Quantum non-linear evolution of inflationary gravitational waves

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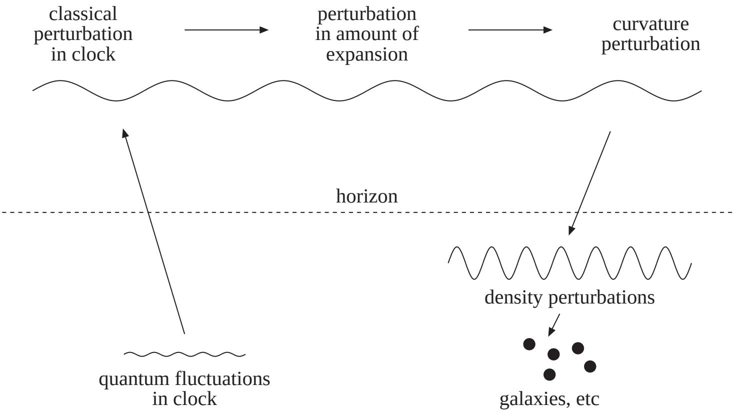
Outline

Generation and evolution of perturbations



Everything seems to be clearly understood

Quantum aspects of perturbations?

If the inflationary picture is the case...

- Quantum-to-classical transition?
- Quantum signature of perturbations?
- Effective theory description?

Important to test the inflationary paradigm

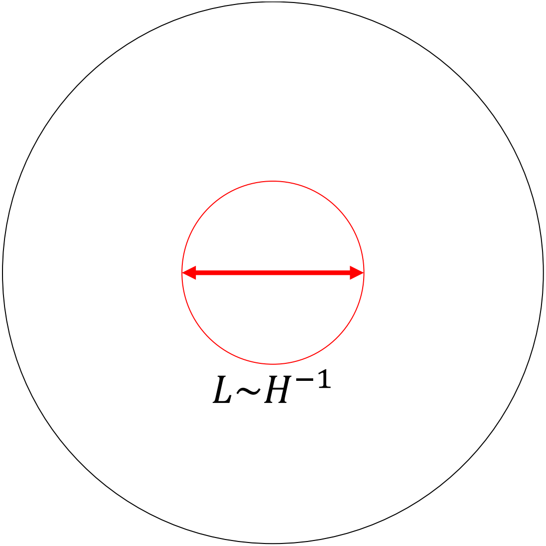
Why tensor perturbations?

- Persistent
- Well defined even during dS
- (For pure tensor) free from gauge

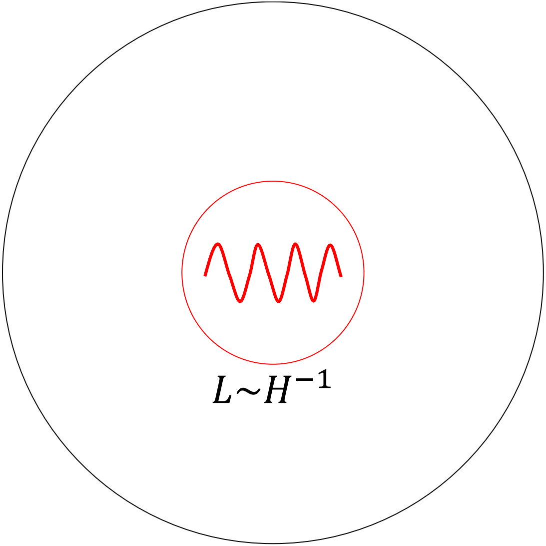
How pure tensor modes behave

1. on super-horizon scales,
2. keeping quantum nature?

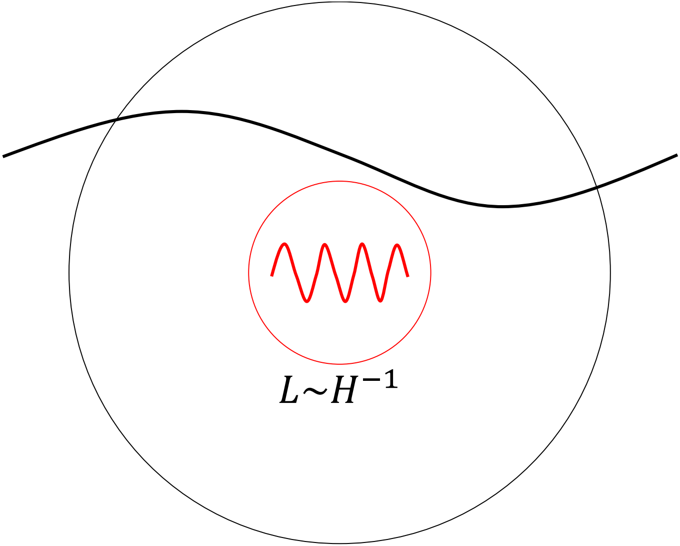
System and environment



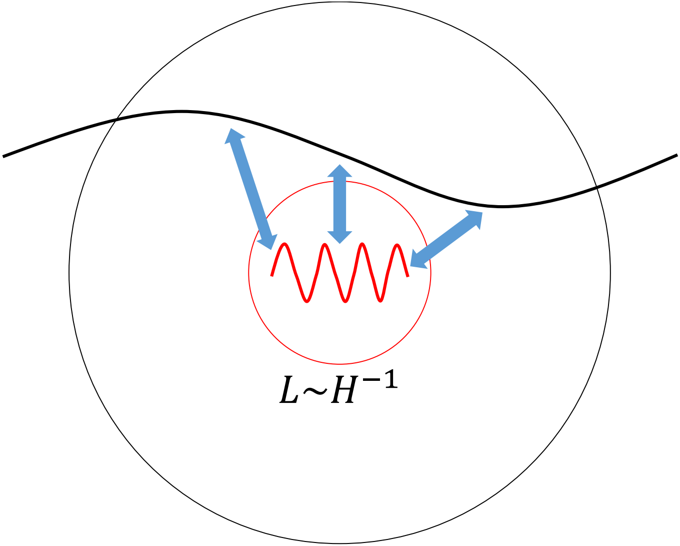
System and environment



System and environment



System and environment



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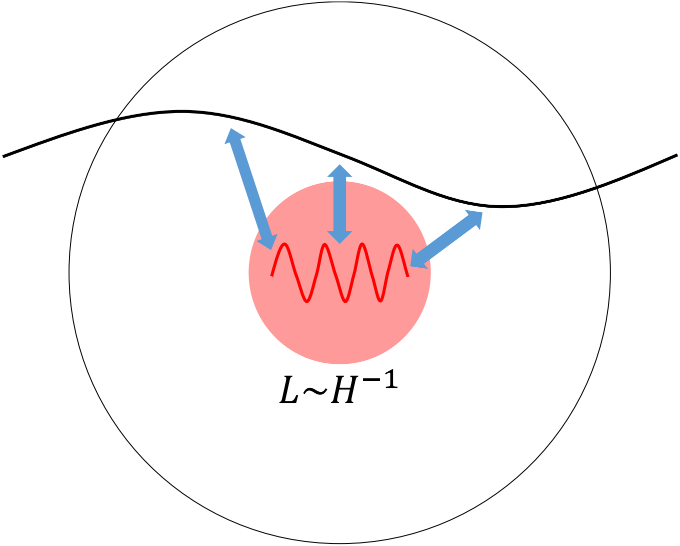

Lindblad equation

$$\frac{d\rho_{\rm red}}{d\tau} = -i[H, \rho_{\rm red}] - \frac{1}{2}\sum\left(L_\mu^\dagger L_\mu \rho_S + \rho_S L_\mu^\dagger L_\mu - 2L_\mu \rho_S L_\mu^\dagger\right)$$

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- Unitary evolution: von Neumann equation

Lindblad equation

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- Unitary evolution: von Neumann equation
- Non-unitary evolution: Lindblad operators
  1. Due to the interaction between system and environment

     $$L_\mu \sim \left\langle \mathcal{E}_f \middle| H_{\rm int} \middle| \mathcal{E}_i \right\rangle$$

  2. Exponential decay of (some components of) $\rho_{\rm red}$
  3. Effective theory description

Pure tensor interaction

$$S_3^{(t)} = \int d\tau d^3x a^2 m_{\rm Pl}^2 \left[-\frac{1}{2} h_{ij} h'_{jk} h'_{ki} - 2\mathcal{H} h_{ij} h_{jk} h'_{ki} + 2\left(1-\frac{\epsilon}{3}\right) h_{ij} h_{jk} h_{ki} + h_{ij}\left(\frac{1}{4} h_{kl,i} h_{kl,j} + \frac{1}{2} h_{ik,l} h_{jl,k} - \frac{3}{2} h_{ik,l} h_{jk,l} \right) \right]$$

1. Most terms are not slow-roll suppressed
2. Pol tensor products with different combinations of indices

Cubic interaction Hamiltonian

$$H_{\text{int},I}(\tau) = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}(2\pi)^3\delta^{(3)}(\mathbf{k}_{123})\sum_{\lambda_1,\lambda_2,\lambda_3}$$
$$\times\left\{h_0(\tau)a^{\lambda_1}_{\mathbf{k}_1}a^{\lambda_2}_{\mathbf{k}_2}a^{\lambda_3}_{\mathbf{k}_3}(\tau_0) + h_1(\tau)\left[a^{\lambda_1\dagger}_{-\mathbf{k}_1}a^{\lambda_2}_{\mathbf{k}_2}a^{\lambda_3}_{\mathbf{k}_3}(\tau_0) + 2\text{ perm}\right] + h.c.\right\}$$

- Coefficients at τ, operators at τ_0
- Sandwiched between $|0\rangle_0$, some operators directly work
- System-environment splitting

$$\int\frac{d^3k}{(2\pi)^3}a_{\mathbf{k}} = \underbrace{\int_{\mathbf{k}\in\mathbf{k}_{\mathscr{S}}}\frac{d^3k}{(2\pi)^3}a_{\mathbf{k}\in\mathbf{k}_{\mathscr{S}}}}_{\text{On } |0\rangle_{\mathscr{S}}\equiv|0\rangle_{k<aH \text{ at } \tau_0}} + \underbrace{\int_{\mathbf{k}\in\mathbf{k}_{\mathscr{E}}}\frac{d^3k}{(2\pi)^3}a_{\mathbf{k}\in\mathbf{k}_{\mathscr{E}}}}_{\text{On } |0\rangle_{\mathscr{E}}\equiv|0\rangle_{k>aH \text{ at } \tau_0}}$$

Structure of Lindblad terms

Splitting the system and environment sectors of H_{int}

$$H_{\text{int}}(\tau) = h^{mn}(\tau) H_{\mathscr{E},m}(\tau_0) H_{\mathscr{S},n}(\tau_0)$$

$$\int_{\tau_0}^{\tau} H_{\text{int}}(\tau_1) d\tau_1 = \underbrace{\left[\int_{\tau_0}^{\tau} h^{mn}(\tau_1)\right]}_{\equiv g^{mn}(\tau)} G_{\mathscr{E},m}(\tau_0) G_{\mathscr{S},n}(\tau_0)$$

Lindblad terms become very systematic as

$$\text{Lindblad terms} = \langle 0|_{\mathscr{E}} (\text{environment sector}) |0\rangle_{\mathscr{E}} \times (\text{system sector})$$

Squeezed state as the pointer basis

Lindblad equation can be schematically written as

$$\frac{d\rho_{\text{red}}}{d\tau} = \sum_{m,n\leq 6} \rho_{mn} U_0 a_1^\dagger a_2^\dagger \cdots a_m^\dagger |0\rangle_{\mathscr{S}} \underbrace{\langle 0|_{\mathscr{S}} a_{1'} a_{2'} \cdots a_n U_0^\dagger}_{=\left(U_0 a_{n'}^\dagger \cdots a_{2'}^\dagger a_{1'}^\dagger |0\rangle_{\mathscr{S}}\right)^\dagger}$$

Seems natural basis: $\left\{ U_0|0\rangle_{\mathscr{S}}, U_0 a_1^\dagger|0\rangle_{\mathscr{S}}, U_0 a_1^\dagger a_2^\dagger|0\rangle_{\mathscr{S}} \cdots \right\}$

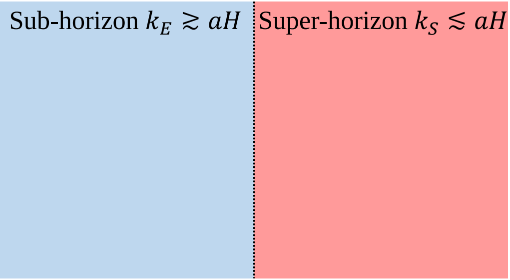
- We do not directly observe primordial perturbations

$$C_\ell^{BB} \sim \int (\text{transfer function}) \times P_h(k)$$

- Classicality not on individual solution but on stat properties

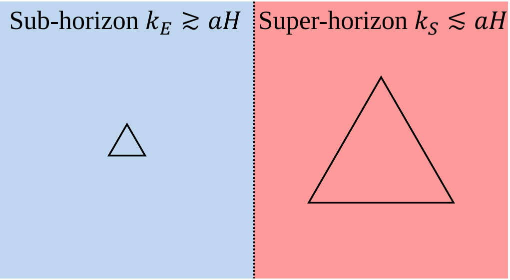
(Guth & Pi 1985, Albrecht et al. 1994, Polarski & Starobinsky 1996)

Cubic interactions



$$L = 1/(aH)$$

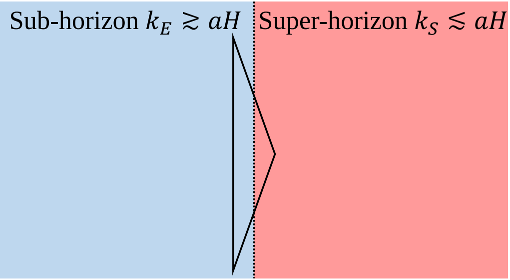
Cubic interactions



$$L = 1/(aH)$$

- All modes are in the environment or system sector

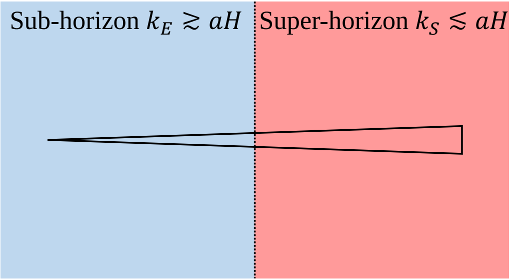
Cubic interactions



$L = 1/(aH)$

- All modes are in the environment or system sector
- 2 system and 1 environment: $\boldsymbol{k}_1 \approx \boldsymbol{k}_2$ and $|\boldsymbol{k}_3| \approx 2|\boldsymbol{k}_1|$

Cubic interactions



$$L = 1/(aH)$$

- All modes are in the environment or system sector
- 2 system and 1 environment: $\boldsymbol{k}_1 \approx \boldsymbol{k}_2$ and $|\boldsymbol{k}_3| \approx 2|\boldsymbol{k}_1|$
- 1 system and 2 environment: $\boldsymbol{k}_1 \approx -\boldsymbol{k}_2$ and $k_3 \ll k_1 \approx k_2$

Triangular contributions

- $\mathscr{E}\mathscr{E}\mathscr{E}$: 0 from the beginning
- $\mathscr{S}\mathscr{S}\mathscr{S}$: Absorbed into unitary evolution
- $\mathscr{E}\mathscr{S}\mathscr{S}$: Flattened triangle
  - No clear distinction (we want $k_{\mathscr{E}} \gg aH$ and $k_{\mathscr{S}} \ll aH$)
  - Disappear in the enfolded limit
- $(\mathscr{E}\mathscr{E}\mathscr{S})_{\rm sq}$: Squeezed triangle
  - At least $\mathscr{O}\left(q^2/\mathscr{H}^2\right)$
  - Disappear at leading order
- $\mathscr{E}\mathscr{E}\mathscr{S}$: Only non-zero contribution

Matrix notation of Lindblad equation

$$\frac{d\rho_{\text{red}}}{d\tau} = -\left(\begin{array}{ccc|c} \mathfrak{E}_{00} & 0 & \mathfrak{E}_{02} & \\ 0 & \mathfrak{E}_{11} & 0 & 0_{3\times 4} \\ \mathfrak{E}_{20} & 0 & 0 & \\ \hline & 0_{4\times 3} & & 0_{4\times 4} \end{array}\right)$$

- $\mathfrak{E}_{00} = \dfrac{2}{(2\pi)^3}\delta^{(3)}(\boldsymbol{q})\dfrac{H^2}{m_{\text{Pl}}^2}\dfrac{4}{3\tau^4}8\pi^2\mathscr{C}_{\mathscr{S}\mathscr{E}}$
- $\mathfrak{E}_{11} = -2\delta^{(3)}(\boldsymbol{q}_{ab})\delta_{\lambda_a\lambda_b}\dfrac{H^2}{m_{\text{Pl}}^2}\dfrac{4}{3\tau^4}2\pi\dfrac{\mathscr{C}_{\mathscr{E}}}{q^3} \qquad (\boldsymbol{q}_a = -\boldsymbol{q}_b \equiv \boldsymbol{q})$
- $\mathfrak{E}_{20} = -3\delta^{(3)}(\boldsymbol{q}_{ab})\delta_{\lambda_a\lambda_b}12\dfrac{H^2}{m_{\text{Pl}}^2}\dfrac{4}{3\tau^4}2\pi e^{2iq\tau}\dfrac{\mathscr{C}_{\mathscr{E}}}{q^3} = \mathfrak{E}_{02}^*$

Consistency of each elements

1. $\mathfrak{E}_{00}$: no excitation for in- and out-states

$$\mathscr{C}_{\mathscr{S}\mathscr{E}} \approx 0.577148 - \frac{32}{45}\log\varepsilon \quad (\varepsilon \ll 1: \text{ cutoff})$$

2. $\mathfrak{E}_{11}$: one-particle excitation for both in- and out-states

$$\mathscr{C}_{\mathscr{S}\mathscr{E}} = \int_{\varepsilon\ll 1}^{1} \bar{q}^2 d\bar{q} \frac{\mathscr{C}_{\mathscr{E}}}{\bar{q}^3} \quad \left(\bar{q} \equiv \frac{q}{aH}\right)$$

Trace of $\mathfrak{E}_{00}$ and $\mathfrak{E}_{11}$ guarantees no divergence

3. $\mathfrak{E}_{20}$ and $\mathfrak{E}_{02}$: 2-particle and no excitation for in- and out-states
   - $e^{-iq\tau}$ and $e^{iq\tau}$ are the phase factors for in- and out-states
   - Phase factor cancel in $\mathfrak{E}_{11}$, but squared in $\mathfrak{E}_{20}$ and $\mathfrak{E}_{02}$

Deviation from unitary evolution

00 element of $\rho_{\rm red}$, i.e. evolution w.r.t. $U_{0,\mathscr{S}}|0\rangle_{\mathscr{S}}$

$$\rho_{\rm red}\big|_{00} = 1 - \frac{2}{(2\pi)^3}\delta^{(3)}(\boldsymbol{q})\frac{H^2}{m_{\rm Pl}^2}\frac{4}{9|\tau|^3}8\pi^2\mathscr{C}_{\mathscr{S}\mathscr{E}}$$

Interaction Hamiltonian $H_{\rm int}$ generates:

1. Unitary evolution in the system sector
2. Non-unitary evolution through the Lindblad terms
   - $\rho_{\rm red}\big|_{00}$ is reduced from 1
   - The probability of keeping the pure squeezed state is reduced
   - Classical probability for other processes involving 2 excitations

Decoherence rate

Exponentiating $\rho_{\text{red}}|_{00}$ with $(2\pi)^3\delta^{(3)}(\boldsymbol{q})$ = comoving 3D volume,

$$\rho_{\text{red}}|_{00} = \exp\left(-V\int\Gamma d\tau\right) \quad \text{with} \quad \Gamma = \frac{\Delta_{\mathcal{R}}^2}{(2\pi)^2}\frac{2}{3\tau^4}r\mathscr{C}_{\mathscr{S}\mathscr{E}}$$

- Γ grows as the phys vol a^3, is suppressed as $r\mathscr{C}_{\mathscr{S}\mathscr{E}} \sim H^2/m_{\text{Pl}}^2$
- Inflation scale $\ll$ QG scale, irrespective of inf dynamics

Number of e-folds for decoherence

For an interval δN, the change in $\rho_{\rm red}|_{00}$ is

$$\exp\left[-\frac{\Delta^2_{\mathscr{R}}}{(2\pi)^2}\frac{2}{9}e^{3\Delta N}r\mathscr{C}_{\mathscr{S}\mathscr{E}}\right]$$

The e-fold for $\rho_{\rm red}|_{00}$ to change by e^{-1}

$$\Delta N_{\rm dec} \approx \frac{1}{3}\log\left[\frac{(2\pi)^2}{\Delta^2_{\mathscr{R}}}\frac{9}{2}\left(r\mathscr{C}_{\mathscr{S}\mathscr{E}}\right)^{-1}\right] \approx 8.38689 - \frac{1}{3}\log\left(r\mathscr{C}_{\mathscr{S}\mathscr{E}}\right)$$

Typically $5 \lesssim \Delta N_{\rm dec} \lesssim 10$ for a wide range of r and $\mathscr{C}_{\mathscr{S}\mathscr{E}}$
c.f. $\Delta N_{\rm dec} \gtrsim 10$ for $\mathscr{R}$ (Nelson 2016)

Choice of pointer basis

Quote from W. H. Zurek (1981):

> … *observable of the measured quantum system can be considered "recorded" by the apparatus. The basis that contains this record – the* ***pointer basis*** *of the apparatus – consists of the eigenvectors of the operator which commutes with the apparatus-environment interaction Hamiltonian.*

$$\left.\frac{d\rho_{\rm red}}{d\tau}\right|_{ab} \equiv \left\langle a \left| \frac{d\rho_{\rm red}}{d\tau} \right| b \right\rangle$$

Choosing pointer basis is related to the mechanism of apparatus

- Are squeezed states best regarding optical apparatus?
- Different states for apparatus w/ obs principles?

Classicality of squeezed state

All the matrix elements of $\rho_{\rm red}$ denote classical probability?

$$\mathfrak{E}_{11} \sim \underbrace{\langle 0|_{\mathscr{S}} a_a}_{=\left(a_a^\dagger|0\rangle_{\mathscr{S}}\right)^\dagger} U_{0,\mathscr{S}}^\dagger \times \frac{d\rho_{\rm red}}{d\tau} \times U_{0,\mathscr{S}} \underbrace{a_b^\dagger|0\rangle_{\mathscr{S}}}_{\text{1-ptl excitation}}$$

$U_{0,\mathscr{S}} a_b^\dagger|0\rangle_{\mathscr{S}} \neq$ Gaussian wave function!

- Is the Wigner function negative?
- How to observe $\mathfrak{E}_{11}$?

Conclusions

1. Studying quantum origin may be relevant
2. Pure tensor perturbations are of physical interest
3. Non-linear evolution allows system-environment interactions
    1. Lindblad equation: evolution of reduced density matrix
    2. Exponential decay of (some components of) $\rho_{\rm red}$
    3. Generation of classical probability for other states
4. Many more questions to be answered