Symmetries and Ward-Takahashi Identities in Cosmology

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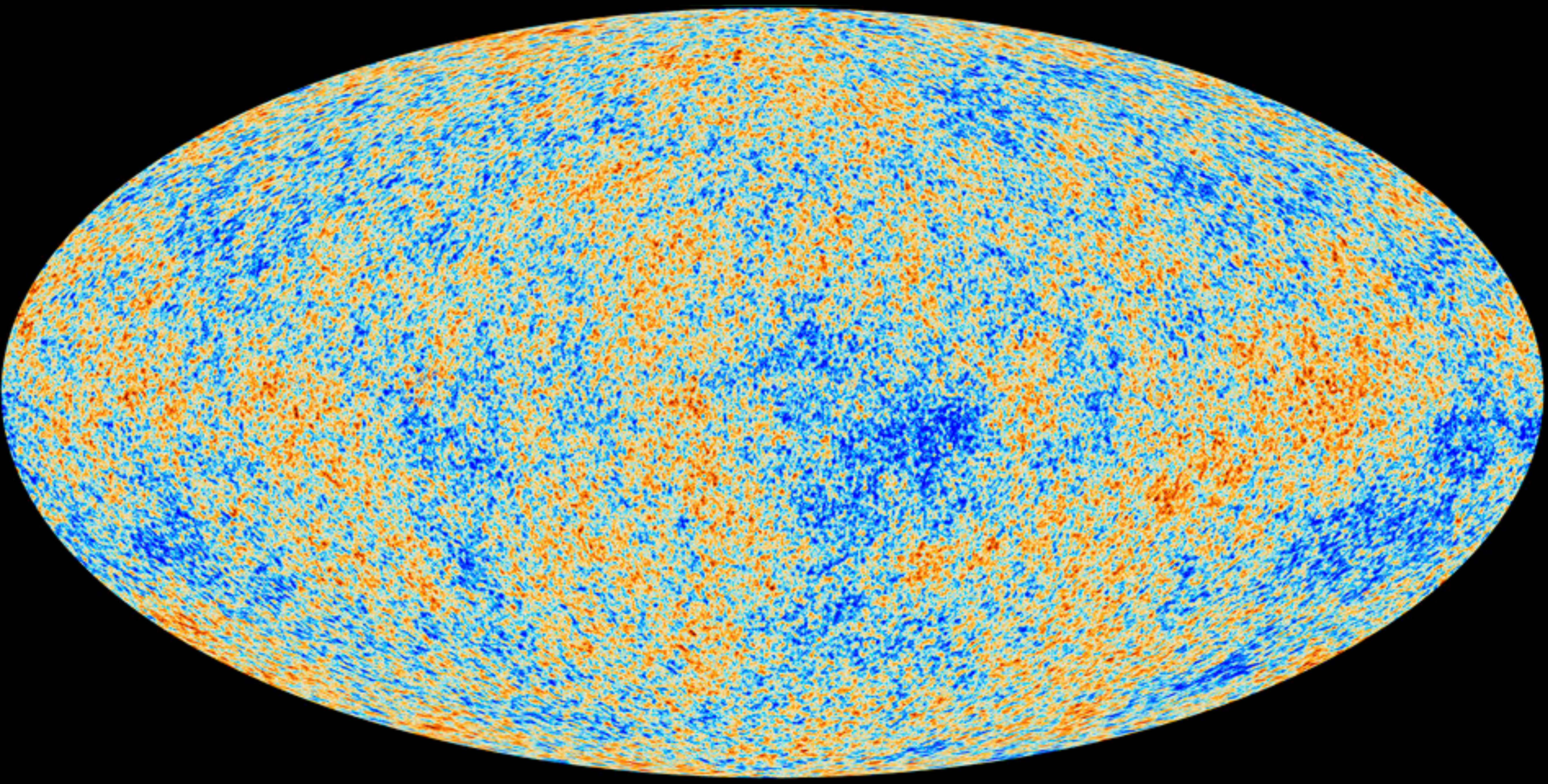
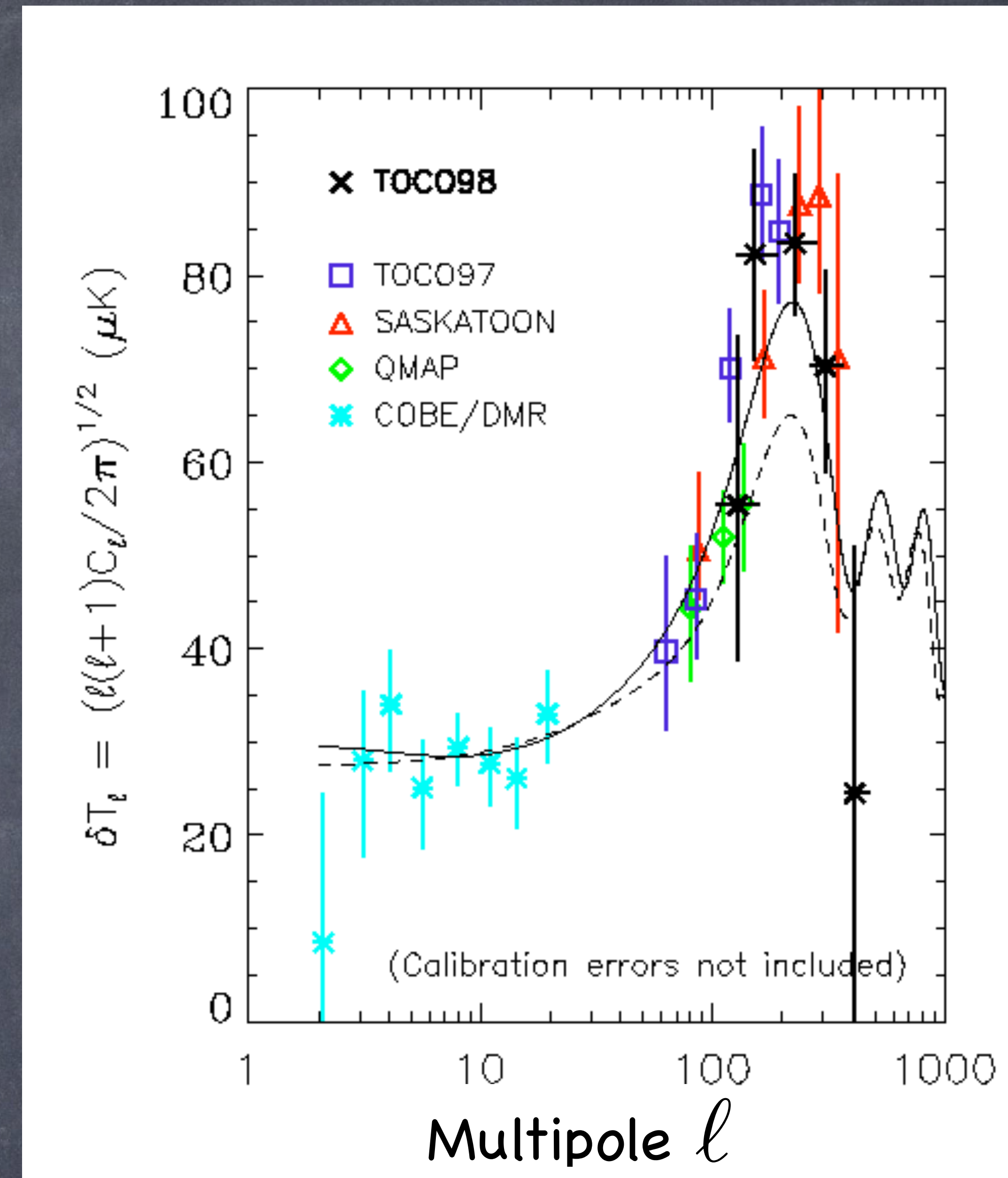


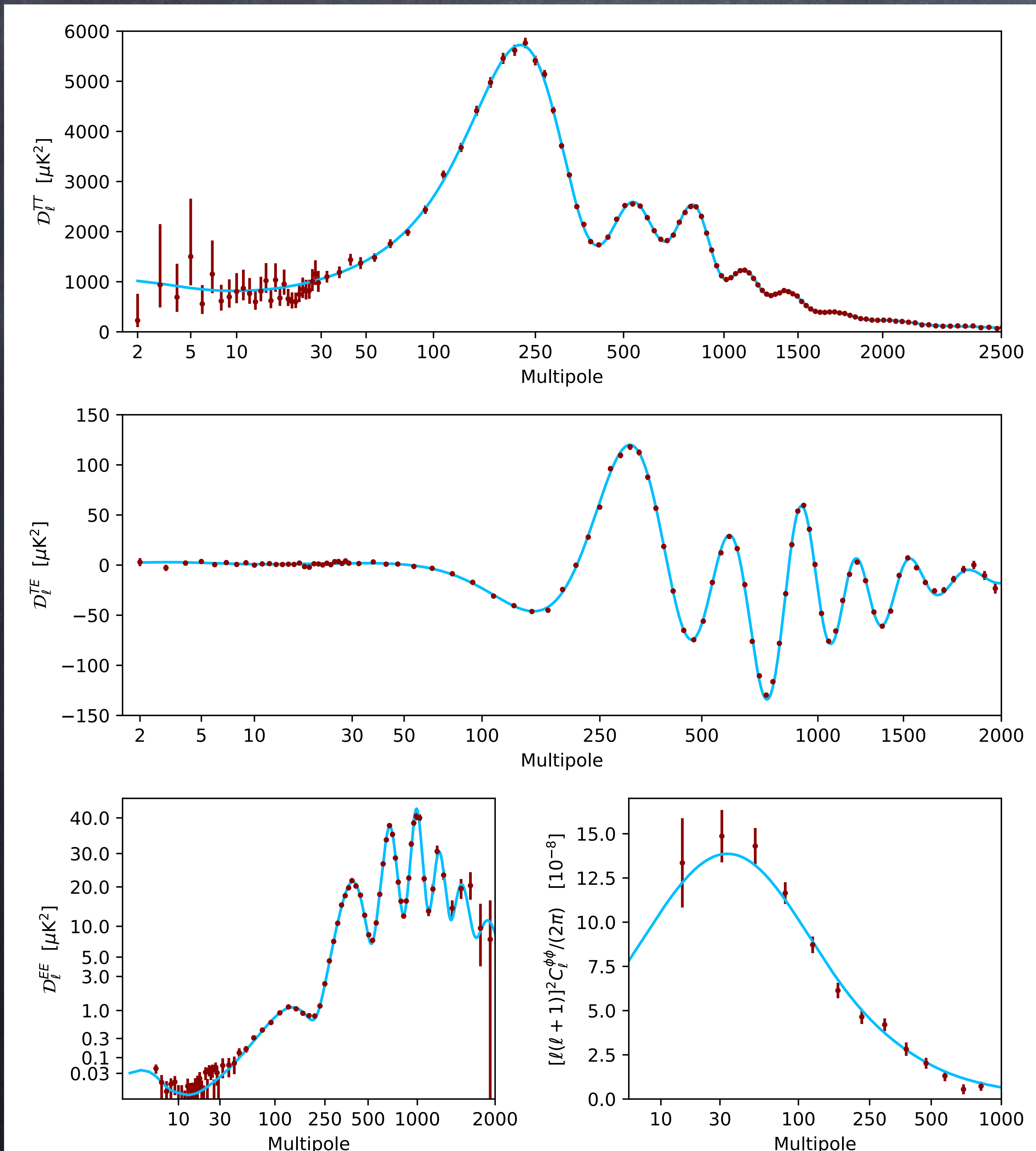
ESA / Planck Science Team

NASA / WMAP Science Team

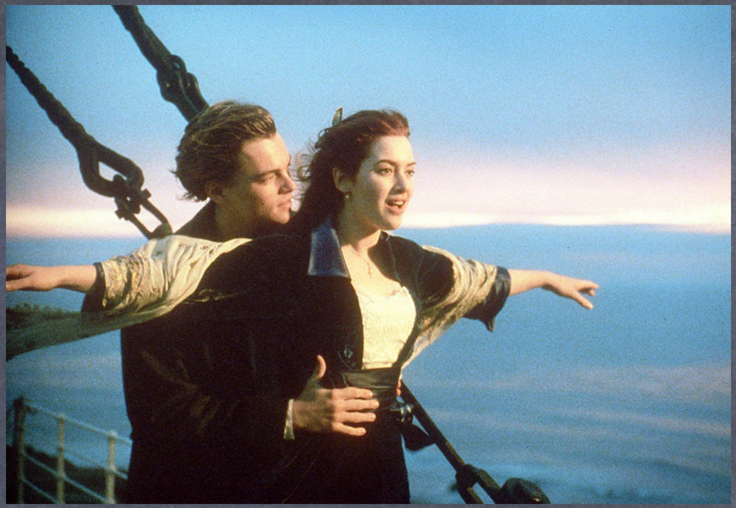
TOCO 1998



Planck 2018



- Not to reminisce about the past



- Not to be complacent about the present

- Look boldly into the future!

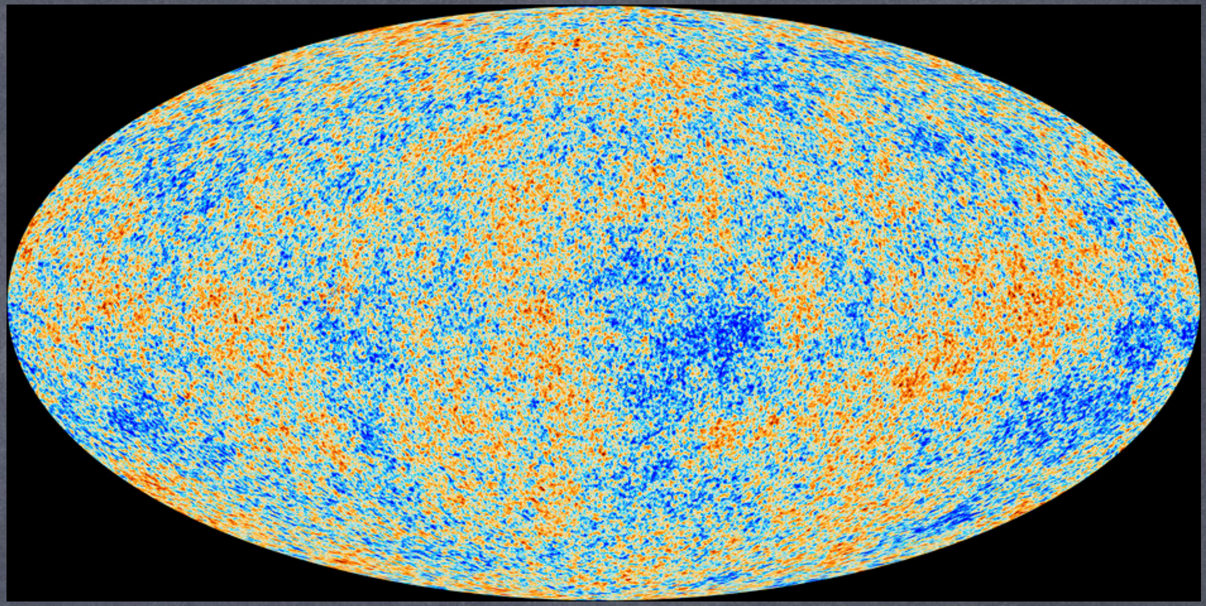
- Fundamental questions: Did inflation really occur? If so, how many fields were participating? Or is some other dynamics responsible for the observations?

- Focus on broad class of theories, distinguished by their symmetries.

Symmetries of Density Perturbations

Primordial inhomogeneities are the simplest imaginable

- Approximately scale invariant



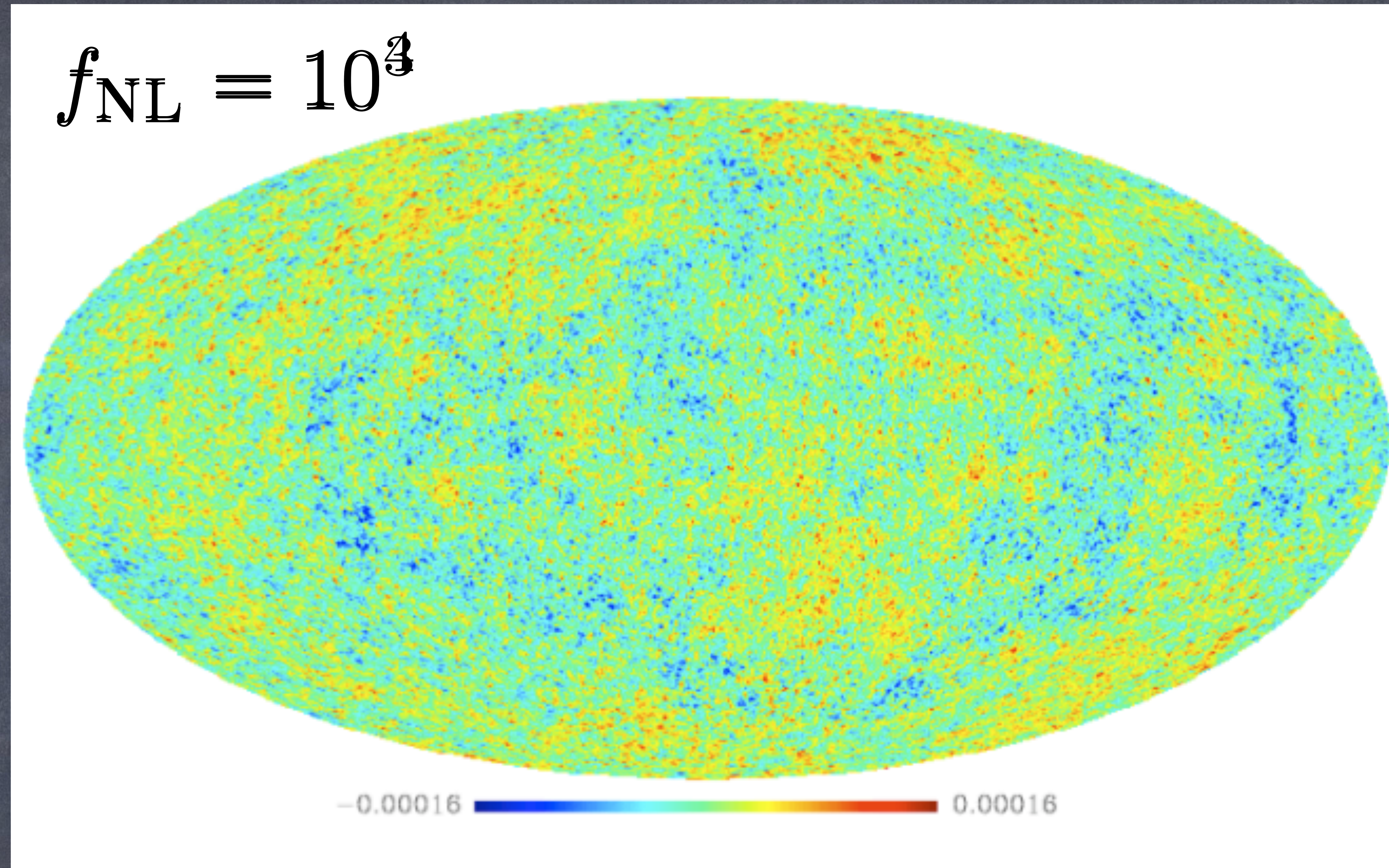
$$\left\langle \frac{\delta T}{T}(\vec{x}_1)\frac{\delta T}{T}(\vec{x}_2) \right\rangle = \int \frac{\mathrm{d}^3 k}{(2\pi)^3 k^3} e^{-i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)} P(k)$$

with $\boxed{P(k) \sim k^{2(n_s-1)}}$

$$\Longrightarrow \quad \boxed{n_s = 0.9649 \pm 0.0042} \quad \text{Planck (2018)}$$

Harrison-Zeldovich spectrum ruled out at $\gtrsim 8\sigma$

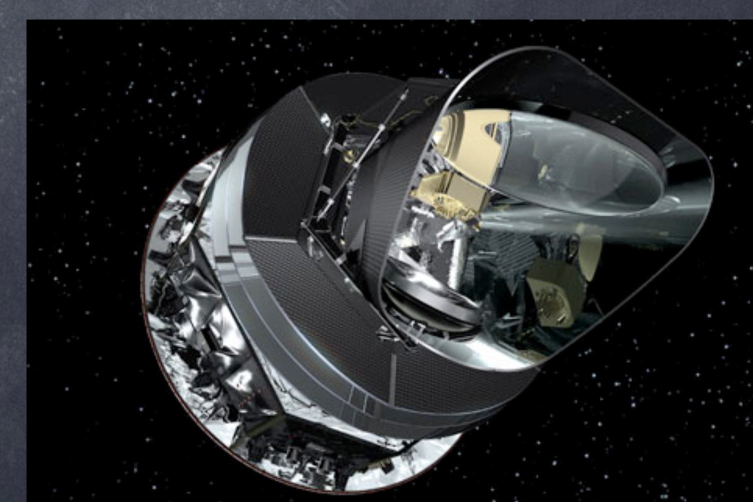
- Approximately Gaussian



$$f_{\rm NL} \equiv -\frac{\langle \frac{\delta T}{T}\frac{\delta T}{T}\frac{\delta T}{T} \rangle}{\langle \frac{\delta T}{T}\frac{\delta T}{T} \rangle^2}$$

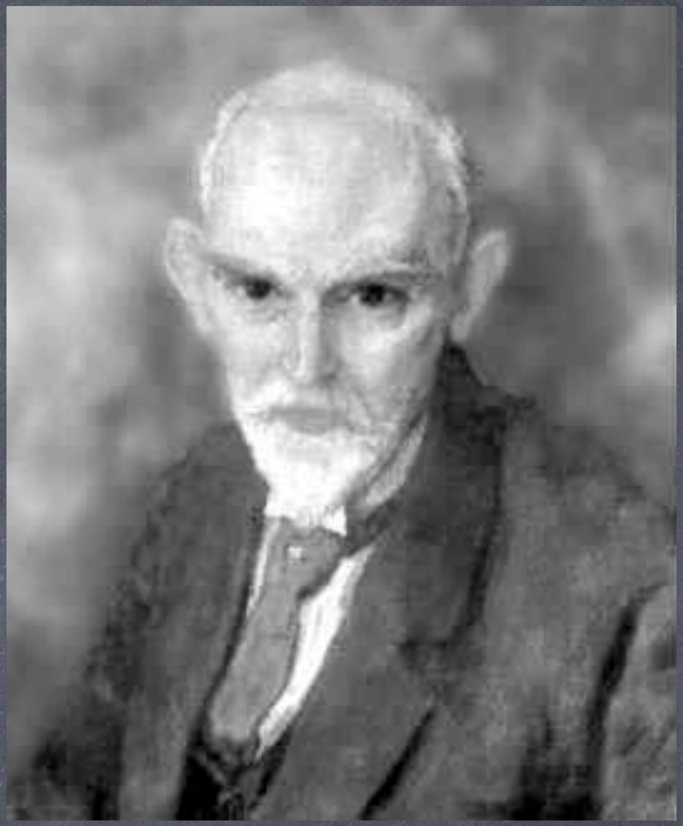
Liguori et al. (2007)

- Planck 2018: $\boxed{f_{\rm NL}^{\rm local} = -0.9 \pm 5.1}$



Gaussian at 10^{-5} level

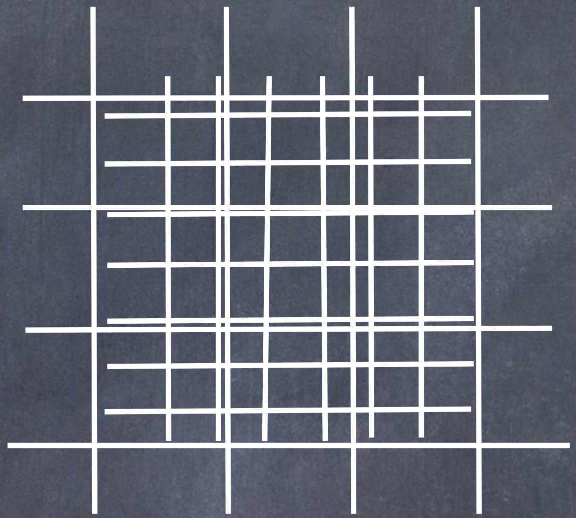
Inflation $\simeq$ de Sitter space



$$\boxed{\mathrm{d}s^2 = \frac{1}{H^2\tau^2}\left(-\mathrm{d}\tau^2 + \mathrm{d}\vec{x}^2\right)}$$

At late times, de Sitter isometries reduce to conformal transformations on R^3

- Transl'ns + Rot'ns
- Dilation:



- Special conformal transformations:

Inversion

$$\vec{x}' = \frac{\vec{x}}{x^2}$$

Translation

$$\vec{y} = \vec{x}' - \vec{b}$$

Inversion

$$\vec{y}' = \frac{\vec{y}}{y^2}$$

Their commutation relations form the $so(4,1)$ algebra

Conformal correlators

Antoniadis, Mazur & Mottola (1997); Maldacena (2011); Creminelli (2011).

We are interested in correlation functions at late times during inflation $\Longrightarrow$ invariance under so(4,1) conformal symmetries.

2-point function (except inflaton):

$$\boxed{\langle \chi(\vec{x},\tau)\chi(\vec{x}',\tau)\rangle \sim |\vec{x}-\vec{x}'|^{-2\Delta}}$$

with scaling dimension $\Delta = \frac{m_\chi^2}{3H^2}$

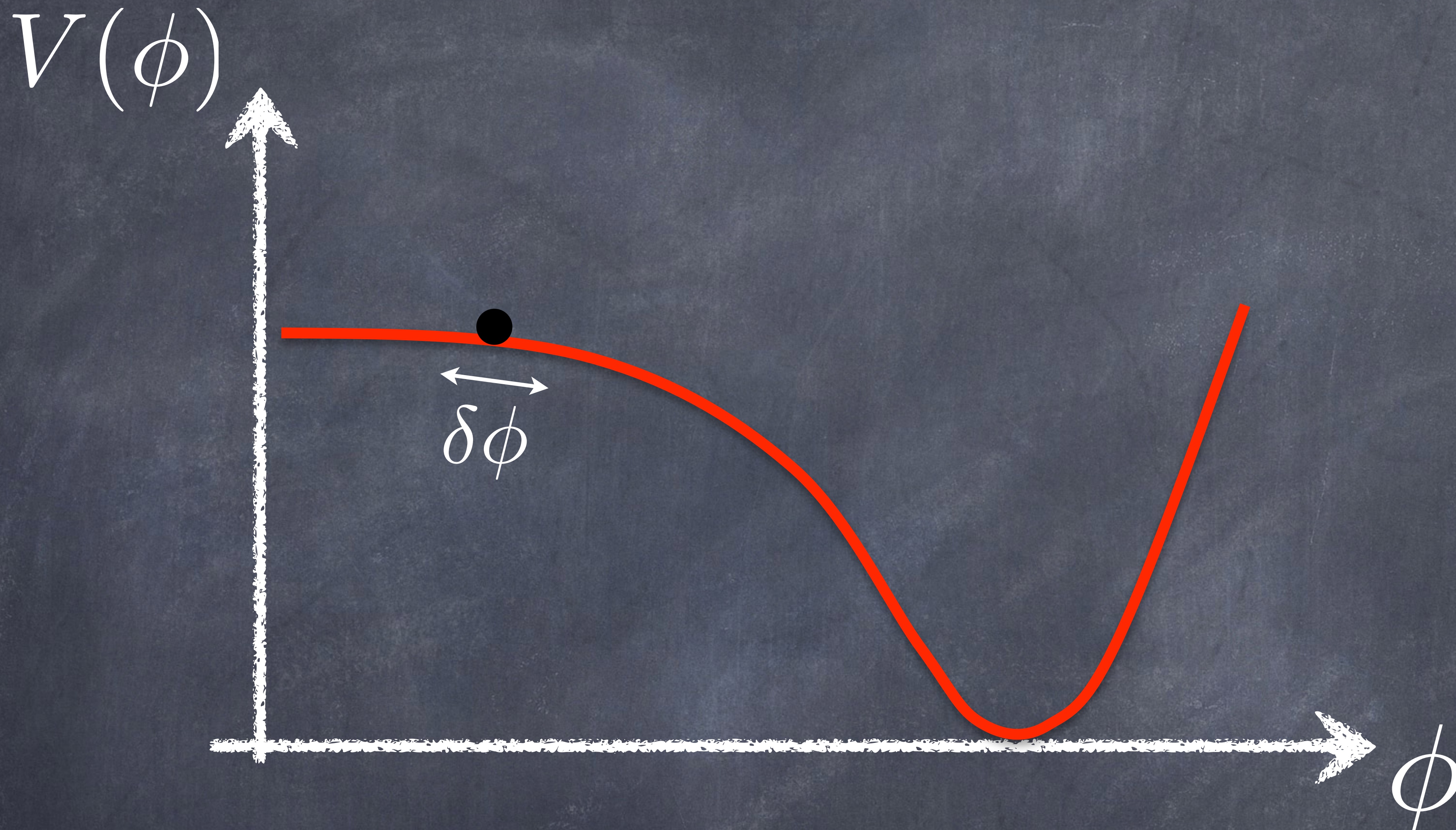
Any field with $m_\chi \ll H$ acquires nearly scale invariant spectrum (including gravitational waves)

3-point function also fixed by symmetries:

$$\boxed{\langle \chi(\vec{x}_1,t)\chi(\vec{x}_2,t)\chi(\vec{x}_3,t)\rangle = \frac{C}{|\vec{x}_1-\vec{x}_2|^{\Delta}|\vec{x}_2-\vec{x}_3|^{\Delta}|\vec{x}_1-\vec{x}_3|^{\Delta}}}$$

Single-Field Inflation

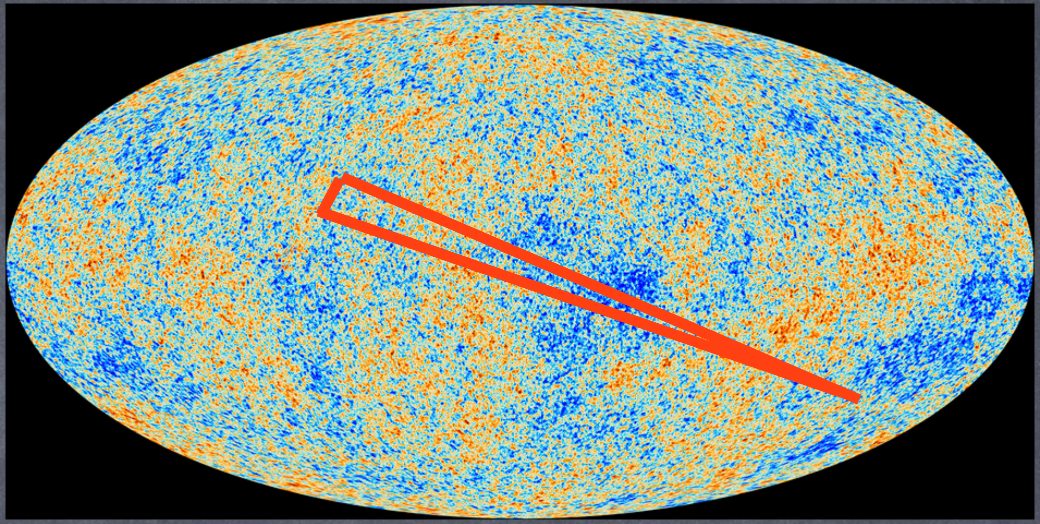
Single Field Inflation



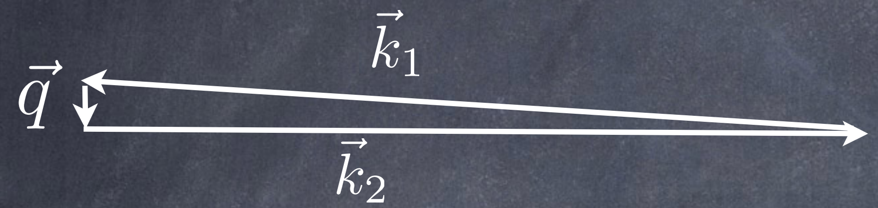
- Economical $\Longrightarrow$ Predictive
- Constrained by <u>infinitely-many</u> relations

(indep. of slow-roll, c_s , ϕ fundamental or not)

Single-field consistency relations



$$\boxed{\lim_{\vec{q}\to 0} \frac{\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle}{P_\zeta(q)} = -(n_s - 1) P_\zeta(k_1)}$$



Maldacena (2002); Creminelli & Zaldarriaga (2004); Cheung, Fitzpatrick, Kaplan & Senatore (2007).

- Holds in all inflationary models, under the assumptions:
  - single "clock"
  - Bunch-Davies vacuum
  - background is attractor $\zeta \to \text{const.}$

- Measuring (primordial) 3-point function in this limit

  $\Longrightarrow$ automatically rules out all standard single-field models

- We will see this is consequence of symmetry: Ward identity for dilation

Background wave

Maldacena (2002); Creminelli & Zaldarriaga (2004)

$$\begin{aligned}
\langle \zeta_S \zeta_S \rangle_{\zeta_L} &= \langle \zeta_S \zeta_S \rangle_0 + \zeta_L \frac{\mathrm{d}}{\mathrm{d}\zeta_L} \langle \zeta_S \zeta_S \rangle \Big|_0 \\
&= \langle \zeta_S \zeta_S \rangle_0 + \zeta_L \frac{\mathrm{d}}{\mathrm{d} \ln |\vec{x}_1 - \vec{x}_2|} \langle \zeta_S \zeta_S \rangle \Big|_0
\end{aligned}$$

Multiply by ζ_L and take expectation value:

$$\langle \zeta_L \langle \zeta_S \zeta_S \rangle_{\zeta_L} \rangle = \langle \zeta_L \zeta_L \rangle \frac{\mathrm{d}}{\mathrm{d} \ln |\vec{x}_1 - \vec{x}_2|} \langle \zeta_S \zeta_S \rangle$$

$$\Longrightarrow \quad \boxed{\lim_{\vec{q} \to 0} \frac{\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle}{P_\zeta(q)} = -(n_s - 1) P_\zeta(k_1)}$$

"Background wave" argument is intuitive and compelling, but...

- Semi-classical
- Technically challenging for other symmetries
- Dependence on initial state unclear

The upshot of field theoretic method:

- Non-perturbative
- Easily generalizes to other symmetries
- Dependence on initial state is explicit

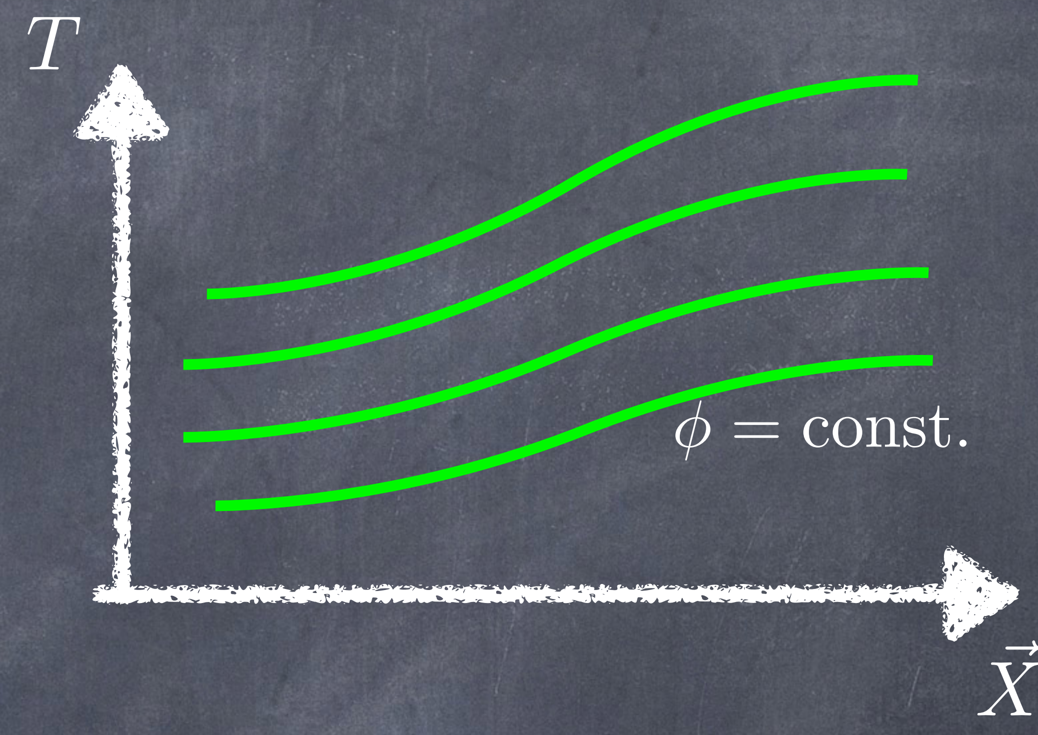
Conformal Symmetries of Scalars

Creminelli, Norena & Simonovic, 1203.4595; Hinterbichler, Hui & Khoury, 1203.6351

Uniform-density gauge:

$$\boxed{\begin{aligned} \phi &= \phi(t)\,; \\ h_{ij} &= a^2(t)e^{2\zeta(t,\vec{x})}\delta_{ij} \end{aligned}}$$

Bardeen, Steinhardt & Turner (1982);
Bond & Salopek (1990)



This completely fixes the gauge, as long as we restrict to diffs that fall off at infinity. $\Longrightarrow$ Focus on diffs that do not fall off.

e.g. Spatial dilation:

$$\vec{x} \to e^{\lambda}\vec{x}$$
$$\zeta \to \zeta + \lambda$$

leaves h_{ij} invariant.

More generally, $h_{ij} = a^2(t) e^{2\zeta(t,\vec{x})} \delta_{ij}$ is preserved by

$$\boxed{\text{Conformal transf'n: } \delta_{ij} \to e^{2\Omega(x)} \delta_{ij} \quad + \quad \text{Shift: } \zeta \to \zeta + \Omega}$$

Conformal transf'ns on R^3 form the group $SO(4,1)$:

- Rotations + Translations $\delta\zeta = 0$

Unbroken
(linearly realized)

- Dilation $x^i \to (1+\lambda) x^i$
  $\delta\zeta = \lambda$

Spontaneously broken
(non-linearly realized)

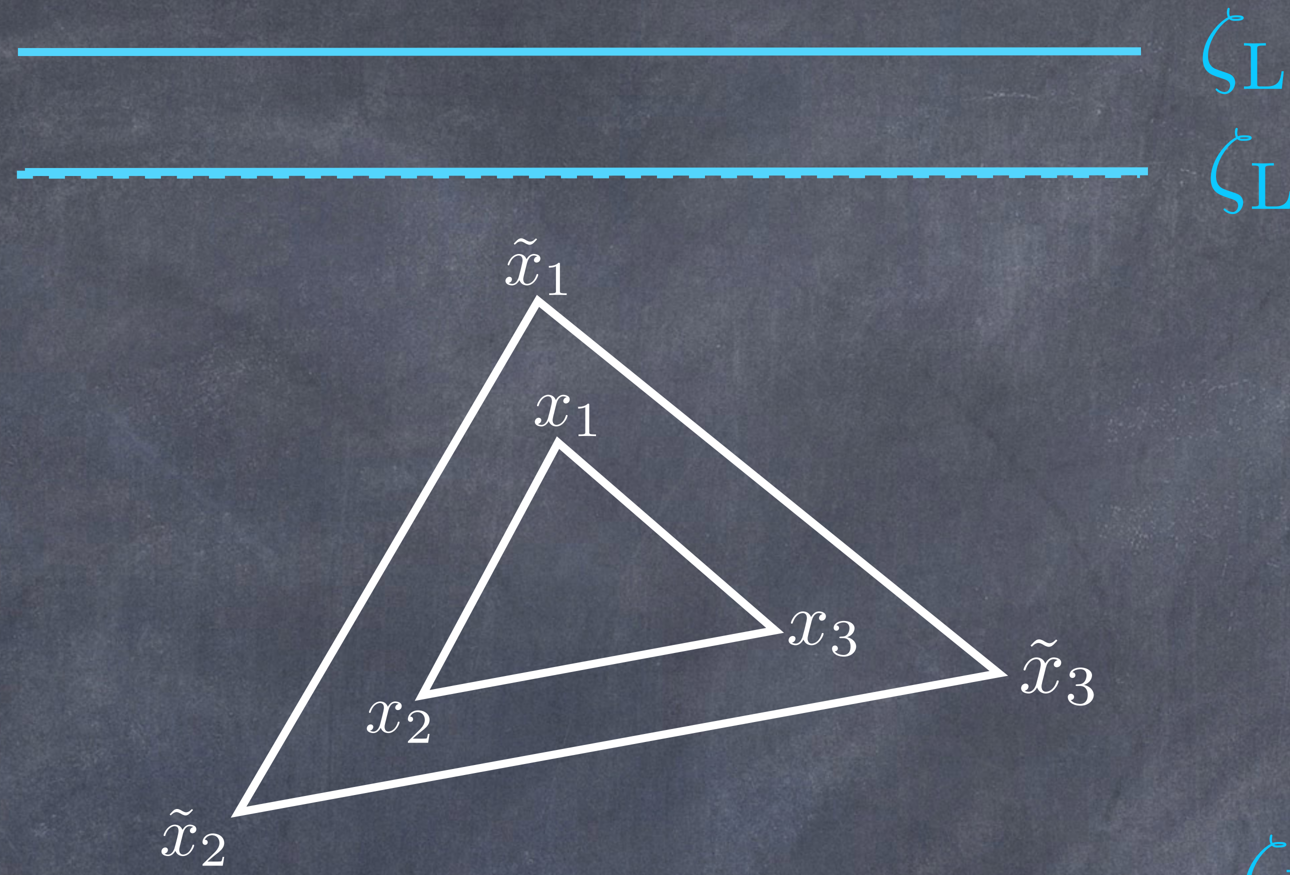
- Special conformal transformations (SCTs) $x^i \to x^i + 2\,\vec{x}\cdot\vec{b}\,x^i - b^i \vec{x}^2$
  $\delta\zeta = -2\vec{b}\cdot\vec{x}$

$$\therefore \quad \boxed{so(4,1) \to \text{rotations} + \text{translations}}$$

ζ is Goldstone boson (dilaton) for the broken symmetries

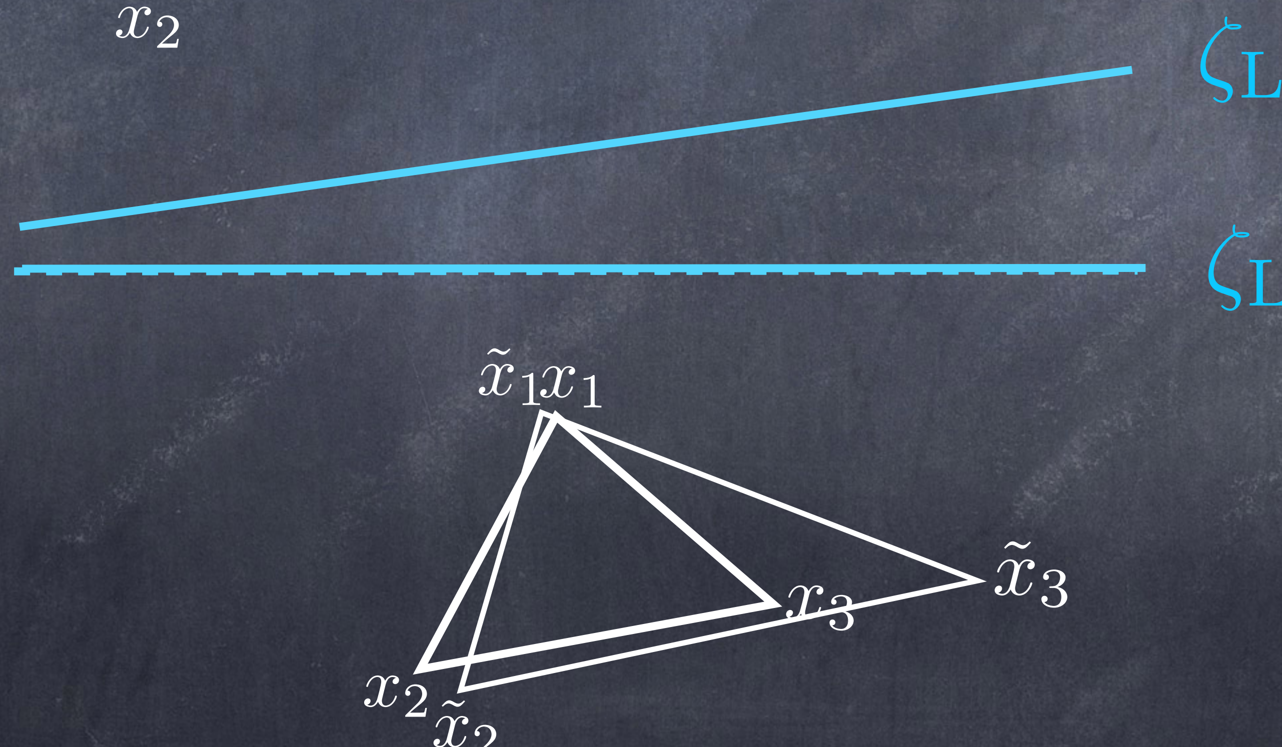
Inflation = spontaneously broken dS

- Dilation



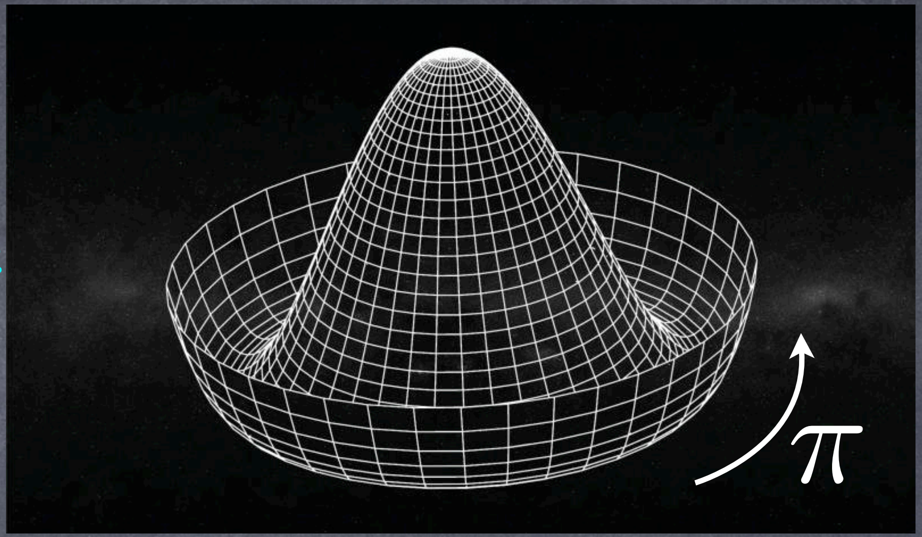
$$\delta\zeta = \lambda$$

- Special conf.



$$\delta\zeta = -2\vec{b}\cdot\vec{x}$$

Ward identities for broken symmetries



Homogeneous Goldstone π is equivalent to change of the vacuum, i.e. to a broken symmetry transformation.

Soft pion thms:

$$\boxed{\lim_{\vec{q}\to 0}\langle \pi(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle \sim \langle \delta\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle}$$

e.g. Strong interactions

Consistency relations as Ward identities

Assasi, Baumann and Green, 1204.4207
Hinterbichler, Hui and Khoury, 1304.5527
Goldberger, Hui and Nicolis, 1303.1193

Dilation:

$$\lim_{\vec{q}\to 0}\frac{1}{P_\zeta(q)}\langle \zeta(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c = -\left(3(N-1)+\sum_{a=1}^{N}\vec{k}_a\cdot\frac{\partial}{\partial\vec{k}_a}\right)\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c$$

Special conformal:

$$\lim_{\vec{q}\to 0}\frac{\partial}{\partial q^i}\left(\frac{1}{P_\zeta(q)}\langle \zeta(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c\right) = -\frac{1}{2}\sum_{a=1}^{N}\left(6\frac{\partial}{\partial k_a^i}-k_a^i\frac{\partial^2}{\partial k_a^j\partial k_a^j}+2k_a^j\frac{\partial^2}{\partial k_a^j\partial k_a^i}\right)\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c+\ldots$$

Creminelli, Norena & Simonovic, 1203.4595

General Ward Identities

Hinterbichler, Hui and Khoury, 1304.5527

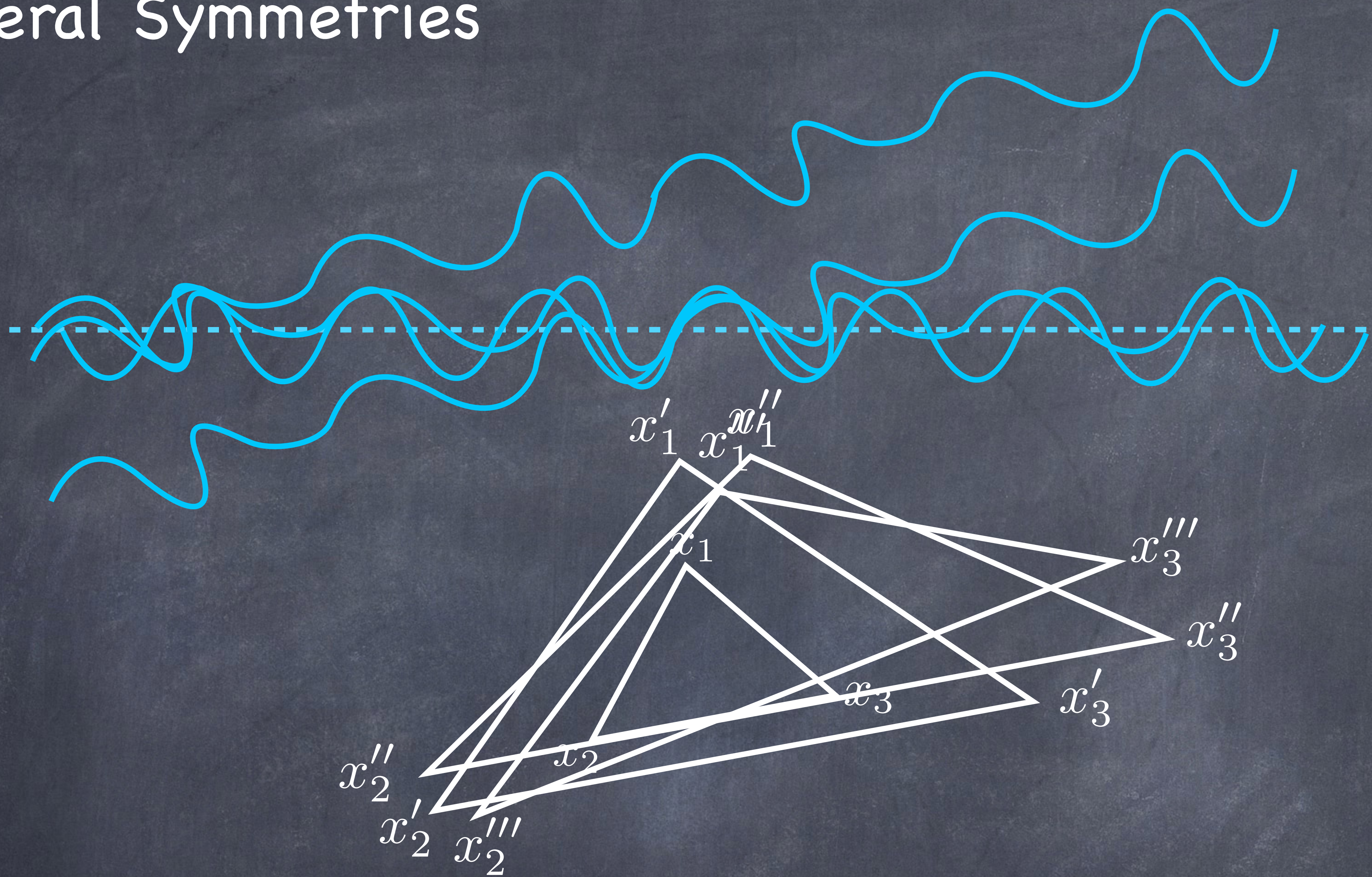
Single-field inflation constrained by infinite number of symmetries, corresponding to an infinite number of consistency relations:

$$\boxed{\lim_{\vec{q}\to 0}\frac{\partial^n}{\partial q^n}\left(\frac{\langle\zeta_{\vec{q}}\mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N}\rangle}{P_\zeta(q)}+\frac{\langle\gamma_{\vec{q}}\mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N}\rangle}{P_\gamma(q)}\right)\sim\frac{\partial^n}{\partial k^n}\langle\mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N}\rangle}$$

- q^0 and q behavior completely fixed
- q^n , $n \geq 2$, behavior partially fixed
- These are physical statements (i.e., can be violated)
- Hold on any spatially-flat FRW background (no slow-roll)

$\Longrightarrow$ Complete checklist for testing single-field inflation

General Symmetries



$$h_{ij} = \bar{\delta}_{ij} + \partial_k \bar{h}_{ij} x^k + \frac{1}{2}\partial_k \partial_\ell \bar{h}_{ij} x^k x^\ell + \dots$$

6 diffs at $\mathcal{O}(x^2)$

General Ward Identities

Hinterbichler, Hui and Khoury, 1304.5527

$$\lim_{\vec{q}\to 0} M_{i\ell_0\ldots\ell_n}(\hat{q})\frac{\partial^n}{\partial q_{\ell_1}\cdots\partial q_{\ell_n}}\left(\frac{1}{P_\gamma(q)}\langle\gamma^{i\ell_0}(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c' + \frac{\delta^{i\ell_0}}{3P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c'\right)$$

$$= -M_{i\ell_0\ldots\ell_n}(\hat{q})\Bigg\{\sum_{a=1}^{N}\left(\delta^{i\ell_0}\frac{\partial^n}{\partial k^a_{\ell_1}\cdots\partial k^a_{\ell_n}} - \frac{\delta_{n0}}{N}\delta^{i\ell_0} + \frac{k^i_a}{n+1}\frac{\partial^{n+1}}{\partial k^a_{\ell_0}\cdots\partial k^a_{\ell_n}}\right)\langle\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c'$$

$$-\sum_{a=1}^{M}\Upsilon^{i\ell_0 i_a j_a}(\hat{k}_a)\frac{\partial^n}{\partial k^a_{\ell_1}\cdots\partial k^a_{\ell_n}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_{a-1},\vec{k}_{a+1},\ldots\vec{k}_M)\gamma_{i_a j_a}(\vec{k}_a)\mathcal{O}^\gamma(\vec{k}_{M+1},\ldots,\vec{k}_N)\rangle_c'$$

$$-\sum_{b=M+1}^{N}\Gamma^{i\ell_0}{}_{i_b j_b}{}^{k_b\ell_b}(\hat{k}_b)\frac{\partial^n}{\partial k^b_{\ell_1}\cdots\partial k^b_{\ell_n}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_M)\mathcal{O}^\gamma_{i_{M+1}j_{M+1},\ldots,k_b\ell_b,\ldots i_N j_N}(\vec{k}_{M+1},\ldots,\vec{k}_N)\rangle_c'\Bigg\} + \ldots$$

where

$$\Upsilon_{abcd}(\hat{k}) \equiv \frac{1}{4}\delta_{ab}\hat{k}_c\hat{k}_d - \frac{1}{8}\delta_{ac}\hat{k}_b\hat{k}_d - \frac{1}{8}\delta_{ad}\hat{k}_b\hat{k}_c\,;$$

$$\Gamma_{abijk\ell}(\hat{k}) \equiv -\frac{1}{2}\left(\delta_{ij}+\hat{k}_i\hat{k}_j\right)\left(\delta_{ab}\hat{k}_k\hat{k}_\ell - \frac{1}{2}\delta_{ak}\hat{k}_\ell\hat{k}_b - \frac{1}{2}\delta_{a\ell}\hat{k}_k\hat{k}_b\right) + \delta_{b(i}\delta_{j)(k}\delta_{\ell)a} - \delta_{a(i}\delta_{j)(k}\delta_{\ell)b}$$

$$-\delta_{b(i}\hat{k}_{j)}\delta_{a(k}\hat{k}_{\ell)} + \delta_{a(i}\hat{k}_{j)}\delta_{b(k}\hat{k}_{\ell)} - \delta_{a(k}\delta_{\ell)(i}\hat{k}_{j)}\hat{k}_b - \delta_{b(k}\delta_{\ell)(i}\hat{k}_{j)}\hat{k}_a + 2\delta_{ab}\hat{k}_{(i}\delta_{j)(k}\hat{k}_{\ell)}$$

Multiple Soft Limits

Joyce, Khoury & Simonovic, 1409.6318

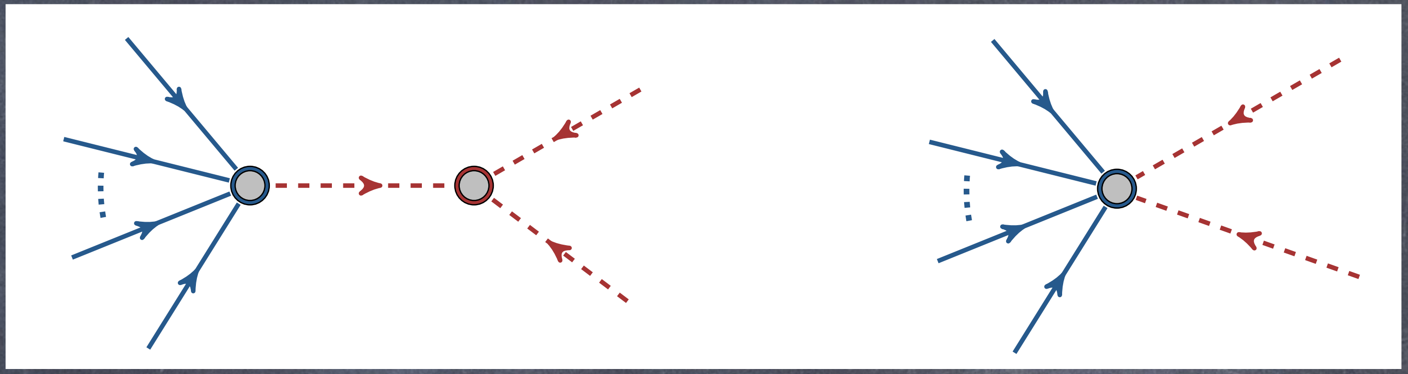
Another probe of higher-q dependence.

e.g. QCD: Double-soft limit probes non-Abelian algebra

Weinberg (1966);
Arkani-Hamed et al. (2008)

$$\lim_{q_a,q_b\to 0}\langle \pi^a(q_a)\pi^b(q_b)\pi^{i_1}(k_1)\cdots\pi^{i_n}(k_n)\rangle = \frac{1}{2}\sum_j \frac{(q_a-q_b)\cdot k_j}{(q_a+q_b)\cdot k_j}\epsilon^{abc}\langle\pi^{i_1}(k_1)\cdots T_c\pi^{i_j}(k_j)\cdots\pi^{i_n}(k_n)\rangle$$

Double-soft result:



$$\lim_{\vec q_1,\vec q_2\to 0}\frac{\langle\zeta_{\vec q_1}\zeta_{\vec q_2}\zeta_{\vec k_1}\cdots\zeta_{\vec k_N}\rangle'}{P_\zeta(q_1)P_\zeta(q_2)} = \frac{\langle\zeta_{\vec q_1}\zeta_{\vec q_2}\zeta_{-\vec q}\rangle'}{P_\zeta(q_1)P_\zeta(q_2)}\left(\delta_{\mathcal{D}}+\frac{1}{2}\vec q_1\cdot\delta_{\vec{\mathcal{K}}}\right)\langle\zeta_{\vec k_1}\cdots\zeta_{\vec k_N}\rangle'$$
$$+\left(\delta_{\mathcal{D}}^2+\frac{1}{2}\vec q_1\cdot\delta_{\vec{\mathcal{K}}}\delta_{\mathcal{D}}+\frac{1}{4}q_1^i q_2^j\delta_{\mathcal{K}^i}\delta_{\mathcal{K}^j}\right)\langle\zeta_{\vec k_1}\cdots\zeta_{\vec k_N}\rangle'$$
$$+\lim_{\vec q\to 0}\left[\frac{1}{2}\left(\vec q^{\,2}\nabla_q^2-2q_iq_j\nabla_q^i\nabla_q^j\right)\langle\zeta_{\vec q}\zeta_{\vec k_1}\cdots\zeta_{\vec k_N}\rangle'+\frac{\langle\zeta_{\vec q_1}\zeta_{\vec q_2}\zeta_{-\vec q}\rangle'}{P_\zeta(q_1)P_\zeta(q_2)}q_iq_j\nabla_q^i\nabla_q^j\frac{\langle\zeta_{\vec q}\zeta_{\vec k_1}\cdots\zeta_{\vec k_N}\rangle'}{P_\zeta(q)}\right]$$

$$\delta_{\mathcal{D}}\equiv\text{dilation}\qquad \delta_{\mathcal{K}}\equiv\text{SCT}$$

Master Consistency Relation

Berezhiani and Khoury, 1309.4461

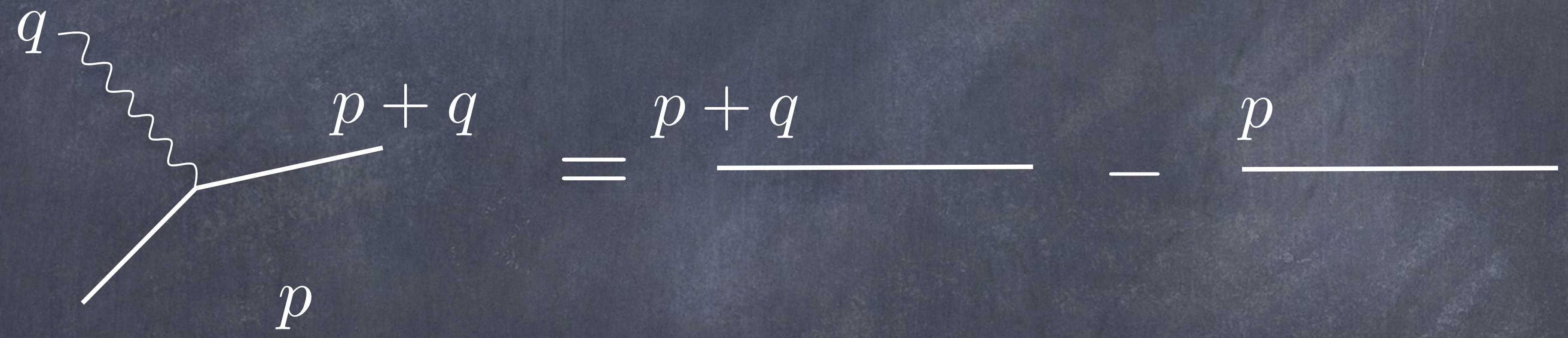
Since symmetries of interest are subset spatial diffeomorphism, consistency relations must be consequence of gauge symmetry (Slavnov-Taylor identity).

Master consistency relation

Berezhiani and Khoury, 1309.4461

Gauge invariance in EM implies Ward-Takahashi identity:

$$q^\mu \Gamma_\mu^{A\psi\psi}(q,p,p+q) = e\left(\Gamma^\psi(p+q) - \Gamma^\psi(p)\right) .$$



Similarly, spatial diffeomorphisms should give rise to a Slavnov-Taylor identity.

E&M warm-up

Berezhiani and Khoury, 1309.4461

$$\boxed{Z[J,\eta] = \int \mathcal{D}A_\mu \mathcal{D}\psi e^{iS_{\rm QED} - \frac{i}{2\xi}\int(\partial^\mu A_\mu)^2 + i\int(J^\mu A_\mu + \eta\psi)}}$$

Field redefinition: $A_\mu \to A_\mu + \partial_\mu \Lambda\,; \qquad \psi \to \psi - i\Lambda\psi$

Z must be invariant: $\left[\frac{i\Box}{\xi}\partial^\mu \frac{\delta}{\delta J^\mu} - \partial^\mu J_\mu + \eta\frac{\delta}{\delta\eta}\right] Z[J,\eta] = 0$

$\delta Z = 0$

Legendre transform ($J^\mu = -\frac{\delta\Gamma}{\delta A_\mu}$ etc.) :

$$\boxed{-\frac{\Box}{\xi}\partial^\mu A_\mu + \partial_\mu \frac{\delta\Gamma}{\delta A_\mu} + i\psi\frac{\delta\Gamma}{\delta\psi} = 0}$$

Can differentiate a number of times, e.g. $\Gamma_\mu^{A\bar\psi\psi} = \frac{\delta^3\Gamma}{\delta A^\mu \delta^2\psi}$,

$$\boxed{q^\mu \Gamma_\mu^{A\bar\psi\psi}(q,p,-p-q) = \Gamma^\psi(p+q) - \Gamma^\psi(p)}$$

(Ward-Takahashi)

$$\boxed{q^\mu \Gamma_\mu^{A\bar{\psi}\psi}(q,p,-p-q) = \Gamma^\psi(p+q) - \Gamma^\psi(p)}$$

General solution is power series:

$$\Gamma_\mu^{A\bar{\psi}\psi}(q,p,-p-q) = \sum_{n=0}^{\infty} q^{\alpha_1} \ldots q^{\alpha_n} \frac{\partial^n \Gamma^\psi(p)}{\partial p^\mu \partial p^{\alpha_1} \ldots \partial p^{\alpha_n}} + C_\mu$$

physical piece

$q^\mu C_\mu = 0$

If C_μ is analytic in q_μ (locality), then it drops out at $\mathcal{O}(q^0)$:

$$\boxed{\Gamma_\mu^{A\bar{\psi}\psi}(0,p,-p) = \frac{\partial \Gamma_\psi(p)}{\partial p^\mu}}$$

(QED analogue of Maldacena)

It can contribute at $\mathcal{O}(q^1)$, e.g. $C^\mu = q_\nu[\gamma^\nu, \gamma^\mu]$:

$$F_{\mu\nu}\bar{\psi}\gamma^\mu\gamma^\nu\psi$$

$\therefore$ C_μ encodes physical info about non-minimal couplings

Cosmological Slavnov-Taylor Identity

Berezhiani & Khoury, 1309.4461

Following similar steps,

$$\boxed{2\partial_j\left(\frac{1}{6}\delta_{ij}\frac{\delta\Gamma}{\delta\zeta}+\frac{\delta\Gamma}{\delta\gamma_{ij}}\right)=\partial_i\zeta\frac{\delta\Gamma}{\delta\zeta}+\mathrm{G.F.}}$$

Can vary this a number of times wrt the fields,
e.g. vary twice wrt ζ,

$$\boxed{q^j\left(\frac{1}{3}\delta_{ij}\Gamma^{\zeta\zeta\zeta}+2\Gamma_{ij}^{\gamma\zeta\zeta}\right)=q_i\Gamma_\zeta(p)-p_i\Big(\Gamma_\zeta(|\vec{q}+\vec{p}|)-\Gamma_\zeta(p)\Big)}$$

(Exact in q)

Analogue of W-T identity in E&M

General schematic solution:

$$\frac{1}{3}\delta_{ij}\Gamma^{\zeta\zeta\zeta}+2\Gamma_{ij}^{\gamma\zeta\zeta}=\sum_{n=0}^{\infty}q^n\frac{\partial^n}{\partial p^n}P_\zeta(p)+A_{ij}(\vec{p},\vec{q})$$

physical piece $q^jA_{ij}(\vec{p},\vec{q})=0$

Whether or not consistency relation holds hinges on model-dependent piece A_{ij} . Most general form:

$$\boxed{A_{ij}(\vec{p},\vec{q}) = \epsilon_{ikm}\epsilon_{j\ell n}q^k q^\ell \Big(a(\vec{p},\vec{q})\delta^{mn} + b(\vec{p},\vec{q})p^m p^n \Big)}$$

arbitrary scalar functions

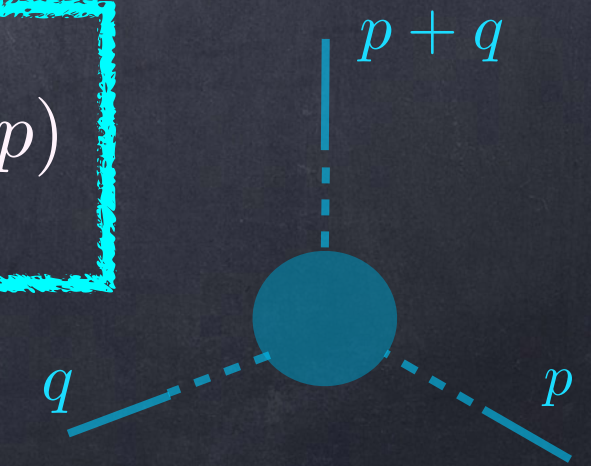
Key assumption: Suppose a and b are analytic in q, such that

$$\boxed{A_{ij} = \mathcal{O}(q^2)}$$

(Locality condition)

Then Maldacena's relation holds. Moreover, at each order in q can project out A_{ij} :

$$\boxed{\lim_{\vec{q}\to 0}\frac{\partial^n}{\partial q^n}\left(\frac{\langle\zeta_{\vec{q}}\zeta_{\vec{p}}\zeta_{-\vec{q}-\vec{p}}\rangle}{P_\zeta(q)} + \frac{\langle\gamma_{\vec{q}}\zeta_{\vec{p}}\zeta_{-\vec{q}-\vec{p}}\rangle}{P_\gamma(q)}\right) \sim -\frac{\partial^n}{\partial p^n}P_\zeta(p)}$$



General consistency relations

The Conformal Scenario

An Old Idea...

Could scale invariance observed in CMB/LSS have originated from (space-time) conformal invariance in early universe?

Conformal Scenario

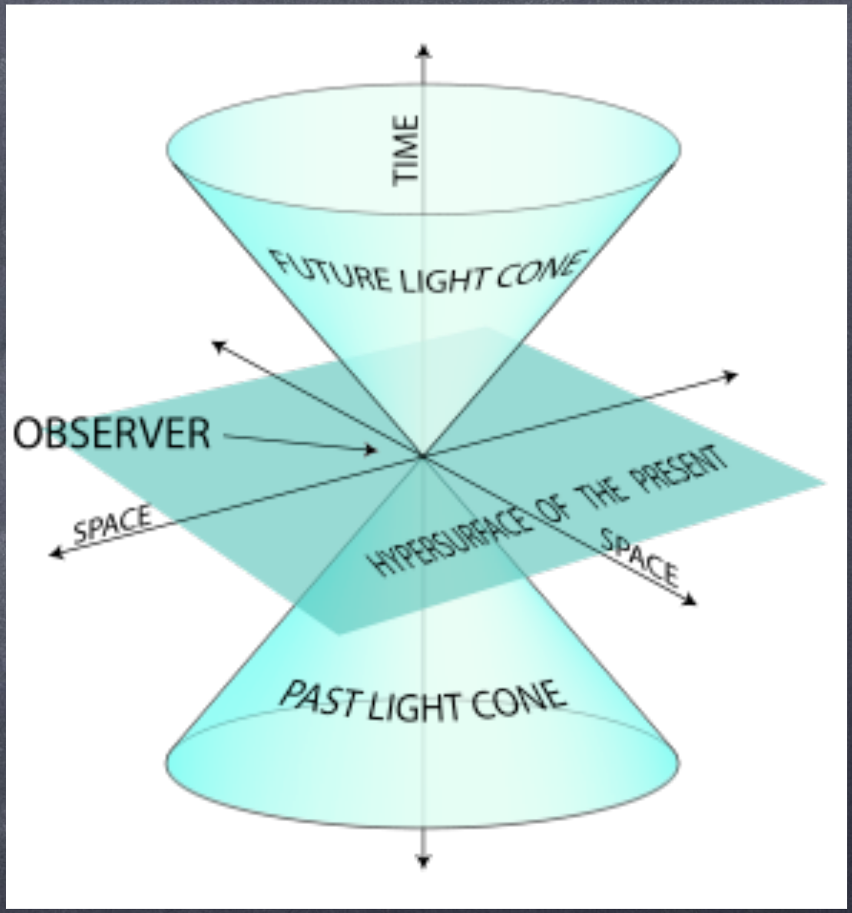
Rubakov (2009); Creminelli, Nicolis & Trincherini (2010);
Hinterbichler & Khoury (2011) ; Hinterbichler, Khoury & Joyce (2012)

- Non-inflationary scenario, takes place before the big bang

- Space-time is nearly static, i.e. $\approx$ flat, Minkowski space

- Relies on approximate conformal invariance in flat space:

$$so(4,2)$$



- Conformal invariance is spontaneously broken:

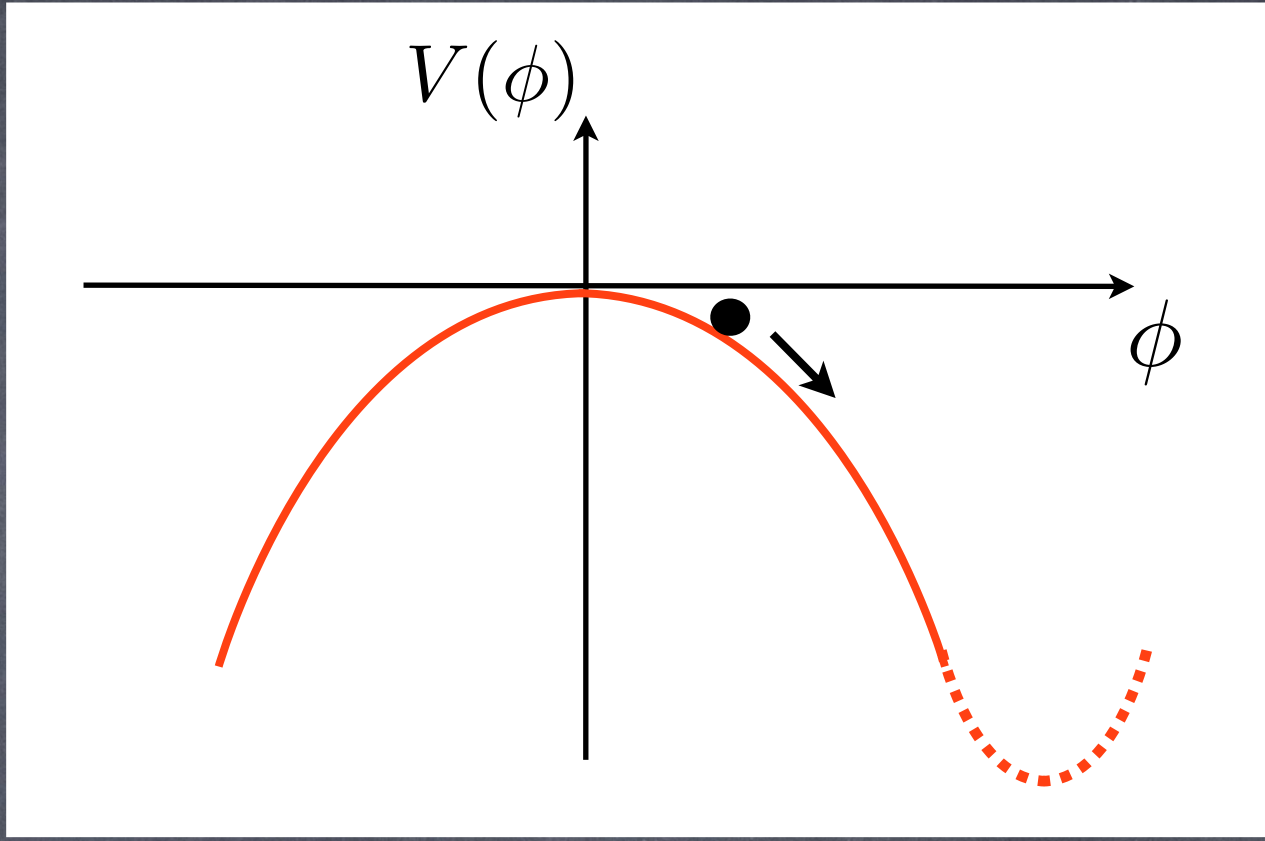
$$so(4,2) \to so(4,1)$$

same as de Sitter algebra
(but not isometries of space-time)

- Essential physics fixed by symmetry breaking pattern, irrespective of microphysics

Simplest Example

Rubakov (2009); Craps, Hertog & Turok (2007); Hinterbichler & Khoury, 1106.1428



$$\boxed{V(\phi) = -\frac{\lambda}{4}\phi^4}$$

$\lambda > 0 \implies$ asymptotically free

As time goes on, ϕ rolls off:

$$E = \frac{1}{2}\dot{\phi}^2 - \frac{\lambda}{4}\phi^4$$

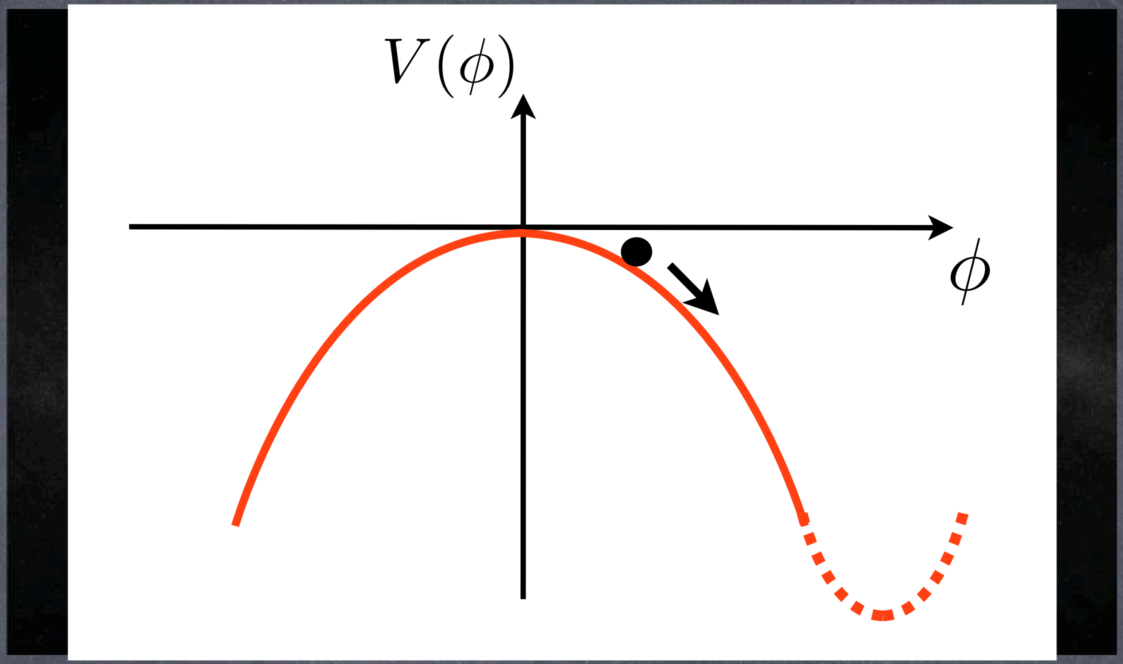
Particular solution is $E = 0$:

$$\boxed{\phi(t) = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}} \qquad -\infty < t < 0$$

This is an <u>attractor</u>: Growing mode = time shift.

$$\boxed{\phi(t) = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}}$$

Preserves dilation



15 original symmetries $\longrightarrow$ 10 unbroken symmetries

$$so(4,2) \qquad\qquad so(4,1)$$

(de Sitter symmetries)

Angular field acquires scale invariant spectrum:

$$\mathcal{L}_\theta = -\frac{1}{2}\phi^2(\partial\theta)^2 \sim \frac{1}{t^2}(\partial\theta)^2 + \ldots$$

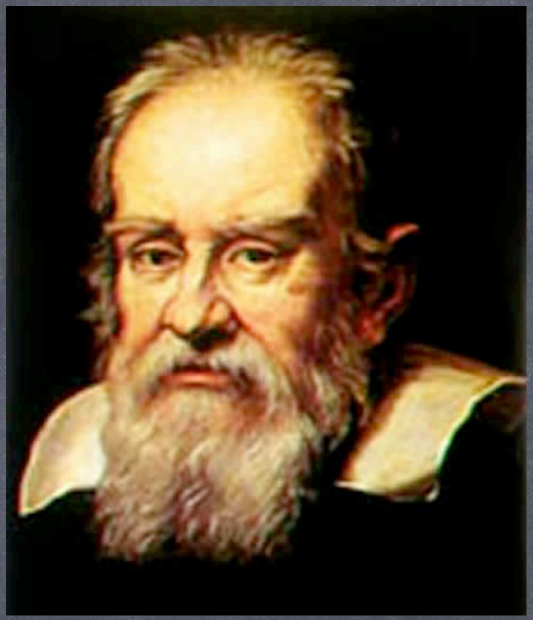
Exactly like massless field in de Sitter!

Other Realizations

- Galilean Genesis

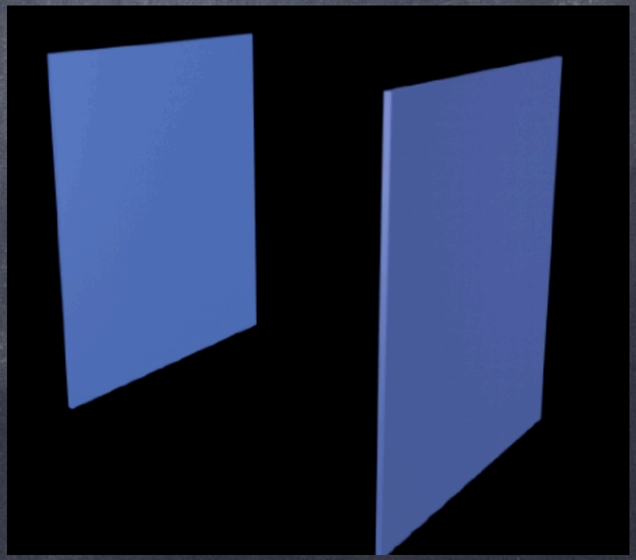
  Creminelli, Nicolis & Trincherini (2010);
  Creminelli, Hinterbichler, Khoury, Nicolis & Trincherini (2012)

  Universe is slowly expanding from asymptotically static past.



- Brane-world (DBI) realizations

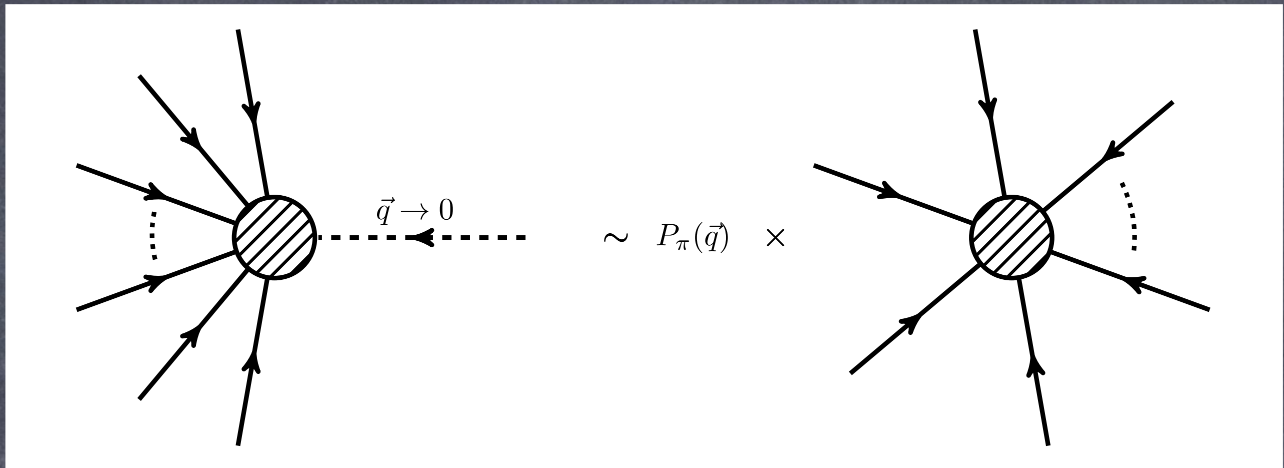
  Hinterbichler & Khoury (2011);
  Hinterbichler, Joyce, Khoury & Miller (2012)



Model-independent predictions

Creminelli, Joyce, Khoury & Simonovic, 1212.3329

- Have additional consistency relations (Ward identities) from the 5 broken symmetries $so(4,2) \to so(4,1)$



$$\boxed{\lim_{\vec{q}\to 0} \frac{1}{P_\pi(q)} \langle \pi(\vec{q}) \mathcal{O}(\vec{k}_a)\rangle = -\left(1 + \frac{1}{N}\sum_a \vec{q}\cdot\frac{\partial}{\partial \vec{k}_a} + \frac{q^2}{6N}\sum_a \frac{\partial^2}{\partial k_a^2}\right) t\frac{\partial}{\partial t}\langle \mathcal{O}(\vec{k}_a)\rangle}$$

- Goldstone spectrum is very red (though contribution to ζ is blue)

$$\boxed{q^3 P_\pi(q) = \frac{A_\pi^2}{q^2 t^2}}$$

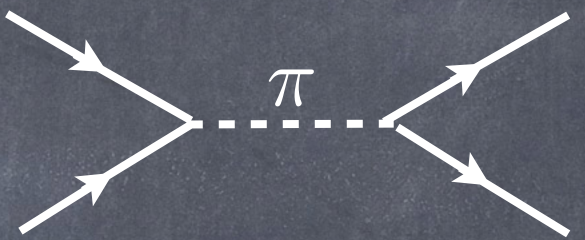
Observational Signatures

Creminelli, Joyce, Khoury & Simonovic, 1212.3329

- Soft internal lines: Libanov, Mironov & Rubakov (2011)

$$\langle \chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}\chi_{\vec{k}_4}\rangle_{q\to 0} = \frac{1}{P_\pi(q)}\langle \pi_{-\vec{q}}\chi_{\vec{k}_1}\chi_{\vec{k}_2}\rangle_{q\to 0}\langle \pi_{\vec{q}}\chi_{\vec{k}_3}\chi_{\vec{k}_4}\rangle_{q\to 0}$$

$$\sim \frac{1}{q}\left(3(\hat{k}_1\cdot\hat{q})^2-1\right)\left(3(\hat{k}_3\cdot\hat{q})^2-1\right) .$$

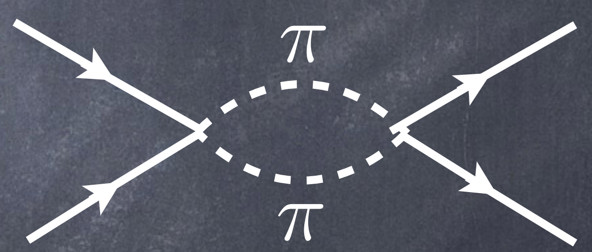


Diverges as $q \to 0$

(Vanishes as q^2 in inflation)

- Loop contribution:

$$\boxed{\tau_{\rm NL} \sim \log\frac{q}{\Lambda}}$$



- Anisotropy: Realization-dependent from super-Hubble π mode

Libanov & Rubakov (2010)

$$\boxed{\langle \chi_{\vec{k}}\chi_{-\vec{k}}\rangle_{\pi_{\vec{q}}} = \langle \chi_{\vec{k}}\chi_{-\vec{k}}\rangle\left(1 + c_1\frac{A_\pi}{2\pi}\frac{H_0}{k}\left(3\cos^2\theta-1\right) + c_2\frac{3A_\pi^2}{4\pi^2}\cos^2\theta\log\frac{H_0}{\Lambda}\right)}$$

Conclusions

- Nearly scale invariant and gaussian primordial density inhomogeneities:
  - Multi-field inflation: $so(4,1)$
  - Single-field inflation: $so(4,1) \rightarrow$ rotations + translations
  - Conformal mechanism: $so(4,2) \rightarrow so(4,1)$

Symmetries reflected in soft limits of correlation functions.

- Single-field inflation constrained by <u>infinitely-many</u> relations (indep. of slow-roll, c_s, ϕ fundamental or not)

$$\lim_{\vec{q}\to 0} \frac{\partial^n}{\partial q^n}\left(\frac{\langle \zeta_{\vec{q}} \mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N}\rangle}{P_\zeta(q)} + \frac{\langle \gamma_{\vec{q}} \mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N}\rangle}{P_\gamma(q)}\right) \sim \frac{\partial^n}{\partial k^n}\langle \mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N}\rangle\,.$$

- All follow from <u>Slavnov-Taylor identity</u> for spatial diffs