Scale invariant models for BSM physics and cosmology

Mikhail Shaposhnikov

SUSY: Model-building and Phenomenology

IPMU, 4 December 2013

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Outline

- The proposal in short
- Diff versus TDiff
- Field theory: classical scale invariance and its spontaneous breakdown
- Unimodular gravity
- Scale invariance, unimodular gravity, cosmological constant, inflation and dark energy
- Quantum scale invariance
- Dilaton as a part of the metric in TDiff gravity
- Conclusions

An alternative to SUSY, large extra dimensions, technicolor, etc

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An alternative to SUSY, large extra dimensions, technicolor, etc

Effective

Theory

Of

Everything

Definitions

“Effective”: valid up to the Planck scale, quantum gravity problem is not addressed. No new particles heavier than the Higgs boson.

“Everything”:

- neutrino masses and oscillations
- dark matter
- baryon asymmetry of the Universe
- inflation
- dark energy

Particle content of ETOE

Particles of the SM

+

graviton

+

dilaton

+

3 Majorana leptons

Symmetries of ETOE: gauge

- SU(3)×SU(2)×U(1) – the same as in the Standard Model
- Restricted coordinate transformations: TDIFF, $\boldsymbol{det[-g] = 1}$ (Unimodular Gravity).

Symmetries of ETOE

- Exact quantum scale invariance
  - No dimensionful parameters
  - Cosmological constant is zero
  - Higgs mass is zero
  - these parameters cannot be generated radiatively, if renormalisation respects this symmetry
- Scale invariance must be spontaneously broken
  - Newton constant is nonzero
  - W-mass is nonzero
  - Λ_{QCD} is nonzero

Roles of different particles

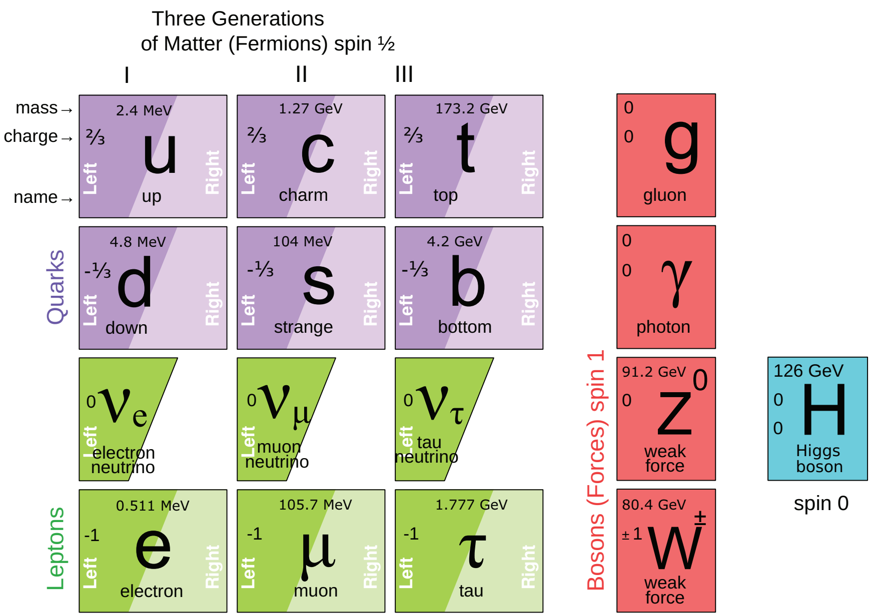
The roles of dilaton:

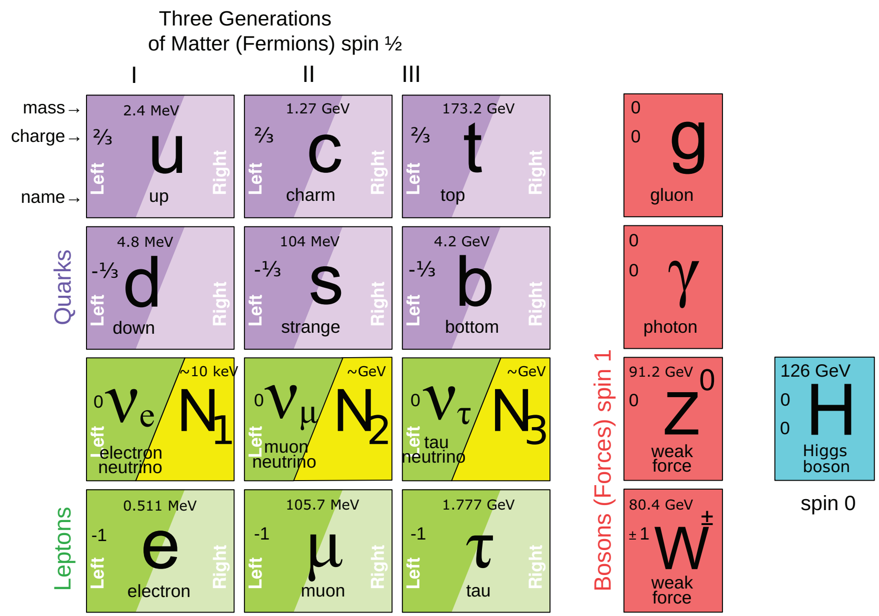
- determine the Planck mass
- give mass to the Higgs
- give masses to 3 Majorana leptons
- lead to dynamical dark energy

Roles of the Higgs boson:

- give masses to fermions and vector bosons of the SM
- provide inflation

New physics below the Fermi scale: the νMSM





Role of N_1 with mass in keV region: dark matter. Search - with the use of X-ray telescopes

Role of N_2, N_3 with mass in 100 MeV – GeV region: “give” masses to neutrinos and produce baryon asymmetry of the Universe. Search - intensity and precision frontier, SPS at CERN.

The couplings of the νMSM

Particle physics part, accessible to low energy experiments: the νMSM. Mass scales of the νMSM:

$M_I < M_W$ (No see-saw)

Consequence: small Yukawa couplings,

$$F_{\alpha I} \sim \frac{\sqrt{m_{atm} M_I}}{v} \sim (10^{-6} - 10^{-13}),$$

here $v \simeq 174$ GeV is the VEV of the Higgs field,

$m_{atm} \simeq 0.05$ eV is the atmospheric neutrino mass difference.

Small Yukawas are also necessary for stability of dark matter and baryogenesis (out of equilibrium at the EW temperature).

GR

Einstein gravity is a theory which is invariant under all diffeomorphisms, Diff: $x^\mu \to f^\mu(x^\nu)$.

Pros - consistence with all tests of GR. One of the main predictions - existence of massless graviton.

Problems of GR

- Large dimensionfull coupling constant $G_N^{-1} = M_P^2$, leading to hierarchy problem $m_H \ll M_P$.
- Extra arbitrary fundamental parameter - cosmological constant - which is known to be very small.
- Quantum gravity?

Our proposal

$$Diff \to TDiff \times Dilatations$$

TDiff - volume conserving coordinate transformations,

$$det\left[\frac{\partial f^{\mu}}{\partial x^{\nu}}\right] = 1 \,.$$

Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \;\to\; \sigma^{n}\Psi(\sigma x) \,,$$

$n = 0$ for the metric, $n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

Similarity with GR: consistency with all tests

Differences with GR:

- Dynamical origin of all mass scales
- Hierarchy problem gets a different meaning - an alternative (to SUSY, techicolor, little Higgs or large extra dimensions) solution of it may be possible.
- Cosmological constant problem acquires another formulation.
- Natural chaotic cosmological inflation
- Low energy sector contains a massless dilaton
- There is Dark Energy even without cosmological constant
- Quantum gravity?

Scale invariance

Trivial statement: multiply all mass parameters in the theory

$$M_W, \quad \Lambda_{QCD}, \quad M_H, M_{Pl}, ...$$

by one and the same number : $M \to \sigma M$. Physics is not changed!

Indeed, this change, supplemented by a dilatation of space-time coordinates $x^\mu \to \sigma x^\mu$ and an appropriate redefinition of the fields does not change the complete quantum effective action of the theory.

This transformation, however, is not a symmetry of the theory (symmetry = transformation of dynamical variables which does not change the action)

Dilatation symmetry

Dilatations:

$$\phi(x) \;\to\; \sigma^n \phi(\sigma x)$$

n-canonical dimension of the field: $n = 1$ for scalars and vectors, $n = 3/2$ for fermions, while the metric transforms as $g_{\mu\nu}(x) \;\to\; g_{\mu\nu}(\sigma x)$.

Dilatation symmetry forbids all dimensionfull couplings: Higgs mass, Newton gravity constant, cosmological constant, etc

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Ruled out by observations?

No, if it is spontaneously broken!

First step: consider classical physics only (no parameters like Λ_{QCD}), just tree explicit mass parameters such as M_H, M_W, M_{Pl}.

Lagrangian of ETOE

Scale-invariant Lagrangian

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{SM}[\mathrm{M}\to 0]} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi,\chi)$$
$$+ \left(\bar{N}_I i\gamma^\mu \partial_\mu N_I - h_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\varphi} - f_I \bar{N_I}^c N_I \chi + \mathrm{h.c.}\right)\ ,$$

Potential (χ - dilaton, φ - Higgs, $\varphi^\dagger\varphi = 2h^2$):

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4,$$

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger\varphi\right)\frac{R}{2}\ ,$$

Spontaneous breaking of scale invariance

Forget first about gravity. Consider scalar potential

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4,$$

Requirements: vacuum state exists if $\lambda \geq 0, \quad \beta \geq 0$

For $\lambda > 0, \;\; \beta > 0$ the vacuum state is unique: $\chi = 0, \;\; \varphi = 0$ and scale invariance is exact.

Field propagators: scalar $1/p^2$, fermion $\not{p}/p^2$. Greenberg, 1961:

free quantum field theory!!

If not - theory does not describe particles !!

Gravity included - argument for $\beta = 0$ gets weaker:

- for $\beta > 0$ there is a de Sitter solution
- for $\beta < 0$ there is an AdS solution

However,

- dS solution is not stable in the presence of massless scalar - no dS invariant ground sate exists
- AdS solution has different pathologies

For $\lambda > 0, \;\; \beta = 0$ the scale invariance can be spontaneously broken.

The vacuum manifold:

$$h_0^2 = \frac{\alpha}{\lambda}\chi_0^2$$

Particles are massive, Planck constant is non-zero:

$$M_H^2 \sim M_W \sim M_t \sim M_N \propto \chi_0, \;\; M_{Pl} \sim \chi_0$$

Phenomenological requirements: $\xi_\chi \ll 1, \;\; \xi_h \gg 1$

$$\alpha \sim \frac{v^2}{M_{Pl}^2} \sim 10^{-38} \lll 1$$

Approximate shift symmetry $\chi \to \chi + const$

Good news: cosmological constant may be zero due to scale invariance and requirement of presence of particles

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Universe is in the state of accelerated expansion, $\mathbf{\Omega_{DE} \simeq 0.7}$!

Unimodular gravity

Ordinary gravity:

the metric $g_{\mu\nu}$ is an arbitrary function of space-time coordinates.

Invariant under general coordinate transformations

Unimodular gravity:

the metric $g_{\mu\nu}$ is an arbitrary function of space-time coordinates with set$[g] = -1$. Invariant under general coordinate transformations which conserve the 4-volume.

van der Bij, van Dam, Ng

Origin of UG: Field theory describing spin 2 massless particles is either GR or UG

Number of physical degrees of freedom is the same.

Unimodular gravity and cosmological constant

Theories are equivalent everywhere except the way the cosmological constant appears

GR. Λ is the fundamental constant:

$$S = -\frac{1}{M_P^2}\int d^4x\sqrt{-g}\,[R+\Lambda]$$

UG. Λ does not appear in the action:

$$S = -\frac{1}{M_P^2}\int d^4x R$$

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Cosmological constant problem is solved in UG??!!

Wilczek, Zee: NO!

UG is equivalent to

$$S = -\frac{1}{M_P^2}\int d^4x\sqrt{-g}\left[R + \Lambda(x)\left(1 - \frac{1}{\sqrt{-g}}\right)\right]$$

Equations of motion ($G_{\mu\nu}$ - Einstein tensor):

$$G_{\mu\nu} = -\Lambda(x)\, g_{\mu\nu}\ ,\ \sqrt{-g} = 1$$

Bianchi identity: $\Lambda(x)_{;} = 0 \rightarrow \Lambda(x) = const.$

Solutions of UG are the same as solutions of GR with an arbitrary cosmological constant.

Conclusion: in UG cosmological constant reappears, but as an integral of motion, related to initial conditions

However: quantum matter fluctuations do not contribute to Λ - no need for fine-tuning of quartic divergences! Weinberg, Smolin

Equations of motion for Unimodular Gravity:

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = 8\pi G_N (T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T)$$

Perfect example of “degravitation" - the “$g_{\mu\nu}$" part of energy-momentum tensor does not gravitate. Solution of the “technical part" of cosmological constant problem - quartically divergent matter loops do not change the geometry. But - no solution of the “main" cosmological constant problem - why $\Lambda \ll M_P^4$? Scale invariance can help!

Scale invariance + unimodular gravity

Solutions of scale-invariant UG are the same as the solutions of scale-invariant GR with the action

$$S = -\int d^4x\sqrt{-g}\left[\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi\right)\frac{R}{2} + \Lambda + ...\right] \ ,$$

Physical interpretation: Einstein frame

$$g_{\mu\nu} = \Omega(x)^2 \tilde{g}_{\mu\nu} \ , \ \ (\xi_\chi \chi^2 + \xi_h h^2)\Omega^2 = M_P^2$$

Λ is not a cosmological constant, it is the strength of a peculiar potential!

Relevant part of the Lagrangian (scalars + gravity) in Einstein frame:

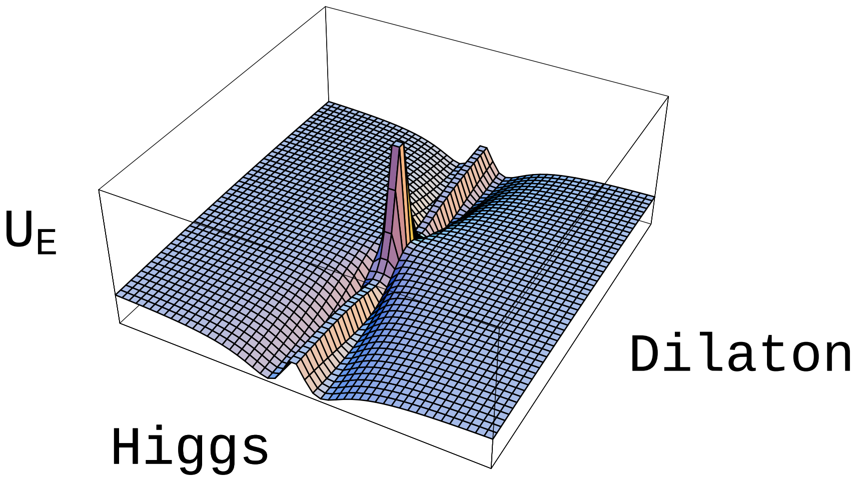
$$\mathcal{L}_E = \sqrt{-\tilde{g}}\left(-M_P^2\frac{\tilde{R}}{2} + K - U_E(h,\chi)\right) \,,$$

K - complicated non-linear kinetic term for the scalar fields,

$$K = \Omega^2\left(\frac{1}{2}(\partial_\mu\chi)^2 + \frac{1}{2}(\partial_\mu h)^2)\right) - 3M_P^2(\partial_\mu\Omega)^2 \,.$$

The Einstein-frame potential $U_E(h,\chi)$:

$$U_E(h,\chi) = M_P^4\left[\frac{\lambda\left(h^2 - \frac{\alpha}{\lambda}\chi^2\right)^2}{4(\xi_\chi\chi^2 + \xi_h h^2)^2} + \frac{\Lambda}{(\xi_\chi\chi^2 + \xi_h h^2)^2}\right] \,,$$



Potential for the Higgs field and dilaton in the Einstein frame.

Left: $\mathbf{\Lambda > 0}$, right $\mathbf{\Lambda < 0}$.

$\mathbf{50\%}$ chance ($\mathbf{\Lambda < 0}$): inflation + late collapse

$\mathbf{50\%}$ chance ($\mathbf{\Lambda > 0}$): inflation + late acceleration

Inflation

Chaotic initial condition: fields χ and h are away from their equilibrium values.

Choice of parameters: $\xi_h \gg 1, \;\; \xi_\chi \ll 1$ (will be justified later)

Then - dynamics of the Higgs field is more essential, $\chi \simeq const$ and is frozen. Denote $\xi_\chi \chi^2 = M_P^2$.

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\tilde{h}}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \quad \begin{cases} h \simeq \tilde{h} & \text{for } h < M_P/\xi \\ h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\tilde{h}}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\sqrt{\xi} \end{cases}$$

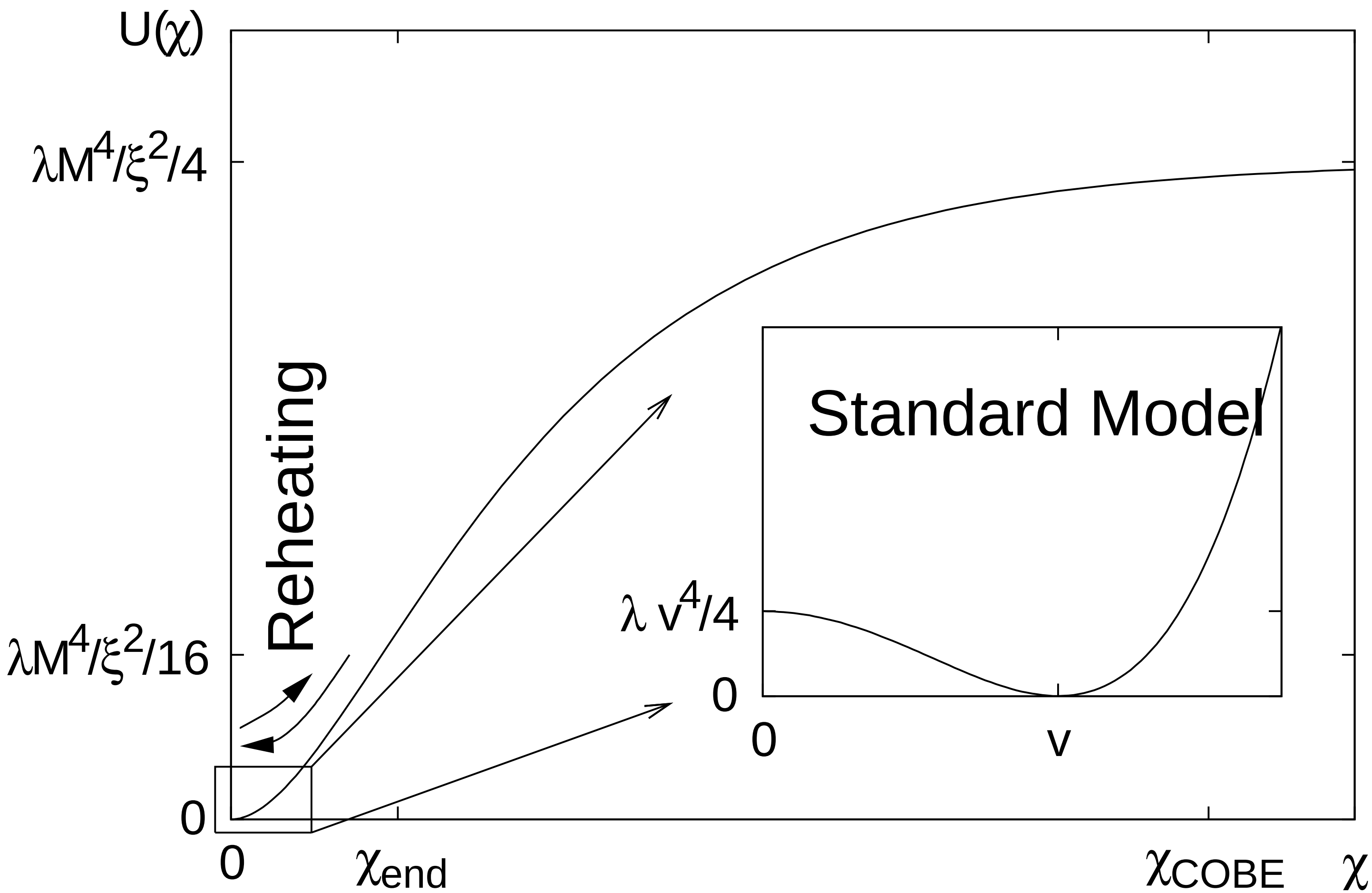
Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \tilde{h} \partial^\mu \tilde{h}}{2} - \frac{1}{\Omega(\tilde{h})^4}\frac{\lambda}{4} h(\tilde{h})^4 \right\}$$

Potential:

$$U(\tilde{h}) = \begin{cases} \frac{\lambda}{4}\tilde{h}^4 & \text{for } h < M_P/\xi \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\tilde{h}}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases} .$$

Potential in Einstein frame



Slow roll stage

$$\epsilon = \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4}{3}\exp\left(-\frac{4\chi}{\sqrt{6}M_P}\right)$$

$$\eta = M_P^2\frac{d^2U/d\chi^2}{U} \simeq -\frac{4}{3}\exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)$$

Slow roll ends at $\chi_{\rm end} \simeq M_P$

Number of e-folds of inflation at the moment h_N is $N \simeq \frac{6}{8}\frac{h_N^2 - h_{\rm end}^2}{M_P^2/\xi}$

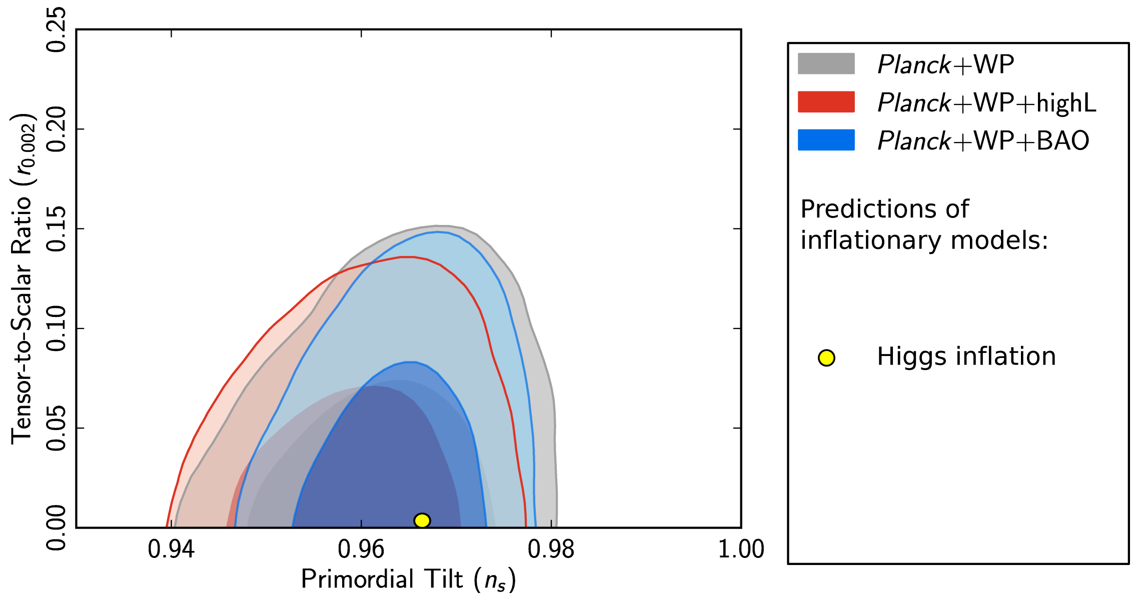
$$\chi_{60} \simeq 5M_P$$

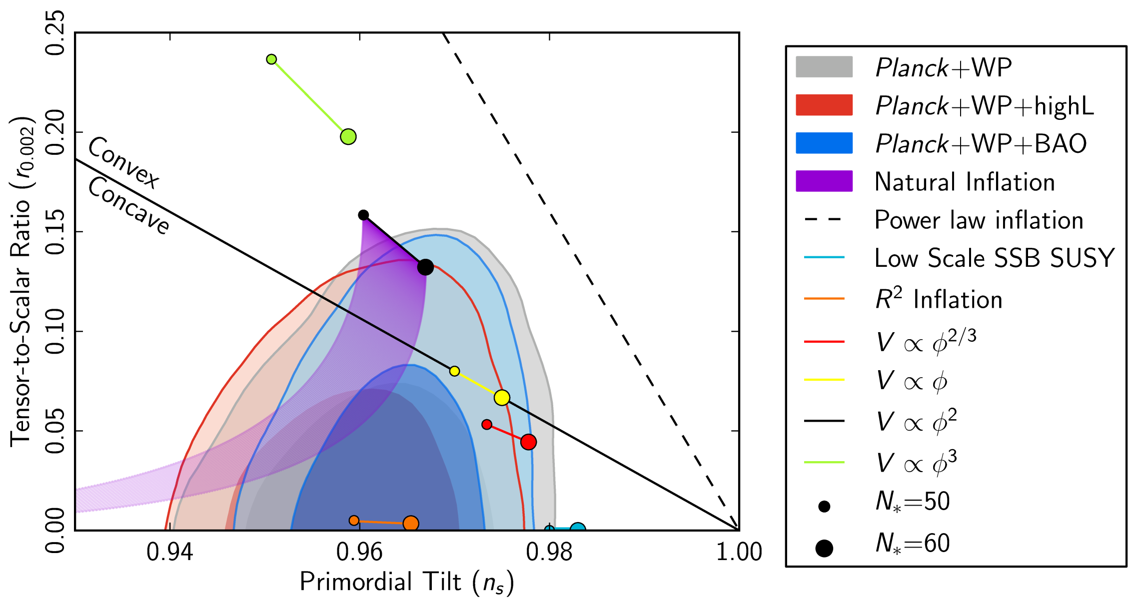
COBE normalization $U/\epsilon = (0.027M_P)^4$ gives

$$\xi \simeq \sqrt{\frac{\lambda}{3}}\frac{N_{\rm COBE}}{0.027^2} \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v}$$

Connection of ξ and the Higgs mass!

CMB parameters—spectrum and tensor modes





Higgs mass from inflation

Previous consideration tells nothing about the Higgs mass: change λ as $\propto \xi^2$ - no modifications!

However: λ is not a constant, it depends on the energy. Typical scale at inflation $\sim M_P/\sqrt{\xi}$.

Therefore, SM must be a valid quantum field theory up to the inflation scale $M_P/\sqrt{\xi}$.

$$M_{crit} - 0.4\,\mathrm{GeV} < m_H < m_{\max}$$

$$M_{crit} = [129.3 + \frac{y_t(M_t) - 0.9361}{0.0058} \times 2.0 - \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \times 0.5]\,\mathrm{GeV}$$

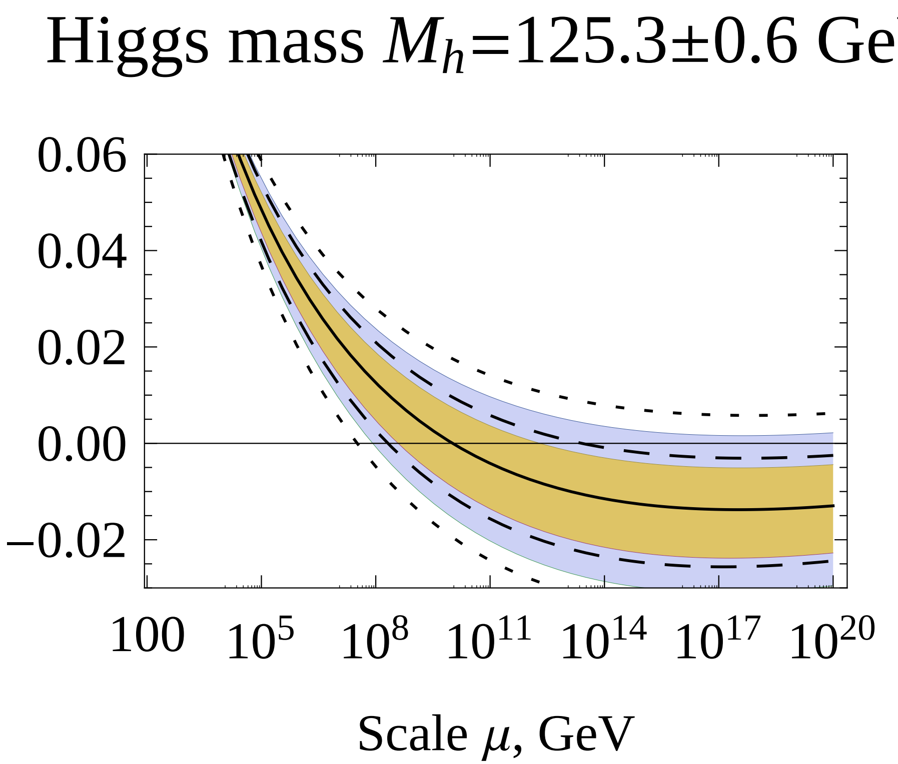
$y_t(M_t)$ - top Yukawa in $\overline{\mathrm{MS}}$ scheme

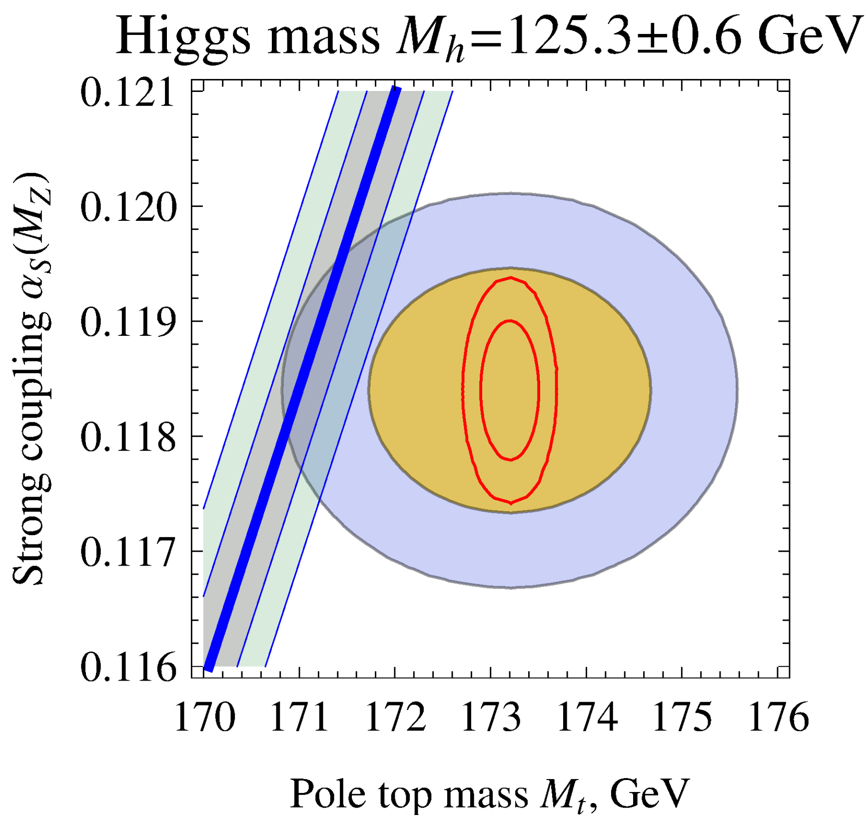
Matching at EW scale	Central value	theor. error
Bezrukov et al, $\mathcal{O}(\alpha\alpha_s)$	129.4 GeV	1.0 GeV
Degrassi et al, $\mathcal{O}(\alpha\alpha_s, y_t^2\alpha_s, \lambda^2, \lambda\alpha_s)$	129.6 GeV	0.7 GeV
Buttazzo et al, complete 2-loop	129.3 GeV	0.07 GeV

Chetyrkin et al, Mihaila et al, Bednyakov et al, 3 loop running to high energies

$$m_{\max} = [173.5 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.118}{0.002} \times 0.1]\ \mathrm{GeV}$$

Comparison with experiment





errors in y_t: theory + experiment

Tevatron: $M_t = \mathbf{173.2 \pm 0.51 \pm 0.71}$ GeV

ATLAS and CMS: $M_t = \mathbf{173.4 \pm 0.4 \pm 0.9}$ GeV

$\alpha_s = \mathbf{0.1184 \pm 0.0007}$

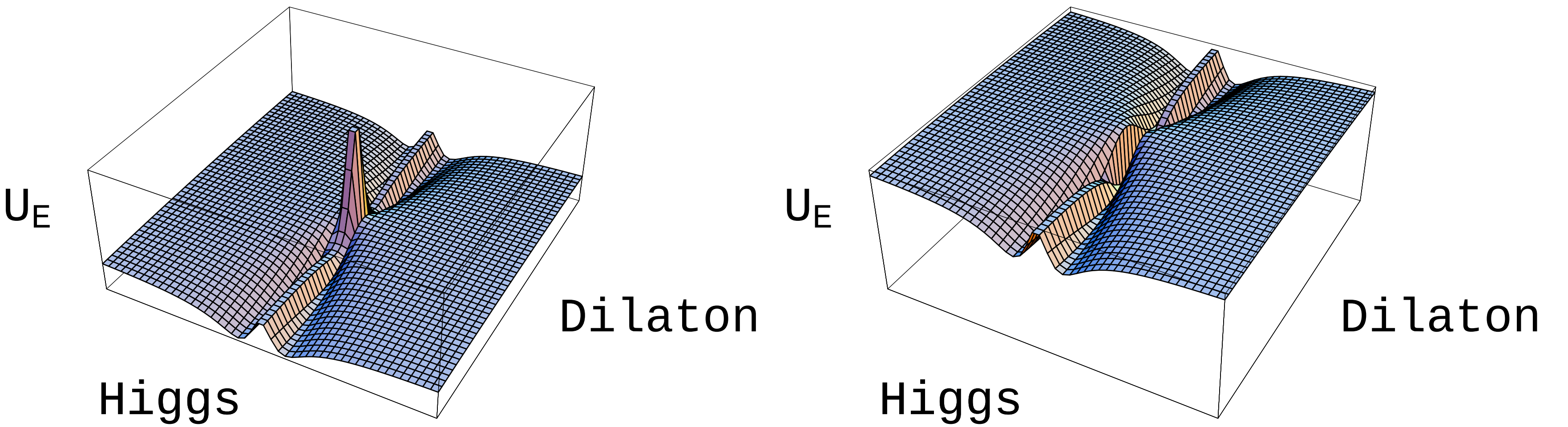
Main uncertainty - top Yukawa coupling.

- 1 GeV experimental error in M_t leads to 2 GeV error in M_{crit}.
- Perturbation theory, $\mathcal{O}(\alpha_s^4)$. Estimate of Kataev and Kim: $\delta y_t/y_t \simeq -750(\alpha_s/\pi)^4 \simeq -0.0015$, $\delta M_{crit} \simeq -0.5$ GeV
- Non-perturbative QCD effects, $\delta M_t \simeq \pm\Lambda_{QCD} \simeq \pm 300$ MeV, $\delta M_{crit} \simeq \pm 0.6$ GeV
- Alekhin et al. Theoretically clean is the extraction of y_t from $t\bar{t}$ cross-section. However, the experimental errors in $p\bar{p} \to t\bar{t} + X$ are quite large, leading to $\delta M_t \simeq \pm 2.8$ GeV, $\delta M_{crit} \simeq \pm 5.6$ GeV.

Precision measurements of m_H, y_t and α_s are needed.

e^+e^- Higgs and top factory!

Dark energy



Potential for the Higgs field and dilaton in the Einstein frame.

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Dark energy

Late time evolution of dilaton ρ along the valley, related to χ as

$$\chi = M_P \exp\left(\frac{\gamma\rho}{4M_P}\right), \quad \gamma = \frac{4}{\sqrt{6+\frac{1}{\xi_\chi}}} .$$

Potential: Wetterich; Ratra, Peebles

$$U_\rho = \frac{\Lambda}{\xi_\chi^2} \exp\left(-\frac{\gamma\rho}{M_P}\right) .$$

From observed equation of state: $0 < \xi_\chi < 0.09$

Result: equation of state parameter $\omega = P/E$ for dark energy must be different from that of the cosmological constant, but $\omega < -1$ is not allowed.

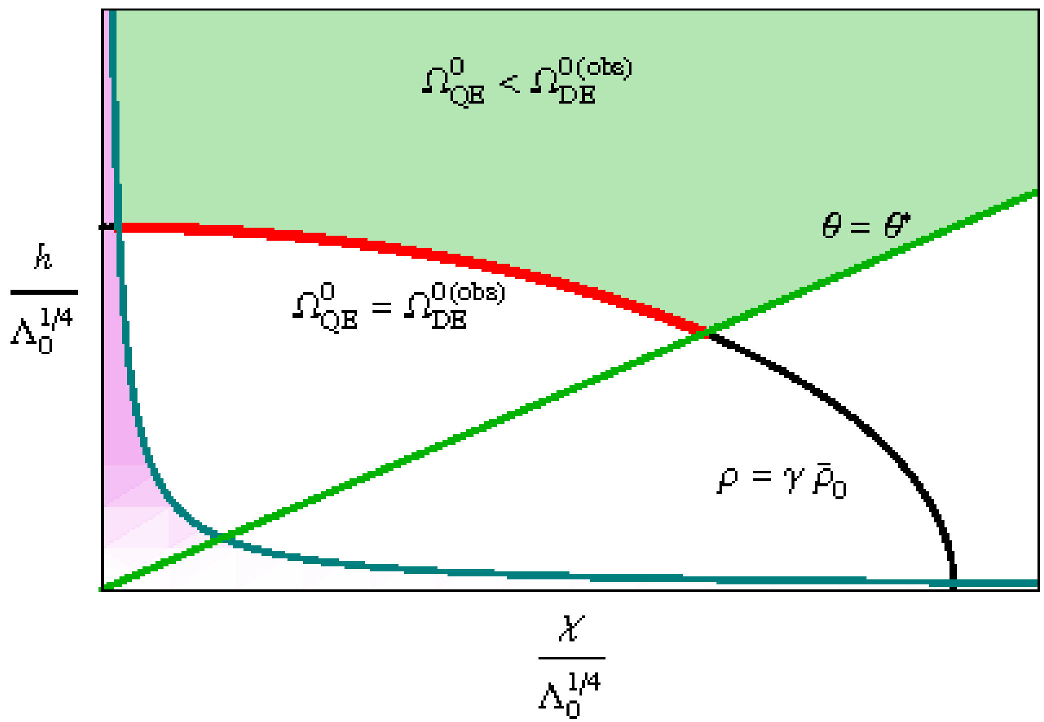
Higgs-dilaton cosmology: Strategy

Juan García-Bellido, Javier Rubio, M.S., Daniel Zenhäusern

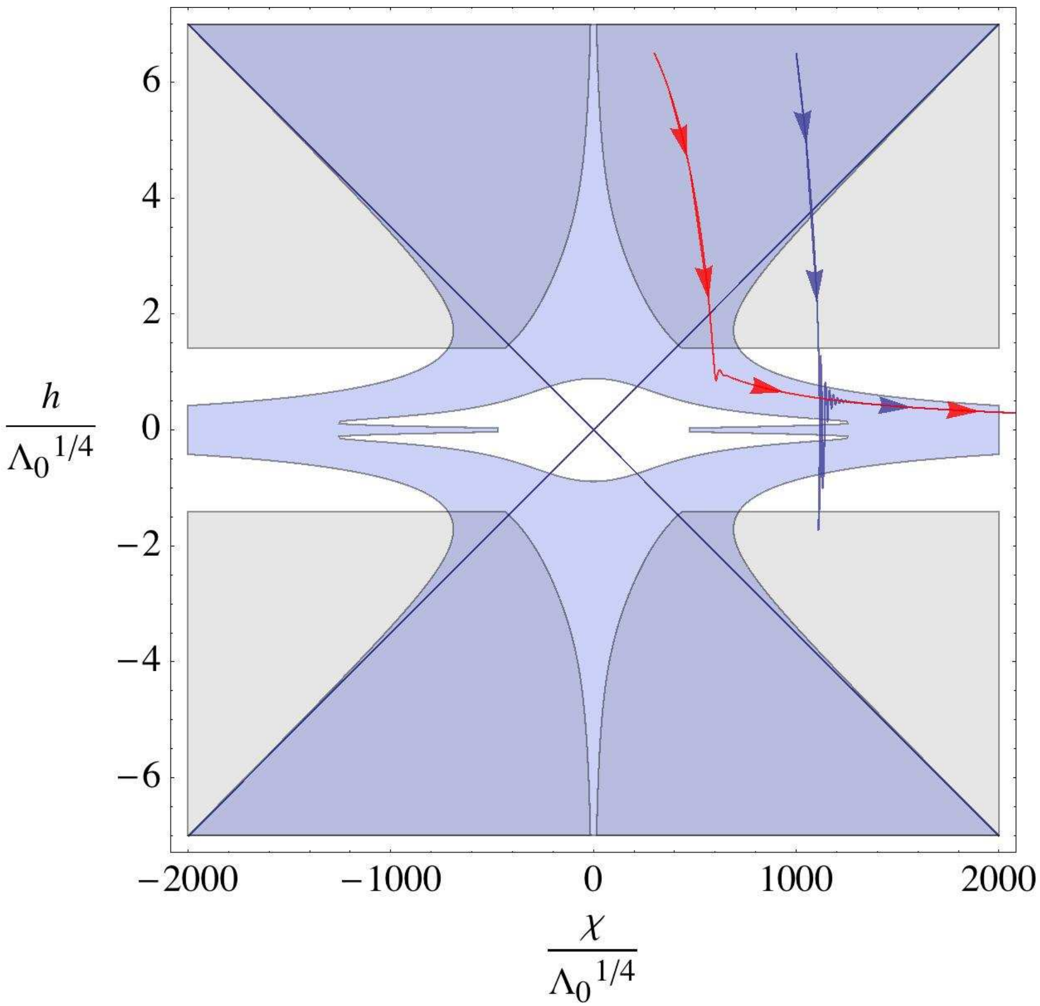
Both fields together:

- Take arbitrary initial conditions for the Higgs and the dilaton
- Find the region on the $\{\chi, h\}$ plane that lead to inflation
- Find the region on the $\{\chi, h\}$ plane that lead to exit from inflation
- Find the region on the $\{\chi, h\}$ plane that lead to observed abundance of Dark Energy

Initial conditions



Trajectories



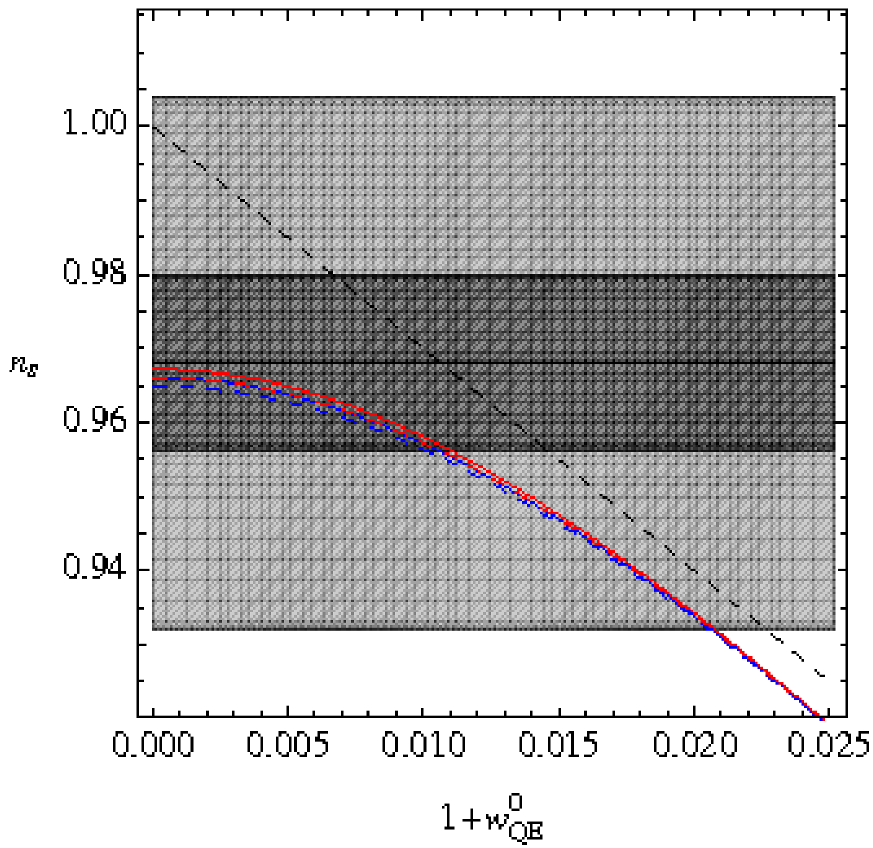
Generic semiclassical initial conditions lead to:

- the Universe, which was inflating in the past
- the Universe with the Dark Energy abundance smaller, than observed

Quantum initial state to explain the DM-DE coincidence problem?

Inflation-dark energy relation

Value of n_s is determined by ξ_h and ξ_χ, and equation of state of DE ω by $\xi_\chi \Longrightarrow n_s - \omega$ relation:



Intermediate summary

- Spontaneously broken scale invariance :
  - All mass scales originate from one and the same source - vev of the massless dilaton
  - Zero cosmological constant – $\beta = 0$ – existence of particles
  - Scale invariance naturally leads to flat potentials and thus to cosmological inflation
- TDiff or Unimodular gravity:
  - New parameter – strength of a particular potential for the dilaton
  - Dynamical Dark Energy

Quantum scale invariance

Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} ,$$

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Everything above does not make any sense???!!!

Standard reasoning

Dimensional regularisation $d = 4 - 2\epsilon$, $\overline{MS}$ subtraction scheme:

mass dimension of the scalar fields: $1 - \epsilon$,

mass dimension of the coupling constant: 2ϵ

Counter-terms:

$$\lambda = \mu^{2\epsilon} \left[\lambda_R + \sum_{k=1}^{\infty} \frac{a_n}{\epsilon^n} \right] ,$$

μ is a dimensionfull parameter!!

One-loop effective potential along the flat direction:

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2} \left[\log \frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2} \right] ,$$

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Reason: mismatch in mass dimensions of bare (λ) and renormalized couplings (λ_R)

Idea: Replace $\mu^{2\epsilon}$ by combinations of fields χ and h, which have the correct mass dimension:

$$\mu^{2\epsilon} \to \chi^{\frac{2\epsilon}{1-\epsilon}} F_\epsilon(x) \ ,$$

where $x = h/\chi$. $F_\epsilon(x)$ is a function depending on the parameter ϵ with the property $F_0(x) = 1$.

Zenhäusern, M.S

Englert, Truffin, Gastmans, 1976

Example of computation

Natural choice:

$$\mu^{2\epsilon} \to \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \;, (\xi_\chi \chi^2 + \xi_h h^2) \equiv \omega^2$$

Potential:

$$U = \frac{\lambda_R}{4}\left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}}\left[h^2 - \zeta_R^2 \chi^2\right]^2 \;,$$

Counter-terms

$$U_{cc} = \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}}\left[A h^2 \chi^2 \left(\frac{1}{\bar{\epsilon}} + a\right) + B\chi^4 \left(\frac{1}{\bar{\epsilon}} + b\right) + C h^4 \left(\frac{1}{\bar{\epsilon}} + c\right)\right],$$

To be fixed from conditions of absence of divergences and presence of spontaneous breaking of scale-invariance

$$
\begin{aligned}
U_1 = & \quad \frac{m^4(h)}{64\pi^2}\left[\log\frac{m^2(h)}{v^2} + \mathcal{O}\left(\zeta_R^2\right)\right] \\
+ & \quad \frac{\lambda_R^2}{64\pi^2}\left[C_0 v^4 + C_2 v^2 h^2 + C_4 h^4\right] + \mathcal{O}\left(\frac{h^6}{\chi^2}\right),
\end{aligned}
$$

where $m^2(h) = \lambda_R(3h^2 - v^2)$ and

$$
\begin{aligned}
C_0 &= \frac{3}{2}\left[2a - 1 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + \frac{4}{3}\log 2\lambda_R + O(\zeta_R^2)\right] , \\
C_2 &= -3\left[2a - 3 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + O(\zeta_R^2)\right] , \\
C_4 &= \frac{3}{2}\left[2a - 5 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) - 4\log 2\lambda_R + O(\zeta_R^2)\right] .
\end{aligned}
$$

Origin of Λ_{QCD}

Consider the high energy ($\sqrt{s} \gg v$ but $\sqrt{s} \ll \chi_0$) behaviour of scattering amplitudes on the example of Higgs-Higgs scattering (assuming, that $\zeta_R \ll 1$). In one-loop approximation

$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2}\left[\log\left(\frac{s}{\xi_\chi \chi_0^2}\right) + \mathrm{const}\right] + \mathcal{O}\left(\zeta_R^2\right) \ .$$

This implies that at $v \ll \sqrt{s} \ll \chi_0$ the effective Higgs self-coupling runs in a way prescribed by the ordinary renormalization group!

For QCD:

$$\Lambda_{QCD} = \chi_0 e^{-\frac{1}{2b_0\alpha_s}}, \quad \beta(\alpha_s) = b_0\alpha_s^2$$

Almost trivial statement - by construction: Quantum effective action is scale invariant in all orders of perturbation theory!

Less trivial statement: Quantum effective action is conformally invariant in all orders of perturbation theory!

Hierarchy problem without gravity: no large perturbative corrections to the Higgs mass: those proportionnal to χ contain necessarily α (shift symmetry $\chi \to \chi + const$), those proportionnal to λ contain only logs of χ.

Hierarchy problem with gravity: M_H is the mass of the particle but M_P is associated with the strength of the gravitational interaction. The graviton is massless.

Perturbative computations of gravitational corrections to the Higgs mass in scale-invariant regularisation : all corrections are suppressed by M_P, and there are no corrections proportional to M_P!

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- Higgs mass is stable against radiative corrections
- Requirement of spontaneous breakdown of scale invariance - cosmological constant is tuned to zero in all orders of perturbation theory

Problems

- Renormalizability: Can we remove all divergences with the similar structure counter-terms? The answer is “no" (Tkachov, MS). However, this is not essential for the issue of scale invariance. We get scale-invariant effective theory. In any event, gravity is not renormalizable
- Unitarity and high-energy behaviour: What is the high-energy behaviour $(E > M_{Pl})$ of the scattering amplitudes? Is the theory unitary? Can it have a scale-invariant UV completion?
- What happens non-perturbatively is an open question.

Dilaton as a part of the metric

Previous discussion - ad hoc introduction of scalar field χ. It is massless, as is the graviton. Can it come from gravity?

Yes - it automatically appears in scale-invariant TDiff gravity as a part of the metric!

Consider arbitrary metric $g_{\mu\nu}$ (no constraints). Determinant g of $g_{\mu\nu}$ is TDiff invariant. Generic scale-invariant action for scalar field and gravity:

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\phi^2 f(-g)R - \frac{1}{2}\phi^2\mathcal{G}_{gg}(-g)(\partial g)^2$$

$$-\frac{1}{2}\mathcal{G}_{\phi\phi}(-g)(\partial\phi)^2 + \mathcal{G}_{g\phi}(-g)\phi\,\partial g\cdot\partial\phi - \phi^4 v(-g)\Big]\,.$$

Equivalence theorem

This TDiff theory is equivalent (at the classical level) to the following Diff scalar tensor theory:

$$\frac{\mathcal{L}_e}{\sqrt{-g}} = -\frac{1}{2}\phi^2 f(\sigma) R - \frac{1}{2}\phi^2 \mathcal{G}_{gg}(\sigma)(\partial\sigma)^2 - \frac{1}{2}\mathcal{G}_{\phi\phi}(\sigma)(\partial\phi)^2$$

$$-\mathcal{G}_{g\phi}(\sigma)\phi\, \partial\sigma \cdot \partial\phi - \phi^4 v(\sigma) - \frac{\Lambda_0}{\sqrt{\sigma}}\,.$$

Transformation to Einstein frame:

$$\frac{\mathcal{L}_e}{\sqrt{-\tilde{g}}} = -\frac{1}{2}M^2\tilde{R} - \frac{1}{2}M^2\mathcal{K}_{\sigma\sigma}(\sigma)(\partial\sigma)^2 - \frac{1}{2}M^2\mathcal{K}_{\phi\phi}(\sigma)(\partial\ln(\phi/M))^2$$

$$-M^2\mathcal{K}_{\sigma\phi}(\sigma)\,\partial\sigma\cdot\partial\ln(\phi/M) - M^4V(\sigma) - \frac{M^4\Lambda_0}{\phi^4 f(\sigma)^2\sqrt{\sigma}}\,,$$

As expected, ϕ is a Goldstone boson with derivative couplings only (except the term containing Λ_0).

So, TDiff scale invariant theory automatically contains a massless dilaton. All previous results can be reproduced in it.

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  - The massless sector of the theory contains dilaton, which has only derivative couplings to matter and can be a part of the metric.
  - All observational drawbacs of the SM can be solved by new physics below the Fermi scale

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