PBHs and GWs

Antonio Riotto
University of Geneva

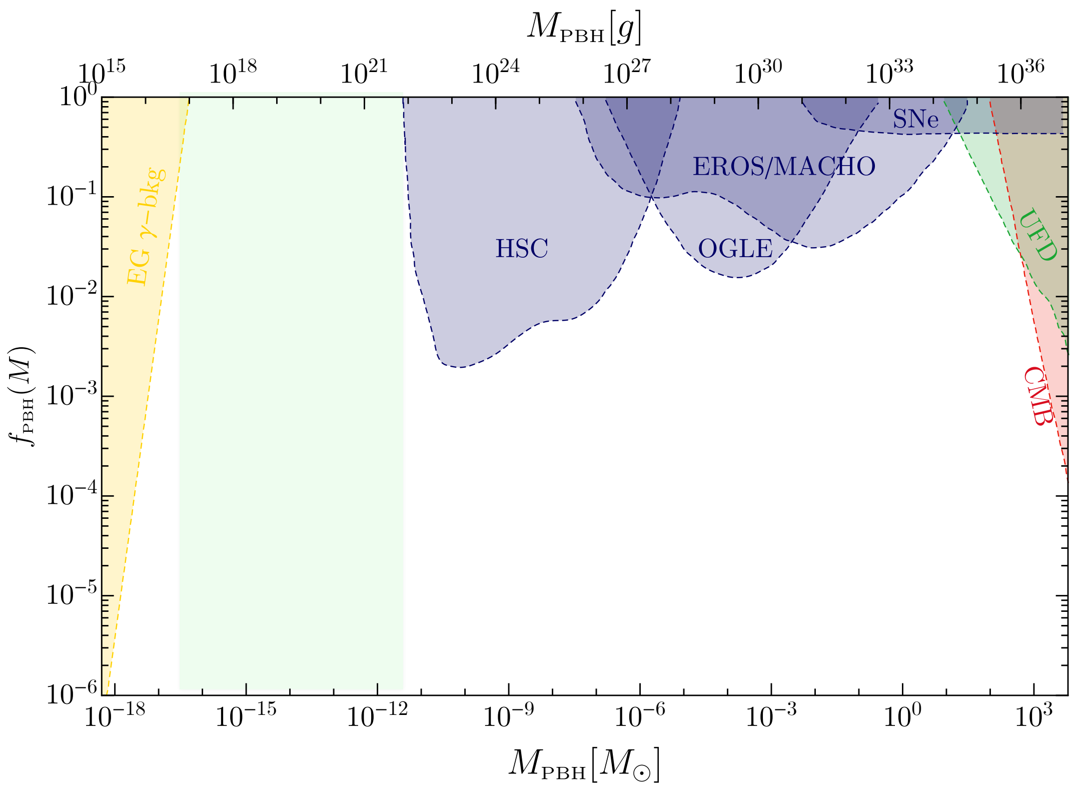
Plan of the talk

- Some comments about the formation of PBHs and their spins at formation
- GWs from PBHs, their characterisation: NG, anisotropies & gauge invariance
- Conclusions

In collaboration with V. De Luca, V. Desjaques, G. Franciolini, A. Kehagias, I. Musco + the Padova group

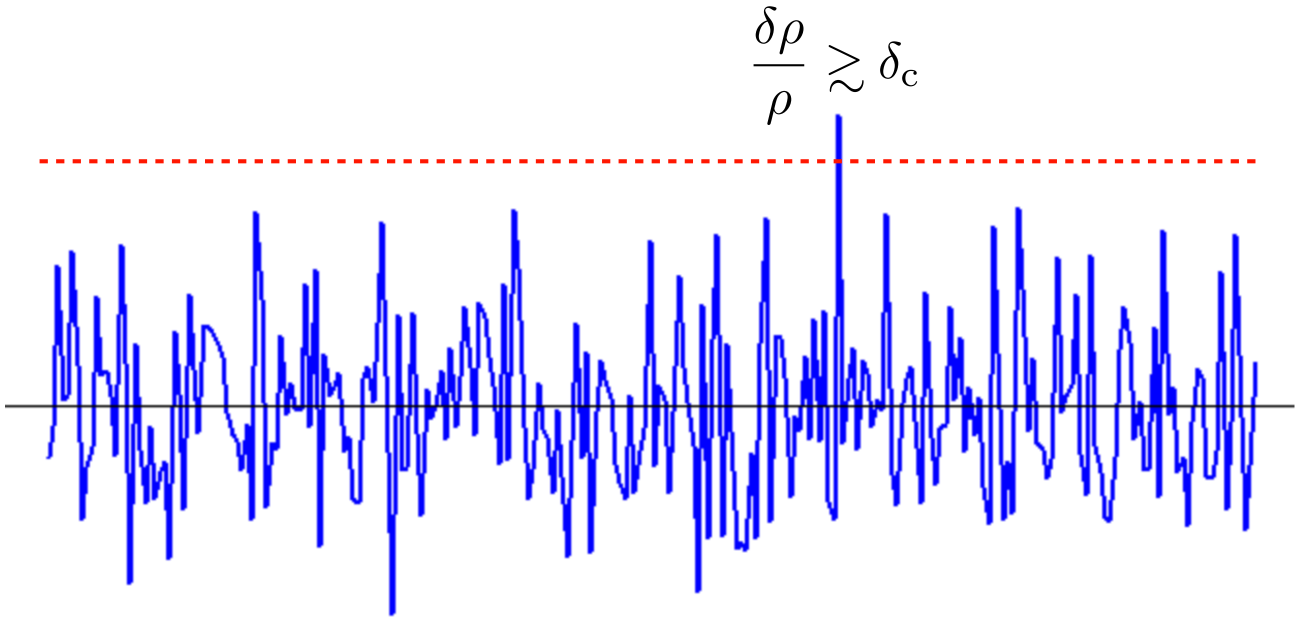
PBHs

If black holes are of primordial origin they can compose all the dark matter (or a fraction of it)



Range mass up to $10^{-12} M_\odot$ still available for DM

PBHs are originated from peaks of the density contrast



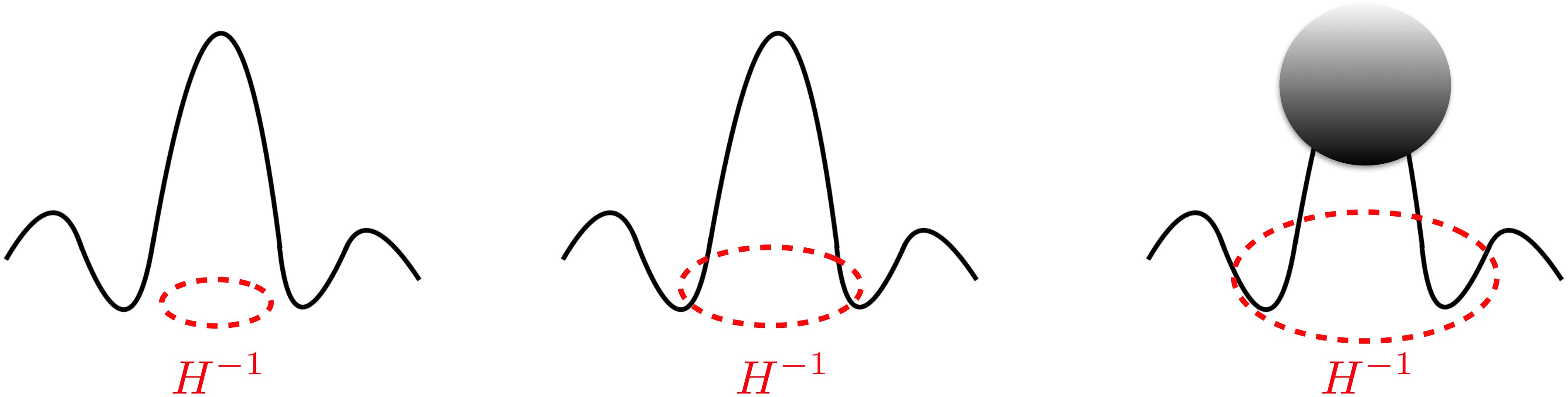
PBHs are rare events, tail of the distribution

One possible mechanism: large fluctuations from inflation

PBHs are originated from peaks of the density contrast

$$\frac{\delta\rho}{\rho} \gtrsim \delta_{\rm c}$$

$$M_{\rm PBH} \sim M_{\rm H}$$

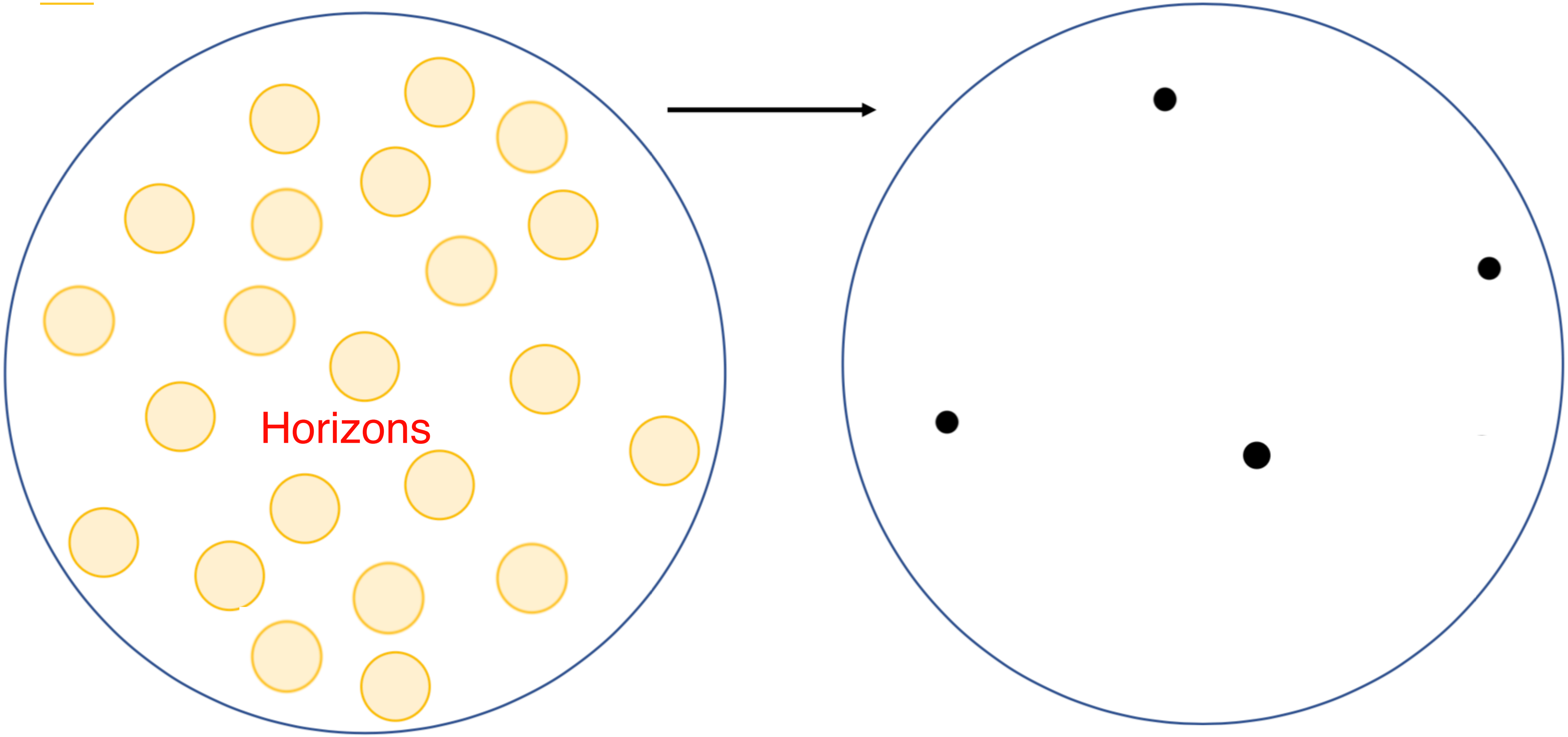


$$\beta(M) = \int_{\delta_{\rm c}}^{\infty} \frac{{\rm d}\delta}{\sqrt{2\pi}\sigma_\delta} e^{-\delta^2/2\sigma_\delta^2}$$

$$\sigma_\delta^2 = \int_0^\infty {\rm d}\ln k W^2(k, R_H)\mathcal{P}_\delta(k)$$

Standard lore: take the probability to be Gaussian

PBHs are originated from peaks of the density contrast



PBHs are not clustered at formation (in the absence of local NG)

$$\frac{x_\xi}{\bar{x}} \sim 10^{-2}\left(\frac{M_{\rm PBH}}{M_\odot}\right)^{1/6}$$

with V. Desjaques (2018)

Unavoidable non-Gaussianity

Even if one assumes Gaussian comoving curvature perturbation

$$\delta(\vec{x},t) = -\frac{4}{9a^2H^2}e^{-2\zeta(\vec{x})}\left[\nabla^2\zeta(\vec{x}) + \frac{1}{2}\zeta_i(\vec{x})\zeta^i(\vec{x})\right]$$

Correspondence between peaks of ζ and δ

Threshold only changed by a few percent

A. Kehagias, I. Musco and A.R. (2019)

De Luca et al. (2019), Byrnes et al. (2019)

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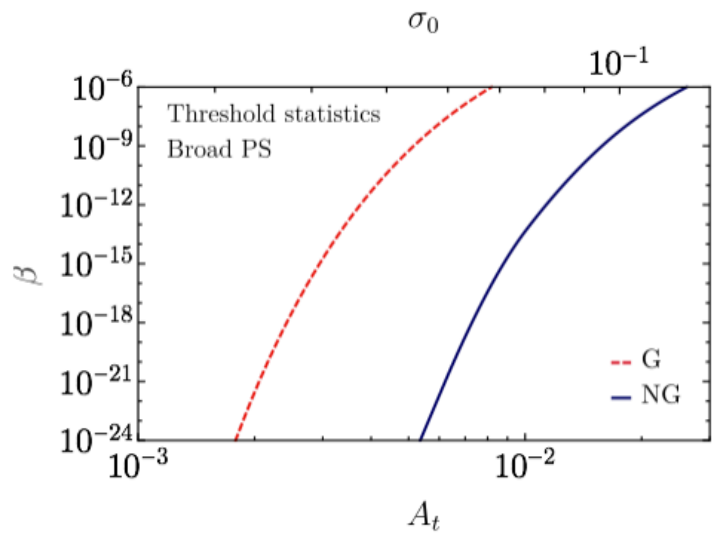
Bardeen (1986)

$\langle\ldots\rangle$	ζ	ζ_i	ζ_{ij}
ζ	σ_0^2		$-\sigma_1^2/3$
ζ_m		$\sigma_1^2/3$	
ζ_{mn}	$-\sigma_1^2/3$		$\sigma_2^2/15$

(times symm. tensor structure)

Gaussian Multivariate PDF

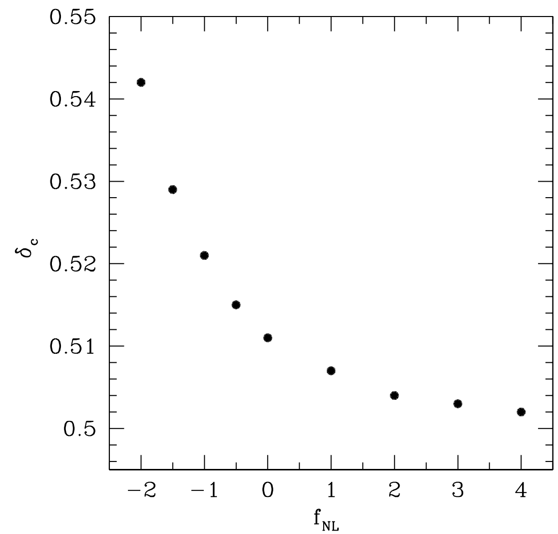
$$\int [\mathrm{d}\zeta][\mathrm{d}\zeta_i][\mathrm{d}\zeta_{ij}] P(\zeta,\zeta_i,\zeta_{ij})\Theta\left[\delta(\zeta,\zeta_i,\zeta_{ij}) - \delta_c\right]$$



De Luca et al. (2019), Byrnes et al. (2019)

Primordial non-Gaussianity

$$\zeta = \zeta_{\rm g} + \frac{3}{5} f_{\rm NL} \zeta_{\rm g}^2$$



- NG affects the threshold for collapse

  A. Kehagias, I. Musco and A.R. (2019)

- NGs affect the abundance (adopting threshold statistics and using the cluster expansion technique)

$$P(\delta > \delta_{\rm c}) = \left\langle \Theta(\delta - \delta_{\rm c}) \right\rangle = \int [D\delta(\vec{x})] P[\delta(\vec{x})] \Theta\Big(\delta(\vec{x}) - \delta_{\rm c}\Big)$$

Assuming: $\delta_c/\sigma_\delta \gg 1$

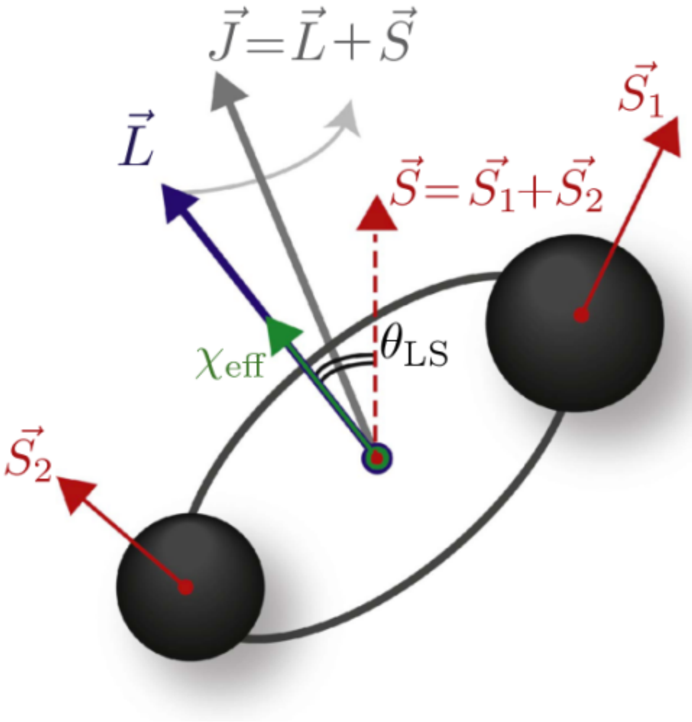
Fully NG PDF which can be written in terms of the higher-order correlators $\xi_n(\vec{x}_1, \cdots, \vec{x}_n) = \left\langle \delta(\vec{x}_1) \cdots \delta(\vec{x}_n) \right\rangle_c$

$$P(\delta > \delta_{\rm c}) = \beta(M) = \frac{1}{\sqrt{2\pi(\delta_c/\sigma_\delta)^2}} \exp\left\{ -(\delta_c/\sigma_\delta)^2/2 + \sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \xi_n(0) \left(\delta_c/\sigma_\delta^2\right)^n \right\}$$

G. Franciolini et al. (2019)

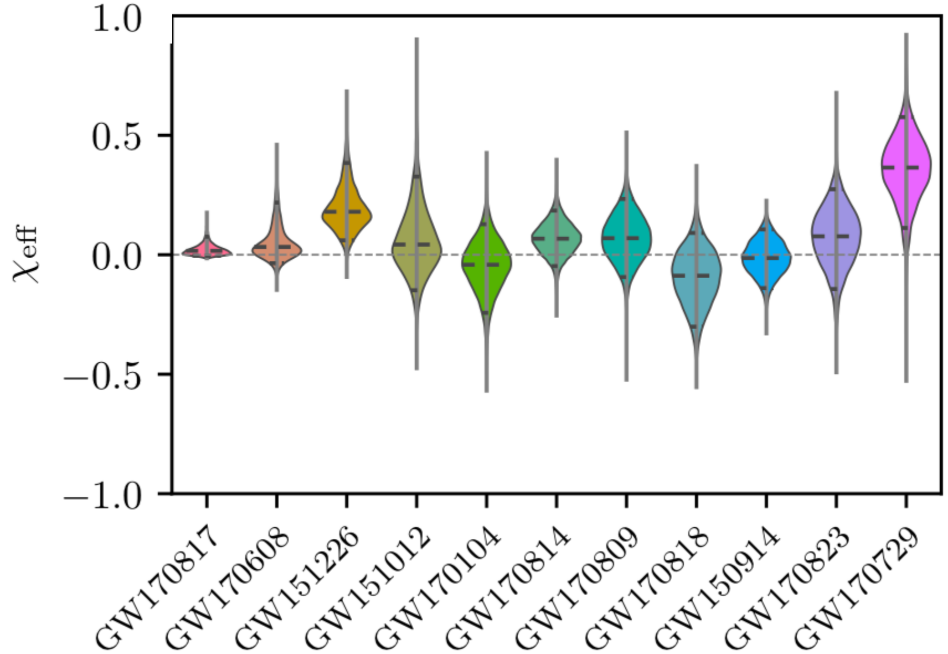
The spin of PBHs

In black hole merging, gravitational waveforms are sensitive to the final spin state and to the orbital projection



$$\chi_{eff} = \frac{M_1 \vec{S}_1 + M_2 \vec{S}_1}{M_1 + M_2} \cdot \hat{L}$$

vanishes if initial spins anti-aligned
or lie on the orbital plane, or initial spins vanish



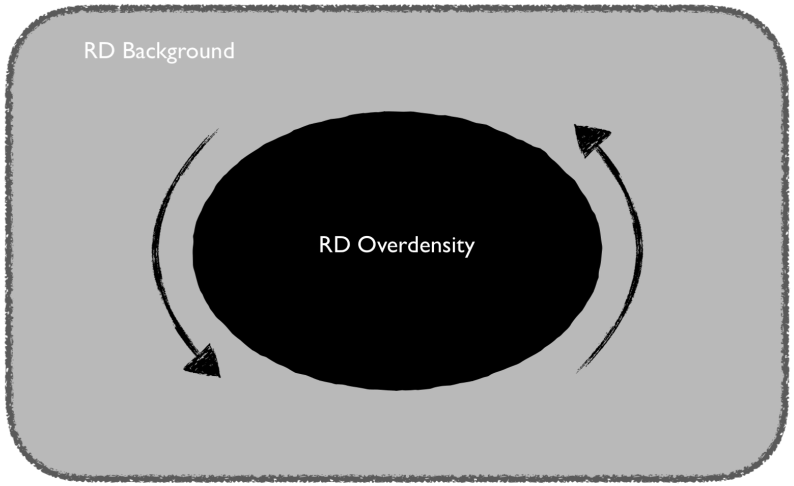
Ligo Scientific, Virgo (2018)

The spin of PBHs at formation

- PBHs originate from peaks, that is from *maxima* of the local density contrast. Need peak theory to obtain the probability distribution of the spin

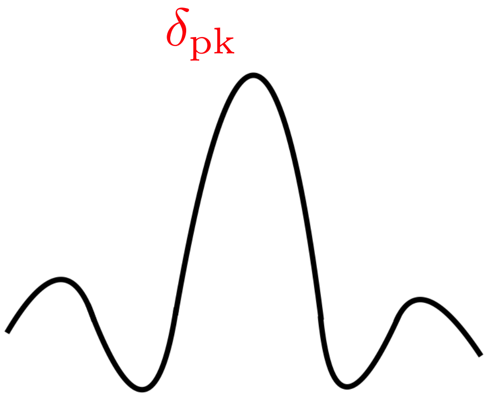
- The spin results from the action of the torques generated by the gravitational tidal forces upon horizon crossing

- It is a *first-order* effect in perturbation theory when accounting for the fact that the collapse is not spherical



De Luca et al. (2018)

Peak theory



$$\delta(\vec{x}) = \frac{\delta\rho(\vec{x})}{\bar{\rho}} \simeq \delta_{\rm pk} + \frac{1}{2}\zeta_{ij}(x - x_{\rm pk})^i (x - x_{\rm pk})^j$$

$$\zeta_{ij} = \left.\frac{\partial^2\delta}{\partial x^i \partial x^j}\right|_{\rm pk}$$

Rotate the coordinate axis to be aligned
with the principal axes of the constant density ellipsoid

$$\delta \simeq \delta_{\rm pk} - \frac{1}{2}\sigma_\zeta \sum_1^3 \lambda_i (x^i - x^i_{\rm pk})^2 \qquad a_i^2 \simeq 2\frac{\sigma_\delta}{\sigma_\zeta}\frac{1}{\lambda_i}\nu. \qquad \lambda_i = \frac{\gamma\nu}{3} + \mathcal{O}(1)$$

ellipsoidal collapse

$$\nu = \frac{\delta_{\rm pk}}{\sigma_\delta}, \quad \gamma = \frac{\sigma_\times^2}{\sigma_\delta \sigma_\zeta}$$

$\sigma_\times^2$ → cross-correlation of δ and ζ_{ij}

σ_ζ → rms of ζ_{ij}

ν → for PBHs $\mathcal{O}(7 \div 8)$

γ → close to unity for steep power spectra

BBKS (1986)

The spin of the non-spherical collapse

$$S_i = \frac{4}{3} a^4(\eta) \epsilon_{ijk} \int \mathrm{d}^3 x \rho(\vec{x}, \eta)(x - x_{\rm pk})^j (v - v_{\rm pk})^k$$

Expand around the peak at first-order

$$S_i = \frac{4}{3} a^4(\eta) \epsilon_{ijk} \overline{\rho}_{\rm rad}(\eta) v_{kl} \int_{V_{\rm e}} \mathrm{d}^3 x \, x^j x^l$$

$$v_{ij} = \left. \frac{\partial v_i}{\partial x^j} \right|_{\rm pk}$$

velocity shear

$$\vec{S} = \frac{16\pi}{45} a^4(\eta) \overline{\rho}_{\rm rad}(\eta) a_1 a_2 a_3 ([a_2^2 - a_3^2] v_{23}, [a_3^2 - a_1^2] v_{13}, [a_1^2 - a_2^2] v_{12})$$

The spin of the collapsing mass is non-zero if

- The collapse is not spherical
- The off-diagonal components of the velocity shear are non-zero and misaligned with the inertia tensor

$$\vec{S} = \frac{16\pi}{45} a^4(\eta)\bar{\rho}_{\rm rad}(\eta) a_1 a_2 a_3 ([a_2^2 - a_3^2]v_{23}, [a_3^2 - a_1^2]v_{13}, [a_1^2 - a_2^2]v_{12})$$

non-spherical collapse

non-zero off-diagonal shear

Heavens and Peacock (1988)

Rough estimate

$$S_i \sim a^4(\eta_{\rm H})\bar{\rho}_{\rm rad}(\eta_{\rm H})a_1 a_2 a_3[a_i^2 - a_j^2]v_{ij}$$

$$a_1 a_2 a_3 \sim (\sigma_\delta/\sigma_\zeta)^{3/2} \sim \mathcal{H}^{-3}(\eta_{\rm H})$$

$$\lambda_i = \frac{\gamma\nu}{3} + \mathcal{O}(1)$$

$$a_i^2 - a_j^2 \sim (\sigma_\delta/\nu\sigma_\zeta) \sim \mathcal{H}^{-2}(\eta_{\rm H})$$

$$a_i^2 \simeq 2\frac{\sigma_\delta}{\sigma_\zeta}\frac{1}{\lambda_i}\nu.$$

$$v_{ij} \sim \mathcal{H}(\eta_{\rm H})\sigma_\delta(\eta_{\rm H})\sqrt{1-\gamma^2}$$

$$\mathcal{H}(\eta_{\rm H}) = a^{1/4}(\eta_{\rm eq})\sqrt{\frac{\mathcal{H}_0}{2GM}}$$

$$a = \frac{S}{G_N M^2} \sim \frac{\bar{\rho}_{\rm rad}(\eta_0)}{\mathcal{H}^4(\eta_{\rm H})}\frac{\sigma_\delta(\eta_{\rm H})}{G_N M^2}\sqrt{1-\gamma^2} \sim \frac{\bar{\rho}_{\rm rad}(\eta_0)}{a(\eta_{\rm eq})}\frac{G_N\sigma_\delta(\eta_{\rm H})}{\mathcal{H}_0^2}\sqrt{1-\gamma^2}$$

$$a = \frac{S}{G_N M^2} \sim \frac{\Omega_{\rm dm}}{\pi}\sigma_\delta\sqrt{1-\gamma^2} \sim 10^{-2}\sqrt{1-\gamma^2}$$

The probability distribution of the PBH spin

Starting point: the joint distribution of sixteen correlated variables

$$V_i:\ \ \zeta_i = \frac{\partial\delta}{\partial x_i}, \zeta_{ij} = \frac{\partial^2\delta}{\partial x_i \partial x_j}, v_i^j = \frac{\partial v^j}{\partial x^i},$$

$$f(V_i)\mathrm{d}^{16}V_i = \frac{1}{(2\pi)^8|\mathbf{M}|^{1/2}} e^{-\frac{1}{2}(V_i - \langle V_i\rangle)\mathbf{M}_{ij}^{-1}(V_j - \langle V_j\rangle)}\mathrm{d}^{16}V_i,$$

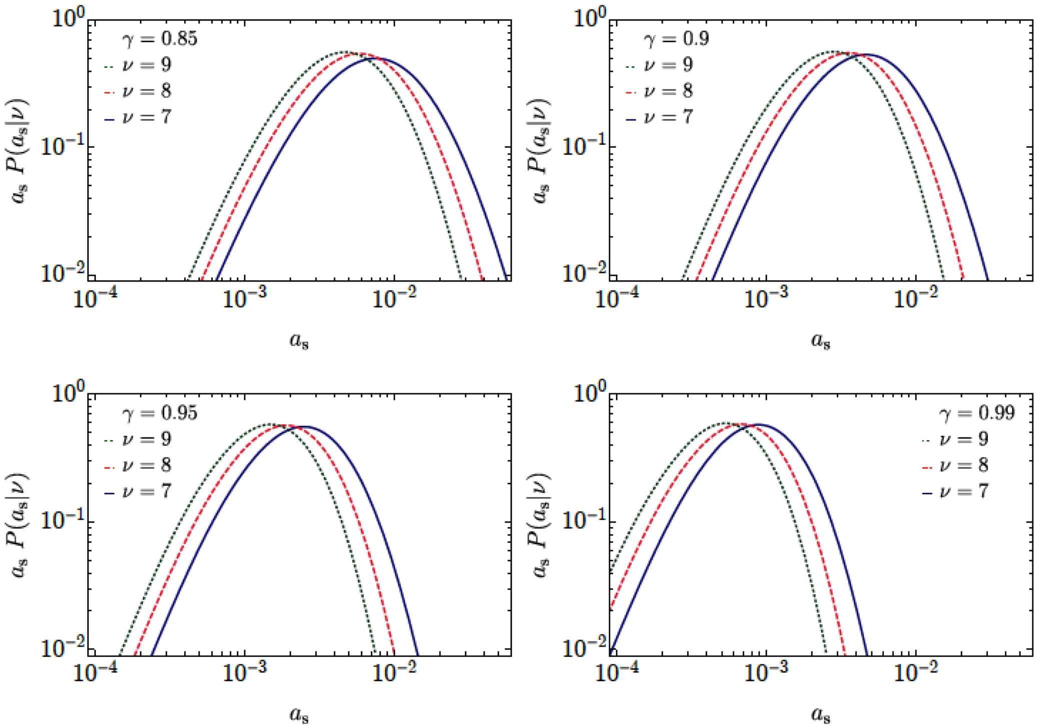
$$\mathbf{M}_{ij} = \langle (V_i - \langle V_i\rangle)(V_j - \langle V_j\rangle)\rangle.$$

$$\underset{i\neq j}{f(v_{ij})} \sim \frac{1}{\sqrt{1-\gamma^2}} \exp\left[-15 v_{ij}^2/2(1-\gamma^2)\right] \underset{\gamma\to 1}{\sim} \delta_{\mathrm{D}}(v_{ij})$$

The PBH spin is proportional to the velocity shear
and goes to zero for monochromatic power spectra:
the cosmic velocity shear is *totally* aligned
with the inertia tensor of the ellipsoidal collapsing object

The conditional probability distribution

Several changes of variables to reduce the covariance matrix plus several integrations

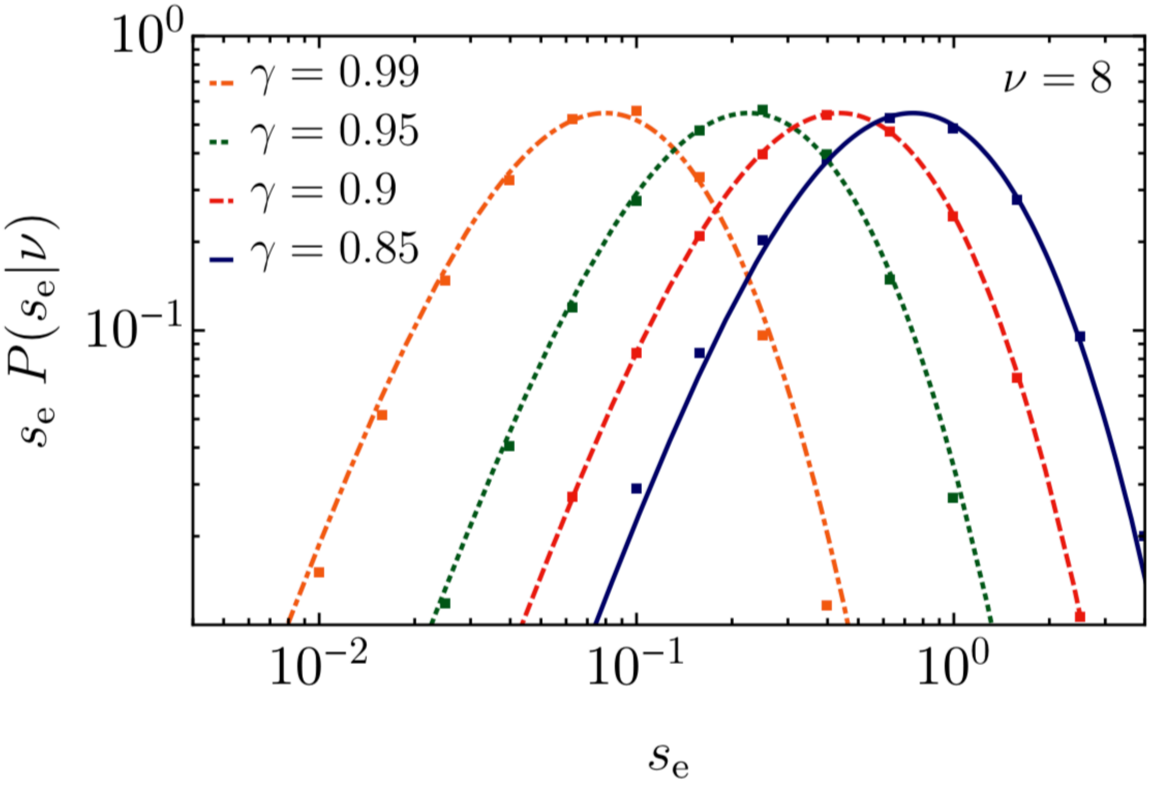


- The typical value of the PBH spin shifts to smaller values for higher peaks (more spherical collapse)

- The PBH spin goes to zero for extreme peaked power spectra

- Typical power spectra for the curvature perturbation used in the literature to give the PBHs as dark matter have $\gamma = \mathcal{O}(0.8 \div 0.9)$

The conditional probability distribution



$$P(h) = \exp(-2.37 - 4.12 \ln h - 1.53 \ln^2 h - 0.13 \ln^3 h)$$

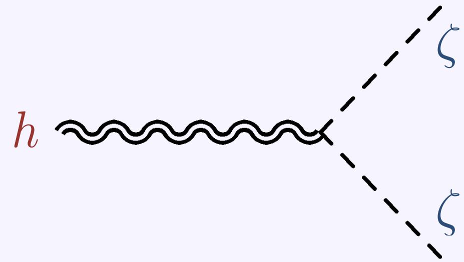
$$a = \frac{\Omega_{\mathrm{dm}} \delta(\eta_{\mathrm{H}})}{\pi} s_\mathrm{e}, \quad s_\mathrm{e} = \frac{2^{9/2}\pi}{5\gamma^6 \nu} \sqrt{1-\gamma^2} h$$

GWs from PBHs

GWs from PBHs

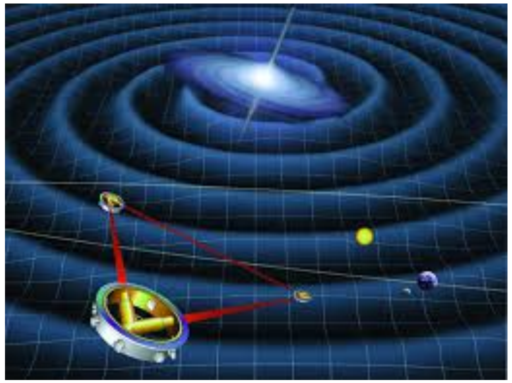
The same curvature perturbations giving rise to PBHs are unavoidably a source for GWs at *second-order* in perturbation theory

$$h''_{ij} + 2\mathcal{H}h''_{ij} - \nabla^2 h_{ij} = \mathcal{O}(\partial_i\zeta\partial_j\zeta)$$



Potentially observable at current and future GW observatories

(LIGO, Virgo, LISA,…)



GWs from PBHs

The same curvature perturbations giving rise to PBHs are unavoidably a source for GWs at *second-order* in perturbation theory

GI from the Poisson gauge

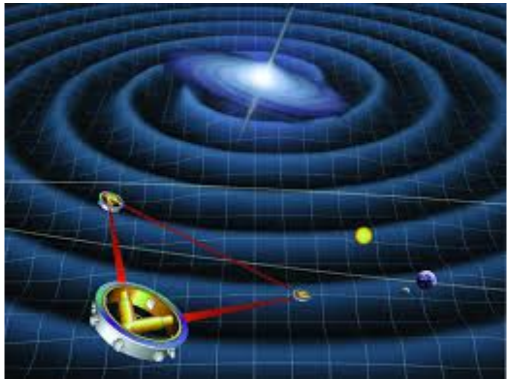
$$h_{2ij}^{\mathrm{GI,P}\prime\prime}+2\mathcal{H}h_{2ij}^{\mathrm{GI,P}\prime}-h_{2ij,kk}^{\mathrm{GI,P}}=-4\mathcal{T}_{ij}^{lm}\left[4\Psi_1\Psi_{1,lm}+2\Psi_{1,l}\Psi_{1,m}-\partial_l\left(\frac{\Psi_1'}{\mathcal{H}}+\Psi_1\right)\partial_m\left(\frac{\Psi_1'}{\mathcal{H}}+\Psi_1\right)\right]$$

TT gauge

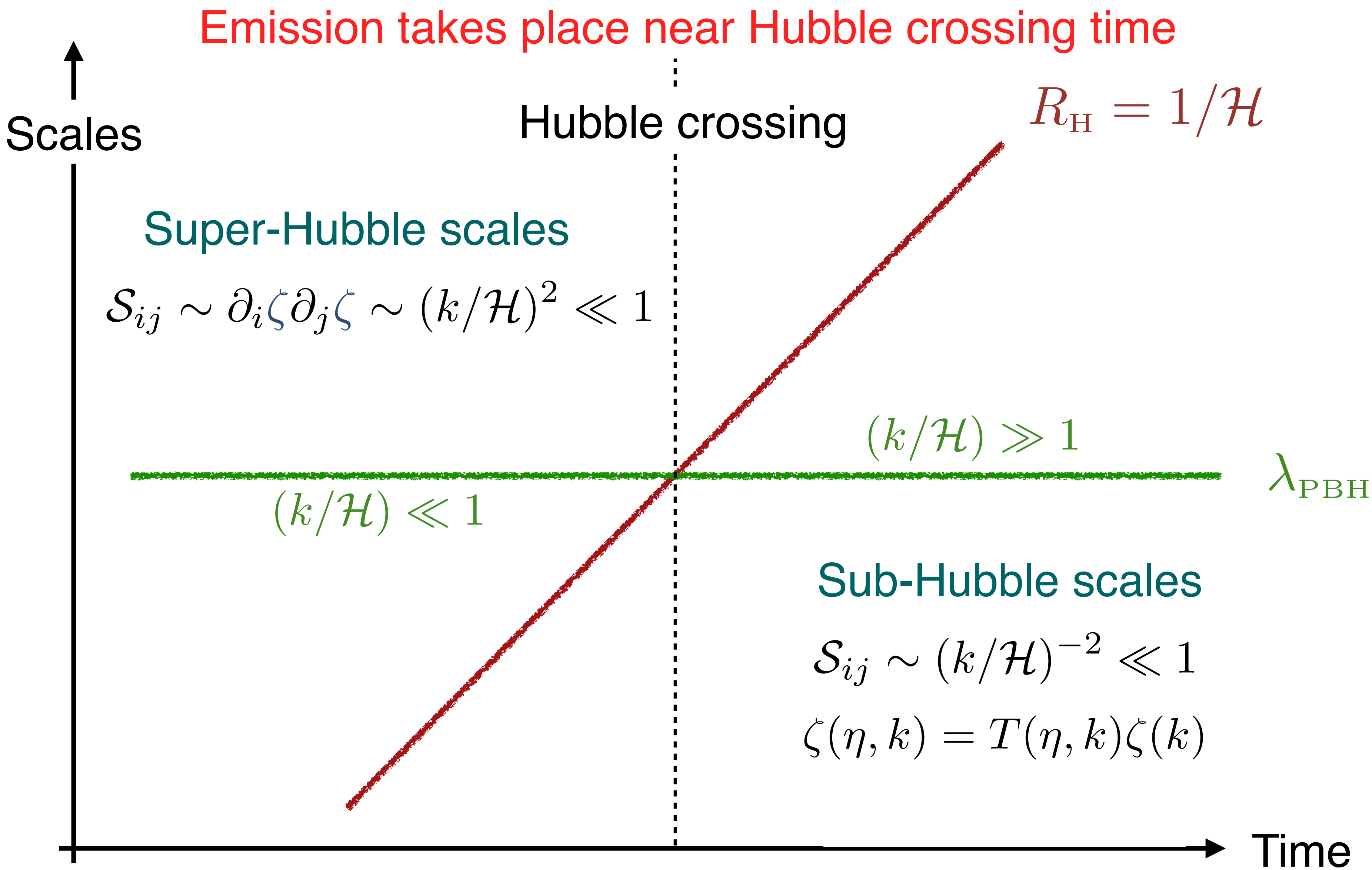
$$\begin{aligned}h_\lambda^{\mathrm{TT}\prime\prime}+2\mathcal{H}h_\lambda^{\mathrm{TT}\prime}+k^2h_\lambda^{\mathrm{TT}}=-4e_\lambda^{ij}(\vec{k})\Big[&2\psi_1^{\mathrm{TT}}\psi_{1,ij}^{\mathrm{TT}}+2\psi_{1,(i}^{\mathrm{TT}\prime}\sigma_{1,j)}^{\mathrm{TT}}+4\psi_{1,(i}^{\mathrm{TT}}\psi_{1,j)}^{\mathrm{TT}}\\&-\psi_1^{\mathrm{TT}\prime}\sigma_{1,ij}^{\mathrm{TT}}+\sigma_{1,ij}^{\mathrm{TT}}\sigma_{1,mm}^{\mathrm{TT}}-\sigma_{1,im}^{\mathrm{TT}}\sigma_{1,jm}^{\mathrm{TT}}-\frac{1}{\mathcal{H}^2}\psi_{1,i}^{\mathrm{TT}\prime}\psi_{1,j}^{\mathrm{TT}\prime}\Big]\end{aligned}$$

Potentially observable at current and future GW observatories

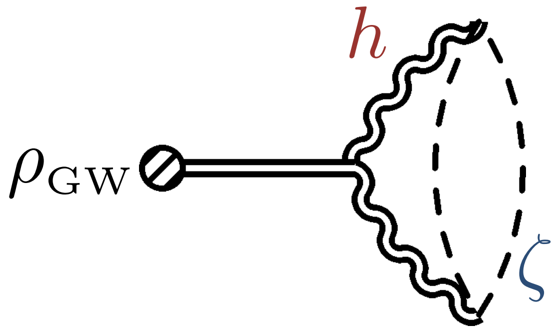
(LIGO, Virgo, LISA,…)



GWs from PBHs at second-order



GW abundance



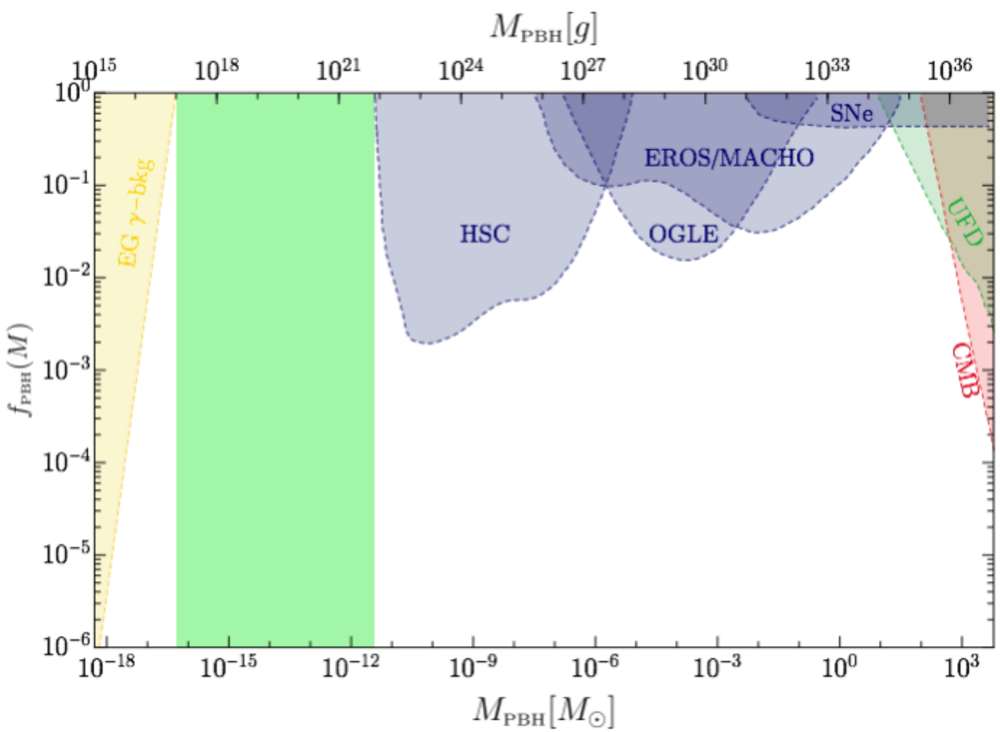
The energy density of GWs is given by the time average over several cycles

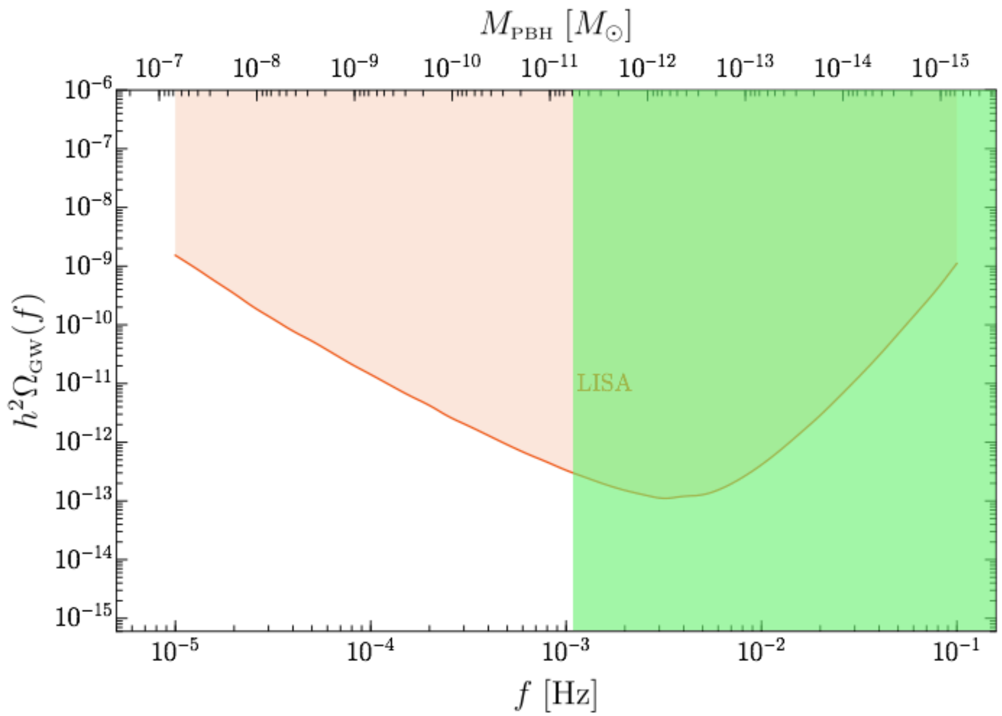
$$\Omega_{\rm GW}(\eta,\vec{x}) = \frac{\rho_{\rm GW}(\eta,\vec{x})}{\overline{\rho}(\eta)} = \frac{M_p^2}{4a^2\overline{\rho}(\eta)} \langle h'_{ab}\,(\eta,\,\vec{x})\, h'_{ab}\,(\eta,\,\vec{x})\rangle_{\rm t.a.}$$

The characteristic frequency of the GWs is similar to the frequency of the scalar perturbations, related to the PBH mass

$$M \simeq 10^{-12} M_\odot \left(\frac{f_{\rm LISA}}{f}\right)^2$$

The PBH dark matter-LISA serendipity





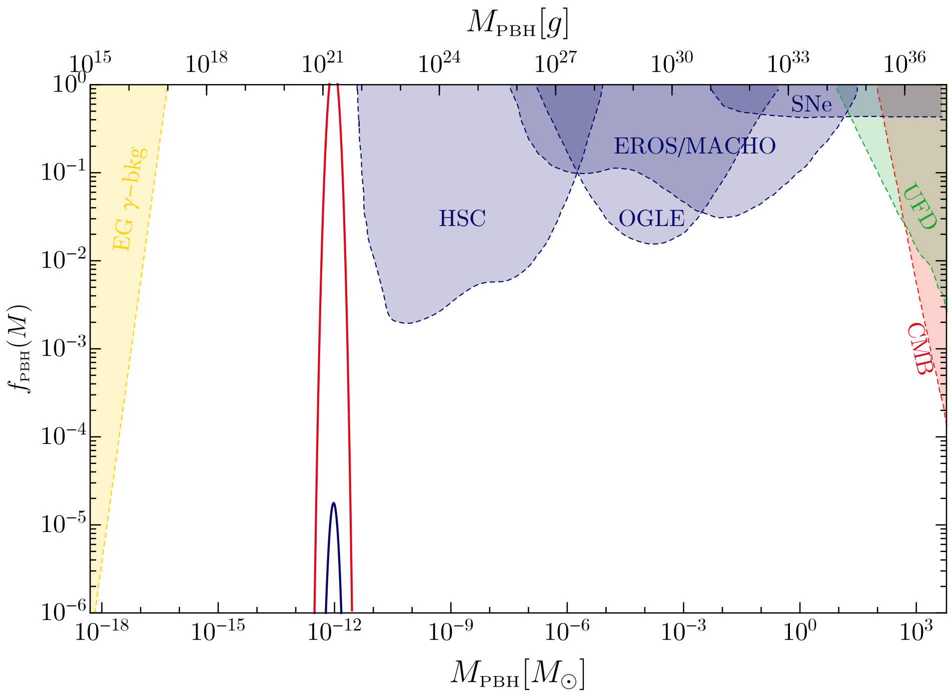
$$M \simeq 10^{-12} M_\odot \left(\frac{f_{\rm LISA}}{f}\right)^2$$

$$f_{\rm LISA} = 3.4 \text{ mHz}$$

$$M \approx 10^{-12} M_\odot$$

Bartolo et al., PRL (2019)

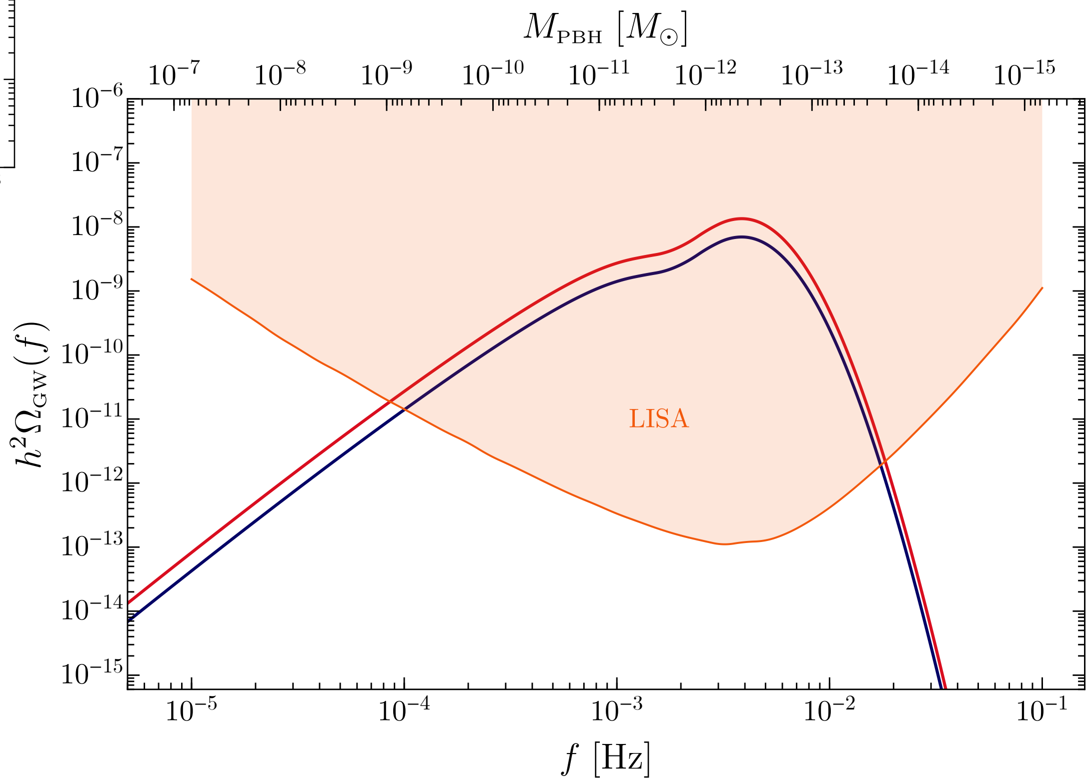
GWs even if PBHs not the DM



$$f_{\rm PBH} \sim {\rm e}^{-\delta_c^2/2\mathcal{A}^2}$$

$$\mathcal{P}_\delta \sim \mathcal{A}^2$$

$$\Omega_{\rm GW} \sim \mathcal{A}^4$$



GW characterization

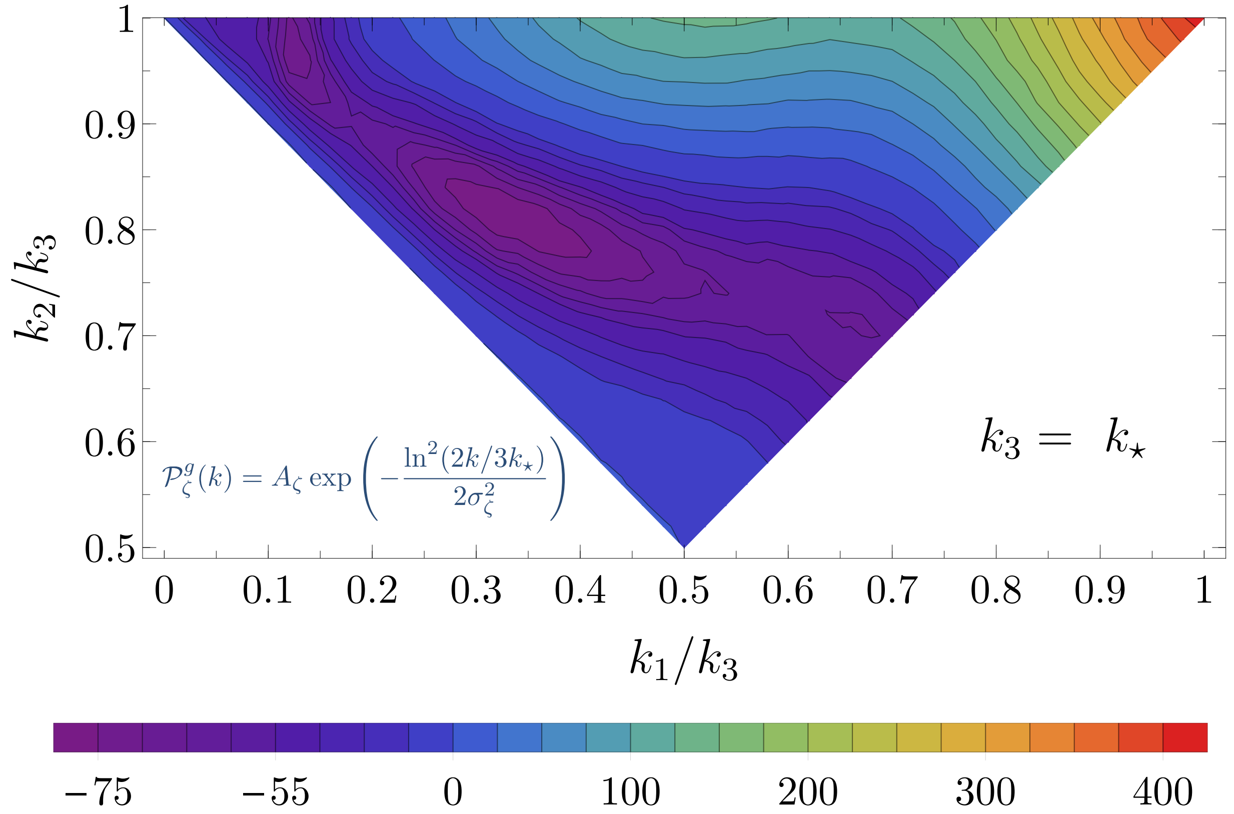
NG & anisotropies

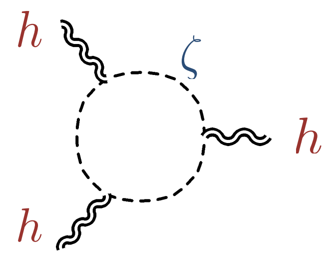
GW non-Gaussianity

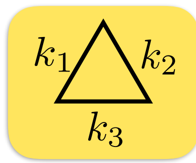
Since sourced at second-order, the emitted GWs are NG at formation

$$\left\langle h_{\lambda_1}\left(\eta_1,\vec{k}_1\right) h_{\lambda_2}\left(\eta_2,\vec{k}_2\right) h_{\lambda_3}\left(\eta_3,\vec{k}_3\right)\right\rangle' \approx \mathcal{P}_\zeta \mathcal{P}_\zeta \mathcal{P}_\zeta$$

$$\mathcal{S}_h^{\lambda_1\lambda_2\lambda_3}(\vec{k}_1,\vec{k}_2,\vec{k}_3) = k_1^2 k_2^2 k_3^2 \frac{\langle h_{\lambda_1}(\eta,\vec{k}_1) h_{\lambda_2}(\eta,\vec{k}_2) h_{\lambda_3}(\eta,\vec{k}_3)\rangle'}{\sqrt{\mathcal{P}_h(\eta,k_1)\mathcal{P}_h(\eta,k_2)\mathcal{P}_h(\eta,k_3)}}$$

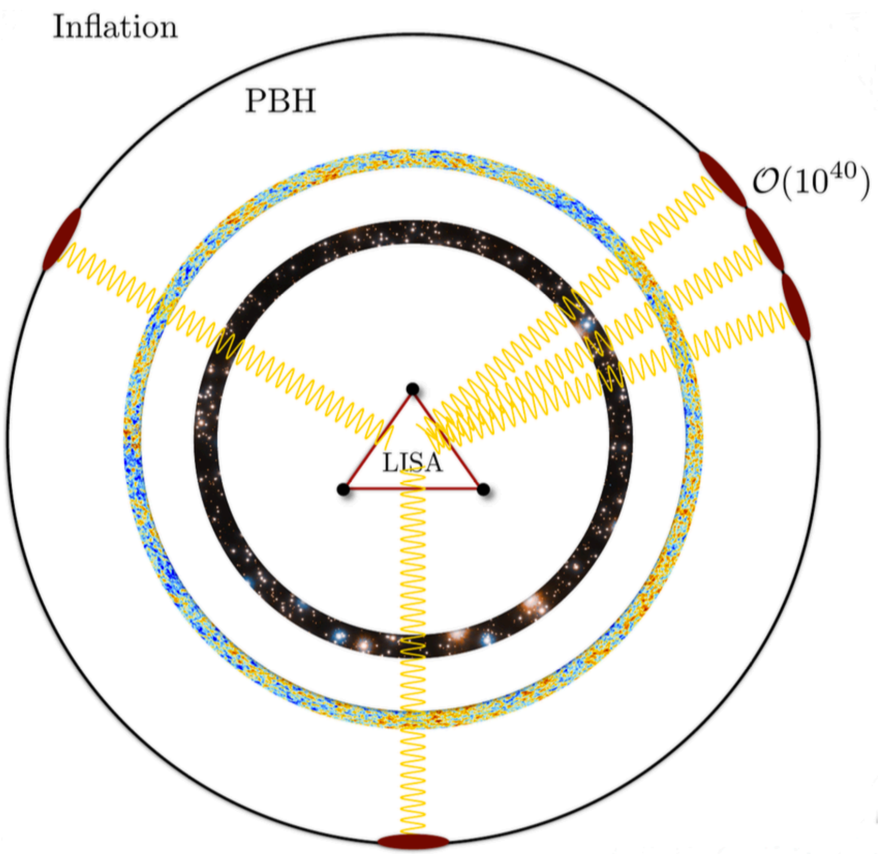






These GWs are born non-Gaussian, but the non-Gaussianity is unobservable:

Non-Gaussianity = phase correlation



Think about the Central Limit Theorem

Bartolo et al., PRL (2019)

Finite frequency resolution

Finite frequency resolution produce dephasing even though inflation generated phase correlation

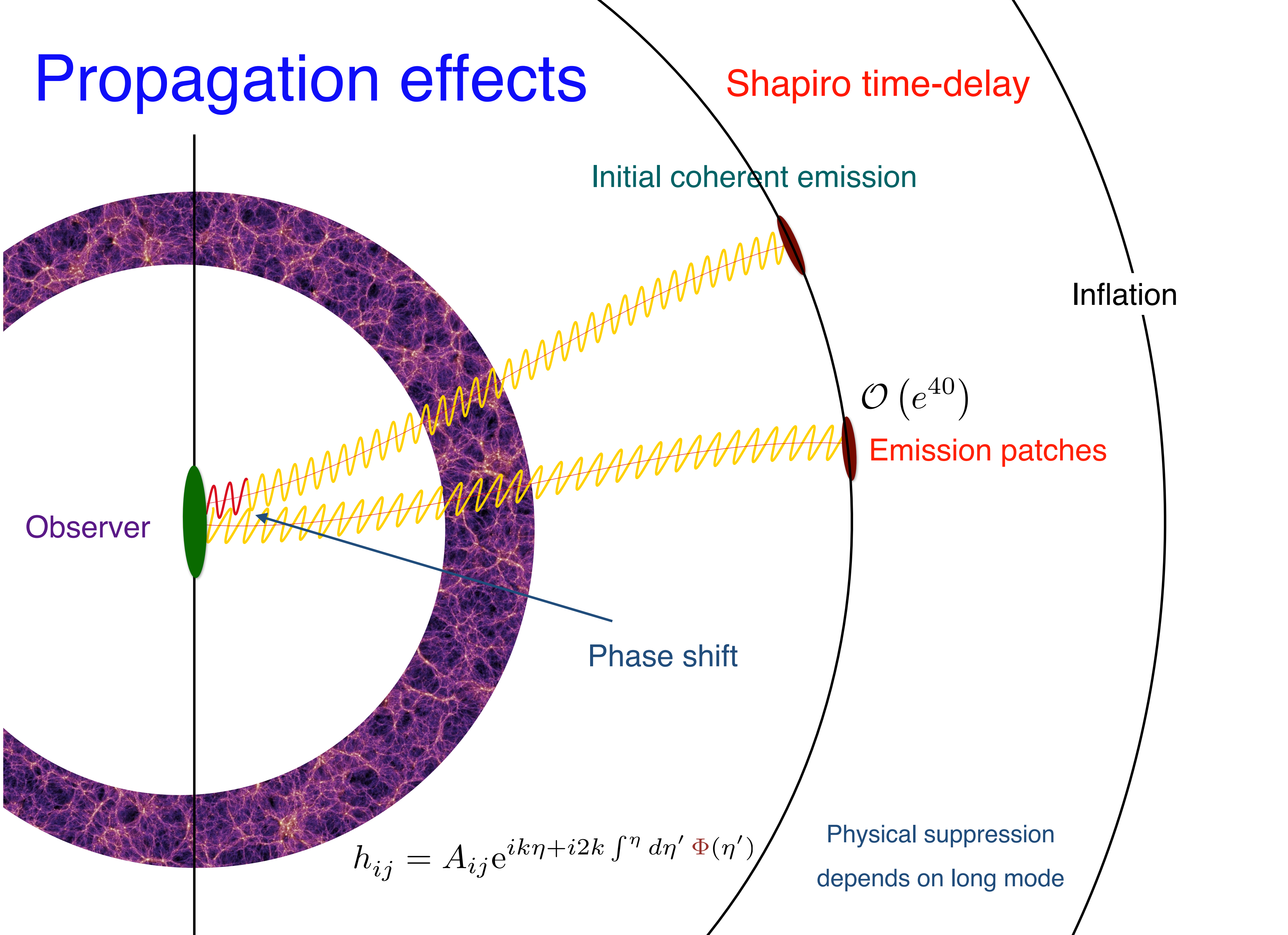
$$\delta k \sim 1/\delta\eta_{\rm exp}$$

$$\delta k\, \eta_0 \gg 1$$

Modes of nearby frequencies get confused due to finite resolution giving rise to dephasing

Bartolo et al., PRL (2019)

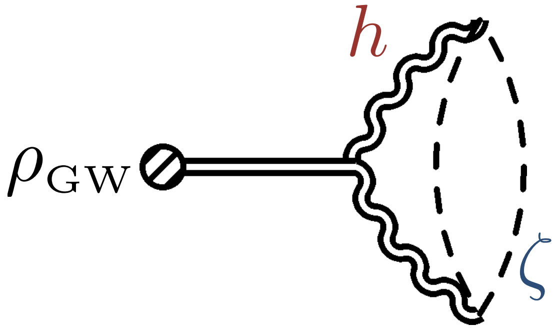
Propagation effects



Non-Gaussianity = phase correlation

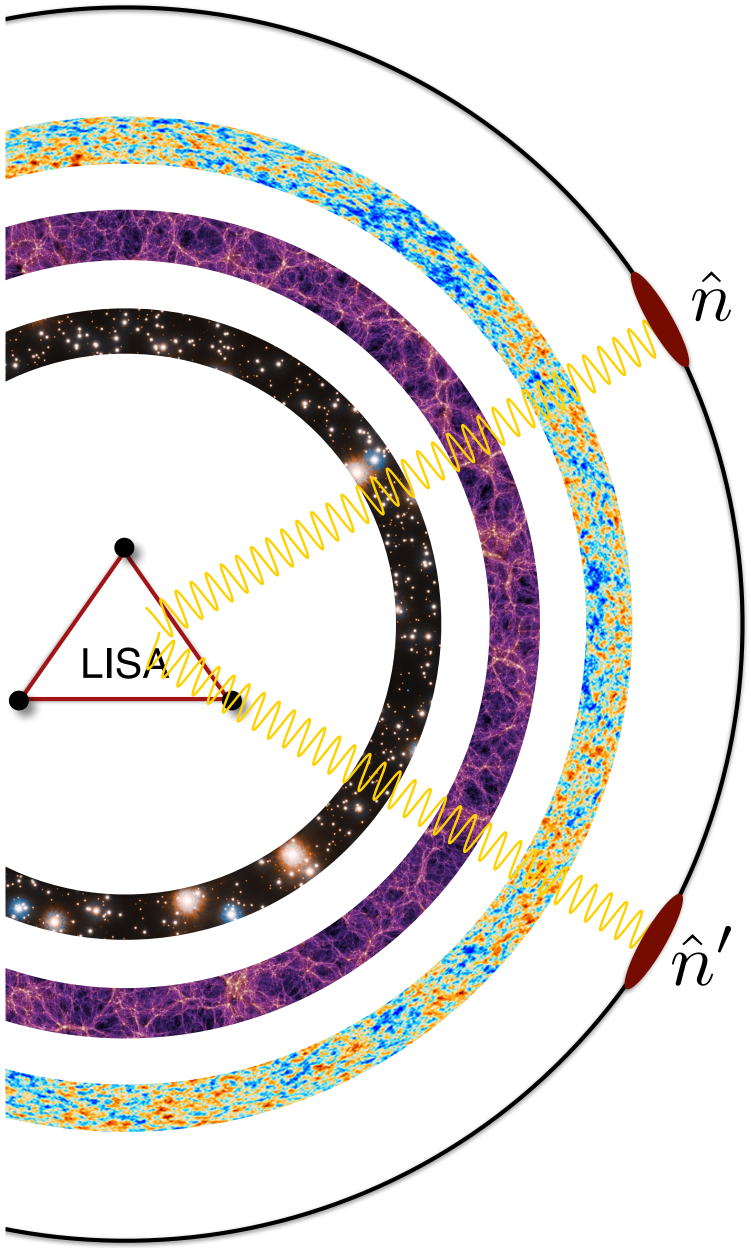
Need to cancel the phases, e.g use the spatial distribution of the energy density

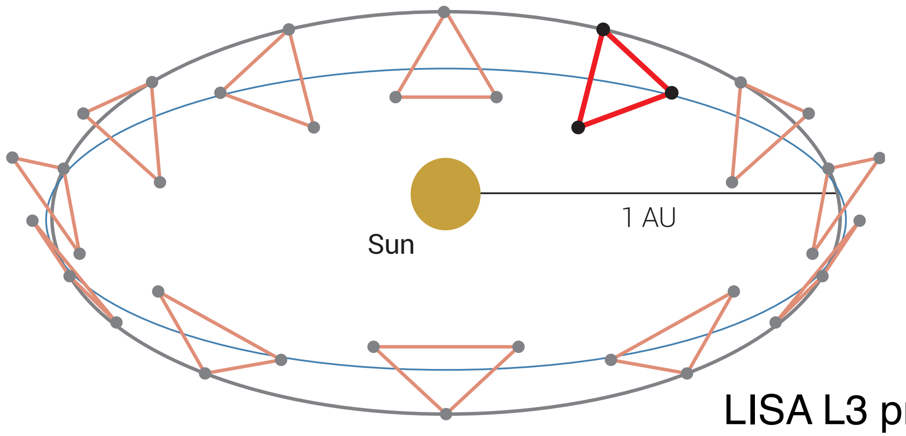
$$\Omega_{\rm GW}(\eta, \vec{x}) = \frac{\rho_{\rm GW}(\eta, \vec{x})}{\overline{\rho}(\eta)} = \frac{M_p^2}{4a^2\overline{\rho}(\eta)} \left\langle h'_{ab}\,(\eta,\,\vec{x})\, h'_{ab}\,(\eta,\,\vec{x})\right\rangle_{\rm t.a.}$$



Bartolo et al., (2019)

GW anisotropies





LISA L3 proposal (2017)

In addition LISA will probably have an angular resolution up to around $\ell \approx 10$

$$\Omega(\eta_0, k, \hat{n}) \equiv \Omega(\eta_0, k) + \delta\Omega(\eta_0, k, \hat{n})$$

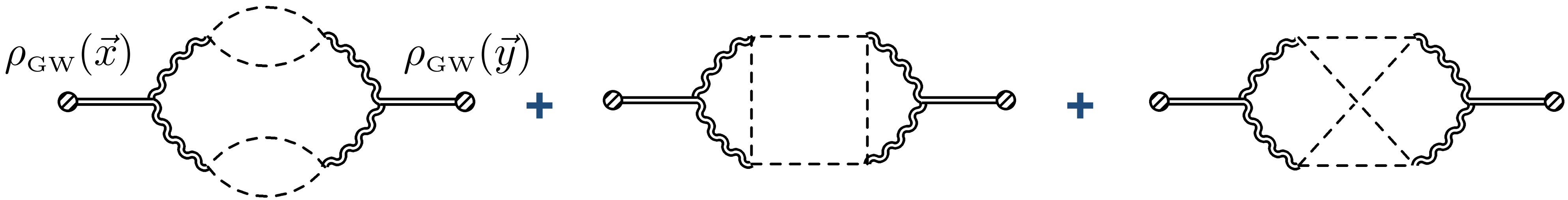
Small perturbations (direction dependent) in the abundance

1) Anisotropies at emission

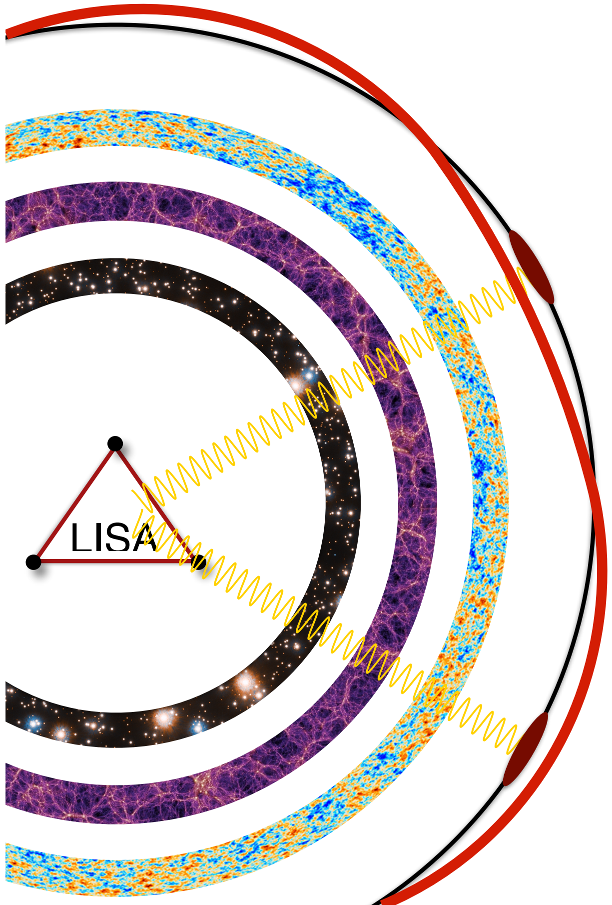
2) Anisotropies due to propagation

Bartolo et al., PRD (2019)

GW anisotropies at emission



$$\langle\, \delta\Omega(\eta_{\rm e}, \vec{x})\, \delta\Omega(\eta_{\rm e}, \vec{y})\, \rangle \approx \mathcal{P}_\zeta \mathcal{P}_\zeta \mathcal{P}_\zeta \mathcal{P}_\zeta$$



At unreachable scales:

$$\langle \delta\Omega(\eta_{\rm e}, \vec{x}) \delta\Omega(\eta_{\rm e}, \vec{y}) \rangle \sim (k_* |\vec{x} - \vec{y}|)^{-2}$$

$$k_* \sim H_{\rm e}^{-1}$$

Large scale perturbations do not impact the emission of GWs since they are on super-Hubble scales (Equivalence Principle)

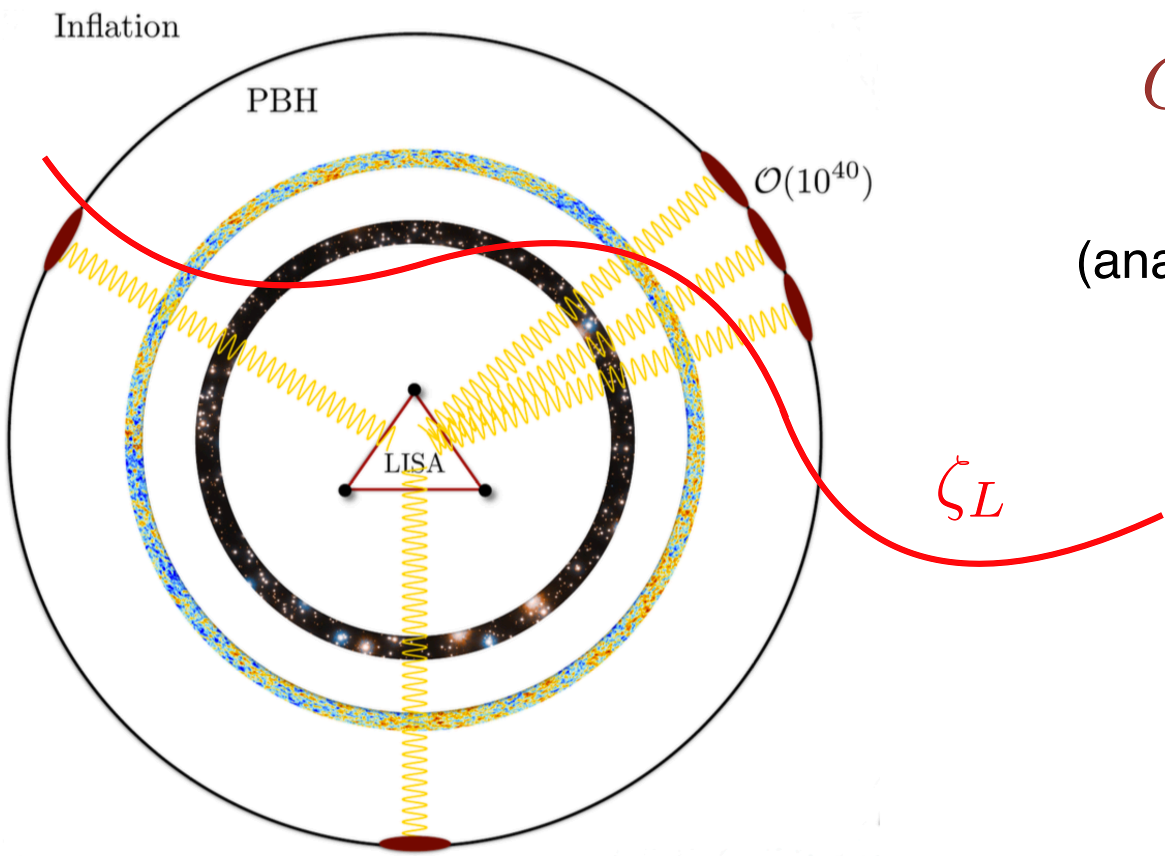
Need long and short mode coupling: primordial NG

Bartolo et al. (2019)

GW anisotropies due to propagation

Non-vanishing even if there is no primordial NG

$$\langle\, \delta\Omega_{\ell m}(\eta_0, k)\, \delta\Omega_{\ell' m'}(\eta_0, k)\,\rangle = \delta_{\ell\ell'}\delta_{mm'}\mathcal{F}(k) C_{\ell,S}$$



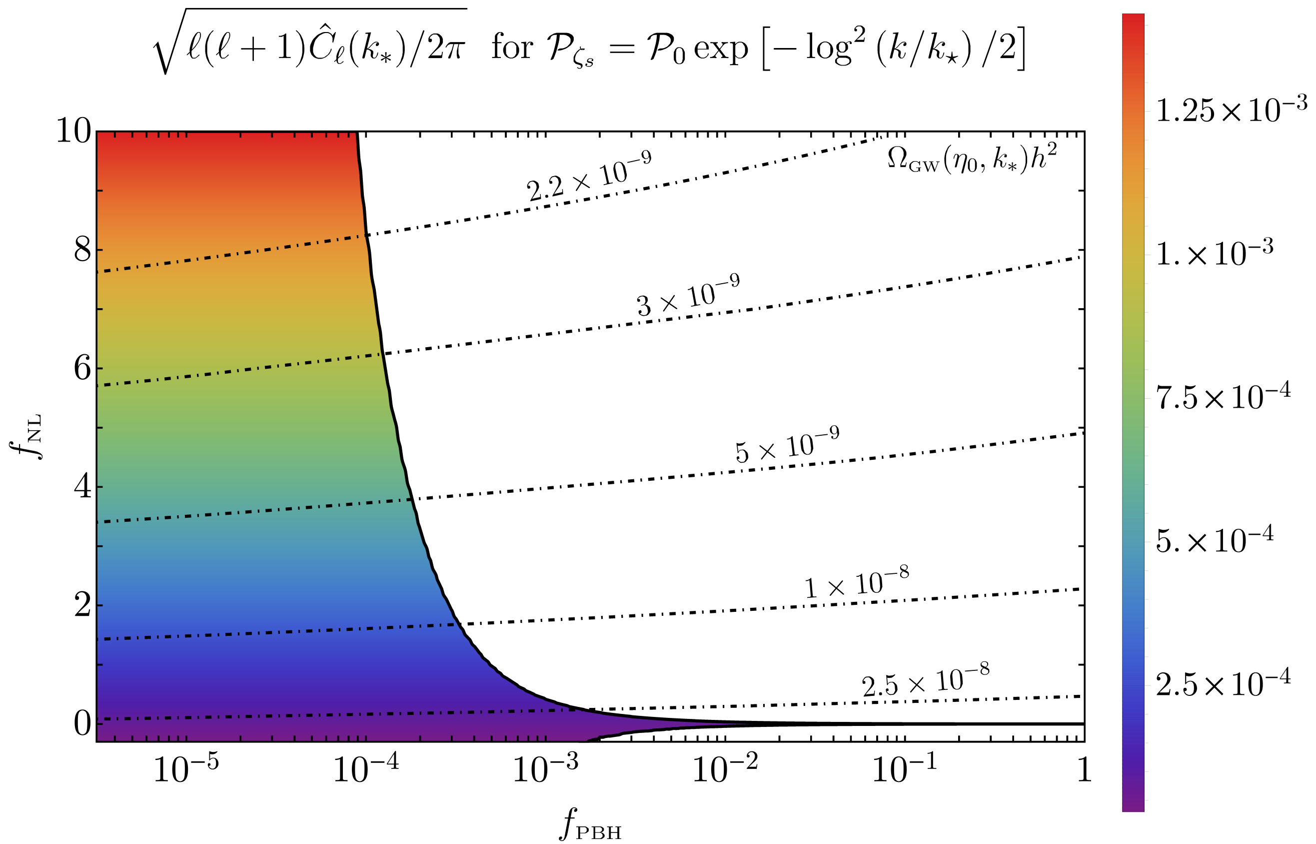
$$C_{\ell,S} = \frac{2\pi}{\ell(\ell+1)}\mathcal{P}_{\zeta_L}$$

(analogous to SW in the CMB)

Bartolo et al. (2019)

Anisotropies of GWs from PBHs

$$\zeta = \zeta_{\rm g} + \frac{3}{5} f_{\rm NL} \zeta_{\rm g}^2$$



A detection of anisotropies will mean PBHs may not form the dark matter

Bartolo et al. (2019)

Gauge invariance of the GWs

At first-order GWs are gauge invariant,

this is not so at second-order

1) GI at the measurement

2) GI emission and propagation

(inside the horizon, GI description)

V. De Luca et al. (2019)

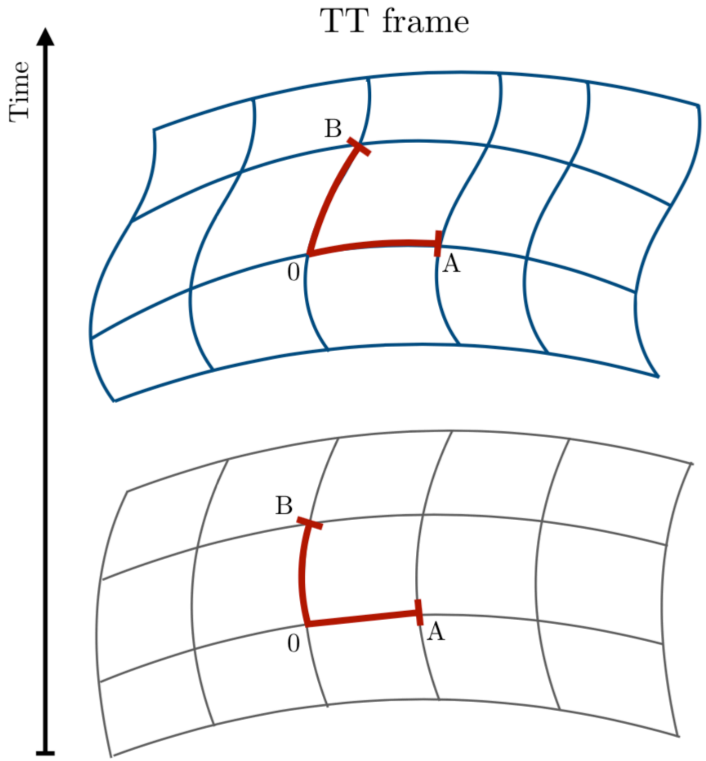
Gong (2019)
Tomikawa and Kobayashi (2019)
Inomata and Terada (2019)
Yuan et al. (2019)

Measurement

GWs affect time shifts in interferometers in two ways:

1) change the photon geodesic,

2) change the coordinate position of the mirrors.



coordinates are fixed with the position of the mirrors

(in the proper detector frame the geodesic is fixed, basically in flat space and coordinates change)

Measurement for $\omega_{\rm GW} L \ll 1$

A single frame can be defined in which the metric
is approximately flat and encompasses the all apparatus

The time shift coincides with the proper time

$$\ddot{L}_i(t) = -R_{i0j0} L_j(t)$$

The detector frame coincides with the TT frame thanks to the gauge invariance of R_{i0j0} at linear order

Measurement for $\omega_{\rm GW} L \sim 1$

Need a general frame description

$$\Delta t^{\rm GI}_{\rm A,B} = \Delta t_{\rm A,B} + \int_{t_0}^{t_0+2L_{\rm A,B}} {\rm d}t\, \phi(t)$$

coincides with the TT result at linear order

At second-order, Riemann tensor not GI, general frame description?

V. De Luca et al. (2019)

Conclusions

- PBHs do not need physics BSM and may form all the DM, intrinsic NG relevant, small spin at formation
- GWs from PBHs are detectable by LISA
- NG observable only in energy density if some primordial NG is present
- GW anisotropies are a way to characterise the source: a detection will imply PBHs may not form the all DM