On the Fukaya-Seidel categories of surface Lefschetz fibrations

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Contents

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- We show that we can define the Fukaya-Seidel category for some LF (not necessarily exact!), and prove the invariance of its derived category.

Existance of structure of exact Lefschetz fibration

Theorem 1.1

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Then, the derived Fukaya-Seidel category $D\mathcal{F}(\pi)$ is an invariant of π, i.e. the equivalent class of $D\mathcal{F}(\pi)$ as triangulated category is independent of the choice of the structure of exact Lefschetz fibration.

Fukaya-Seidel category for closed LF with boudary data

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Let π be a closed LF over S^2
and s be a section of π that does not pass through the critical points of π.
We can define a Fukaya-Seidel category for a pair (π, s) and its derived category is an invariant of (π, s).

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Definition 2.1 (Lefschetz fibration)

Let D be the closed unit disc in $\mathbb{C}$. $\pi\colon E^4 \to S^2$ (resp. D) is a Lefschetz fibration over S^2 $(\mathrm{resp.}\, D)$ if the following conditions are satisfied.

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   Crit$(\pi) \subset$ Interior(E).

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1. *Crit*$(\pi) \xrightarrow{\pi}$ *Critv*(π) *is a bijection between finite sets.*
   Crit$(\pi) \subset$ *Interior*(E).
2. $\pi|_{\pi^{-1}(S^2 \setminus \mathit{Critv}(\pi))}$ (*resp.* $\pi|_{\pi^{-1}(D \setminus \mathit{Critv}(\pi))}$) *is a smooth fibre bundle.*
   The fiber M is a manifold with boundary.

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4. *∂E is trivial in some sense.*

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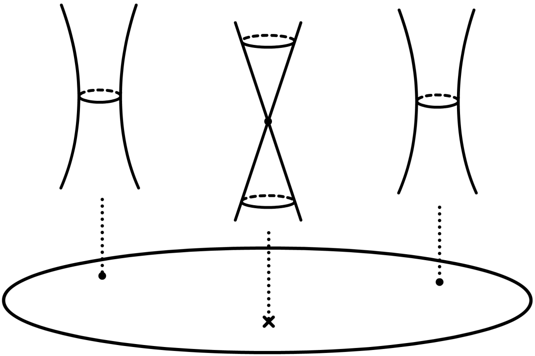

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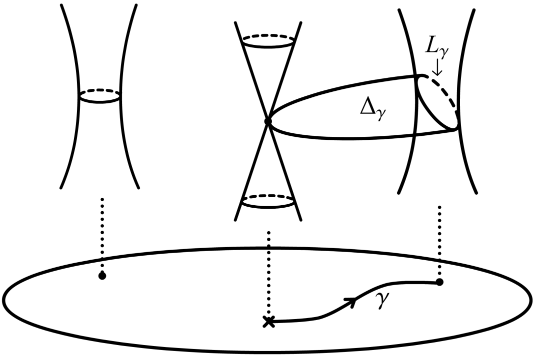

exact symplectic manifold

Definition 2.2 (exact symplectic manifold)

A four-tuple (M, ω, θ, J) is called an exact symplectic manifold if the following conditions hold.

1. (M, ω) *is a compact symplectc manifold with corner.*
2. θ *is a 1-form on* M *and satisfies* $d\theta = \omega$.
3. *The negative Liouville vector field* X_θ *points strictly inwards on* ∂M.
4. J *is an* ω*-compatible almost complex structure on* M.
5. ∂M *is weakly* J*-convex.*

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1. *j is a complex structure on D.*
   π is a (J, j)-holomorphic Lefschetz fibration with regular fibre M.

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   π *is a* (J, j)*-holomorphic Lefschetz fibration with regular fibre* M.
2. J *is integrable around Crit*(π).
3. ω *is canonical one around* $\partial_h E := \partial M \times D \subset U \cong (\bigsqcup S^1) \times [0, \varepsilon) \times D$

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Proof.

- A symplectic form ω is constructed by Gompf's method.

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Proof.

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- We can construct an exact symplectic form if all vanishing cycles are non-separating.
- We can also construct θ, J, j that makes π an exact LF.

□

Contents

Fukaya-Seidel categories: objects

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Fukaya-Seidel categories: objects

Let $\pi\colon E \to D$ be an exact LF.
$\to$ gather vanishing cycles $L_1, L_2, \ldots, L_N$ to a regular fibre M in some coherent way. (We use the symplectic form on E in this procedure.)
Then, we set $Ob(\mathcal{F}(\pi)) := \{L_i\}$.

Fukaya-Seidel categories: morphisms

We assume that $L_i \pitchfork L_j$ if $i \neq j$. Then we define:

$$\mathrm{Hom}(L_i, L_j) := \begin{cases} \bigoplus_{p \in L_i \cap L_j} \mathbb{K}[p] & (i < j) \\ \mathbb{K} e_i & (i = j) \\ 0 & (i > j) \end{cases}.$$

Here, $\mathbb{K}$ is an arbitrary coefficient field.
$\bigoplus_{p \in L_i \cap L_j} \mathbb{K}[p]$ is also written as $CF(L_i, L_j)$.
We can give $\mathrm{Hom}(L_i, L_j)$ a grading when we fix grading datum of L_i's.

Fukaya-Seidel categories: A_∞-structure

The A_∞-structure is a collection of maps $\{\mu^d\}_{d\geq 1}$:

$$\mu^d : \mathrm{Hom}(L_{i_{d-1}}, L_{i_d}) \otimes \mathrm{Hom}(L_{i_{d-2}}, L_{i_{d-1}}) \otimes \cdots \otimes \mathrm{Hom}(L_{i_0}, L_{i_1}) \to \mathrm{Hom}(L_{i_0}, L_{i_d}).$$

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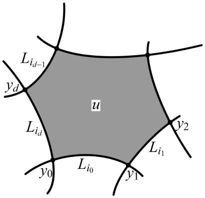
These are defined by counting such discs.



$$\mu^d([y_d],[y_{d-1}],\ldots,[y_1]) := \sum_u \pm[y_0]$$

Fukaya-Seidel categories: A_∞-relation

The A_∞-structure $\{\mu^d\}_{d\geq 1}$ satisfies the A_∞-relation:

$$\sum \pm\mu^l(y_d, y_{d-1}, \ldots, y_{j+k+1}, \mu^k(y_{j+k}, \ldots, y_{j+1}), y_j, \ldots, y_1) = 0.$$

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Remark 3.1

Because of exactness, the A_∞-structure starts from μ^1.

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- *We can assume that* $[L_i] = [L'_i] \in [S^1, M]$.

preparation(2)

Theorem 4.2 (S.)

Let $M := (\Sigma_{g,1}, \omega, \theta, j)$ *be an exact smplectic manifold, and* L, L' *be Lagrangian* S^1*'s satisfying the following conditions.*

- $\int_L \theta = \int_{L'} \theta$.
- $M \setminus L, M \setminus L'$ *are connected.*
- $[L] = [L'] \in [S^1, M]$.

Then, there exists a Hamiltonian diffeomorphism $\phi \in$ *Ham*$(M, \partial M, \omega)$ *s.t.* $\phi(L') = L$

preparation(3)

Proposition 4.3 (S.)

In general case, there exist L_i'''s satisfy the following conditions.

- $\int_{L_i''} \theta = \int_{L_i''} \theta' = 0$.
- $M \setminus L_i''$ *is connected.*
- $[L_i''] = [L_i] = [L_i'] \in [S^1, M]$.

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  $D\mathcal{F}(\boldsymbol{L})_\omega \cong D\mathcal{F}(\boldsymbol{L}'')_\omega$ by Seidel.
  $D\mathcal{F}(\boldsymbol{L}')_{\omega'} \cong D\mathcal{F}(\boldsymbol{L}'')_{\omega'}$ by Seidel.
  $\mathcal{F}(\boldsymbol{L}'')_\omega \cong D\mathcal{F}(\boldsymbol{L}'')_\omega$ since the A_∞-structure does not depend on the symplectic structure in the case of the Riemann surface.

That completes the proof.

Problems

- Application to computing the Fukaya-Seidel categories.

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- Application to the 4-dimensional topology.

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- Application to computing the Fukaya-Seidel categories.
- Application to the 4-dimensional topology.
- Relation between 4-dimensional topology and Mirror symmetry.

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Thank you for your listening!!(≧ ▽ ≦)