Durham University

Primordial Black Holes and Higgs Vacuum Decay

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Outline

- Quantum Tunnelling
- Gravity in Tunnelling
- Black Holes
- Constraining SM with PBH

Exploring Quantum Gravity?

Although we do not have an uncontested theory of quantum gravity, we do have ideas on how quantum effects in gravity behave below the Planck scale.

Quantum Effects in Gravity

Below the Planck scale, we expect that spacetime is essentially classical, but that gravity can contribute to quantum effects through the wave functions of fields, and through the back-reaction of quantum fields on the spacetime.

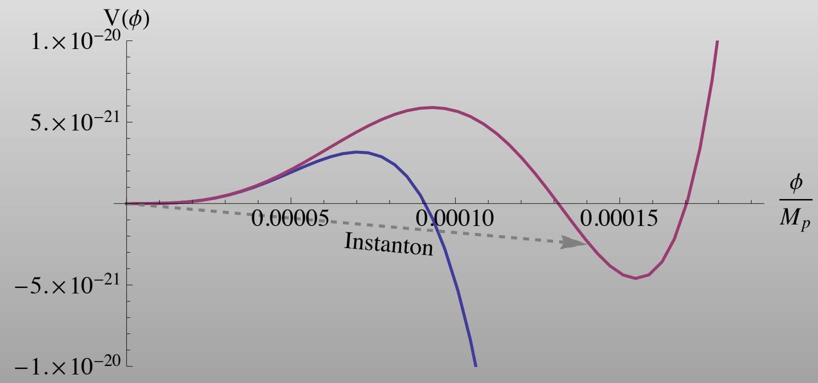
We use this in black hole thermodynamics, cosmological perturbation theory, and for non-perturbative solutions in field theory, this method is particularly unambiguous, but can we test these ideas in a broader sense?

Quantum Tunnelling

Developed by Coleman and others in the 1970's.

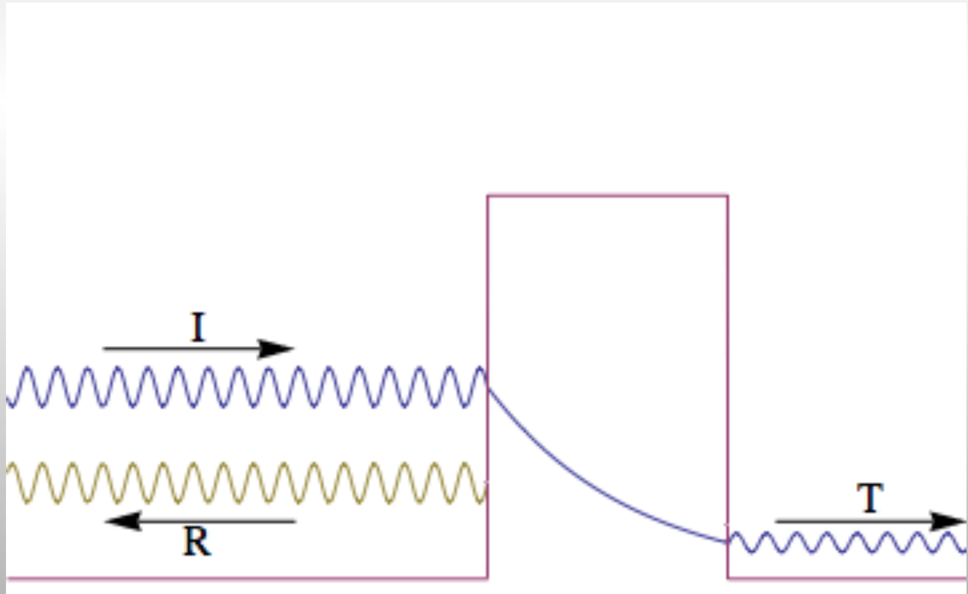
Vacuum understood as an effective state, defined by the minimum of a potential.

The potential itself depends on temperature and scale



MOTIVATION

To motivate the calculation, step back to 1st year QM.
First meet tunneling in the Schrodinger equation. Standard 1+1 Schrodinger tunneling exactly soluble. Recall tunnelling probabilities exponentially suppressed.



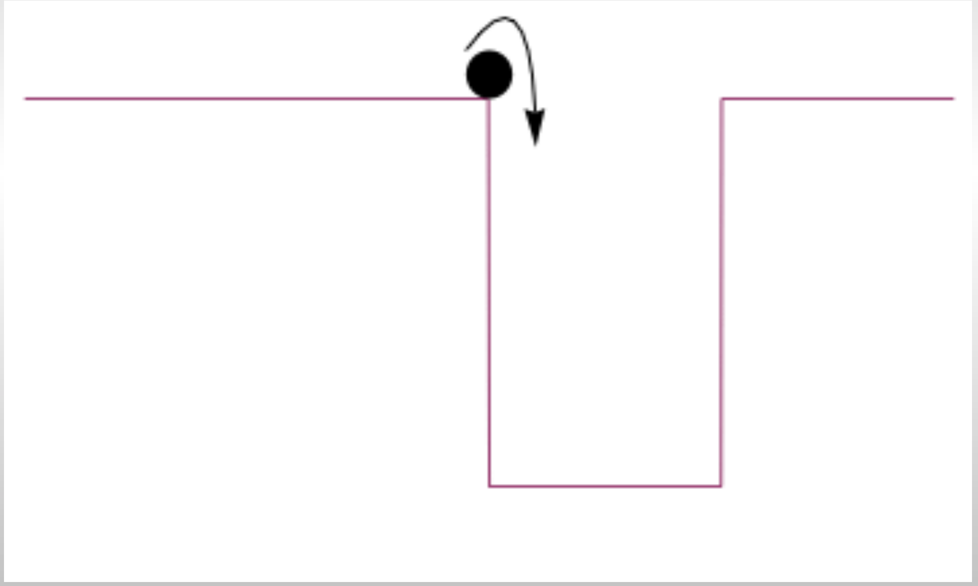
$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \approx e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\Omega d = \frac{1}{\hbar} \int_0^d \sqrt{2m(V_0 - E)}dx$$

Euclidean Trick

A simple and intuitive way of extracting this leading order behaviour is to take "classical" motion in Euclidean time:



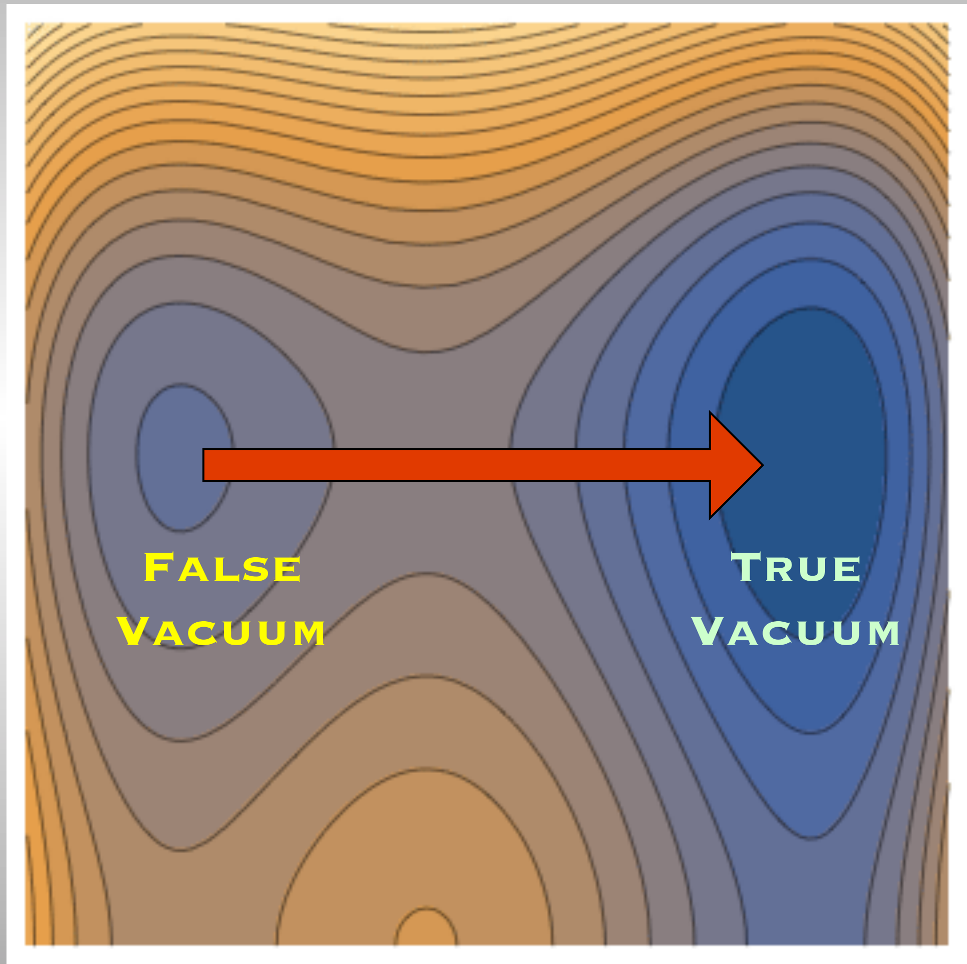
$$\boxed{t \to i\tau}$$

$$\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 = \Delta V$$

$$\int \sqrt{2\Delta V}\, dx = \int 2\Delta V\, d\tau = \int \left(\Delta V + \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2\right) d\tau = \int L_E\, d\tau$$

Most Probable Escape Paths

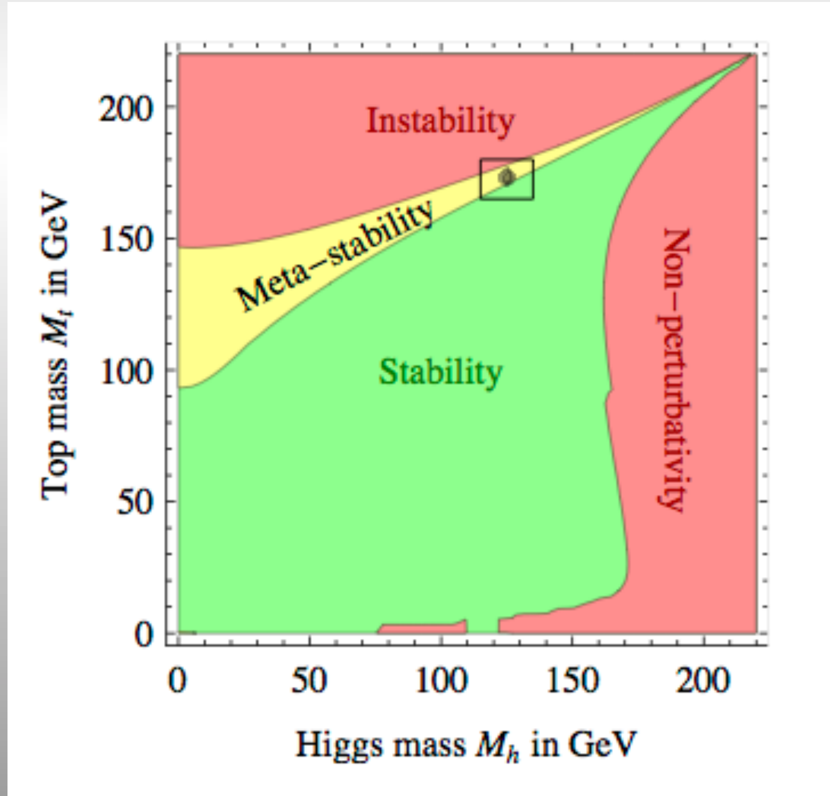
This picture was generalised by Banks Bender and Wu to describe multi-dimensional tunnelling, that then motivates the field theory Euclidean approach.

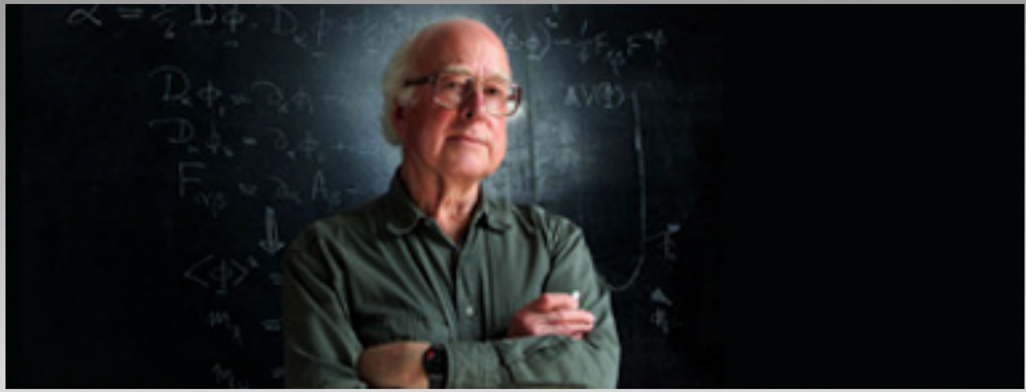


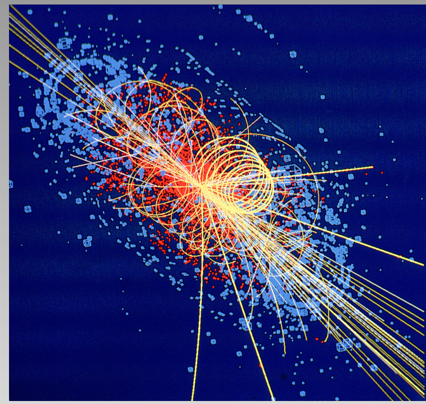
Higgs Vacuum

But the self-coupling of the Higgs changes with energy scale, so this value – the Higgs at its lowest energy state – may not be where we currently are!

The calculation depends on the masses of other fundamental particles (mainly top quark).

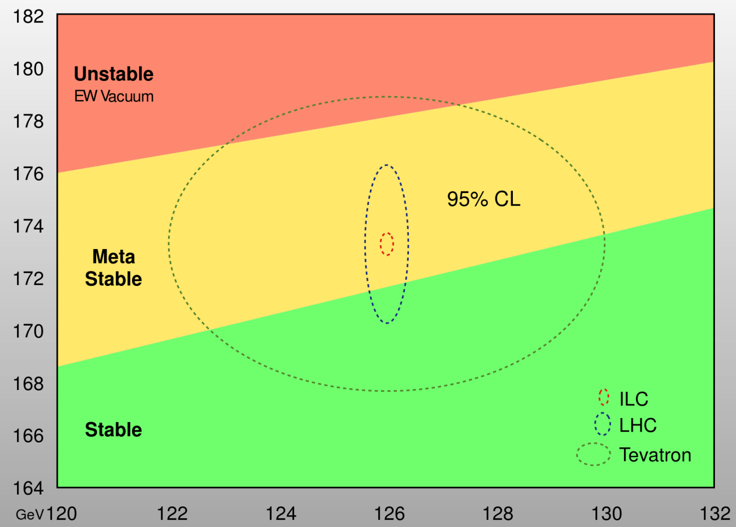






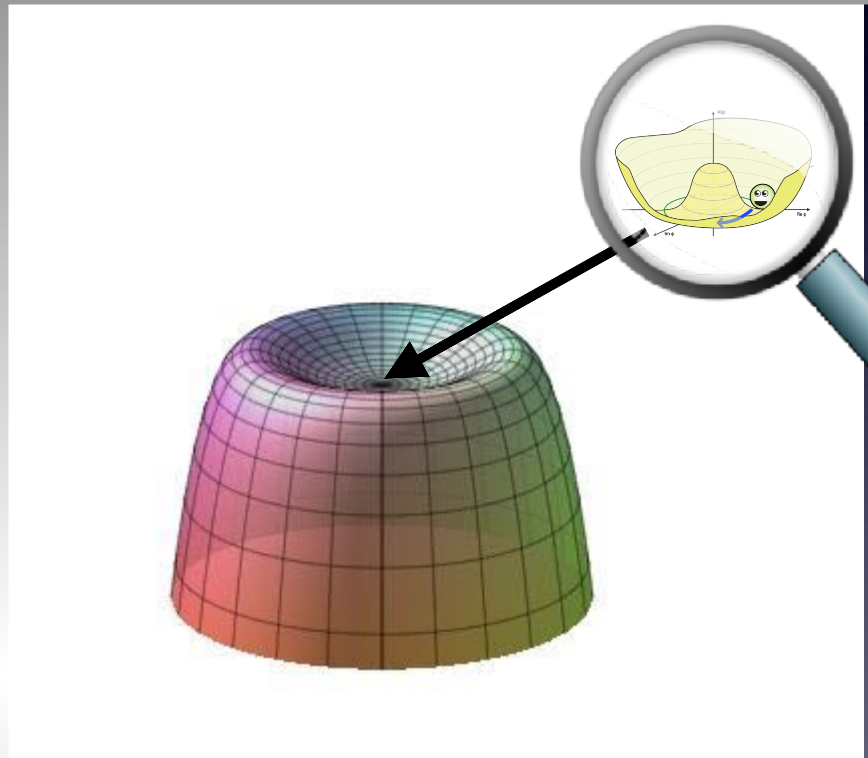
Is Our Vacuum Stable?

Calculating the running of the Higgs coupling tells us that we seem to be very close to a sweet spot between stability and instability – metastability.



Though see 1904.05237 (CMS) & 1905.02302 (ATLAS).

The bigger picture from the standard model tells us our universe may be…



….not entirely stable!

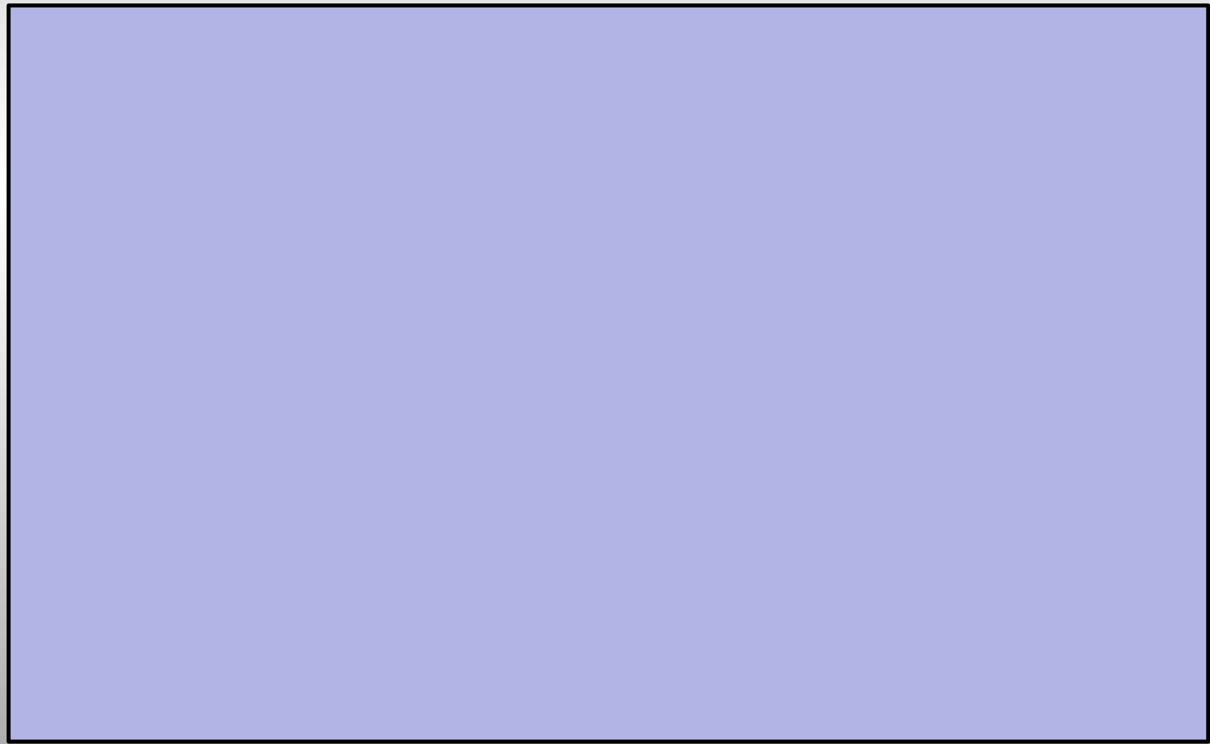
We call this local – not global – minimum a false vacuum, and expect there is a tunneling process to the true minimum / true vacuum. This gives a first order phase transition.

First Order Phase Transition

A first order phase transition proceeds by bubble nucleation – in this case of true vacuum within false. This is described by quantum mechanical tunnelling, and was explored by Coleman and collaborators in the 70's and 80's.

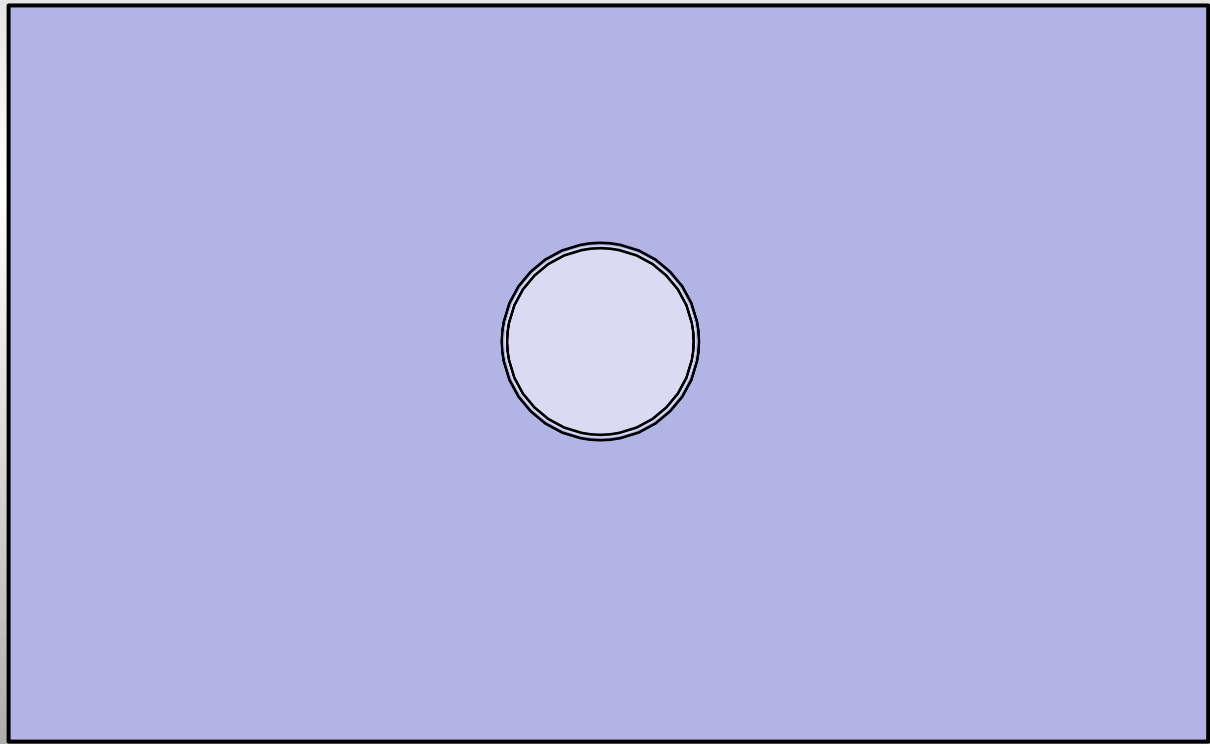
Coleman, 1977

Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum



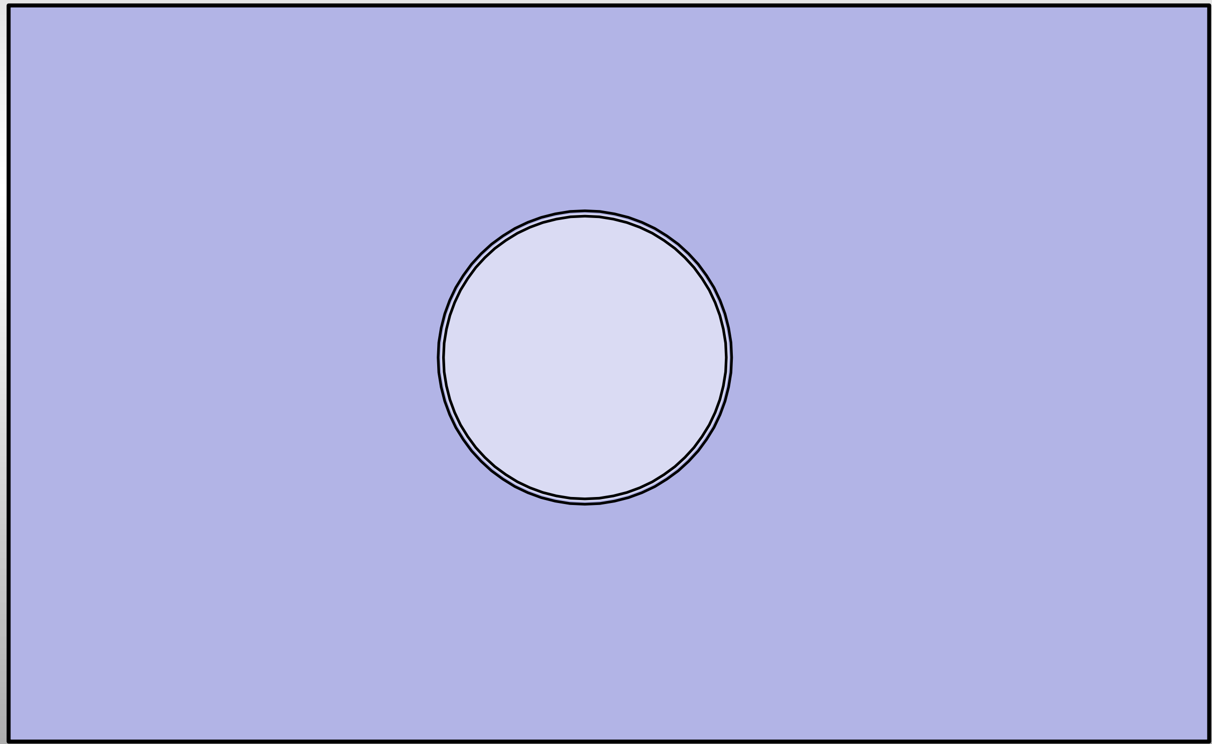
Coleman, 1977

Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum,



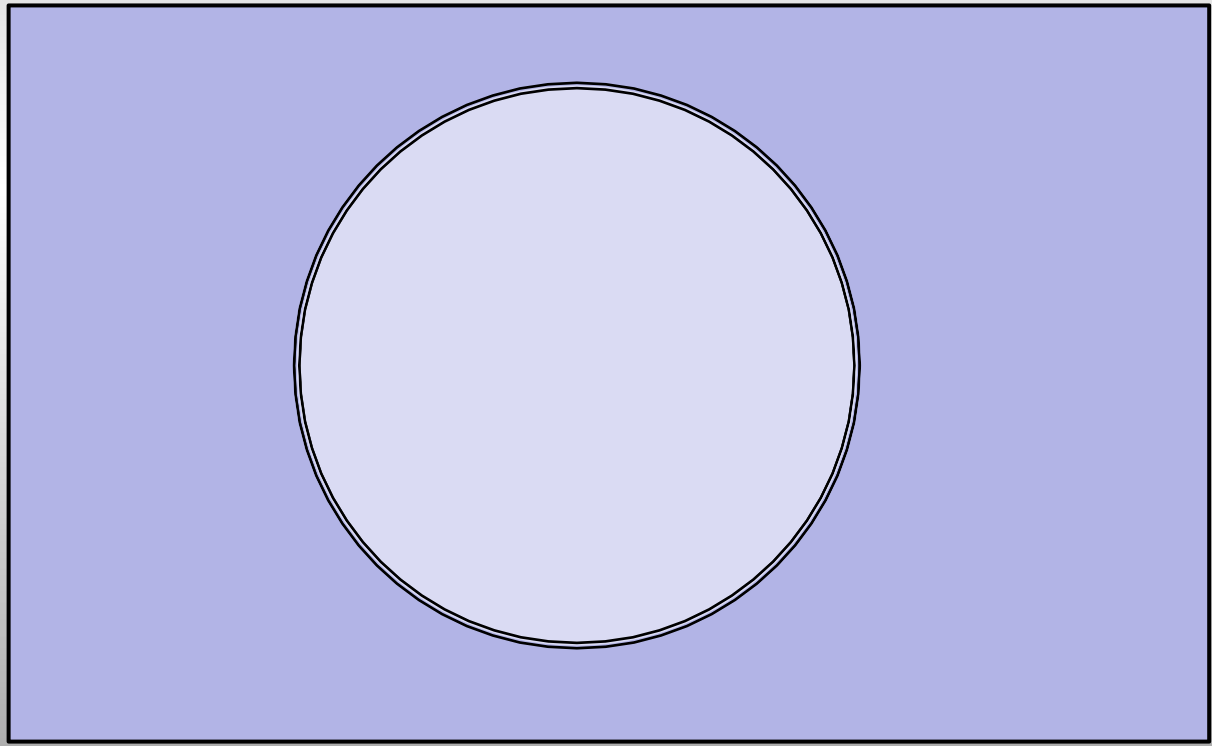
Coleman, 1977

Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum, then



Coleman, 1977

Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum, then expands.



“Goldilocks Bubble”

If a bubble fluctuates into existence, we gain energy from moving to true vacuum, but the bubble wall costs energy.

Too small and the bubble has too much surface area – recollapses.

Too large and it is too expensive to form.

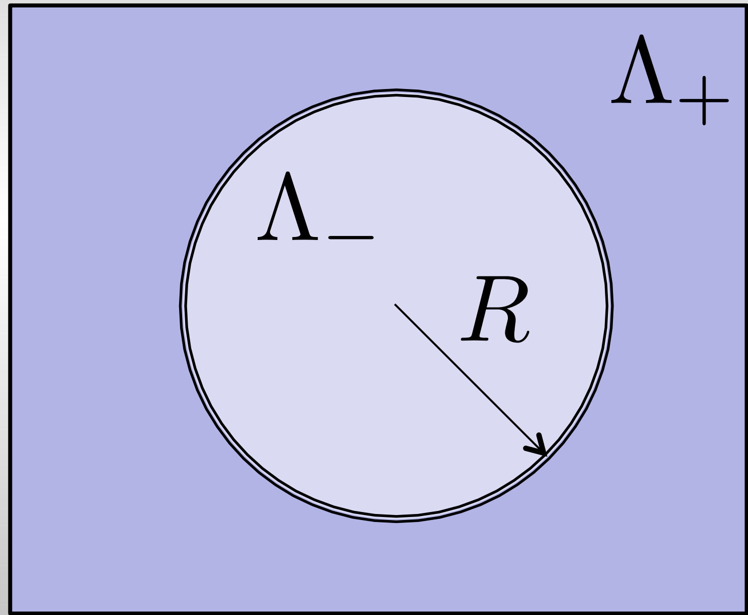
“Just Right” means the bubble will not recollapse, but is still “cheap enough” to form.

Euclidean Action

This corresponds beautifully to the Euclidean calculation of the tunneling solution: "The Bounce"

ENERGY COST $\sigma \times 2\pi^2 R^3$

ENERGY GAIN $\varepsilon \times \pi^2 R^4/2$



Solution stationary wrt R,

$$\Rightarrow R = 3\sigma/\varepsilon$$

Coleman Bounce

This gives us the bubble radius, and the amplitude for the decay – backed up by full field theory calculations.

$$\mathcal{B} = \frac{\pi^2 R^3}{2}\left(-\sigma + \varepsilon R\right) \sim \frac{27\pi^2}{2}\frac{\sigma^4}{\varepsilon^3}$$

Tunneling amplitude, leading order:

$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

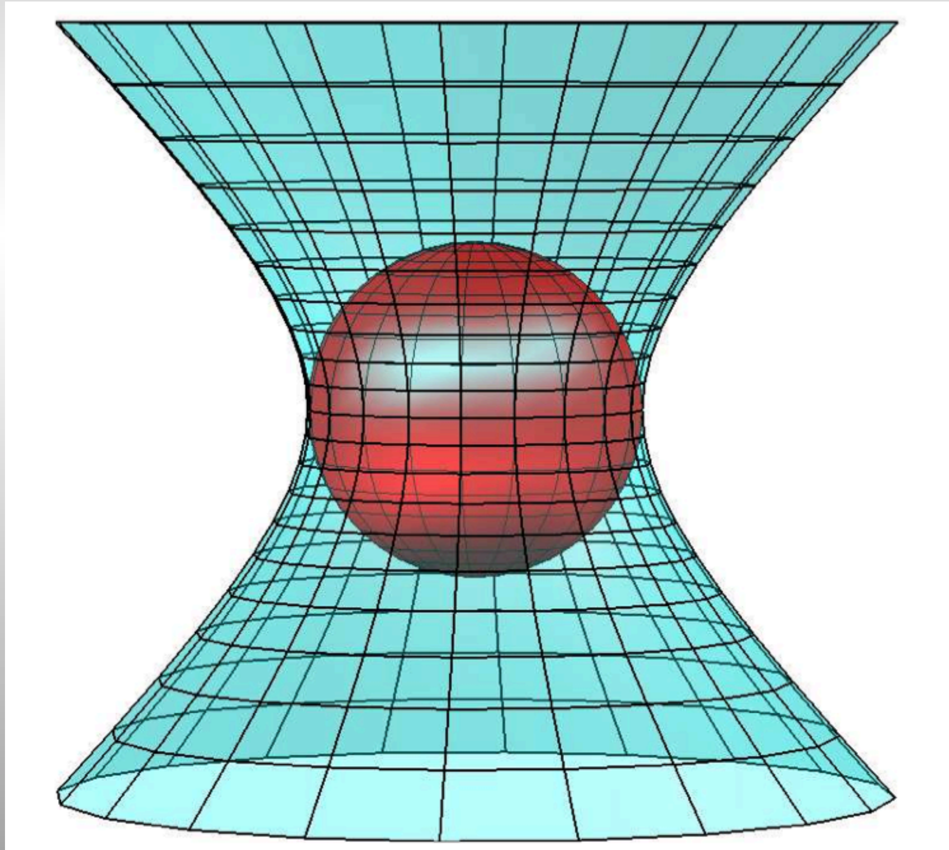
This gives the leading order or saddle point approximation to the amplitude. We must also include fluctuations:

$$\frac{\Gamma}{V} = \left| \frac{\det\, S''[\phi_{FV}]}{\det' S''[\phi_B]} \right|^{1/2} \left(\frac{\mathcal{B}}{2\pi} \right)^{D/2} e^{-\mathcal{B}/\hbar}$$

To get the nett decay rate per unit volume, per unit time.

Does this Euclidean calculation ***mean*** anything real?
Conventional answer is to rotate back to real time: $i\tau \to t$

$$r^2 + \tau^2 = R^2 \to r^2 - t^2 = R^2$$

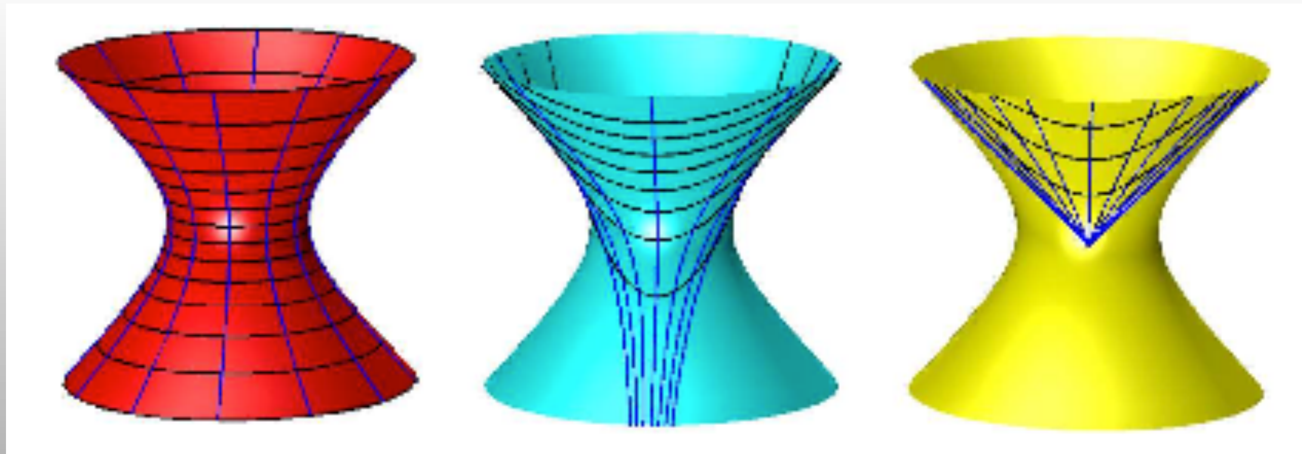


Real time picture is that the bubble expands rapidly.

$$r^2 = R^2 + t^2$$

Gravity and the Vacuum

This is not the full story! Vacuum energy gravitates – e.g. a positive cosmological constant gives us de Sitter spacetime – so we must add gravity to this picture



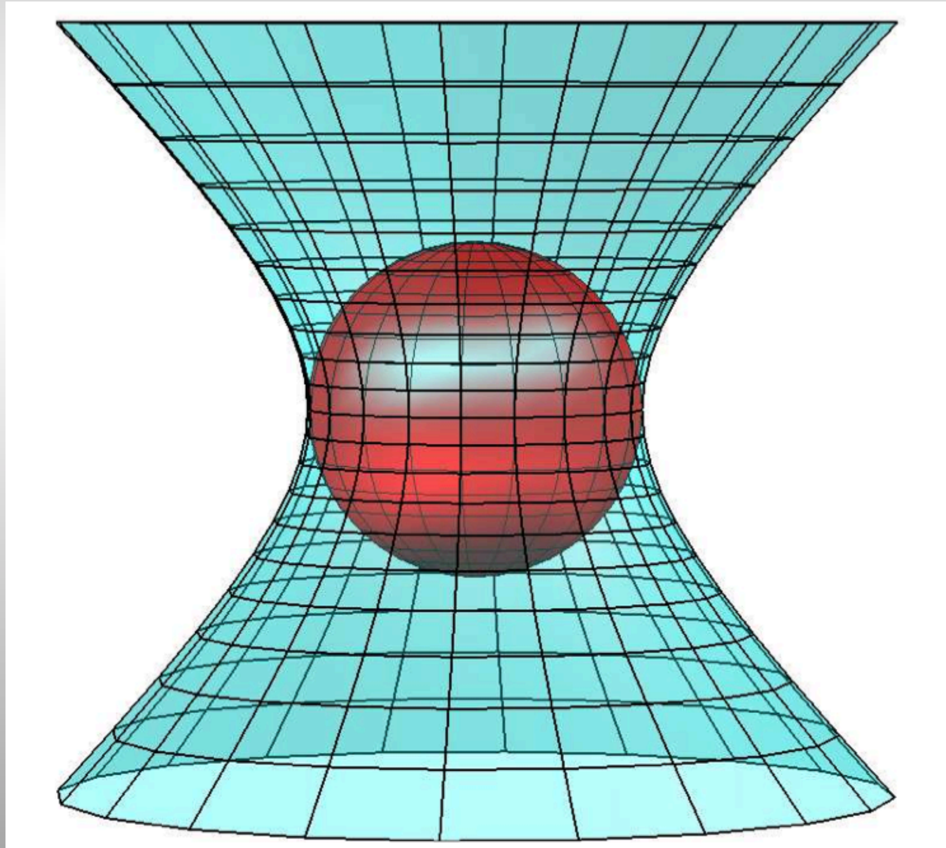
Gibbons-Hawking Euclidean Approach

Extend partition function description to include the Einstein-Hilbert action – at finite temperature we take finite periodicity of Euclidean time.

$$S = -\frac{M_p^2}{2}\int d^4x\sqrt{|g|}R + \int d^4x\mathcal{L}_{SM}$$

Fluctuations treated with caution, but saddle points unambiguous.

De Sitter spacetime has a Lorentzian (real time) and Euclidean (imaginary time) spacetime. The real time expanding universe looks like a hyperboloid and the Euclidean a sphere:

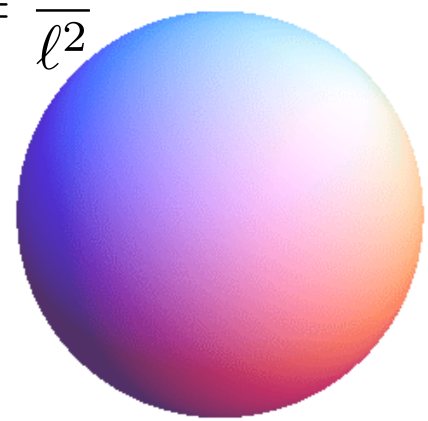


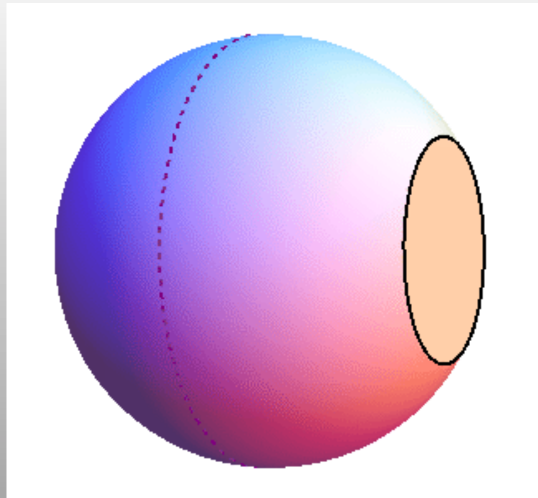
Our instanton must cut the sphere and replace it with flat space (true vacuum).

Coleman de Luccia (CDL)

Coleman and de Luccia showed how to do this with a bubble wall: Euclidean de Sitter space is a sphere, of radius ℓ related to the cosmological constant. The true vacuum has zero cosmological constant, so must be flat.

$$\Lambda = \frac{3}{\ell^2}$$





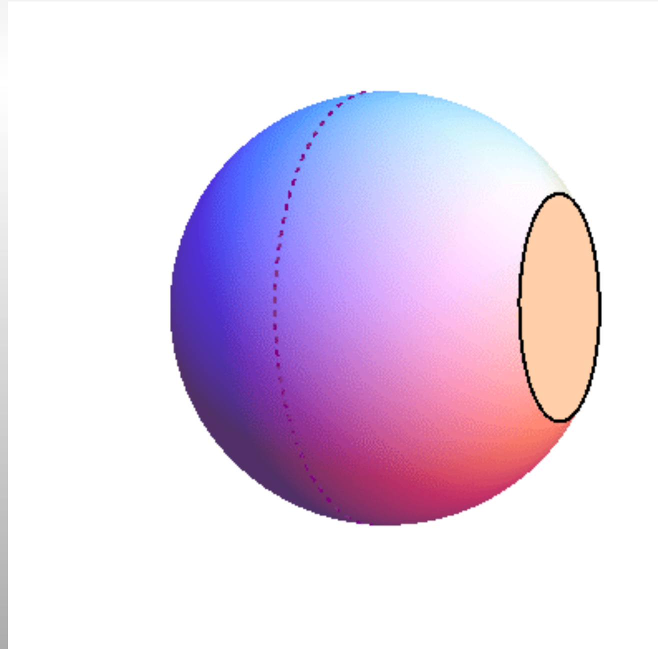
The bounce looks like a truncated sphere.

Coleman and de Luccia, PRD21 3305 (1980)

Goldilocks with Gravity

We can play the same "Goldilocks bubble" game – finding the cost of making this truncated sphere, but adding in the effect of gravity.

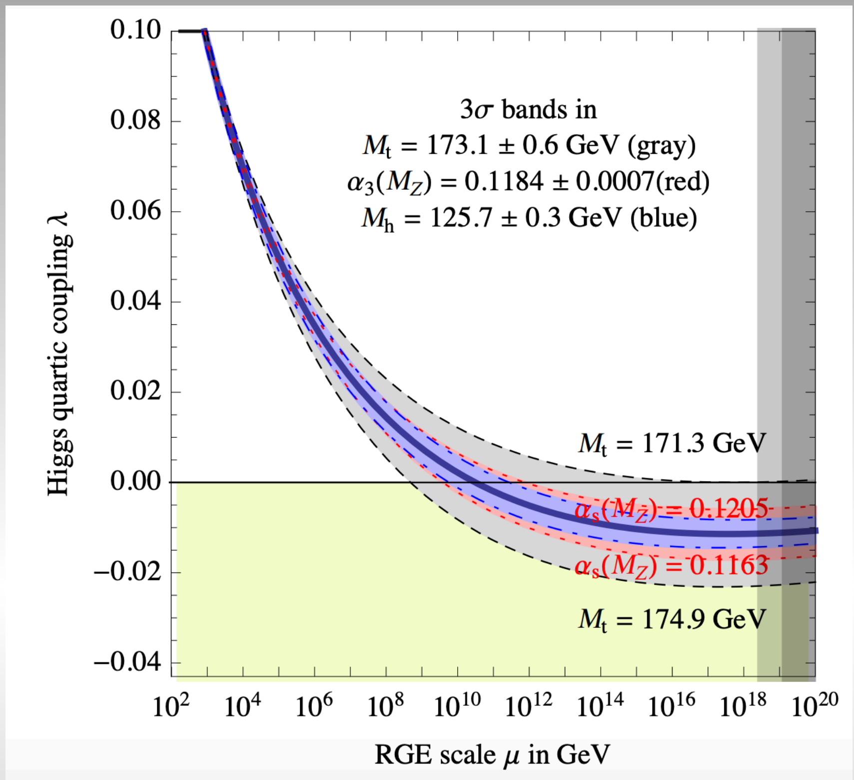
$$\mathcal{B}(R) = \frac{4}{3}\pi^2 \varepsilon \ell^4 \left[1 - \left(1 - \frac{R^2}{\ell^2} \right)^{\frac{3}{2}} \right] - 2\pi^2 \varepsilon \ell^2 R^2 + 2\pi^2 \sigma R^3$$



CDL Action

Once again, too small a bubble will recollapse, and large bubbles are harder to make, so there is a “just right” bubble that corresponds to a solution of the Euclidean Einstein equations that we can find either numerically with the full field theory, or analytically if we take our bubble wall to be thin, and we can find our instanton action.

$$
\begin{aligned}
\mathcal{B} &= -\frac{\Lambda}{8\pi G}\int_{\text{int}} d^4x\sqrt{g} - \frac{\sigma}{2}\int_{\mathcal{W}} d^3x\sqrt{h} \\
&= \frac{\pi\ell^2}{4G}(1-\cos\chi_0)^2 = \frac{\pi\ell^2}{G}\frac{16\bar{\sigma}^4\ell^4}{(1+4\bar{\sigma}^2\ell^2)^2}
\end{aligned}
$$



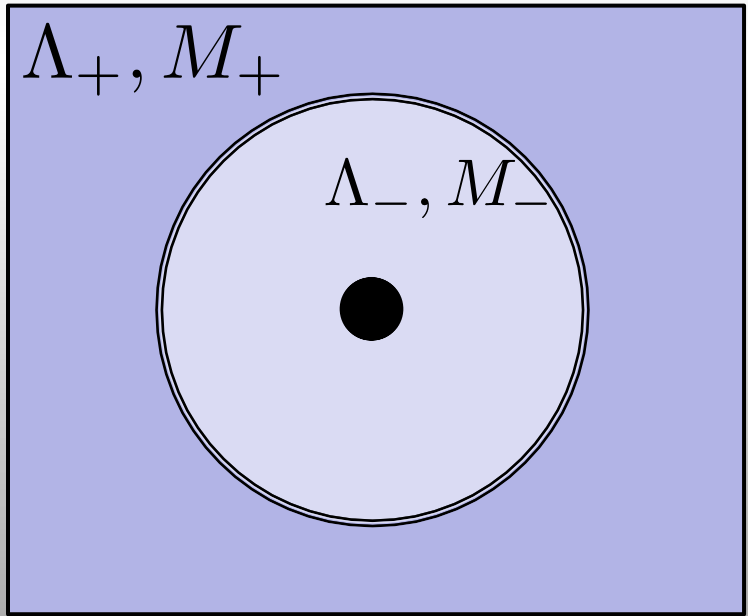
For the Higgs, this gives a half-life of many hundreds of billions of years.

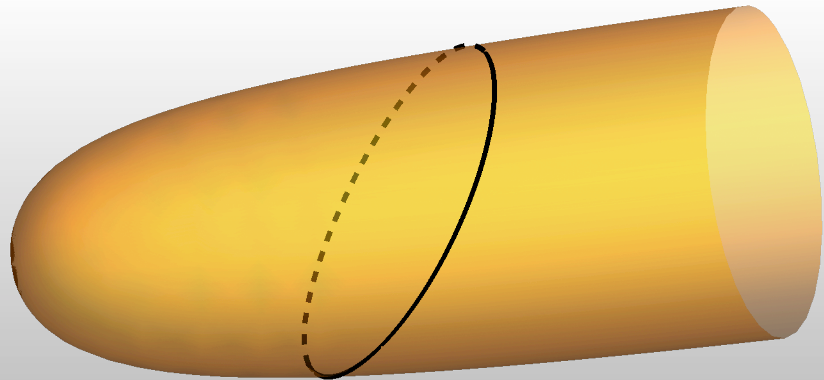
BUT

Most first order phase transitions do not proceed by ideal bubble nucleation, but by seeds.
These calculations are very idealised – an empty and featureless background – what if we throw in a little impurity?

Tweaking CDL

A black hole is an inhomogeneity, and also exactly soluble:





RG, Moss & Withers, 1401.0017

Goldilocks Black Hole Bubbles

- The bubble with a black hole inside, can have a different mass term outside (seed).
- The solution in general depends on time, but for each seed mass there is a unique bubble with lowest action.
- For small seed masses this is time, but the bubble has no black hole inside it – no remnant black hole.

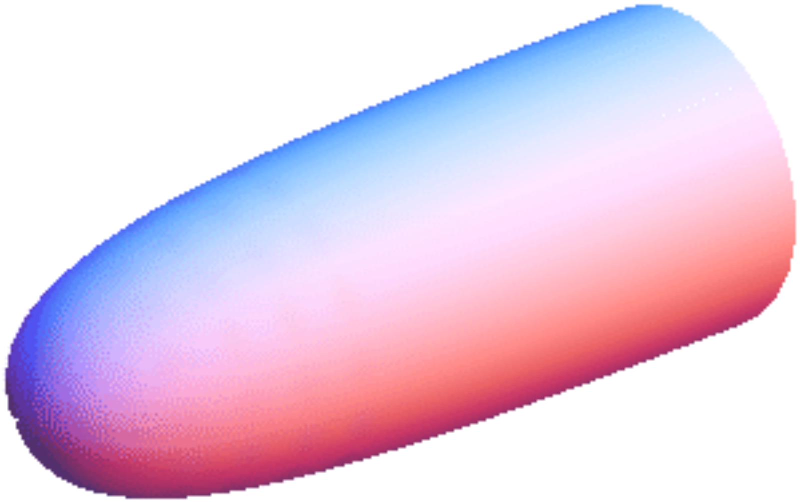
- For larger seed masses the bubble does not depend on Euclidean time, and has a remnant black hole.

This last case is the relevant one – the action is the difference in entropy (area) between the seed and remnant black holes!

Technical Aside:

Euclidean Black Holes

In Euclidean Schwarzschild, to make the black hole horizon regular, we must have **τ** periodic. This "explains" black hole temperature, but also sets a specific value, 8πGM.



$$ds^2 = \left(1 - \frac{2GM}{r}\right) d\tau^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$$

$$\sim \rho^2 d\left(\frac{\tau}{4GM}\right)^2 + d\rho^2 + (2GM)^2 d\Omega_{II}^2$$

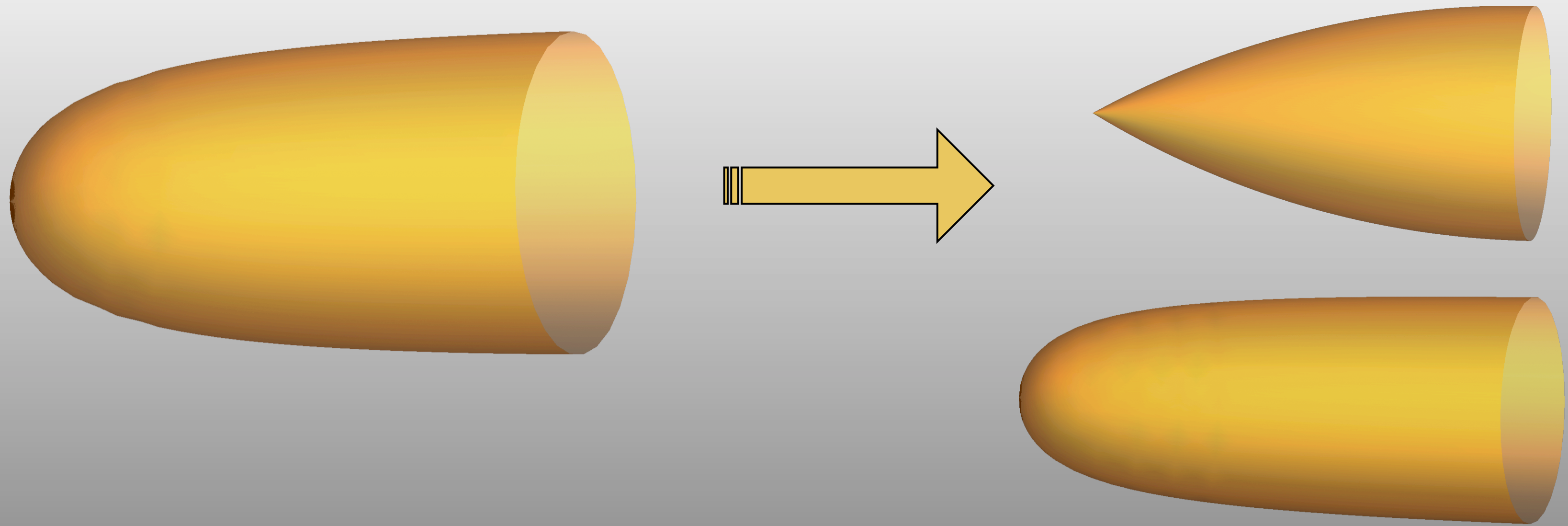
$$\rho^2 = 8GM(r - 2GM) \qquad \tau \sim \tau + 8\pi GM$$

Technical Aside:

Conical Deficits

For different seed and remnant masses the periodicity is different – we need to deal with conical deficits. This technicality is **crucial** to the calculation, and give a much lower instanton action.

To subtract off the false vacuum background, we must shrink the time circles to fit



Black Hole Bounces

Balance of action changes because of periodic time:

$$B \sim \sigma \times 4\pi R^2 L - \varepsilon \times \frac{4}{3}\pi R^3 L$$

$$R \sim 2\sigma/\varepsilon$$

$$B \sim \frac{\sigma^3}{\varepsilon^2} L$$

The result is that the action is the difference in entropy of the seed and remnant black hole masses:

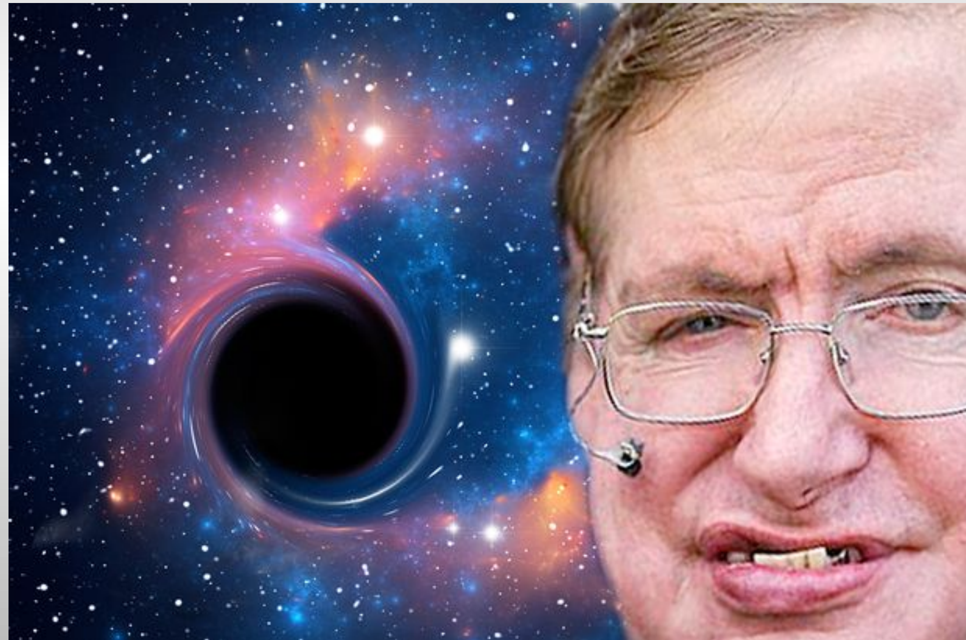
$$B \sim \mathcal{A}_+ - \mathcal{A}_-$$

Seeded tunneling is much more likely than CDL!

The fate of the black hole?

Vacuum decay is not all that can happen! Hawking tells us that black holes are black bodies, and radiate:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$



So we must compare evaporation rate to tunneling half-life.

Tunneling v Evaporation

Although we have computed bubble actions in full, we can estimate the dependence of the action on mass using input from our solutions which show that the seed and remnant masses are very close:

$$\begin{aligned} \mathcal{B} &= \pi(r_s^2 - r_r^2) \\ &\sim 4\pi(M_s + M_r)(M_s - M_r) \\ &\sim 8\pi M_s \delta M \quad \Rightarrow \Gamma_D \propto e^{-8\pi M_s \delta M} \end{aligned}$$

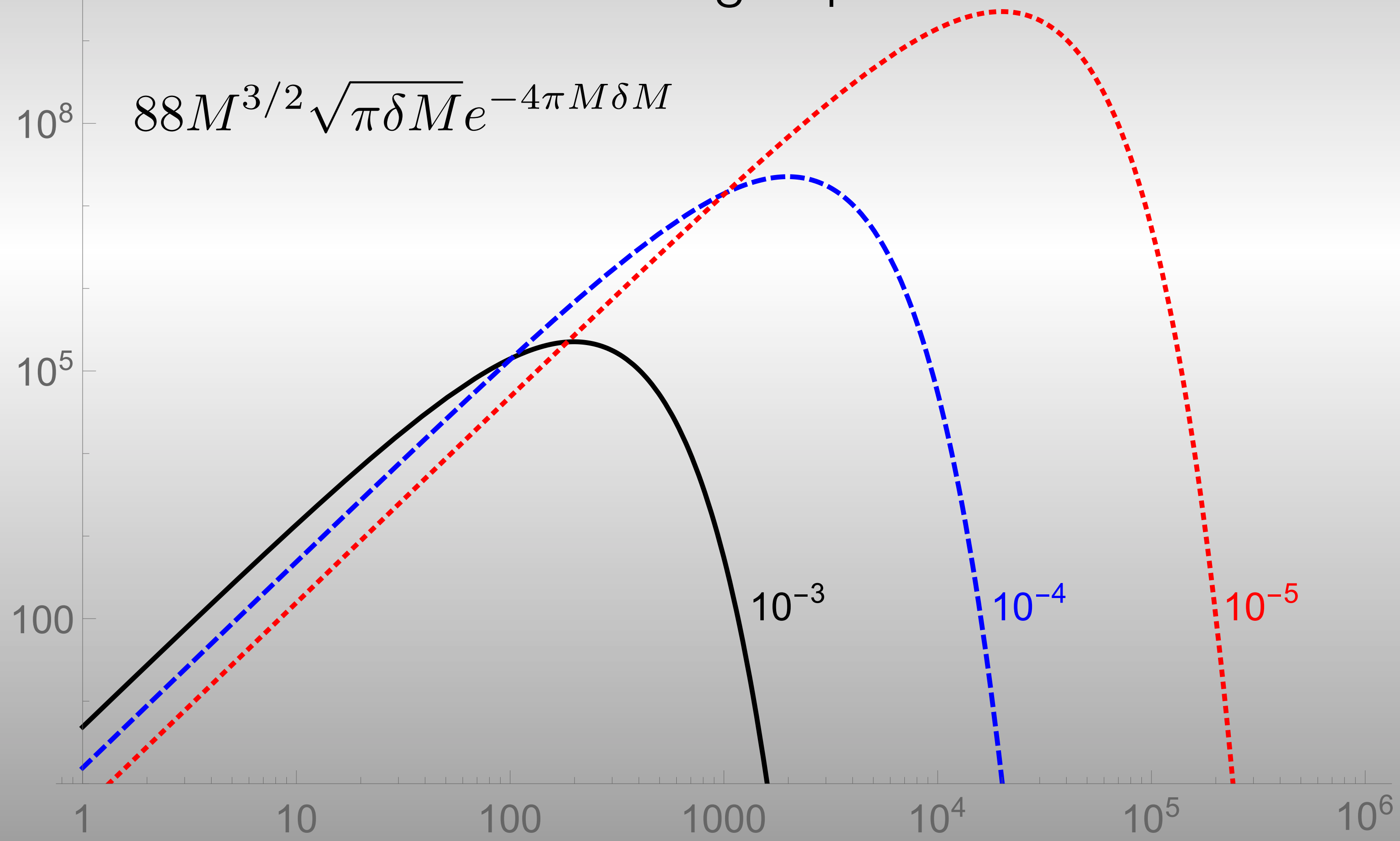
So our decay rate depends on an exponential of M_s, whereas evaporation depends on an inverse power of M – tunneling becomes important for smaller M

$$\Gamma_H \approx 3.6 \times 10^{-4} M^{-3}$$

Page, PRD-76

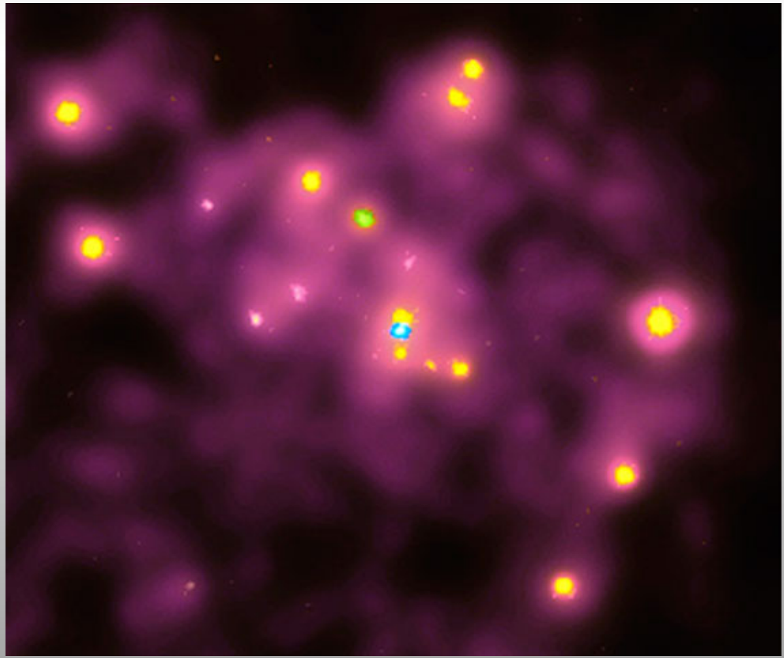
Tunneling v Evaporation

Taking this branching ratio estimate (in Planck units) shows how the dominance of tunnelling depends on δM and M:

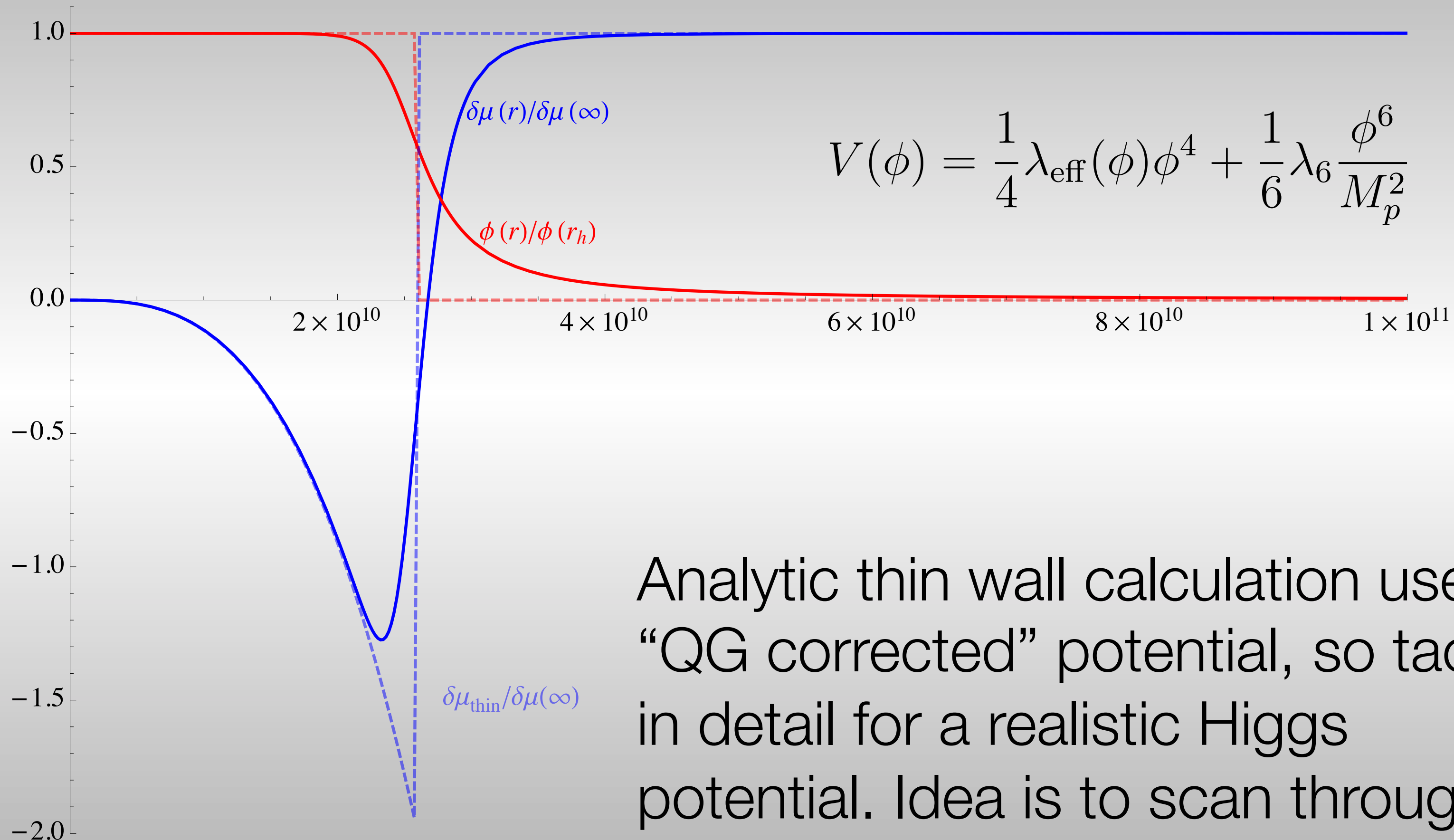


Primordial Black Holes

In other words – only a primordial black hole will catalyse vacuum decay! Those primordial black holes that start out with small enough mass to evaporate will eventually hit these curves!



Numerical Results

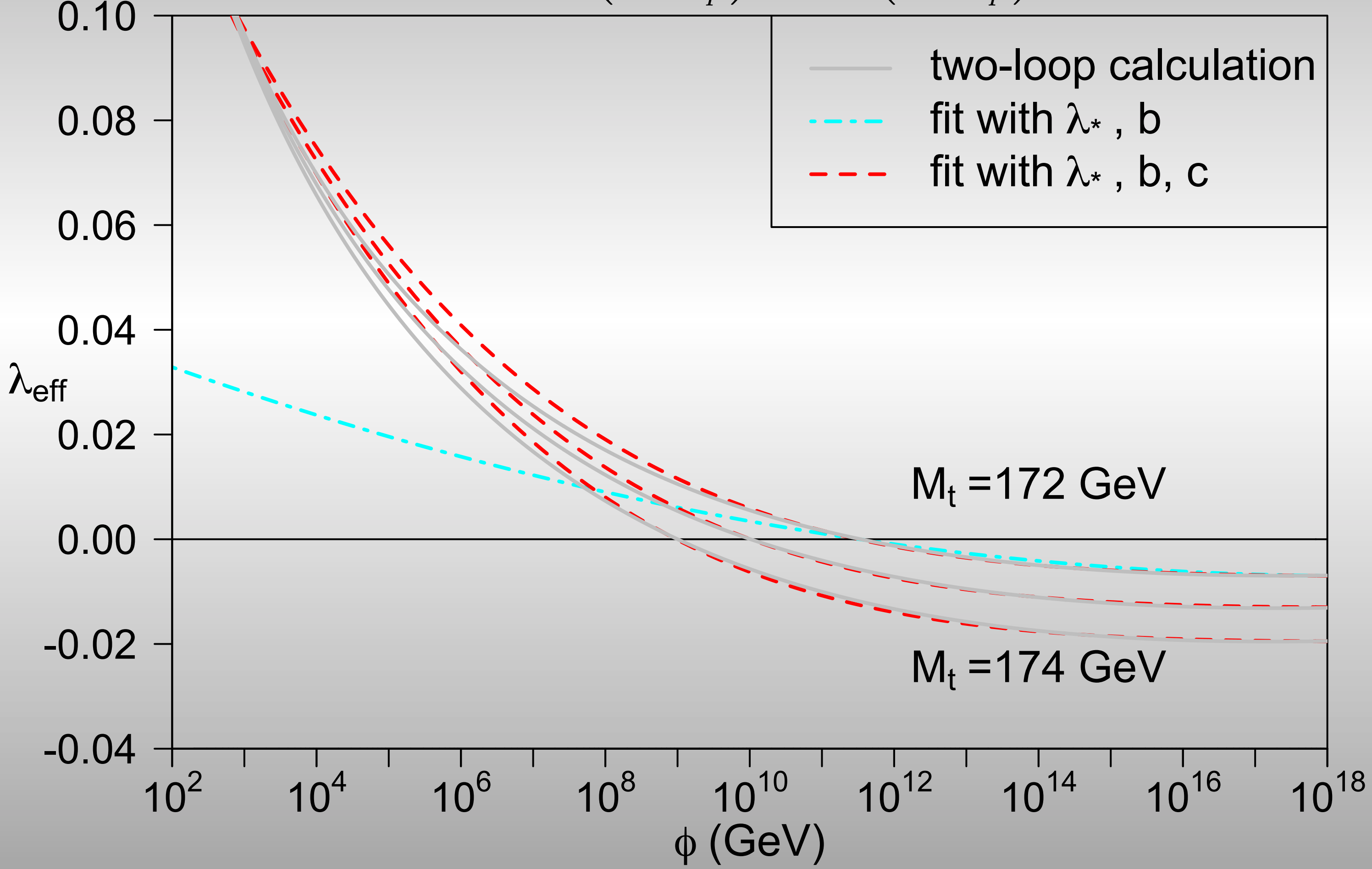


$$V(\phi) = \frac{1}{4}\lambda_{\rm eff}(\phi)\phi^4 + \frac{1}{6}\lambda_6\frac{\phi^6}{M_p^2}$$

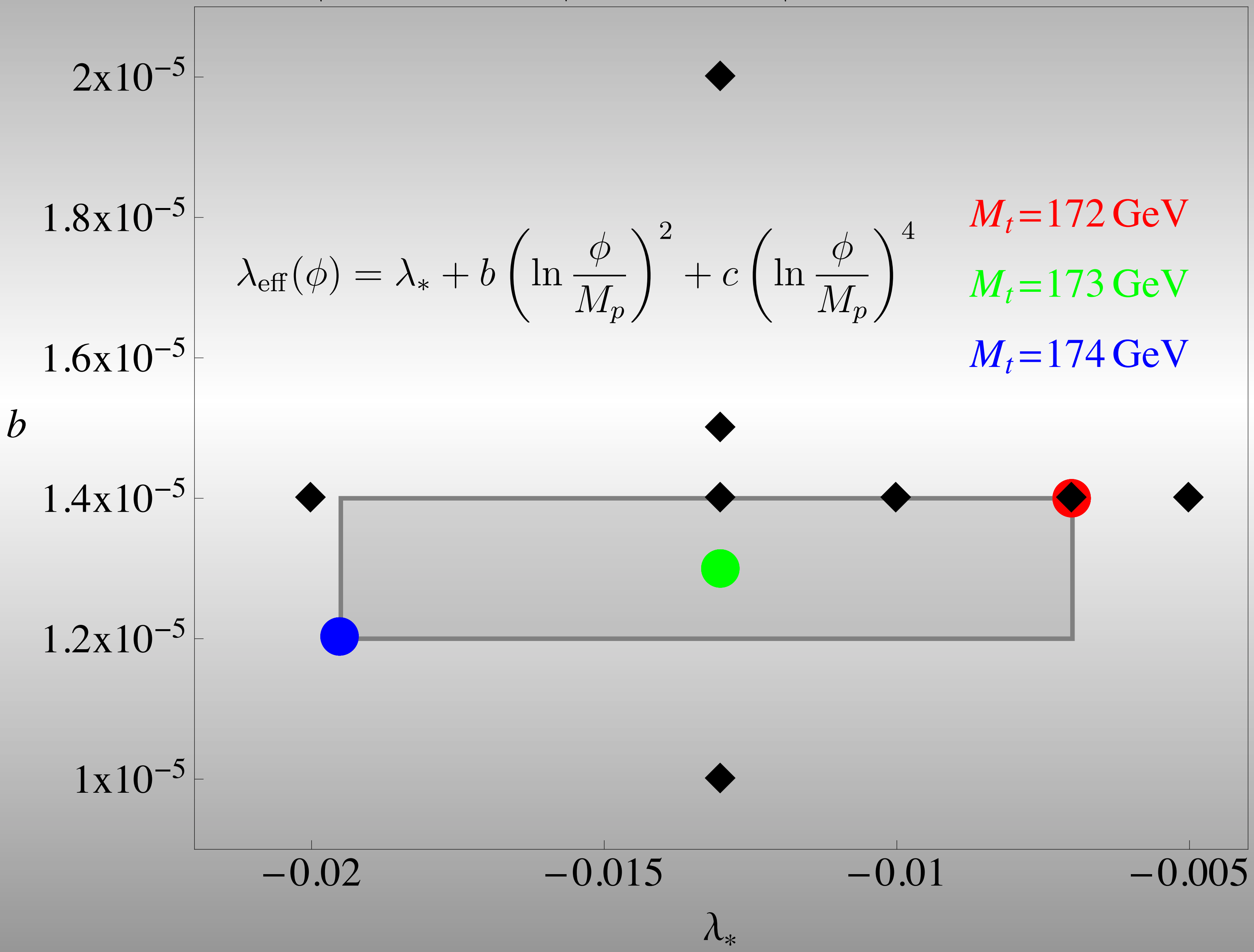
Analytic thin wall calculation uses a "QG corrected" potential, so tackle in detail for a realistic Higgs potential. Idea is to scan through parameter space (beyond standard model) to see how robust result it.

Fitting the Potential

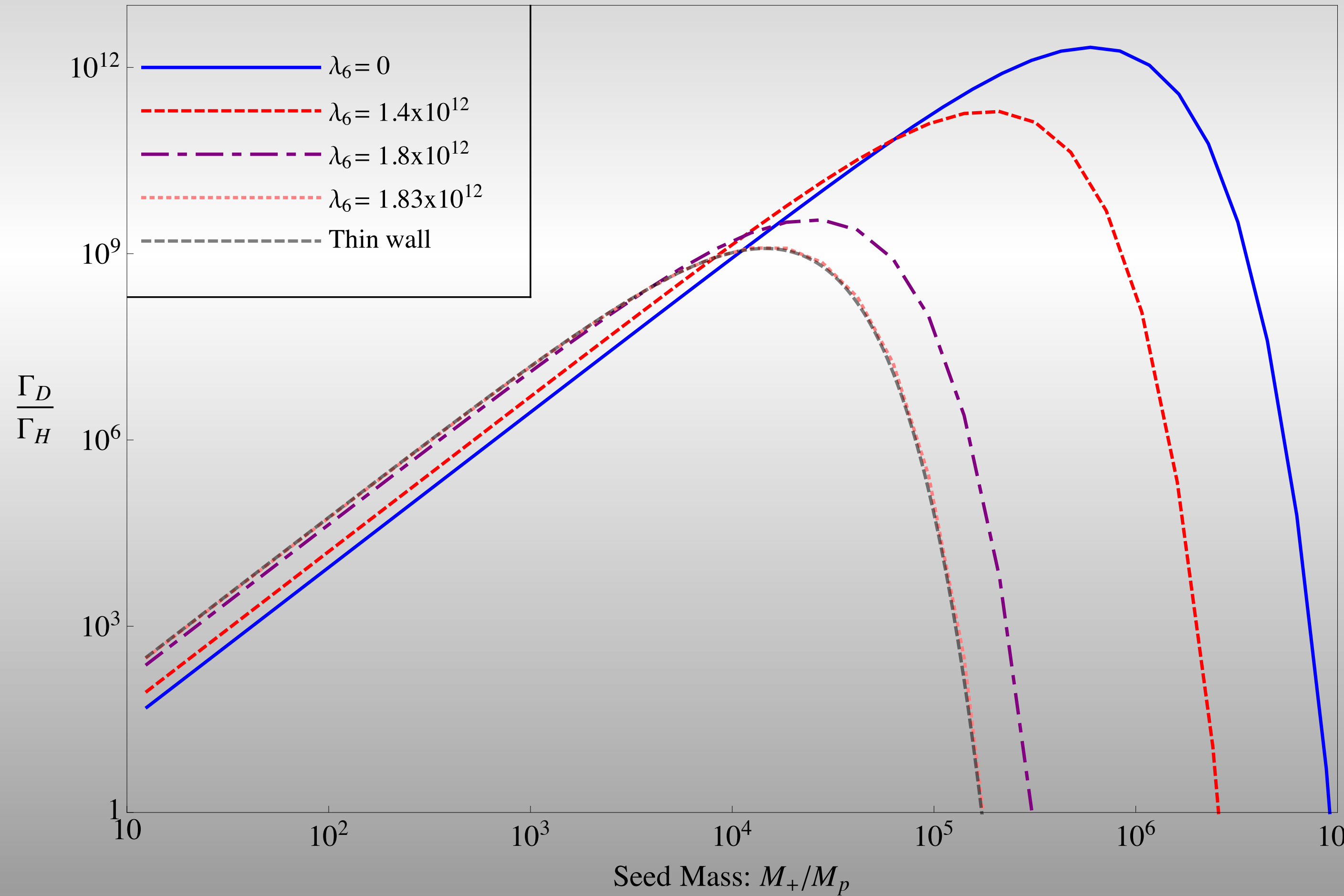
$$\lambda_{\text{eff}}(\phi) = \lambda_* + b\left(\ln\frac{\phi}{M_p}\right)^2 + c\left(\ln\frac{\phi}{M_p}\right)^4$$



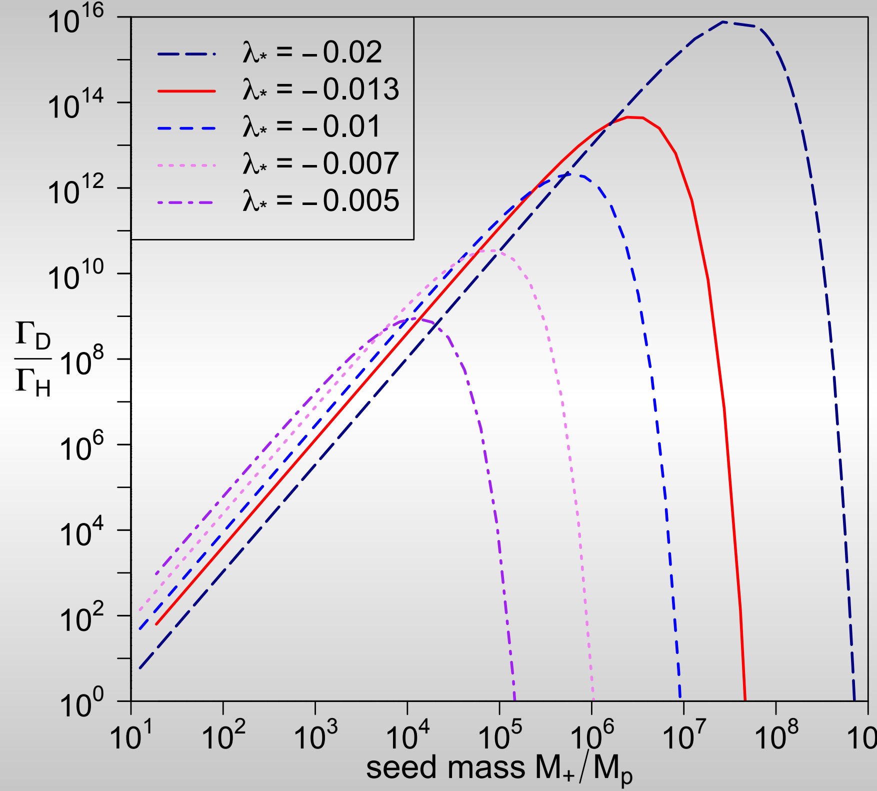
Numerical Integration

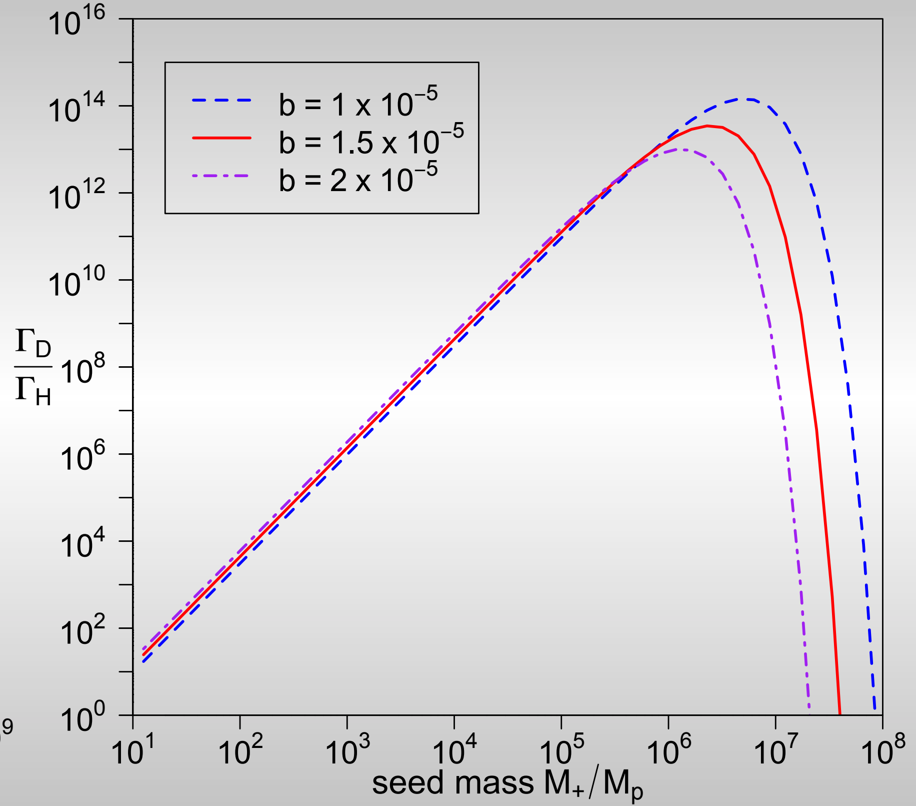


First check thin wall, by increasing λ_6. Thickening the wall increases the effectiveness of the instanton – the primordial black hole will hit the danger zone much sooner, and the decay will proceed rapidly.



Scanning through parameter space for pure SM potential shows main dependence on λ_*:





And because we are at such extreme scales, the lifetime of the universe drops to around 10^{-17}s!

PBH Seeded Decay

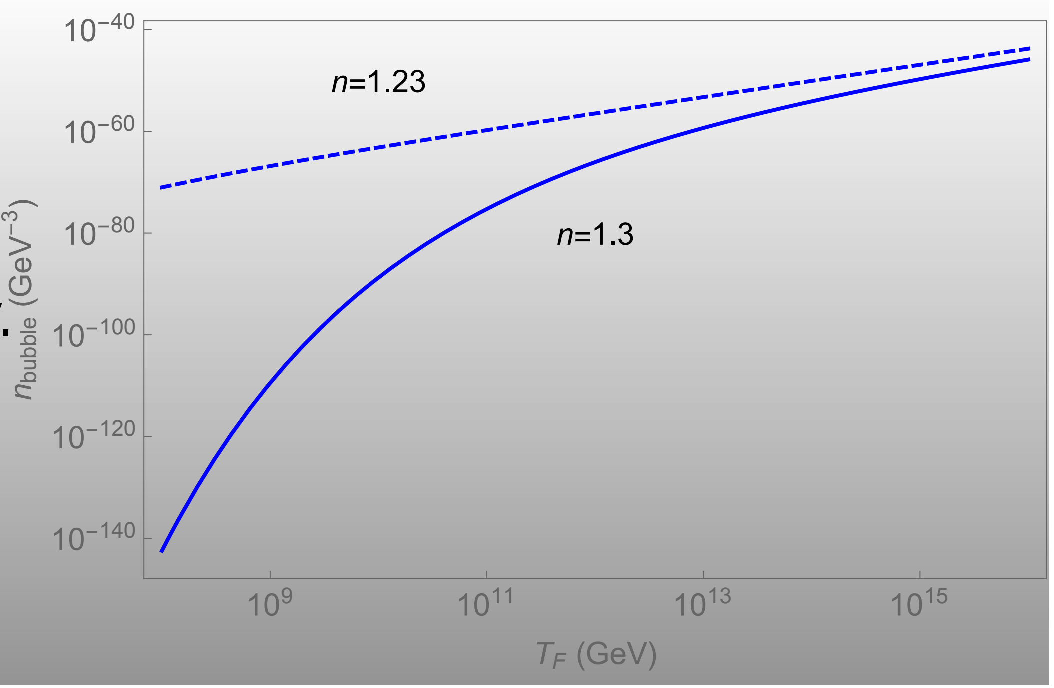
If a primordial black hole evaporates sufficiently, it will catalyse vacuum decay. The bubble expands, rapidly approaching the speed of light. Whether our universe decays then depends on whether the black hole has evaporated into the mass range, and whether the true vacuum bubble has had time to expand sufficiently.

PBH Seeded Decay

Assume black holes are created at a given temperature T_F, with a given mass M_F, and mass fraction

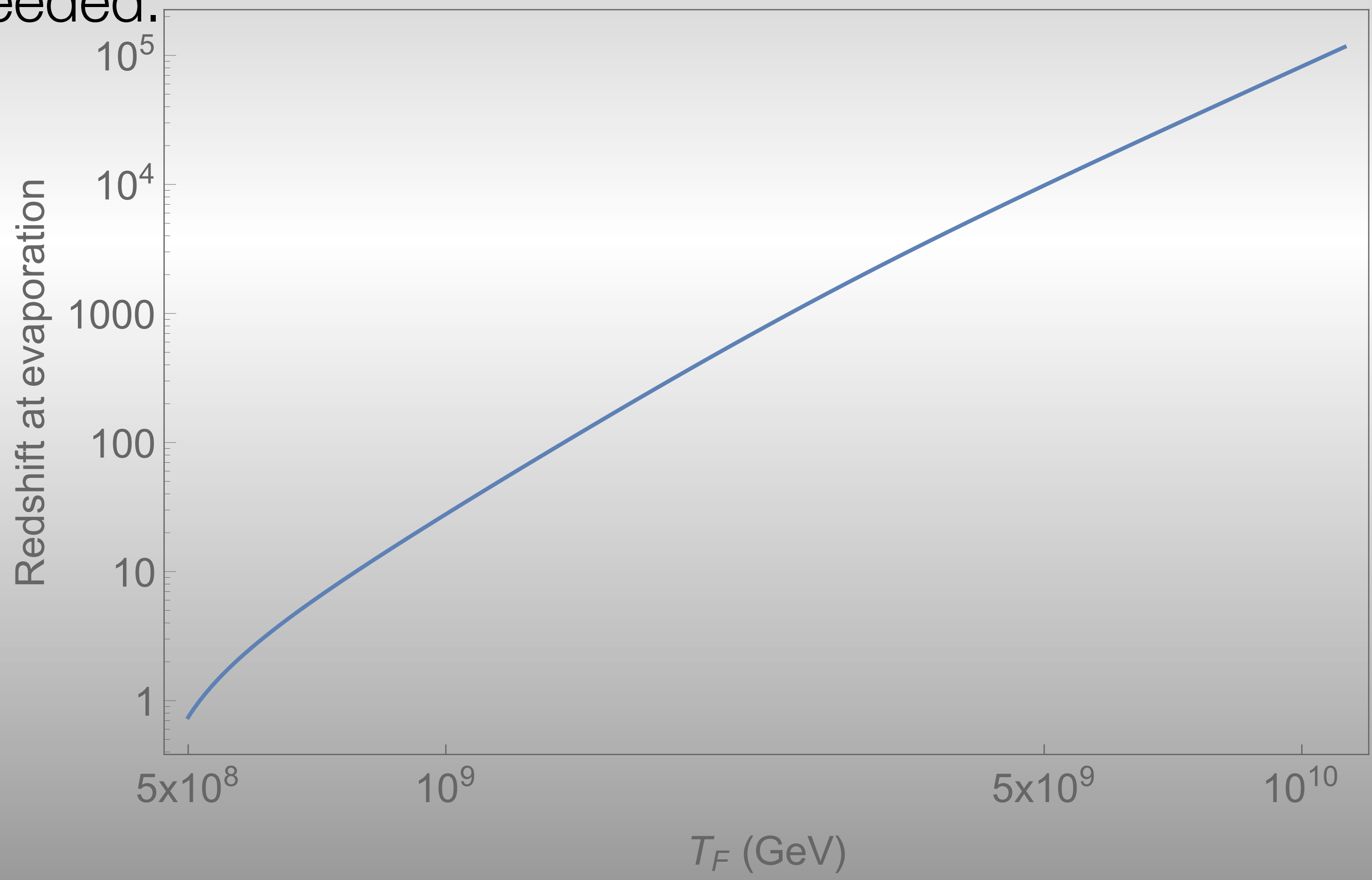
$$\beta_i \approx \frac{\sigma_H(T_F)}{\sqrt{2\pi}\delta_{\min}}\, e^{-\delta_{\min}^2/2\sigma_H^2(T_F)}$$

That allows us to calculate the current number density.



PBH Seeded Decay

M_F (or T_F) tells us the lifetime of the black hole, which translates to a redshift Z_B at which a bubble of true vacuum is seeded.



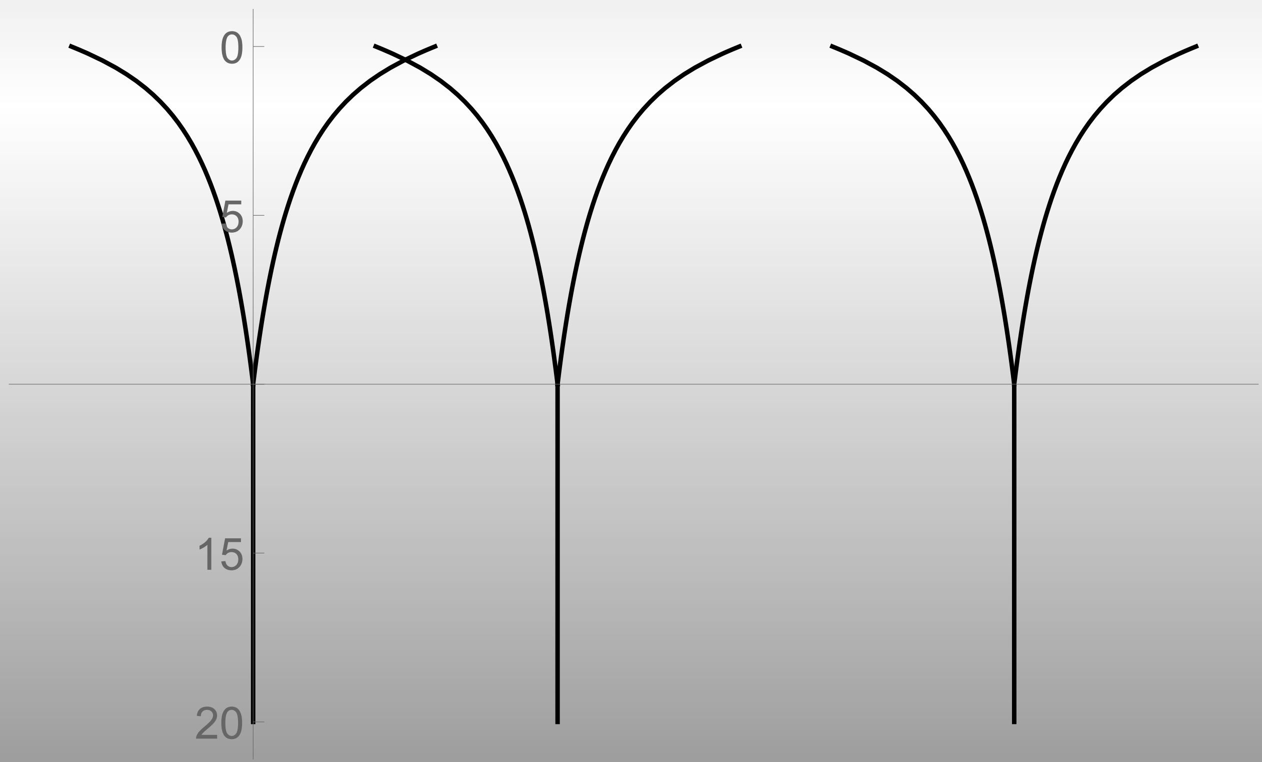
PBH Bubbles

This bubble expands to a current radius:

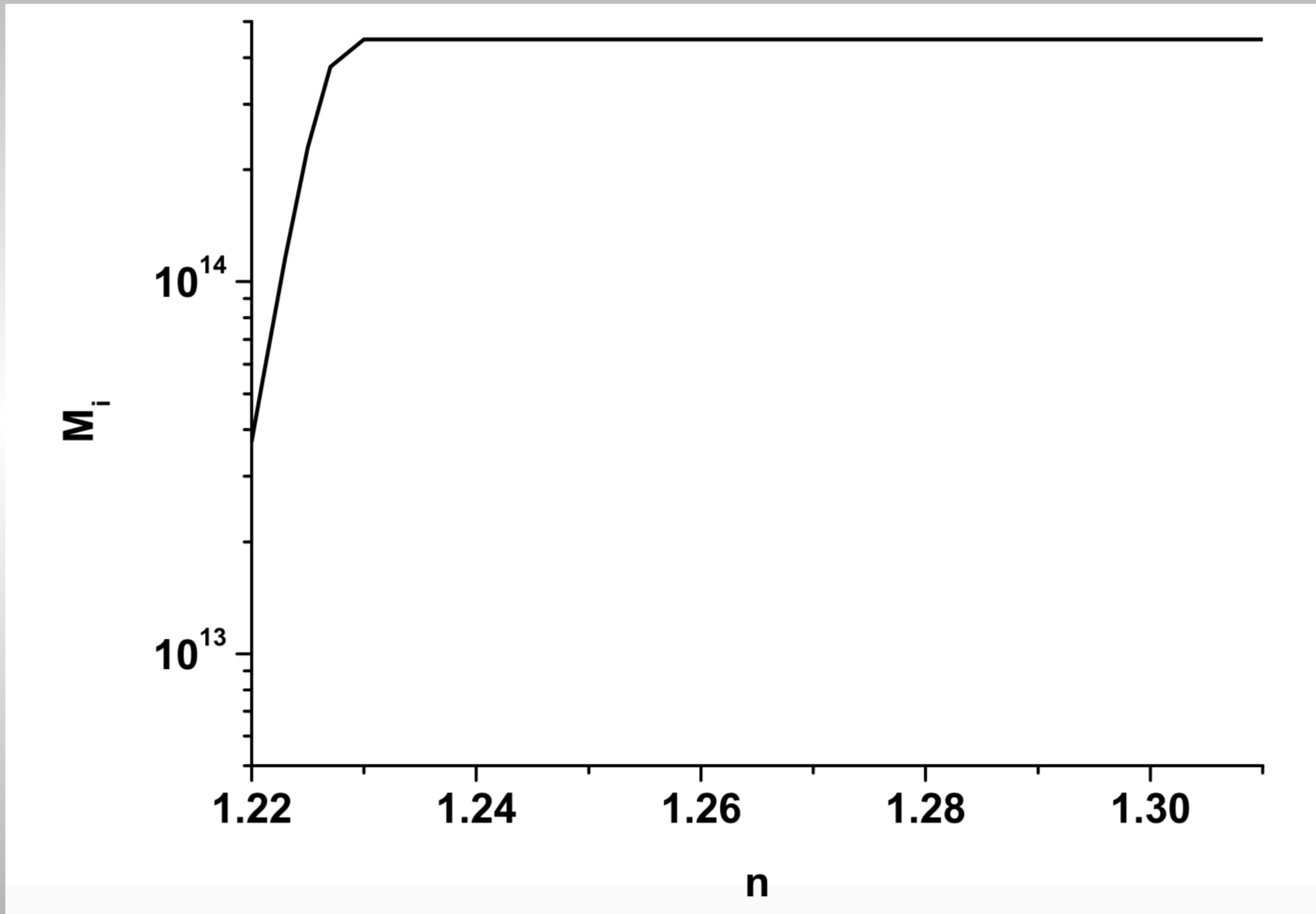
$$R = \frac{c}{H_0} \int_0^{Z_B} \frac{dz}{\hat{H}}$$

Fraction of current volume in true vacuum

$$\mathcal{P} = \frac{4\pi}{3} R^3 n_b(T_0)$$



When P=1 – we are doomed!



Summary

- Vacuum decay is an example of quantum effects in action with gravity – we have good tools, but they are idealised.
- Tunneling amplitudes significantly enhanced in the presence of a black hole – bubble forms around black hole and can remove it altogether. Important if Higgs vacuum metastable.
- Not a problem for PBH models of dark matter, but if there is a spread of M_F then there may be a constraint.