

# **PLANCK mission, Starobinsky inflation and its realization in old-minimal supergravity**

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- PLANCK mission results (March 2013)
- Starobinsky inflation and supergravity setup
- Starobinsky inflation in old-minimal supergravity
- Reheating and leptogenesis in supergravity

## Our References

arXiv:1309.7494 with Takahiro Terada, to appear in JHEP

arXiv:1309.0293, to appear in PTEP

arXiv:1201.2239, invited review in Int.J.Mod.Phys. A28 (2013) 1330021

## Inflation in Early Universe

- Cosmological **inflation** ('rapid' accelerated expansion) predicts **homogeneity** of our Universe at large scales, its spatial **flatness**, **large** size and entropy, and the almost **scale-invariant** spectrum of cosmological perturbations (in good agreement with the WMAP/PLANCK measurements of the CMB radiation spectrum).
- Inflation is a paradigm, not a theory! Known theoretical **mechanisms** of inflation use a **slow-roll** scalar field (called **inflaton**) with the proper scalar potential.
- The **scale** of inflation (about  $10^{13}$  *GeV*) is well beyond the electro-weak scale (i.e. beyond the SM). Inflation is a **great window** to the HEP.
- The **nature** of inflaton and the **origin** of its scalar potential are **unknown**.

## PLANCK mission data (March 2013) on CMB

combined with the WMAP9 and lensing data: Ade et al (PLANCK Collaboration), arXiv:1303.5082, and G. Hinshaw et al [WMAP Collaboration], arXiv:1212.5226:

- $n_s = 0.960 \pm 0.007$  for the CMB spectral index
- $r < 0.08$  for the CMB tensor-to-scalar-ratio (with 95% CL),
- $f_{NL} = 2.7 \pm 5.8$  (with 68% CL)

It favours the inflationary models with relatively low  $r$  and low non-Gaussianity.

## Chaotic Inflation in the Starobinsky model

Viable inflationary models are easily constructed in  $f(R)$ -gravity theories,

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} f(R) \quad (1)$$

The simplest (Starobinsky, 1980) model is given by ( $M_{\text{Pl}} = 1$ )

$$f_S(R) = R - \frac{R^2}{6M^2} \quad (2)$$

and is known as the excellent model of **chaotic** inflation.  $M$  coincides with the (scalaron/inflaton) mass. The model fits the observed amplitude of scalar perturbations if  $M/M_{\text{Pl}} \approx 3.0 \cdot 10^{-6}(50/N_e)$ , and gives rise to  $n_s - 1 \approx -2/N_e \approx -0.04(50/N_e)$  and  $r \approx 12/N_e^2 \approx 0.004(50/N_e)^2$ , in terms of the e-foldings number  $N_e \approx (50 \div 55)$ . It perfectly fits the PLANCK data too!

## Starobinsky model and quintessence

Any  $f(R)$  gravity model is classically equivalent to a quintessence model. In the special case of  $f_S(R) = R - \frac{1}{6M^2}R^2$  one finds the inflaton scalar potential

$$V(\varphi) = \frac{3}{4}M^2 \left( 1 - \exp \left[ -\sqrt{\frac{2}{3}}\varphi \right] \right)^2 \quad (3)$$

having a *plateau* needed for slow roll. It results in the global **scale invariance** in the large  $\varphi$  limit.

The scale invariance is *not* exact for finite values of  $\varphi$ , and its *violation* is measured by the *slow-roll parameters*, in full correspondence to the **nearly conformal** spectrum of the CMB perturbations associated with the inflaton field  $\varphi$ .

## Pre-Comments (Introduction)

- The Starobinsky inflation is **preferred** by the PLANCK data. The Starobinsky model is truly non-perturbative and **phenomenological**.
- In the Starobinsky inflation, inflaton is **scalaron** (spin-0 part of metric).
- The Starobinsky inflation automatically leads to the **universal reheating mechanism** after inflation, in a matter-coupled  $f(R)$  gravity after a transformation to the Einstein frame, with the reheating temperature  $T_{\text{reh}} \approx 10^9$  GeV.
- The **Higgs inflation** (preferred by the PLANCK data also) leads to the *same* quintessence scalar potential but the *higher* reheating temperature ( $10^{13}$  GeV).

## Some Motivation for Supergravity

- Combining SUSY with inflation **requires** SUGRA which is more restrictive (harder to get inflation) but more general (more particles and fields) framework.
- Deriving the Starobinsky inflation from a **fundamental** theory of Quantum Gravity (Superstrings), in order to upgrade its present (phenomenological) status.
- SUGRA is the LEEA of (closed) **superstrings**.
- Including **Dark Matter** (the LSP) and connecting to the **MSSM**.



## Remarks about higher-derivative supergravity

- Available tools: (i) superconformal tensor calculus, (ii) **curved superspace**.
- The **old-minimal** and new-minimal off-shell SUGRAs are **inequivalent**.
- A **minimal** SUGRA extension of the  $(R + R^2)$  gravity has  $12_B + 12_F$  d.o.f. off-shell and, generically,  $6_B + 6_F$  d.o.f. on-shell, though the latter can be reduced to  $4_B + 4_F$  on-shell d.o.f. in a *chiral*  $F(\mathcal{R})$  supergravity.

**Note:** the standard SUGRA (i.e. the extension of the  $R$ -gravity) has  $12_B + 12_F$  d.o.f. off-shell but only  $2_B + 2_F$  on-shell.

## Old-minimal SUGRA fields and superfields

The supergravity superfield (off-shell gravity supermultiplet)  $\mathcal{R}$  containing the **Ricci scalar** curvature  $R$  amongst its field components is covariantly chiral,  $\bar{\nabla}_{\alpha} \bullet \mathcal{R} = 0$ , and obeys other off-shell constraints. Its bosonic field components are

$$\mathcal{R}| = X, \quad \nabla_{\alpha} \nabla_{\beta} \mathcal{R}| = \frac{1}{2} \varepsilon_{\alpha\beta} \left( -\frac{1}{3} R + 16 \bar{X} X + \frac{2}{9} b_a b^a + \dots \right), \quad (4)$$

The old-minimal supergravity fields (in a WZ-like gauge) are  $(e_m^a, \psi_m^{\alpha}; X, b_a)$ .

There exist an invariant **chiral** superspace with the chiral density  $\mathcal{E}$ , so that

$$\mathcal{E}| = \frac{1}{2} e, \quad \nabla^2 \mathcal{E}| = -12 e \bar{X} + \dots, \quad (5)$$

where  $e = \sqrt{-g}$ . One can define the covariantly chiral **projection** of the  $\bar{R}$  too,

$$\Sigma(\bar{R}) = -\frac{1}{8} \left( \bar{\nabla}^2 - 8R \right) \bar{R}, \quad \bar{\nabla}_{\alpha} \bullet \Sigma = 0. \quad (6)$$

## The SUGRA invariant actions

There are **three** different types of the  $\mathcal{R}$ -dependent invariants (Cecotti, SVK):

- of D-type, leading to the  $g(X)R + h(X)R^2$  gravity,

$$S_N = \int d^4x d^4\theta E^{-1} N(\mathcal{R}, \overline{\mathcal{R}}) \ , \quad (7)$$

- of F-type, leading to  $g(X) + h(X)R$  gravity,

$$S_F = \int d^4x d^2\Theta 2\mathcal{E}F(\mathcal{R}) + \text{H.c.} \ , \quad (8)$$

- and their extensions with a  $\Sigma(\overline{R})$ -dependence,

$$S_{N+F} = \int d^4x d^4\theta E^{-1} N(\mathcal{R}, \overline{\mathcal{R}}; \Sigma, \overline{\Sigma}) + \left[ \int d^4x d^2\Theta 2\mathcal{E}F(\mathcal{R}, \Sigma) + \text{H.c.} \right] \quad (9)$$

## Duality with matter-coupled supergravity (I)

A generic action above is **dual** (classically equivalent) to the standard matter-coupled SUGRA (without higher derivatives):

$$S = \int d^4x d^4\theta E^{-1} N(J, \bar{J}; H, \bar{H}) + \left\{ \int d^4x d^2\Theta 2\mathcal{E} \left[ F(J, H) + \Lambda(J - \mathcal{R}) + \Xi(H - \Sigma(\bar{R})) \right] + \text{H.c.} \right\}, \quad (10)$$

where we have introduced two independent (covariantly) chiral superfields,  $J$  and  $H$ , and the covariantly chiral Lagrange multiplier superfields  $\Lambda$  and  $\Xi$ . Varying the action (10) with respect to  $\Lambda$  and  $\Xi$  yield

$$J = \mathcal{R} \quad \text{and} \quad H = \Sigma(\bar{R}) \quad (11)$$

respectively, which gives back the original action.

## Duality with matter-coupled supergravity (II)

It its turn, the action (10) can be rewritten to

$$S = \int d^4x d^4\theta E^{-1} \left[ N(J, \bar{J}; H, \bar{H}) - (\Lambda + \bar{\Lambda}) - (\Xi \bar{J} + \bar{\Xi} J) \right] + \left\{ \int d^4x d^2\Theta 2\mathcal{E} [F(J, H) + \Lambda J + \Xi H] + \text{H.c.} \right\} \quad (12)$$

Comparing it with the standard matter-coupled SUGRA action in the form

$$S_{\text{msg}} = -3 \int d^4x d^4\Theta E^{-1} e^{-\frac{1}{3}K} + \left[ \int d^4x d^2\Theta 2\mathcal{E}W + \text{H.c.} \right] , \quad (13)$$

in terms of the **Kähler** potential  $K(\Phi, \bar{\Phi})$  and the **superpotential**  $W(\Phi)$ , we find

$$K = -3 \ln \left[ \frac{\Lambda + \bar{\Lambda} + \Xi \bar{J} + \bar{\Xi} J - N}{3} \right] \quad (14)$$

and

$$W = F(J, H) + \Lambda J + \Xi H . \quad (15)$$

## Kinetic terms and the scalar potential

The **bosonic** sector of the theory is given by

$$e^{-1}L_{\text{bos.}} = -\frac{1}{2}R + K_{i,\bar{j}}\partial_{\mu}z^i\partial^{\mu}\bar{z}^{\bar{j}} - V \quad (16)$$

with the scalar potential (Cremmer et al.)

$$V(z, \bar{z}) = e^G \left[ G_{,i} \left( \frac{\partial^2 G}{\partial z^i \partial \bar{z}^{\bar{j}}} \right)^{-1} G_{,\bar{j}} - 3 \right] \quad (17)$$

where we have introduced the notation

$$\Lambda| = z^1, \quad J| = z^2, \quad \Xi| = z^3, \quad H| = z^4, \quad i, j = 1, 2, 3, 4 \quad (18)$$

and the Kähler gauge-invariant function

$$G = K + \ln(\bar{W}W), \quad G_{,i} = \frac{\partial G}{\partial z^i}, \quad G_{,\bar{j}} = \frac{\partial G}{\partial \bar{z}^{\bar{j}}}. \quad (19)$$

## Some results of our detailed studies

- The Kähler potential in Eq. (14) is of the **no-scale** type, favoured in many superstring compactifications, and often used for a dynamical solution to the hierarchy problem between  $M_H$  and  $M_{Pl}$ .
- A non-trivial  $\Sigma(\overline{R})$ -dependence leads to **ghosts** and should be discarded.
- An F-type action can be included into a D-type action, **except of** a constant.
- An F-type constant triggers spontaneous SUSY breaking of **arbitrary** scale.

## Embedding the Starobinsky inflation into supergravity

- Choosing  $N = \lambda \bar{J} J$  with  $F'(J) = -1/2$  generates the Starobinsky scalar potential in the parametrization  $\Lambda = \frac{1}{2} \exp[\sqrt{\frac{2}{3}}\phi] + ib$  along the inflationary trajectory  $J = b = 0$  (Cecotti). However, this trajectory is **unstable** w.r.t.  $J$ .
- Stabilization can be achieved by **modifying**  $N(\bar{J}J) = \lambda \bar{J} J - \zeta(\bar{J}J)^2$  for appropriate values of the positive parameter  $\zeta$  (Kallosh, Linde et al).
- The  $F(\mathcal{R})$  supergravity model with a **cubic**  $F$  function leads to the high-curvature inflation (Ketov, Starobinsky). However, it is **unstable** w.r.t.  $R$ .
- No-scale supergravities with the  $SU(N, 1)/SU(N) \times U(1)$  Kähler potential and special superpotential also allow the Starobinsky inflation (Ellis et al).



## Reheating and gravitino leptogenesis

- Scalaron is **universally** coupled to ALL matter, so that reheating can proceed through the out-of-equilibrium scalaron decay.
- The scale of SUSY breaking is **directly** related to the gravitino mass.
- Gravitino interactions have the gravitational strength and can generate CP-asymmetry carried by sleptons. Hence, gravitino may produce L-asymmetry too. It requires the high scale of SUSY breaking,  $M_S \geq 10^{13}$  GeV (Krauss et al).
- Then gravitino is heavy,  $m_{3/2} \geq 10^8$  GeV, so the gravitino problem does **not** arise. The **reheating temperature** after inflation is  $T_{reh} \geq 10^{12}$  GeV.

## Conclusion and Outlook

The story of the Starobinsky inflation, reheating, and leptogenesis in supergravity is fascinating and still unfinished.

Thank you for your attention!