Formation Threshold of Rotating Primordial Black Holes

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Collaborator: Teruaki Suyama (Tokyo Institute of Technology)

Based on: Phys. Rev. D100 (2019) 063520 [arXiv:1906.10987]

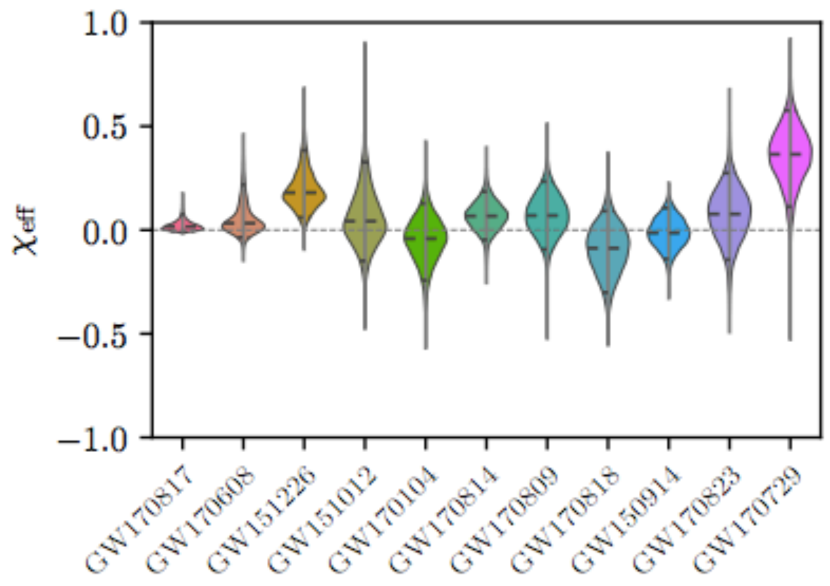
20191205@IPMU

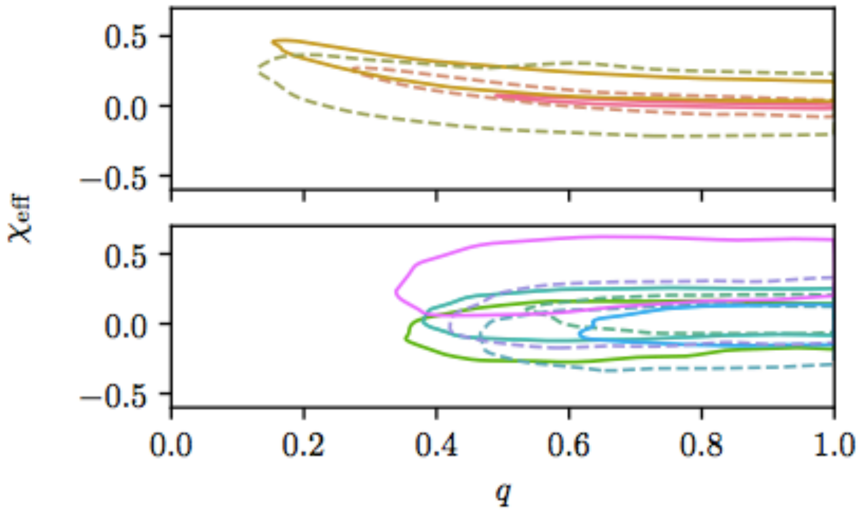
Contents

- Motivation
- Non-rotating primordial black holes (PBH) formation revisit
- Role of angular momentum
- Formation criterion of PBHs with angular momentum

Motivation

$$\chi_{\text{eff}} = \frac{(m_1\vec{\chi}_1 + m_2\vec{\chi}_2)\cdot\hat{L}_N}{M}$$





B. P. Abbott et al, arXiv:1811.12907

The whole picture of spin of PBHs

Press-Schechter formalism (here we ignore the critical phenomena)

Spin distribution $$W(J,t) = \int dJ' \, \underline{Q(J,J',t)} \int_{\underline{\delta_{\rm th}(J')}} \underline{P(\delta_M, J')} d\delta_M$$

The probability distribution of δ_M and J' of the overdense region

The evolution of the spin of PBHs at time t after formation

The threshold beyond which the PBH forms with initial angular momentum J'

J. C. Niemeyer and K. Jedamzik, Phys. Rev. Lett., 80:5481-5484, 1998

J. Yokoyama, Phys. Rev., D58:107502, 1998

T. Chiba, and S. Yokoyama, PTEP, 2017(8):083E01, 2017

M. Mirbabayi, A. Gruzinov, and J. Norea, arXiv:1901.05963

V. De Luca, V. Desjacques, G. Franciolini, A. Malhotra, and A. Riotto, JCAP 1905, 018 (2019)

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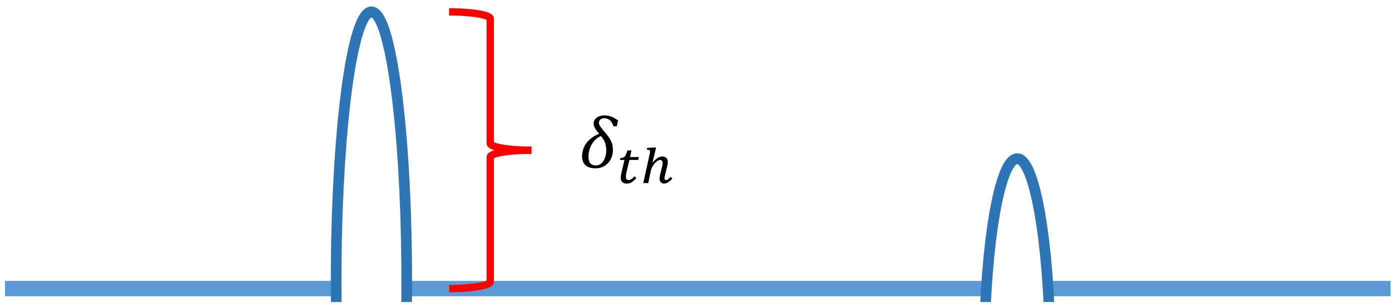
Non-rotating PBH formation

- Radiation-dominated epoch, $c_s^2 = w = 1/3$, $\rho \propto a^{-4}$
- Jeans instability = threshold δ_{th}



B. J. Carr, Astrophys. J. 201:1-19, 1975

The physical process

1. The large curvature perturbation is on super-horizon scale

2. After inflation ends, the large peak re-enters the horizon

3. The overdense region begins to evolve as a closed FLRW and reaches a maximum expansion

4. If the maximum size of the region is larger than Jeans radius, it experiences gravitational collapse to form a PBH

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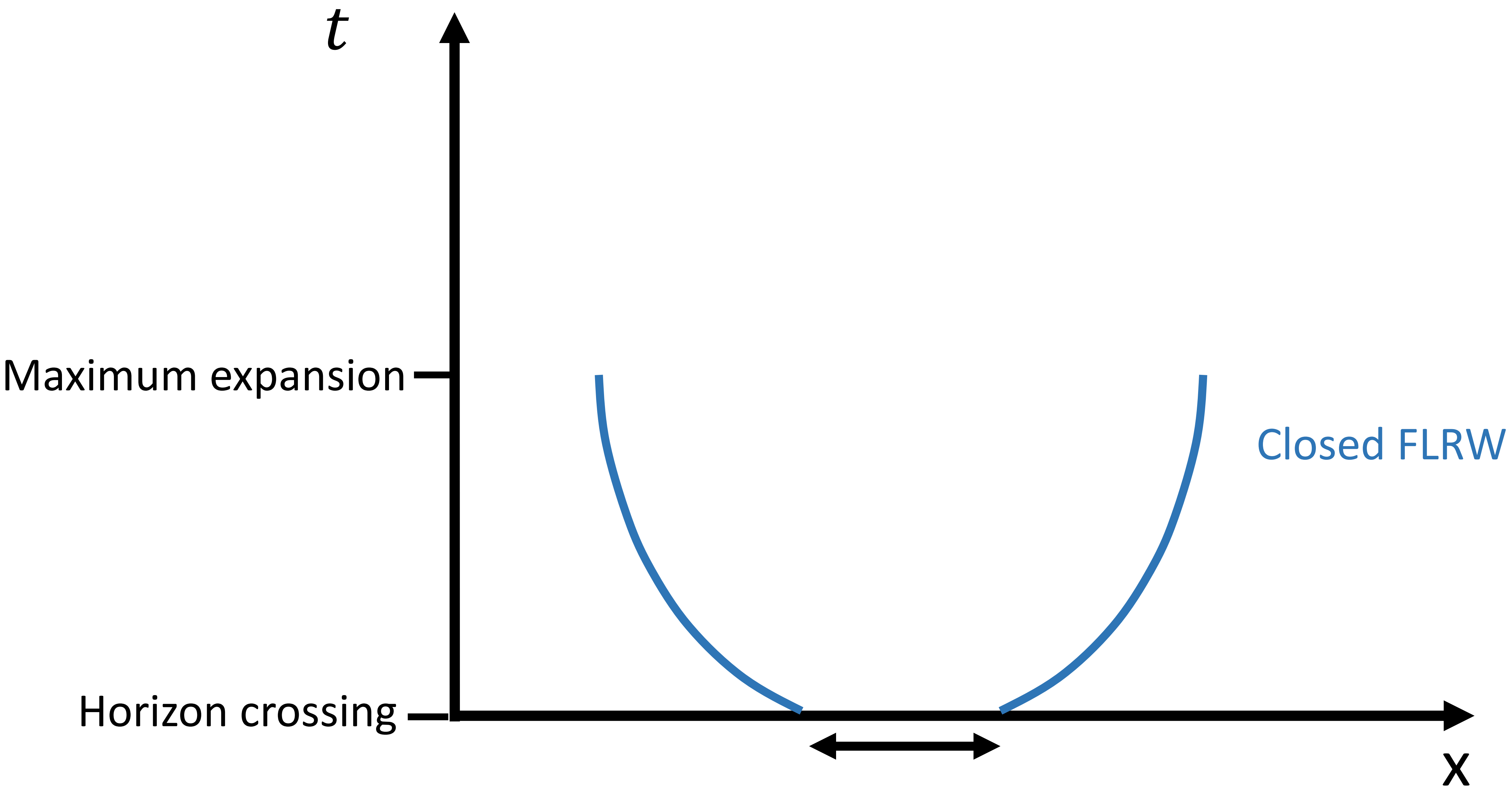
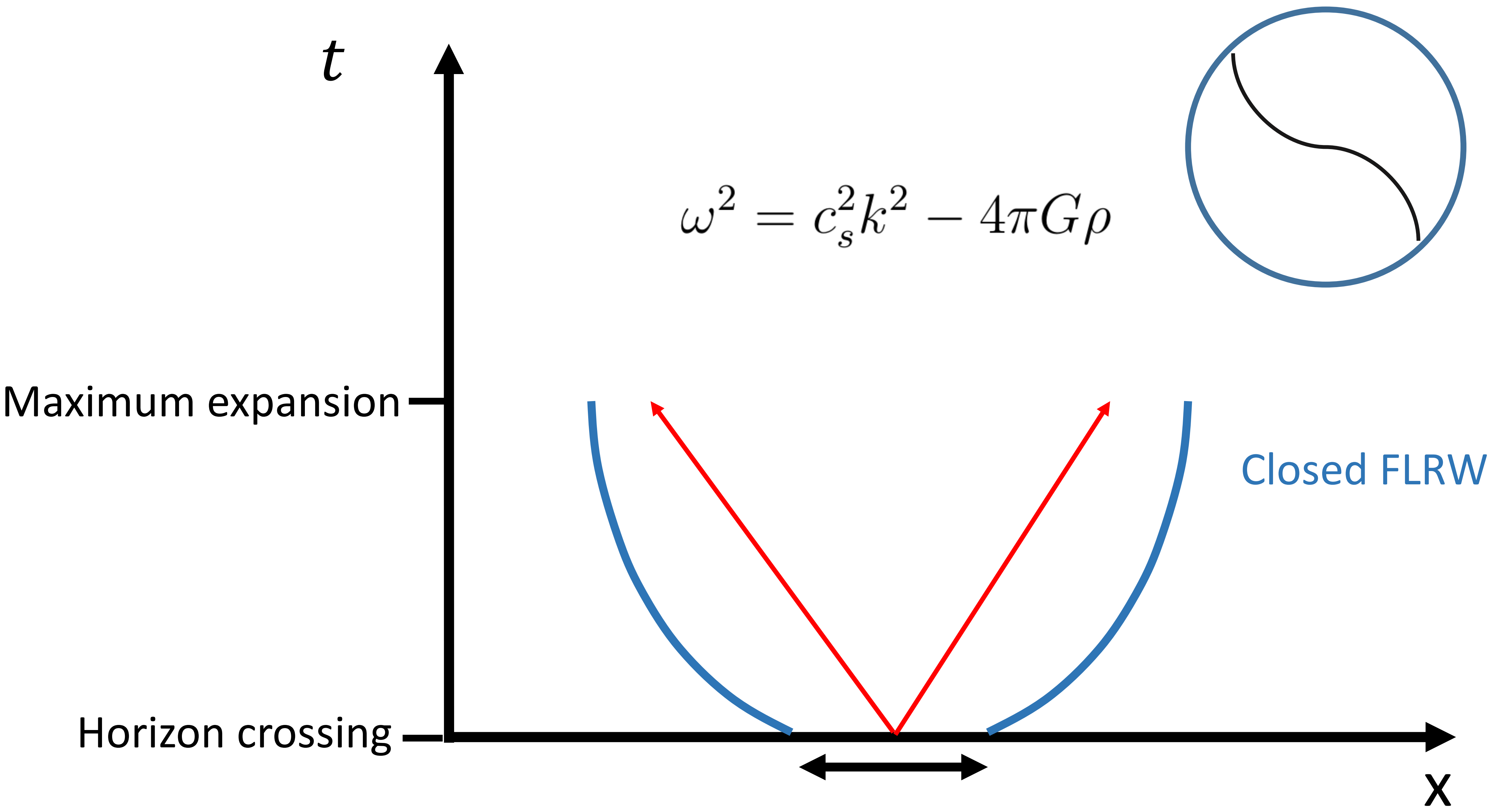
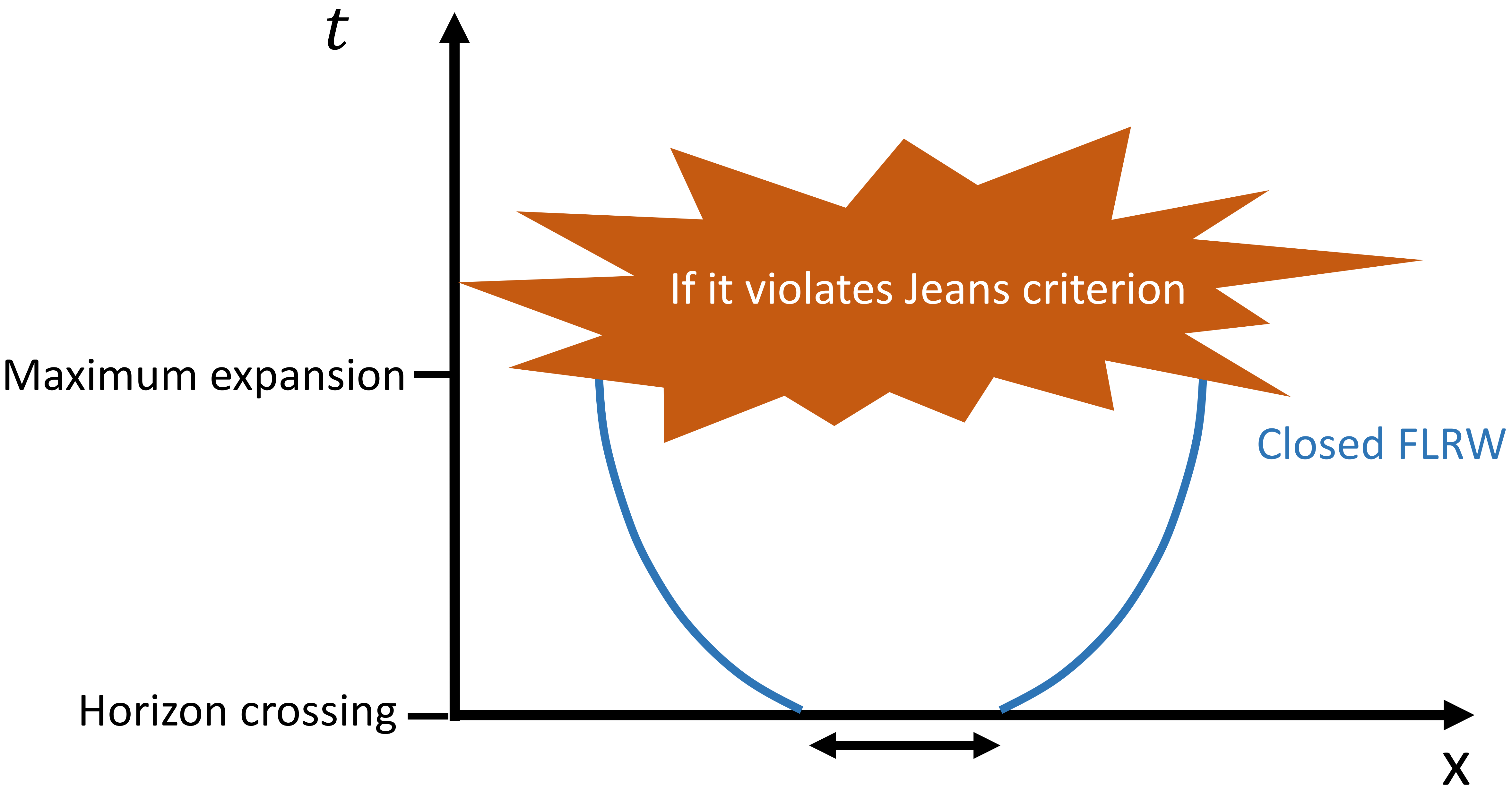


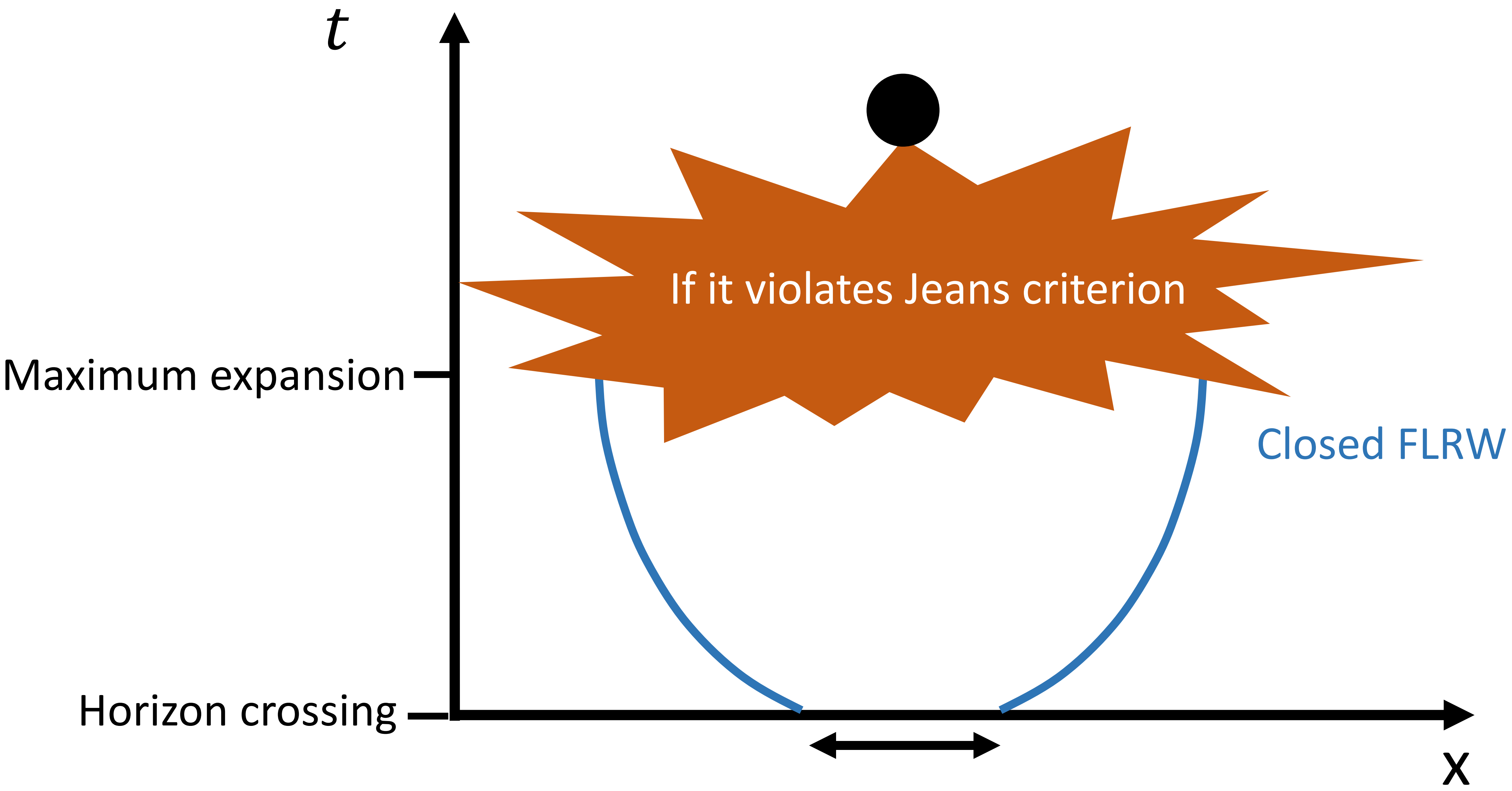
t
Maximum expansion
Horizon crossing
Closed FLRW
x



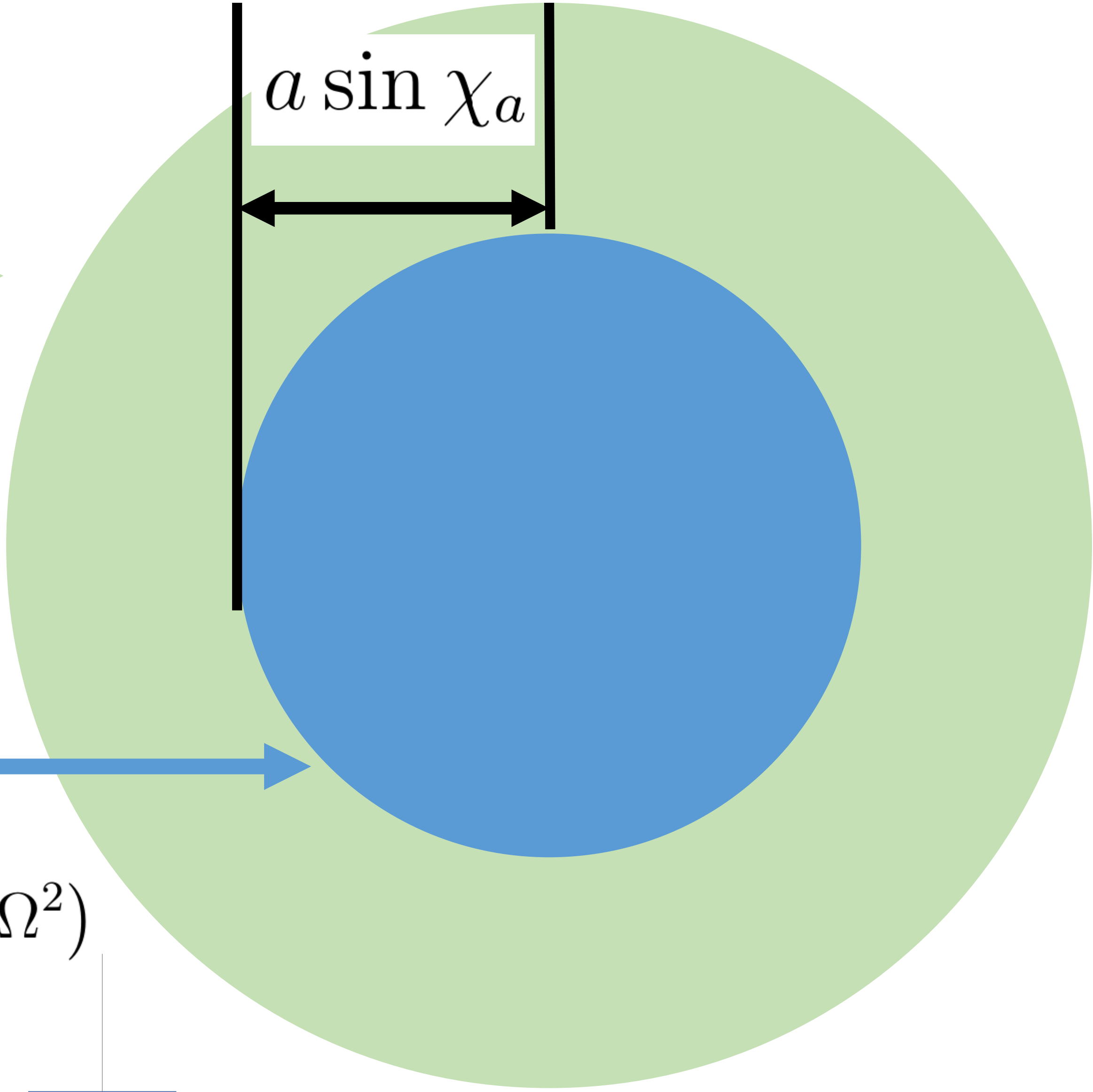
t
$\omega^2 = c_s^2 k^2 - 4\pi G \rho$
Maximum expansion
Closed FLRW
Horizon crossing
x



t
If it violates Jeans criterion
Maximum expansion
Closed FLRW
Horizon crossing
x


t
If it violates Jeans criterion
Maximum expansion
Closed FLRW
Horizon crossing
x

- Flat FLRW background

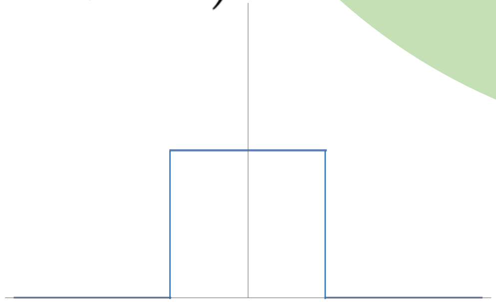


$$0 < \chi < \chi_a$$

- Closed FLRW

$$ds^2 = -dt^2 + a^2(t)\left(\underline{d\chi^2} + \sin^2 \chi d\Omega^2\right)$$

$$H_{\rm od}^2 = \frac{8\pi G}{3}\bar{\rho}(1+\underline{\delta}) - \frac{1}{a^2}$$



B. J. Carr, Astrophys. J. 201:1-19, 1975

Jeans instability

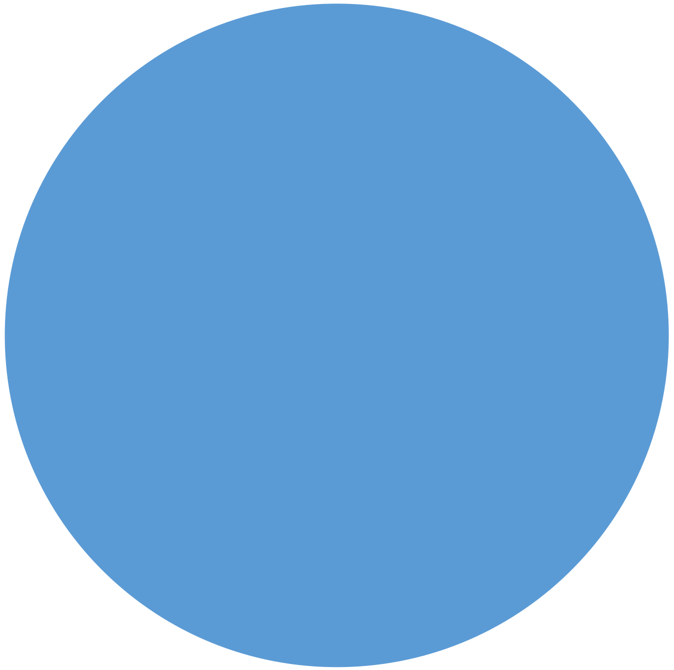
- Newtonian Jeans length:

sound speed × free-fall time

$$R_J = \frac{c_s}{\sqrt{G\bar{\rho}_{\rm max}(1+\delta_{\rm max})}} \simeq c_s a_{\rm max}$$

- Size at maximum expansion:

$$\frac{a_{\rm max}}{a_{\rm hc}} H_{\rm hc}^{-1}$$



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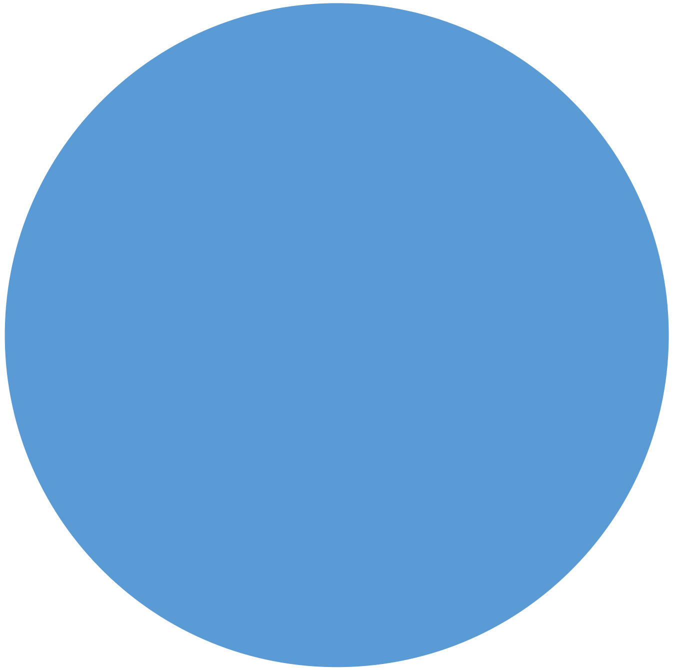

- Size at maximum expansion:

$$\frac{a_{\max}}{a_{\mathrm{hc}}} H_{\mathrm{hc}}^{-1} \quad > \quad R_J$$

Jeans criterion

$$\delta_{\mathrm{hc}} > \delta_{\mathrm{th}} = c_s^2 = \frac{1}{3}$$

B. J. Carr, Astrophys. J. 201:1-19, 1975

Refined relativistic analysis

- Go beyond the Newtonian Jeans length

- Full relativistic calculation of the propagation of the sound wave on a closed FLRW

$$\delta_{Hc}^{\mathrm{UH}} = \sin^2 \frac{\pi}{2\sqrt{3}} \simeq 0.62$$

This result is larger than the original one and fits the numerical result better

T. Harada, C.-M. Yoo, and K. Kohri, Phys. Rev., D88(8):084051, 2013

Role of angular momentum

- Intuitively, the centrifugal force due to rotation will resist gravity

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- Mathematically speaking, $G^{(0)}_{\mu\nu} = 8\pi G T^{(0)}_{\mu\nu} + 8\pi G \left(-\frac{1}{8\pi G} \left\langle G^{(2)}_{\mu\nu} \right\rangle + \left\langle T^{(2)}_{\mu\nu} \right\rangle \right)$

Effective energy-momentum tensor

- Therefore, the evolution of the scale factor is modified

→ the propagating distance of the sound wave changes

$$\sqrt{3} \int_0^{\chi_s} d\chi = \int_0^{t_{\max}} \frac{dt}{\tilde{a}}$$

Main physical assumptions

- The existence of rotational perturbations, especially vector-type, $(l, m) = (1,0)$

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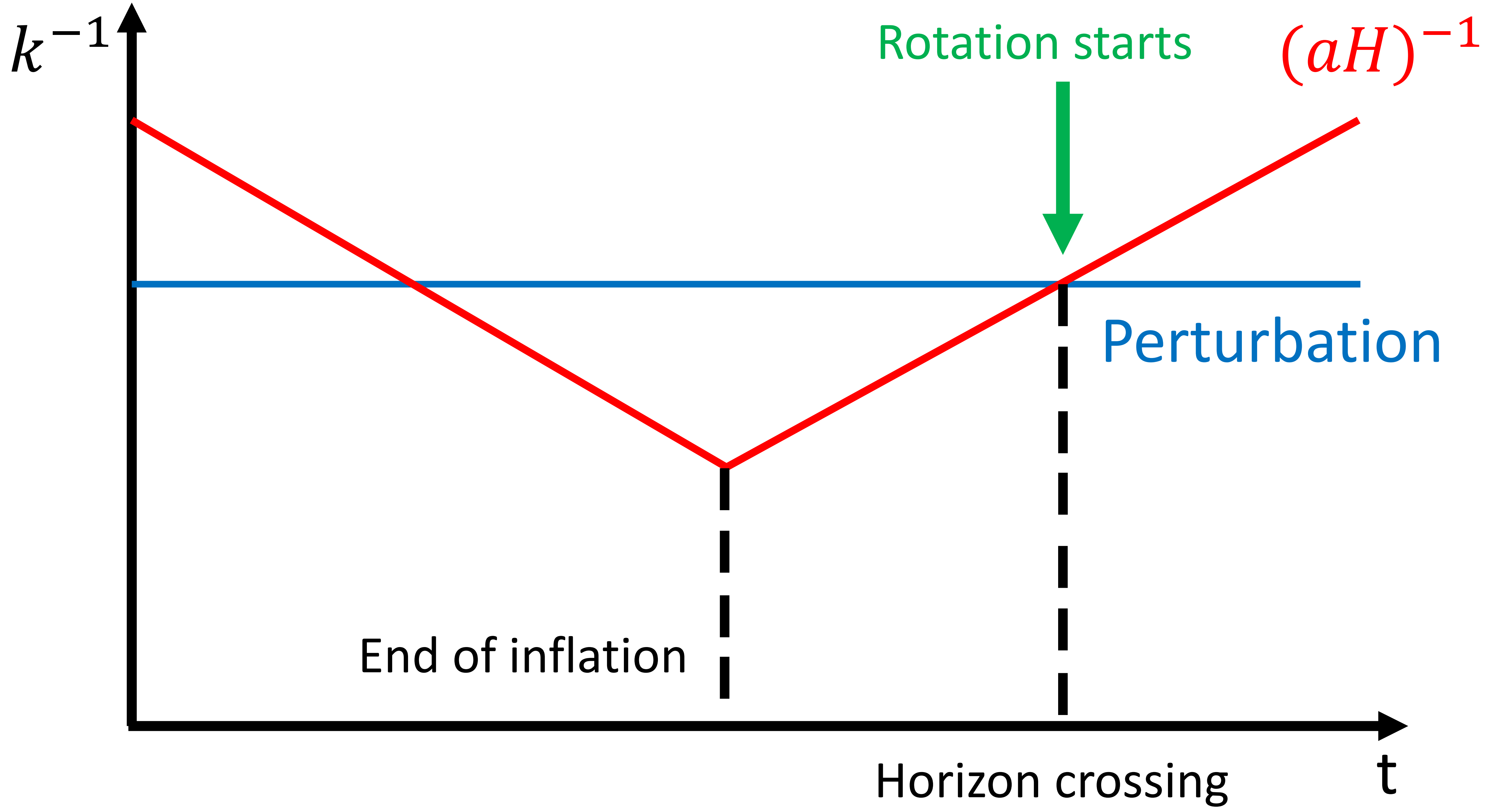
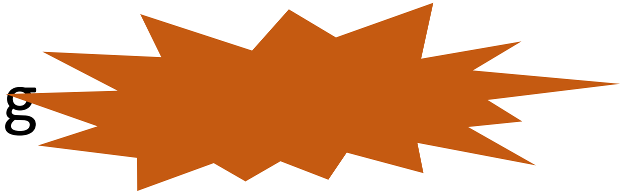
- Angular momentum switches on as soon as horizon crossing


k^{-1}
Rotation starts
$(aH)^{-1}$
Perturbation
End of inflation
Horizon crossing
t

Main physical assumptions

- The existence of rotational perturbations, especially vector-type,
$$(l, m) = (1,0)$$

- Angular momentum switches on as soon as horizon crossing

- The mass and angular momentum are conserved during 
  - Evaluate M and J at the maximum expansion time

- Collapse occurs only when all directions violate Jeans stability

The physical process revisit

1. The large curvature perturbation is on super-horizon scale

2. After inflation ends, the large peak re-enters the horizon

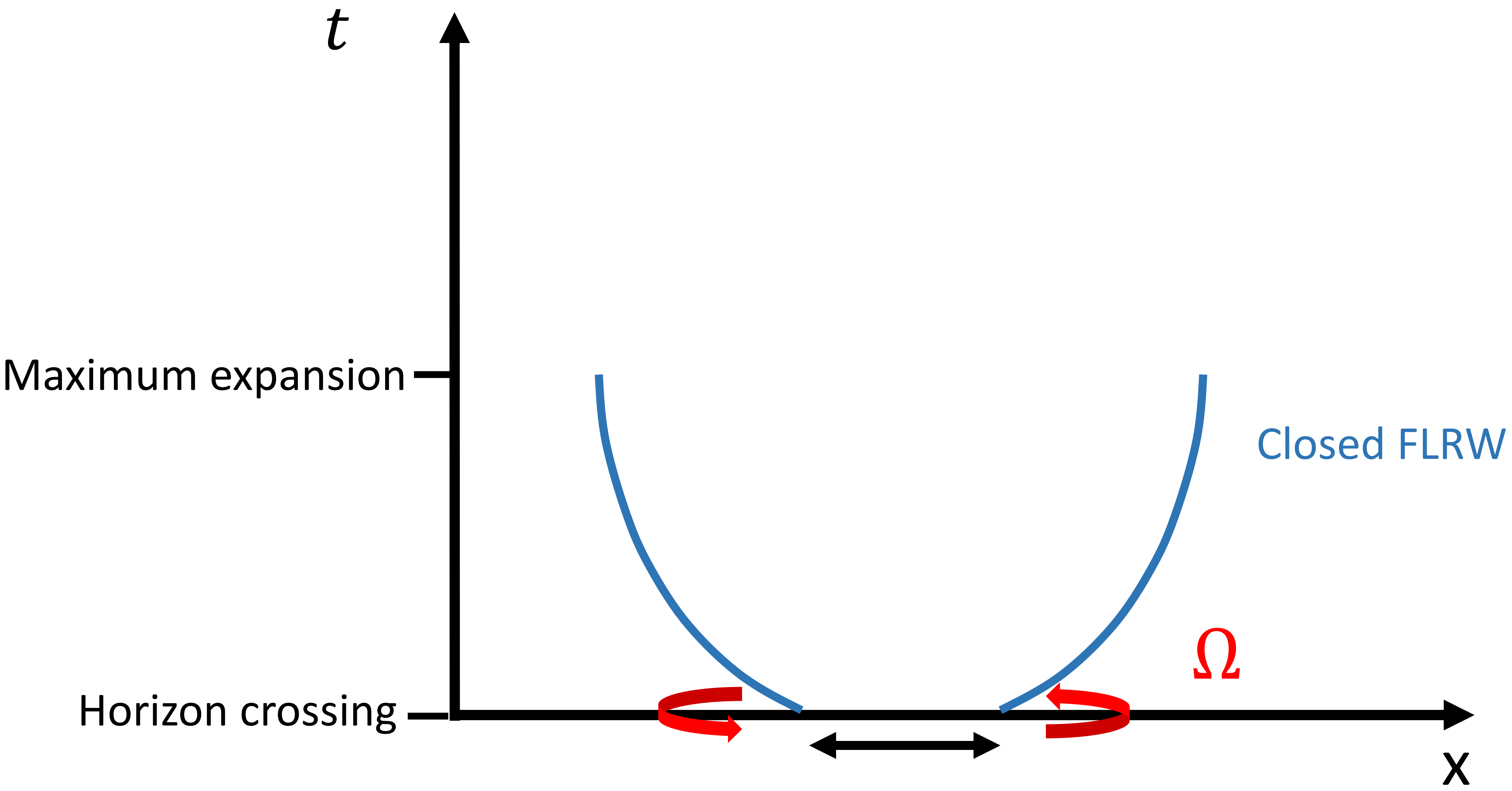
3. The overdense region begins to evolve as a closed FLRW and reaches a maximum expansion

4. If the maximum size of the region is larger than Jeans radius, it experiences gravitational collapse to form a PBH

B. J. Carr, Astrophys. J. 201:1-19, 1975

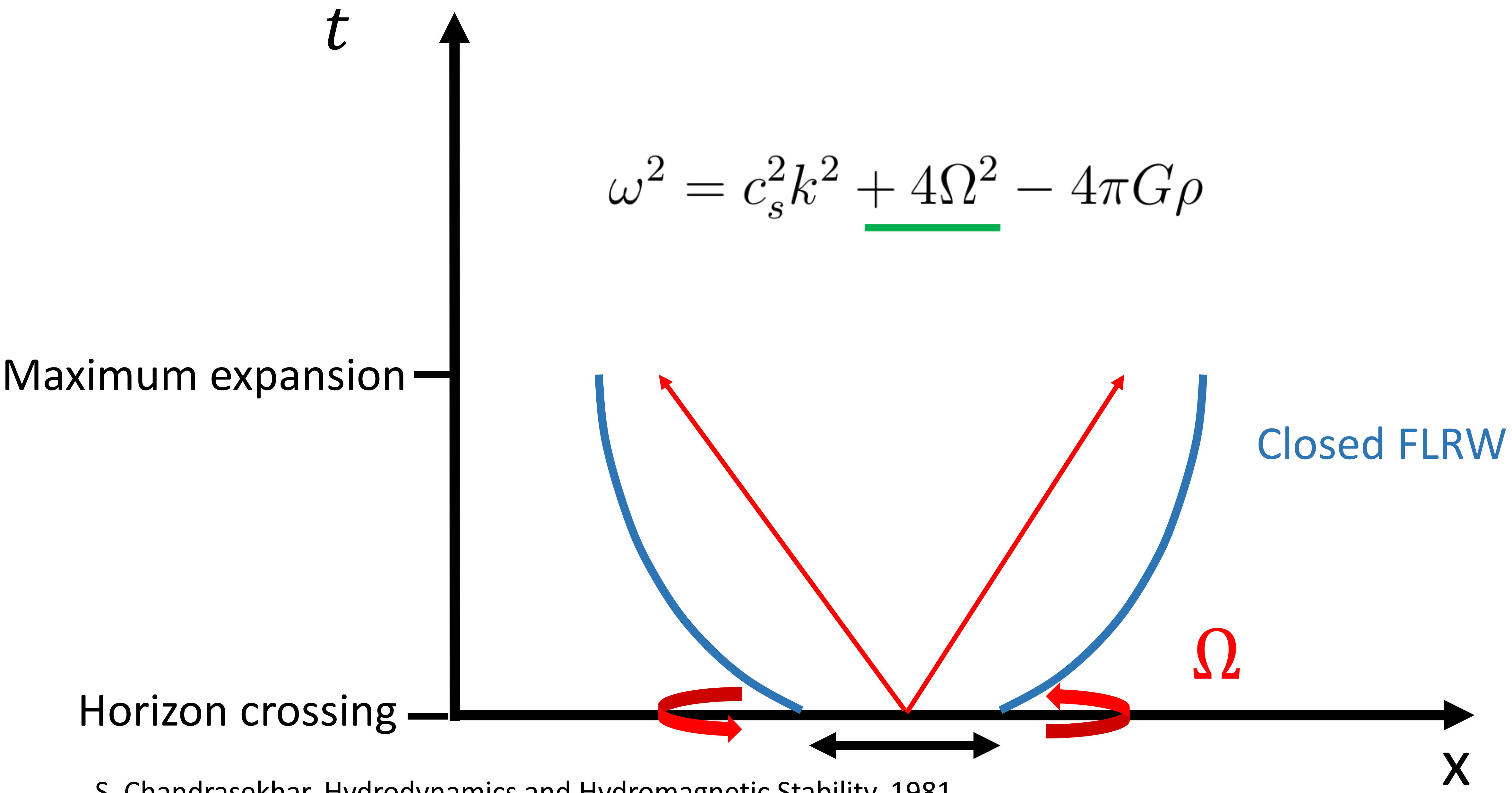
The physical process revisit

1. The large curvature perturbation is on super-horizon scale
2. After inflation ends, the large peak re-enters the horizon and angular momentum switches on
3. The overdense region begins to evolve as a closed FLRW with vector perturbations and rotating fluid and reaches a maximum expansion
4. If the maximum size of the region is larger than the new Jeans radius, it experiences gravitational collapse to form a PBH

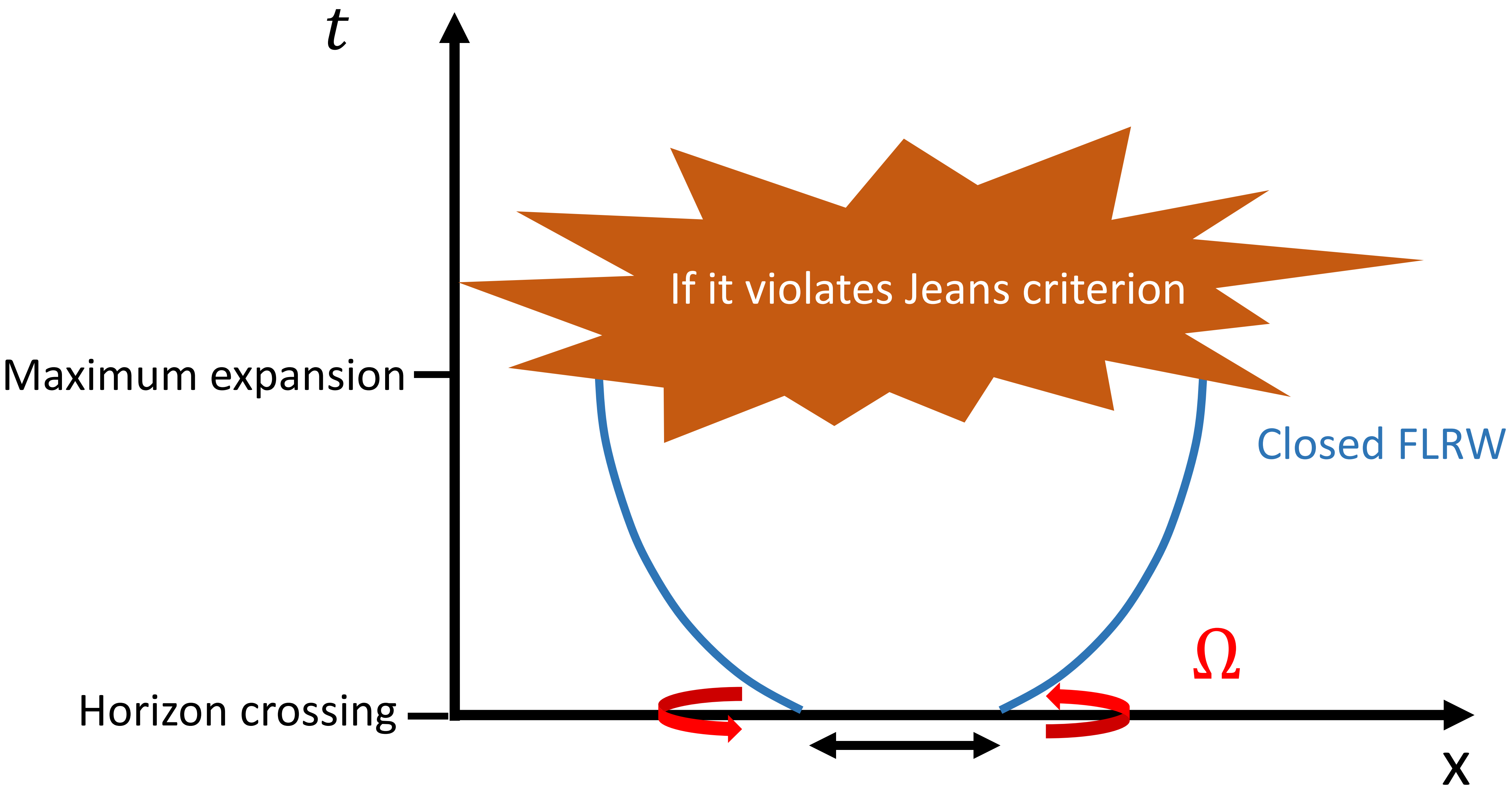


t
Maximum expansion
Horizon crossing
Closed FLRW
Ω
x

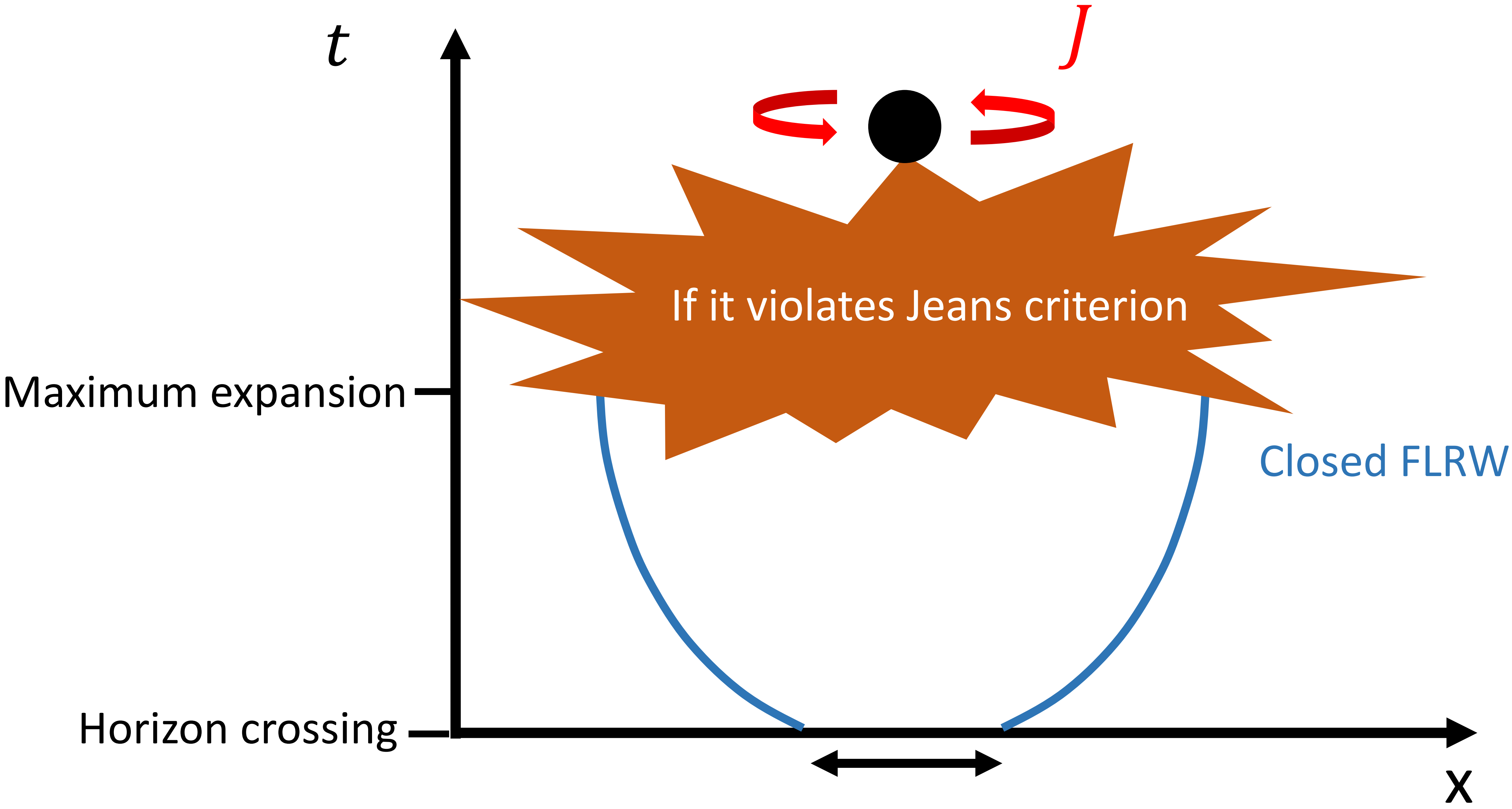
$$\omega^2 = c_s^2 k^2 + 4\Omega^2 - 4\pi G\rho$$



S. Chandrasekhar, Hydrodynamics and Hydromagnetic Stability, 1981



t
If it violates Jeans criterion
Maximum expansion
Closed FLRW
Ω
Horizon crossing
x



t
J
If it violates Jeans criterion
Maximum expansion
Closed FLRW
Horizon crossing
x

$$J = \frac{2(1 + c_s^2) M R^2 \Omega}{5}$$

$$a_K = \frac{J}{G M^2}$$

Non-rotating	(Slowly) rotating
$\omega^2 = c_s^2 k^2 - 4\pi G\rho$	$\omega^2 = c_s^2 k^2 + 4\Omega^2 - 4\pi G\rho$
$R_J \simeq \frac{c_s}{\sqrt{G\rho}}$	$R_J \simeq \frac{c_s}{\sqrt{G\rho - \frac{\Omega^2}{\pi}}} \simeq \frac{c_s}{\sqrt{G\rho}}\left(1 + \frac{\Omega^2}{2\pi G\rho}\right)$
$\delta_{\mathrm{hc}} > \delta_{\mathrm{th}} = c_s^2 = \frac{1}{3}$	$\delta_{\mathrm{hc}} > \delta_{\mathrm{th}} = c_s^2\left(1 + \frac{25 c_s^2 a_K^2}{6(1+c_s^2)^3}\right)$

$$\frac{2GM_{hc}}{H_{hc}^{-1}} = 1$$

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$\omega^2 = c_s^2 k^2 - 4\pi G\rho$	$\omega^2 = c_s^2 k^2 + 4\Omega^2 - 4\pi G\rho$
$R_J \simeq \frac{c_s}{\sqrt{G\rho}}$	$R_J \simeq \frac{c_s}{\sqrt{G\rho - \frac{\Omega^2}{\pi}}} \simeq \frac{c_s}{\sqrt{G\rho}}\left(1 + \frac{\Omega^2}{2\pi G\rho}\right)$
$\delta_{\rm hc} > \delta_{\rm th} = c_s^2 = \frac{1}{3}$	$\delta_{\rm hc} > \delta_{\rm th} = c_s^2\left(1 + \frac{25 c_s^2 a_K^2}{6(1+c_s^2)^3}\right)$

$$\frac{2GM_{hc}}{H_{hc}^{-1}} = 1$$

$(l,m) = (1,0)$ $\quad h_{t\phi} = \frac{a_{\max}}{a(t)} j(\chi) \sin^2\theta, \qquad \delta u_\phi = \left(\frac{a(t)}{a_{\max}}\right)^{3w} V_{\max}(\chi) \sin^2\theta$

Uniform rotation $\quad V_{\max}(\chi) = V_f \sin^2\chi$ where $V_f = \text{const}$

$$\delta_{Hc}^{\rm UH} = \sin^2\left(\frac{\sqrt{3}\pi}{6}\right) + \frac{5\sqrt{3}}{64}\sin\left(\frac{\sqrt{3}\pi}{3}\right)\left(\frac{2GM_{\rm hc}}{a_{\rm hc}\chi_a}\right)^2 \frac{\sin\chi_a}{(2+\cot^2\chi_a)^2} a_K^2$$

For small rotation, we take $\chi_a = \frac{\sqrt{3}\pi}{6}$, and assume $2GM_{\rm hc}/(a_{\rm hc}\chi_a) = 1$

$$\longrightarrow \quad \delta_{Hc}^{\rm UH} \simeq 0.62 + \boxed{0.015 a_K^2}$$

Correction from the angular momentum to the formation threshold

$$W(J,t) = \int dJ' \; Q(J,J',t) \underbrace{\int_{\delta_{\rm th}(J')} P(\delta_M, J') d\delta_M}_{\text{At formation}}$$

At formation

V. De Luca, V. Desjacques, G. Franciolini, A. Malhotra, and A. Riotto, JCAP 1905, 018 (2019)

Discussion

- We derive the threshold in the presence of small angular momentum by the Jeans criterion.
- The leading-order correction is proportional to square of a_K .
- Ambiguities:
  1. Simple rotation, vector perturbations
  2. Angular momentum switches on at horizon crossing
  3. Backreaction of vector perturbations on a closed FLRW
  4. Ignoring the effects of scalar perturbations to the rotation
  5. Conservation of mass and angular momentum during collapse
- The result could be used to contrast the experiment when we accumulate enough black hole merger events.

Thank you for your attention!

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Vector-type perturbations on the closed FLRW (below w=1/3)

$$ds^2 = \left(g^{(0)}_{\mu\nu} + h_{\mu\nu}\right) dx^\mu dx^\nu \qquad u_\mu = \bar{u}_\mu + \delta u_\mu$$

$$h_{t\phi} = \frac{a_{\max}}{a(t)} j(\chi) \sin^2\theta, \qquad \delta u_\phi = \frac{a(t)}{a_{\max}} V_{\max}(\chi) \sin^2\theta$$

$$j(\chi) \equiv 8 \sin^2\chi \int_0^\chi \frac{d\chi'}{\sin^4\chi'} \int_0^{\chi'} d\chi'' \sin^2\chi'' V_{\max}(\chi'')$$

Here we choose the gauge where $h_{\chi\phi} = 0$

- Comment:

The 3-geometry of a closed FLRW is a 3-sphere. A uniform vector field, for example, on a sphere will lead to singularity. However, the overdense region only covers only part of the sphere which is charaterised by χ_a so it is okay.

D. Lynden-Bell, J. Katz, and J. Bicak, Mon. Not. Roy. Astron. Soc., 272:150-160, 1995
T. Regge and J. A. Wheeler, Phys. Rev., 108:1063-1069, 1957

$$G_{\mu\nu}^{(0)} = 8\pi G T_{\mu\nu}^{(0)} + 8\pi G \left(-\frac{1}{8\pi G} \left\langle G_{\mu\nu}^{(2)} \right\rangle + \left\langle T_{\mu\nu}^{(2)} \right\rangle \right)$$

Time-time component $$\Delta\rho(t) = -\frac{1}{8\pi G} \left\langle G_{00}^{(2)} \right\rangle + \left\langle T_{00}^{(2)} \right\rangle$$

$$\left\langle T_{00}^{(2)} \right\rangle = \frac{16\bar{\rho}}{9a^4(\chi_a - \sin\chi_a \cos\chi_a)} \int_0^{\chi_a} d\chi \left(a_{\max} j - \frac{a^2}{a_{\max}} V_{\max} \right)^2 ,$$

$$\left\langle G_{00}^{(2)} \right\rangle = -\frac{a_{\max}^2}{3a^6(\chi_a - \sin\chi_a \cos\chi_a)} \int_0^{\chi_a} d\chi \left[(j' - 2j\cot\chi)^2 + 32 j V_{\max} - 4(4 + \dot{a}^2) j^2 \right]$$

We choose a simple case, uniform rotation $V_{\max}(\chi) = V_f \sin^2 \chi$ where $V_f = \text{const.}$

$$\left\langle T_{00}^{(2)} \right\rangle = \frac{8V_f^2 \bar{\rho}}{15a_{\max}^2}\chi_a^2 - \frac{8V_f^2 \bar{\rho} a^{-2}}{1575 a_{\max}}\left[120 a_{\max} + 29\frac{a^2}{a_{\max}}\right]\chi_a^4$$

$$\left\langle G_{00}^{(2)} \right\rangle = -\frac{352 a_{\max}^2 V_f^2}{175 a^6}\chi_a^4$$

To leading order, $\Delta\rho = \epsilon_0 \bar{\rho}$ with $\epsilon_0 \equiv \frac{8}{15a_{\max}^2} V_f^2 \chi_a^2$

Friedmann equation with new scale factor

$$\frac{1 + \dot{\tilde{a}}^2}{\tilde{a}^2} = \frac{8\pi G}{3}\bar{\rho}(\tilde{a})(1 + \epsilon(\tilde{a}))$$

with $\epsilon(a) = \epsilon_0 \Theta(a - a_{hc})$

Switch on after horizon crossing

$$\underline{\sqrt{3}\int_0^{\chi_s} d\chi} = \int_0^{t_{\max}} \frac{dt}{\tilde{a}} = \int_0^1 \frac{du}{u\sqrt{\frac{1+\epsilon(\tilde{a}_{\max}u)}{1+\epsilon}\frac{1}{u^2} - 1}}$$

New Jeans length

$$u \equiv \tilde{a}/\tilde{a}_{\max}$$

$$\chi_a > \chi_s$$

The final result becomes

$$\delta_{Hc}^{\mathrm{UH}} = \sin^2\left(\frac{\sqrt{3}\pi}{6}\right) + \frac{5\sqrt{3}}{64}\sin\left(\frac{\sqrt{3}\pi}{3}\right)\left(\frac{2GM_{\mathrm{hc}}}{a_{\mathrm{hc}}\chi_a}\right)^2 \frac{\sin\chi_a}{(2+\cot^2\chi_a)^2} a_K^2$$

Quantites defined at maximum expansion

$$a_K = J/(GM_{\mathrm{BH}}^2) \quad M_{\mathrm{BH}} \equiv 4\pi a_{\max}^3 \bar{\rho}_{\max} \chi_a^3/3 \quad J = \frac{32\pi}{45}\frac{a^4\bar{\rho}}{a_{\max}} V_f \chi_a^5$$

Relation with curvature perturbation

$$\sqrt{3}\int_0^{\chi_s} d\chi = \int_0^{t_{\max}} \frac{dt}{\tilde{a}}$$

$$\downarrow$$

$$\begin{aligned}\zeta_c &\simeq -2\ln\cos\frac{\chi_s}{2}\\ &\simeq 0.21 + 0.008 a_K^2\end{aligned}$$

M. Kopp, S. Hofmann, and J. Weller, Phys. Rev. D83:124025, 2011

- Expansion in χ_a
  - From non-rotating and radiation: $\chi_a = \sqrt{3}\pi/6 \approx 0.91$ is not so small

$$\Delta\rho = \underbrace{\frac{8V_f^2\bar{\rho}}{15a_{\rm max}^2}\chi_a^2}_{\Delta\rho^{(2)}} + \underbrace{\frac{4V_f^2}{4725\pi G a^6}\left[297a_{\rm max}^2 - 8\pi G a^4\left(90 + \frac{87}{4}\frac{a^2}{a_{\rm max}^2}\right)\bar{\rho}\right]\chi_a^4}_{\Delta\rho^{(4)}}$$

However, for $a_{\rm hc} \le a \le a_{\rm max}$

$$\frac{\Delta\rho^{(4)}}{\Delta\rho^{(2)}} = \frac{1}{105}\left(-29 + 12\frac{a_{\rm max}^2}{a^2}\right)\chi_a^2 < 20\%$$

We neglect higher order terms