Vacuum state as entangled state between Left, Right, Future and Past

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This talk is based on collaborations:

A. Higuchi, S. Iso, K. Ueda, K. Yamamoto (PRD 2017)
A. Higuchi, K. Yamamoto (PRD 2018)
A. Higuchi, Y. Nan, K. Ueda, K. Yamamoto, in preparation
T. Kobayashi, Y. Sugiyama, K. Yamamoto, in progress

Outline

1. *Introduction*
2. *Scalar field in Minkowski space*
3. *De Sitter space*
4. *Dirac field*
5. *Gravitational wave*
6. *Discussions - Application*
7. *Summary and Conclusions*

1. Introduction

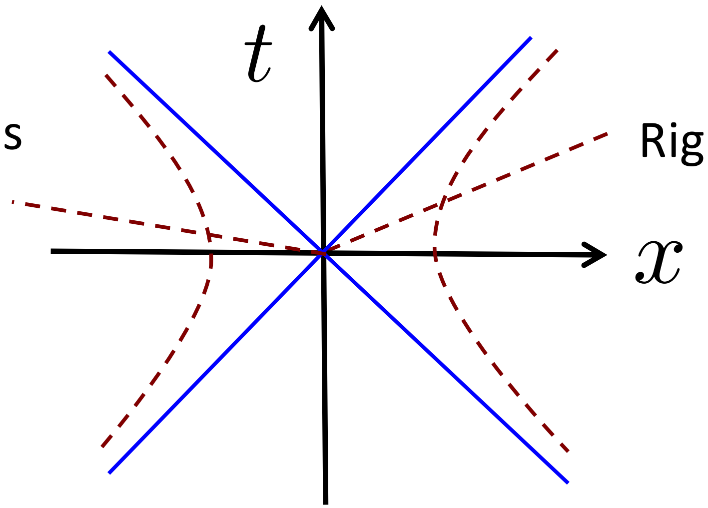
Minkowski vacuum is expressed as an entangled state between the right and left Rindler wedges

Unruh & Wald (1984)

$$|0, \mathrm{M}\rangle = N \prod_j \Big[\sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j / a} |n_j, \mathrm{R}\rangle \otimes |n_j, \mathrm{L}\rangle \Big]$$

Left Rindler wedges

$|n_j, \mathrm{L}\rangle$



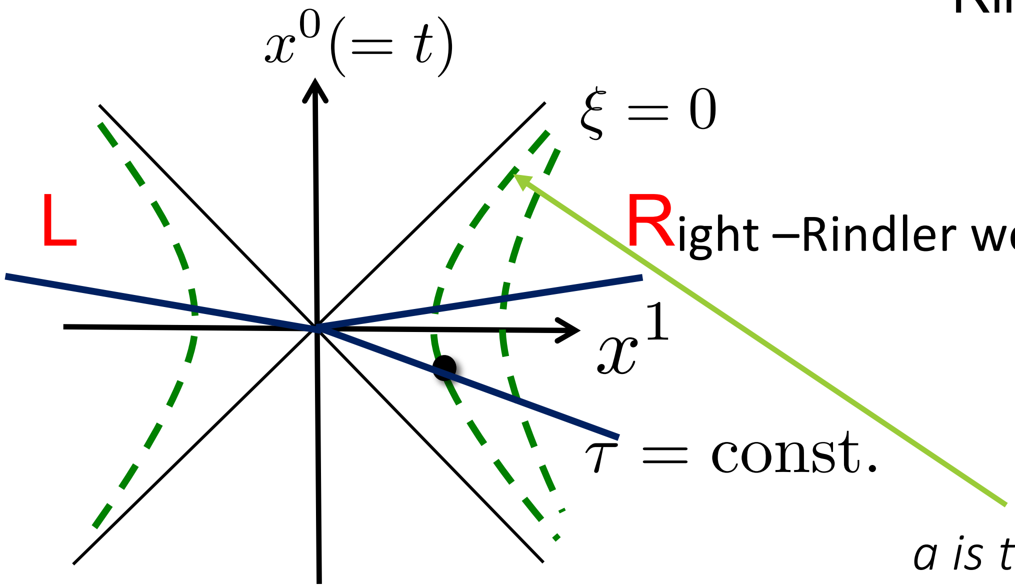
Right –Rindler wedge

$|n_j, \mathrm{R}\rangle$

j denotes a mode of a field in Rindler space, and *n*th excited state of the mode j

(Right) Rindler spacetime

Coordinate for an observer in a uniform acceleratedmotion



Rindler coordinate (τ, ξ)

$$t = \frac{e^{a\xi}}{a} \sinh a\tau \qquad x = \frac{e^{a\xi}}{a} \cosh a\tau$$

$$ds^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - d\mathbf{x}_\perp^2$$

a is the acceleration for an observer fixed at xi=0

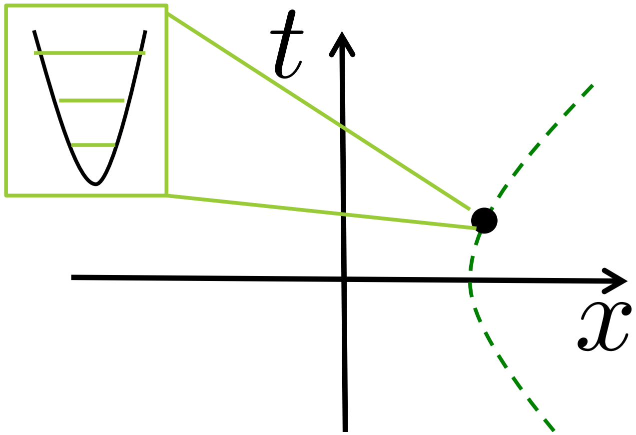
We use the unit $a = 1$ without notice

Origin of the thermality of the Unruh effect

A observer in an uniformly accelerated motion sees the Minkowski vacuum as a thermally excited state.

Partial trace of the density matrix of the Minkowski vacuum state w.r.t. the left Rindler space →the thermal state appears in the right Rindler wedge

$$\mathrm{Tr}_{\mathrm{L}}\left(|0,M\rangle\langle 0,M|\right) = N' \prod_j \sum_{n_j=0,1,\cdots} e^{-2\pi n_j \omega_j/a} |n_j,R\rangle\langle n_j,R|$$



Thermal state of temperature

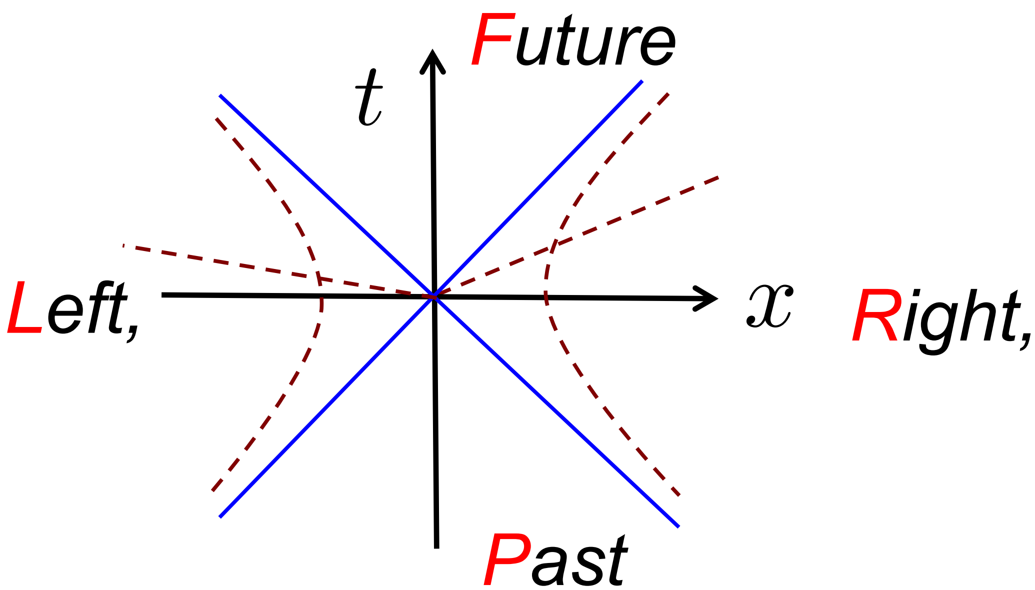
$$T = \frac{a}{2\pi}$$

Thermal excitations of a detector (and a particle) in a uniform accelerated motion coupled to vacuum fluctuations: Unruh effect

$$|0,\mathrm{M}\rangle = N\prod_j\Big[\sum_{n_j=0}^{\infty} e^{-\pi n_j\omega_j/a}|n_j,\mathrm{R}\rangle\otimes|n_j,\mathrm{L}\rangle\Big]$$

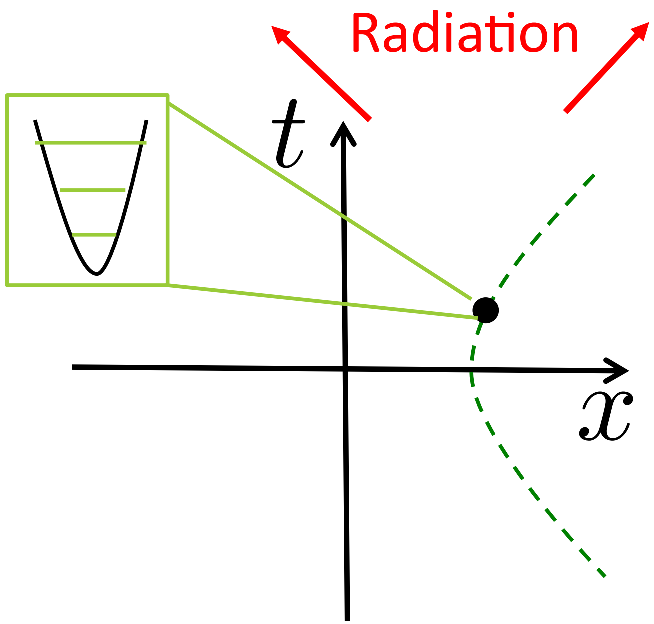
A. Higuchi, S. Iso, K. Ueda, K. Yamamoto (201

Vacuum state as an entangled state between Left, Right, Future (and past) regions



Useful to understand the origin of quantum radiation produced by a detector (and particle) in uniformly accelerating motion, thermally excited due to the Unruh effect.

Uniformly accelerating detector (particle) coupled vacuum fluctuations is thermally excited due to the Unruh effect and it may produce quantum radiation.
We have shown that he quantum radiation is not the thermal radiation locally generated by the thermal excited detector, but the quantum radiation is related to the nonlocal correlation reflecting the entanglement structure.



Iso, Oshita, Tatsukawa,Yamamoto, Zhang (2017)
Higuchi, Iso, Ueda, Yamamoto (2017)

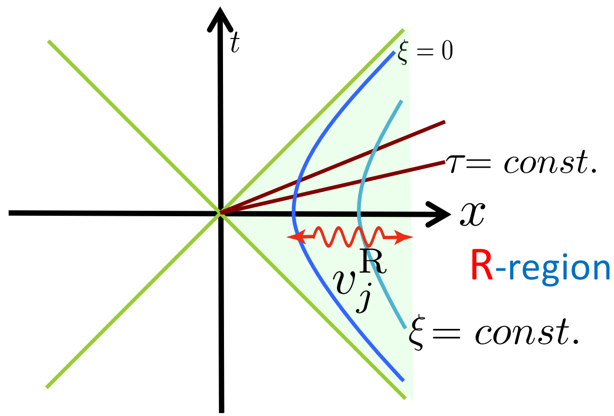
Today's talk is not about the quantum radiation, but about the structure of the vacuum state

2. Quantum field theory of scalar field in Right Rindler space

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right) = \int d\tau L$$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi - m^2\phi = 0 \qquad \pi = \frac{\delta L}{\delta\dot{\phi}}$$

$$[\phi(\tau,\xi,\mathbf{x}_\perp),\pi(\tau,\xi',\mathbf{x}'_\perp)] = i\delta(\xi-\xi')\delta(\mathbf{x}_\perp-\mathbf{x}'_\perp)$$



$$\hat{\phi}(x) = \int_0^\infty d\omega\int d^2k_\perp\left(\hat{a}^I_{p,\mathrm{k}_\perp}v^R_{p,\mathbf{k}_\perp}(x^R) + \mathrm{h.c.}\right)$$

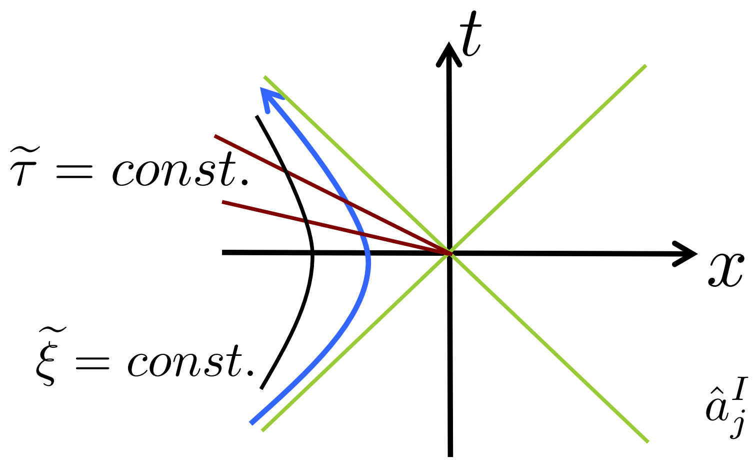
$$v_j^R(x) = \sqrt{\frac{\sinh\pi\omega}{4\pi^4}}e^{-i\omega\tau}K_{i\omega}(\kappa e^\xi)e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp} \qquad \omega>0$$

$$\kappa = \sqrt{k_\perp^2+m^2}$$

Right Rindler vacuum state : in accelerated frame: $\hat{a}^I_j|0,\mathrm{R}\rangle = 0$ for any $j=(\omega,\mathbf{k}_\perp)$

excited particle state : $|n_j\,\mathrm{R}\rangle = \frac{1}{\sqrt{n_j!}}(\hat{a}^{I\dagger}_j)^{n_j}|0,\mathrm{R}\rangle$

Left Rindler coordinate :



$$ds^2 = dt^2 - dx^2 - d\mathbf{x}_\perp^2$$

$$t = \frac{e^{a\widetilde{\xi}}}{a} \sinh a\widetilde{\tau}$$

$$x = -\frac{e^{a\widetilde{\xi}}}{a} \cosh a\widetilde{\tau}$$

$$ds^2 = e^{2a\widetilde{\xi}}(d\widetilde{\tau}^2 - d\widetilde{\xi}^2) - d\mathbf{x}_\perp^2$$

$$\hat{a}_j^I |0, \mathrm{II}\rangle = 0$$

Quantized scalar field

$$\hat{\phi}(x) = \int_0^\infty d\omega \int d^2k_\perp \left(\hat{a}^{II}_{\omega,\mathbf{k}_\perp} v^L_{\omega,\mathbf{k}_\perp}(x_L) + \mathrm{h.c.}\right)$$

Left Rindler mode

$$v^L_\omega(x_L) = \sqrt{\frac{\sinh \pi\omega/a}{4\pi^4 a}} e^{-i\omega\widetilde{\tau}} K_{i\omega/a}\left(\frac{\kappa e^{a\widetilde{\xi}}}{a}\right) e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp},$$

$$\omega \geq 0$$

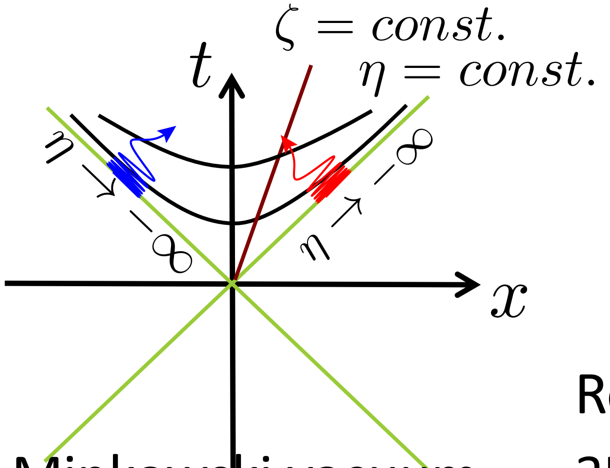
Left Rindler vacuum state :

$$\hat{a}_j^{II} |0, \mathrm{L}\rangle = 0$$

Excited particle state

$$|n_j, \mathrm{L}\rangle = \frac{1}{\sqrt{n_j!}} (\hat{a}_j^{II\dagger})^{n_j} |0, \mathrm{L}\rangle$$

Degenerate Kasner space: the Future-region



$$ds^2 = dt^2 - dx^2 - d\mathbf{x}_\perp^2$$

$$t = \frac{e^{a\zeta}}{a}\cosh a\eta \quad x = \frac{e^{a\zeta}}{a}\sinh a\eta$$

$$ds^2 = e^{2a\eta}(d\eta^2 - d\zeta^2) - d\mathbf{x}_\perp^2$$

Related to the Rindler space by the analytic continuation of coordinate variables

$$\mathrm{F} \longleftrightarrow \mathrm{R} \quad \zeta = \tau + \frac{\pi}{2a}i, \qquad \eta = \xi - \frac{\pi}{2a}i$$

Description of the Minkowski vacuum

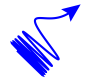
$$\hat{\phi}(x) = \int_{-\infty}^{\infty} d\omega \int d^2k_\perp \left(\hat{a}_{\omega,\mathbf{k}_\perp} v^F_{\omega,\mathbf{k}_\perp}(x_F) + \mathrm{h.c.}\right)$$

$$v^F_{\omega,\mathbf{k}_\perp}(x_F) = -\frac{i}{4\pi\sqrt{a\sinh(\pi a|\omega|)}} J_{-i|\omega|/a}(k_\perp e^{a\eta}/a)e^{-i\omega\zeta}e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}$$ Kasner mode

$$\propto e^{-i|\omega|\eta - i\omega\zeta} \quad (\eta \to -\infty) \begin{cases} \omega > 0 \\ \omega < 0 \end{cases}$$

Asymptotic form near the horizon

$\omega > 0$  Left moving wave mode

$\omega < 0$  right moving wave mode

Kasner vacuum $\hat{a}_{\omega,\mathbf{k}_\perp}|0\rangle_K = 0$

Excited state $|n_j\rangle_K = \frac{1}{\sqrt{n_j!}}(\hat{a}_j{}^\dagger)^n_j|0\rangle_K$

We can choose the different mode function :

$$u^F_{\omega,\mathbf{k}_\perp}(x_F) = -\frac{i}{4\pi\sqrt{2a}} e^{\frac{\pi|\omega|}{2a}} H^{(2)}_{i|\omega|/a}(k_\perp e^{a\eta}/a) e^{-i\omega\zeta} e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}$$

$$\phi(x) = \int_{-\infty}^{\infty} d\omega \int d^2k_\perp \left(\hat{b}_{\omega,\mathbf{k}_\perp} u^F_{\omega,\mathbf{k}_\perp}(x_F) + \text{h.c.}\right)$$

Mode function for the Minkowsiki vacuum

$\hat{b}_{\omega,\mathbf{k}_\perp}|0\rangle_M = 0$ for any mode

$$J_{-i|p|}(y) = \frac{1}{2}\left(e^{\pi|p|} H^{(2)}_{i|p|}(y) + \overline{H^{(2)}_{i|p|}(y)}\right)$$

$$v^F_{\omega,\mathbf{k}_\perp}(x_F) = \frac{1}{\sqrt{1-e^{-2\pi|\omega|}}}\left(u^F_{\omega,\mathbf{k}_\perp}(x_F) - e^{-\pi|\omega|}\overline{u^F_{-\omega,-\mathbf{k}_\perp}(x_F)}\right)$$

$$\hat{b}_{\omega,\mathbf{k}_\perp} = \frac{1}{\sqrt{1-e^{-2\pi|\omega|}}}\left(\hat{a}_{\omega,\mathbf{k}_\perp} - e^{-\pi|\omega|}\hat{a}^\dagger_{-\omega,-\mathbf{k}_\perp}\right)$$

$$|0\rangle_M = \mathcal{N}\exp\left[\int_0^\infty d\omega \int d\mathbf{k}_\perp\, e^{-\pi\omega} \hat{a}^\dagger_{\omega,\mathbf{k}_\perp} \hat{a}^\dagger_{-\omega,-\mathbf{k}_\perp}\right]|0\rangle_{\mathrm{K}}$$

$$= \mathcal{N} \prod_{\omega>0,\mathbf{k}_\perp} \sum_{n_j} e^{-\pi\omega n_j} |\omega, \mathbf{k}_\perp, n_j\rangle_K \otimes |-\omega, -\mathbf{k}_\perp, n_j\rangle_K$$

Minkowski vacuum state is expressed as a two mode entangled state of left moving wave particle state and the right-moving wave particle state.

Analytic continuation of mode between the F-region and the R-region

Simply analytic continuation of the Right Rindler mode into the F-region gives

Minkowsiki vacuum (positive frequency) mode

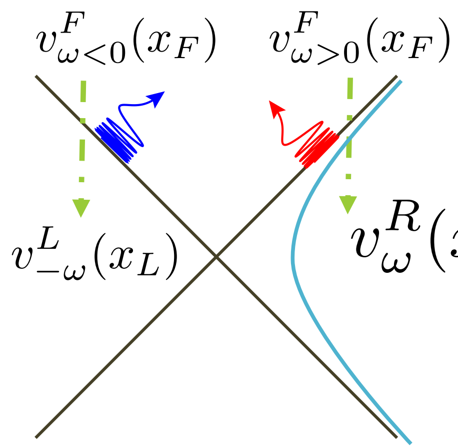
$$u_\omega^F(x_F) = e^{-i\omega\zeta} H^{(2)}_{i|\omega|}(\kappa e^{\eta})$$

$$\tau = \zeta - \frac{\pi}{2} i$$

$$\xi = \eta + \frac{\pi}{2} i$$

Kasner mode

$$v_\omega^F(x_F) = e^{-i\omega\zeta} J_{-i|\omega|}(\kappa e^{\eta})$$



$$v_\omega^R(x_R) = e^{-i\omega\tau} K_{i\omega}(\kappa e^{\xi})$$

For proper analytic continuation of mode function between the F-region and the Rindler region, we express the Kasner mode function in terms of the positive frequency of the Minkowski mode

$$v_\omega^F(x_F) = \frac{1}{\sqrt{1-e^{-2\pi|\omega|}}}\left(u_\omega^F(x_F) - e^{-\pi|\omega|}\overline{u^F_{-\omega}(x_F)}\right)$$

Analytic continuation is not trivial, it should be performed carefully taking the convergence of the Minkowski mode function in a consistent way.

For the Minkowski mode function in the global coordinate, the prescription for the convergence is different for the positive frequency mode and the negative frequency mode.

$$e^{-ik_0 t} = e^{-ik_0(t-i\varepsilon)} \quad \text{for the convergence at } k_0 \longrightarrow \infty$$

$$e^{ik_0 t} = e^{ik_0(t+i\varepsilon)}$$

Continuation for the positive frequency mode is different from that for the negative frequency mode.

For the consistent prescription, express the negative frequency mode of the Minkowsiki vacuum as the complex conjugate of the positive frequency mode, then the continuation can be performed as a consistent procedure.

$$e^{ik_0 t} = \overline{e^{-ik_0 t}} = \overline{e^{-ik_0(t-i\varepsilon)}}$$

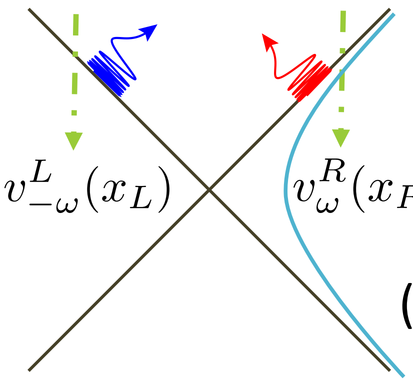
$$v_\omega^F(x_F) = \frac{1}{\sqrt{1-e^{-2\pi|\omega|}}}\left(u_\omega^F(x_F) - e^{-\pi|\omega|}\overline{u_{-\omega}^F(x_F)}\right)$$

Continuation of the mode functions from the F-region to the R-region and the L-region

Right moving wave Kanser mode $v^F_{\omega<0}(x_F)$

$$\mathrm{F} \longleftrightarrow \mathrm{L} \quad \zeta = -\tilde{\tau} - \frac{\pi}{2}i, \quad \eta = \tilde{\xi} - \frac{\pi}{2}i$$

Left Rindler mode $v^L_{-\omega}(x_L)$



$v^F_{\omega>0}(x_F)$ Left moving wave Kasner mode

$$\mathrm{F} \longleftrightarrow \mathrm{R} \quad \zeta = \tau + \frac{\pi}{2}i, \quad \eta = \xi - \frac{\pi}{2}i$$

$v^R_{\omega}(x_R)$ Right Rindler mode

(Normalization factor is different slightly)

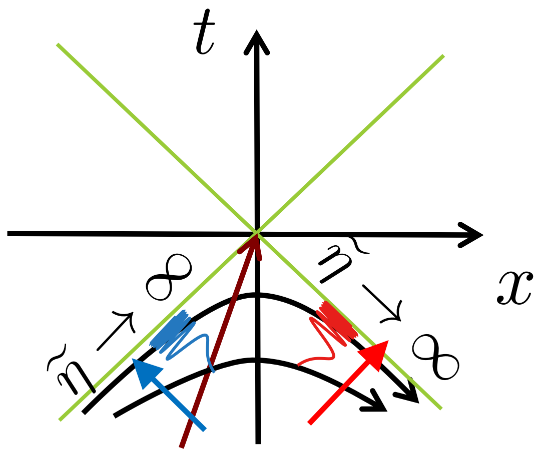
$$|0\rangle_{\mathrm{M}} = \mathcal{N} \prod_{\omega>0,\mathbf{k}_\perp} \sum_{n_j} e^{-\pi\omega n_j} |\omega, \mathbf{k}_\perp, n_j\rangle_K \otimes |-\omega, -\mathbf{k}_\perp, n_j\rangle_K$$

$$|0\rangle_{\mathrm{M}} = \mathcal{N} \prod_{\omega>0,\mathbf{k}_\perp} \sum_{n_j} e^{-\pi\omega n_j} |\omega, \mathbf{k}_\perp, n_j\rangle_R \otimes |\omega, \mathbf{k}_\perp, n_j\rangle_L$$

(Unruh & Wald 1984)

Minkowski vacuum state is expressed as an entangled state between the rigth-Rindler particle states of the left-Rindler particle state.

Past-region (degenerate shrinking Kasner coordinate)



$$ds^2 = dt^2 - dx^2 - d\mathbf{x}_\perp^2$$

$$t = -\frac{e^{-a\tilde{\eta}}}{a}\cosh a\tilde{\zeta}$$

$$x = \frac{e^{-a\tilde{\eta}}}{a}\sinh a\tilde{\zeta}$$

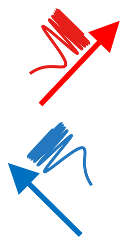
$$ds^2 = e^{-2a\tilde{\eta}}(d\tilde{\eta}^2 - d\tilde{\zeta}^2) - d\mathbf{x}_\perp^2$$

$$\mathrm{P} \longleftrightarrow \mathrm{R} \quad \tilde{\eta} = -\xi - \frac{\pi}{2a}i, \qquad \tilde{\zeta} = -\tau - \frac{\pi}{2a}i$$

$$\hat{\phi}(x) = \int_{-\infty}^{\infty} d\omega \int d^2k_\perp \left(\hat{a}^P_{\omega,\mathbf{k}_\perp} v^P_{\omega,\mathbf{k}_\perp}(x_P) + \mathrm{h.c.}\right)$$

$$v^P_{\omega,\mathbf{k}_\perp}(x_P) = \frac{i}{4\pi\sqrt{a\sinh(\pi a|\omega|)}} J_{i|\omega|/a}(k_\perp e^{-a\tilde{\eta}}/a) e^{i\omega\tilde{\zeta}} e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}$$

$$\propto e^{-i|\omega|\tilde{\eta}+i\omega\zeta} \quad (\tilde{\eta} \to +\infty) \begin{cases} \omega > 0 \\ \omega < 0 \end{cases}$$

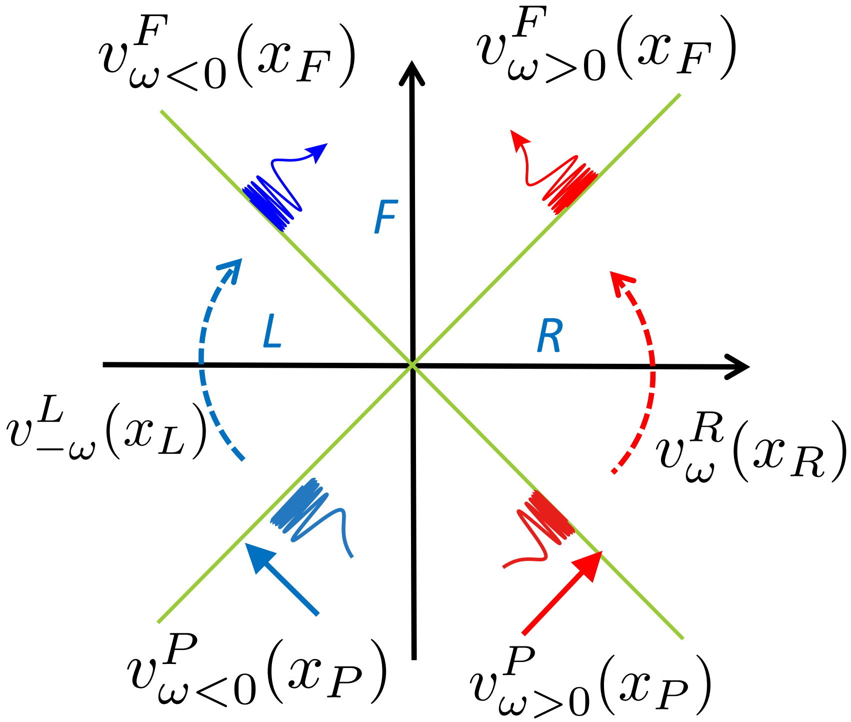


$\omega > 0$: right moving wave mode

$\omega < 0$: left moving wave mode

vacuum $\quad \hat{a}^P_{\omega,\mathbf{k}_\perp}|0\rangle_K = 0 \qquad$ Excited state $\quad |n_j\rangle_K = \frac{1}{\sqrt{n_j!}}(\hat{a}_j{}^{P\dagger})^n_j|0\rangle_K$

Description of the Minkowski vacuum state with entanglement



$$v^{\mathrm{I}}_\omega(x) = \begin{cases} v^F_{\omega>0}(x_F) & F \\ v^R_{\omega>0}(x_R) & R \\ 0 & L \\ v^P_{\omega>0}(x_P) & P \end{cases}$$

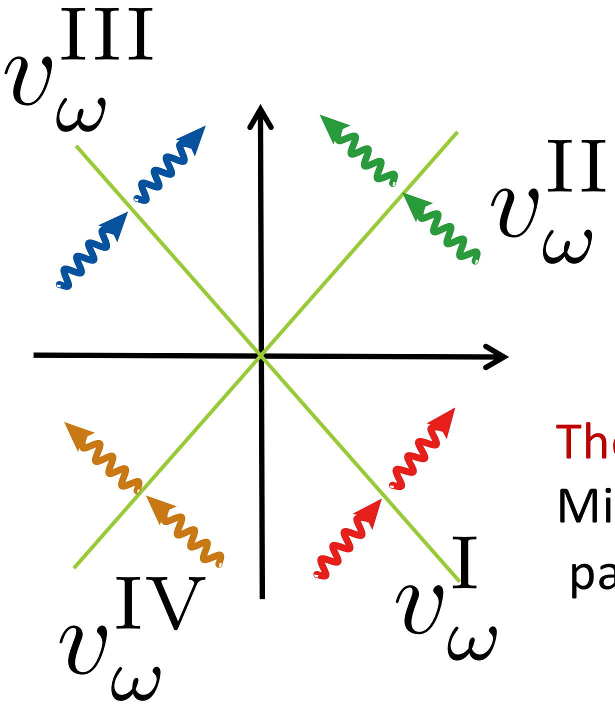
$$v^{\mathrm{II}}_\omega(x) = \begin{cases} v^F_{\omega<0}(x_F) & F \\ 0 & R \\ v^L_{-\omega}(x_L) & L \\ v^P_{\omega<0}(x_P) & P \end{cases}$$

$$|0, M\rangle = \prod_j \left[N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j \omega/a} |n_j, \mathrm{I}\rangle \otimes |n_j, \mathrm{II}\rangle \right]$$

$$\hat{\phi}(x) = \sum_j \left(\hat{a}^{\mathrm{I}}_j v^{\mathrm{I}}_j(x) + \hat{a}^{\mathrm{II}}_j v^{\mathrm{II}}_j(x) + \mathrm{h.c.} \right),$$

The Minkowski vacuum state is described in a unified way including the F-region and the P-region. (4-dimensional case with $\mathbf{k}_\perp \neq 0$)

The structure of the 2-dimensional massless field ($\mathbf{k}_\perp = 0$) is different from that of the 4-dimensional field with $\mathbf{k}_\perp \neq 0$



$$|0,\mathrm{M}\rangle = \prod_{\omega}\Big[N_\omega \sum_{n_\omega=0}^{\infty} e^{-\pi n_\omega \omega/a}|n_\omega,\mathrm{I}\rangle \otimes |n_\omega,\mathrm{III}\rangle\Big]$$
$$\otimes \prod_{\omega'}\Big[N_{\omega'} \sum_{n'_\omega=0}^{\infty} e^{-\pi n_{\omega'} \omega'/a}|n_{\omega'},\mathrm{II}\rangle \otimes |n_{\omega'},\mathrm{IV}\rangle\Big]$$

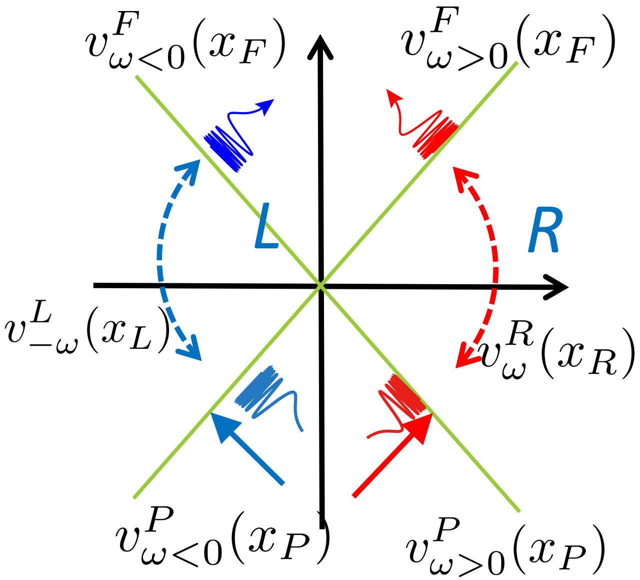
The modes with different direction of momentum are decoupled.

Minkowski vacuum is expressed as an entangled state of the pairs of the excited states of the modes, I and III, and II and IV.

To find the Minkowski vacuum state with states in the two Rindler coordinates

① $u^F_\omega(x_F)$ Positive frequency mode function of the Minkowski vacuum in the F-region

② $v^F_{\omega>0}(x_F)$ $v^F_{\omega<0}(x_F)$ left moving wave and right moving wave near the horizon



③ Bogoliubov transformation and description of Minkowsiki vacuum

$$v^F_\omega(x_F) = \frac{1}{\sqrt{1-e^{-2\pi|\omega|}}}\left(u^F_\omega(x_F) - e^{-\pi|\omega|}\overline{u^F_{-\omega}(x_F)}\right)$$

$$|0\rangle_{\rm M} = \mathcal{N}\prod_{\omega>0}\sum_{n_j} e^{-\pi\omega n_j}|\omega, n_j\rangle_v \otimes |-\omega, n_j\rangle_v$$

④ Continuation of the mode functions in F-region to the right, left Rindler region

$$|0\rangle_{\rm M} = \mathcal{N}\prod_{\omega>0}\sum_{n_j} e^{-\pi\omega n_j}|\omega, n_j\rangle_R \otimes |\omega, n_j\rangle_L$$

1. Introduction

2. Minkowski space

3. De Sitter space

Higuchi and Yamamoto (2018)

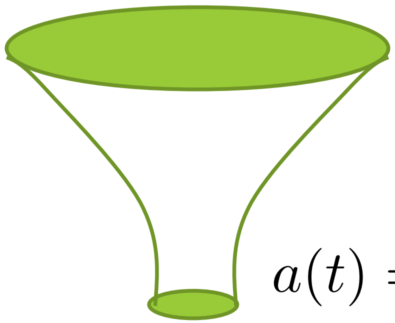
cf. Tanaka and Sasaki (1994)

4. Spinor field

5. Conclusions

3. De Sitter spacetime

maximally symmetric spacetime, various coordinates to cover



Flat chart

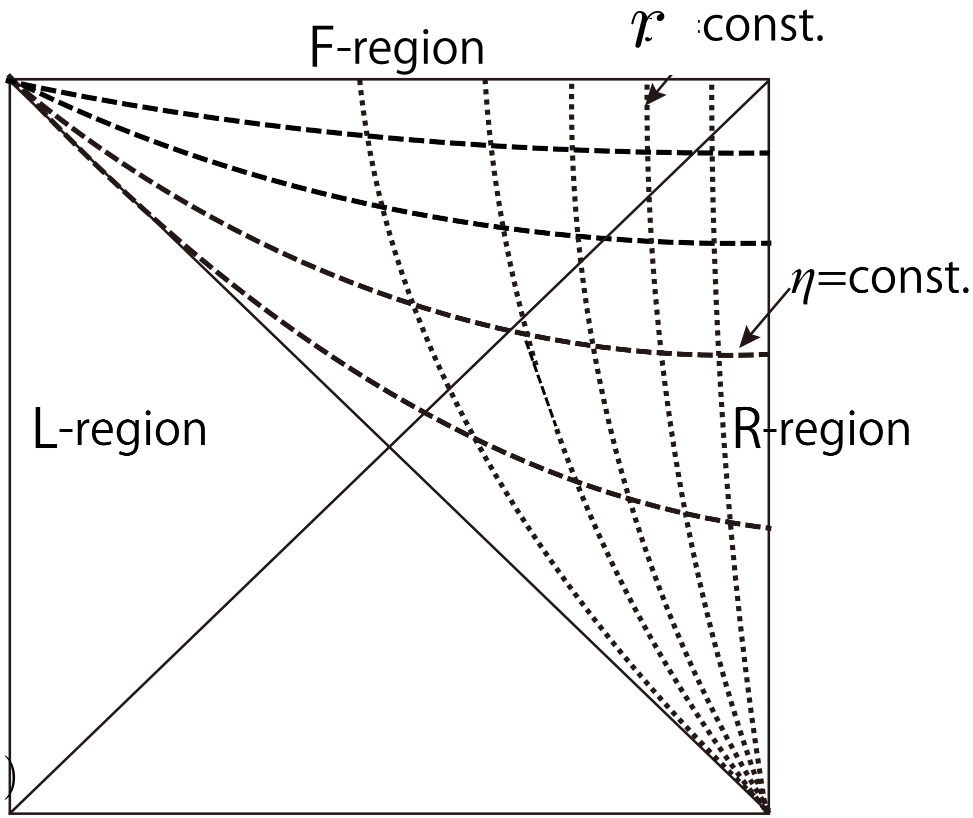
$$a(t) = e^{Ht} = -\frac{1}{H\eta}$$

$$ds^2 = \left(\frac{-1}{H\eta}\right)\left(-d\eta^2 + dr^2 + r^2 d\Omega_{(2)}^2\right)$$

Positive frequency mode for the Bunch-Davies vacuum for a scalar field

$$\varphi_{k\ell}(r,\eta) = \frac{e^{-\frac{i\pi}{2}\left(\ell+\frac{1}{2}\right)}}{\sqrt{2k}}(-k\eta)^{\frac{3}{2}} e^{\frac{i\nu\pi}{2}} H_\nu^{(1)}(-k\eta) j_\ell(kr) Y_{\ell m}(\Omega_{(2)})$$

$$\nu = \sqrt{\frac{9}{4} - m^2}$$

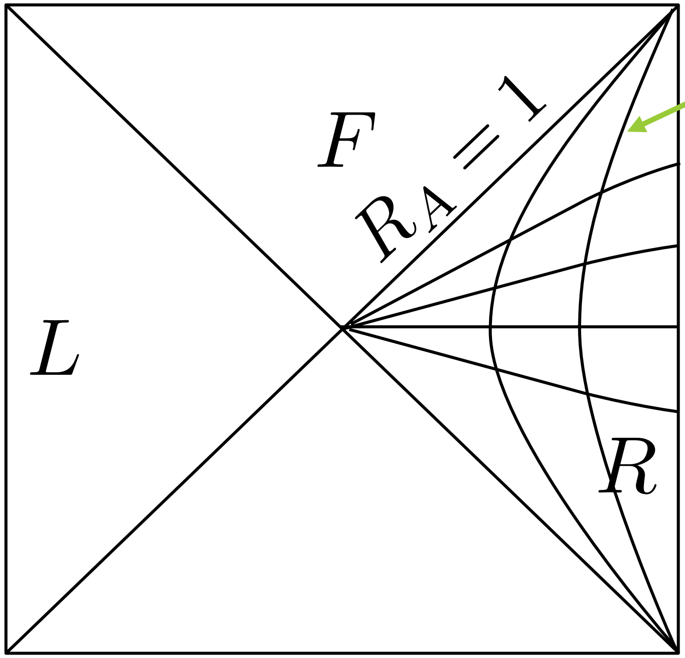


Conformal diagram of de Sitter space

de Sitter spacetime with a static chart

The line element of the static chart in de Sitter spacetime is

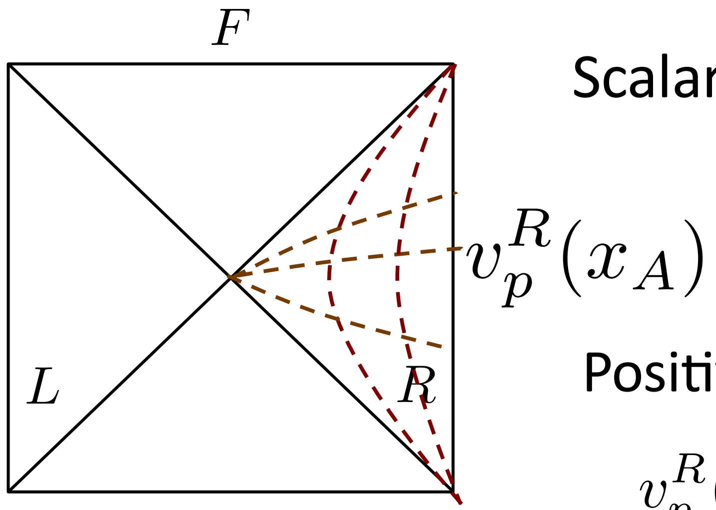
$$ds^2 = -(1 - R_A^2)dT_A^2 + \frac{dR_A^2}{1 - R_A^2} + R_A^2 d\Omega^2 \qquad H = 1 \text{ unit}$$



$R_\mathrm{A} = \text{const.} \qquad 0 \le R_A \le 1$

$T_A = \text{const.}$ This static chart ⇆ R-region .

A particle on a trajectory of constant R_A is in a uniformly accelerated motion,
on which a detector feels thermal excitations.
We considered a problem whether a uniform accelerating detector produce quantum radiation or not.



Scalar field theory on the static chart of de Sitter space

Higuchi (1987)

Positive frequency mode function for a scalar field

$$v_p^R(x_A) = e^{-ipT_A}\mathcal{F}_{p\ell}(R_A)Y_{\ell m}(\Omega)$$

$$\mathcal{F}_{p\ell}(R_A) = \frac{\Gamma((\ell+ip+3/2+\nu)/2)\Gamma((\ell+ip+3/2-\nu)/2)}{(4\pi p)^{1/2}\Gamma(\ell+3/2)\Gamma(ip)}$$

$$\times(R_A)^{\ell}(1-R_A^2)^{ip/2}{}_2F_1\left(\frac{\ell+ip+3/2+\nu}{2},\frac{\ell+ip+3/2-\nu}{2},\ell+\frac{3}{2};(R_A)^2\right)$$

$$\nu=\sqrt{\frac{9}{4}-m^2}$$

$H = 1$ unit

Coordinate system in de Sitter space, Right, Left, Future and Past region

$$T_B = -T_L - \frac{\pi}{2} i$$

$$R_B = R_L$$

$$ds^2 = -\frac{dR_B^2}{R_B^2 - 1} + (R_B^2 - 1) dT_B^2 + R_B^2 d\Omega^2$$

$$T_A = T_B - \frac{\pi}{2} i$$

$$R_A = R_B$$

$R_B = \text{const.}$

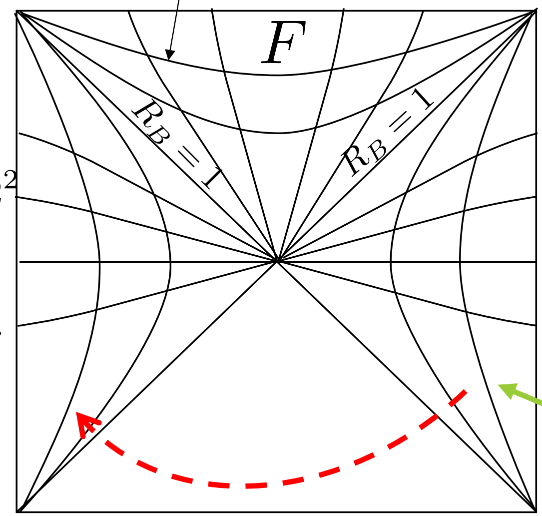
$R_B > 1$

Analytic continuation

$0 < R_A < 1$

$$ds^2 = -(1 - R_L^2) dT_L^2 + \frac{dR_L^2}{1 - R_L^2} + R_L^2 d\Omega^2$$

$$ds^2 = -(1 - R_A^2) dT_A^2 + \frac{dR_A^2}{1 - R_A^2} + R_A^2 d\Omega^2$$



L $\quad$ R

$T_L = \text{const.}$

$T_A = \text{const.}$

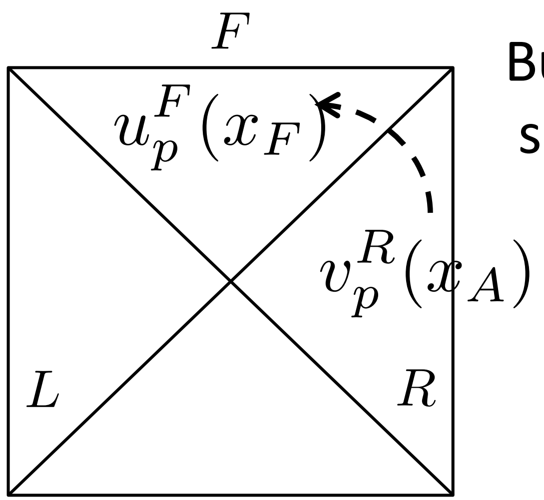
$R_A = \text{const.}$

$$T_A = -T_L - \pi i, R_A = R_L$$

Continuation from R-region to F-region is performed by continuation R_A ($0<R_A<1$) to R_B ($R_B>1$)

In the L-region we introduce the similar coordinate(T_L, R_L)

Cauchy surface with two static charts



Bunch-Davis vacuum positive frequency mode in the F-region:
simple analytic continuation of the R-region to F-region

$$v_p^R(x_A) = e^{-ipT_A}\mathcal{F}_{p\ell}(R_A) \qquad T_A = T_B - \frac{\pi}{2}i$$

$$R_A = R_B$$

$$u_p^F(x_F) = \underbrace{U_{p\ell}(R_B)e^{-ipT_B}}$$

(without constant factor)

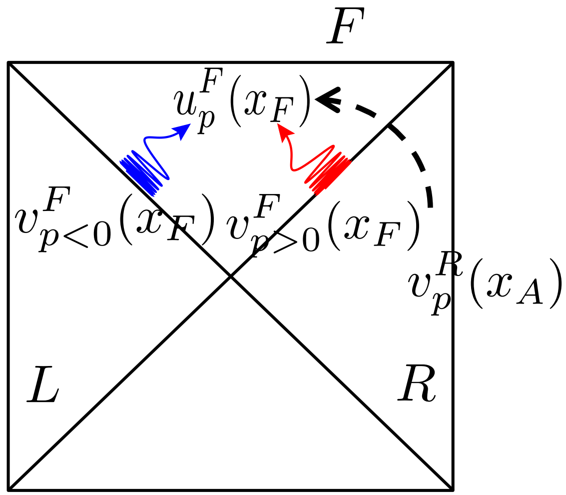
$$U_{|p|\ell}(R_B) = R_B^\ell (R_B^2-1)^{\frac{i|p|}{2}}\,{}_2F_1\left(\frac{\frac{3}{2}+\ell+i|p|+\nu}{2}, \frac{\frac{3}{2}+\ell+i|p|-\nu}{2}, \ell+\frac{3}{2}; R_B^2\right)$$

We can prove that this is the positive frequency mode for the Bunch-Davies vacuum

$$\frac{1}{\sqrt{2\pi}}\int_0^\infty dk\, k^{-ip-\frac{1}{2}}\varphi_{k\ell}(\eta, r) = 2^{-ip}e^{i\delta_p}e^{-\pi|p|/2}N_{p\ell}\overbrace{U_{|p|\ell}(R_B)e^{-ipT_B}}$$

$$\int_0^\infty dz z^\lambda H_\nu^{(1)}(az)J_\mu(bz) = a^{-\lambda-1}e^{(\lambda-\nu+\mu)i\pi/2}\frac{2^\lambda (b/a)^\mu}{\pi\Gamma(\mu+1)}\Gamma\left(\frac{\lambda+\nu+\mu+1}{2}\right)\Gamma\left(\frac{\lambda-\nu+\mu+1}{2}\right)$$

$${}_2F_1\left(\frac{\lambda+\nu+\mu+1}{2}, \frac{\lambda-\nu+\mu+1}{2}, \mu+1; (b/a)^2\right)$$



left moving wave mode and right moving wave mode

$$
{}_2F_1\left(a,b,c;z\right)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}{}_2F_1\left(a,b,a+b-c+1;1-z\right)
$$

$$
+(1-z)^{c-a-b}\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}{}_2F_1\left(c-a,c-b,c-a-b+1;1-z\right)
$$

$$
V_{|p|\ell}(R_B)=\frac{\Gamma\left(\frac{\frac{3}{2}+\ell+i|p|+\nu}{2}\right)\Gamma\left(\frac{\frac{3}{2}+\ell+i|p|-\nu}{2}\right)}{2\sinh(\pi|p|)\Gamma(\ell+\frac{3}{2})\Gamma(i|p|)}\left[U_{|p|\ell}(R_B)-e^{-\pi|p|}\overline{U_{|p|\ell}(R_B)}\right]
$$

$$
V_{|p|\ell}(R_B)=R_B^{\ell}(R_B^2-1)^{-\frac{i|p|}{2}}{}_2F_1\left(\frac{\frac{3}{2}+\ell-i|p|+\nu}{2},\frac{\frac{3}{2}+\ell-i|p|-\nu}{2},1-i|p|;1-R_B^2\right)
$$

Near the cosmological horizon

$$
R_B\to 1 \qquad V_{p\ell}(R_B)\cong 2^{-i|p|}e^{i|p|R_{B*}}
$$

$$
v^F_{p>0}(x_F)\sim V_{p(>0)\ell}e^{-ipT_B}\propto e^{ip(R_{B*}-T_B)}
$$

Left moving wave mode

$$
v^F_{p<0}(x_F)\sim V_{p(<0)\ell}e^{-ipT_B}\propto e^{-ip(R_{B*}+T_B)}
$$

Right moving wave mode

Bogoliubov transformation in the F-region

$$V_{|p|\ell}(R_B) = \frac{\Gamma\left(\frac{\frac{3}{2}+\ell+i|p|+\nu}{2}\right)\Gamma\left(\frac{\frac{3}{2}+\ell+i|p|-\nu}{2}\right)}{2\sinh(\pi|p|)\Gamma(\ell+\frac{3}{2})\Gamma(i|p|)}\left[U_{|p|\ell}(R_B) - e^{-\pi|p|}\overline{U_{|p|\ell}(R_B)}\right]$$

Right (left) moving wave mode function

Bunch-Davies vacuum mode function

$$\hat{\phi} = \int_{-\infty}^{\infty} dp \sum_{\ell m} \left[V_{|p|\ell}(R_B)e^{-ipT_B}Y_{\ell m}(\Omega)\hat{a}_{p\ell m} + \text{h.c.}\right]$$

$$\hat{a}_{p\ell m}|0\rangle_{\text{vF}} = 0 \qquad |p\ell m, n\rangle_{\text{vF}} = \frac{\hat{a}^{\dagger n}_{p\ell m}}{\sqrt{n!}}|0\rangle_{\text{vF}}$$

$$\hat{\phi} = \int_{-\infty}^{\infty} dp \sum_{\ell m} \left[U_{|p|\ell}(R_B)e^{-ipT_B}Y_{\ell m}(\Omega)\hat{b}_{p\ell m} + \text{h.c.}\right]$$

$\hat{b}_{p\ell m}|0\rangle_{\text{BD}} = 0$ for any $p\ell m$

Bunch-Davies vacuum is expressed as the entangled state in the F-region

$$|0\rangle_{\text{BD}} = \mathcal{N} \prod_{p>0,\ell,m} \sum_{n=0} e^{-\pi p n}|p\ell m, n\rangle_{\text{vF}} \otimes |-p\ell m, n\rangle_{\text{vF}}$$

Continuation from the F-region to R-static chart and L-static chart

$$V_{|p|\ell}(R_B) = \frac{\Gamma\left(\frac{\frac{3}{2}+\ell+i|p|+\nu}{2}\right)\Gamma\left(\frac{\frac{3}{2}+\ell+i|p|-\nu}{2}\right)}{2\sinh(\pi|p|)\Gamma(\ell+\frac{3}{2})\Gamma(i|p|)}\left[U_{|p|\ell}(R_B) - e^{-\pi|p|}\overline{U_{|p|\ell}(R_B)}\right]$$

Left moving wave mode

$$v^F_{p>0,\ell}(x_B) \longrightarrow \begin{cases} v^R_{p\ell}(x_A) & \text{(R-region)} \\ 0 & \text{(L-region)} \end{cases} \qquad T_A = T_B - \frac{\pi}{2}i, \quad R_A = R_B$$

Right moving mode

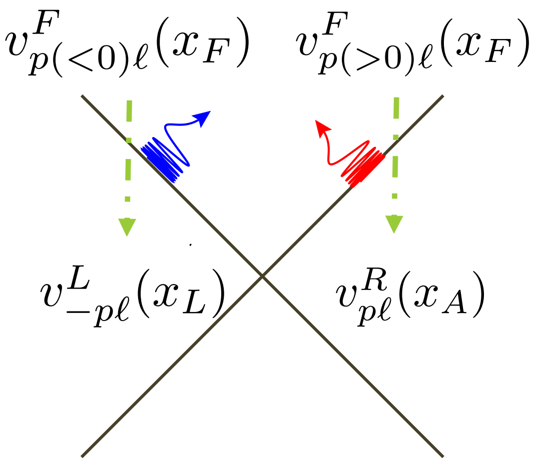
$$v^F_{p<0,\ell}(x_B) \longrightarrow \begin{cases} 0 & \text{(R-region)} \\ v^L_{-p\ell}(x_L) & \text{(L-region)} \end{cases} \qquad T_B = -T_L - \frac{\pi}{2}i, \quad R_B = R_L$$

Bunch-Davies vacuum is expressed as the two mode entangled state of the Right static chart and in the Left static chart

$$|0\rangle_{\rm BD} = \mathcal{N} \prod_{p>0,\ell,m} \sum_{n=0} e^{-\pi p n} |p\ell m, n\rangle_{\rm R} \otimes |p\ell m, n\rangle_{\rm L}$$

Description of the de Sitter vacuum(Bunch-Davies vacuum) state with the two static charts (Same procedure as that of the Rindler space and the Mikowski space)

① $u^F_{p\ell}(x_F)$ Positive frequency mode function of the Bunch-Davies vacuum in F-region



② left moving wave and right moving wave near the horizon

$v^F_{p(>0)\ell}(x_F)$ $\quad$ $v^F_{p(<0)\ell}(x_F)$

③ Bogoliubov transformation and description of BD vacuum state

$$v^F_{p\ell}(x_F) = \frac{1}{\sqrt{1-e^{-2\pi|p|}}}\left(u^F_{p\ell}(x_F) - e^{-\pi|p|}\overline{u^F_{-p\ell}(x_F)}\right)$$

$$|0\rangle_{\rm BD} = \mathcal{N}\prod_{p>0,\ell,m}\sum_{n=0} e^{-\pi p n}|p\ell m, n\rangle_{\rm vF}\otimes|-p\ell m, n\rangle_{\rm vF}$$

④ Continuation of the mode functions in F-region to the right, left static charts

$$|0\rangle_{\rm BD} = \mathcal{N}\prod_{p>0,\ell,m}\sum_{n=0} e^{-\pi p n}|p\ell m, n\rangle_{\rm R}\otimes|p\ell m, n\rangle_{\rm L}$$

Some implication

$$|0\rangle_{\mathrm{BD}} = \mathcal{N} \prod_{p>0,\ell,m} \sum_{n=0} e^{-\pi p n} |p\ell m, n\rangle_{\mathrm{R}} \otimes |p\ell m, n\rangle_{\mathrm{L}}$$

Tanaka, Sasaki (94)

Two mode entangled state

n_j *th* excited state in the right and left static de Sitter charts

Entanglement entropy for a pair of entangled two modes

$$s_{p\ell m} = -Tr_L[\hat{\rho}_{p\ell m} \log \hat{\rho}_{p\ell m}] = -log(1 - e^{-2\pi p}) - \frac{e^{-2\pi p}}{1 - e^{-2\pi p}} \log e^{-2\pi p}$$

This does not depend on the mass of the field.

α-vacuum state in de Sitter spacetime

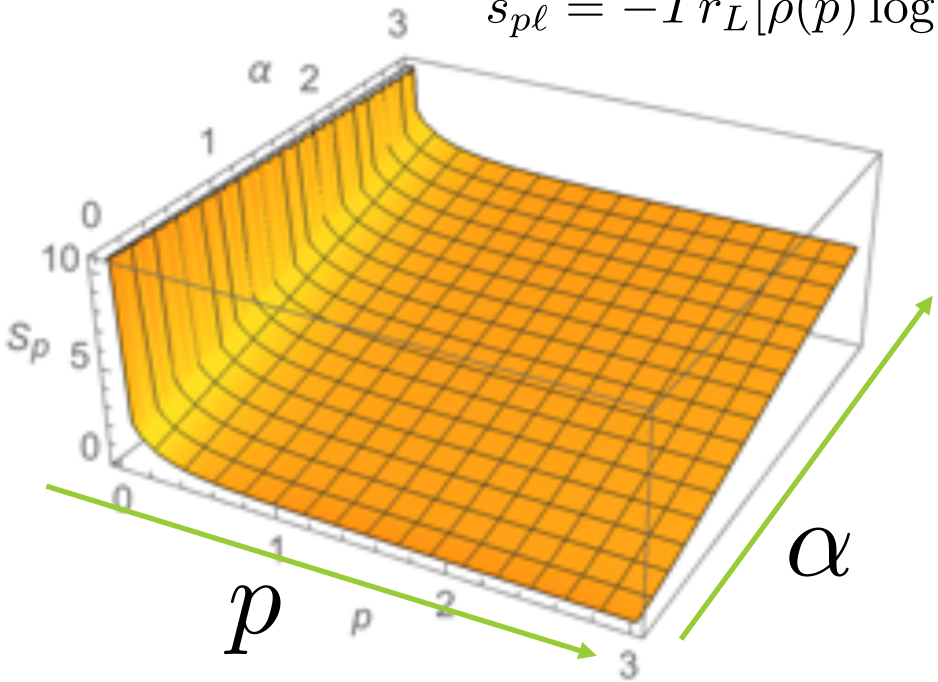
$$\mathcal{U}^F_{p\ell}(x_B) = \cosh\alpha\, u^F_{p\ell}(x_B) + e^{i\theta}\sinh\alpha\, \overline{u^F_{-p\ell}(x_B)}$$

$$|0\rangle_\alpha \propto \prod_{p>0,\ell,m} \sum_{n=0}^{\infty} [\gamma_p(\alpha,\theta)]^n |p\ell m, n\rangle_{\mathrm{vF}} \otimes |-p\ell m, n\rangle_{\mathrm{vF}}$$

Entanglement entropy for a pair of entangled two modes

$$s_{p\ell} = -Tr_L[\hat\rho(p)\log\hat\rho(p)] = -\log(1-|\gamma_p|^2) - \frac{|\gamma_p|^2}{1-|\gamma_p|^2}\log|\gamma_p|^2$$

$$\gamma_p(\alpha,\theta) = \frac{e^{-i\theta}\sinh\alpha + e^{-\pi p}\cosh\alpha}{\cosh\alpha + e^{-\pi p}e^{-i\theta}\sinh\alpha}$$



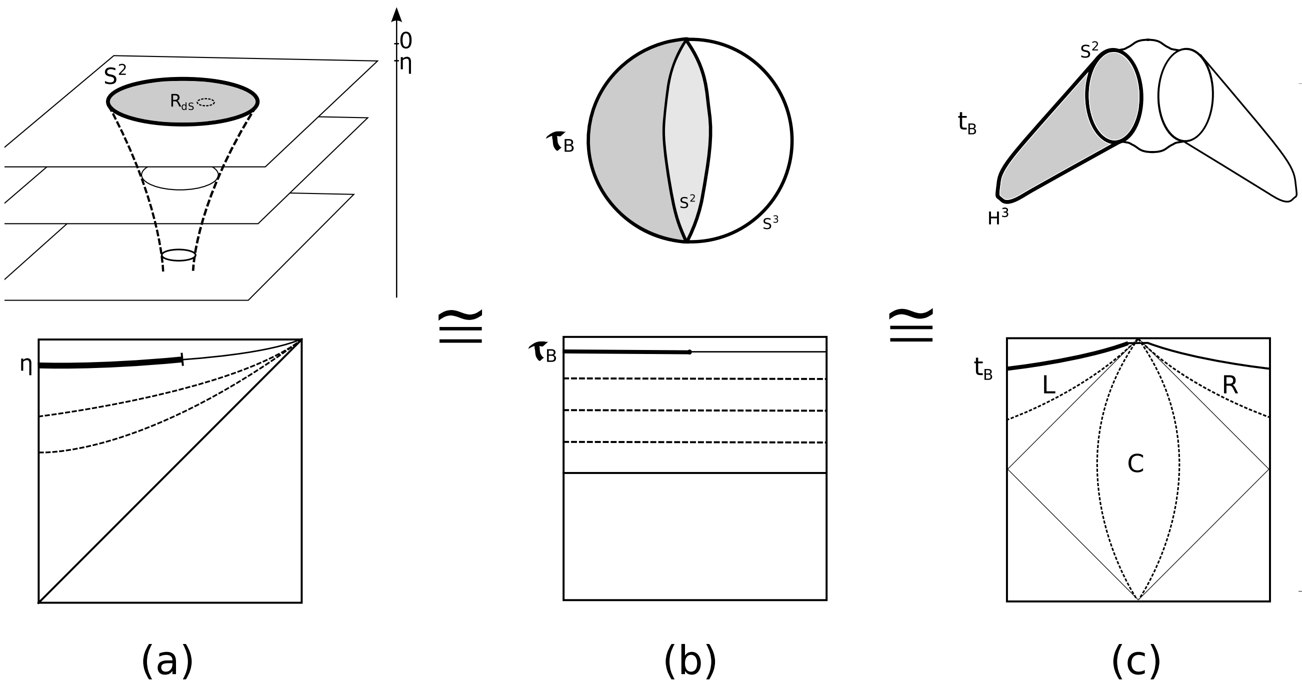
The results do not depend on ϑ unless ϑ takes values around $-\pi$.

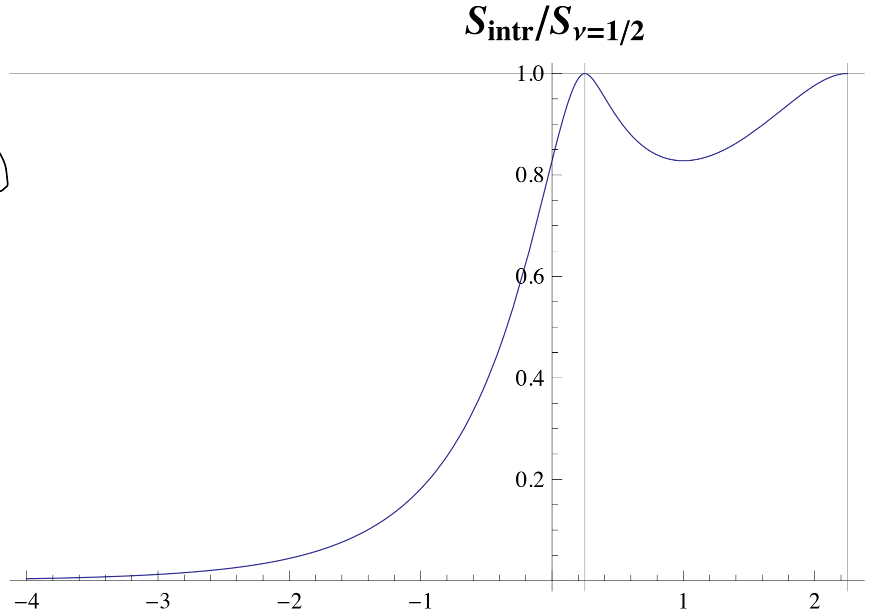
This does not depend on the mass of field.

Entanglement entropy $s(p,\nu) = -Tr_L[\hat{\rho}(p)\log\hat{\rho}(p)] = -\log(1-|\gamma_p|^2) - \frac{|\gamma_p|^2}{1-|\gamma_p|^2}\log|\gamma_p|^2$

Between two open chart

$$S(\nu) \propto \int_0^\infty dp p^2 s(p,\nu)$$





Maldacena, Pimentel (2013)
Kanno, Shock, Soda (2015)

The results depend on the mass of the field

4. Massive Dirac field in 2 dimensional Rindler space

With K. **Ueda**, (Higuchi, Nan)

$$[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m]\psi = 0 \qquad \Gamma_\mu = \frac{1}{4}\gamma_\nu\left(\frac{\partial\gamma^\nu}{\partial x^\mu} + \Gamma^\nu_{\lambda\mu}\gamma^\lambda\right)$$

Solution in Rindler space in two dimension (e.g., Cripspino et al. 2008)

$$\hat{\Psi}(\tau,\xi) = \sum_{\sigma=\pm}\int_0^\infty d\omega\left(\hat{c}_{\omega\sigma}\psi^{\mathrm{R}}_{\omega\sigma}(\tau,\xi,) + \hat{d}^\dagger_{\omega\sigma}\psi^{\mathrm{R}}_{-\omega-\sigma}(\tau,\xi)\right)$$

Two dimensional solution $\psi^{\mathrm{R}}_{\omega\sigma} \equiv f^{\mathrm{R}}_{\omega\sigma}(\xi)e^{-i\omega\tau}$

Up spin solution

$$f^{\mathrm{R}}_{\omega+} = \mathrm{A}^{\mathrm{R}}_{+}\begin{pmatrix} K_{i\omega/a+1/2}(\kappa\rho) + iK_{i\omega/a-1/2}(\kappa\rho) \\ 0 \\ -K_{i\omega/a+1/2}(\kappa\rho) + iK_{i\omega/a-1/2}(\kappa\rho) \\ 0 \end{pmatrix}$$

Down spin solution

$$f^{\mathrm{R}}_{\omega-} = \mathrm{A}^{\mathrm{R}}_{-}\begin{pmatrix} 0 \\ K_{i\omega/a+1/2}(\kappa\rho) + iK_{i\omega/a-1/2}(\kappa\rho) \\ 0 \\ K_{i\omega/a+1/2}(\kappa\rho) - iK_{i\omega/a-1/2}(\kappa\rho) \end{pmatrix}$$

$$\kappa = m \qquad \rho = \frac{1}{a}e^{a\xi}$$

7. Summary and conclusions

- ☑ We investigated the description of the Minkowski vacuum as an entangled state of the right-Rindler particle states and the left-Rindler states, the extension to the future (past) Kanser region.
- ☑ Bunch-Davies vacuum state and α-vacuum state in de Sitter space as entangled states of the right static chart and the left static chart, Similar structure to those in the case of Minkowski spacetime.
- ☑ Spinor field, electromagnetic field, tensor field
- ☑ Useful for computation of quantum radiation associated with Unruh effect
- ☑ Application to cosmology