Part I
PBHs from Inflation and GWs

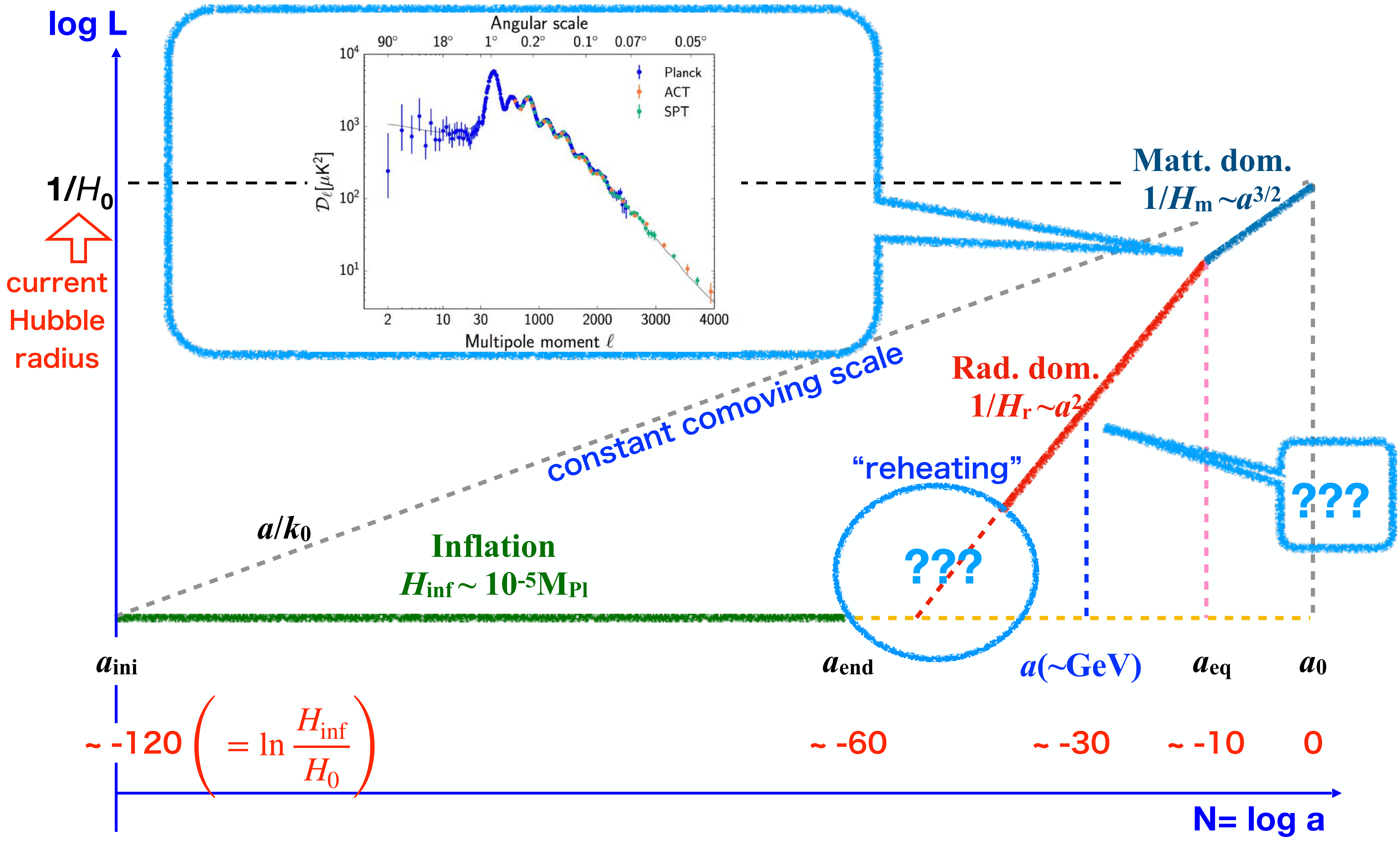
Misao Sasaki

Kavli IPMU, University of Tokyo
YITP, Kyoto University
LeCosPA, National Taiwan University
CAS Key Lab Theor. Phys., ITP-CAS

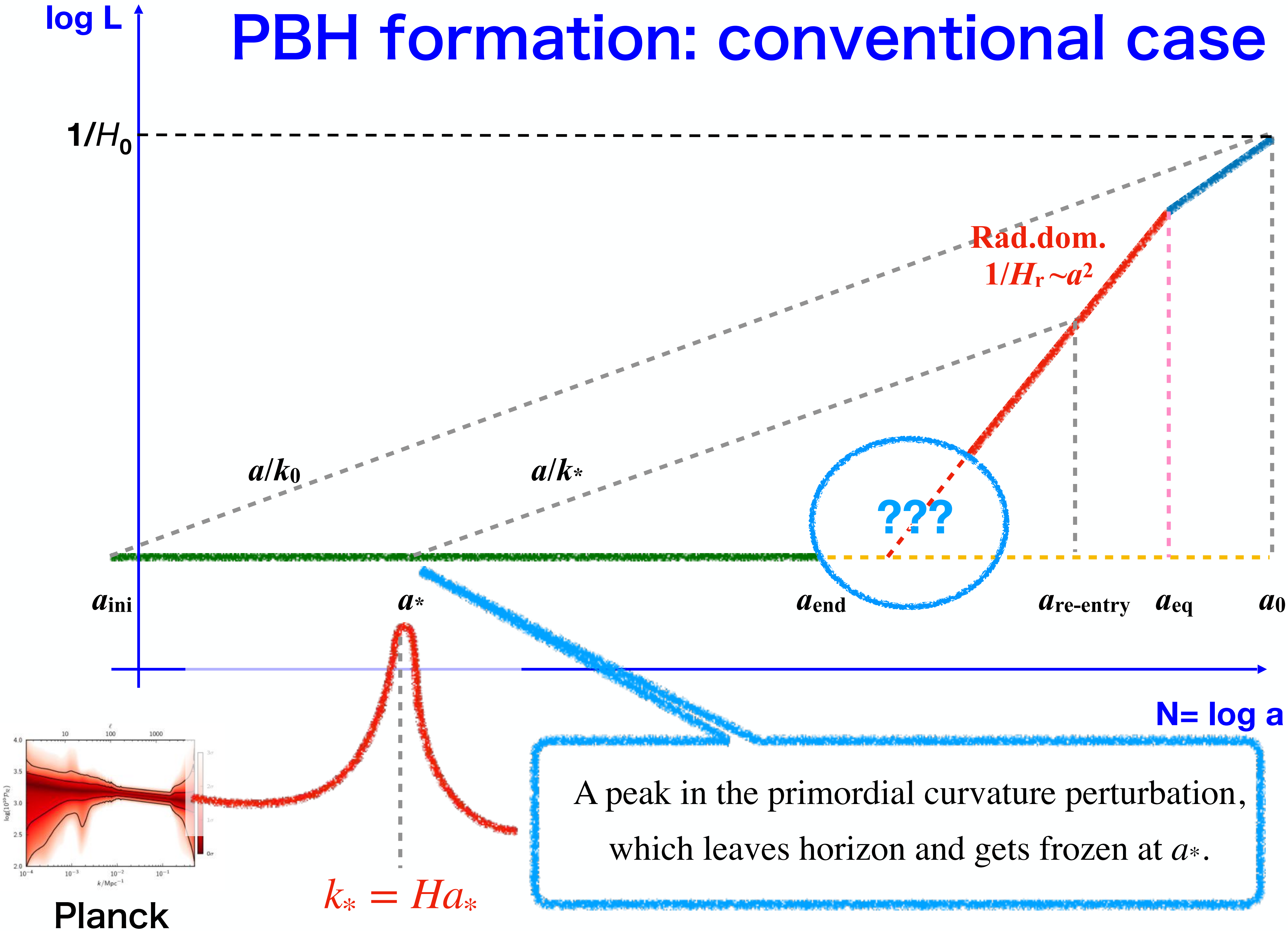
based on

S. Pi, Y.l. Zhang, Q.G. Huang & MS, arXiv:1712.09896, JCAP 1805 (2018) 042.
R.G. Cai, S. Pi & MS, arXiv:1810.11000, PRL 122 (2019) 201101.
R.G. Cai, S. Pi & MS, arXiv:1909.13728.

cosmic spacetime diagram



PBH formation: conventional case



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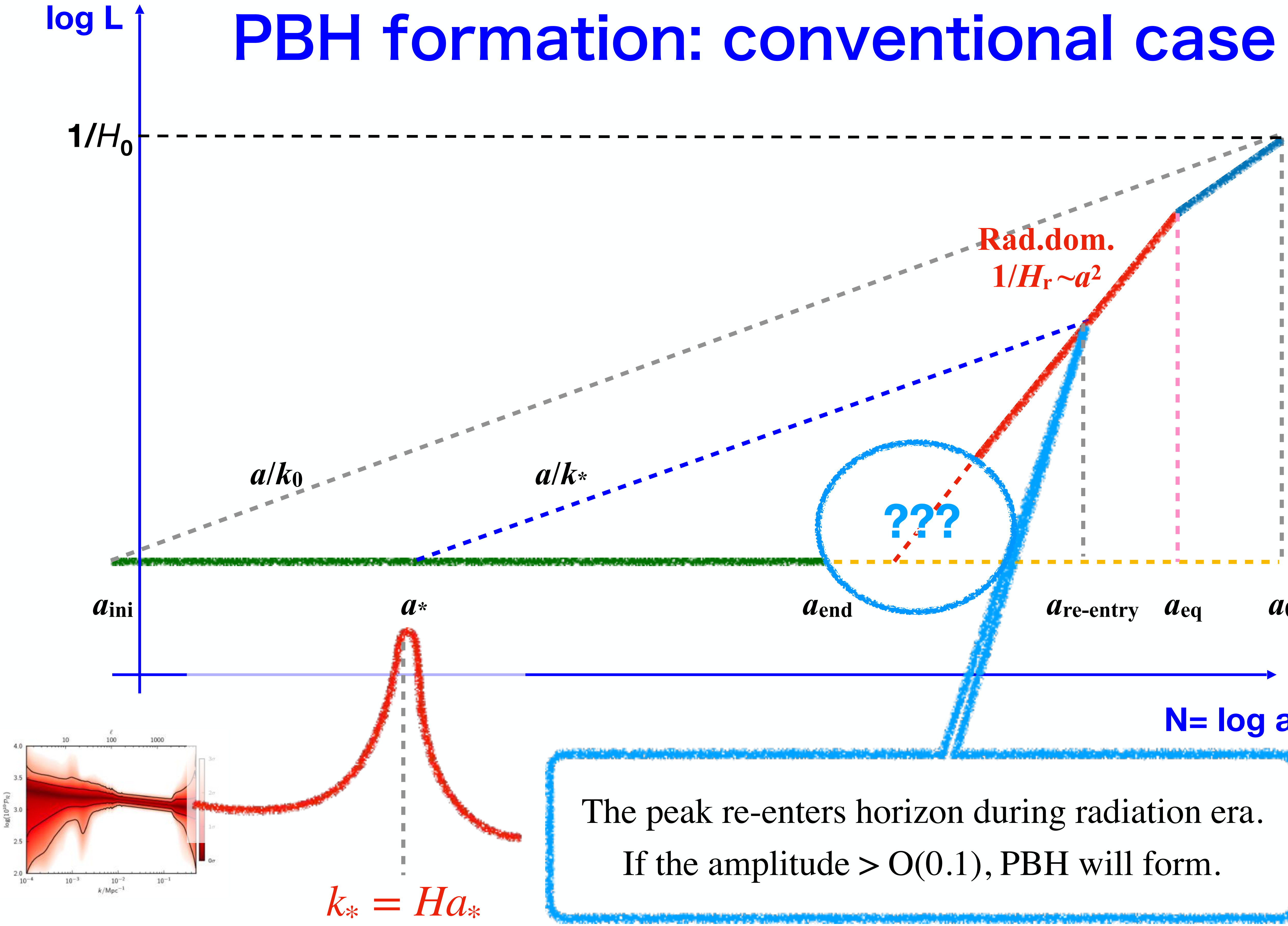
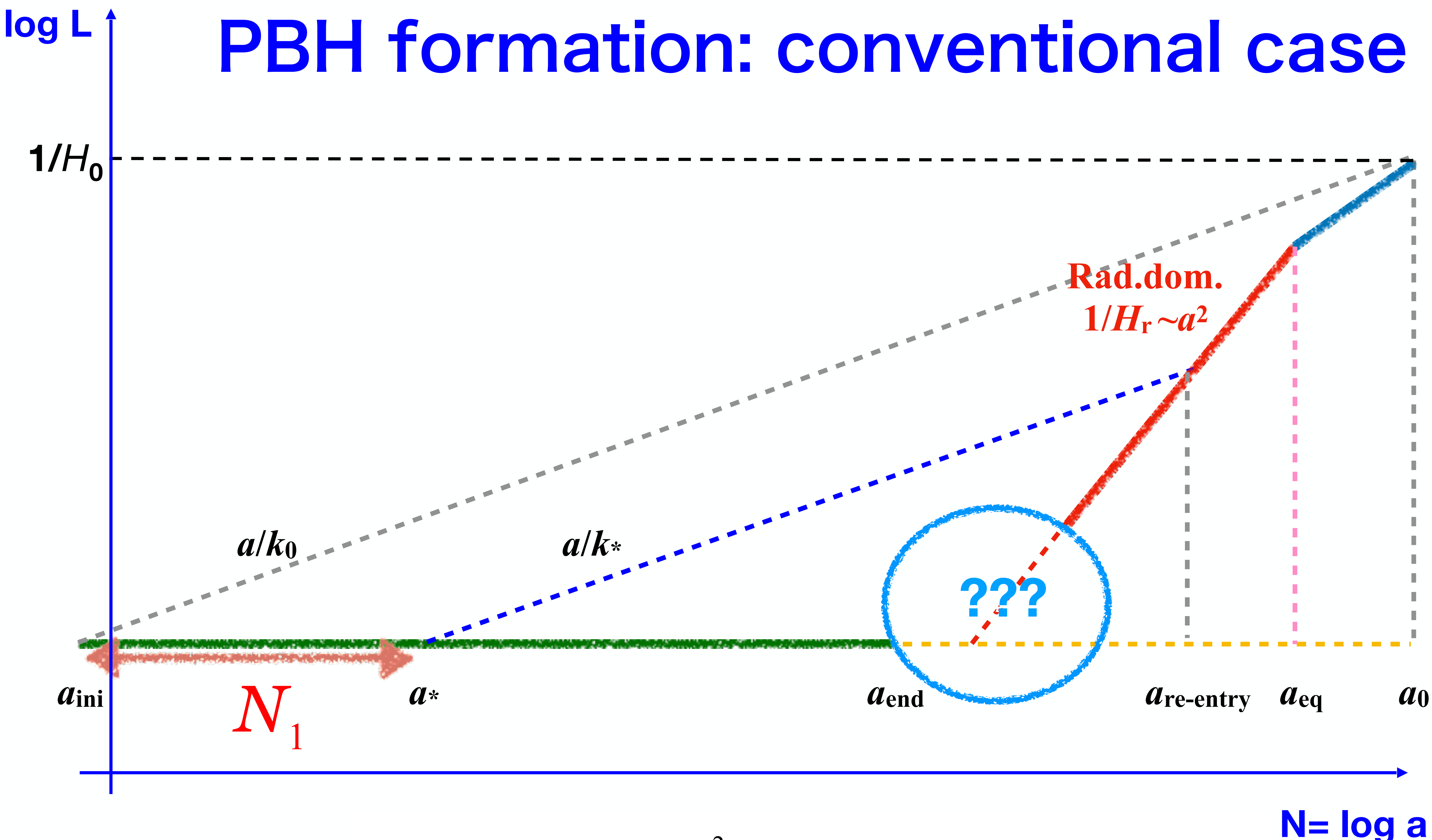


PBH formation: conventional case

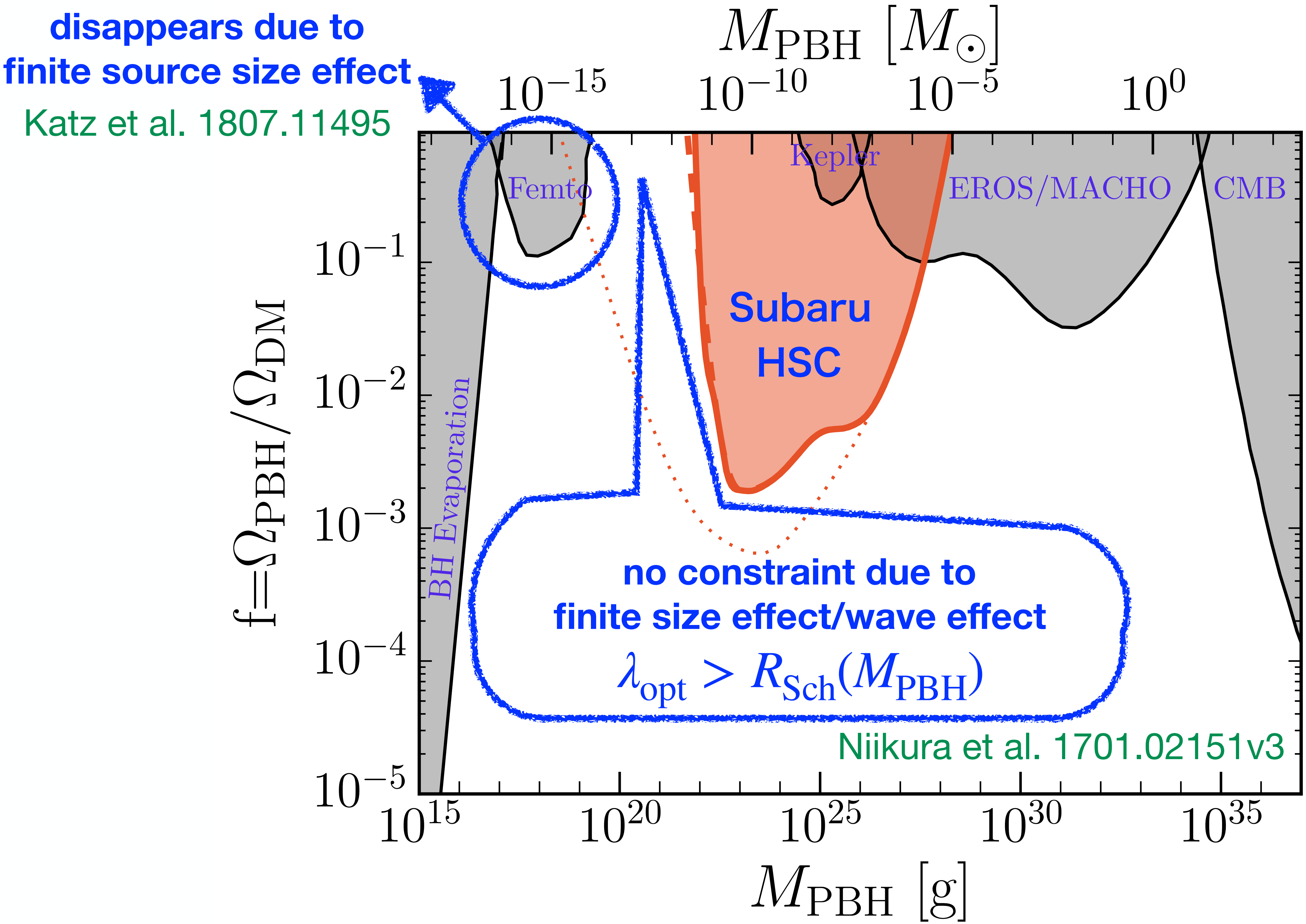


PBH mass: $M_{PBH} = \gamma M_H \sim \dfrac{M_{Pl}^2}{H} = 10^{58} M_{Pl} e^{-2N_1} = M_{Pl} 10^{58-0.87N_1}$

Inverse relation: $N_1 = 44.4 - \dfrac{1}{2}\ln\left(\dfrac{M_{PBH}}{10^{16}\,\mathrm{g}}\right)$

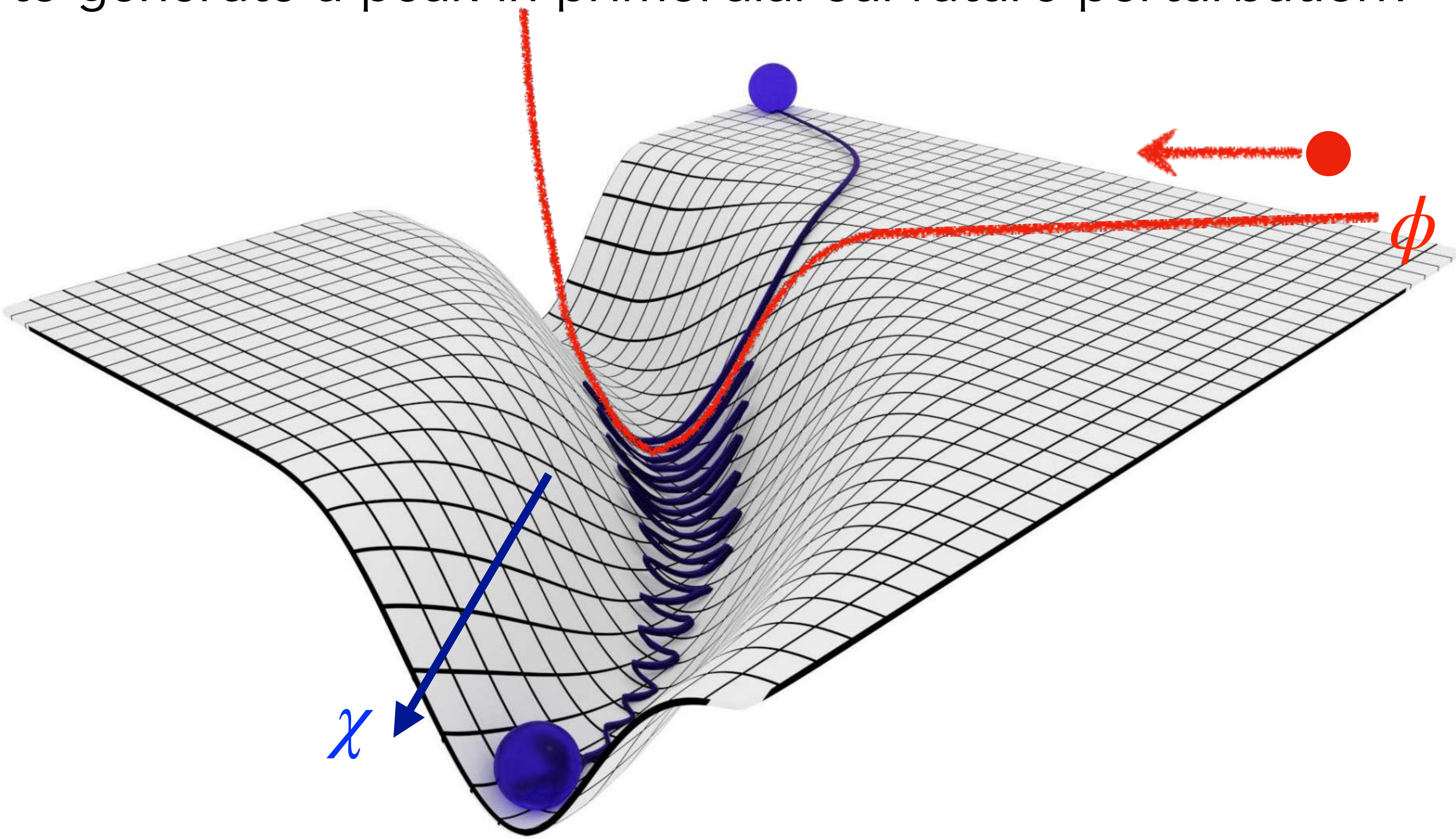
PBH mass scale does NOT depend on the reheating physics

PBH constaints: recent updates



big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$ g ⟷ $T_{\text{re-entry}} \sim 10^4$ -10^8 GeV

How to generate a peak in primordial curvature perturbation?



- We start with Starobinsky model: $R + \dfrac{R^2}{6M^2} \Rightarrow V \sim \left(1 - e^{-\phi/M_{\rm Pl}}\right)^2$
- Then add another field: $-(\partial\chi)^2/2 - V(\chi)$

simple 2-field model

Pi, Zhang, Huang & MS '17

- Starobinsky R^2 gravity plus a scalar field χ, non-minimally coupled to gravity: (→scalaron ϕ)

$$S_J = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\left(R+\frac{R^2}{6M^2}\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-V(\chi)-\frac{1}{2}\xi R\chi^2\right\}.$$

conformal transformation

$$S_E = \int d^4x\sqrt{-\tilde{g}}\cdot\left\{\frac{M_{\rm Pl}^2}{2}\tilde{R}-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right.$$

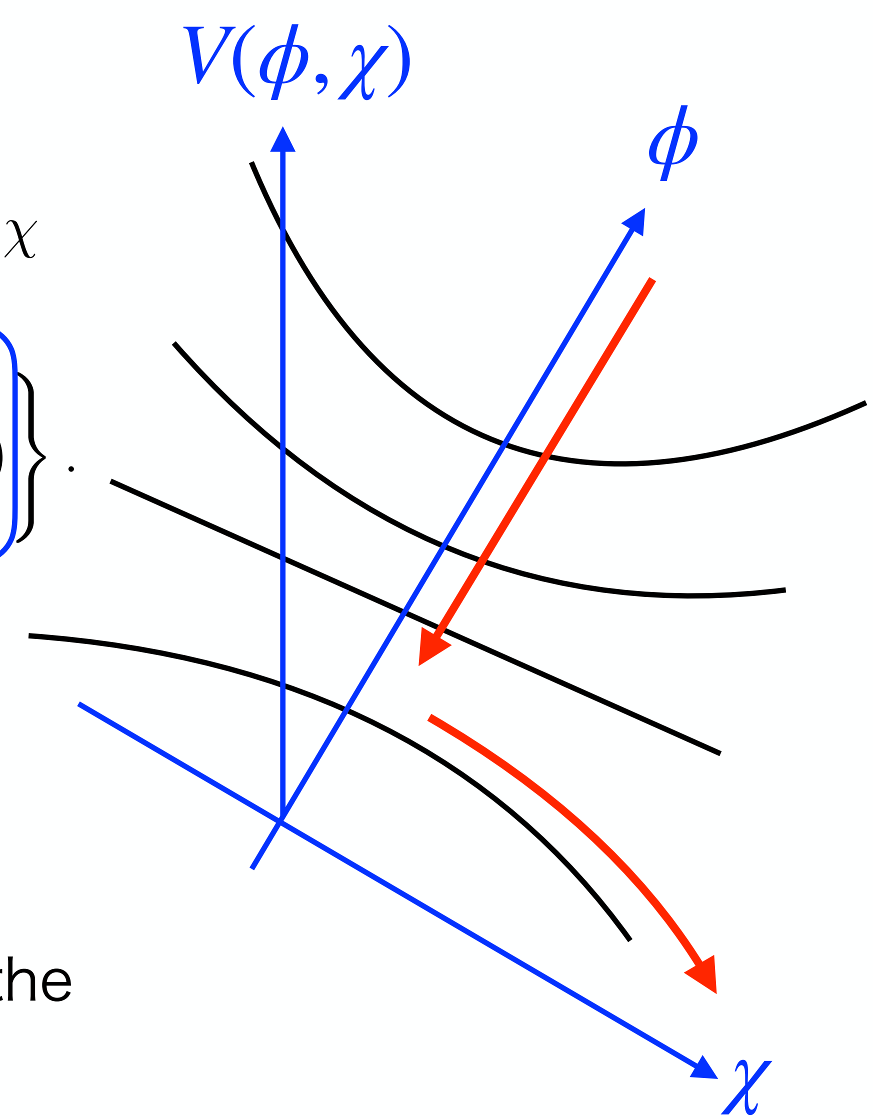
$$\left.-\frac{3}{4}M^2M_{\rm Pl}^2\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}+e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\xi\frac{\chi^2}{M_{\rm Pl}^2}\right)^2-e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}V(\chi)\right\}.$$

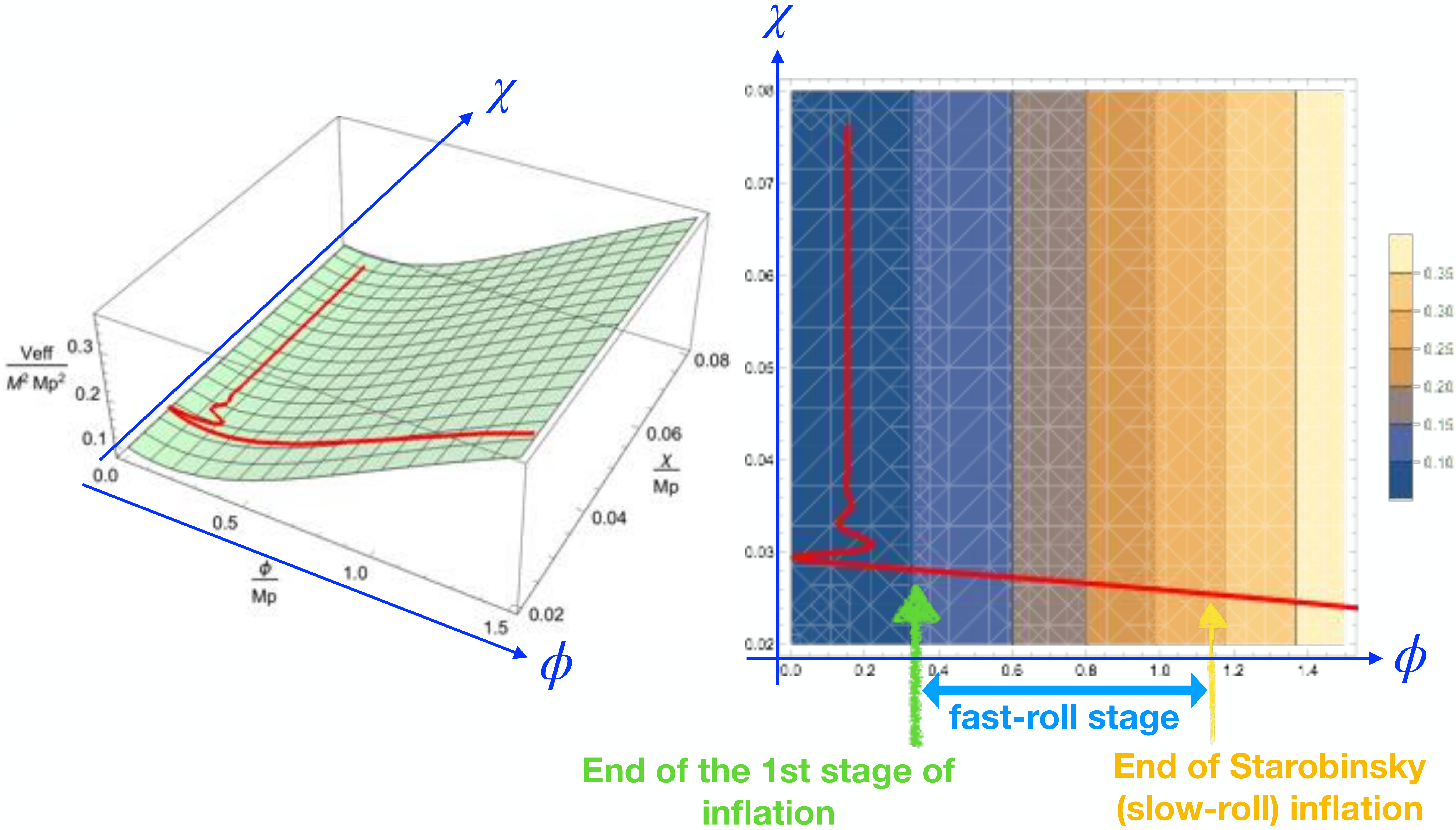
$= V(\phi,\chi)$

- for $V(\chi)$ we pick the small-field form:

$$V(\chi)=V_0-\frac{1}{2}m^2\chi^2+\cdots$$

- ξ-term is the non-minimally coupled term to stabilize the initial condition problem.
  A version of SSB in χ direction.

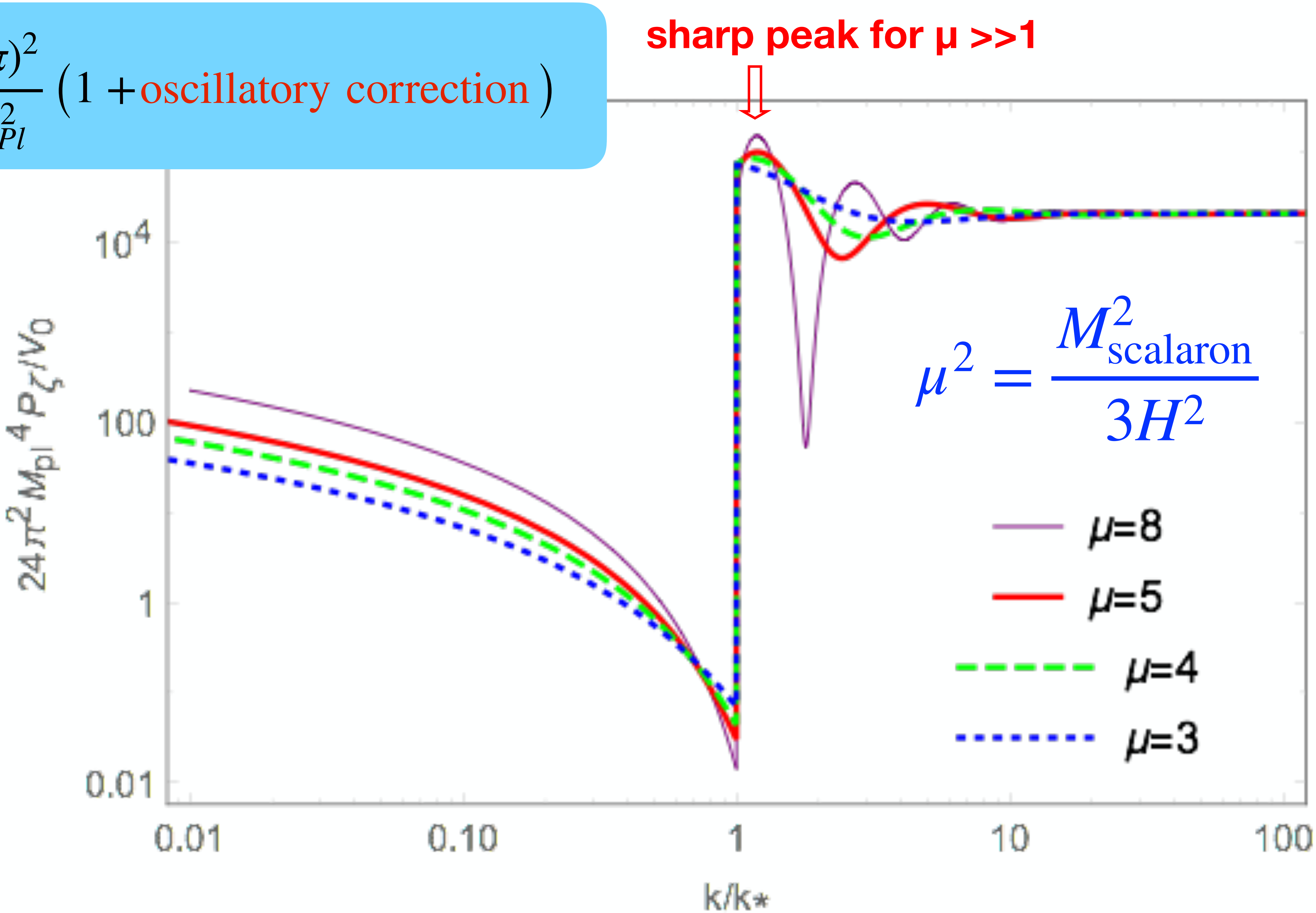




- Scalaron ϕ becomes massive at the end of the 1st stage.
- Field χ plays the role of inflaton at the 2nd stage.

Curvature Perturbation Power Spectrum

$$P \approx \frac{(H/2\pi)^2}{2\epsilon M_{Pl}^2}\left(1 + \text{oscillatory correction}\right)$$



width of P(k) can be tuned by tuning the value of μ

Curvature perturbation to PBH

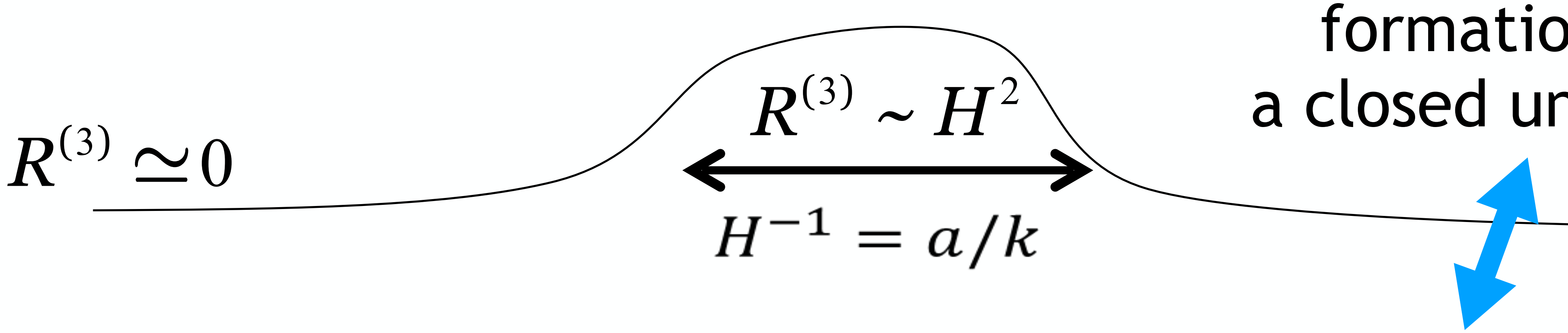
conventional (radiation-dominated) case

➢ gradient expansion/separate universe approach

$$6H^2(t,x) + R^{(3)}(t,x) = 16\pi G\rho(t,x) + \cdots$$

Hamiltonian constraint (Friedmann eq.)

$\Rightarrow$ $R^{(3)} \approx -\frac{4}{a^2}\nabla^2\mathcal{R}_c \approx \frac{8\pi G}{3}\delta\rho_c$ $\Rightarrow$ $\frac{\delta\rho_c}{\rho} \approx \mathcal{R}_c$ at $\frac{k^2}{a^2} = H^2$



formation of a closed universe

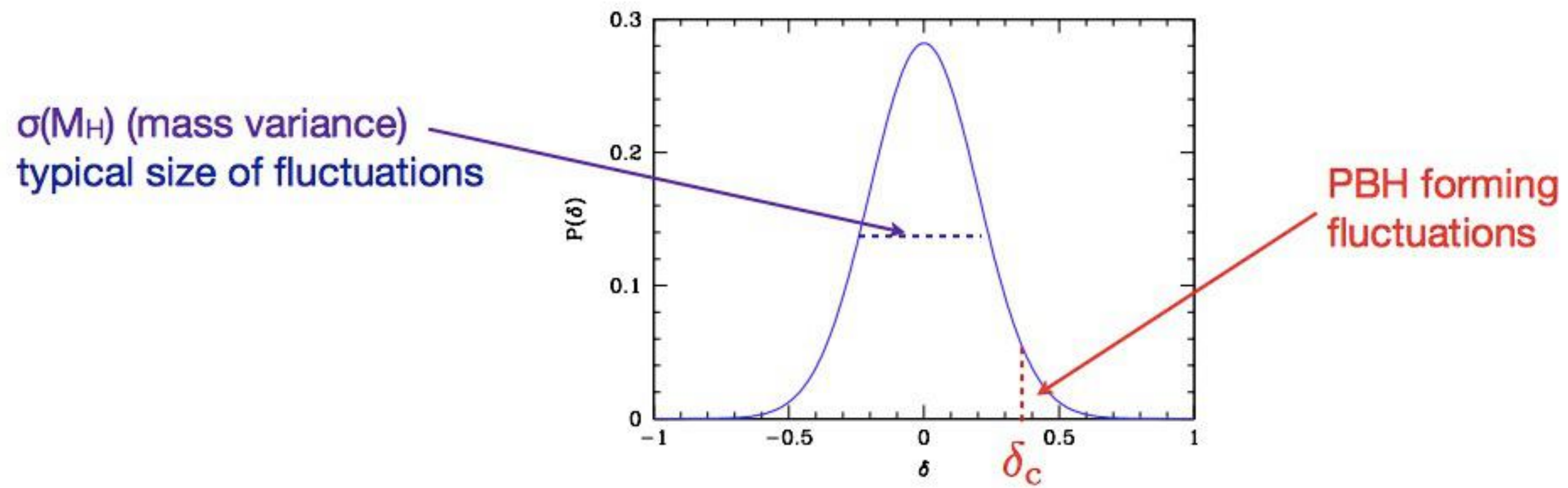
➢ If $R^{(3)} \sim H^2$ $(\Leftrightarrow \delta\rho_c/\rho \sim 1)$, it collapses to form BH

Young, Byrnes & MS '14

➢ Spins of PBHs are expected to be very small

fraction β that turns into PBHs

for Gaussian probability distribution



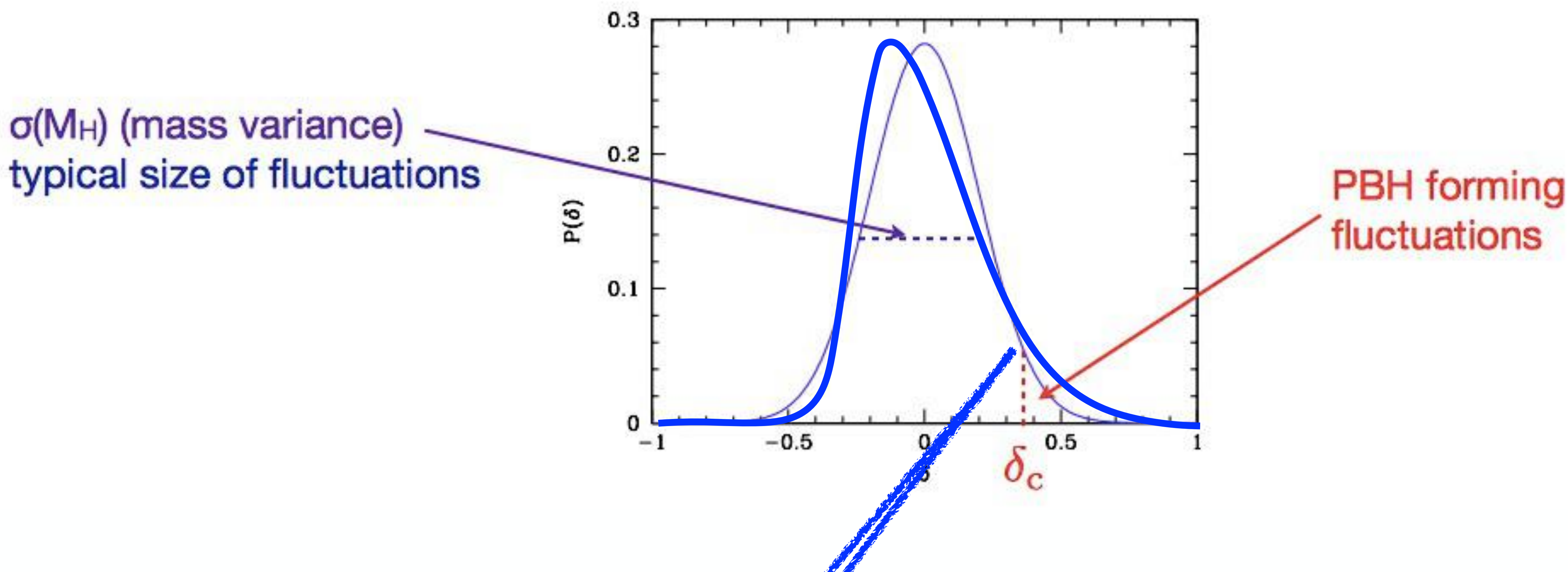
- When $\sigma_M << \delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

$$\delta_c \equiv \left(\frac{\delta\rho_c}{\rho}\right)_{\rm crit} \sim 0.4$$

effect of non-Gaussianity

for a peaked spectrum



Non-Gaussianity can increase (f_{NL}>0) or decrease (f_{NL}<0) the PBH adundances, substantially if $\sigma(M_H) \ll 1$.

Young & Byrnes, 1307.4995

PBH formation

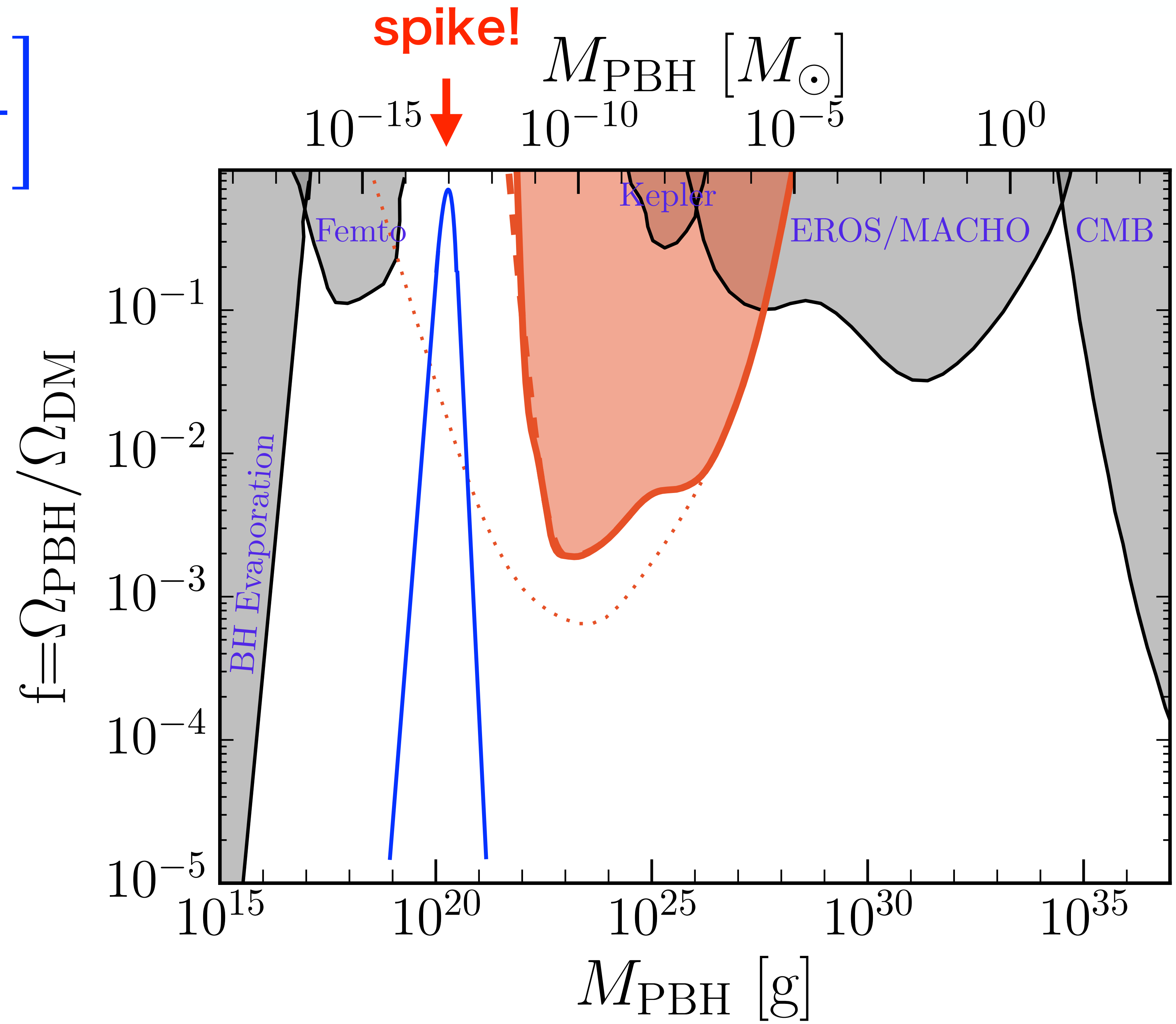
Pi, Zhang, Huang & MS '17

$$f(M) \propto \exp\left[-\frac{O(0.1)}{\mathscr{P}(k)}\right]$$

a sharp peak in $P(k)$

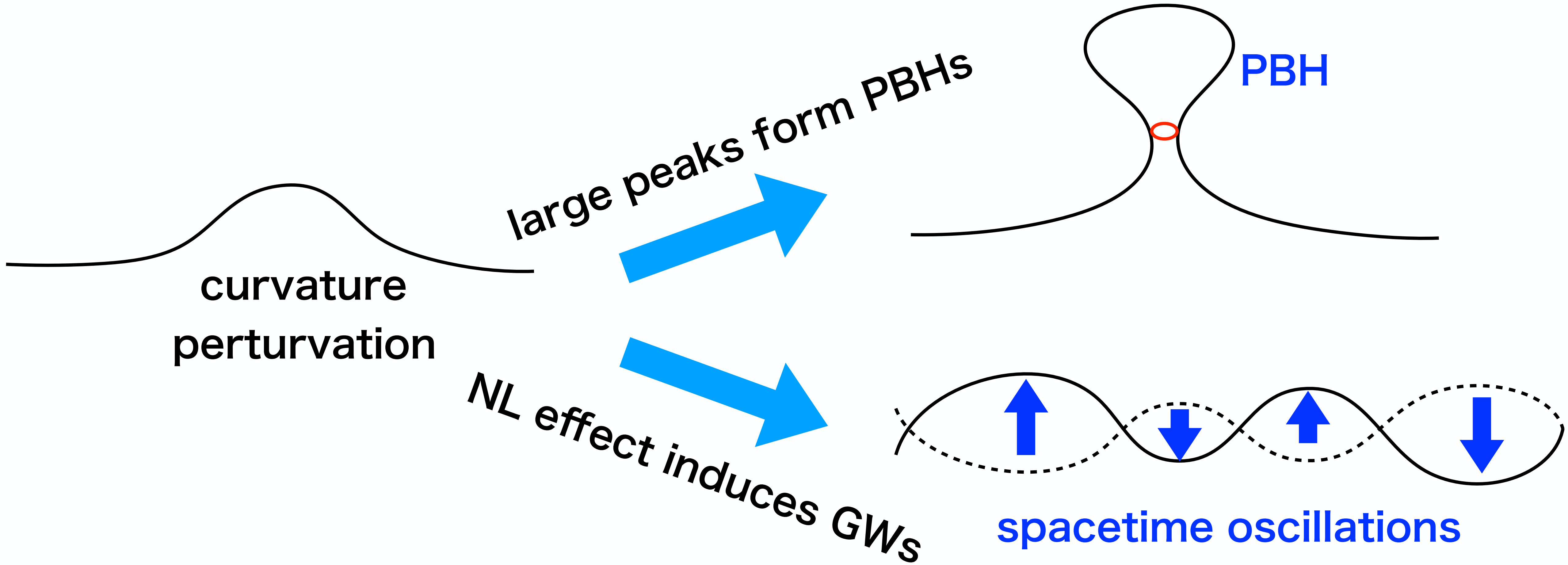
⇩

a spike in $f(M)$



In our 2-field model, we expect nearly monochromatic mass function

GWs can capture PBHs!



PBHs = CDM with M_{PBH} ~10^{21}g
generates GWs with f~10^{-3} Hz

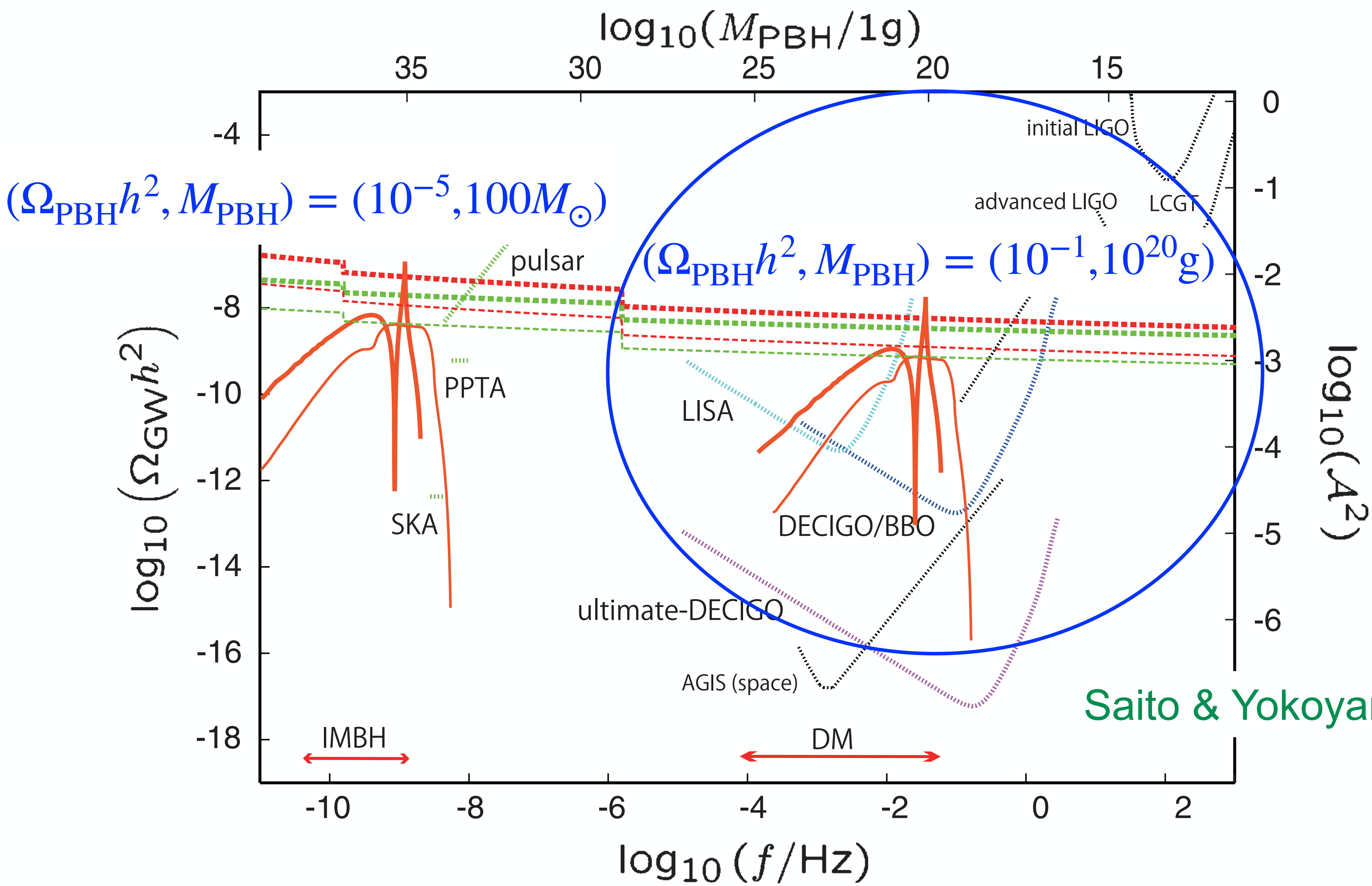
⇒

Background GWs
at LISA band

⇔

LIGO-Virgo :10 - 1000 Hz

GWs test PBH=DM!



Saito & Yokoyama '09

$$M_{\rm PBH} \sim 0.1 M_\odot \left(\frac{1{\rm GeV}}{T}\right)^2 \sim 10 M_\odot \left(\frac{1{\rm pc}^{-1}}{k}\right)^2$$

Induced GWs & Non-Gaussianity

- Tensor perturbation eq. with 2nd order curvature perturbation

$$h''_{\mathbf{k}} + 2\mathscr{H} h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathcal{S}(\mathbf{k}, \eta) \sim \int d^3 l \; l_i l_j \Phi_{\mathbf{l}}(\eta) \Phi_{\mathbf{k}-\mathbf{l}}(\eta)$$

$-\Phi = \Psi$: Newton potential

- The quantity to calculate is

$$\Omega_{\rm GW}(k) \equiv \frac{1}{12} \left(\frac{k}{Ha}\right)^2 \frac{k^3}{\pi^2} \overline{\langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}}(\eta) \rangle}.$$

$$\Rightarrow \quad \Omega_{\rm GW} \sim \langle hh \rangle \sim \langle \mathcal{S}\mathcal{S} \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_{\Phi}^2$$

- Φ may not be Gaussian: Consider a non-Gaussianity:

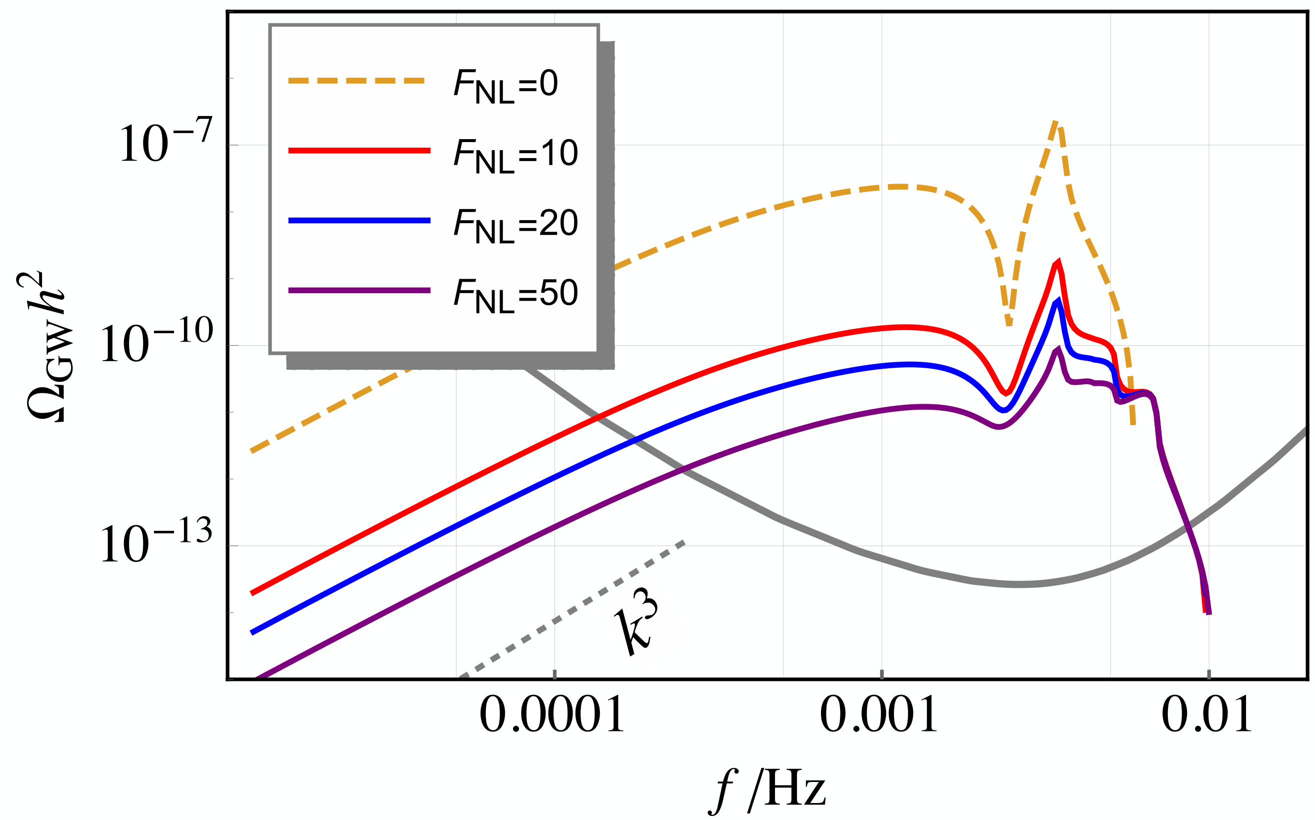
$$\mathscr{R}(\mathbf{x}) = \mathscr{R}_g(\mathbf{x}) + F_{\rm NL}\left[\mathscr{R}_g^2(\mathbf{x}) - \langle \mathscr{R}_g^2(\mathbf{x}) \rangle\right].$$

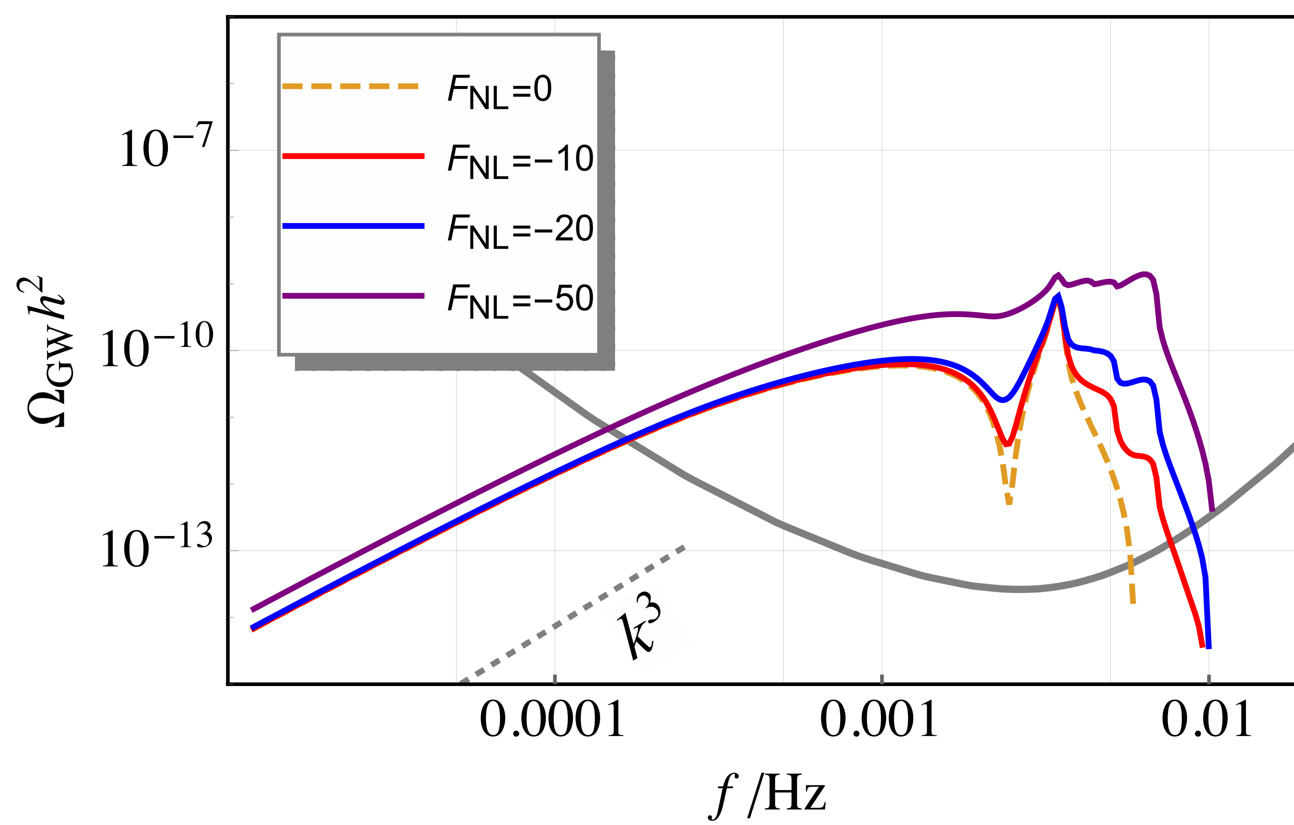
$\Phi = \frac{2}{3}\mathscr{R}$ at radiation-dom stage: $\mathscr{R} =$ conserved curvature perturbation

Effects of non-Gaussianity

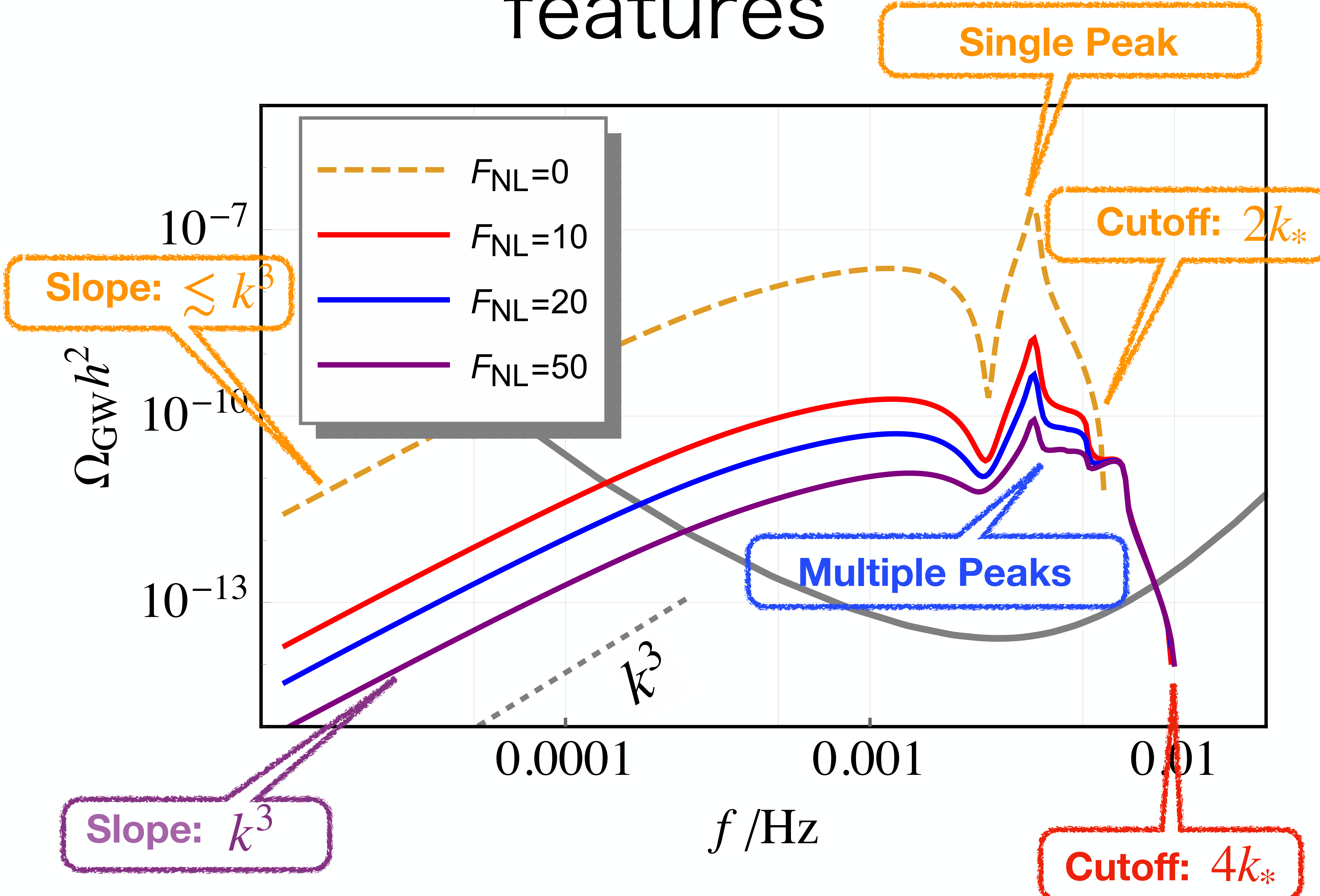
Cai, Pi & MS, '18

- Up: $F_{NL} > 0$, and we fix PBH abundance to be 1.
- Down: $F_{NL} < 0$, and we fix the peak amplitude to be $\mathcal{A}_{\mathcal{R}} = 10^{-2}$
- Gray curve: LISA
- Frequency: PBH window <—> LISA band
- No matter how large F_{NL} is, LISA will detect the induced GWs if PBH=CDM





features



Single Peak
Cutoff: $2k_*$
Slope: $\lesssim k^3$
F_{NL}=0
F_{NL}=10
F_{NL}=20
F_{NL}=50
10^{-7}
10^{-10}
10^{-13}
$\Omega_{GW}h^2$
k^3
Multiple Peaks
0.0001
0.001
0.01
f /Hz
Slope: k^3
Cutoff: $4k_*$

Summary 1

- 2-field inflation models can provide PBH-as-CDM scenario.

$N_1 \sim 35 - 40$ after CMB scale left the horizon

$\Longleftrightarrow M_{\rm PBH} \sim 10^{19} - 10^{22}{\rm g}$

- GWs are generated from large scalar perturbations:

k^3 - slope, multiple peaks, cutoff

this turns out to be common for all peaked scalar spectra

Cai, Pi & MS '19

- If PBHs = CDM, induced GWs willl be detected by LISA, indep of non-Gaussianity $f_{\rm NL}$.

- Conversely if LISA doesn't detect the induced GWs, it constrains the PBH abundances on mass range $M_{\rm PBH} \sim 10^{17}$ -10^{22}g where no other experiment can explore.

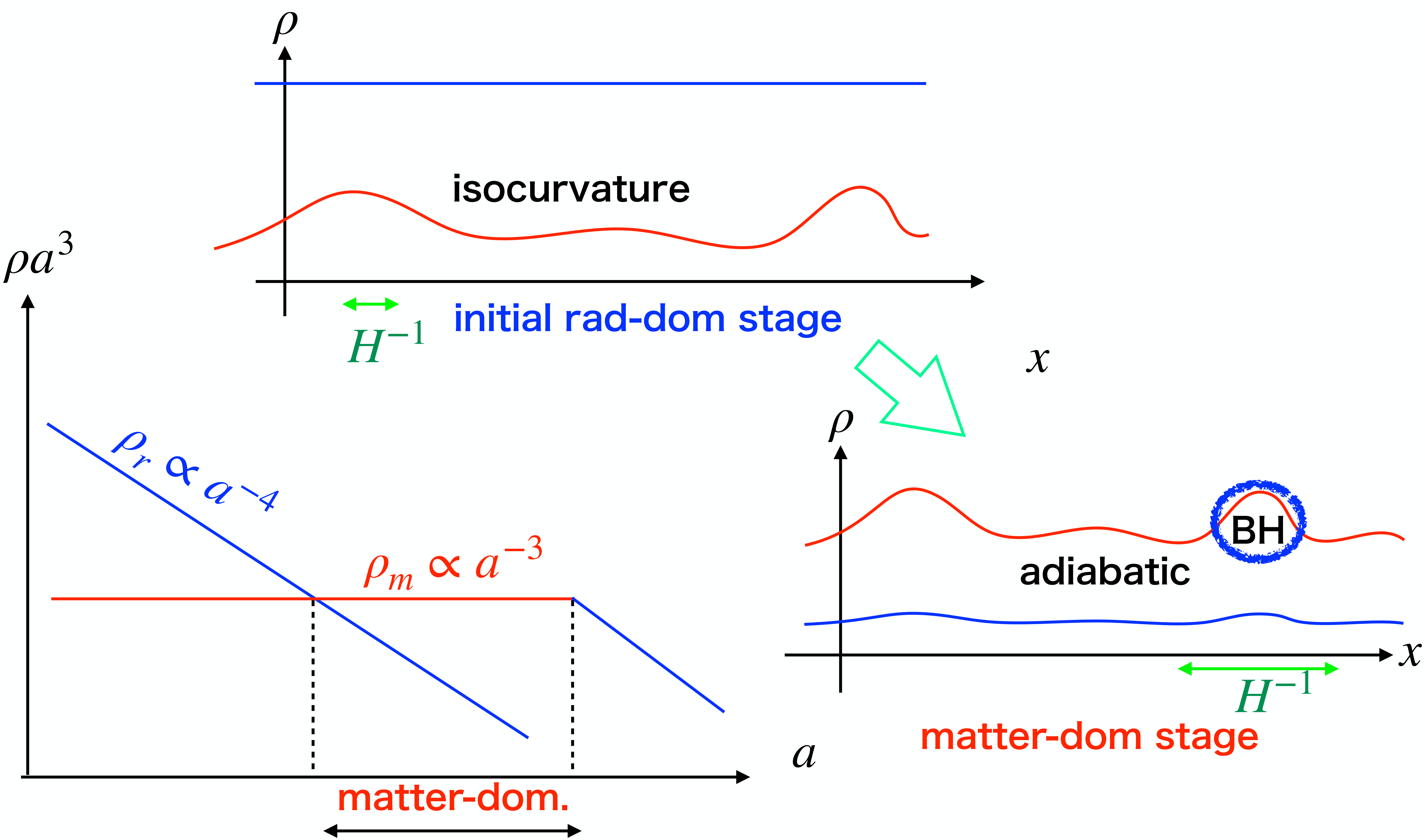
Part II

some random thoughts

sorry, nothing new, though . . .

PBHs from Isocurvature Perterbation

eg, E. Cotner, A. Kusenko, MS & V. Takhistov,1907.10613



linear theory

H. Kodama & MS, '86,'87

matter isocurvature perturbation

$$S \equiv \delta_m - \frac{3}{4}\delta_r \to \delta_m \text{ at } a \to 0$$ (on, say, comoving slice)

evolution: $\omega \ll 1$ $\omega \equiv \left(\frac{k}{Ha}\right)_{eq}$, $R \equiv \frac{a}{a_{eq}}$

$R \ll 1$

$$\begin{cases} \mathscr{R}_c = \frac{R}{4} S & \left(\Phi = \frac{R}{8} S\right) \\ \delta = \frac{1}{6}\omega^2 R^3 S \end{cases}$$

$R \gg 1$

$$\begin{cases} \mathscr{R}_c = \frac{1}{3} S & \left(\Phi = \frac{1}{5} S\right) \\ \delta = \frac{4}{15}\omega^2 R\, S \end{cases}$$

$\mathscr{R}_c$: curv pert on comoving slice

Φ : curv pert on Newton slice

horizon crossing: $\omega^2 R = \frac{1}{2}$

formation criterion: $\delta(k = aH) = \frac{2}{15} S > \delta_{\rm cr}$ **?**

or HYKN criterion?

linear theory continued

evolution: $\omega \gg 1$

$R \ll \omega^{-1}$

$$\begin{cases} \mathscr{R}_c = \dfrac{R}{4} S \\ \delta = \dfrac{1}{6}\omega^2 R^3 S \end{cases}$$

⇨

$\omega^{-1} \ll R \ll 1$

$$\begin{cases} \mathscr{R}_c = \dfrac{3}{4\omega^2 R} S \\ \delta = R\, S \end{cases}$$

⇨

$1 \ll R$

$$\begin{cases} \mathscr{R}_c = \dfrac{5}{4\omega^2} S \\ \delta = \dfrac{3R}{2} S \end{cases}$$

$$\left(\Phi = \frac{3}{4\omega^2} S \right)$$

conventional growth rate at matter-dom stage

horizon crossing: $\omega R = \dfrac{1}{2}$

$$\delta(k = aH) = \frac{1}{2\omega} S \ll S$$

Either $S = O(\omega^2) \gg 1$ or HYKN formation criterion?

need more studies!

Non-Gaussianity

primoridal non-Gaussianity

- generically small for a single-field model.
- need multiple fields for PBH formation, anyway.
- any non-gaussianity is possible for isocurvature case

eg, K. Yamamoto et al., PRD46, 4206 (1992): $n_B \propto \sin\left(\frac{\sigma}{2\pi f}\right)$

toy model

- assume that a field with non-canonical kinetic term ϕ determines PBH formation:

$$L = -\frac{1}{2}K^2(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \cdots$$
$$= -\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \cdots$$

$$K(\phi)d\phi = d\sigma$$

playing with a toy model

consider $K(\phi)$ given by

$$K(\phi) = \frac{x^{2n-1}}{1+x^{2n}}\,; \qquad x \equiv \frac{\phi}{M}$$

$$\longrightarrow \qquad \sigma = \int K(\phi)d\phi = \frac{M}{2n}\ln(1+x^{2n})$$

or

$$y = \frac{1}{2n}\ln(1+x^{2n}) \quad \Leftrightarrow \quad x^{2n} = e^{2ny}-1\,; \quad y \equiv \frac{\sigma}{M}$$

assume σ is Gaussian:

$$P(y) = \frac{1}{\sqrt{2\pi}\sigma_y}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\,; \qquad \langle y^2\rangle \equiv \sigma_y^2$$

$$P(y)dy = \mathscr{P}(x)dx$$

$$\Rightarrow \quad \mathscr{P}(x) = P(y(x))\frac{dy}{dx} = \frac{1}{\sqrt{2\pi}\sigma_y}\frac{x^{2n-1}}{1+x^{2n}}\exp\left[-\frac{\ln^2(1+x^{2n})}{2\sigma_y^2(2n)^2}\right]$$

$$\xrightarrow[x^{2n}\gg 1]{} \frac{1}{\sqrt{2\pi}\sigma_y|x|}\exp\left[-\frac{\ln^2|x|}{2\sigma_y^2}\right]$$

~ log-normal

$\left(\ln(\phi/M) \sim \sigma/M\right)$

σ_x^2

$$\sigma_x^2 = \int x^2\mathscr{P}(x)dx = \int \frac{1}{\sqrt{2\pi}\sigma_y}\frac{x^{2n+1}}{1+x^{2n}}\exp\left[-\frac{\ln^2(1+x^{2n})}{2\sigma_y^2(2n)^2}\right]$$

$$= \int \frac{1}{\sqrt{2\pi}\sigma_y}\left(e^{2ny}-1\right)^{1/n}e^{-y^2/2\sigma_y^2}dy$$

$\approx \exp[\sigma_y^2]$ for $\sigma_y^2 \gg 1$

σ_x^2 can be significantly enhanced

$\beta_{\rm PBH}$ may be further a non-trivial function of x

any realistic model?

Conclusion

It seems there are so many interesting remaining issues.

- inflation: multi-field or multi-verse
- spectrum: peaked or extended
- statistics: gaussian or non-gaussian
- formation criterion: spherical or non-spherical
- formation epoch: radiation- or matter-dominated
- spatial distribution: poisson or clustered
- spins: non-rotating or rotating
- …

so, no conclusion . . .