=const. & Πij=0
g*(T) & Πij,neutrinos
neutrino decoupling
QGP P.T.
end of reheating
log(ΩGW)
log(f0[Hz])
−12
−14
−16
−18
−15
−10
−5
0
5
ΩGW
10−5
10−10
10−15
PTA
SKA
LIGO
DECIGO
primordial tensor perturbation
reheating
10−10
10−5
1
105
f/Hz



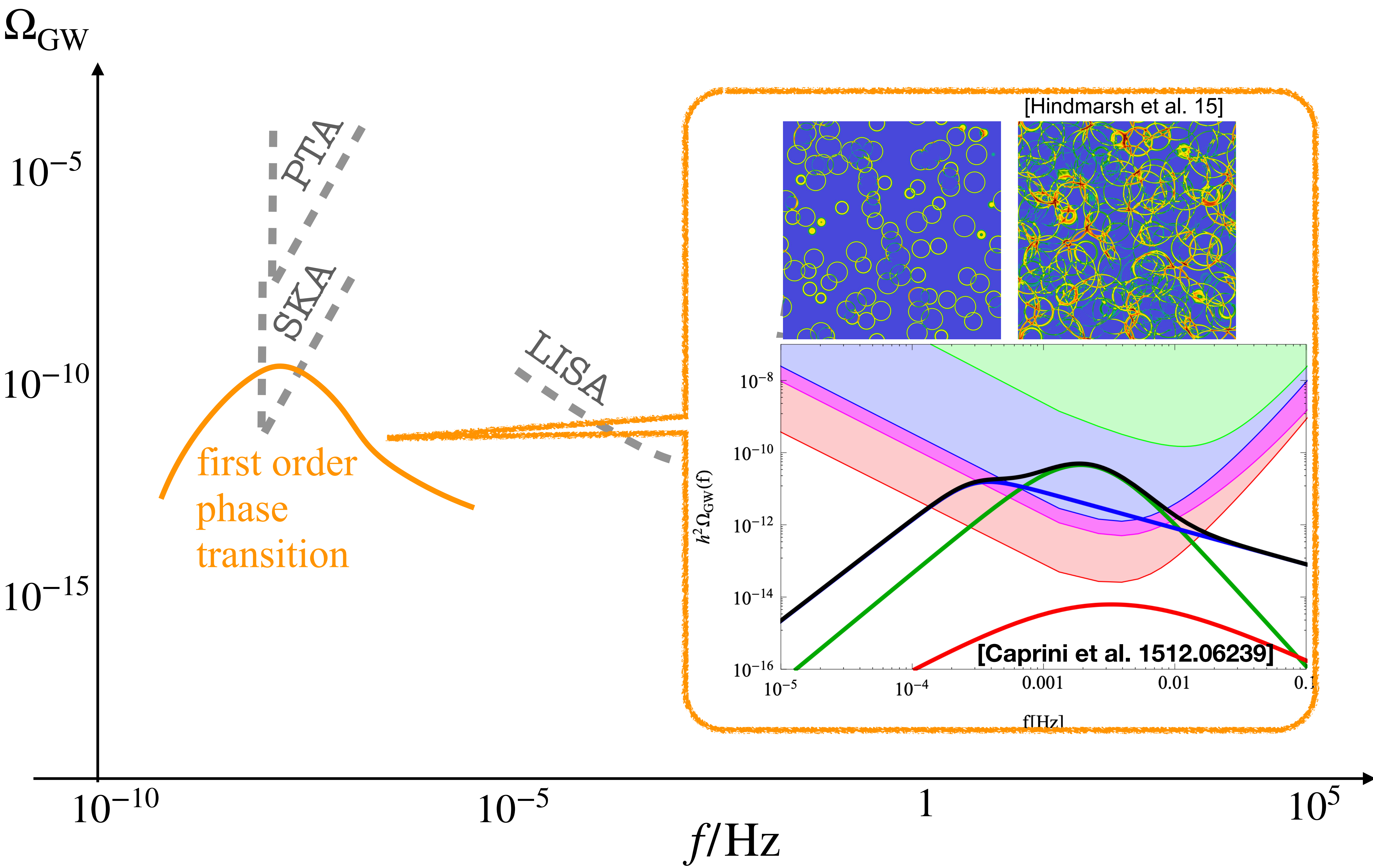
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
primordial tensor perturbation
late reheating
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz
DECIGO/BBO is designed to detect the PGWs and reheating



Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
Blue-tilted PGW may be detected by LIGO
[Fujita et al. 1808.02381]
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz



Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
10^{-10}
10^{-5}
1
10^{5}
f/Hz
Semi-analytical
Numerical
$f^{2/3}$
BNS
BBH
BH–NS
$\Omega_{GW}(f)$
10^{-9}
10^{-10}
10^{-11}
10^{0}
10^{1}
10^{2}
10^{3}
f (Hz)
[Zhu et al. 1209.0595]
$$\Omega_{GW} \approx \frac{(G\pi)^{2/3}}{3\rho_c} M_c^{5/3} f^{2/3} \int_0^{z_{max}} \frac{R(z)}{(1+z)^{4/3} H(z)} .$$
[Peters & Mathews, 1963]
LISA
LIGO
DECIGO
incoherent superposition by BH/NS binaries



Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
first order
phase
transition
[Hindmarsh et al. 15]
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
$h^2\Omega_{GW}(f)$
[Caprini et al. 1512.06239]
10^{-5}
10^{-4}
0.001
0.01
0.1
f[Hz]
10^{-10}
10^{-5}
1
10^{5}
f/Hz

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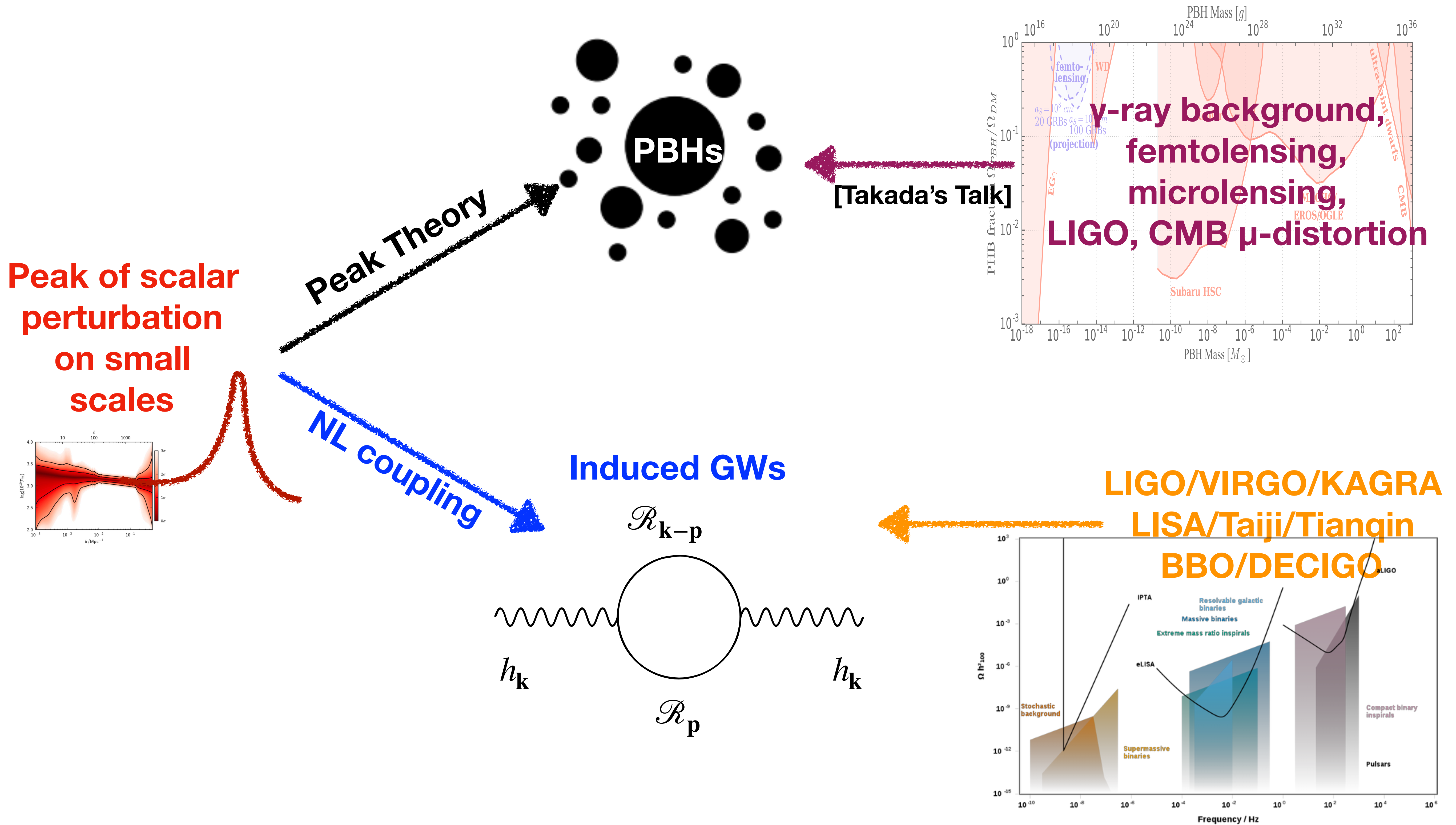
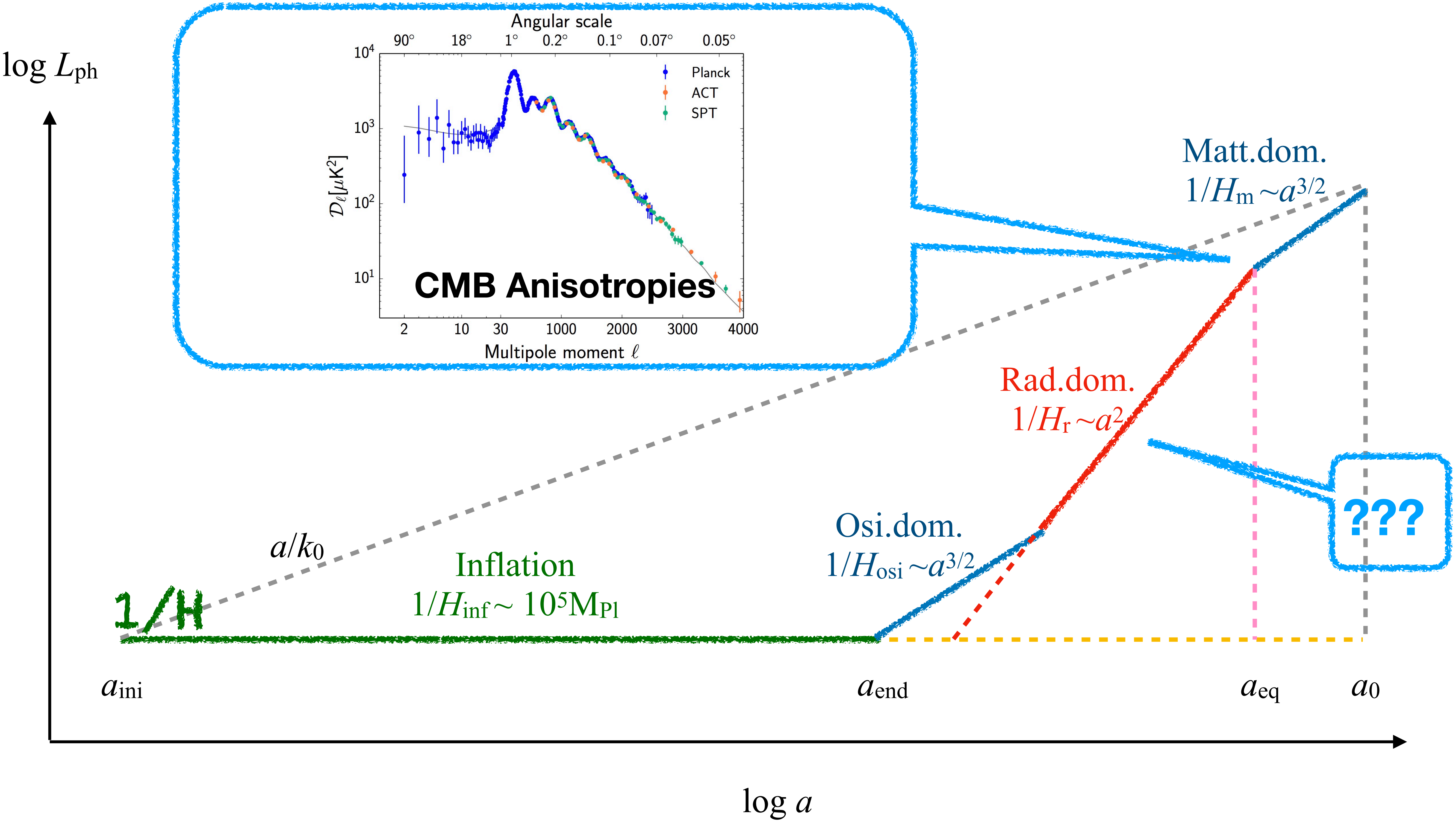
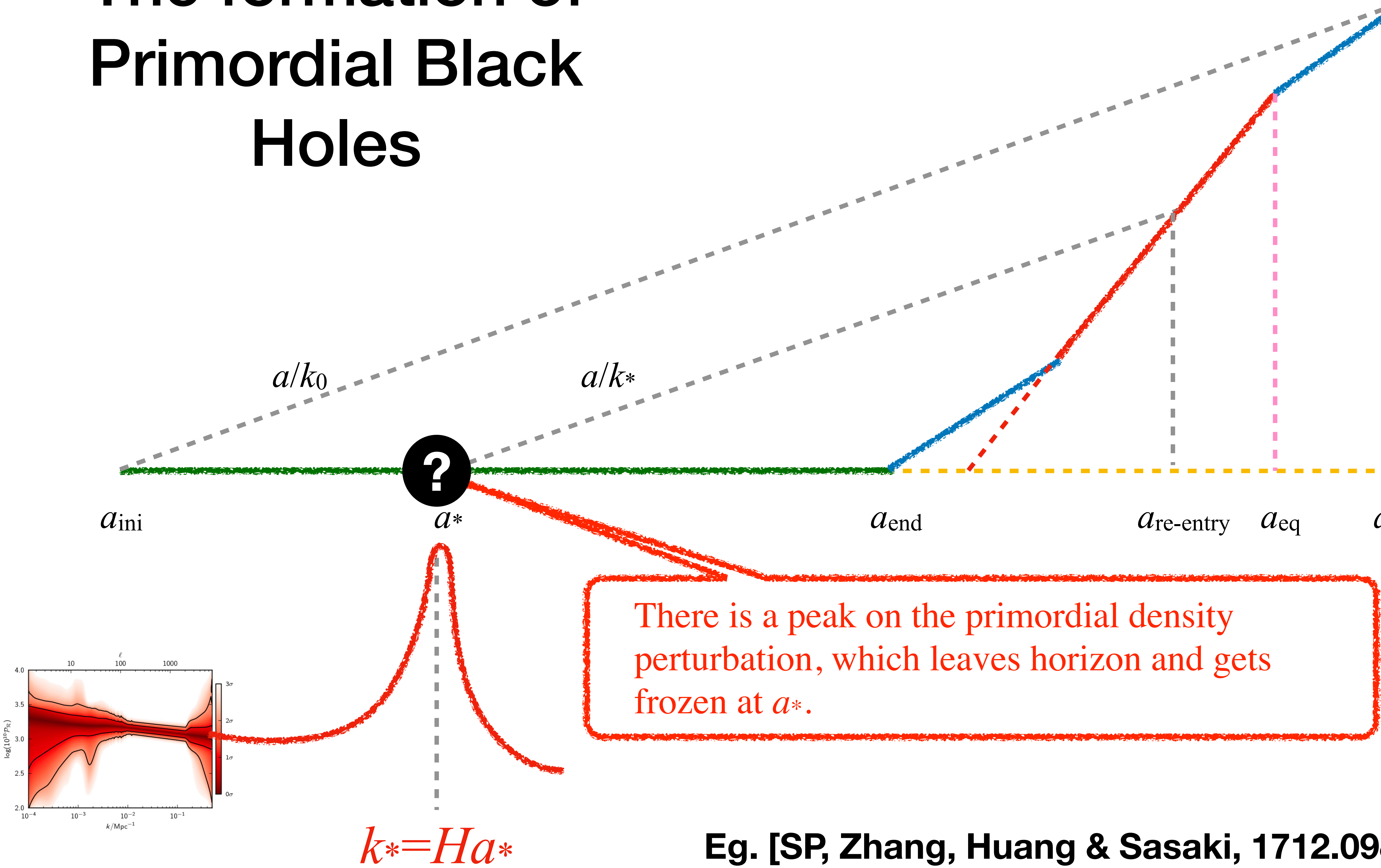
Primordial Black Holes and Induced Gravitational Waves





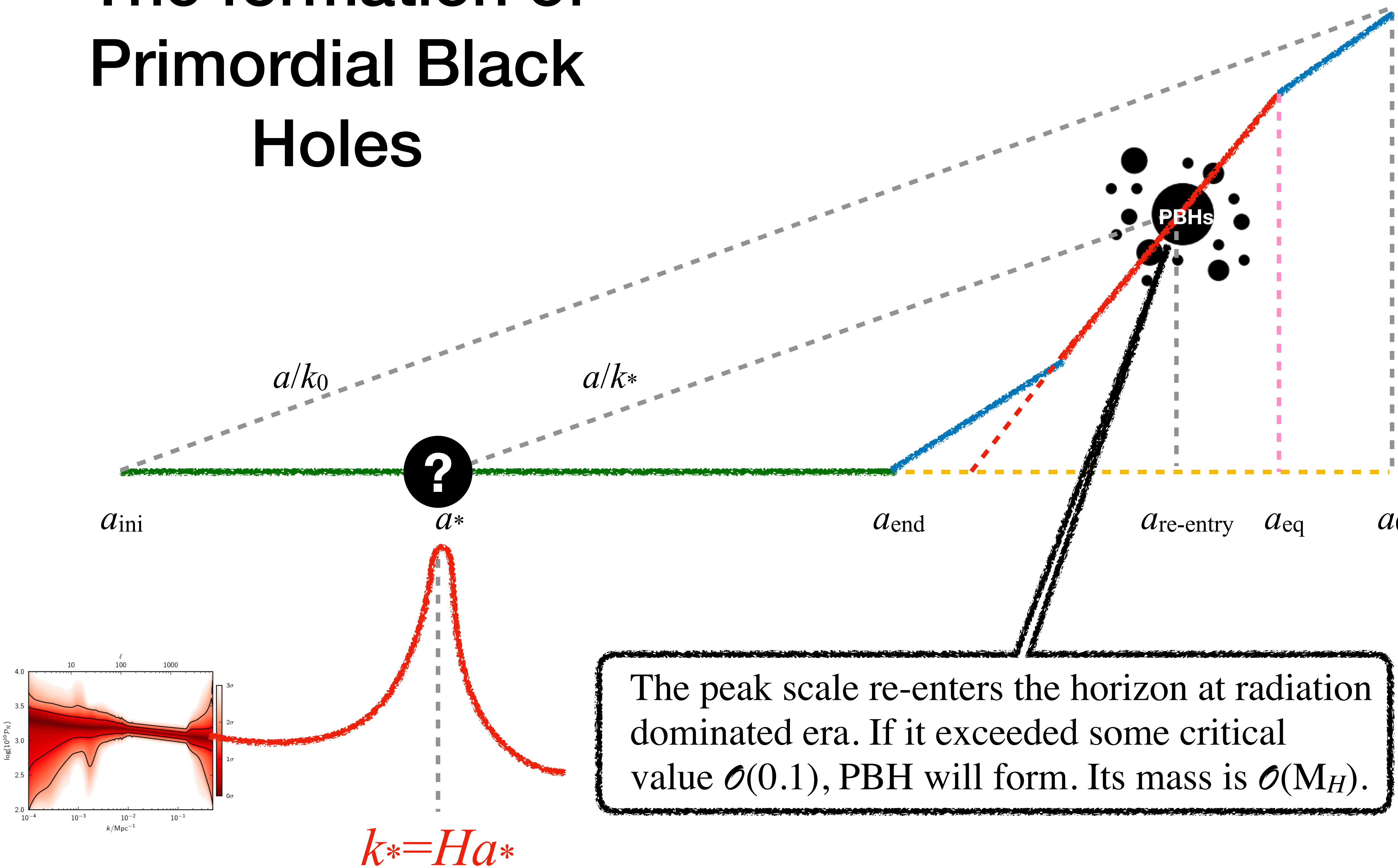
log L_{ph}
Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
10^4 10^3 10^2 10^1
$\mathcal{D}_\ell[\mu K^2]$
Planck
ACT
SPT
CMB Anisotropies
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
Matt.dom.
$1/H_m \sim a^{3/2}$
Rad.dom.
$1/H_r \sim a^2$
???
Osi.dom.
$1/H_{osi} \sim a^{3/2}$
a/k_0
Inflation
$1/H_{inf} \sim 10^5 M_{Pl}$
1/H
a_{ini}
a_{end}
a_{eq}
a_0
log a

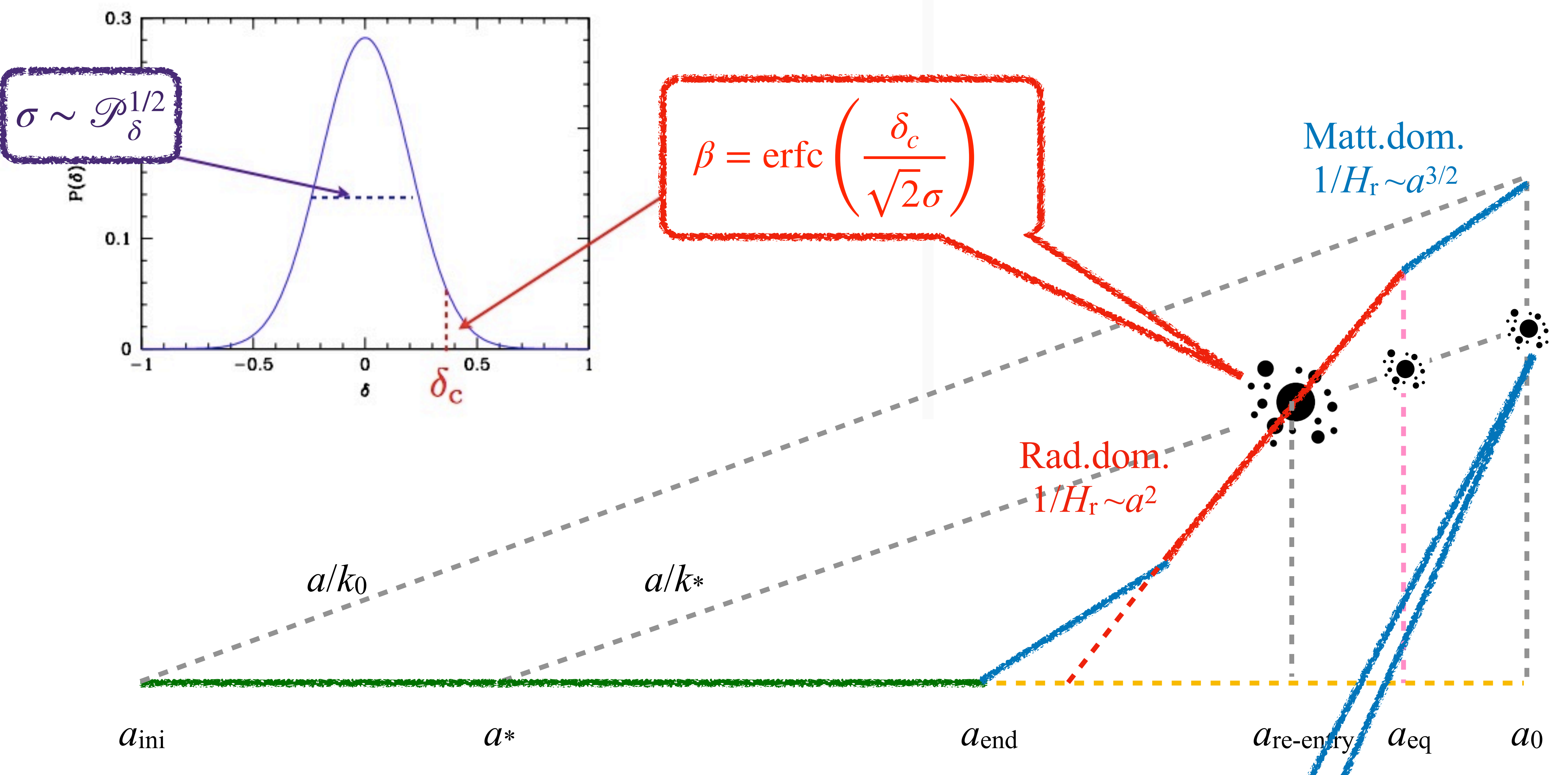
The formation of Primordial Black Holes



Eg. [SP, Zhang, Huang & Sasaki, 1712.09896]

The formation of Primordial Black Holes





$$\Omega_{\rm PBH} = \beta \frac{a_{\rm eq}}{a_{\rm re}} = \beta \frac{a_{\rm eq}}{a_0} \frac{a_0}{a_{\rm re}} \simeq \beta \Omega_r (1 + z_{\rm re}(M))$$

$$M = \frac{c^3}{G H_{\rm re}} = \frac{c^3}{G \Omega_r^{1/2} (1+z)^2 H_0}$$

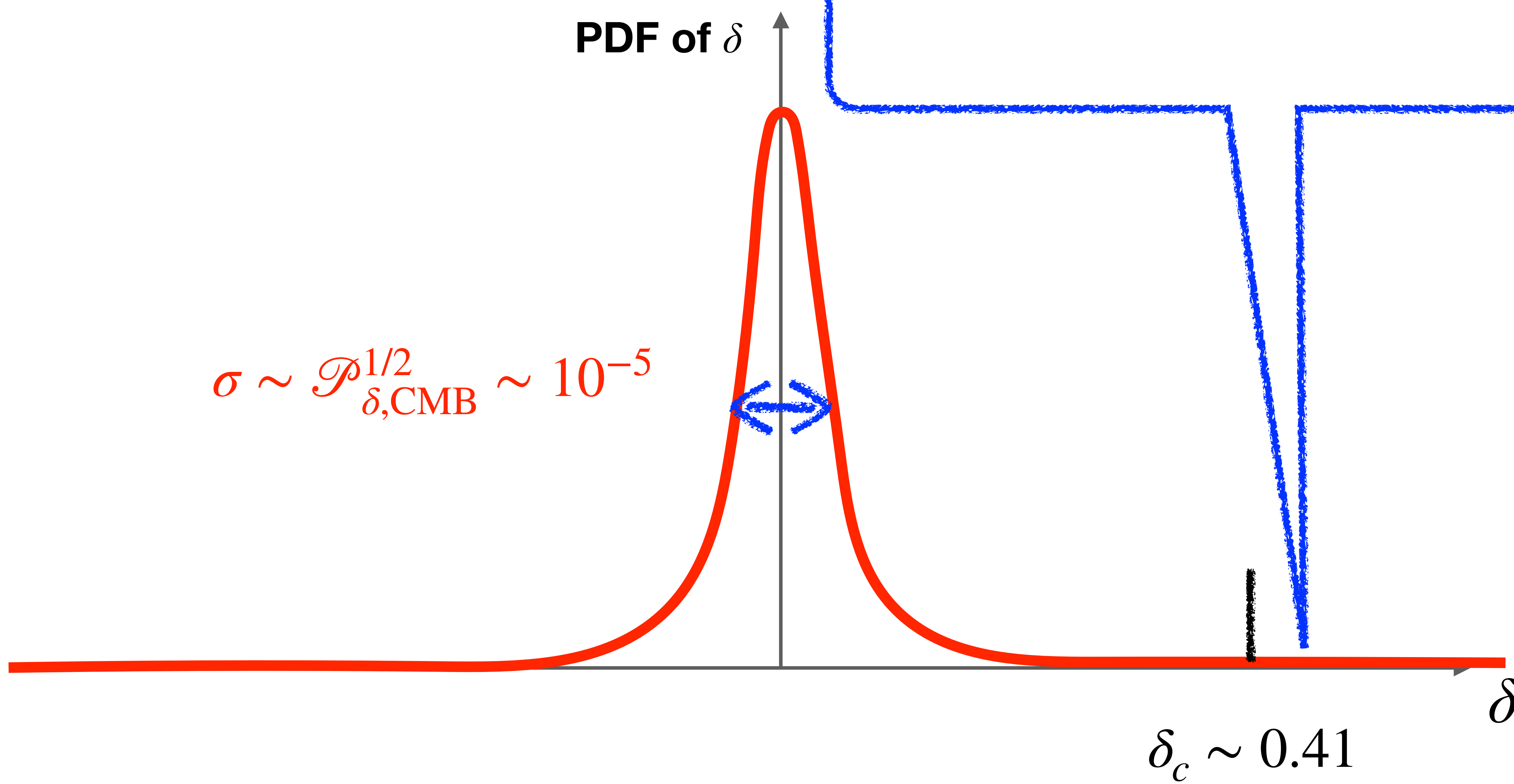
$$f_{\rm PBH} \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm CDM}} = 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot}\right)^{-1/2}$$

CMB-scale perturbations are negligible

PBH formation when $\delta > \delta_c$.

$$\beta = \mathrm{erfc}\frac{\delta_c}{\sqrt{2}\sigma} \simeq \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right) \sim \exp\left(-10^{10}\right).$$

Negligible!



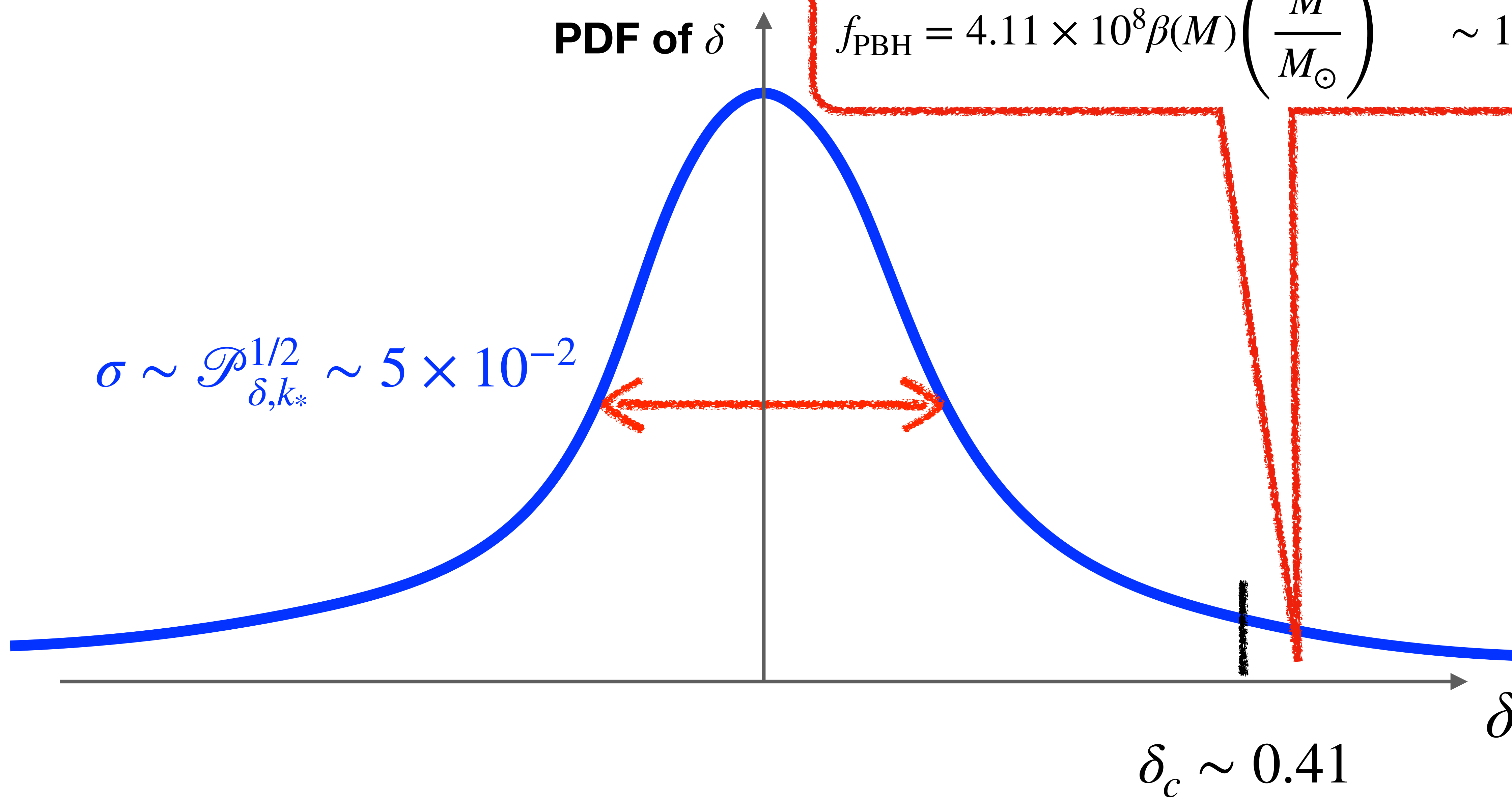
PBH=DM requires large perturbations (on small scales)

PBH formation when $\delta > \delta_c$.

$$\beta = \mathrm{erfc}\frac{\delta_c}{\sqrt{2}\sigma} \simeq \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right) \sim 2.5 \times 10^{-15}$$

PBH=DM (For $M \sim 10^{21}$ g **)**

$$f_{\mathrm{PBH}} = 4.11 \times 10^{8} \beta(M) \left(\frac{M}{M_\odot}\right)^{-1/2} \sim 1$$



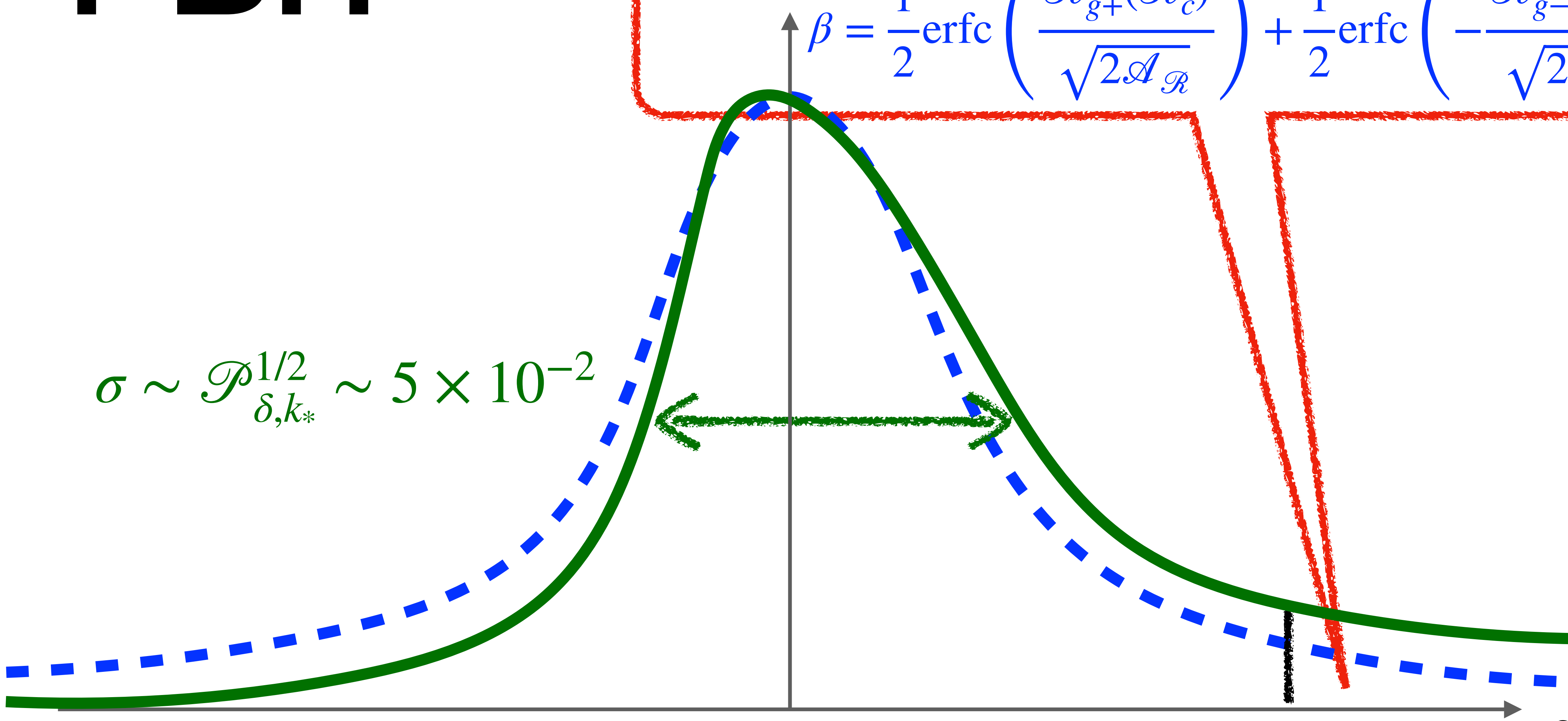
NG to enhance PBH

PBH formation when $\delta > \delta_c$.

(Positive) Non-Gaussianity can increase the PBH production. (If we fix the variance)

$$\mathscr{R}_{g\pm}(\mathscr{R}) = \frac{1}{2} f_{\rm NL}^{-1}\left(-1 \pm \sqrt{1 + 4 f_{\rm NL}\left(f_{\rm NL}\mathscr{A}_{\mathscr{R}} + \mathscr{R}\right)}\right) .$$

$$\beta = \frac{1}{2}{\rm erfc}\left(\frac{\mathscr{R}_{g+}(\mathscr{R}_c)}{\sqrt{2\mathscr{A}_{\mathscr{R}}}}\right) + \frac{1}{2}{\rm erfc}\left(-\frac{\mathscr{R}_{g-}(\mathscr{R}_c)}{\sqrt{2\mathscr{A}_{\mathscr{R}}}}\right) .$$



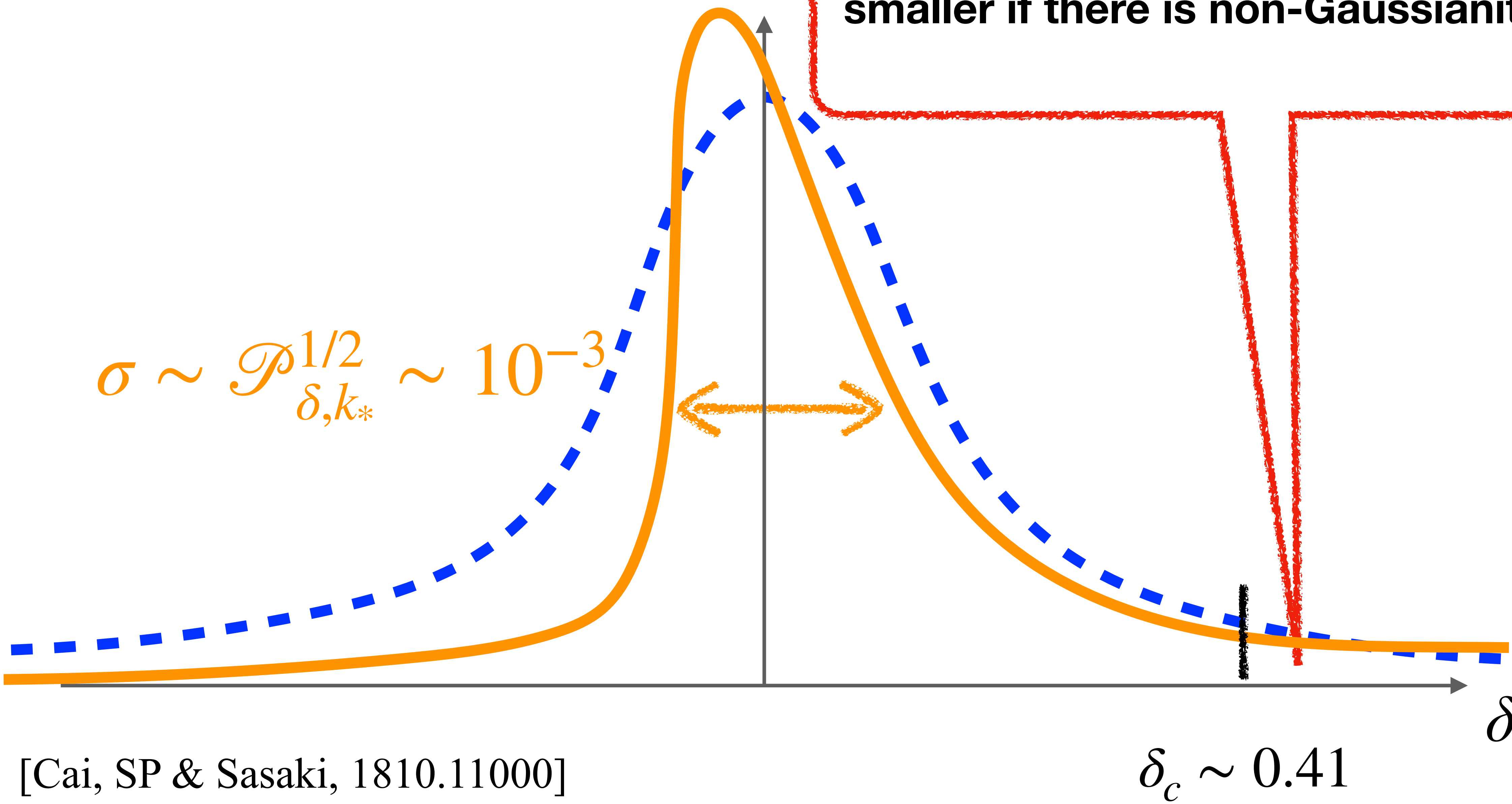
[Young & Byrnes, 1307.4995]

PBH=DM w/. NG

PBH formation when $\delta > \delta_c$.

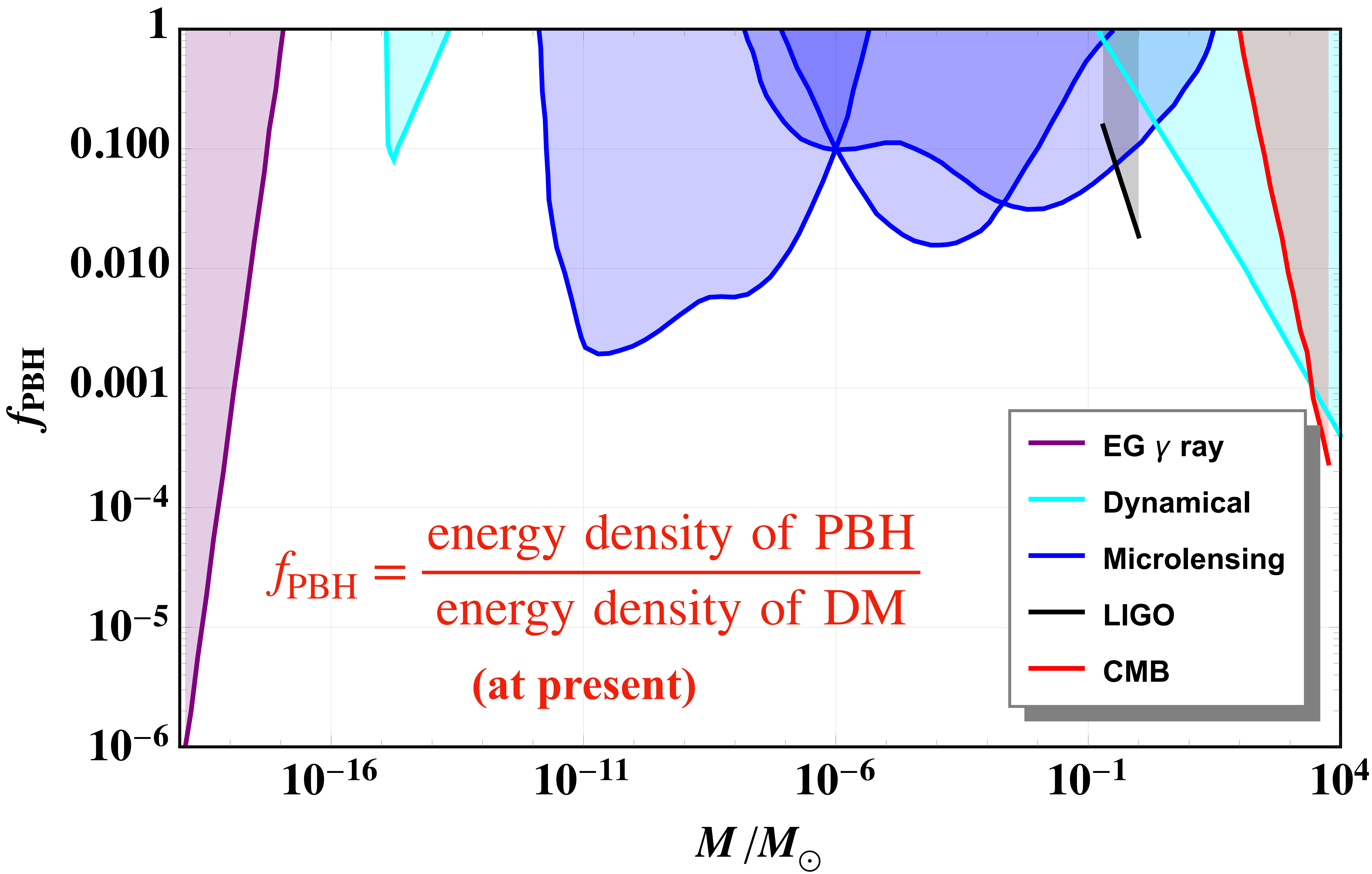
$$f_{\mathrm{PBH}} = 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot}\right)^{-1/2} \sim 1$$

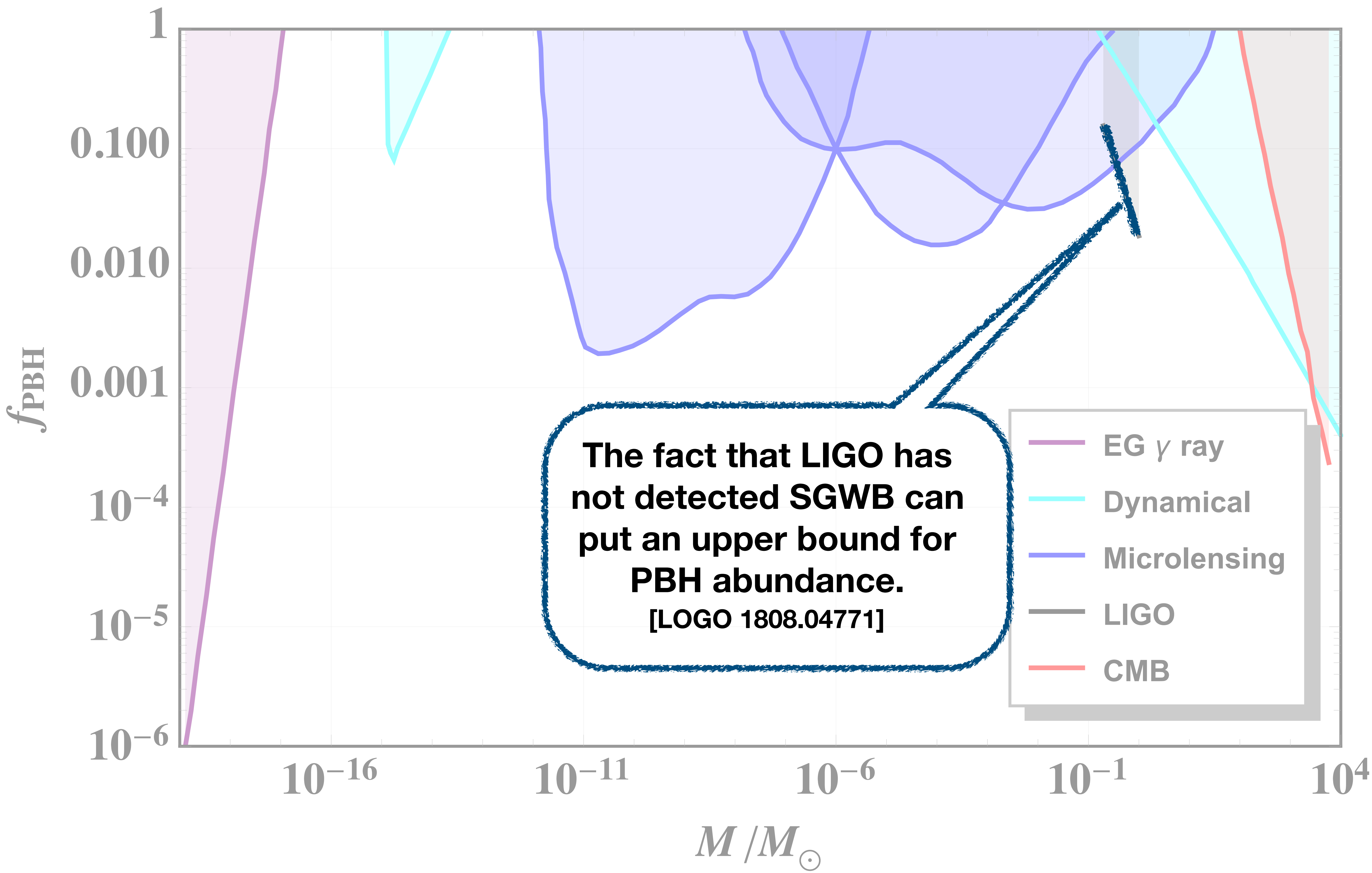
If we fix the PBH abundance (to be all dark matter), the variance must be smaller if there is non-Gaussianity.



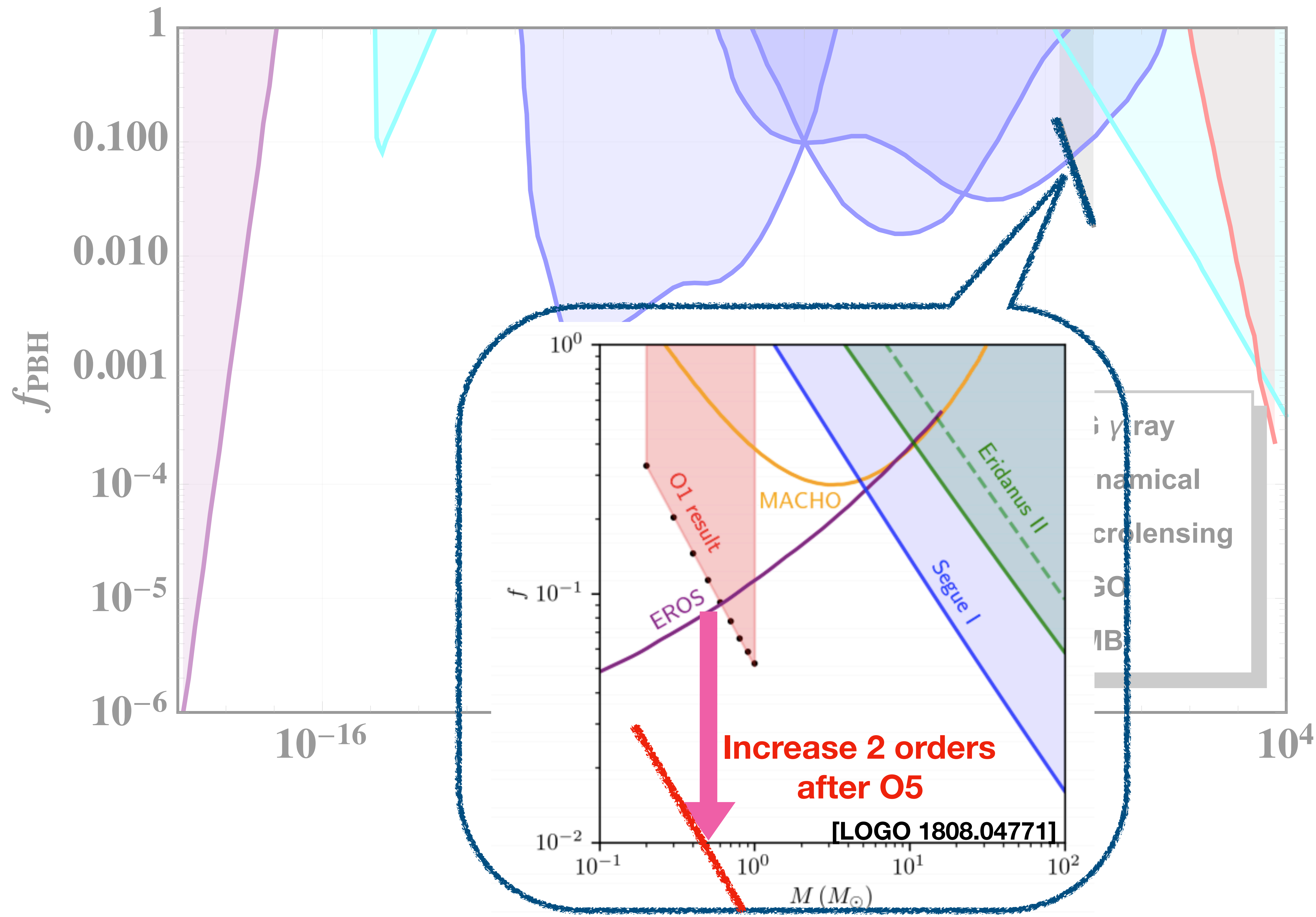
[Cai, SP & Sasaki, 1810.11000]

Observational Constraints on PBH

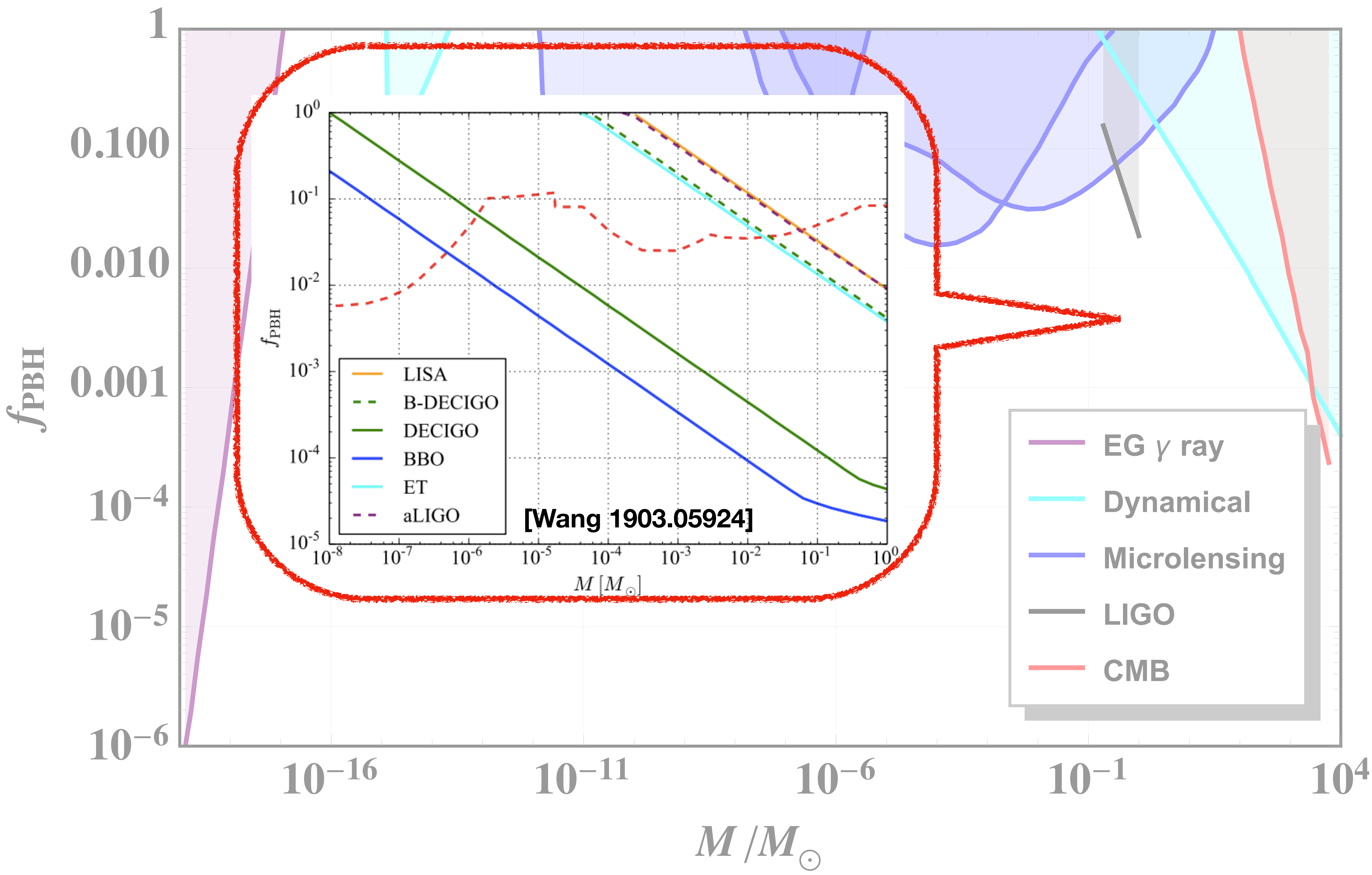

1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$f_{\rm PBH}$
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_\odot$
EG γ ray
Dynamical
Microlensing
LIGO
CMB
$f_{\rm PBH} = \frac{\text{energy density of PBH}}{\text{energy density of DM}}$
(at present)



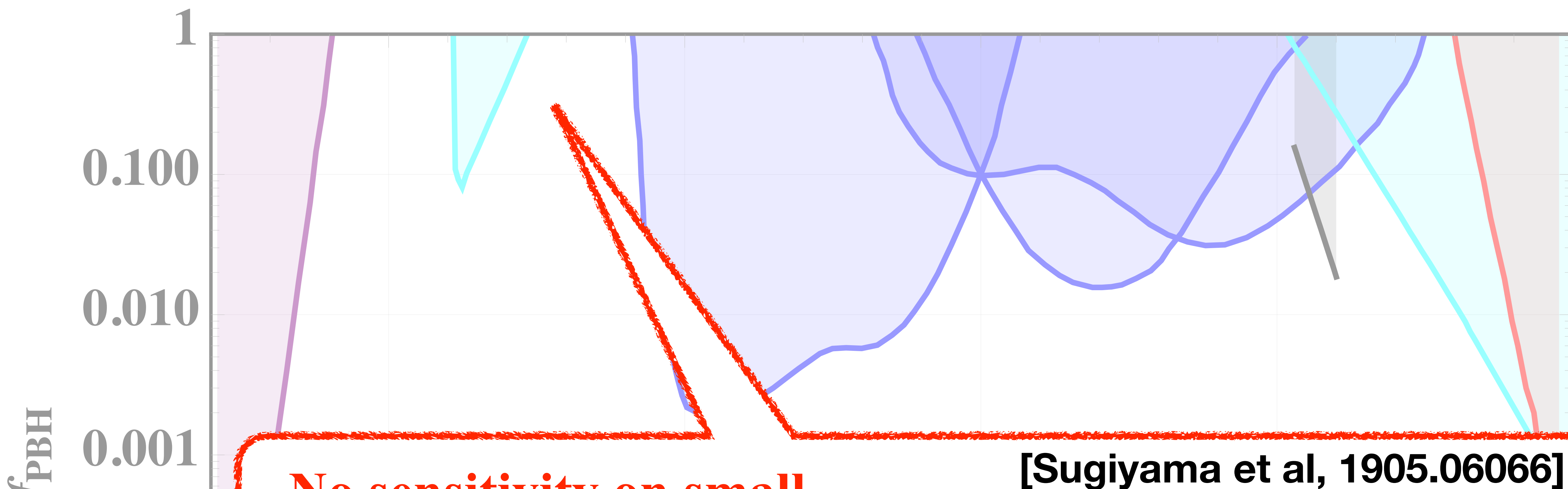
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$f_{\rm PBH}$
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M\,/M_{\odot}$
The fact that LIGO has not detected SGWB can put an upper bound for PBH abundance.
[LOGO 1808.04771]
EG γ ray
Dynamical
Microlensing
LIGO
CMB



1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$f_{\rm PBH}$
10^{-16}
10^{4}
γ ray
namical
crolensing
GO
MB
10^{0}
10^{-1}
10^{-2}
f
10^{-1}
10^{0}
10^{1}
10^{2}
$M\ (M_{\odot})$
O1 result
MACHO
EROS
Eridanus II
Segue I
Increase 2 orders after O5
[LOGO 1808.04771]



1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$f_{\rm PBH}$
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_\odot$
EG γ ray
Dynamical
Microlensing
LIGO
CMB
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$f_{\rm PBH}$
LISA
B-DECIGO
DECIGO
BBO
ET
aLIGO
[Wang 1903.05924]
10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
$M\,[M_\odot]$



No sensitivity on small scales

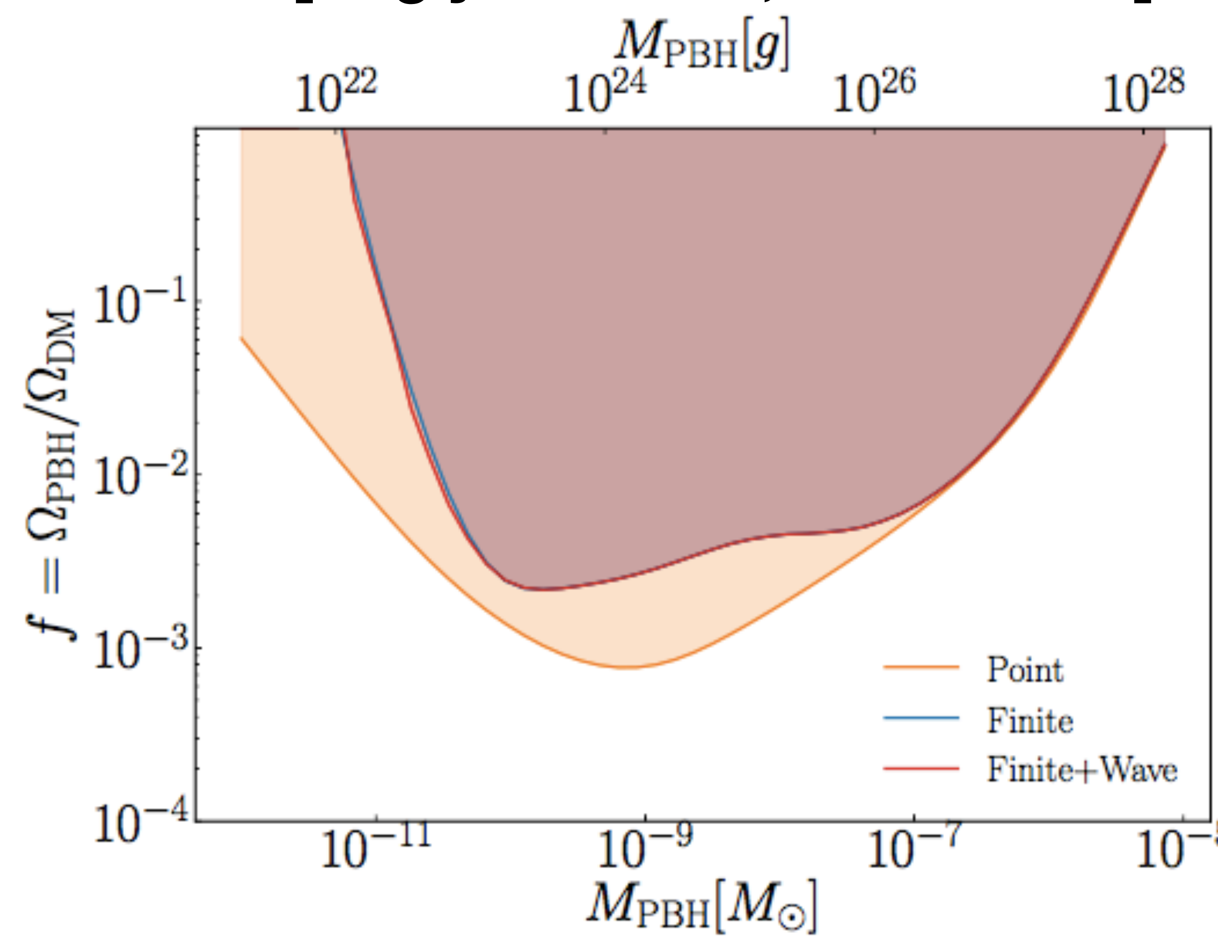
Finite Size Effect:

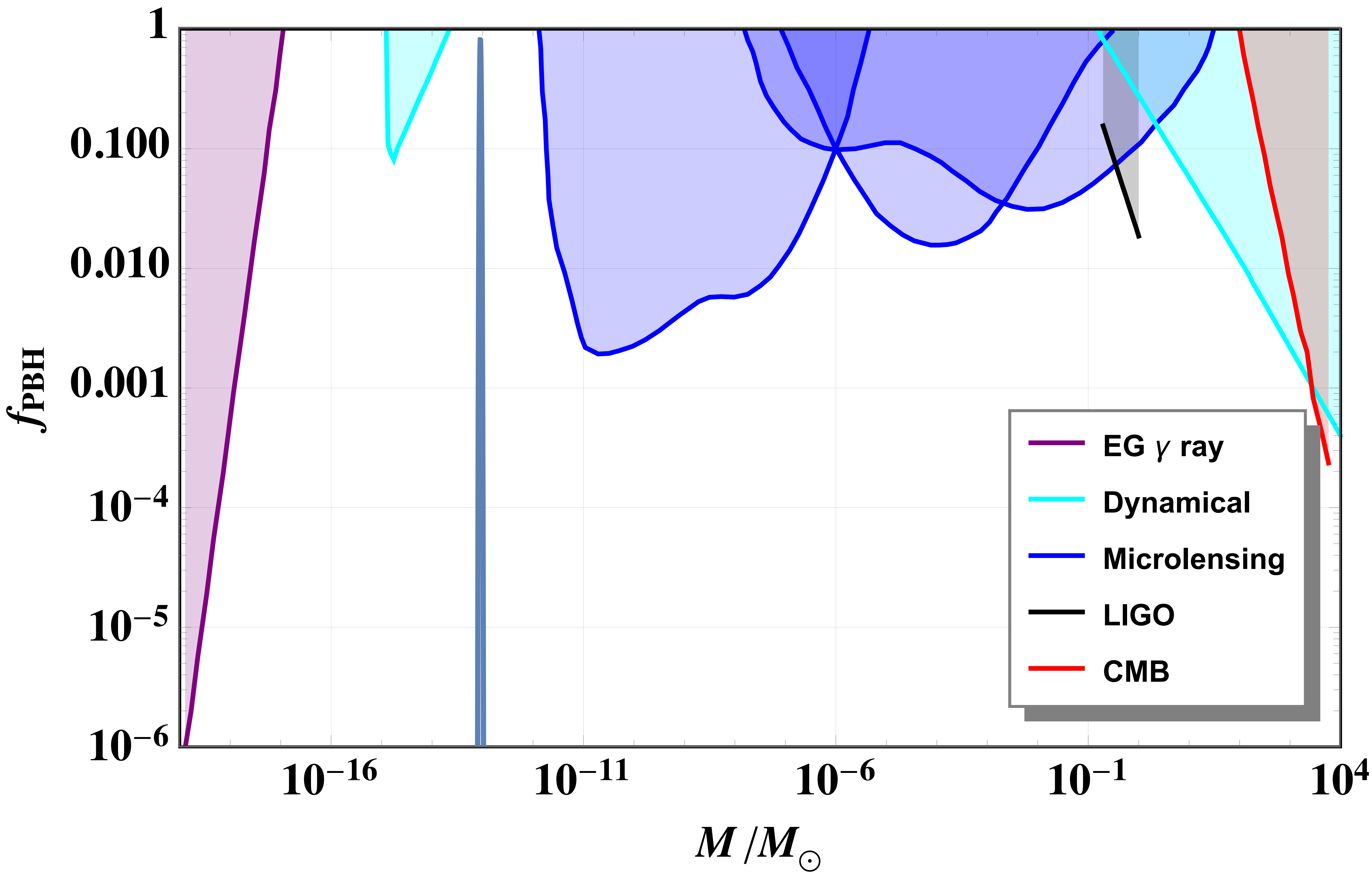
$$M < 5 \times 10^{-10} M_\odot \left(\frac{R_s}{R\odot} \right)^2$$

Wave effect:

$$M < 4 \times 10^{-10} M_\odot \left(\frac{\lambda_s}{\mu \mathrm{m}} \right)$$

[Sugiyama et al, 1905.06066]





[SP, Zhang, Huang & Sasaki, 1712.09896]

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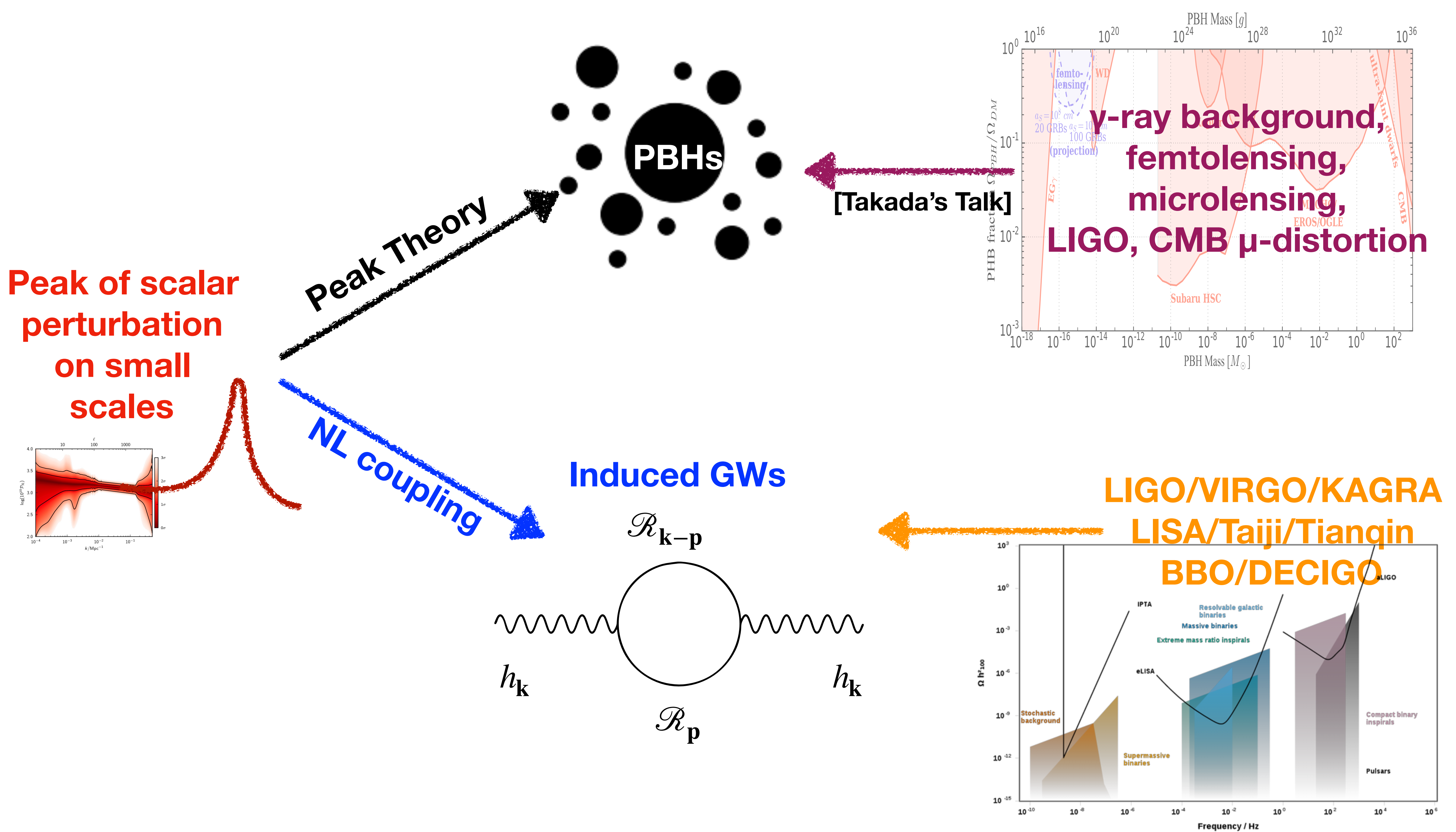
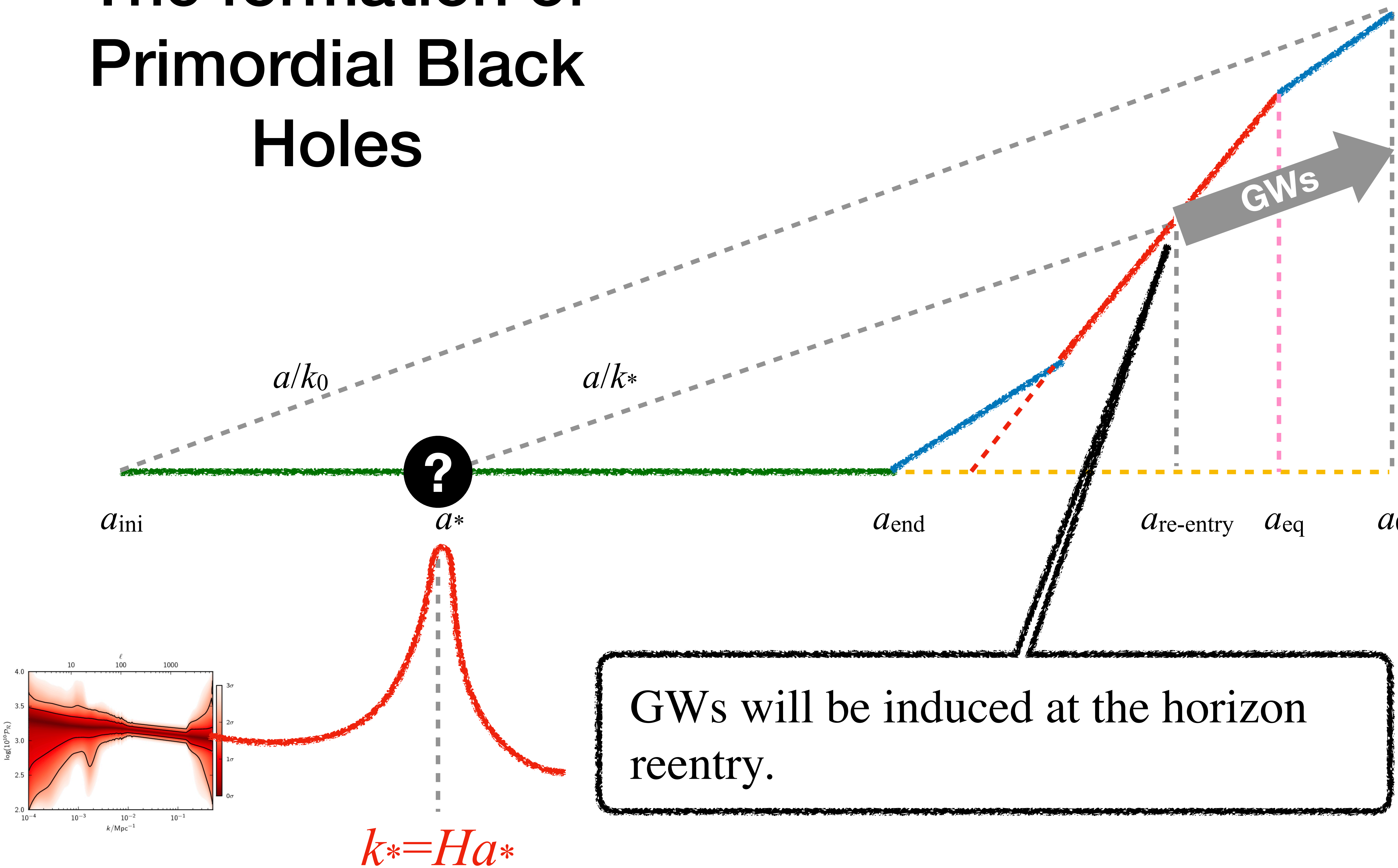
Primordial Black Holes and Induced Gravitational Waves



The formation of Primordial Black Holes



Induced GWs

- The metric is

$$ds^2 = a(\eta)^2\left[-(1-2\Phi)\,d\eta^2 + \left(1+2\Phi+h_{ij}\right)dx^i dx^j\right].$$

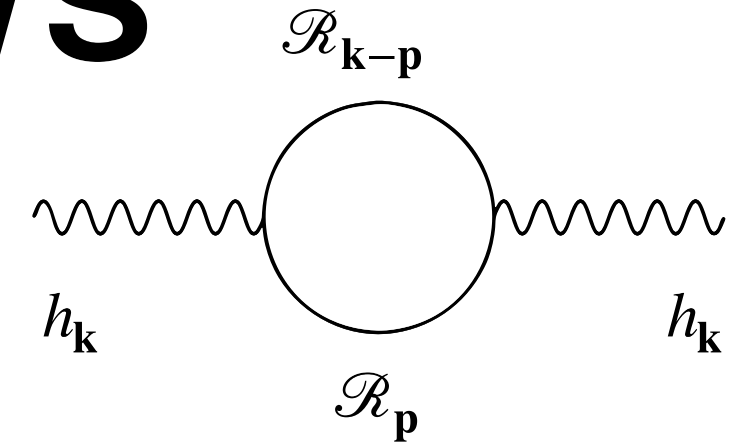
- From the nonlinear equation of motion for the tensor perturbation

$$h''_{\mathbf{k}} + 2\mathscr{H}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathcal{S}(\mathbf{k},\eta)$$

- where the source term is

$$\mathcal{S}(\mathbf{k},\eta) \sim \int d^3p\,\Phi_{\mathbf{p}}\Phi_{\mathbf{k-p}} \times (\text{transfer functions})$$

Induced GWs



- The induced (secondary) GWs are

$$h_{\mathbf{k}} \sim \int d\eta \times (\text{Green function}) \int d^3p \times (\text{Transfer function}) \times \Phi_{\mathbf{p}} \Phi_{\mathbf{k}-\mathbf{p}} \,.$$

- The energy density parameter is then

$$\Omega_{\mathrm{GW}} \sim \langle hh \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_{\Phi}^2 \sim \mathscr{P}_{\mathscr{R}}^2$$

- In the radiation dominated universe we have $\mathscr{R} = \frac{2}{3}\Phi$

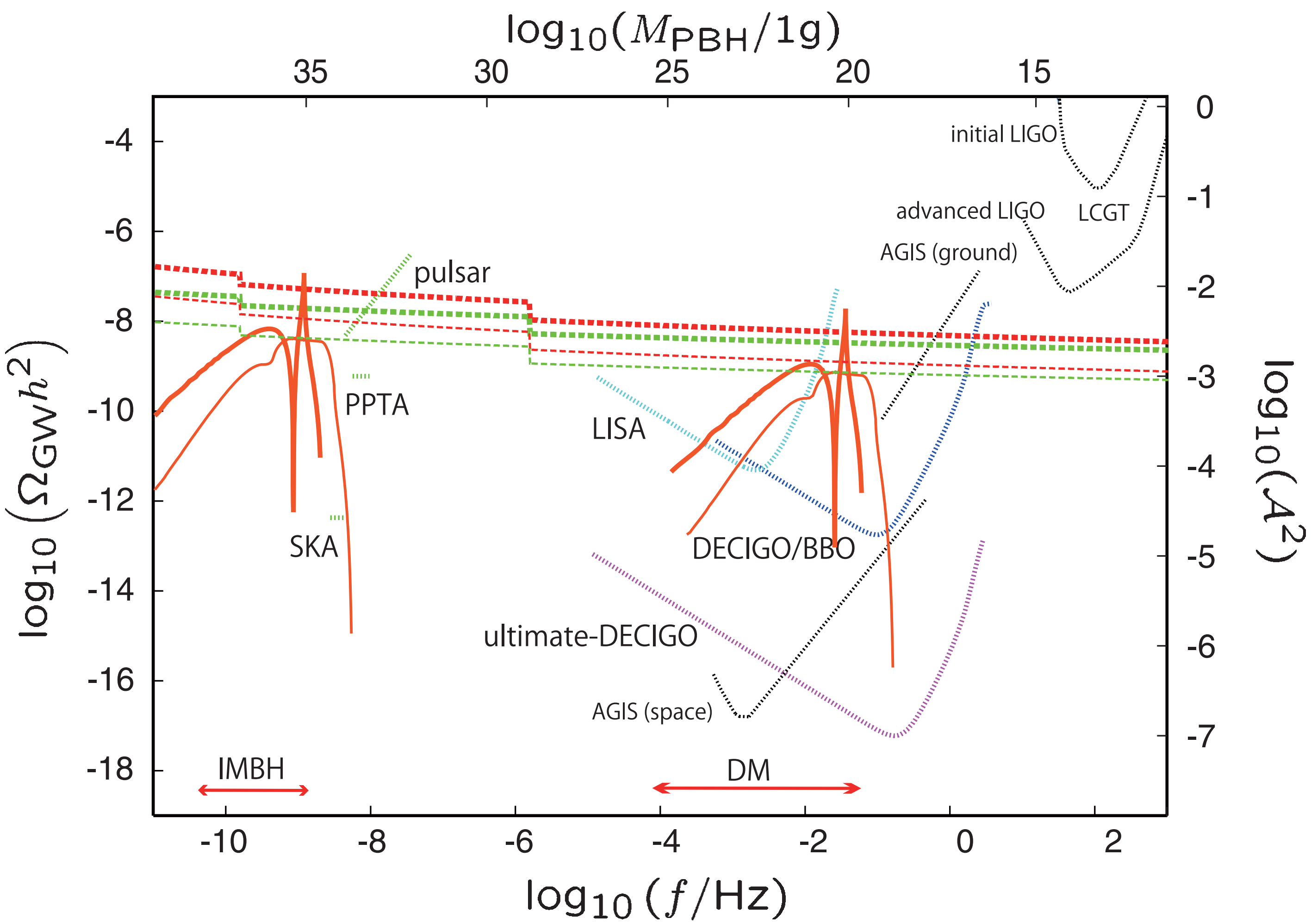
Induced GWs

$$\Omega_{\rm GW} \sim \langle hh \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_\Phi^2 \sim \mathscr{P}_\mathscr{R}^2$$

PBH abundance

$$\beta \sim \mathrm{erfc}\left(\frac{\mathscr{R}_c}{2\mathscr{P}_\mathscr{R}^{1/2}}\right)$$

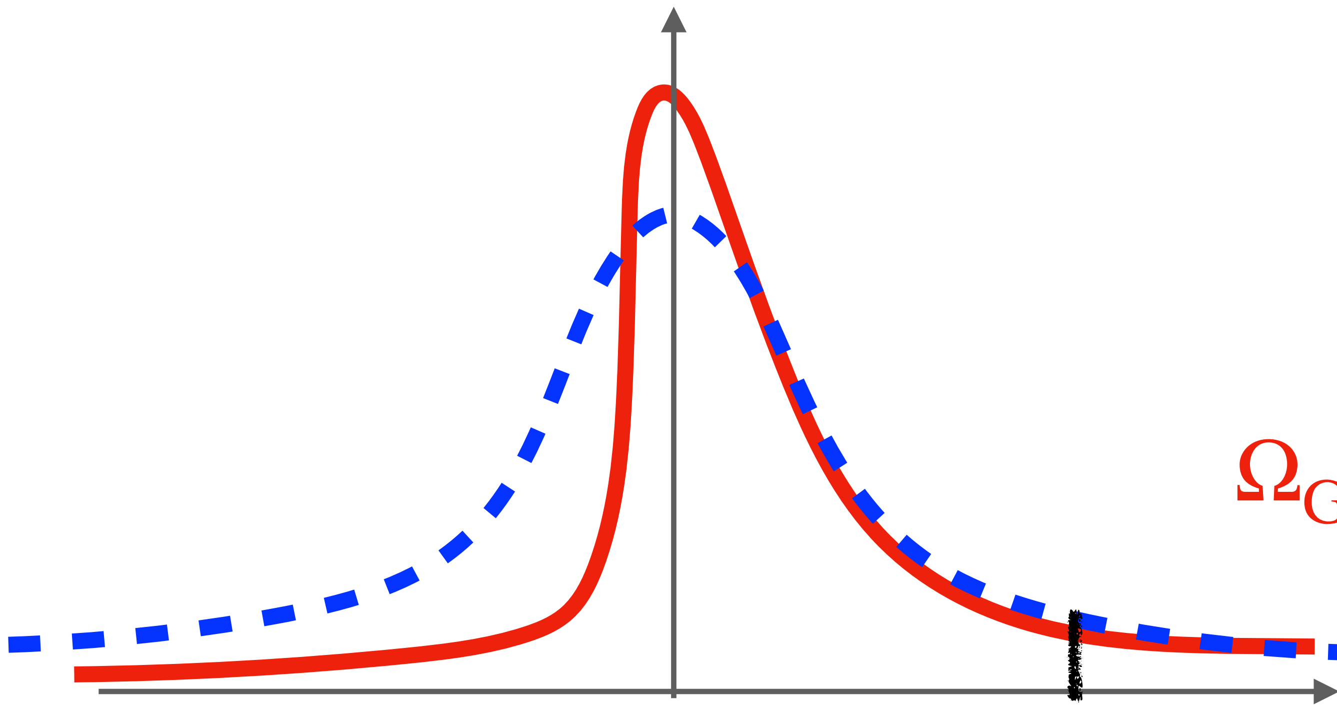
$$f_{\rm PBH} \sim 4.11 \times 10^{-8} \beta(M) \left(\frac{M}{M_\odot}\right)^{-1/2}$$



[Saito and Yokoyama, 0812.4339]

PBH as DM with nG

Gaussian: $\Omega_{\rm GW} \sim \mathcal{P}_{\mathcal{R}}^2 .$ $\beta \sim {\rm erfc}\left(\frac{\mathcal{R}_c}{2\mathcal{P}_{\mathcal{R}}^{1/2}}\right)$



Non-Gaussian:

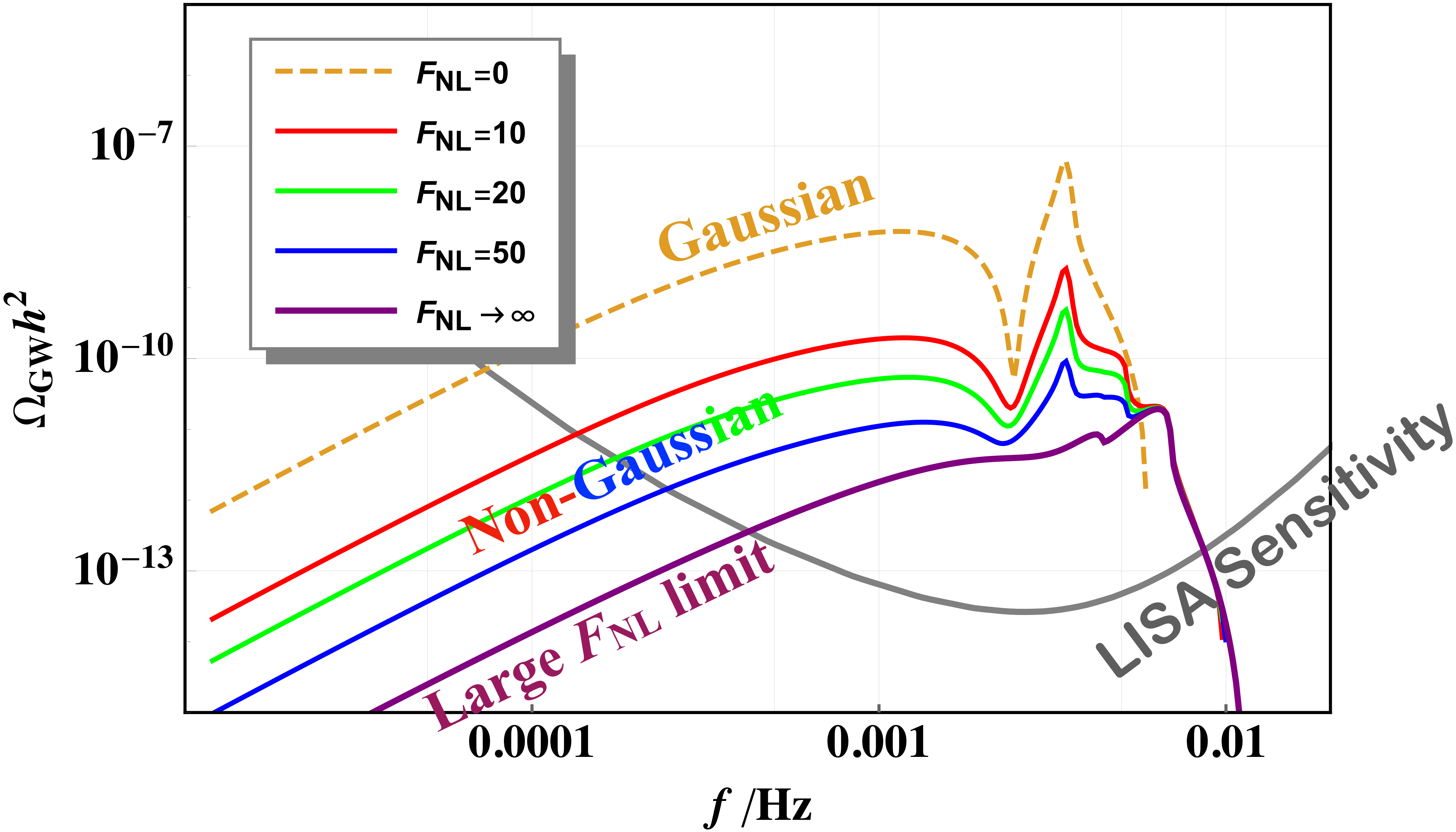
$$\mathcal{R} = \mathcal{R}_G + F_{NL}\mathcal{R}_G^2$$

$$\Omega_{\rm GW} \sim \mathcal{P}_{\mathcal{R},{\rm G}}^2 + F_{\rm NL}^2\mathcal{P}_{\mathcal{R},{\rm G}}^3 + F_{\rm NL}^4\mathcal{P}_{\mathcal{R},{\rm G}}^4$$

$$\mathcal{R}_{g\pm}(\mathcal{R}_c) = \frac{1}{2F_{\rm NL}}\left(-1 \pm \sqrt{1 + 4F_{\rm NL}\left(F_{\rm NL}\mathcal{P}_{\mathcal{R},G} + \mathcal{R}_c\right)}\right) .$$

$$\beta = \frac{1}{2}{\rm erfc}\left(\frac{\mathcal{R}_{g+}(\mathcal{R}_c)}{\sqrt{2\mathcal{P}_{\mathcal{R},G}}}\right) + \frac{1}{2}{\rm erfc}\left(-\frac{\mathcal{R}_{g-}(\mathcal{R}_c)}{\sqrt{2\mathcal{P}_{\mathcal{R},G}}}\right); \quad f_{\rm NL} > 0.$$

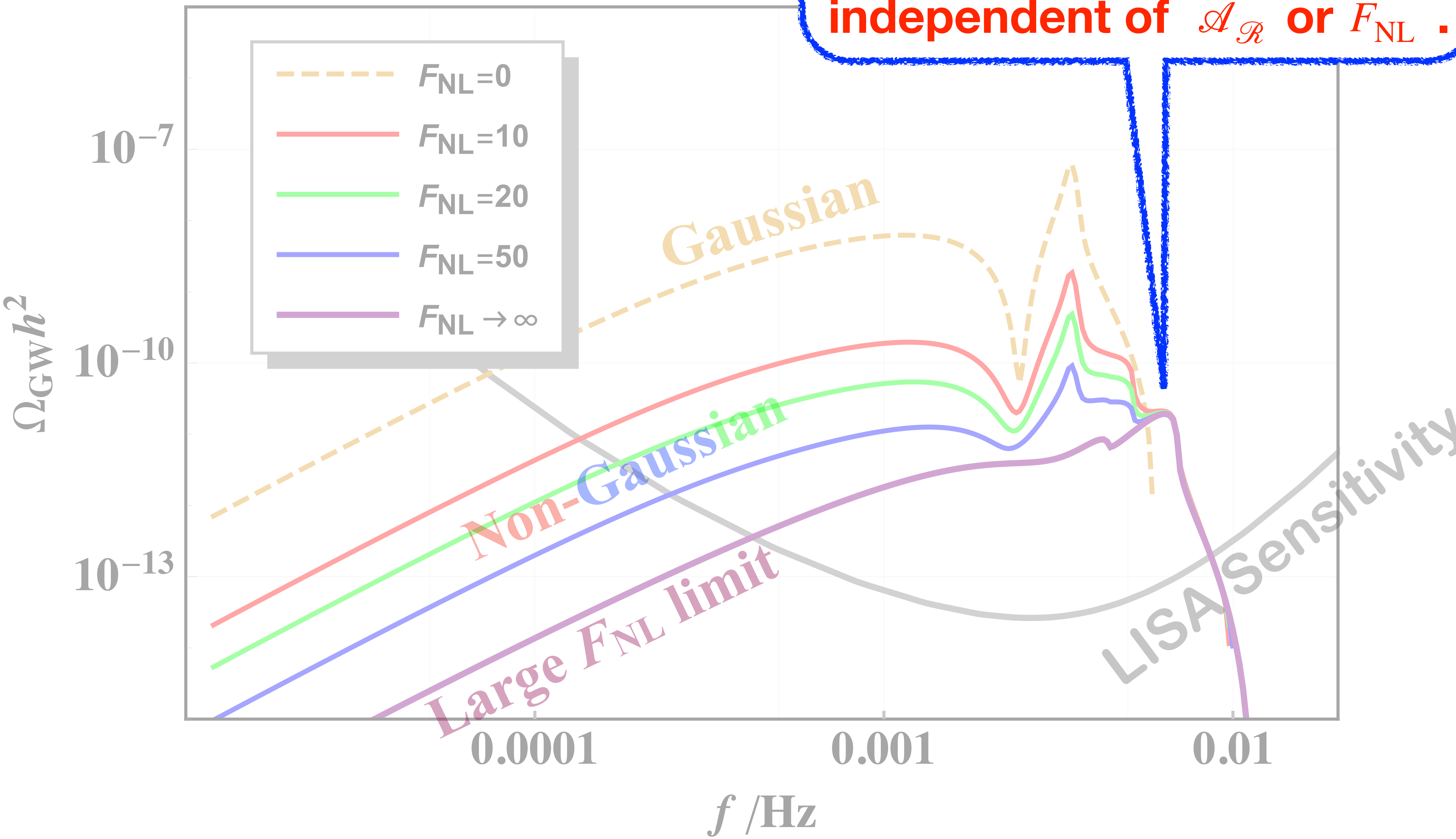
GWs when PBH=DM



[Cai, SP & Sasaki, 1810.11000]

GWs when F

If PBH serves as all DM,
the induced GWs must be
detectable by LISA,
independent of $\mathscr{A}_{\mathscr{R}}$ or $F_{\rm NL}$.



[Cai, SP & Sasaki, 1810.11000]

Summary

- Stochastic Background of GW is one of the important scientific goal of the next generation GW detectors (LISA, Taiji, Tianqin, …)

- Multiple peaks in induced GW spectrum can be used as a smoking gun of non-Gaussianity.

- If PBHs can serve as all the DM, induced GWs must be detectable by LISA/Taiji/Tianqin, no matter how small $\mathscr{A}_{\mathscr{R}}$ or f_{NL} is.

- Conversely if LISA can not detect the induced GWs, we can put an independent constraint on the PBH abundances on mass range 10^{19}g to 10^{22}g where no current experiment can explore.