

Nom-Linear PBH statistics

— with Gaussian curvature perturbations —

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CREDITS:

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Based on:

1907.13311

1912.xxxxx



Disclaimer:

We are going to talk about
rare events



Spherical symmetry

PBH Threshold

@ super-hubble scales (grad exp. approx.)

$$ds^2 \simeq -dt^2 + a^2(t) \left[\frac{dr^2}{1 - \boxed{r^2 K(r)}} + r^2 d\Omega^2 \right]$$

~ "grav. potential"
of the perturbation

When does a BH form?

A Japanese story

1999: PRD, Shibata~Sasaki

→ over-mass

Horizon forms when: $C(r_m) = \frac{2EM}{(R \approx r)} \sim 1$ ($m = M_{\text{MAX}}$)

$$C'(r_m) = 0$$



compaction function
(cute old Schwarzschild)

2014: Nakama et. al IJAP

From simulation it seems only 2 parameters
define the threshold

The Italian...

2018: Musco

Indeed the threshold to trigger
grav. coll. into BH

$$C(r_m) \simeq f(\omega) r_m^2 K(r_m) \geq C_c$$

2 parameters C_c and r_m .

Is C_c a (carr) Universal number? **NO!**

The Threshold :

2018: Musco :

C_c depends upon the
full shape $\frac{C(r)}{C(r_m)}$

The Threshold :

2018: Musco :

C_c depends upon the

gull shepe

$$\frac{C(r)}{C(r_m)}$$

Really?

Well mathematically yes ----

but physically is another story!

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• Regularity $K(r)r^2 \xrightarrow{r \rightarrow 0} 0$



Who cares about the shape $K(r)$ for $r \rightarrow 0$

Well mathematically yes ----

but physically is another story!

- The threshold is the condition for which the only shell with enough over-density to collapse is the one @ r_m

$r > r_m$ diffuse away

$r < r_m$ resisting collapse

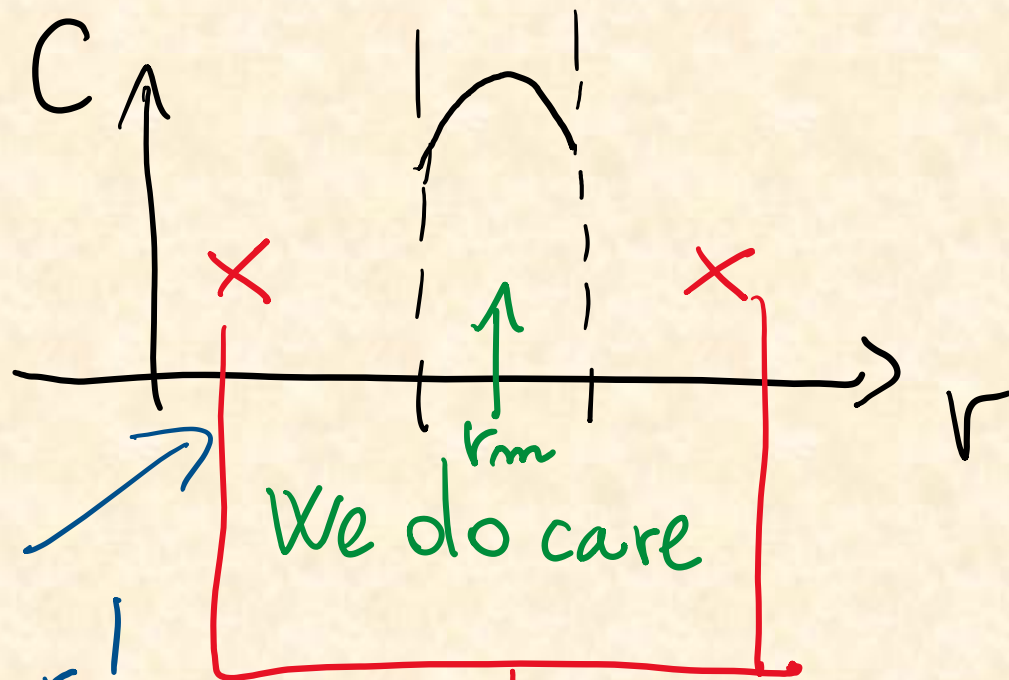


$\Rightarrow$ zero mass BH !

So

(Escrivà, CG, Sheth)

Threshold only cares about
pressure gradient around $r \sim r_m$



as long
as is regular!

We do not care!

Cool!

We just need a base for approximating

$K(r)$ around $r = r_m$

Because of numerical stability we have chosen

$$K_b(r) = \frac{C(r_m)}{f(\omega) r_m^2} e^{\frac{1}{q} \left(1 - \left(\frac{r}{r_m}\right)^{2q}\right)}$$

Exp basis
with peak $r = r_m$

where: $q \equiv - \frac{C'(r_m) r_m^2}{4 C(r_m)}$

Bottom Line:

The threshold only depends upon

$$q = - \frac{C''(r_m) r_m^2}{4 C(r_m)} \quad \leftarrow \text{normalized curvature (shape)}$$

and r_m 2 numbers !!

Back to Japan!

The Indo-European
(me-Ravi-Albert)

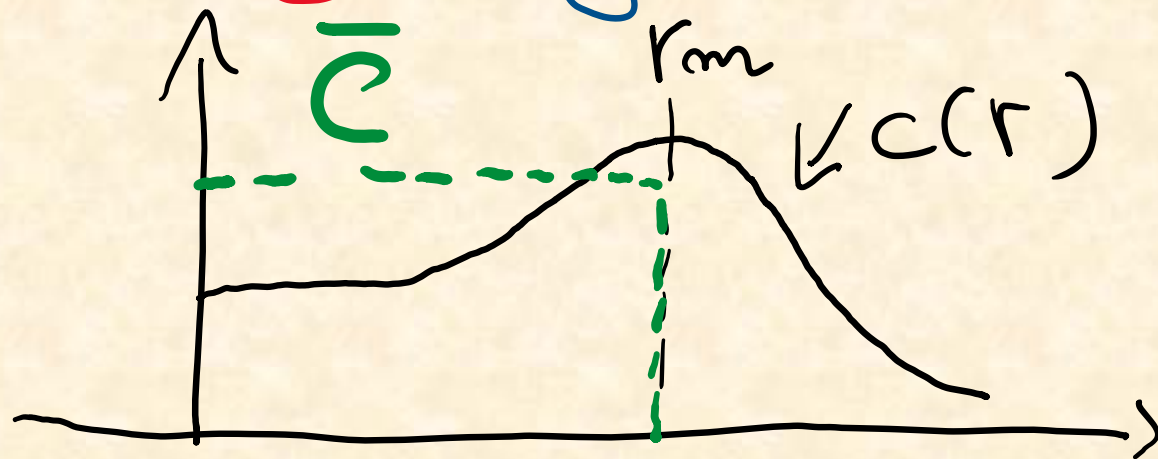
At this level still $C_c(r_m)$ depends
numerically and non-linearly on q .



$C_c(r_m)$ is far from being a
universal number!

Nevertheless,
We (= Indo-European) asked the following
question:

What would be the threshold associated
to the average of $C(r)$?



From now on only in Radiation

Sorry Alex Kusenko

but "easily" generalizable for
any fluid

A compaction function of the square form has only one threshold corresponding to the Harada-Yoo-Kohri limit (HYK) ($q \rightarrow 0$)

$$\text{threshold} \rightarrow \bar{C}_c = \frac{3}{r_m^3} \int_0^{r_m} C_c(r) r^2 dr = 0.4 \underset{\substack{\uparrow \\ \text{HYK}}}{1}$$

$\uparrow$ threshold

In fact $\bar{C} = \frac{2}{5} = 0.4$

as can be seen in $q \rightarrow \infty$ limit

So yes Bernard, after all there is
a shape independent threshold!

Inverting the integral

$$C_c(r_m) = \frac{4}{15} e^{-\frac{1}{q}} \frac{1 - 5/2 q}{\Gamma(\frac{5}{2q}) - \Gamma(\frac{5}{2q}, \frac{1}{q})}$$

↑
physical threshold

Analytical!!!

So yes Bernard, after all there is
a shape independent threshold!

Inverting the integral

$$C(v) = \frac{4}{15} e^{-\frac{1}{q}} \frac{1 - 5/2 q}{\Gamma(\frac{5}{2}q) - \Gamma(\frac{5}{2}q, \frac{1}{q})}$$

physical threshold

Public code

Escrivà

1907.13065

Analytical!!!

Nom - Linear statistics

(me-Ravi, almost baked)

So far we found that the threshold depends upon 2 parameters

$$r_m^2 C''(r_m), \quad r_m$$

Given a point in space $\vec{x}_0$ what is the prob. that the compaction function is maximal at $r_m = |\vec{x}_m - \vec{x}_0|$ with $C(r_m) \geq C_c$?

Preamble

$$\vec{x}_0 \Rightarrow C = C(r, \vec{x}_0)$$

$$\text{where } r = |\vec{x} - \vec{x}_0|$$

Also:

$$r_H = \frac{1}{a_H}$$

$$G = \frac{r^2}{r_H^2} \delta e \left(1 - \frac{3}{8} \frac{r^2}{r_H^2} \delta e \right) \quad [\text{Non-linear!}]$$

No approx!

$$\delta e = \frac{3}{r^3} \int_0^r \frac{\delta S}{S} \Big|_{\text{linear}} \bar{r}^2 d\bar{r}$$

→ smoothing scale!

⇐ Averaged
linear over-density

$$\frac{\delta S}{S} \Big|_{\text{linear}} = \frac{4}{9} \frac{1}{a_H^2} \nabla^2 \zeta$$

curvature perturb.
INFLATION!

In Fourier space

$$\frac{r^2}{r_H^2} \delta_e = \frac{4}{9} \int \frac{dk}{k} (kr)^2 \underbrace{W_{TH}(kr)}_{\text{top-hat window function}} \mathcal{Z}_k$$

top-hat window
function

Assumption here $[ds^2 = -dt^2 + e^{2J} d\vec{x} \cdot d\vec{x}]$

} GAUSSIAN VARIABLE!



C is a non-centrally peaked
 χ^2 distribution!

Conditional Probability

- $\vec{x}_0$ is a center $\equiv$ a peak for $C(r, \vec{x}_0)$
- r_m is a maximum of $C(r, \vec{x}_0)$
- Threshold $C(r_m, \vec{x}_0) \geq C_c(q)$

Define the Gaussian variables:

- $W = -r_m^2 \frac{d^2 \delta e}{dr^2} \Big|_{r=r_m}$ [Related to g]

- $g = \frac{r_m^2}{r_H^2} \delta e$ [Related to C_c]

- $U = r_m \frac{d \delta e}{dr} \Big|_{r=r_m}$ [Related to r_m]

Before

- Missing full non-linear analysis
(was done: statistics of $\frac{\delta S}{S}|_e$, stat of $\mathcal{I}$,
expansions etc.)

- Condition that $r=r_m$ is a MAX was never imposed! (v variable was not there!)

Before r_m inferred as MAX of mean profiles!

Now

We consider all possible configuration
within all possible smoothing scales
 r_m !

i.e. $\int \frac{dr_m}{r_m} \text{prob.}(r_m, w, g \dots)$

I am not going to bother you with statistical details

Results

- Concept of mean profile for given r_m :

PDF peaked @

$$\bar{w} = \frac{\sigma_w}{\sigma_g} \frac{\sigma_{wg} - \sigma_{wv} \sigma_{vg}}{1 - \sigma_{gv}^2} g$$

$$\sigma_i^2 = \langle x_i^2 \rangle, \quad \sigma_{ij} = \frac{\langle x_i x_j \rangle}{\sigma_i \sigma_j}$$

↑↑
Curvature @ r_m

Abundance

$$\frac{d\beta}{d\ln r_m} \sim \text{Exp} \left[- \frac{g_c}{g_g \sqrt{1 - \gamma_{g0}^2}} \right]$$



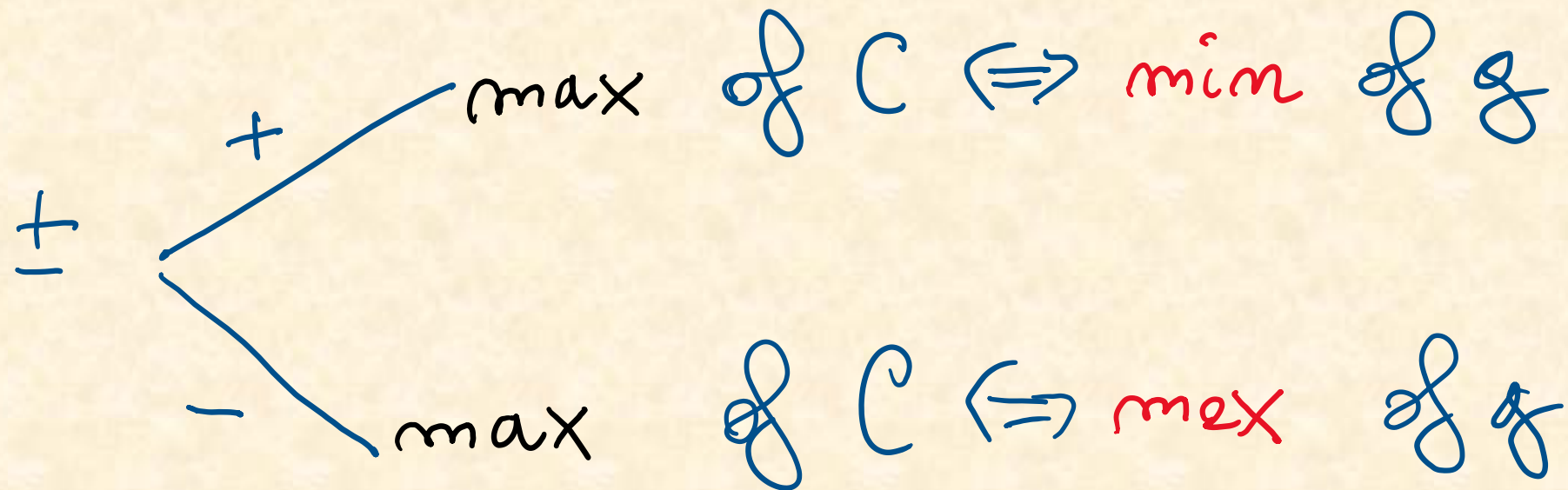
constraint in r_m !

We can consider the cumulative mass distribution!

Interesting things:

$$g_c = \frac{4}{3} \left(1 \pm \sqrt{1 - \frac{3}{2} C_c} \right)$$

↑ "linear threshold" ↑ non-linear threshold



$\bar{\omega} > 0$ selects max. of δ_e

$\bar{\omega} < 0$ selects mins of δ_e

Previous statistics could NOT

de-entangle, MAX was arbitrarily chosen!

Comparison to previous statistics

$$\text{all } \frac{d\beta}{d\ln r_m} \sim e^{-v^2/2}$$

$$v_i = \frac{\#}{\sigma_i}$$

- Non-linear (this work)
- Linear (CG, Musco PRL 2018), peak theory on $\frac{\delta\delta}{\delta^2}|_{\text{lin}}$
- $\mathcal{I}$ (Yoo et al. PTPT 2018), peak theory on $\mathcal{I}$

Comparison to previous statistics

Conditions:

- NL r_m MAX, peak in $\vec{x}_0$ for C , $C(r_m, \vec{x}_0) > C_c$
- L peak in $\vec{x}_0$ for $\frac{\delta \mathcal{L}}{\delta}$ |_{lin}, $\frac{\delta \mathcal{L}}{\delta}$ |_{lin} ($\vec{x}_0$) $\geq \frac{\delta \mathcal{L}}{\delta}$ |_{crit}.
- $\mathcal{I}$ peak in $\vec{x}_0$ for $\mathcal{I}$, $\mathcal{I}(\vec{x}_0) \geq \mathcal{I}_{crit}$

↓
Different Statistics !!!

Comparison to previous statistics

$$\bullet \sigma_{me} = \left[\int \frac{dk}{k} (kr_m)^4 W_{TH}^2(kr_m) P(k) \right]^{\frac{1}{2}} \sqrt{1 - \gamma_{gv}^2}$$

[
extra
constr.

$$\bullet \sigma_e = \frac{\int \frac{dk}{k} (kr_m)^4 \text{Sinc}(kr_m) P(k)}{\left[\int \frac{dk}{k} (kr_m)^4 P(k) \right]^{\frac{1}{2}}}$$

$$\bullet \sigma_z = \frac{\int \frac{dk}{k} (kr_m)^2 W_{TH}(kr_m) P(k)}{\left[\int \frac{dk}{k} P(k) \right]^{\frac{1}{2}}} \leftarrow \text{IR problems ...}$$

Comparison to previous statistics

Extremal NL case

$$\mathcal{P}(k) = a^2 k_0 \delta(k - k_0)$$

$$\frac{\sigma_{nl}}{\sqrt{1 - \gamma_{gv}^2}} \sim \sigma_e \sim \sigma_3$$

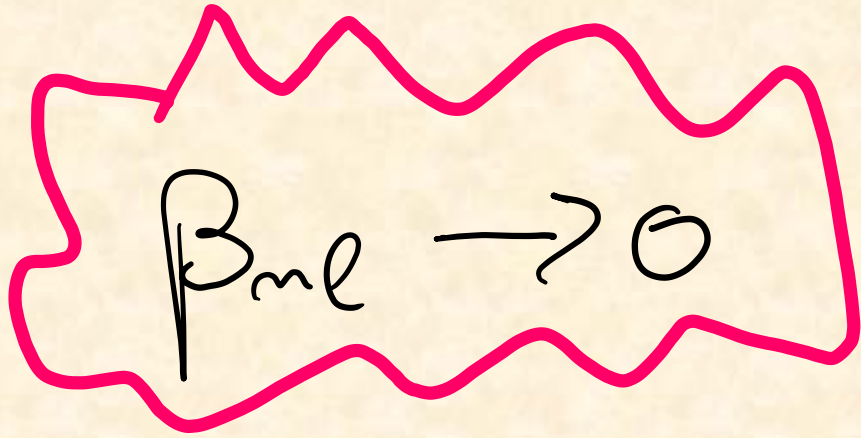
But!

$$\gamma_{gv} \rightarrow 1!!!$$

Comparison to previous statistics

Extremal NL case

$$P(k) = a^2 k_0 \delta(k - k_0)$$


$$\beta_{nl} \rightarrow 0$$



While

$\beta_e \sim \beta_3$ finite !!!

To be taken home:

- BH Threshold is universal in terms of the averaged compaction function $r < r_m$
- Back to school: $\langle x^2 | c_1 \rangle \neq (\langle x | c_2 \rangle)^2$



Non-linear statistics hugely differs
from linear and/or 3 statistics!

To be done @ home:

- Non-Gaussianities!
- Time for new thoughts:
 - Statistics for:
 - Alex Q-Balls
 - Edoardo Bu(bells)

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Statistics for:

- Alex Q-Balls

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- Foot (Balls)

Thanks!