Constant roll and primordial black holes

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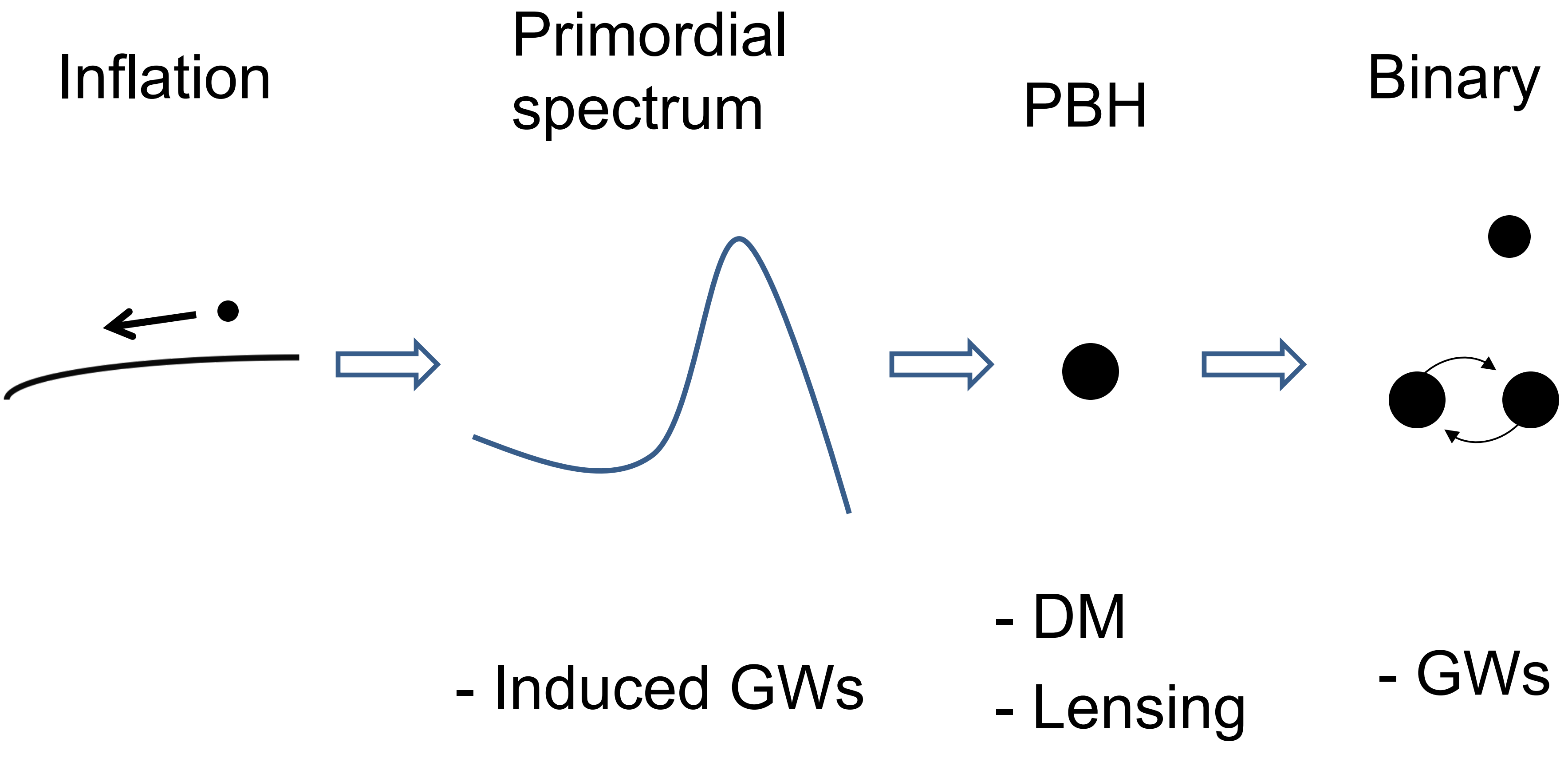
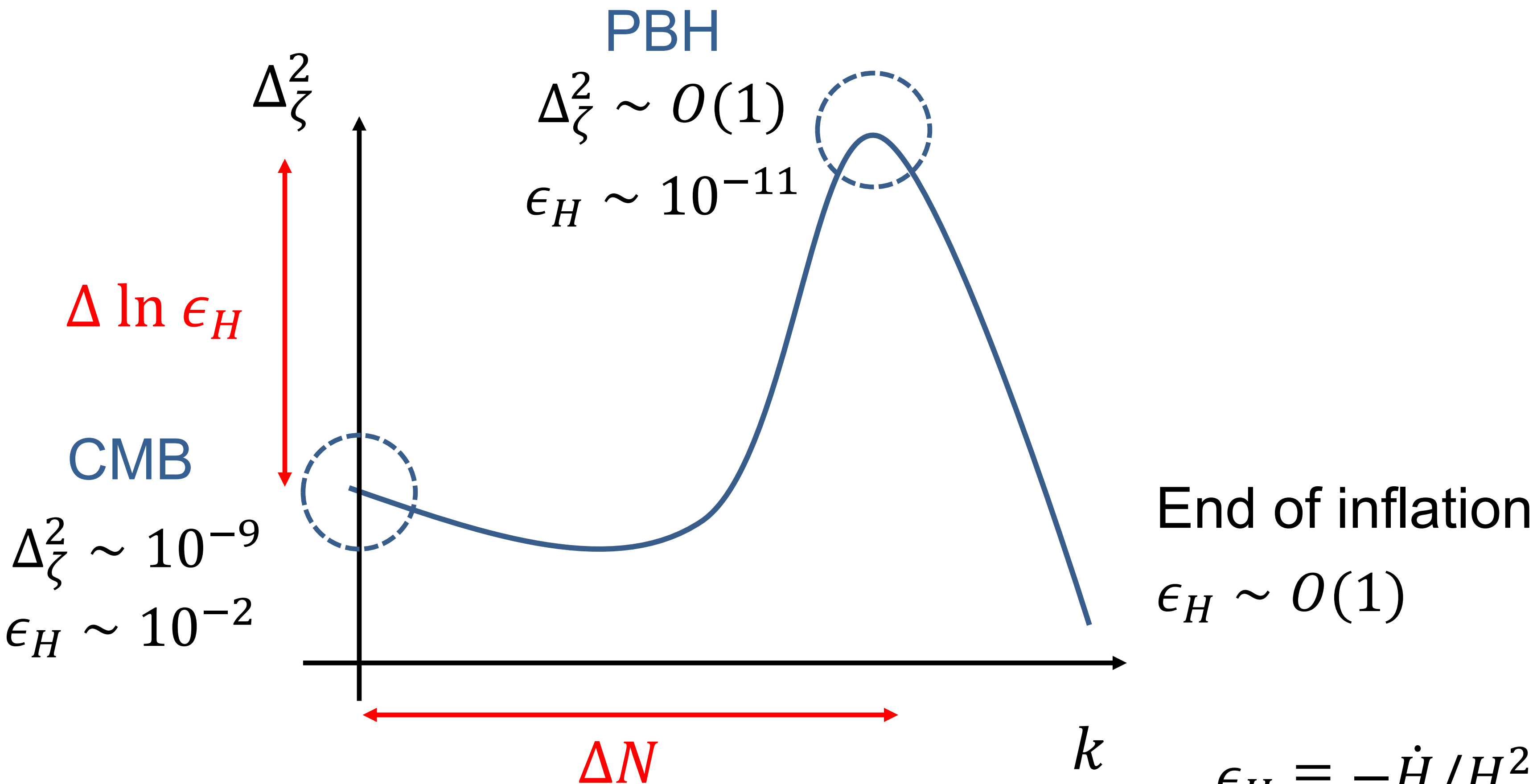
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

Gravitational Wave Physics and Astronomy Genesis

科研費 KAKENHI


Inflation
Primordial spectrum
PBH
Binary
- Induced GWs
- DM
- Lensing
- GWs



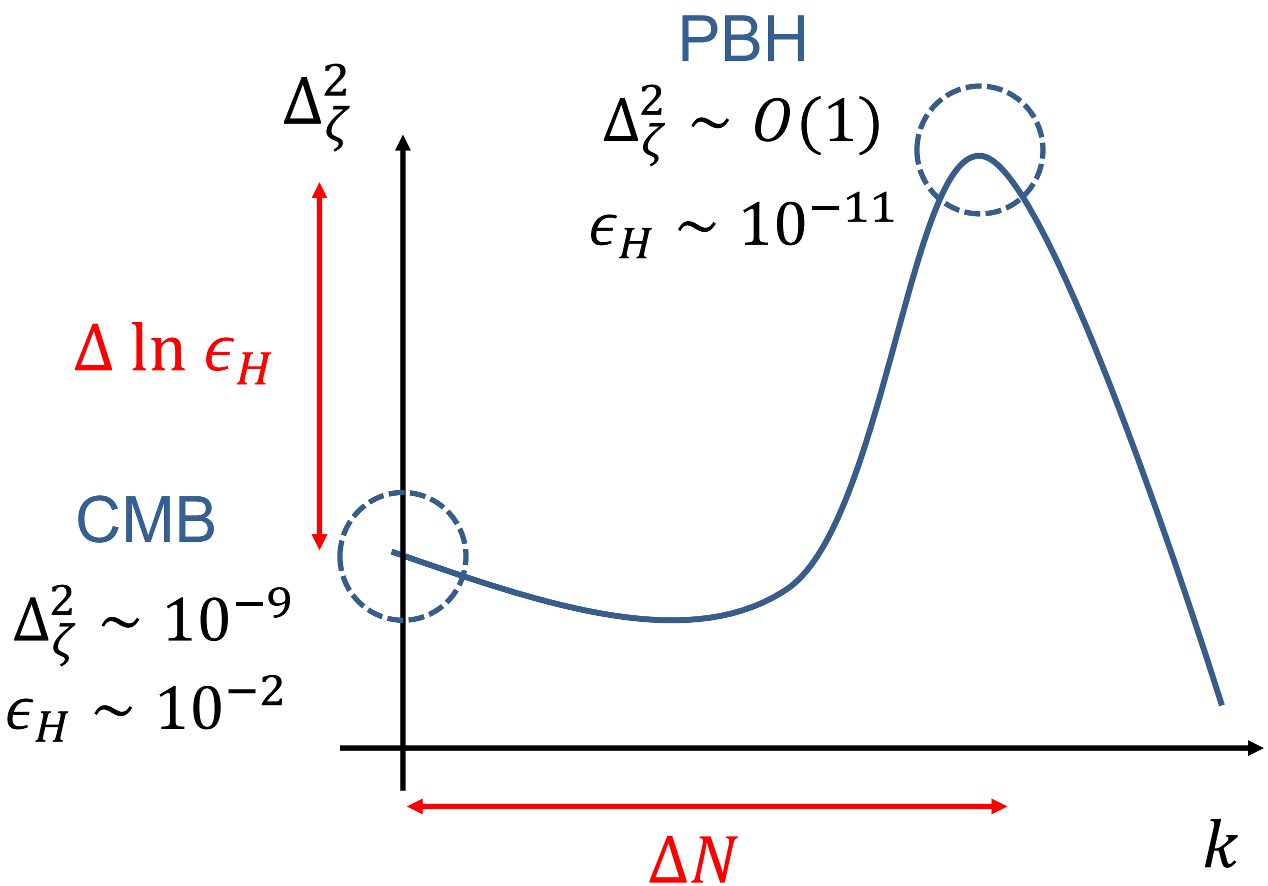
$$\epsilon_H \equiv -\dot{H}/H^2$$

Leading-order slow-roll approx.

$$\Delta_\zeta^2 \approx \frac{H^2}{8\pi^2 \epsilon_H}$$

$\epsilon_H \ll 1$ and

$$\left| \frac{d \ln \epsilon_H}{dN} \right| \ll 1 \,?$$



End of inflation

$\epsilon_H \sim O(1)$

$$\epsilon_H \equiv -\dot{H}/H^2$$

$$\beta \approx \mathrm{erfc}\left(\frac{\zeta_c}{\sqrt{2}\Delta_\zeta}\right) \approx 10^{-9}\left(\frac{\Omega_{\mathrm{PBH}}h^2}{0.12}\right)\left(\frac{M}{M_\odot}\right)^{1/2}$$

$$\Delta N \approx 18 - \frac{1}{2}\ln\frac{M}{M_\odot}$$

$\epsilon_H \ll 1$ and

$$\left|\frac{d\ln\epsilon_H}{dN}\right| \ll 1\,?$$

No go of slow roll for PBH production

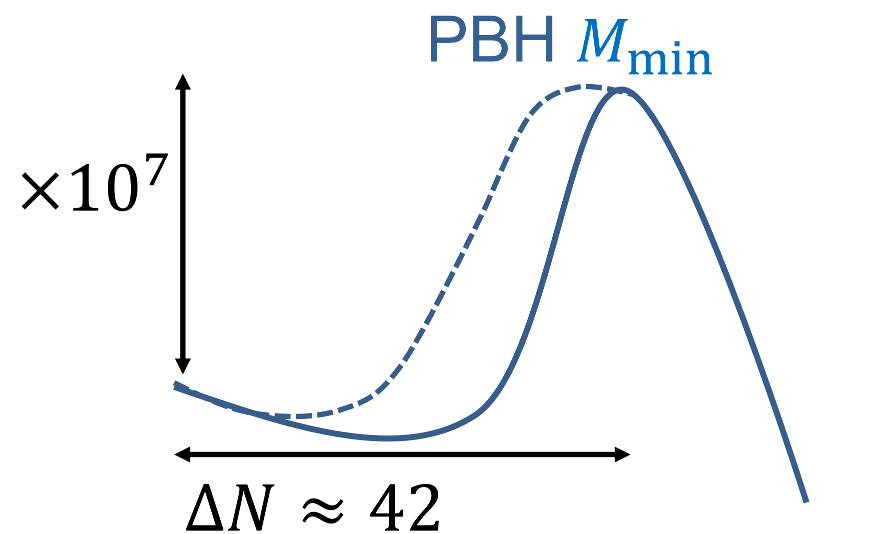
HM, Hu, 1706.06784

$$(\Omega_{\mathrm{PBH}}, M) = \begin{cases} (\Omega_{\mathrm{DM}}, M_{\mathrm{min}}) \quad \Rightarrow \left|\dfrac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38 \\ \\ \end{cases}$$

Slow-roll violation

$$\left|\frac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38$$

for single-field canonical inflation



$M_{\mathrm{min}} \approx 10^{-21} M_{\odot}$: Smallest PBH mass that does not evaporate by matter-radiation equality barring merging and accretion $\Longrightarrow$ Lower bound on SR violation

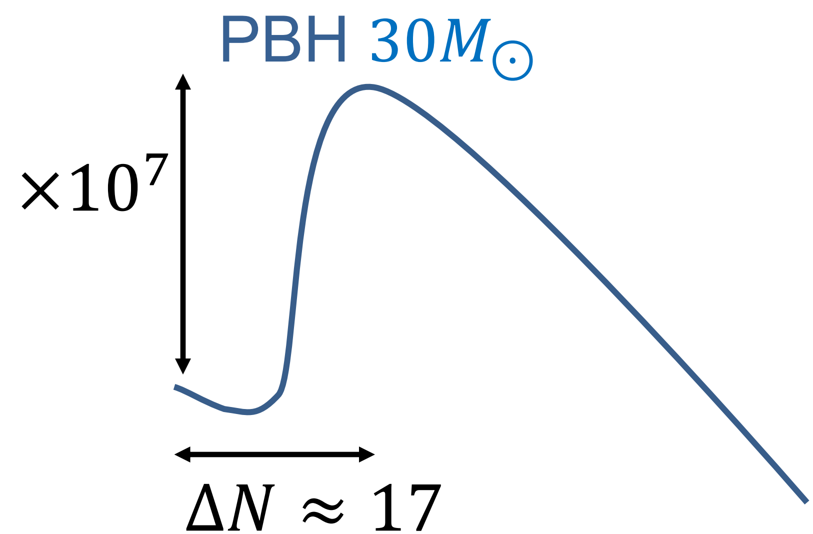
No go of slow roll for PBH production

HM, Hu, 1706.06784

$$
(\Omega_{\rm PBH}, M) = \begin{cases} (\Omega_{\rm DM}, M_{\rm min}) & \Rightarrow \left|\dfrac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38 \\ (10^{-3}\Omega_{\rm DM}, M_{\rm min}) & \Rightarrow \left|\dfrac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.37 \\ (10^{-3}\Omega_{\rm DM}, 30M_\odot) & \Rightarrow \left|\dfrac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.99 \end{cases}
$$

PBH = GW events scenario

Sasaki, Suyama,
Tanaka, Yokoyama, 1603.08338



No go of slow roll for PBH production

HM, Hu, 1706.06784

Passaglia, Hu, HM, 1812.08243

Summary:

PBH production

$\Leftarrow$ Sharp enhancement of $\Delta_\zeta^2(k)$

$\Leftrightarrow$ Quick decay of ϵ_H within $\sim$ 50 efolds

$\Leftrightarrow \; -\frac{\Delta \ln \epsilon_H}{\Delta N} > O(1)$

$\Leftarrow \; \epsilon_H \propto a^{-p}$ with $p = O(1)$

Simple realization:

Constant roll inflation with $p =$ const.

Martin, HM, Suyama, 1211.0083

HM, Starobinsky, Yokoyama, 1411.5021

Constant-roll inflation

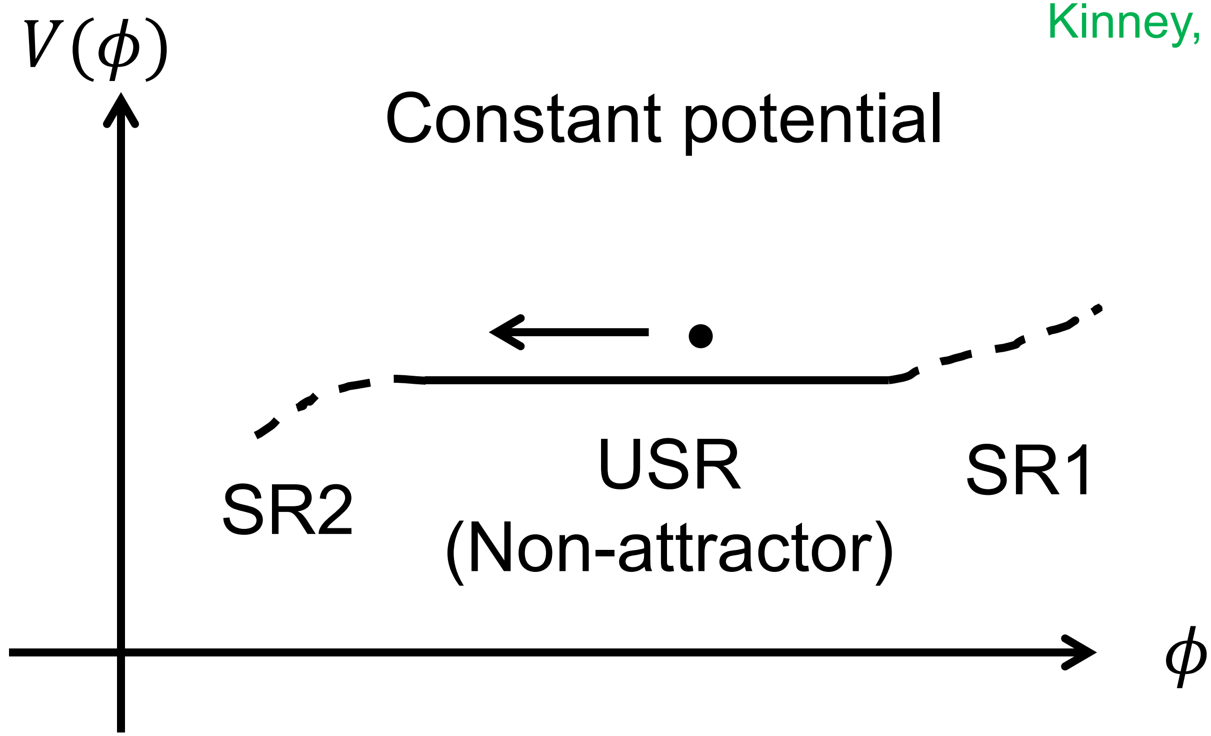
Canonical single filed inflation

	$\simeq 0$	Slow-roll
$\ddot{\phi}/(H\dot{\phi})$	$= -3$	Ultra slow-roll
	$= \beta$ (constant)	Constant-roll

Ultra slow-roll inflation ($\beta = -3$)

Tsamis, Woodard, astro-ph/0307463

Kinney, gr-qc/0503017



KG eq: $\ddot{\phi} = -3H\dot{\phi} \quad \Rightarrow \quad \dot{\phi} \propto a^{-3}$

$\Rightarrow \quad \epsilon_H \propto a^{-6}$

Ultra slow-roll inflation ($\beta = -3$)

Superhorizon solution of MS eq

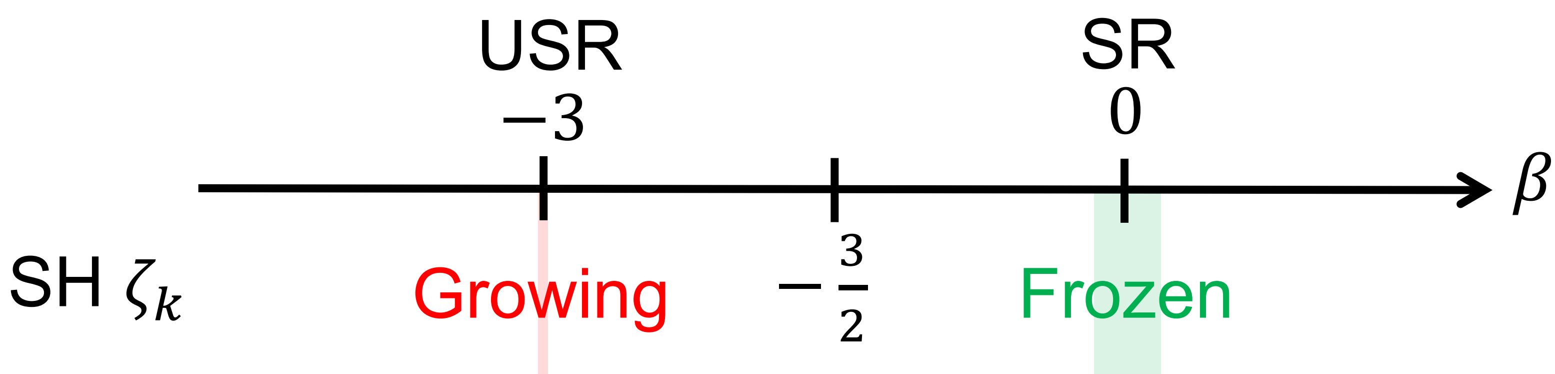
$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_H}$$

Constant mode

Slow-roll $\quad \epsilon_H \simeq \text{const.} \ll 1 \quad$ Decaying mode

Ultra slow-roll $\quad \epsilon_H \propto a^{-6} \quad$ Growing mode

While $\epsilon_H \ll 1$, $d\ln\epsilon_H/dN = -6$ violates slow roll.



Constant-roll inflation (general β)

Superhorizon solution

Martin, HM, Suyama, 1211.0083

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_H}$$

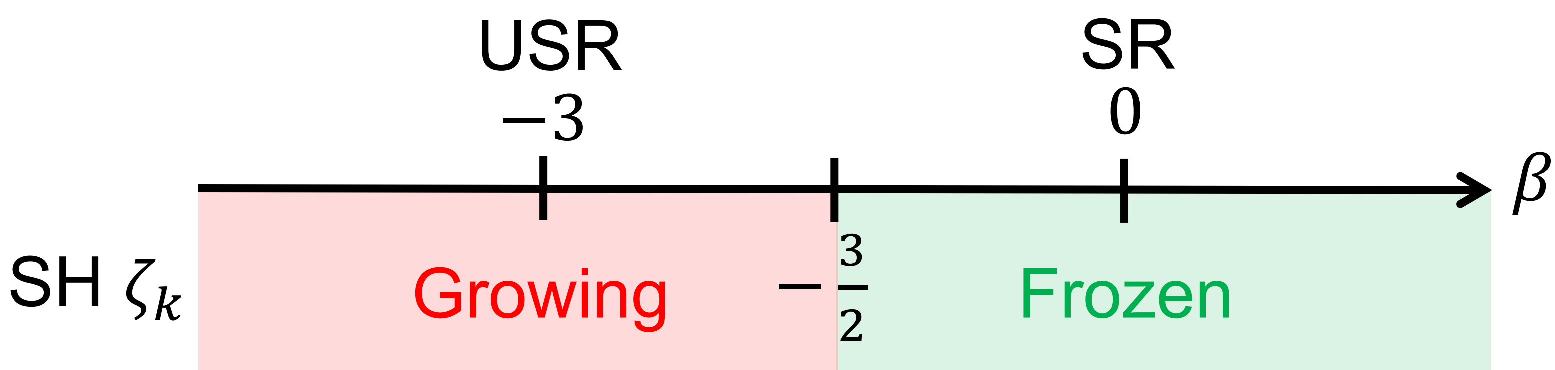
Constant mode

Decaying mode

Growing mode

Constant roll $\epsilon_H \propto a^{2\beta}$

$2\beta \gtrless -3$



Constant-roll inflation (general β)

HM, Starobinsky, Yokoyama, 1411.5021

Constant-roll condition $\ddot{\phi} = \beta H \dot{\phi}$

+ Hamilton-Jacobi formalism $H = H(\phi)$.

$$-2\dot{H} = \dot{\phi}^2 \quad \Rightarrow \quad \dot{\phi} = -2\frac{dH}{d\phi} \quad \Rightarrow \quad \frac{d^2 H}{d\phi^2} = -\frac{\beta}{2} H$$

Analytic solution:

$H(\phi) =$ linear combination of $e^{\pm\sqrt{-\beta/2}\,\phi}$

$\Rightarrow V(\phi) = 3H^2 - \frac{\dot{\phi}^2}{2} = 3H^2 - 2\left(\frac{dH}{d\phi}\right)^2$

$=$ linear combination of $e^{\pm\sqrt{-2\beta}\,\phi}$

Analytic solutions for $\phi(t), H(t), a(t)$

can also be derived.

Constant-roll inflation (general β)

HM, Starobinsky, Yokoyama, 1411.5021

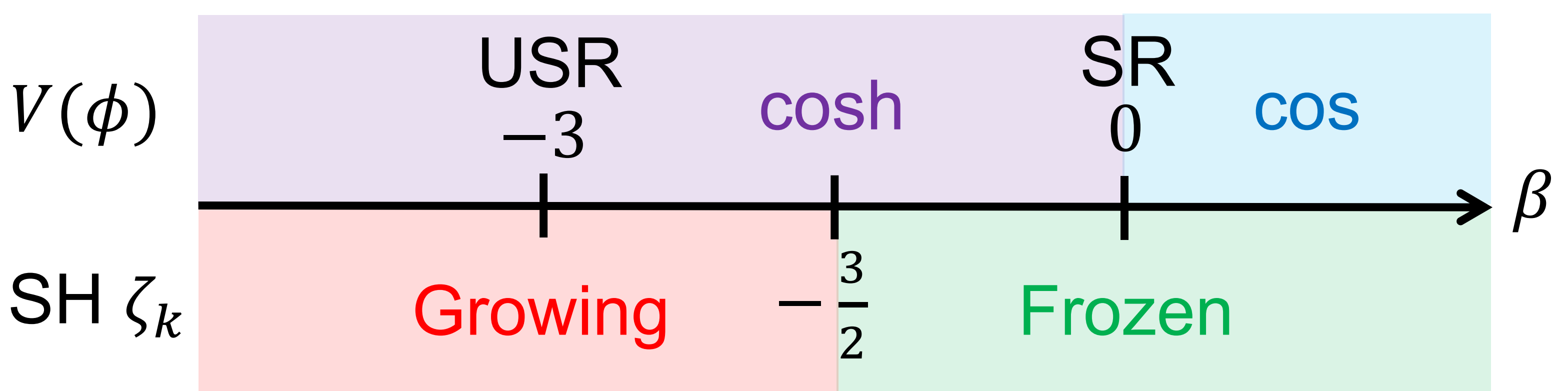
Potentials

a) $V \propto e^{\sqrt{-2\beta}\phi}$ with $\beta < 0$: Power-law inflation

$r = 8(1 - n_s)$

b) $V \propto \cosh(\sqrt{-2\beta}\phi)$ + const. with $\beta < 0$

c) $V \propto \cos(\sqrt{2\beta}\phi)$ + const. with $\beta > 0$



cosh potential $(\beta < 0)$

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = 0$.

Analytic solution

$$\frac{\phi(t)}{M_{Pl}} = \sqrt{\frac{2}{-\beta}} \ln\left[\coth\left(\frac{-\beta}{2} Mt\right)\right] \rightarrow 0 \qquad (t \rightarrow \infty)$$

$$\frac{H(t)}{M} = \coth(-\beta Mt) \rightarrow 1$$

$$a(t) \propto \sinh^{-1/\beta}(-\beta Mt) \rightarrow e^{Mt}$$

$$\epsilon_1 \equiv -\dot{H}/H^2$$

$$\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$$

Slow-roll parameters

$$2\epsilon_1 = \epsilon_{2n+1} = -\beta/\cosh^2(-\beta Mt) \rightarrow 0$$

$$\epsilon_{2n} = 2\beta \tanh^2(-\beta Mt) \rightarrow 2\beta$$

cos potential $(\beta > 0)$

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = \phi_c$.

Analytic solution

$$\frac{\phi(t)}{M_{Pl}} = 2\sqrt{\frac{2}{\beta}}\arctan(e^{\beta M t}) \to 0 \qquad (t \to -\infty)$$

$$\frac{H(t)}{M} = -\tanh(\beta M t) \qquad \to 1$$

$$a(t) \propto \cosh^{-1/\beta}(\beta M t) \qquad \to e^{Mt}$$

$$\epsilon_1 \equiv -\dot{H}/H^2$$
$$\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$$

Slow-roll parameters

$$2\epsilon_1 = \epsilon_{2n+1} = 2\beta/\sinh^2(\beta M t) \to 0$$

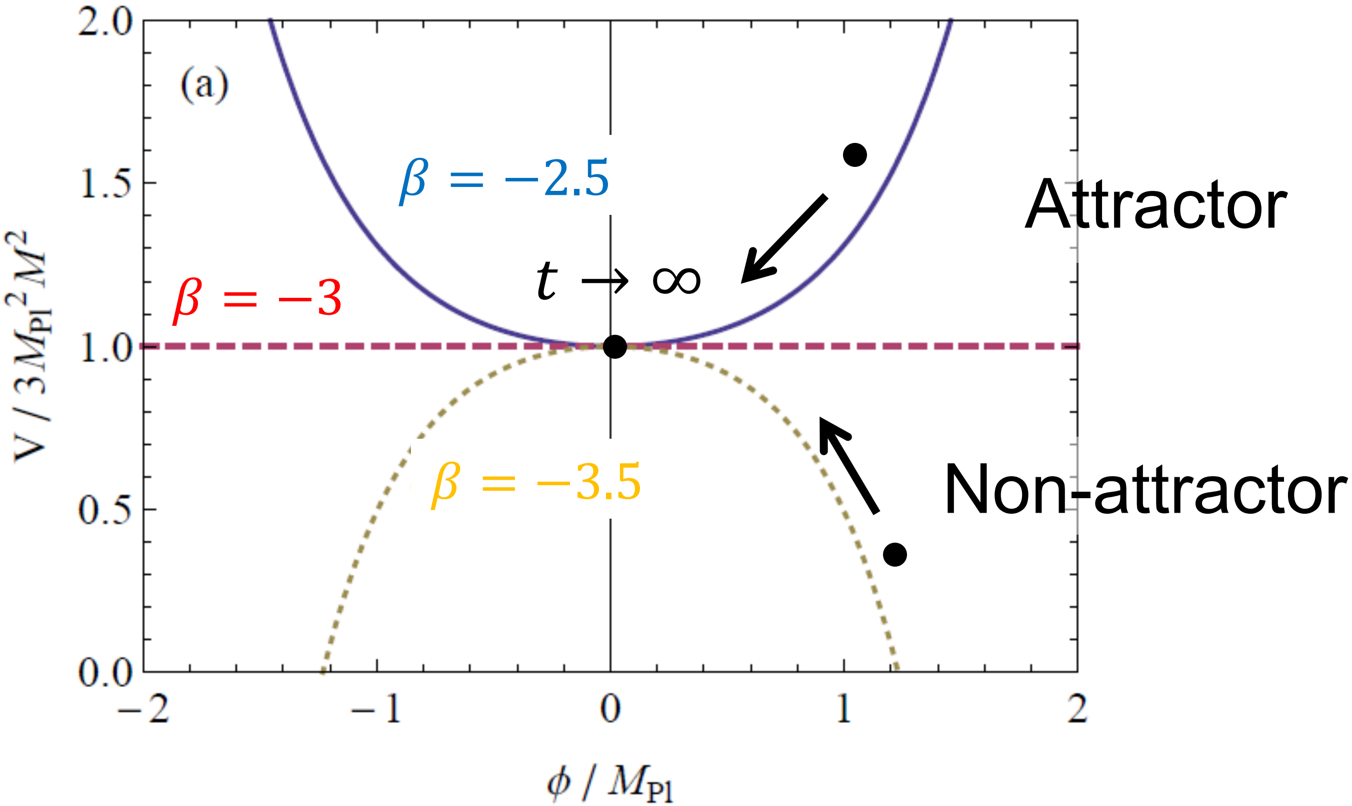
$$\epsilon_{2n} = 2\beta\tanh^2(\beta M t) \qquad \to 2\beta$$

Same asymptotic values

cosh potential ($\beta < 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

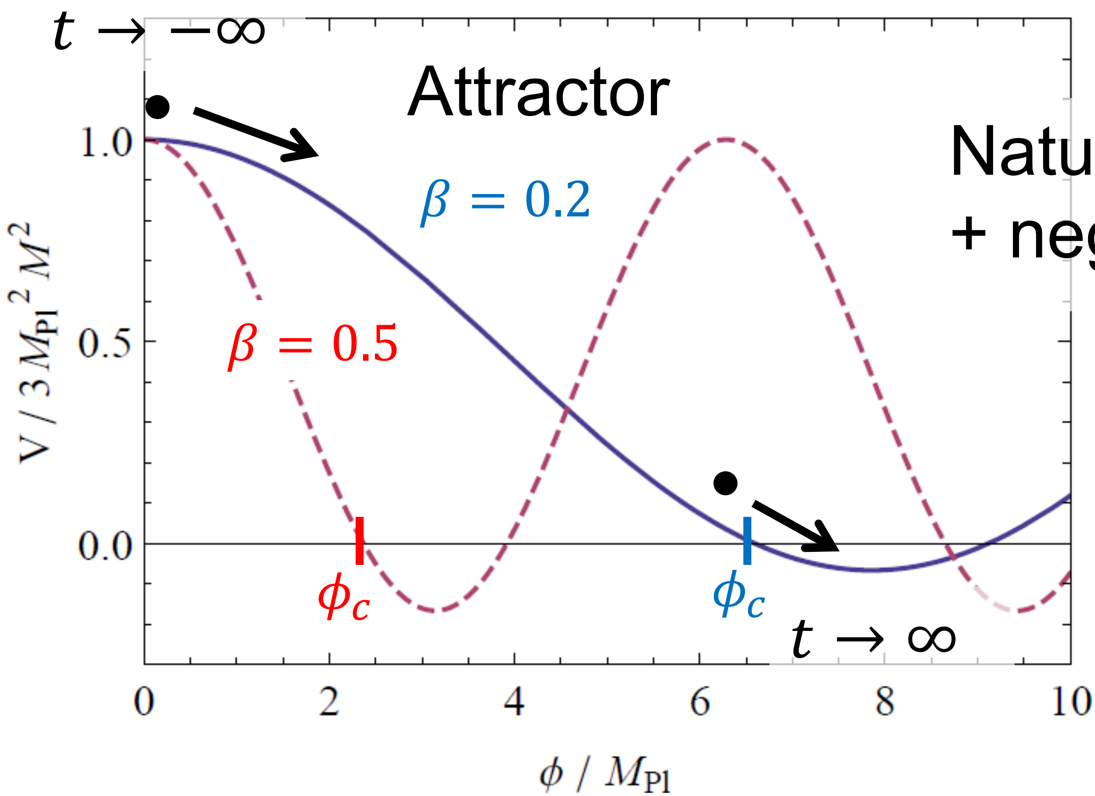
Assume a transition to reheating at $\phi > 0$.



cos potential ($\beta > 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

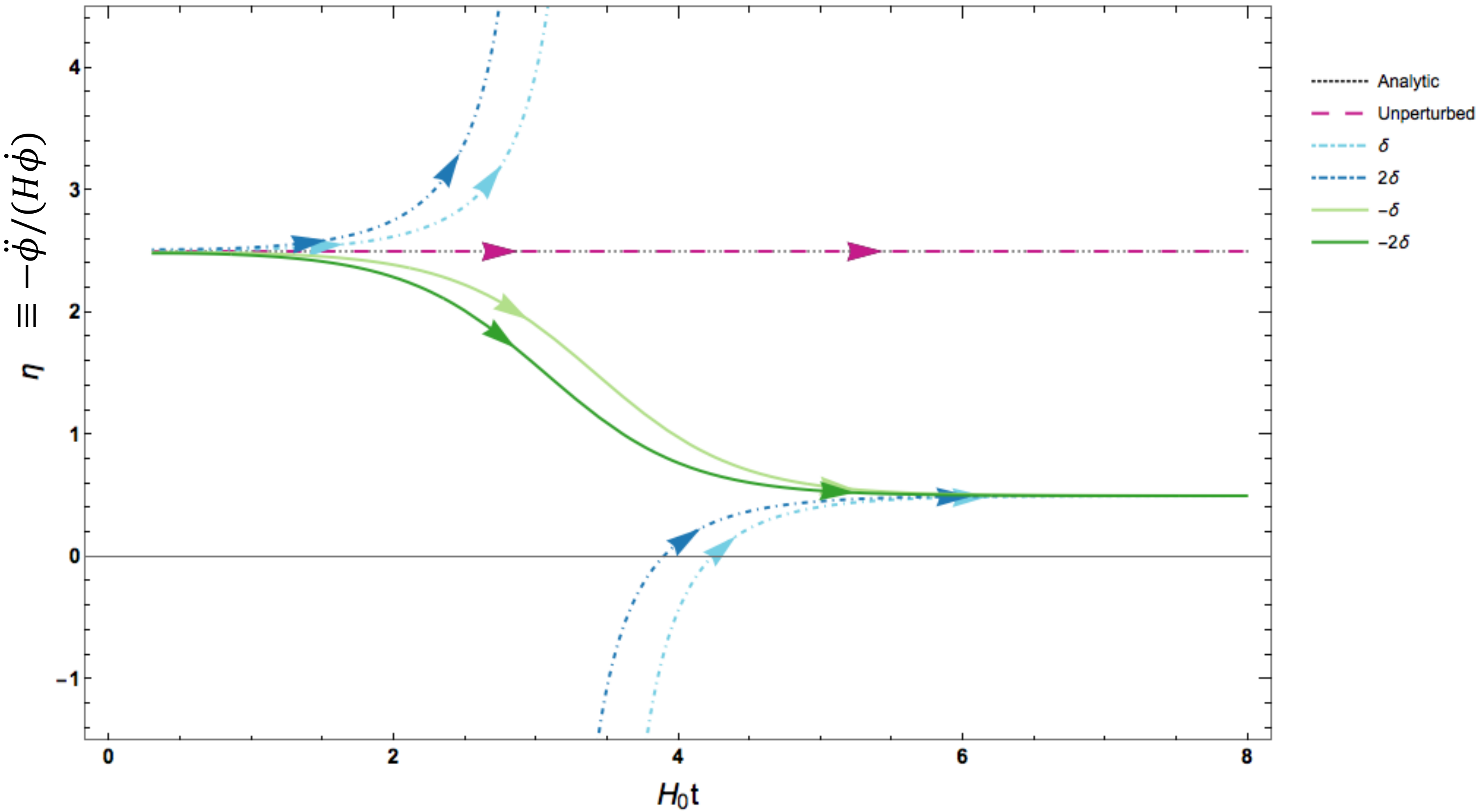
Assume a transition to reheating at $\phi < \phi_c$.



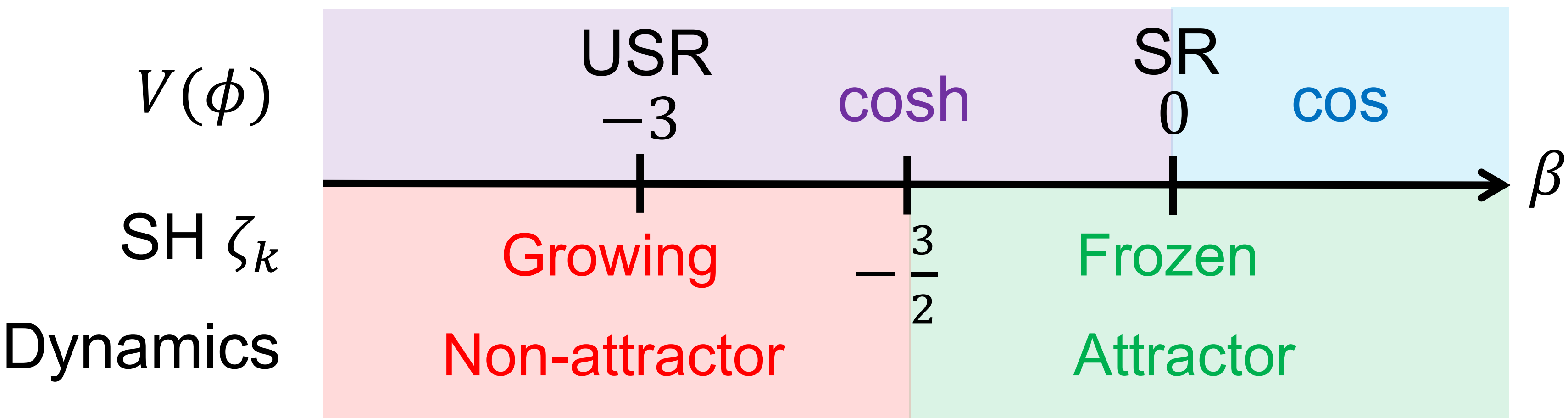
Natural inflation
+ negative CC

Non-attractor behavior

Lin, Morse, Kinney, 1904.06289



$\ddot{\phi}/(H\dot{\phi}) = \beta$ (constant)



Curvature perturbation

HM, Starobinsky, Yokoyama, 1411.5021

Mukhanov-Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

where $v_k = \sqrt{2} z \zeta_k$ with $z = a\sqrt{\epsilon_1}$.

$\epsilon_1 \equiv -\dot{H}/H^2$

$\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$

Without approximation,

$$\frac{z''}{z} = a^2 H^2 \left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{2}\epsilon_2\epsilon_3\right)$$

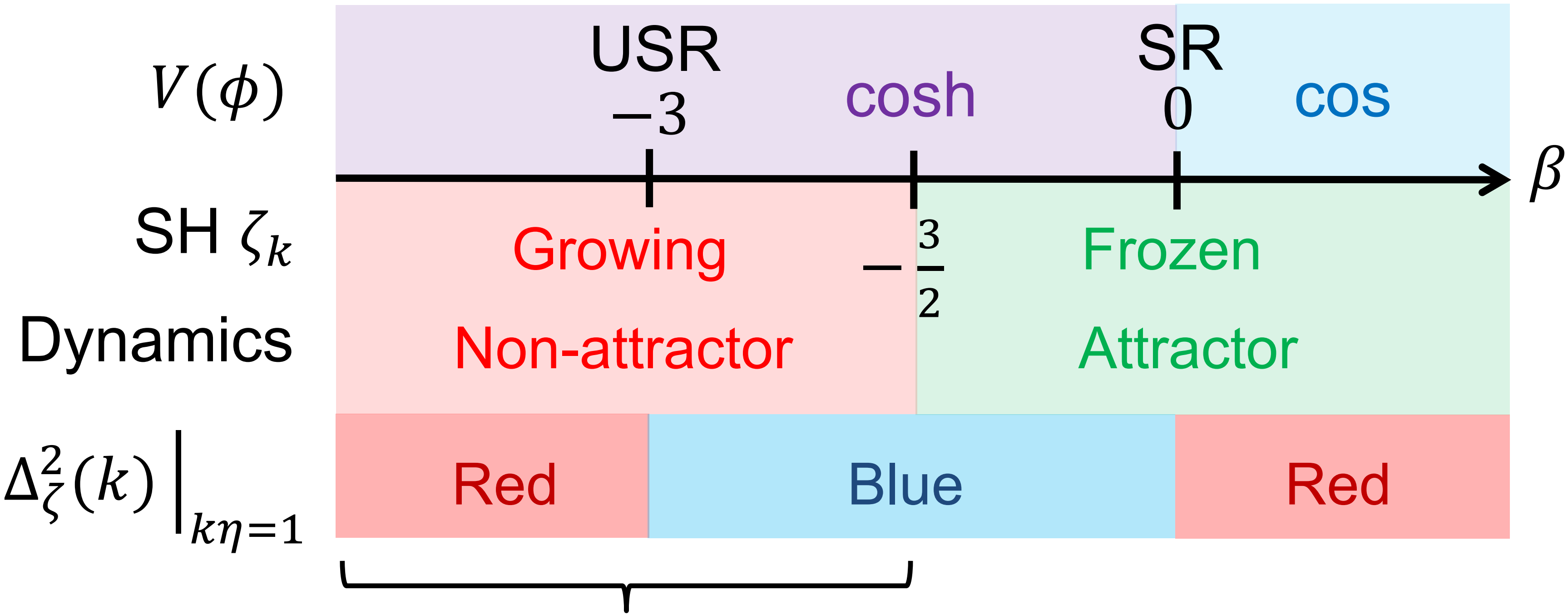
For both cosh potential and cos potential,

$$\frac{z''}{z} \to \frac{(\beta+2)(\beta+1)}{\tau^2} = \frac{\nu^2 - 1/4}{\tau^2}$$

where $\nu \equiv |\beta + 3/2|$ and hence

$$n_s - 1 \Big|_{k\eta=1} = 3 - |2\beta + 3|$$

$$\ddot{\phi}/(H\dot{\phi}) = \beta \text{ (constant)}$$

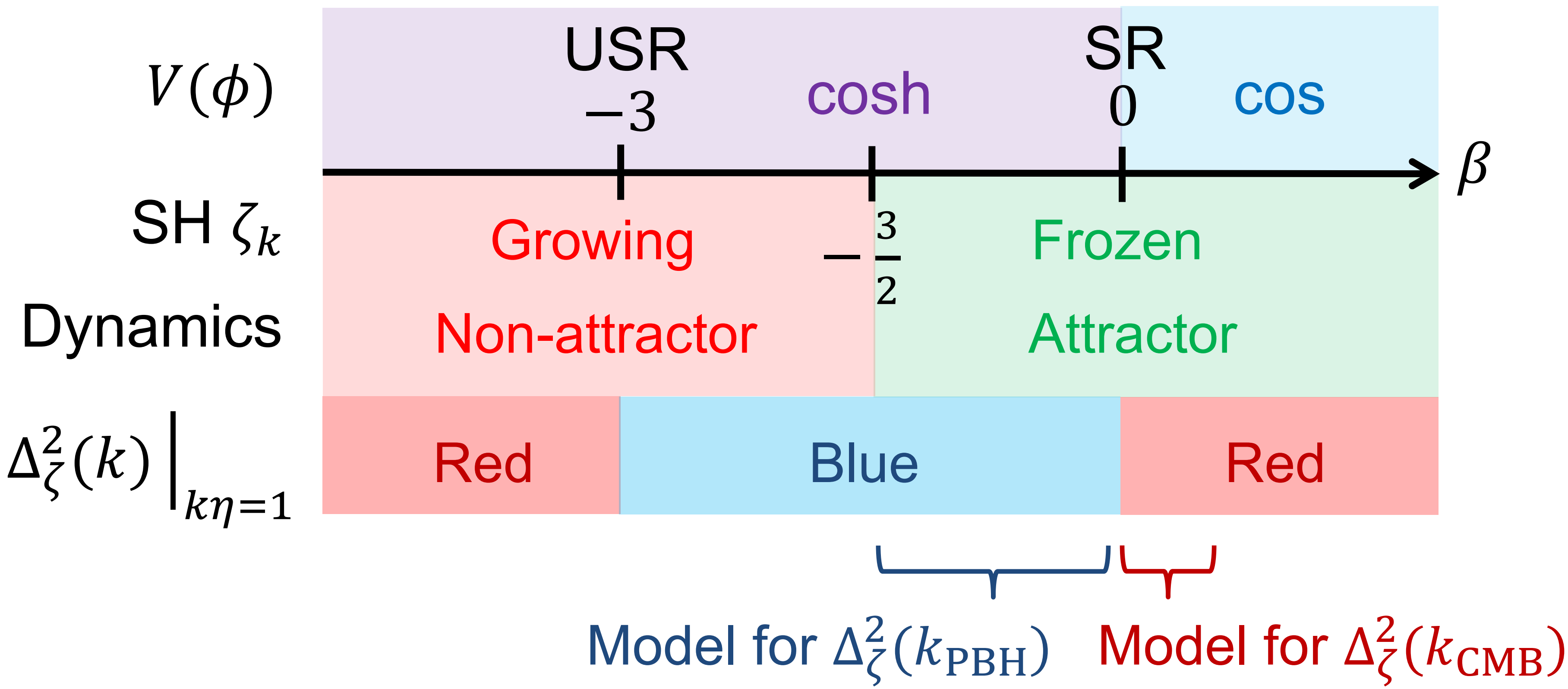


Need to take into account superhorizon growth to evaluate final Δ_ζ^2

$$n_s - 1\big|_{k\eta=1} = 3 - |2\beta + 3|$$

HM, Starobinsky, Yokoyama, 1411.5021

$\ddot{\phi}/(H\dot{\phi}) = \beta$ (constant)



HM, Mukohyama, Oliosi, 1910.13235

$\beta \approx 0.015$

HM, Starobinsky, Yokoyama, 1411.5021

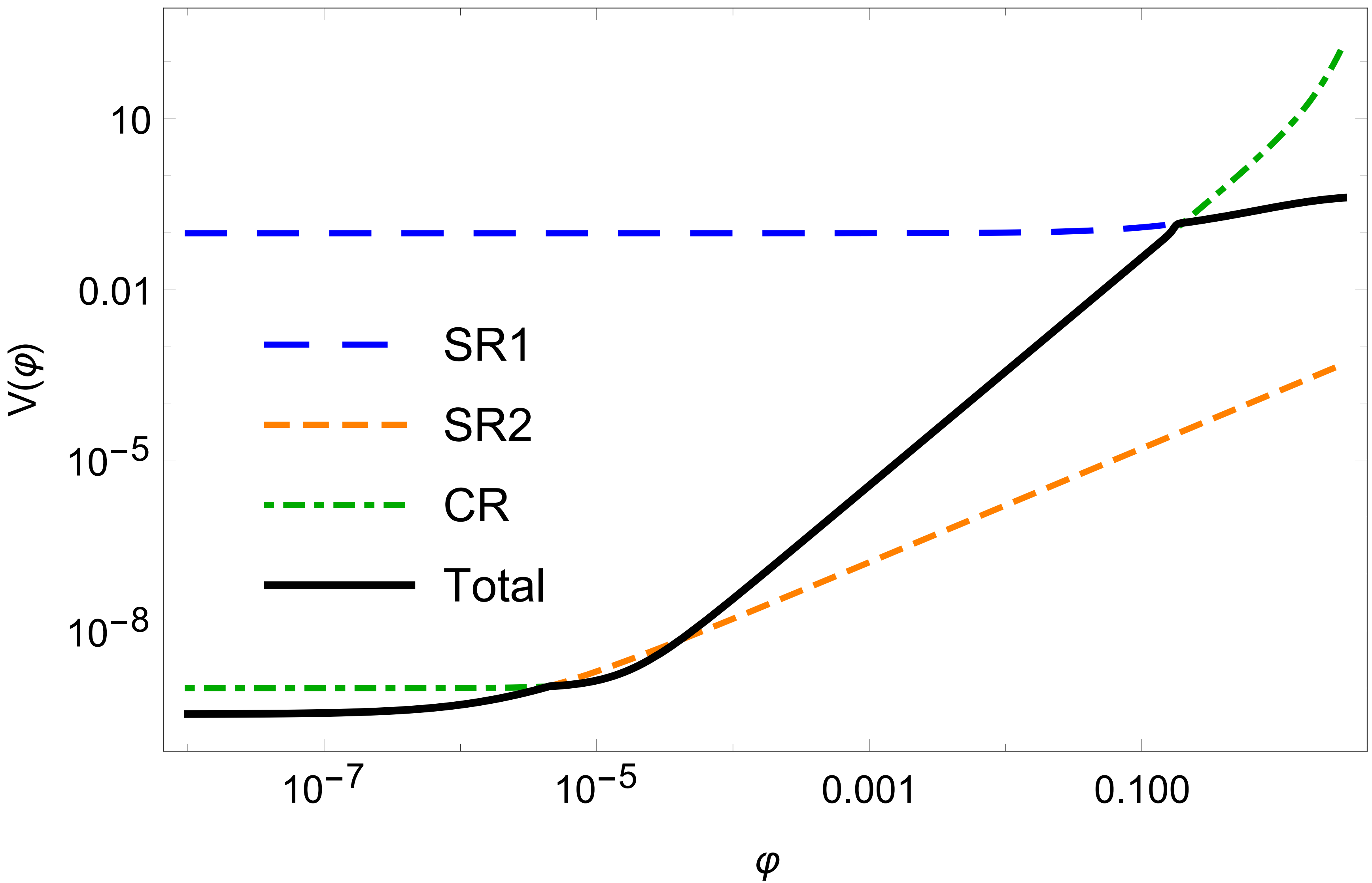
HM, Starobinsky, 1702.05847

Other CR models

HM, Starobinsky, 1704.08188, 1909.10883

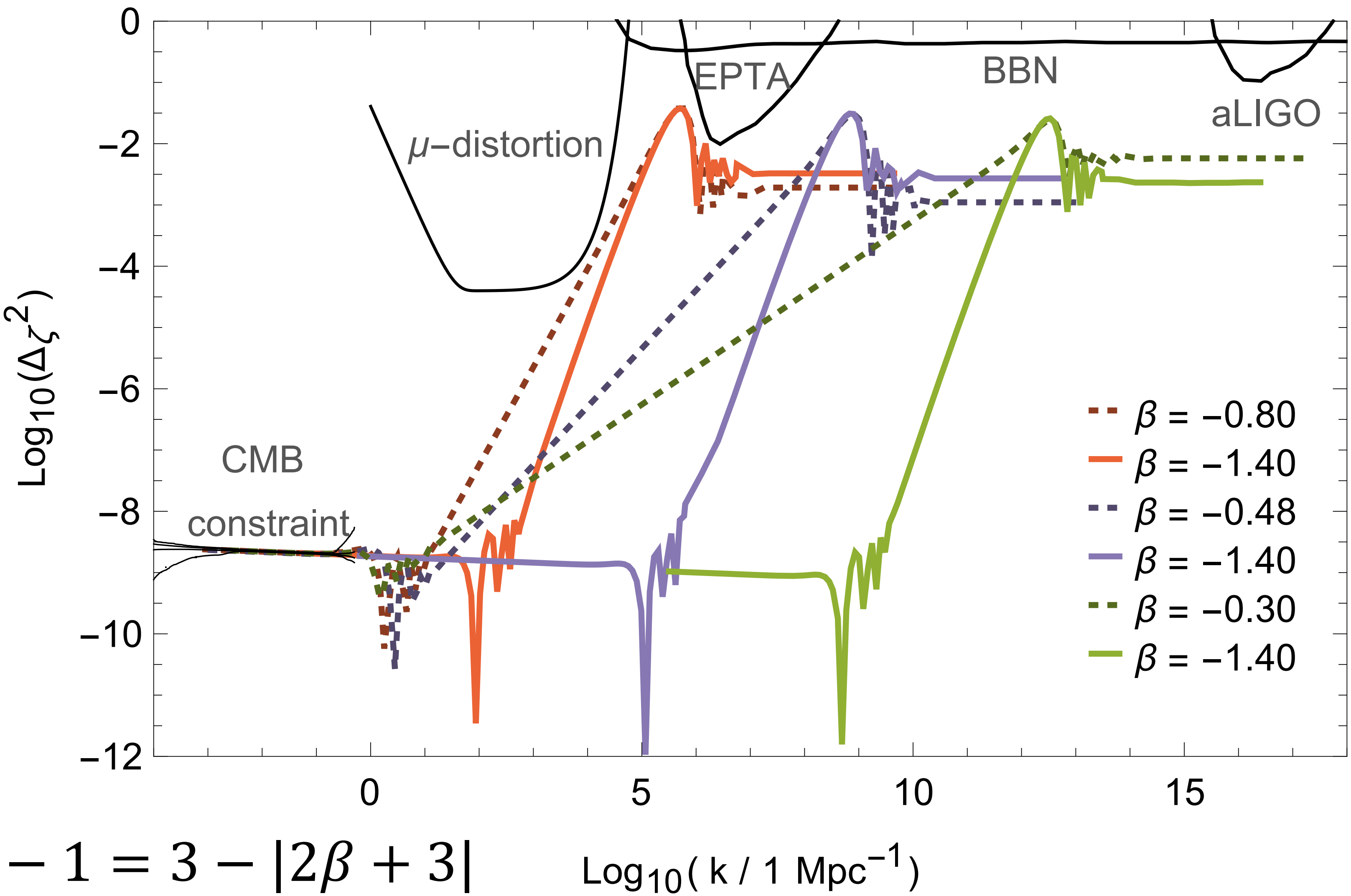
PBH from constant-roll inflation

HM, Mukohyama, Oliosi, 1910.13235



PBH from constant-roll inflation

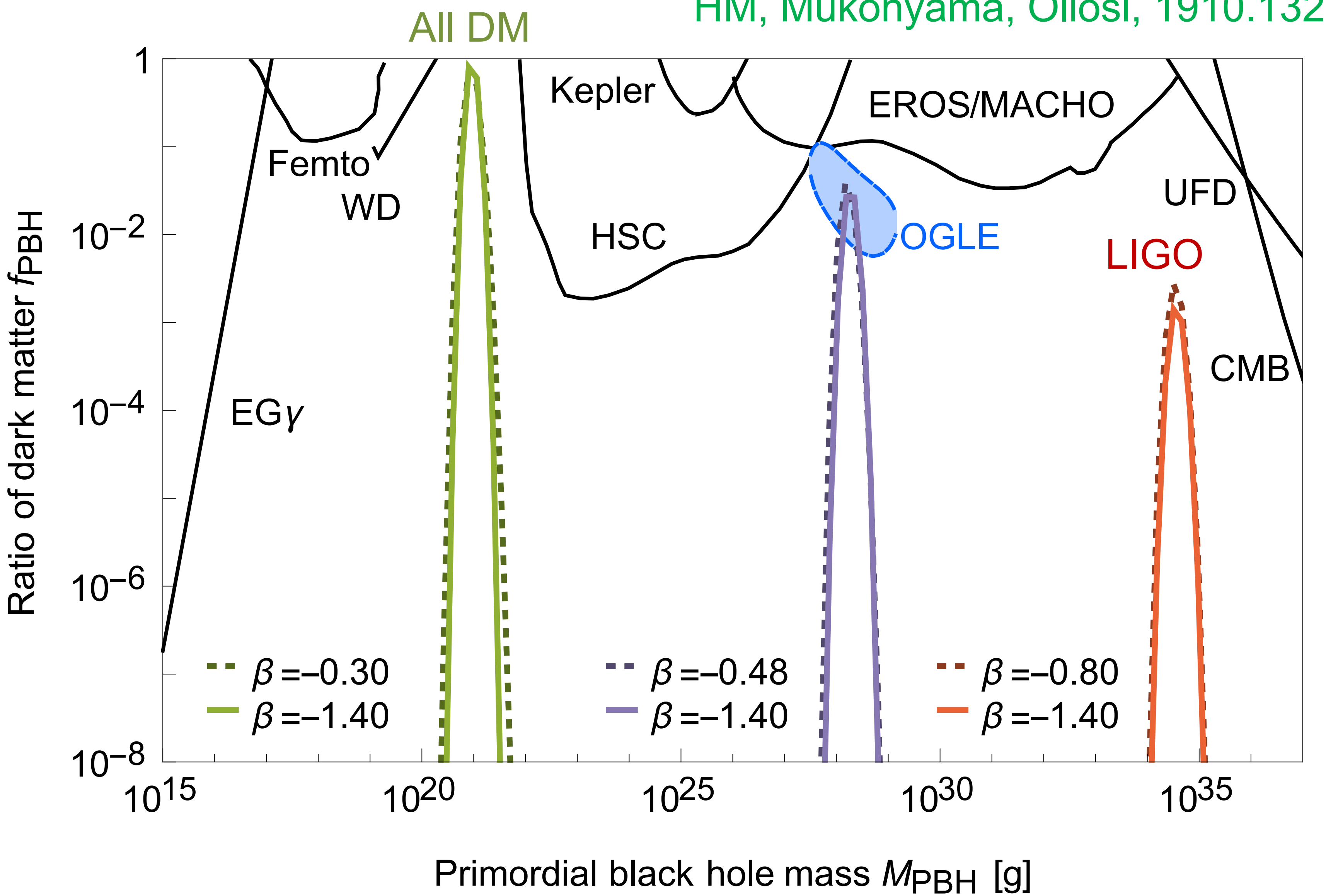
HM, Mukohyama, Oliosi, 1910.13235



$$n_s - 1 = 3 - |2\beta + 3|$$

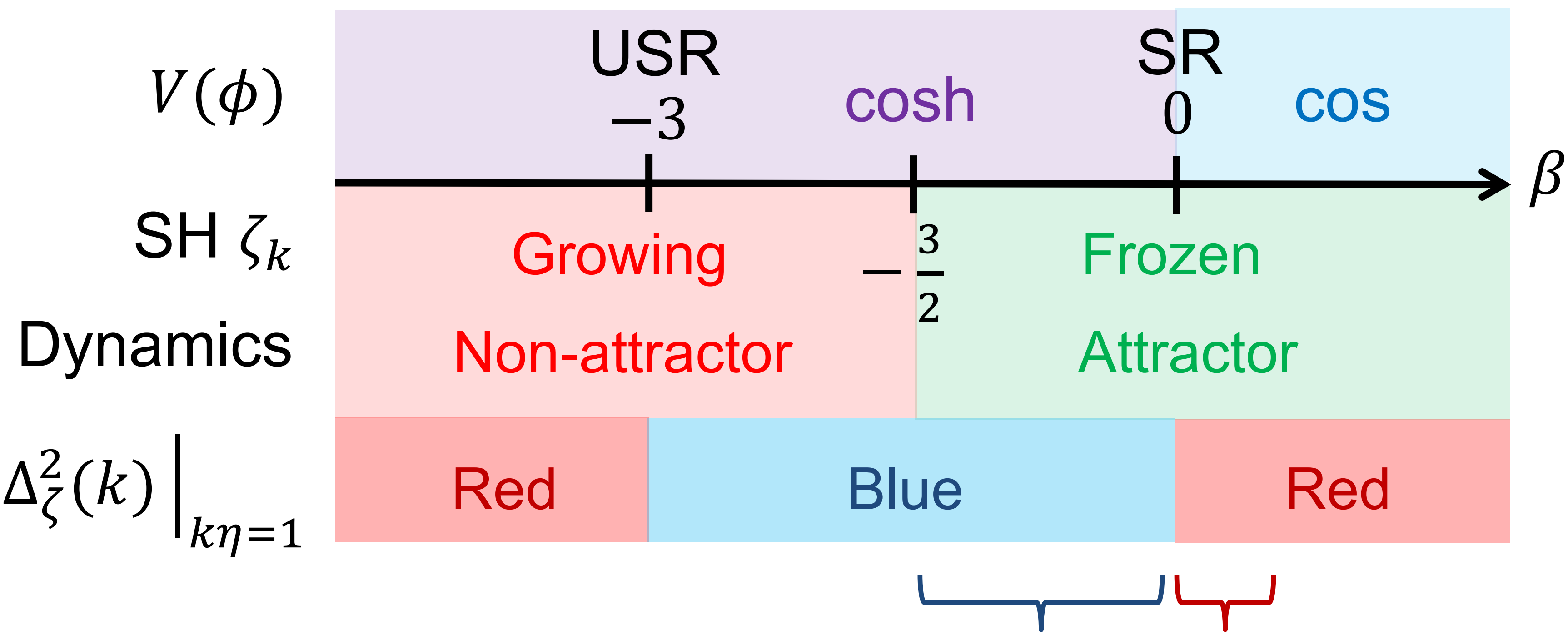
PBH from constant-roll inflation

HM, Mukohyama, Oliosi, 1910.13235



Summary

$$\ddot{\phi}/(H\dot{\phi}) = \beta \text{ (constant)}$$



Model for $\Delta_\zeta^2(k_{\mathrm{PBH}})$ with various values of slope

Model for $\Delta_\zeta^2(k_{\mathrm{CMB}})$

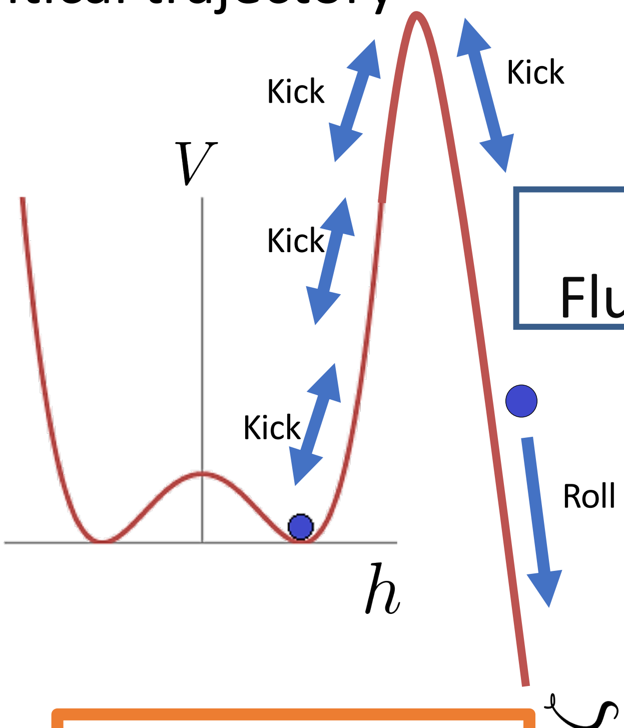
$\beta \approx 0.015$

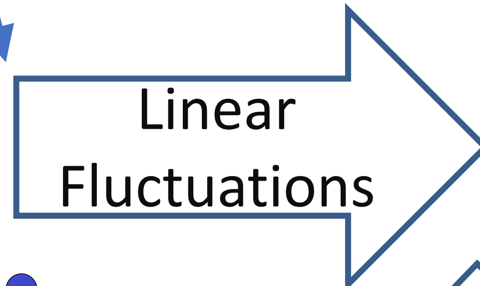
$$n_s - 1 = 3 - |2\beta + 3|$$

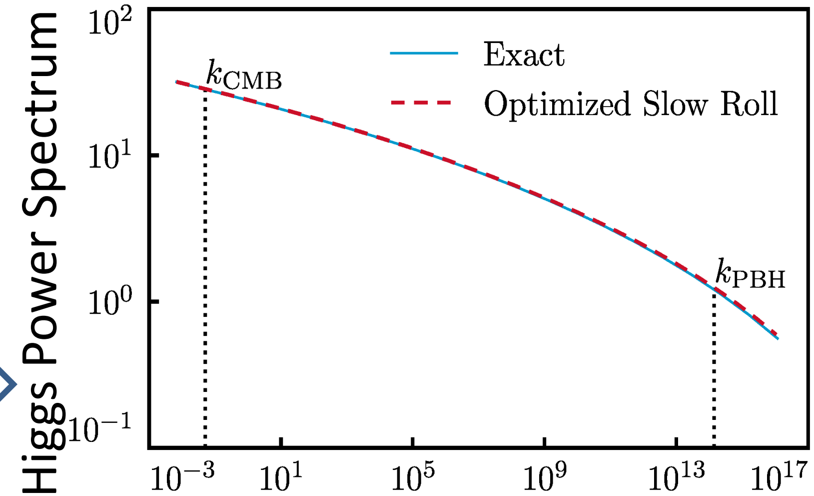
PBH DM from Higgs?

Passaglia, Hu, HM, 1912.02682

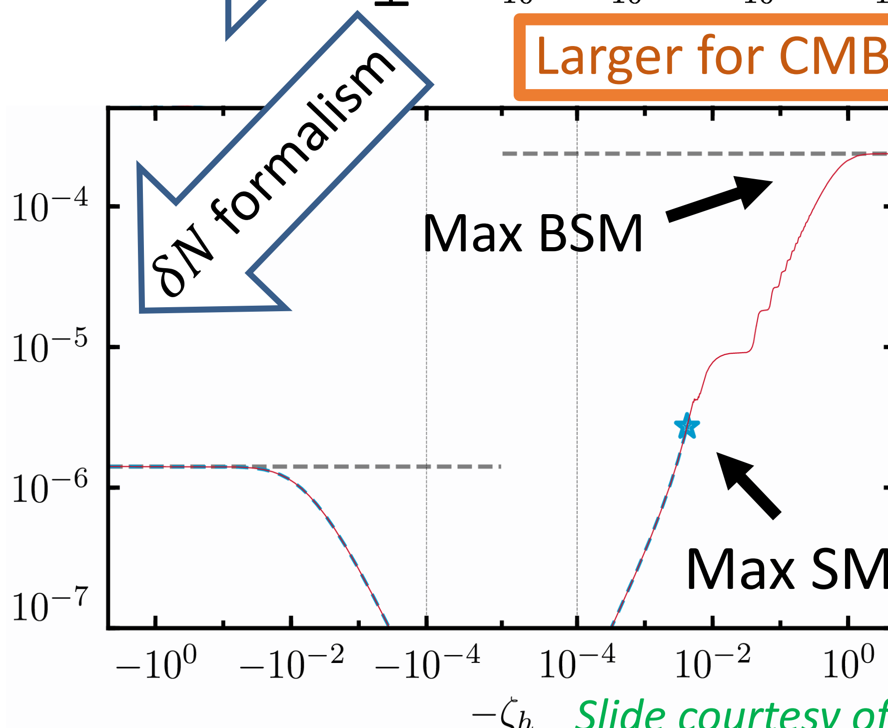
Higgs stochastically reaches a critical trajectory







Larger for CMB than PBH!



Not converted to large curvature at reheating!
No PBHs or GWs

Slide courtesy of Sam Passaglia