Tokyo Tech

Clustering of primordial black holes with non-Gaussian initial fluctuations

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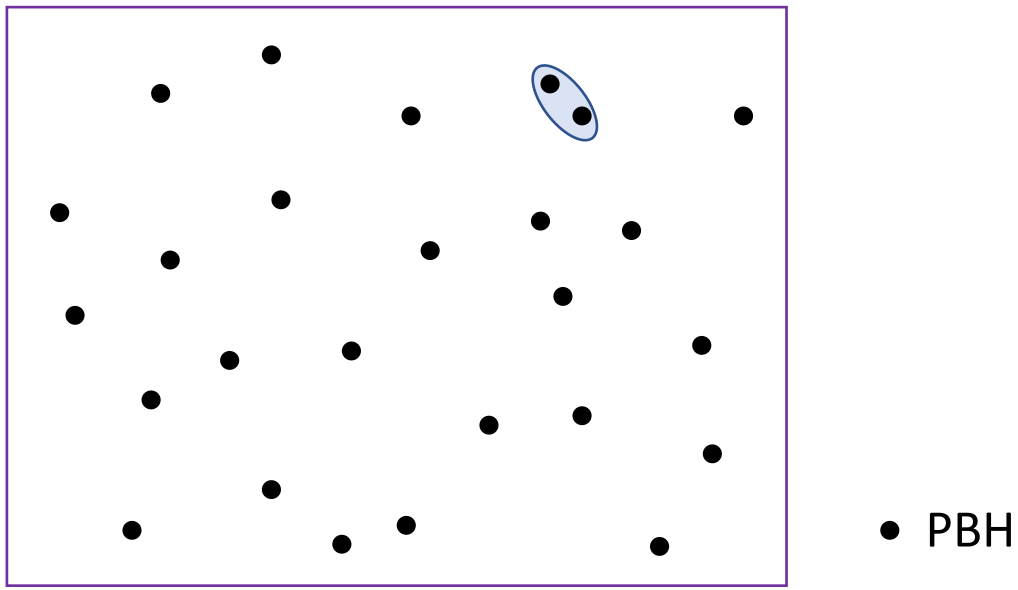
Ref.
PTEP 2019 no.10,103E02(arXiv:1906.04958)
TS and Shuichiro Yokoyama

PBH workshop IPMU, Dec.2-6 2019

Formation of the PBH binaries

Nakamura+ 1997

- Spatial distribution of PBHs in the early universe -



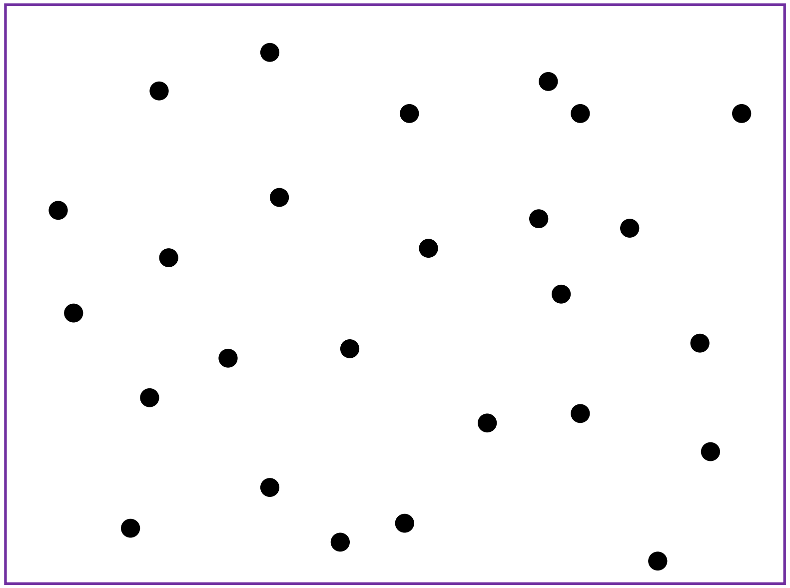
PBHs are initially at rest with respect to comoving coordinate.

Some PBHs form binaries in the radiation dominated era.

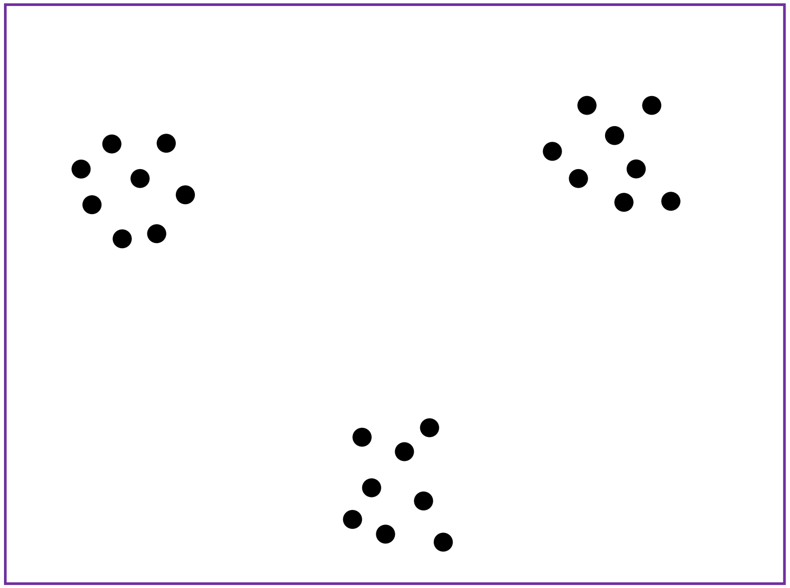
PBH binaries merge at the present time.

LIGO observations → $f_{PBH} \sim 10^{-3}$

Clustering also determines the merger rate.



No clustering

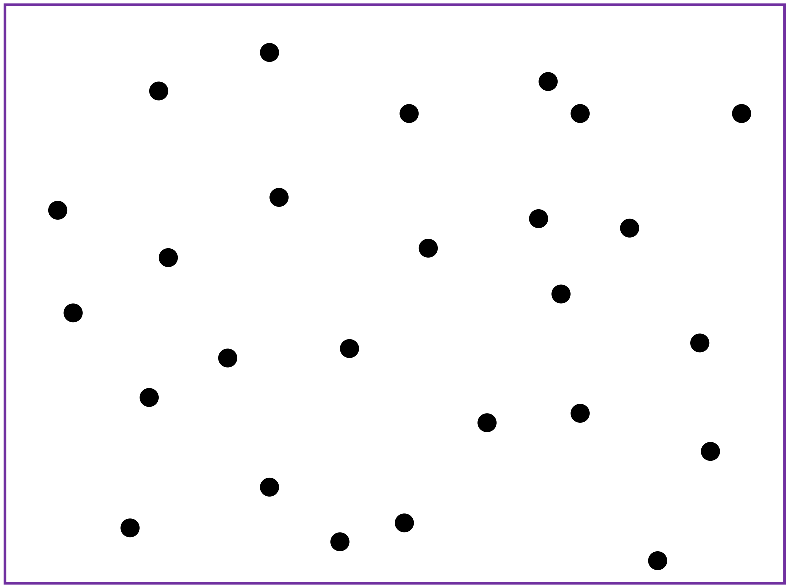


clustering

What determines the clustering of PBHs?

I focus on initial clustering in RD.

Initial clustering in MD: Matsubara, Terada, Kohri, Yokoyama 2019



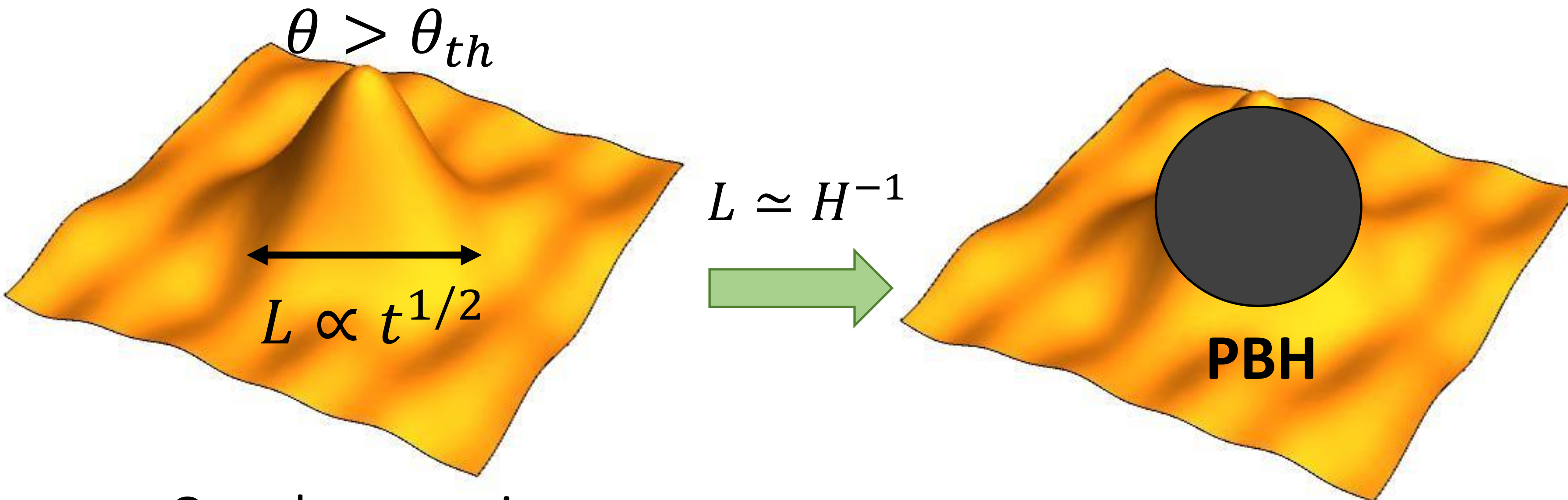
PBHs are rare initially (at formation time).

PBHs are initially separated by super-Hubble distance.

Initial clustering = clustering on super-Hubble scales

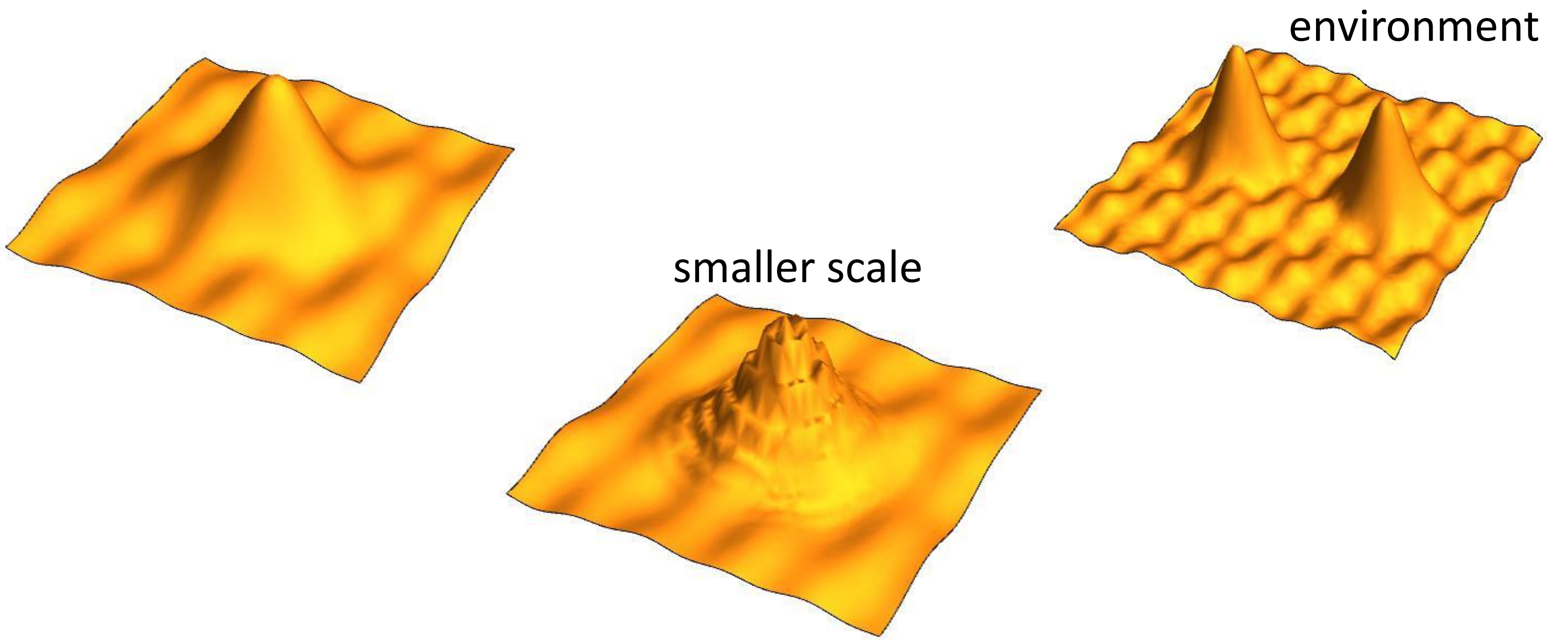
Setup

Super-Hubble scale perturbations θ generated by inflation



Overdense region
(Initially super-Hubble)

PBH is formed soon after the horizon reentry.
(Radiation dominated epoch)



Environment and smaller scale not relevant to PBH formation

PBH formation

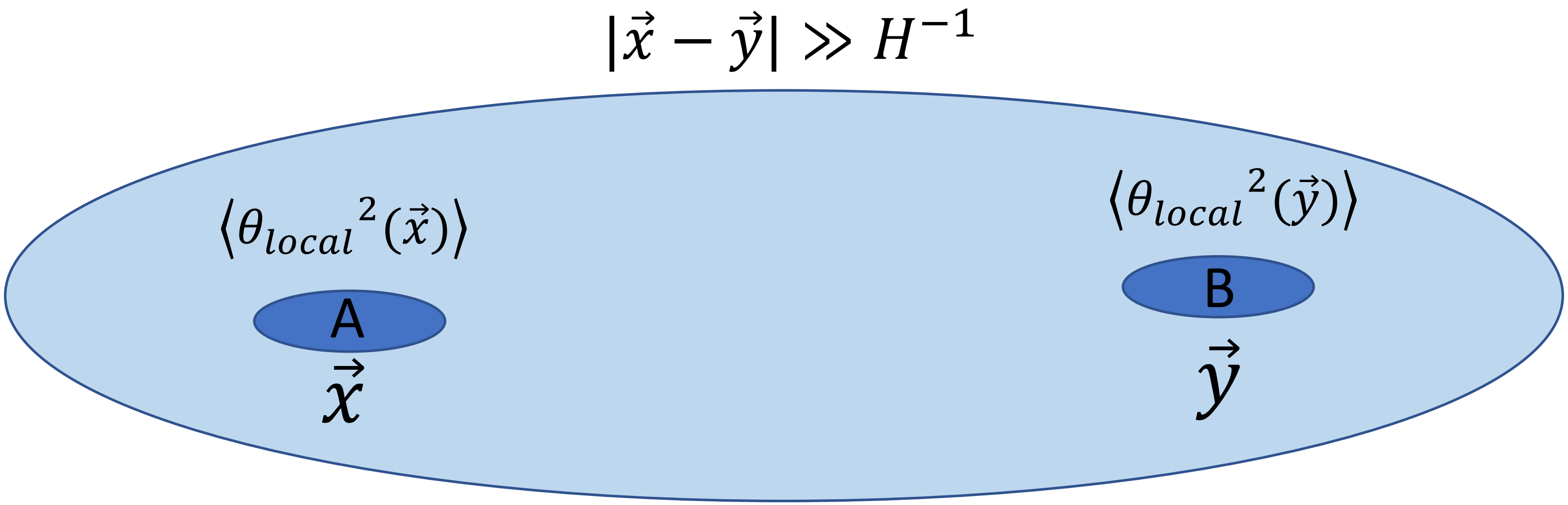
density contrast on the comoving slice

Young+ 2014

$$\theta_{local}(\vec{x}) = \int W(R, \vec{x} - \vec{y})\theta(\vec{y})d^3y$$

$$\theta_{local} > \theta_{th}$$

Clustering of PBHs



PBH abundance in A(B) is determined by $\langle \theta_{local}{}^2(\vec{x}) \rangle$ ($\langle \theta_{local}{}^2(\vec{y}) \rangle$).

$$\boldsymbol{\langle \theta_{local}{}^2(\vec{x})\theta_{local}{}^2(\vec{y}) \rangle_c \neq 0} \textbf{ for } \boldsymbol{|\vec{x}-\vec{y}| \gg H^{-1}}$$

⇩

Super-Hubble scale clustering

Simple toy model

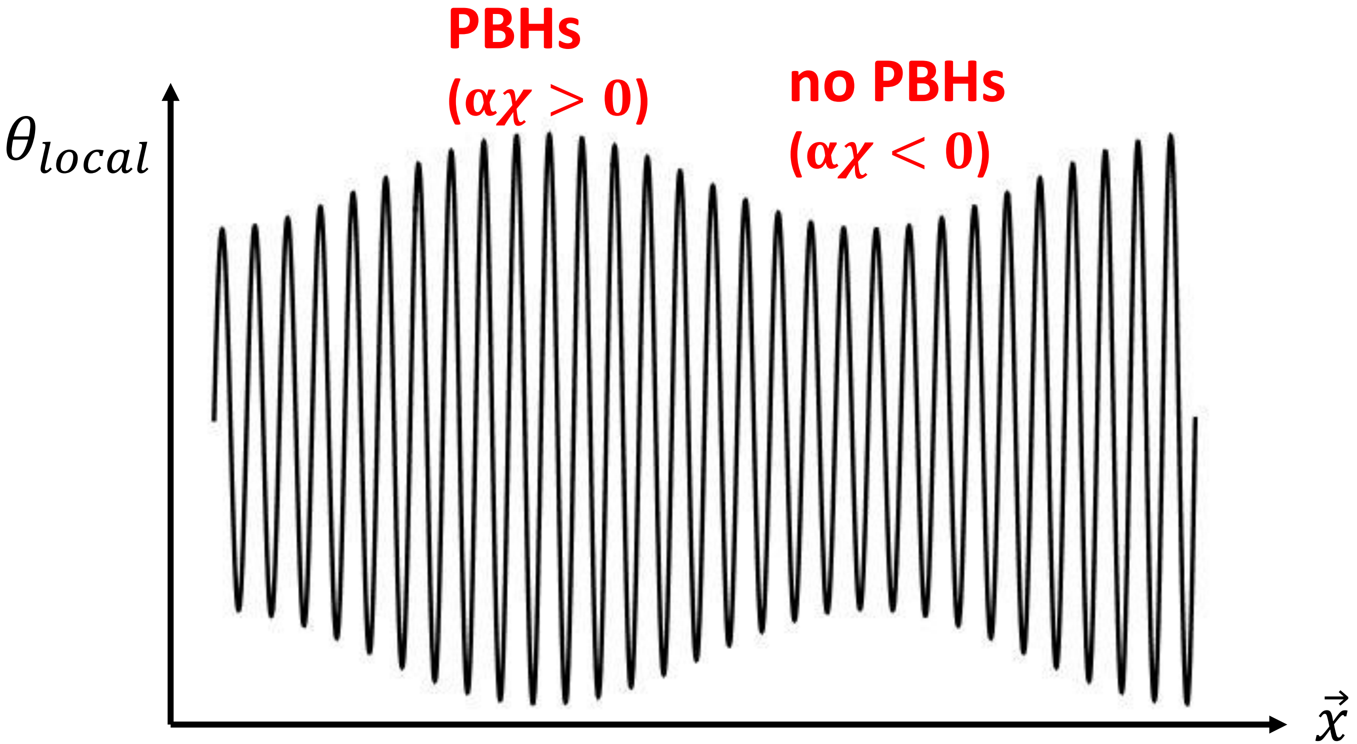
$$\mathcal{R}(\vec{x}) = \big(1 + \alpha\chi(\vec{x})\big)\phi(\vec{x})$$

Super-Hubble

PBH scale

χ, ϕ: uncorrelated Gaussian variables

$$\mathcal{R}(\vec{x}) = \left(1 + \alpha\chi(\vec{x})\right)\phi(\vec{x})$$



$$\frac{\left\langle \boldsymbol{\theta_{local}}^{2}(\vec{\boldsymbol{x}})\boldsymbol{\theta_{local}}^{2}(\vec{\boldsymbol{y}})\right\rangle_c}{\left\langle \boldsymbol{\theta_{local}}^{2}(\vec{\boldsymbol{x}})\right\rangle^2} \approx 4\alpha^2\langle\chi(\vec{x})\chi(\vec{y})\rangle + O(\alpha^4)$$

- Super-Hubble correlation of the local variance is generated by the super-Hubble correlation of χ.
- Clustering is characterized by the four-point function (trispectrum).

$$P_\phi = P_\chi$$

$$\mathcal{R}(\vec{x}) = \big(1 + \alpha\chi(\vec{x})\big)\phi(\vec{x}) \quad \Rightarrow \quad \alpha^2 = \tau_{NL}, \qquad f_{NL} = 0$$

Comparison with the previous studies

Tada&Yokoyama 2015, Young&Byrnes 2015

$$\mathcal{R}(\vec{x}) = \phi(\vec{x}) + \frac{3}{5} f_{NL} \phi^2(\vec{x})$$

ϕ : Gaussian

Presence of f_{NL} yields super-Hubble clustering of PBHs.

Tada&Yokoyama 2015, Young&Byrnes 2015

$\phi = \phi_l + \phi_s$ long mode: ϕ_l, short mode: ϕ_s

$$\Rightarrow \quad \mathcal{R} \approx \left(1 + \frac{6}{5} f_{NL} \phi_l\right) \phi_s$$

$$\tau_{NL} = \frac{36}{25} f_{NL}^2$$

(τ_{NL} is non-zero)

There is no inconsistency.

PBH correlation function

Functional integral approach

e.g. Franciolini+ 2018

$P[\theta]$: probability density of θ

- Probability that point $\boldsymbol{x}$ becomes a PBH

$$P_1(\boldsymbol{x}) = \int [D\theta] P[\theta] \int_{\theta_{\rm th}}^{\infty} d\alpha \, \delta_D(\theta_{\rm local}(\boldsymbol{x}) - \alpha)$$

- Probability that points $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ becomes PBHs

$$P_2(\boldsymbol{x}_1, \, \boldsymbol{x}_2) = \int [D\theta] P[\theta] \int_{\theta_{\rm th}}^{\infty} d\alpha_1 \, \delta_D(\theta_{\rm local}(\boldsymbol{x}_1) - \alpha_1) \int_{\theta_{\rm th}}^{\infty} d\alpha_2 \, \delta_D(\theta_{\rm local}(\boldsymbol{x}_2) - \alpha_2)$$

- PBH correlation function

$$\xi_{\rm PBH}(\boldsymbol{x}_1, \boldsymbol{x}_2) := \frac{P_2(\boldsymbol{x}_1, \boldsymbol{x}_2)}{P_1^2} - 1$$

I assume θ is weakly (local type) non-Gaussian and expand ξ_{PBH} up to trispectrum ($f_{NL}, \tau_{NL}, g_{NL}$).

$$\langle \mathcal{R}_c(\boldsymbol{k}_1)\mathcal{R}_c(\boldsymbol{k}_2)\mathcal{R}_c(\boldsymbol{k}_3)\rangle := (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,\frac{6}{5}f_{\mathrm{NL}}[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)+2\text{ perms.}$$

$$\begin{aligned}\langle \mathcal{R}_c(\boldsymbol{k}_1)\mathcal{R}_c(\boldsymbol{k}_2)\mathcal{R}_c(\boldsymbol{k}_3)\mathcal{R}_c(\boldsymbol{k}_4)\rangle := {}& (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)\\ &\times\left\{\frac{54}{25}g_{\mathrm{NL}}[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)P_{\mathcal{R}_c}(k_3)+3\text{ perms.}]\right.\\ &\left.+\tau_{\mathrm{NL}}[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)P_{\mathcal{R}_c}(|\boldsymbol{k}_1+\boldsymbol{k}_3|)+11\text{ perms.}]\right\}\end{aligned}$$

$$P_1(\boldsymbol{x}) = \int [D\theta] P[\theta] \int_{\theta_{\rm th}}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp\left[i\phi \int d^3y\, W_{\rm local}(\boldsymbol{x}-\boldsymbol{y})\,\theta(\boldsymbol{y}) - i\phi\,\alpha \right]$$

$$Z[J] := \int [D\theta] P[\theta] \exp\left[i \int d^3y J(\boldsymbol{y})\theta(\boldsymbol{y}) \right]$$

$$\xi_{\theta(c)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) := \frac{1}{i^n} \frac{\delta^n \log Z[J]}{\delta J(\boldsymbol{x}_1)\delta J(\boldsymbol{x}_2)\cdots\delta J(\boldsymbol{x}_n)}\bigg|_{J=0}$$

$$\begin{aligned} P_1(\boldsymbol{x}) &= \int_{\theta_{\rm th}}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} e^{-i\phi\,\alpha}\, Z\left[\phi\, W_{\rm local}(\boldsymbol{x}-\boldsymbol{y})\right] \\ &= \int_{\theta_{\rm th}}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp\left[-i\phi\,\alpha\right] \exp\left[\sum_{n=2}^{\infty} \frac{i^n}{n!} \phi^n\, \underline{\xi^{(n)}_{{\rm local}(c)}} \right] \end{aligned}$$

Expand

$$
\begin{aligned}
P_{\rm PBH}(k) \simeq\ & \left(\frac{4\nu}{9\sigma_R}\right)^2 W_{\rm local}(k)^2 P_{\mathcal{R}_c}(k) \\
& + \frac{6}{5} f_{\rm NL} \left(\frac{4\nu}{9\sigma_R}\right)^3 W_{\rm local}(k) \\
& \times \int \frac{d^3p}{(2\pi)^3} W_{\rm local}(p) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}|) \\
& \quad \times [2P_{\mathcal{R}_c}(p)P_{\mathcal{R}_c}(k) + P_{\mathcal{R}_c}(p)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}|)] \\
& + \frac{18}{25} g_{\rm NL} \left(\frac{4\nu}{9\sigma_R}\right)^4 W_{\rm local}(k) \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|) \\
& \qquad \times [3P_{\mathcal{R}_c}(k)P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2) + P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|)] \\
& + \frac{\tau_{\rm NL}}{3} \left(\frac{4\nu}{9\sigma_R}\right)^4 W_{\rm local}(k) \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|) \\
& \quad \times [6P_{\mathcal{R}_c}(k)P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(|\boldsymbol{p}_1+\boldsymbol{p}_2|) + 6P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}_1|)] \\
& + \frac{54}{25} g_{\rm NL} \left(\frac{4\nu}{9\sigma_R}\right)^4 \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}+\boldsymbol{p}_1|) W_{\rm local}(|\boldsymbol{p}_2-\boldsymbol{k}|) \\
& \qquad\qquad \times P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1|) \\
& + \frac{\tau_{\rm NL}}{4} \left(\frac{4\nu}{9\sigma_R}\right)^4 \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}+\boldsymbol{p}_1|) W_{\rm local}(|\boldsymbol{p}_2-\boldsymbol{k}|) \\
& \times [4P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(k) \\
& \quad +4P_{\mathcal{R}_c}(p_1)\left(P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1-\boldsymbol{p}_2|) + P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1|)P_{\mathcal{R}_c}(|\boldsymbol{p}_1+\boldsymbol{p}_2|)\right)]
\end{aligned}
$$

In the super-Hubble ($k \to 0$) limit,

$$
\begin{aligned}
P_{\rm PBH}(k) \simeq\ & \left(\frac{4\nu}{9\sigma_R}\right)^2 W_{\rm local}(k)^2 P_{\mathcal{R}_c}(k) \\
&+\frac{6}{5} f_{\rm NL}\left(\frac{4\nu}{9\sigma_R}\right)^3 W_{\rm local}(k) \\
&\times \int \frac{d^3p}{(2\pi)^3} W_{\rm local}(p) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}|) \\
&\quad \times \left[2P_{\mathcal{R}_c}(p)P_{\mathcal{R}_c}(k) + P_{\mathcal{R}_c}(p)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}|)\right] \\
&+\frac{18}{25} g_{\rm NL}\left(\frac{4\nu}{9\sigma_R}\right)^4 W_{\rm local}(k) \\
&\times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|) \\
&\quad \times \left[3P_{\mathcal{R}_c}(k)P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2) + P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|)\right] \\
&+\frac{\tau_{\rm NL}}{3}\left(\frac{4\nu}{9\sigma_R}\right)^4 W_{\rm local}(k) \\
&\times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|) \\
&\quad \times \left[6P_{\mathcal{R}_c}(k)P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(|\boldsymbol{p}_1+\boldsymbol{p}_2|) + 6P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}_1|)\right] \\
&+\frac{54}{25} g_{\rm NL}\left(\frac{4\nu}{9\sigma_R}\right)^4 \\
&\times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}+\boldsymbol{p}_1|) W_{\rm local}(|\boldsymbol{p}_2-\boldsymbol{k}|) \\
&\qquad \times P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1|) \\
&+\frac{\tau_{\rm NL}}{4}\left(\frac{4\nu}{9\sigma_R}\right)^4 \\
&\times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}+\boldsymbol{p}_1|) W_{\rm local}(|\boldsymbol{p}_2-\boldsymbol{k}|) \\
&\times \left[4P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(k)\right. \\
&\quad \left. +4P_{\mathcal{R}_c}(p_1)\left(P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1-\boldsymbol{p}_2|) + P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1|)P_{\mathcal{R}_c}(|\boldsymbol{p}_1+\boldsymbol{p}_2|)\right)\right]
\end{aligned}
$$

In the super-Hubble ($k \to 0$) limit,

Main result

$$\xi^{(2)}_{\mathrm{PBH}}(\boldsymbol{r}) = \tau_{\mathrm{NL}} \left(\frac{4\nu}{9}\right)^4 \xi^{(2)}_{\mathcal{R}_c}(\boldsymbol{r}) \qquad \nu = \frac{\theta_{th}}{\sigma}$$

PBH clusters produce dark matter isocurvature perturbations.

Tada&Yokoyama 2015, Young&Byrnes 2015

Isocurvature constraint

$$f_{PBH}^2 \tau_{NL} \nu^4 \leq O(10^{-2})$$

Summary

PBHs cluster on super-Hubble scale if the seed perturbation has local-type trispectrum parametrized by τ_{NL}.