Can we Observe Quantum Entanglement in the CMB?

Jerome Martin

CNRS/Institut d'Astrophysique de Paris

Quantum Entanglement in Cosmology
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Outline

- Introduction & Motivations

- Cosmological Perturbations of Quantum-Mechanical Origin

- Signature of the Quantum Origin of the Perturbations in the Sky?

- Discussion & Conclusions

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- But can we find a direct signature (in the sky)?

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The origin of the fluctuations

According to inflation, the perturbations originate from quantum fluctuations of the gravitational and scalar fields then amplified by gravitational instability and stretched by the cosmic expansion

$$g_{\mu\nu} = g_{\mu\nu}(t) + \widehat{\delta g_{\mu\nu}}(t, \mathbf{x}) + \cdots$$

$$\phi = \phi(t) + \widehat{\delta\phi}(t, \mathbf{x}) + \cdots$$

$$\downarrow$$

$$\hat{\zeta}(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}^3\mathbf{k}\, \hat{\zeta}_{\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{x}} = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3\mathbf{k}}{\sqrt{2k}} \left[\hat{c}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{c}^{\dagger}_{\mathbf{k}}(t) e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

Scalar perturbations are described by a single quantity, curvature perturbation, which is a combination of metric and field fluctuations and which directly determines CMB temperature anisotropies

The evolution of scalar inflationary quantum perturbations is governed by the following Hamiltonian

$$\hat{H} = \int \mathrm{d}^3\mathbf{k} \left[\frac{k}{2}\left(\hat{c}_{\mathbf{k}}\hat{c}^{\dagger}_{\mathbf{k}} + \hat{c}_{-\mathbf{k}}\hat{c}^{\dagger}_{-\mathbf{k}}\right) - \frac{i}{2}\frac{(a\sqrt{\epsilon_1})'}{a\sqrt{\epsilon_1}}\left(\hat{c}_{\mathbf{k}}\hat{c}_{-\mathbf{k}} - c^{\dagger}_{-\mathbf{k}}c^{\dagger}_{\mathbf{k}}\right)\right]$$

Derived from first principles: expansion of Einstein-Hilbert action
Plus action of a scalar field at second order

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Perturbations of quantum-mechanical origin

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1- Pump field ~ time dependent coupling constant.
2- Only depends on the scale factor and its derivatives.
3- Vanishes if a'=0

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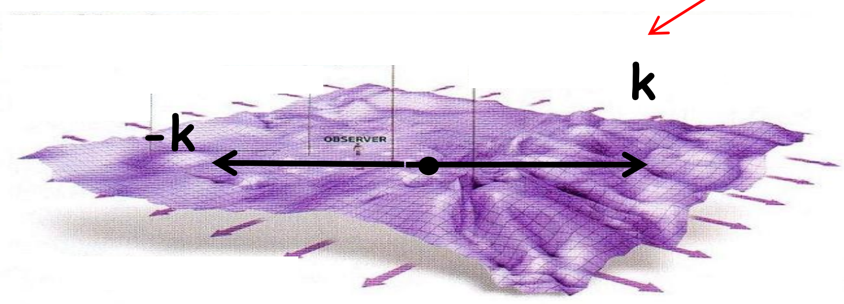


Corresponds to creation of pair of particles

This is similar to the Schwinger effect: interaction of a quantum field with a classical source

J. Martin, Lect. Notes Phys. 738 (2008), 195
arXiv:0704.3540

Schwinger effect

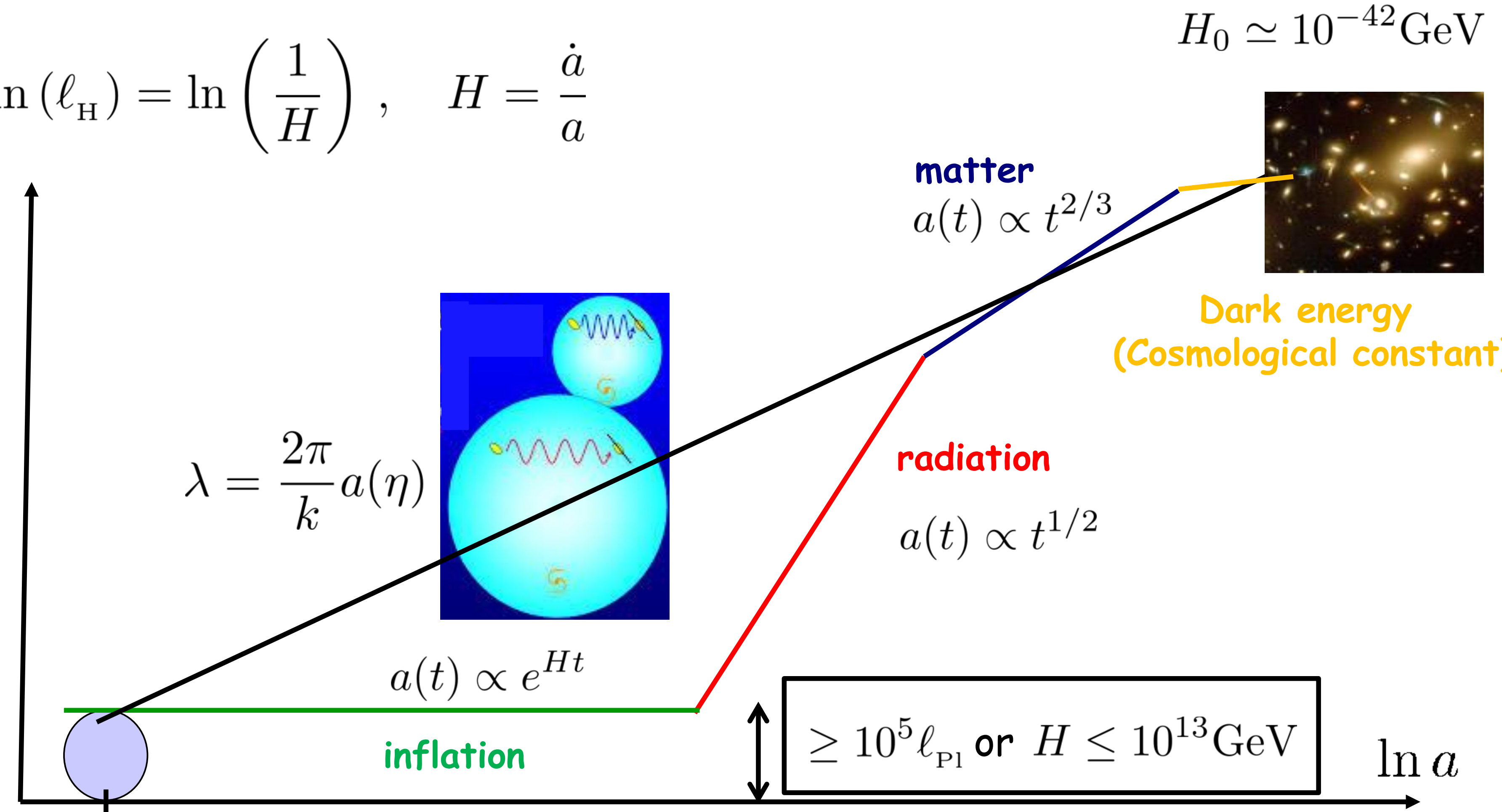
- Electron and positron fields
- Classical electric field
- Amplitude of the effect controlled by E

Inflationary cosmological perturbations

- Inhomogeneous gravity field
- Background gravitational field: scale factor
- Amplitude controlled by the Hubble parameter H

See also dynamical Schwinger effect, dynamical Casimir effect etc …

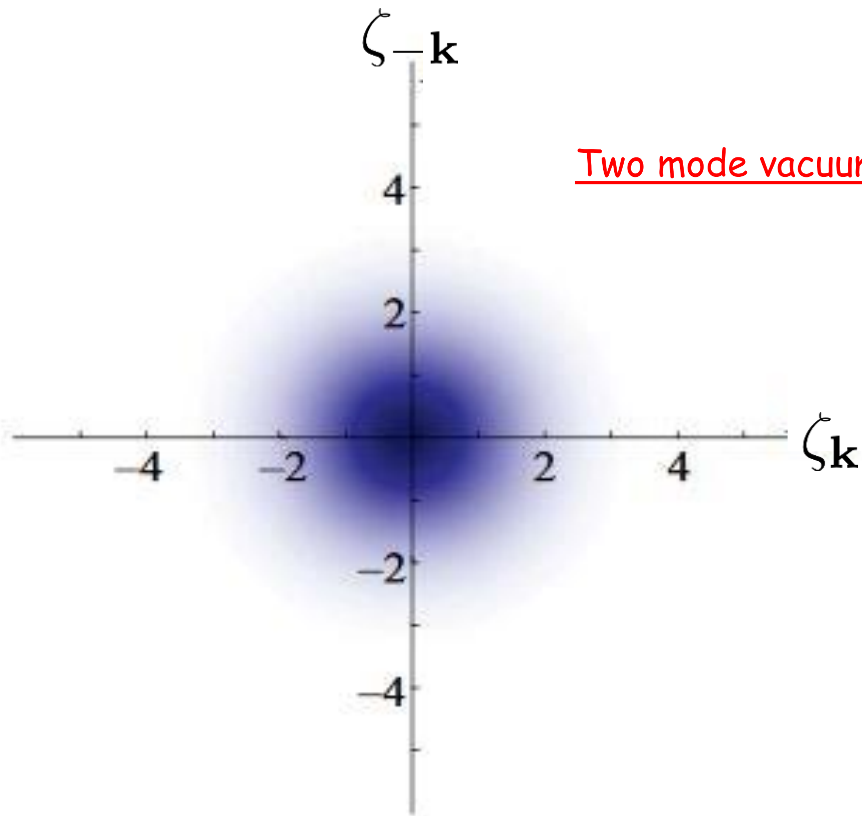
$$\ln\left(\ell_{\mathrm{H}}\right) = \ln\left(\frac{1}{H}\right), \quad H = \frac{\dot{a}}{a}$$



Initially, curvature perturbations are placed in the vacuum state

The wave functional of curvature perturbations takes the form

$$\Psi[\zeta] = \prod_{\mathbf{k}\in \mathrm{R}^{3+}} \Psi(\zeta_{\mathbf{k}}, \zeta_{-\mathbf{k}})$$



Two mode vacuum coherent state

$$\Psi_0\left(\zeta_{\mathbf{k}}, \zeta_{-\mathbf{k}}\right) = \frac{1}{\pi^{1/4}} e^{-\zeta_{\mathbf{k}}^2/2-\zeta_{-\mathbf{k}}^2/2} = \frac{1}{\sqrt{\pi}} e^{-(\zeta_{\mathbf{k}}-\zeta_{-\mathbf{k}})^2/4} e^{-(\zeta_{\mathbf{k}}+\zeta_{-\mathbf{k}})^2/4}$$

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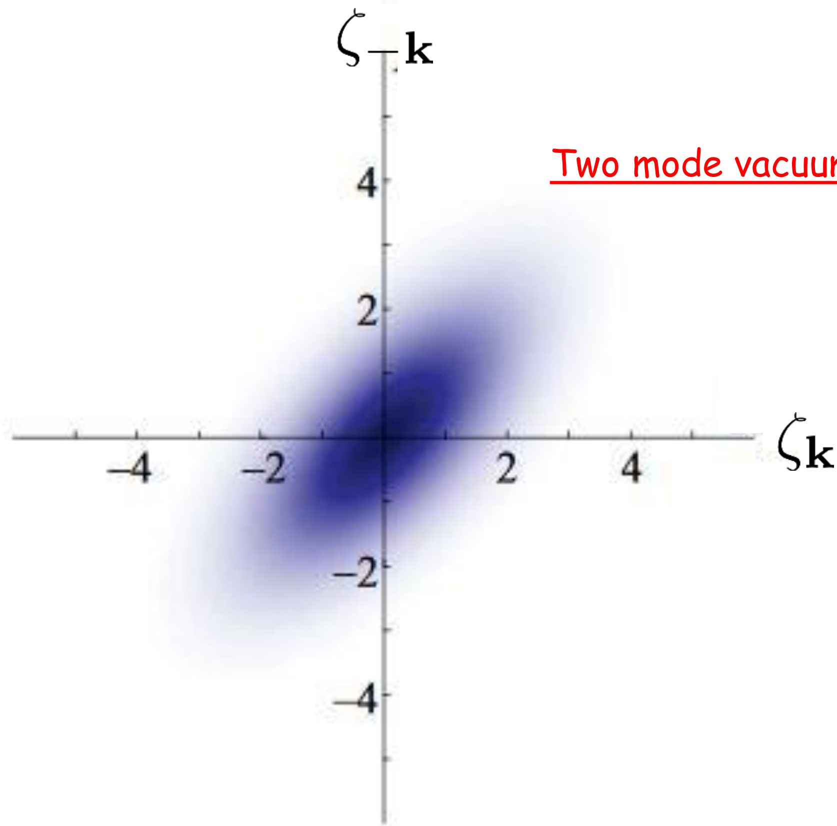

Two mode vacuum squeezed state

$$\Psi_R\left(\zeta_{\mathbf{k}}, \zeta_{-\mathbf{k}}\right) = \frac{1}{\sqrt{\pi}} e^{-(\zeta_{\mathbf{k}}-\zeta_{-\mathbf{k}})^2/(4R^2)} e^{-R^2(\zeta_{\mathbf{k}}+\zeta_{-\mathbf{k}})^2/4}$$

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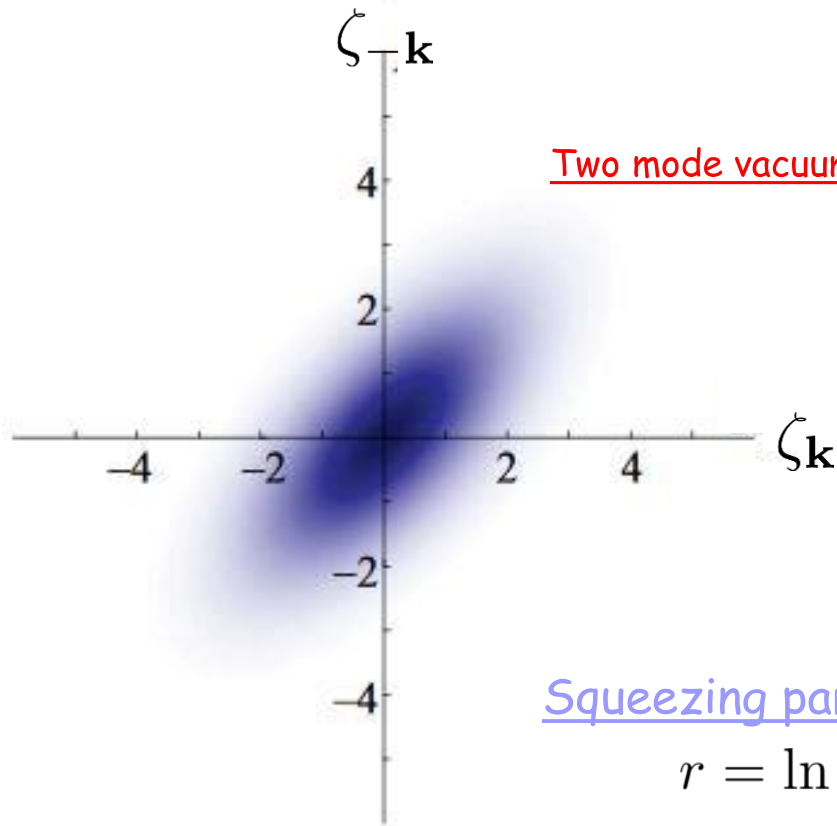

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Squeezing parameter

$$r = \ln R$$

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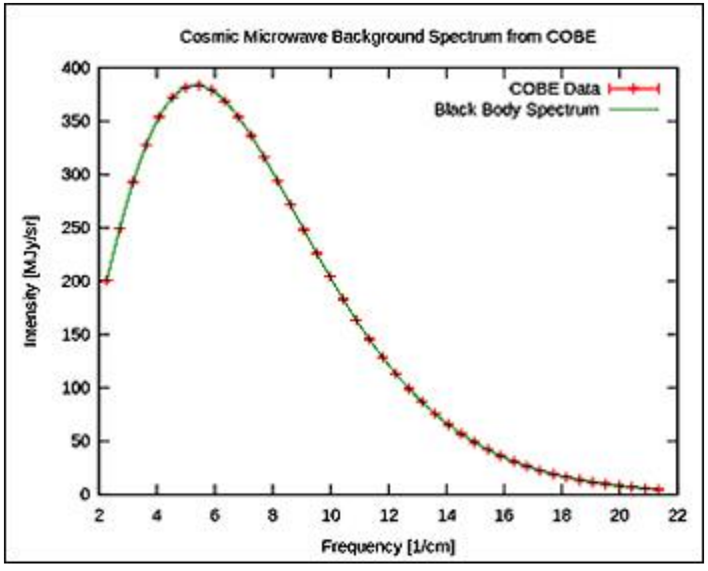
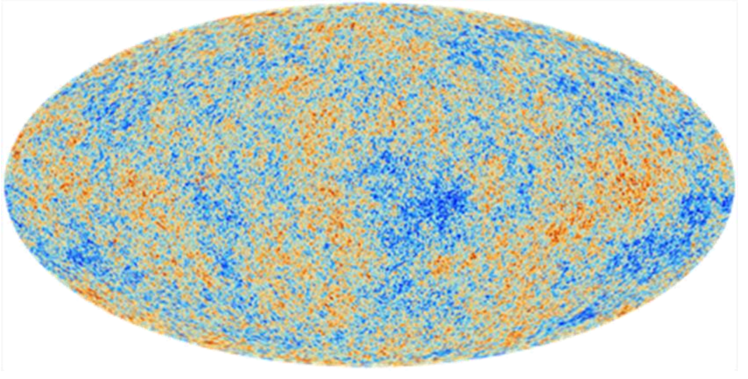


CMB anistropy is the strongest squeezed state ever produced in Nature



$$r_k = \mathcal{O}\left(10^2\right)$$

Quantum state of the CMB: a two-mode squeezed state

- The cosmological two-mode squeezed state is (very!) strongly squeezed

- It is an entangled state (correlations between mode k and -k)

$$|\Psi_{\mathbf{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{+\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle$$

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So the CMB appears to be a highly non-classical object … is it really the case?

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Wigner function: "quantum distribution" in phase space

$$W(q,p) = \int e^{ipu} \left\langle q - \frac{u}{2} \middle| \frac{\hat{\rho}}{2\pi} \middle| q + \frac{u}{2} \right\rangle \mathrm{d}u$$

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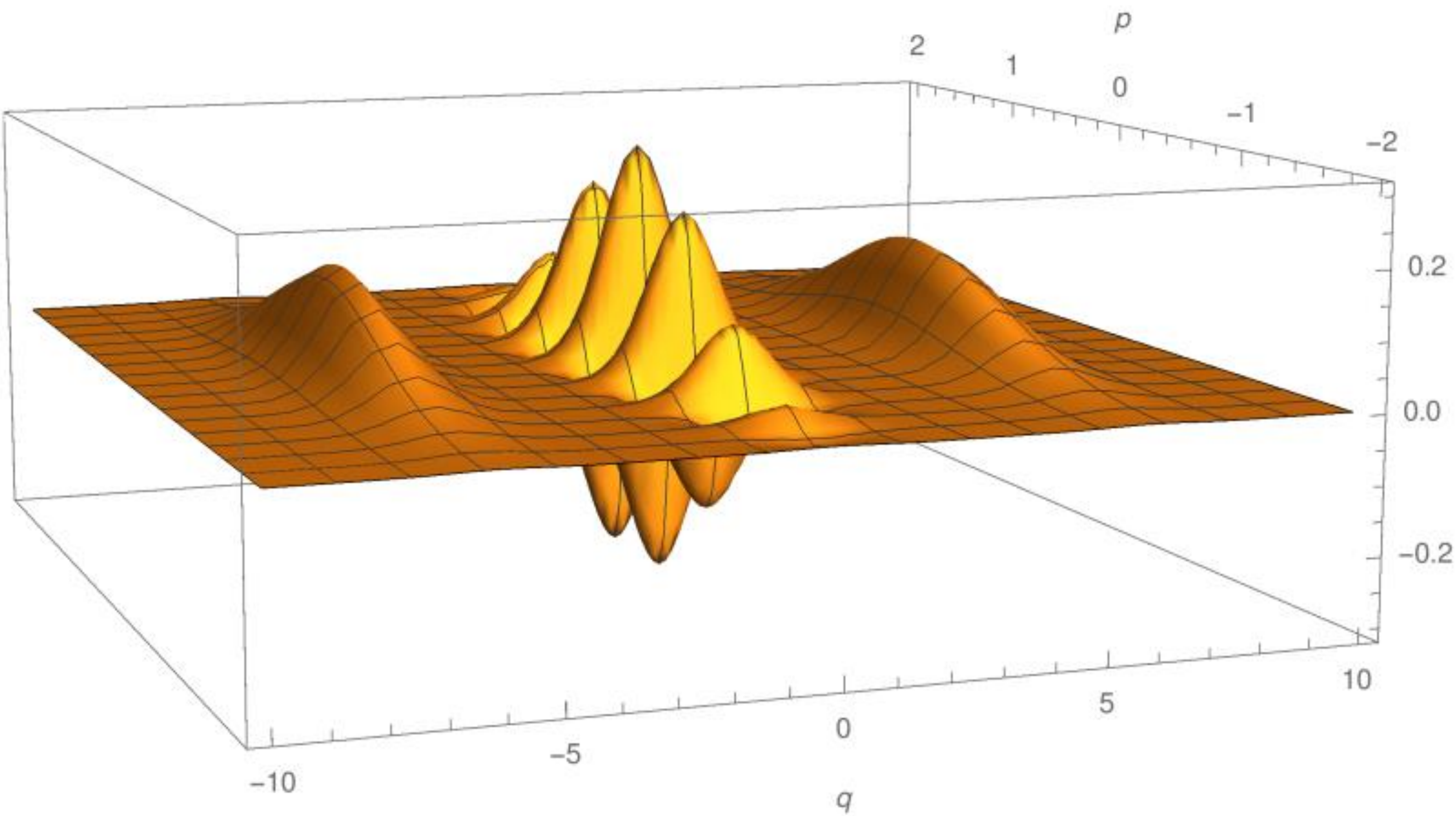
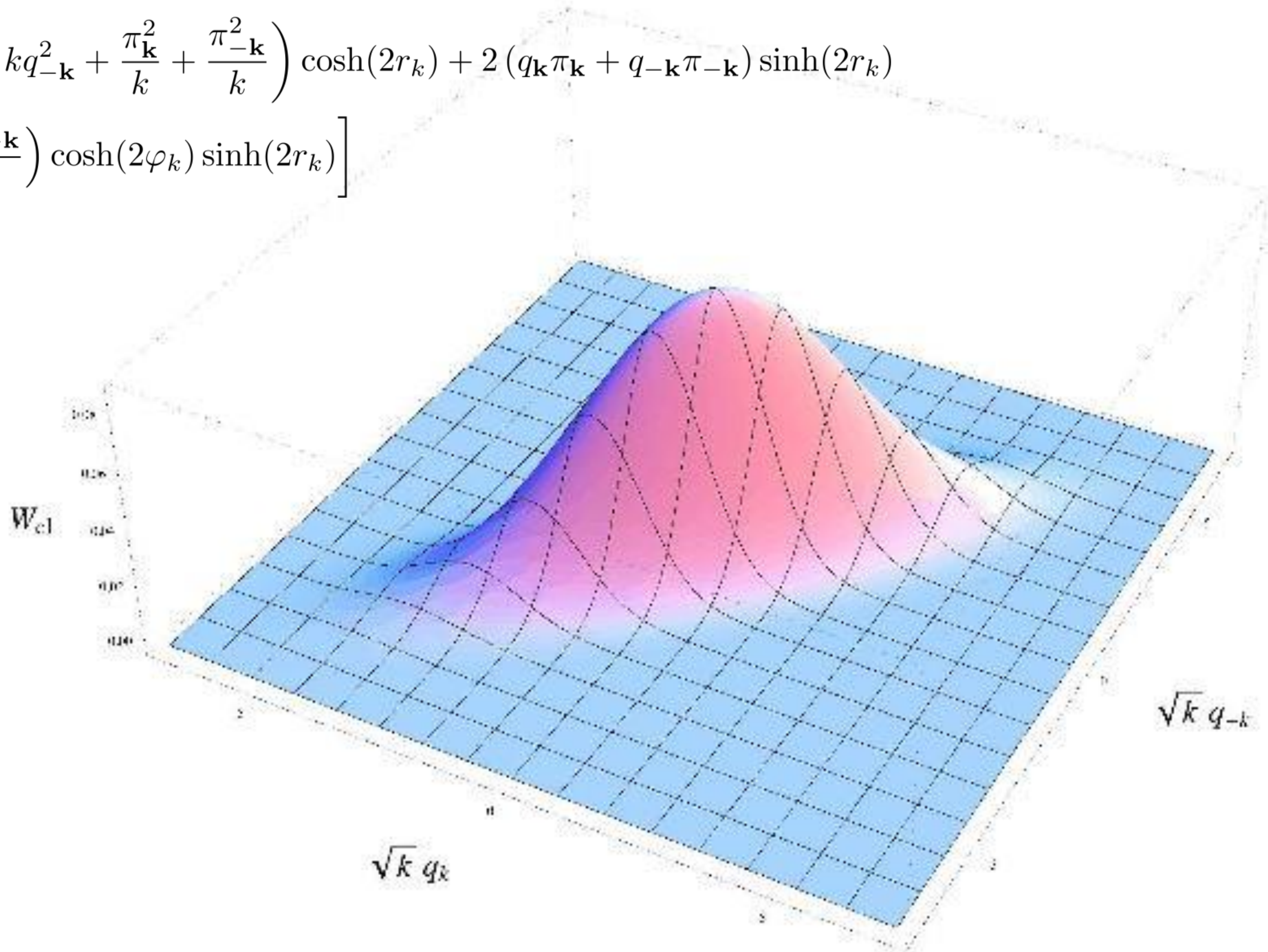
- The proto-typical example is the Schroedinger cat



$$\Psi_{\mathrm{CAT}} = \frac{N_{\mathrm{CAT}}}{\sqrt{2}} \left[\Phi_+(q) + \Phi_-(q)\right]$$

A two mode squeezed state is a Gaussian state and hence has a positive definite Wigner function

$$W=\frac{1}{\pi^2}\exp\Bigg[-\left(kq_{\mathbf{k}}^2+kq_{-\mathbf{k}}^2+\frac{\pi_{\mathbf{k}}^2}{k}+\frac{\pi_{-\mathbf{k}}^2}{k}\right)\cosh(2r_k)+2\left(q_{\mathbf{k}}\pi_{\mathbf{k}}+q_{-\mathbf{k}}\pi_{-\mathbf{k}}\right)\sinh(2r_k)$$
$$+2\left(kq_{\mathbf{k}}q_{-\mathbf{k}}-\frac{\pi_{\mathbf{k}}\pi_{-\mathbf{k}}}{k}\right)\cosh(2\varphi_k)\sinh(2r_k)\Bigg]$$



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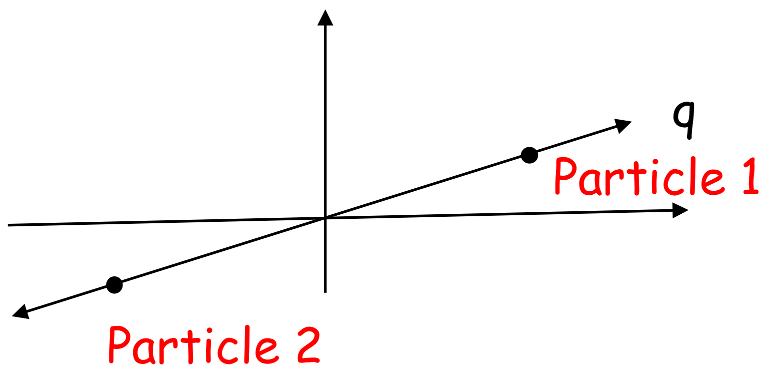
- This idea is also discussed by John Bell in "EPR correlations and EPW distributions" reproduced in "Speakable and unspeakable in Quantum Mechanics" and dedicated to E. Wigner

John Bell: if a state has a positive definite Wigner function, then Bell's inequality cannot be violated



1- Two spinless particles are moving in opposite directions

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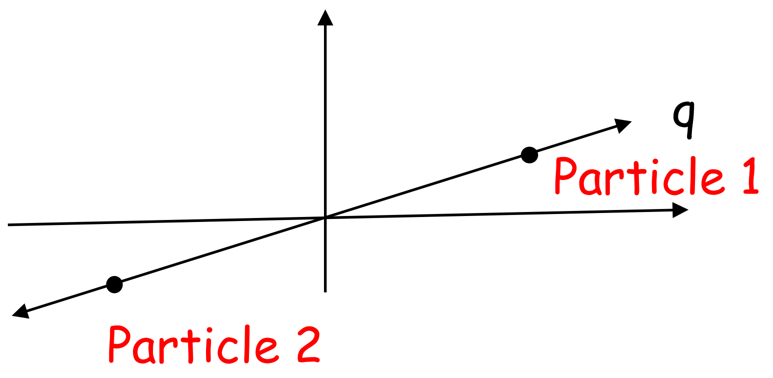

2- Bell discusses his inequality in the CHSH form which involves dichotomic variables +/-1.

Wigner function

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$$\hat{S}_1(t_1) = \mathrm{sgn}\left(\hat{q}_1 + \frac{q_0}{2}, t_1\right)$$

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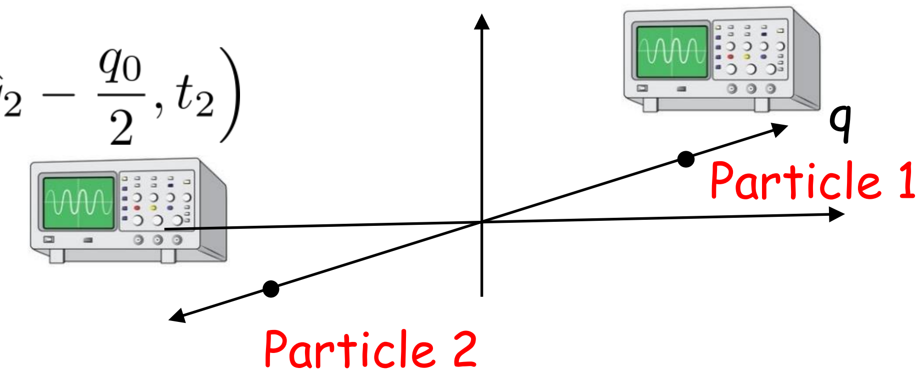

2- Bell discusses his inequality in the CHSH form which involves dichotomic variables +/-1. In order to define dichotomic variables for a system characterized by continuous variables, he introduces fictitious spins operators

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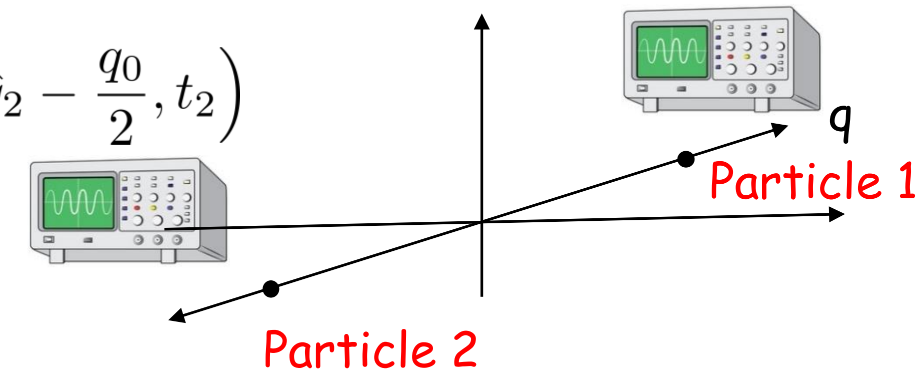

3- Infer correlation functions and Bell inequality

$$E(t_1, t_2) = \left\langle \Psi \left| \hat{S}_1(t_1)\hat{S}_2(t_2) \right| \Psi \right\rangle$$

$$B(t_1, t_2, t_1', t_2') = E(t_1, t_2) + E(t_1, t_2') + E(t_1, t_2') - E(t_1', t_2')$$ >2?

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- But Bell's paper is … wrong as well as the papers criticizing Bell's paper …

This story is told in full detail in J. Martin "Cosmic inflation, quantum information and the pioneering role of JS Bell in cosmology", Universe 2019, 5(4), 92, arXiv:1904.00083

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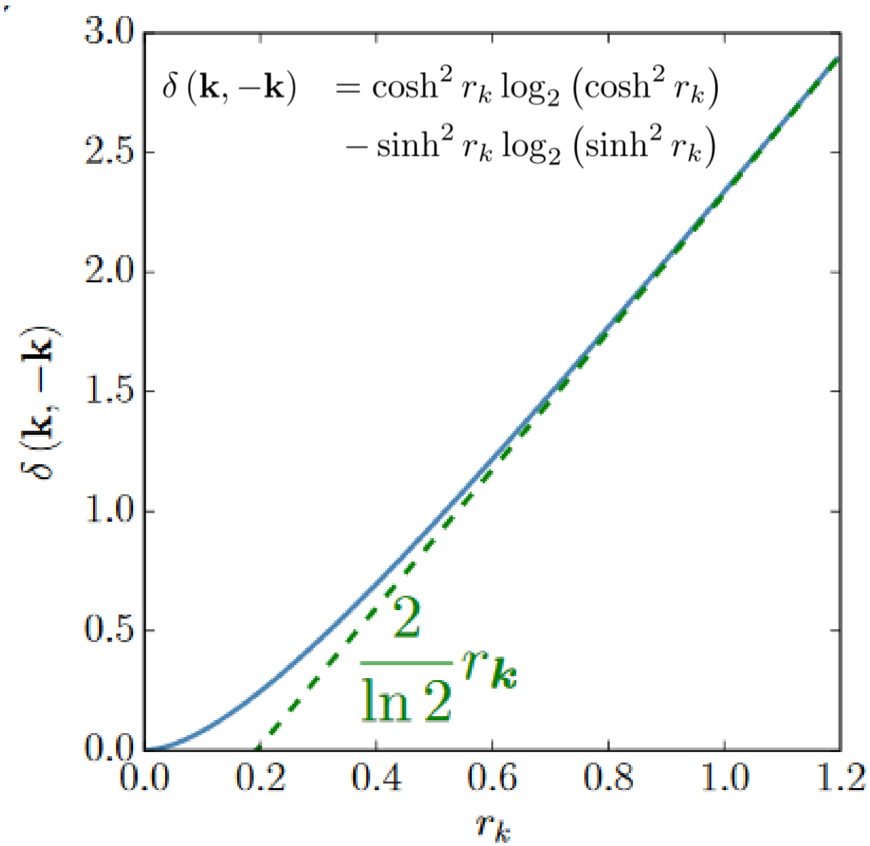
  - Implications for cosmology: CMB improper operators?

Confirmed by an analysis using tools of quantum information

Quantum discord corresponds to two ways to calculate mutual information between two subsystems that coincide classically but not necessarily in quantum systems [Ollivier & Zurek, PRL 88, (2014), 017901]



J. Martin & V. Vennin, PRD 93 (2016), 1023505, arXiv:1510.04038

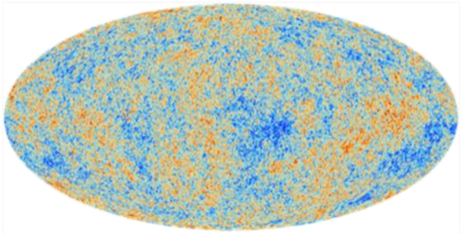
Since there is no no go theorem against a Bell inequality violation with a Gaussian state, can we implement it for the CMB?

- Entangled state: two-mode squeezed state

- Bipartite system: k and –k

- Improper CMB variable? Can we find an operator in the CMB such that Bell inequality is violated … our only chance to see the quantum origin of the perturbations according to Revzen theorem … yes using fictitious spin operators a la John Bell!

We need a dichotomic variable +/-1 while we measure a CV in the CMB



$\longrightarrow$

Continuous variable q_k

$$\hat{q}_{\mathbf{k}} = \frac{\hat{c}_{\mathbf{k}} + \hat{c}_{\mathbf{k}}^{\dagger}}{\sqrt{2k}} = \hat{q}_{\mathbf{k}}^{\dagger}$$

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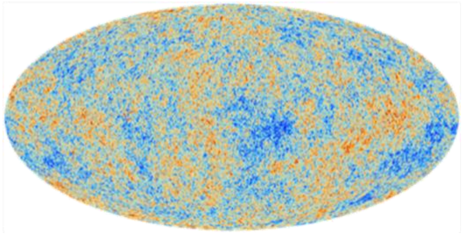

$\longrightarrow$ Continuous variable q_k

q_k

Continuous spectrum

Bell inequalities in the sky?

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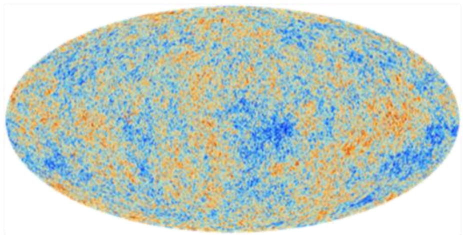

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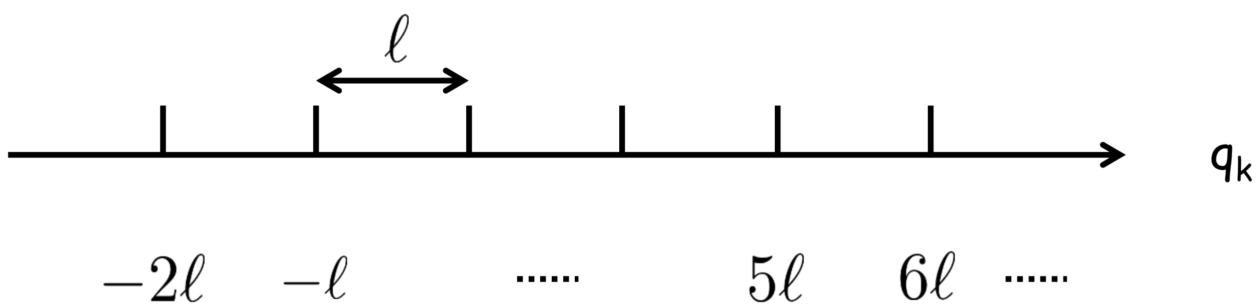

Divide the real axis in an infinite number of interval $[n\ell, (n+1)\ell]$

Bell inequalities in the sky?

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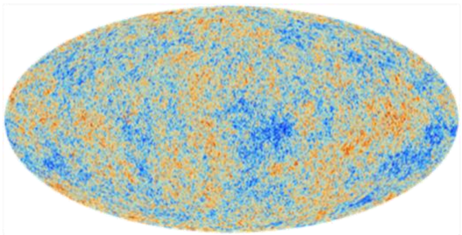
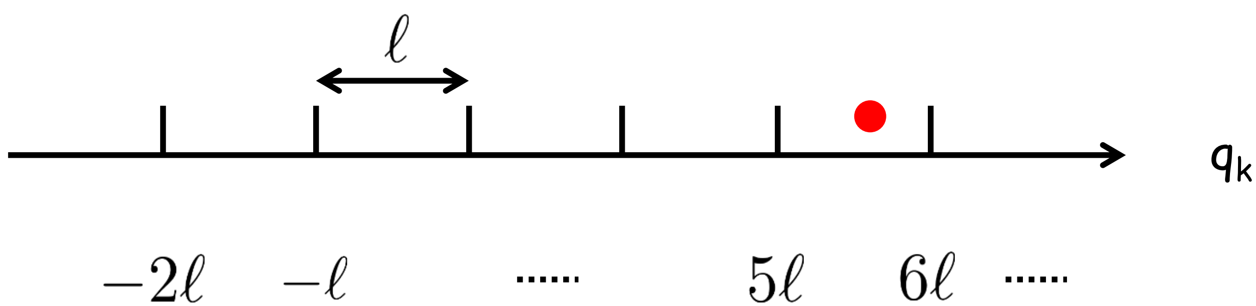


→ Continuous variable q_k



Perform a measurement of q_k

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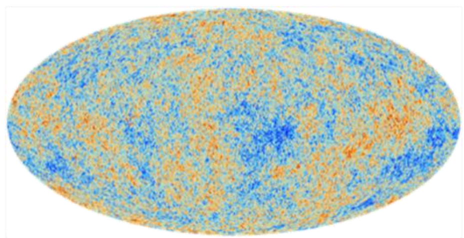
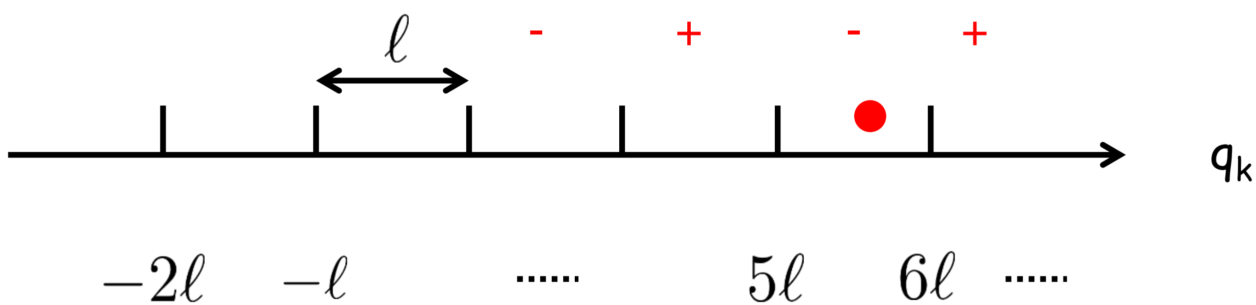
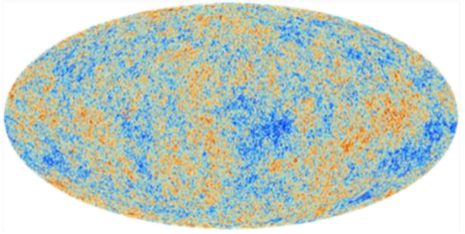


$\longrightarrow$ Continuous variable q_k



Give the "fictitious" spin value $S_z(\ell) = (-1)^n$

We need a dichotomic variable +/-1 while we measure a CV in the CMB



$\longrightarrow$ Continuous variable q_k

This defines the following z-component spin operator

$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} \mathrm{d}q_{\mathbf{k}} |q_{\mathbf{k}}\rangle\langle q_{\mathbf{k}}|$$

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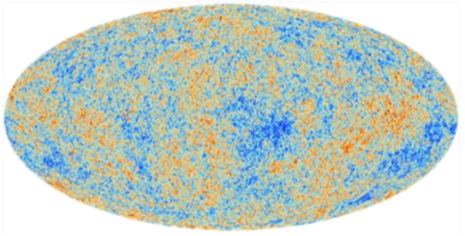

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$$\hat{S}_z^2(\ell) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} (-1)^{n+m} \int_{n\ell}^{(n+1)\ell} \mathrm{d}q_{\mathbf{k}} |q_{\mathbf{k}}\rangle\langle q_{\mathbf{k}}| \int_{m\ell}^{(m+1)\ell} \mathrm{d}\overline{q}_{\mathbf{k}} |\overline{q}_{\mathbf{k}}\rangle\langle \overline{q}_{\mathbf{k}}|$$

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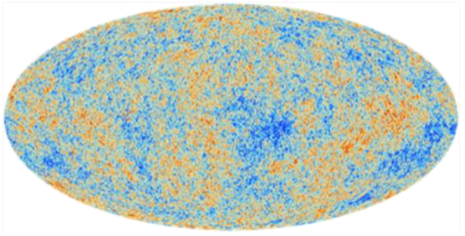

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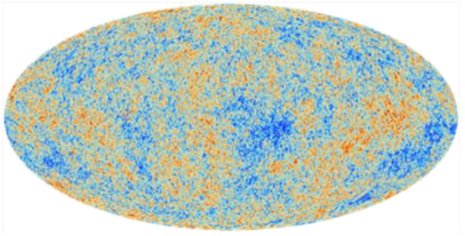
$$= \sum_{n=-\infty}^{+\infty} (-1)^{2n} \int_{n\ell}^{(n+1)\ell} \mathrm{d}q_{\mathbf{k}} |q_{\mathbf{k}}\rangle\langle q_{\mathbf{k}}| = 1$$

It is a spin!

We need a dichotomic variable +/-1 while we measure a CV in the CMB



$\longrightarrow$ Continuous variable q_k

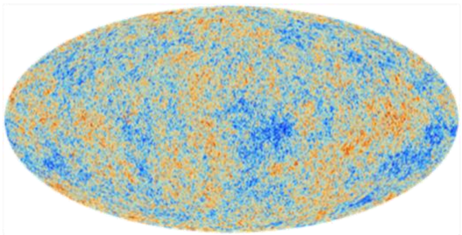
One easily define the two other components

$$\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}q_{\mathbf{k}} |q_{\mathbf{k}}\rangle\langle q_{\mathbf{k}} + \ell|$$

$$\hat{S}_x(\ell) = \hat{S}_+(\ell) + S_+^\dagger(\ell)$$

$$\hat{S}_y(\ell) = -i\left[\hat{S}_+(\ell) - S_+^\dagger(\ell)\right]$$

We need a dichotomic variable +/-1 while we measure a CV in the CMB



$\longrightarrow$ Continuous variable q_k

They satisfy usual the SU(2) commutation relations for a spin

$$\left[\hat{S}_x(\ell), \hat{S}_y(\ell)\right] = 2i\hat{S}_z(\ell)$$

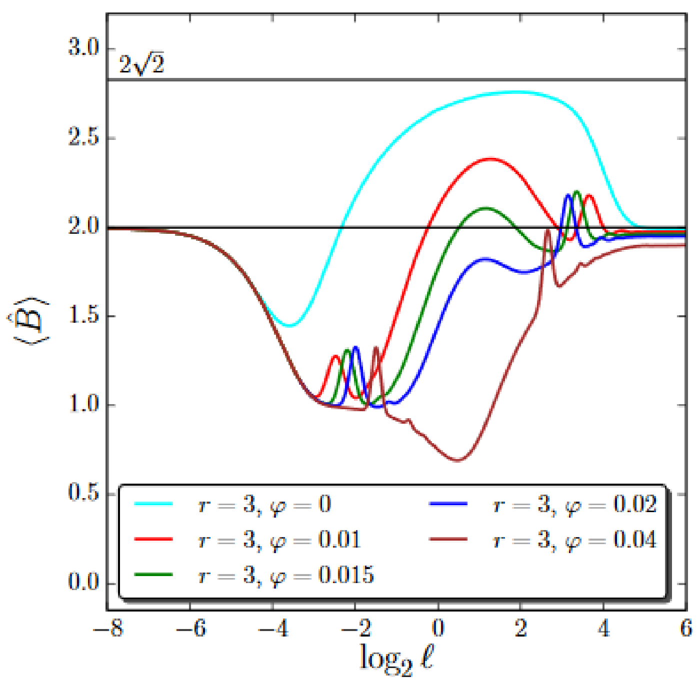
$$\left[\hat{S}_x(\ell), \hat{S}_z(\ell)\right] = -2i\hat{S}_y(\ell)$$

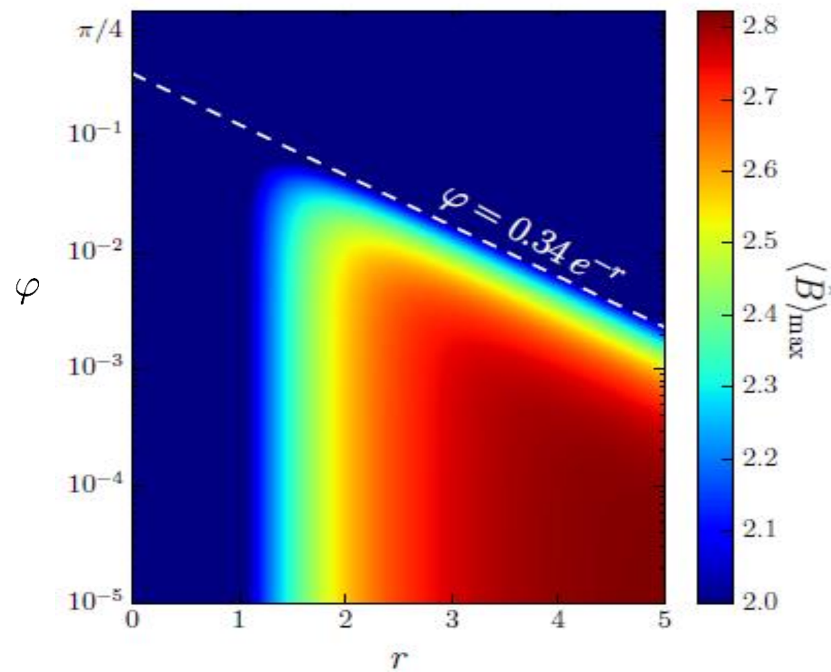
$$\left[\hat{S}_y(\ell), \hat{S}_z(\ell)\right] = 2i\hat{S}_x(\ell)$$

J.-A Larsson, Phys. Rev. A 70, 022102 (2004)

Bell operator for two-mode squeezed state (1 is k and 2 is -k)

$$\begin{aligned}\hat{B}(\ell) &= \left[\mathbf{n}\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]+\left[\mathbf{n}\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}'\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]\\ &+\left[\mathbf{n}'\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]-\left[\mathbf{n}'\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}'\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]\end{aligned}$$





J. Martin & V. Vennin, PRA93 (2016), 062117,
arXiv:1605.02944

The Larsson fictitious spin operators are not unique

Gour-Khanna-Mann-Revzen (GKMR) spin operators

$$\hat{\mathcal{S}}_x = \int_0^{+\infty} \mathrm{d}Q_{\mathbf{k}} \left(|\mathcal{E}\rangle\langle\mathcal{O}| + |\mathcal{O}\rangle\langle\mathcal{E}| \right)$$

$$\hat{\mathcal{S}}_y = i \int_0^{+\infty} \mathrm{d}Q_{\mathbf{k}} \left(|\mathcal{O}\rangle\langle\mathcal{E}| - |\mathcal{E}\rangle\langle\mathcal{O}| \right)$$

$$\hat{\mathcal{S}}_z = - \int_0^{+\infty} \mathrm{d}Q_{\mathbf{k}} \left(|\mathcal{E}\rangle\langle\mathcal{E}| - |\mathcal{O}\rangle\langle\mathcal{O}| \right)$$

with

$$|\mathcal{O}\rangle = \frac{1}{\sqrt{2}} \left(|Q_{\mathbf{k}}\rangle - | - Q_{\mathbf{k}}\rangle \right)$$

$$|\mathcal{E}\rangle = \frac{1}{\sqrt{2}} \left(|Q_{\mathbf{k}}\rangle + | - Q_{\mathbf{k}}\rangle \right)$$

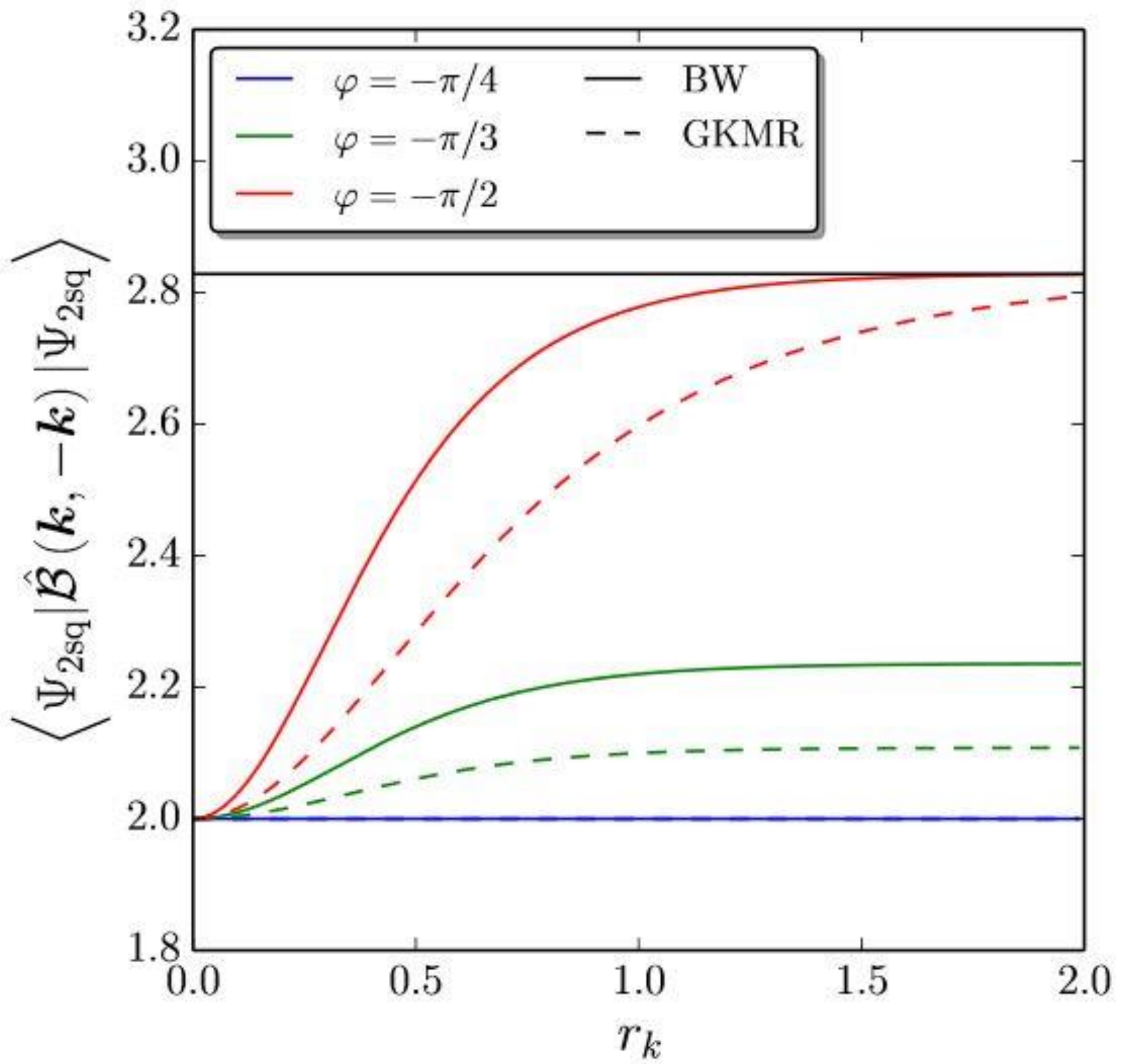
Banaszek-Wodkiewicz (BW) spin operators

$$\hat{s}_x = \sum_{n=0}^{\infty} \left(|2n_{\mathbf{k}} + 1\rangle\langle 2n_{\mathbf{k}}| + |2n_{\mathbf{k}}\rangle\langle 2n_{\mathbf{k}} + 1| \right)$$

$$\hat{s}_y = i \sum_{n=0}^{\infty} \left(|2n_{\mathbf{k}}\rangle\langle 2n_{\mathbf{k}} + 1| - |2n_{\mathbf{k}} + 1\rangle\langle 2n_{\mathbf{k}}| \right)$$

$$\hat{s}_z = \sum_{n=0}^{\infty} \left(|2n_{\mathbf{k}} + 1\rangle\langle 2n_{\mathbf{k}} + 1| - |2n_{\mathbf{k}}\rangle\langle 2n_{\mathbf{k}}| \right)$$

Other spin operators



Discussion

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  - We measure temperature fluctuations (Sachs-Wolfe effect):

$$\widehat{\frac{\delta T}{T}}(\mathbf{e}) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} \left[F(\mathbf{k}) + i\mathbf{k}\cdot\mathbf{e}\, G(\mathbf{k})\right] \hat{\zeta}_{\mathbf{k}}(\eta_{\mathrm{end}}) e^{i\mathbf{k}\cdot\mathbf{e}(\eta_{\mathrm{lss}}-\eta_0)+i\mathbf{k}\cdot\mathbf{x}_0}$$

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$$\downarrow$$

$$\hat{\zeta}_{\mathbf{k}} = \frac{1}{\sqrt{2k}}\left(c_{\mathbf{k}} + c^{\dagger}_{-\mathbf{k}}\right) \neq \hat{\zeta}^{\dagger}_{\mathbf{k}}$$

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  - Seems conservative to assume that one measures "position"

$$\hat{q}_{\mathbf{k}} = \frac{z}{2}\left(\hat{\zeta}_{\mathbf{k}} + \hat{\zeta}_{\mathbf{k}}^{\dagger}\right) + \frac{z}{2k}\left(\zeta'_{\mathbf{k}} - \zeta'_{-\mathbf{k}}\right)$$

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$$\left\langle \left[\hat{\mathcal{S}}_x(\mathbf{k}), \hat{q}_{\mathbf{k}} \right] \right\rangle = 0,$$

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  - Yes for the x-components of the GKMR operators

$$\left\langle \left[\hat{S}_x(\mathbf{k}), \hat{q}_\mathbf{k}\right] \right\rangle = 0,$$

  - But probably no for the other components

$$\left\langle \left[\hat{S}_y(\mathbf{k}), \hat{q}_\mathbf{k}\right] \right\rangle \neq 0,$$

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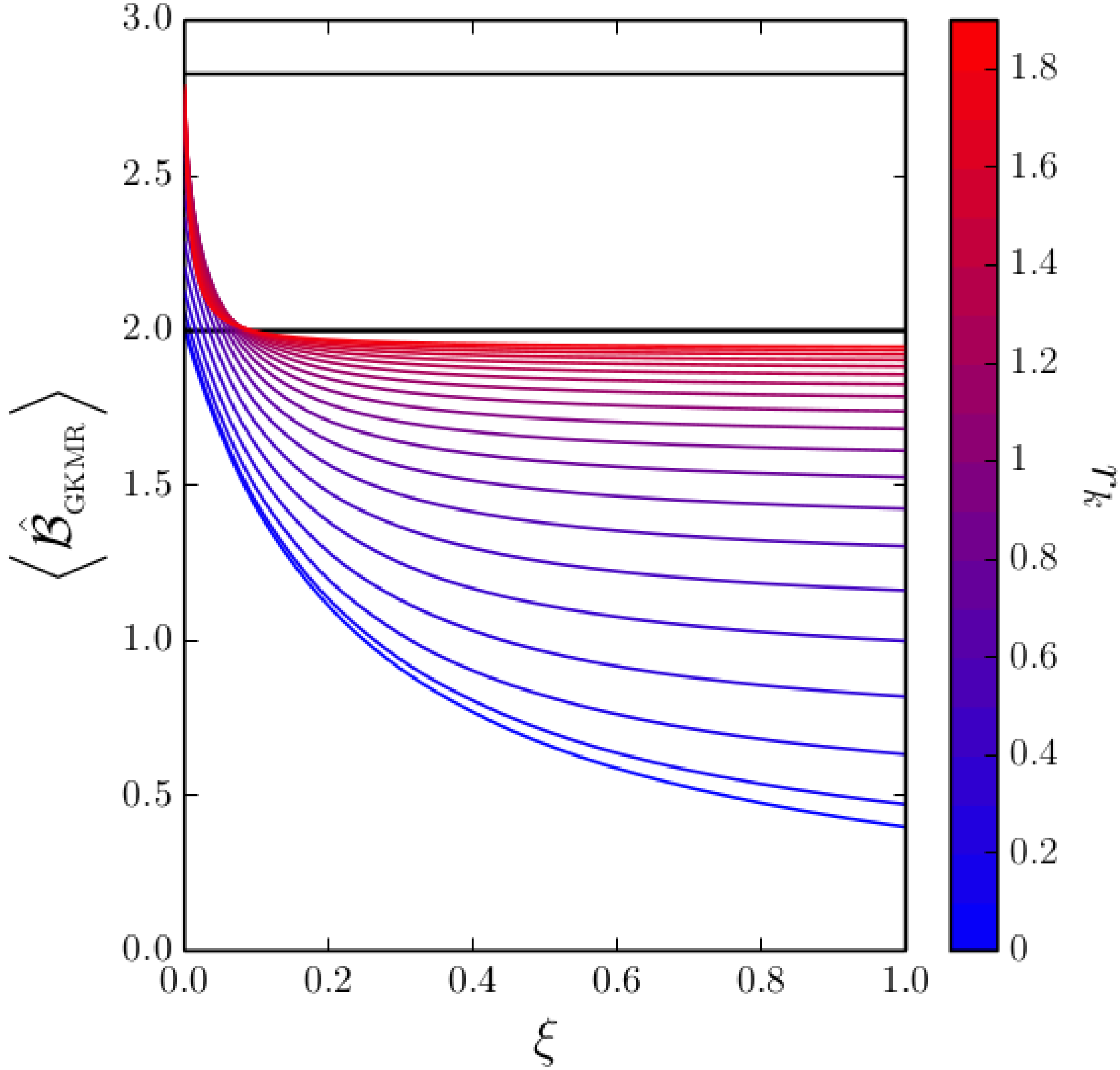
- Decoherence reduces Bell inequality violation

And, of course, there is decoherence …



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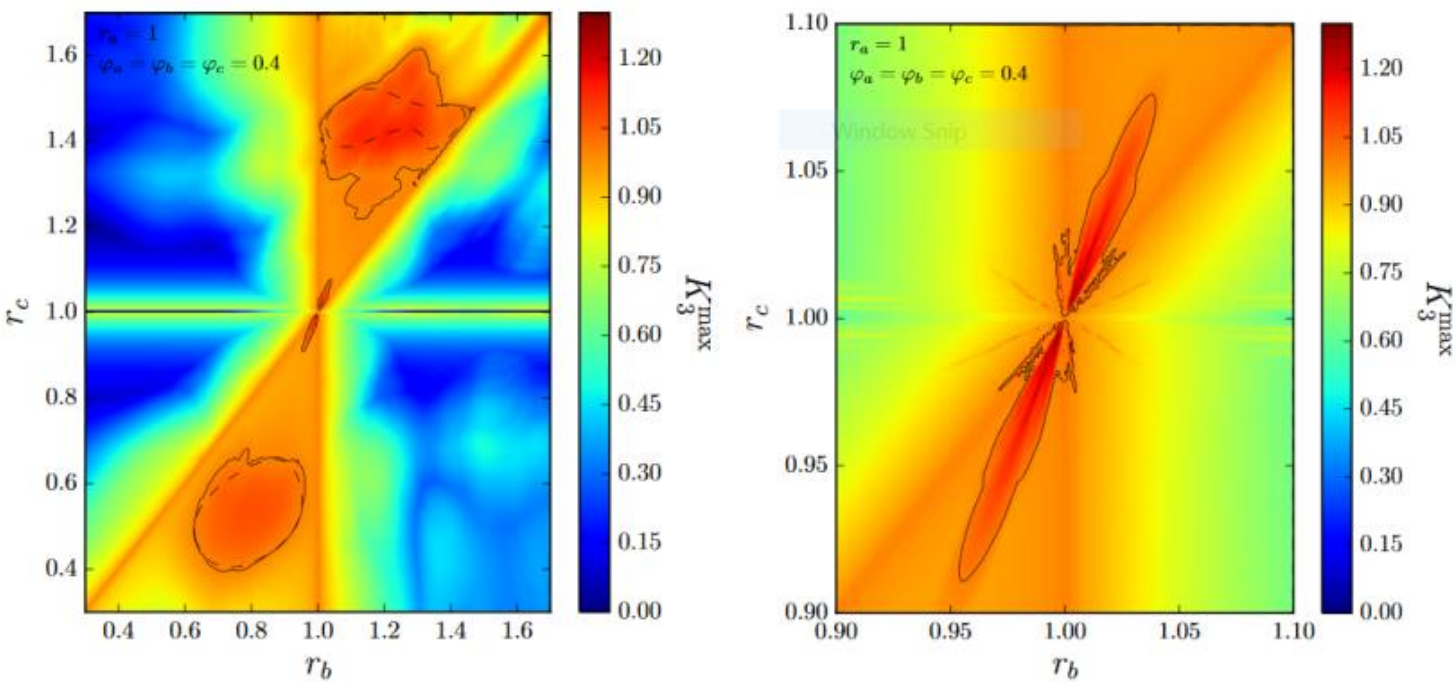
- Way out? Measuring only one component but at different time (redshift)?

Measure the z-component only but at different times= Leggett-Garg inequalities



J. Martin & V. Vennin, PRA94 (2016), 052135,
arXiv:1611.0185

The Leggett-Garg inequalities are violated for a squeezed state

Outline

- Introduction & Motivations

- Cosmological Perturbations of Quantum-Mechanical Origin

- Signature of the Quantum Origin of the Perturbations in the Sky?

- Discussion & Conclusions

Recap

- According to cosmic inflation, the CMB fluctuations are placed in a strongly two- mode squeezed state which is a discordant and entangled state

- However, a quantum mechanical signature in the sky seems to be hidden in our inability to measure more than two non-commutating observables. Pessimistic result: we will never see the quantum origin of perturbations … ??

- Ways out?? Measuring the z-component only but at different times?

- Take away message: inflation is not only a successful scenario of the early Universe, it is also a very interesting playground for foundational issues of quantum mechanics

Inflation is the only known system that uses GR and QM and where high-accuracy data are available