Effect of Inhomogeneity on PBH Formation in the Matter-Dominated Era

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This talk is based on
T. Kokubu (Hunan Normal U), K. Kyutoku (KEK), K. Kohri (KEK) & T. Harada,
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PBH formation in the matter-dominated (MD) era

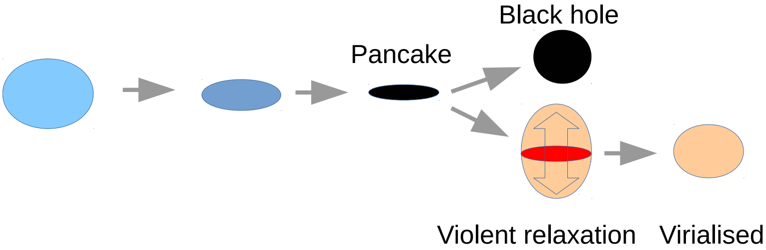
- Pioneered by Khlopov & Polnarev (1980).
- Well motivated by early MD phase scenarios such as inflaton oscillations, phase transitions, and superheavy particles.
- There have been many important works on this topic in this decade.
- Since pressure is neglible and therefore the PBH threshold is small, Newtonian analysis can be useful.
- The standard deviation of the density perturbation at horizon entry, σ_H, can be written in terms of P_ζ as

$$\sigma_H^2 \simeq \left(\frac{2}{5}\right)^2 P_\zeta(k_{BH}).$$

- Nonspherical effects play important roles.

Effect of anisotripic collapse

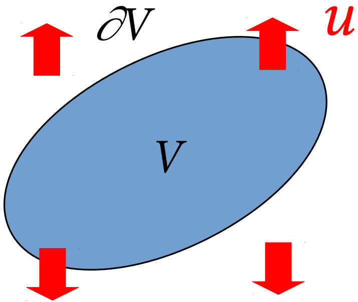
- The triaxial dust ellipsoid leads to a "pancake" singularity. (Lin, Mestel & Shu 1965, Zeldovich 1969)
- The hoop conjecture (Thorne (1972)) gives the condition for BH formation.

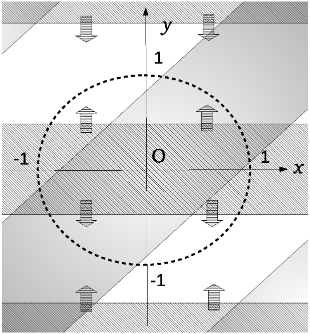


- This effect gives $\beta_0 \simeq 0.05556\sigma_H^5$ for small σ_H (Harada, Yoo, Kohri, Nakao & Jhingan (2016)).

Effect of angular momentum

- A spin may develop due to growing modes and halt collapse.
- The Kerr bound (Kerr (1963)) gives the condition for the PBH formation. (Harada, Yoo, Kohri & Nakao (2017))

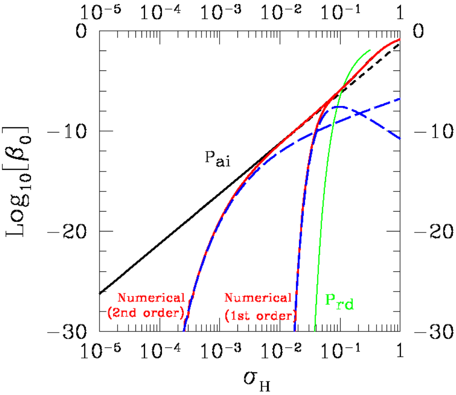




- The spin effect suppresses PBH formation and provides PBHs with near-critical spins for small σ_H.

PBH production probability

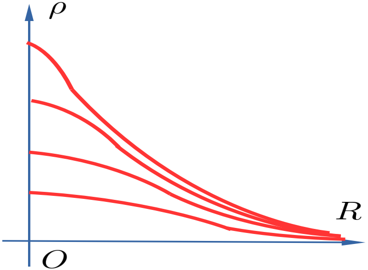
- If we take both anisotropy and spin into account, we obtain (Harada, Yoo, Kohri & Nakao (2017))



- Some uncertainty remains in spin effect because the distribution of initial nonsphericity is not well determined.

BH formation and inhomogeneity

- The evolution of spherical dust (=pressureless fluid) is described by the LTB solution and results in a runaway collapse.



- "Conjecture" by Khlopov & Polnarev (1980)
  - If the central concentration is large, the central density may blow up well before the collapsing region is covered by an apparent horizon.
  - In this case, BH formation may be prohibited because high-density centre may gain strong radiation pressure and/or velocity dispersion.
- It is still unclear what happens in reality.

Lemaitre-Tolman-Bondi (LTB) solution

- Metric

$$
\begin{aligned}
ds^2 &= -dt^2 + \frac{(R'(t,r))^2}{1+f(r)}dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2), \\
t - t_s(r) &= -\sqrt{\frac{R(t,r)^3}{R_g(r)}} F\left[-\frac{f(r)R(t,r)}{R_g(r)}\right], \\
F(y) &= \frac{\sin^{-1}\sqrt{y}}{y^{3/2}} - \frac{\sqrt{1-y}}{y} \quad (0 < y \le 1), \\
\rho(t,r) &= \frac{R'_g(r)}{8\pi R^2(t,r)R'(t,r)}.
\end{aligned}
$$

- There are three arbitrary functions: $R_g(r)$ ($2\times$ Misner-Sharp mass), $f(r)$ (energy fn) and $t_s(r)$ (occurrence of singularity).
- If $f(r) < 0$, the dust particle at r turns around and collapses.
- We assume $\dot{R}(t_i, r) = 0$ for all r and give $\rho(t_i, r)$. We fix the scaling of r so that $R(t_i, r) = r$.

Khlopov-Ponarev condition

- Khlopov & Polnarev (1980) adopt the BH formation condition $t_{\rm ah}(r_1) < t_s(0)$, so that they reach

$$u \lesssim x^{3/2} \quad \text{and} \quad \beta_{0,\rm inhom} \simeq x^{3/2} \quad \text{for} \;\; x \ll 1,$$

where

$$u := \frac{\rho(t_i, 0)}{\bar{\rho}(t_i, r_1)} - 1 \quad \text{(initial inhomogeneiety)}$$

$$x := \frac{R_g(r_1)}{r_1} \sim \delta \quad \text{(initial compactness)}$$

- The above condition can be analytically reinterpreted as

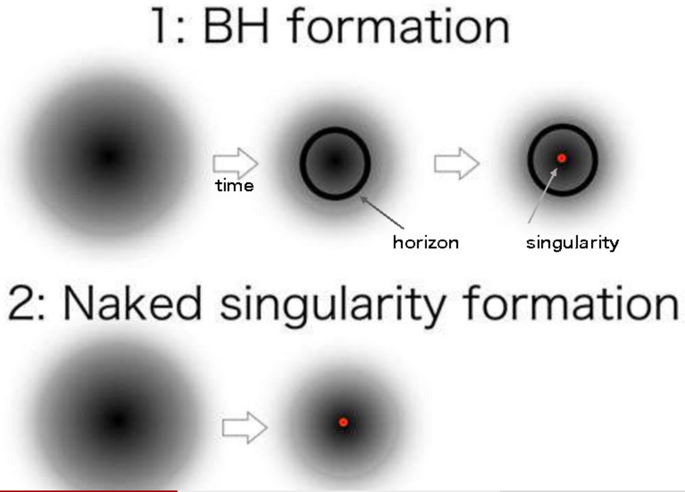
$$u \lesssim \frac{8}{3\pi} x^{3/2} \simeq 0.849 x^{3/2} \quad \text{and} \quad \beta_{0,\rm inhom} \simeq \frac{0.45}{\Sigma} \sigma_H^{3/2},$$

where $\Sigma = O(1)$ is the standard deviation of u.

- However, they compare between the occurrence of singularity $t_s(0)$ at $r = 0$ and the apparent horizon $t_{\rm ah}(r_1)$ at $r = r_1$. Is it legitimate?

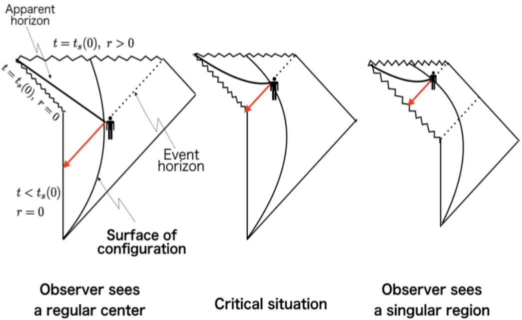
Khlopov-Polnarev condition and naked singularity

Khlopov-Polnarev condition should be understood in the context of naked singulairity formation in the LTB solution.



Naked singularity formation in the LTB solution

- Naked singularity generically appears in the LTB solution. (Eardley & Smarr (1979), Christodoulou (1984), Joshi & Dwivedi (1993))
- Covered singularity, critical situation, naked singularity



- The correct condition for BH formation should be $t_{ah}(r_1) < t_{s,null}(r_1)$, where $t = t_{s,null}(r)$ is the first light ray from the singular centre.

New estimate of the inhomogeneity effect

- We need the numerical integration of the null geodesic equation in the LTB spacetime. The result leads to the increase of β_0 by a factor of 10 than the Khlopov-Polnarev estimate.

$$u \lesssim 7.0x^{3/2}, \quad \text{and} \quad \beta_{0,\text{inhom}} \simeq \frac{3.7}{\Sigma}\sigma_H^{3/2}$$

- Combined with the anisotropy effect, we find

$$\beta_0 \simeq \beta_{0,\text{inhom}}\beta_{0,\text{aniso}} \simeq \frac{0.21}{\Sigma}\sigma_H^{13/2},$$

where the spin effect is neglected.

- Caveat 1: We do NOT know what happens in the central high density region. Therefore, the above estimate is regarded as a lower limit.

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- Caveat 2: I'm NOT claiming the violation of cosmic censorship in the Universe.

Summary

- In the matter-dominated era, nonspherical effects play crucial roles.
  - For relatively large σ_H, the anisotropy effect is dominant and gives $\beta_0 \simeq 0.05556\sigma_H^5$.
  - The effect of angular momentum results in nearcritically spinning PBHs.
- Inhomogeneity may suppress PBH formation. This effect may result in an additonal factor $\sim \sigma_H^{3/2}$ to β_0. The consideration of causality increases β_0 by a factor of 10 than the almost forty-year-old estimate.