Abundance of primordial black holes with local non-Gaussianity in peak theory

Chulmoon Yoo(Nagoya Univ.)

arXiv:1805.03946

with Tomohiro Harada
Jaume Garriga
Kazunori Kohri

arXiv:1906.06790

with Jinn-Ouk Gong
Shuichiro Yokoyama

What we have done

A new procedure to estimate PBH abundance

◎Better motivated than Press-Schechter formalism

◎Non-linearity is taken into account

◎Optimized criterion is implemented

◎No window function dependence for a narrow spectrum

CY,Harada,Garriga,Kohri[1805.03946]

◎Larger PBH abundance for a positive F_{NL} as expected

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No Introduction

Press-Schechter formalism and the point at issue

Perturbation variables

◎Spatial metric

$$\mathrm{d}l^2 = a^2 \mathrm{e}^{-2\zeta} \tilde{\gamma}_{ij} \mathrm{d}x^i \mathrm{d}x^j$$

◎Relation between ζ and density perturbation δ

$$\delta = -\frac{4(1+w)}{3w+5} \frac{1}{a^2 H^2} e^{5/2\zeta} \Delta \mathrm{e}^{-\zeta/2}$$

with long wave-length approx. comoving slicing, $p = w\rho$

◎Newtonian counterpart

$\zeta \sim \phi$:Newton potential, $\delta \sim \rho$: density

◎Problems

- δ is far from Gaussian distribution
- value of ζ is sensitive to environments which is not essential

δ_{th} and statistics of ζ

◎Threshold should be set based on δ

◎Statistical properties are well known for ζ

◎What we have to do

- Statistics of ζ ⇒ probability of δ ⇒ PBH formation prob.
- w/ long-wavelength approx. and w/o linear approx.

◎Relation between ζ and δ w/ long-wavelength approx.

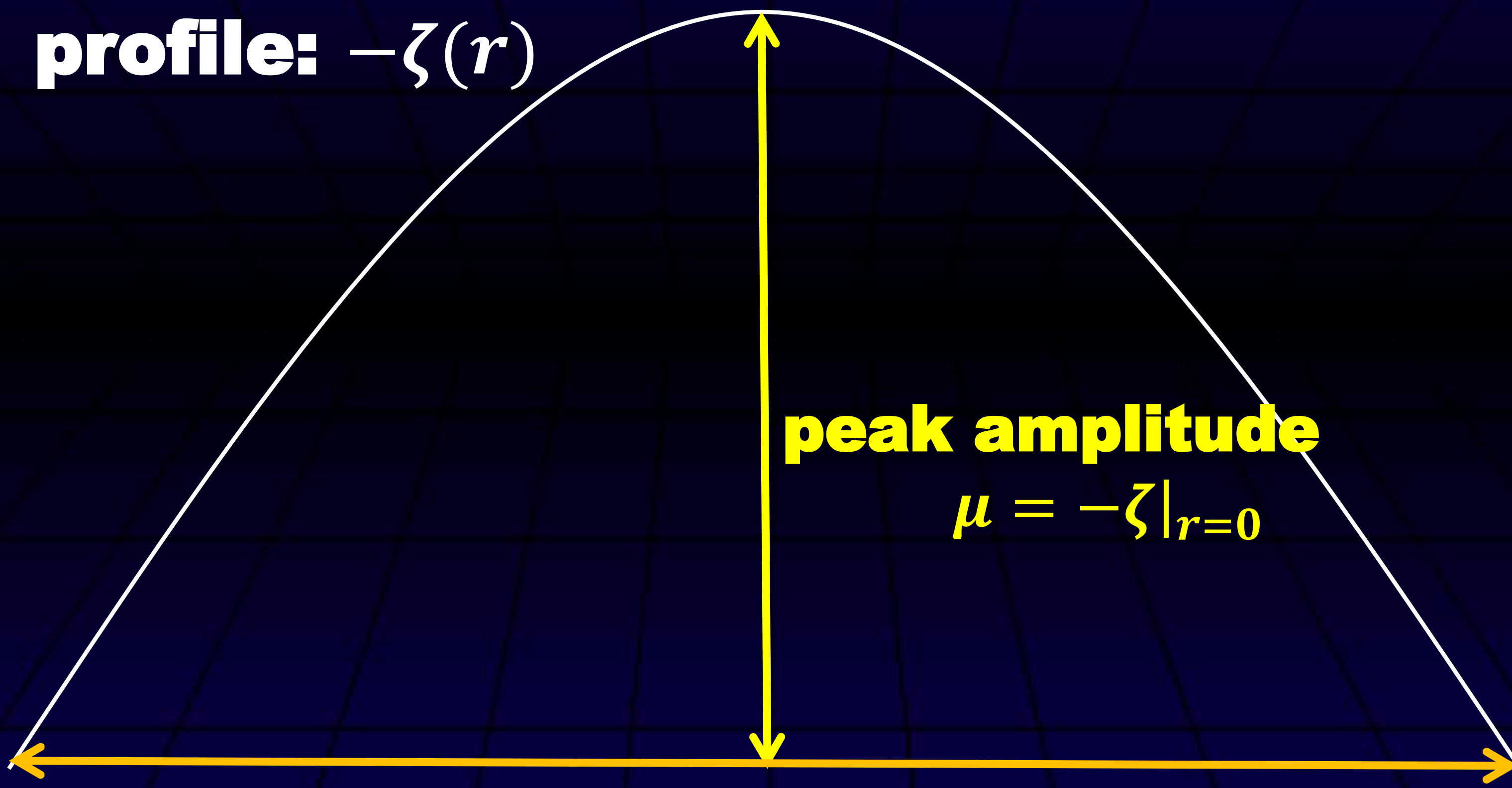
$$\delta = -\frac{4(1+w)}{3w+5}\frac{1}{a^2H^2}\, e^{5/2\zeta}\Delta e^{-\zeta/2}$$

comoving slicing, $p = w\rho$

Our Procedure

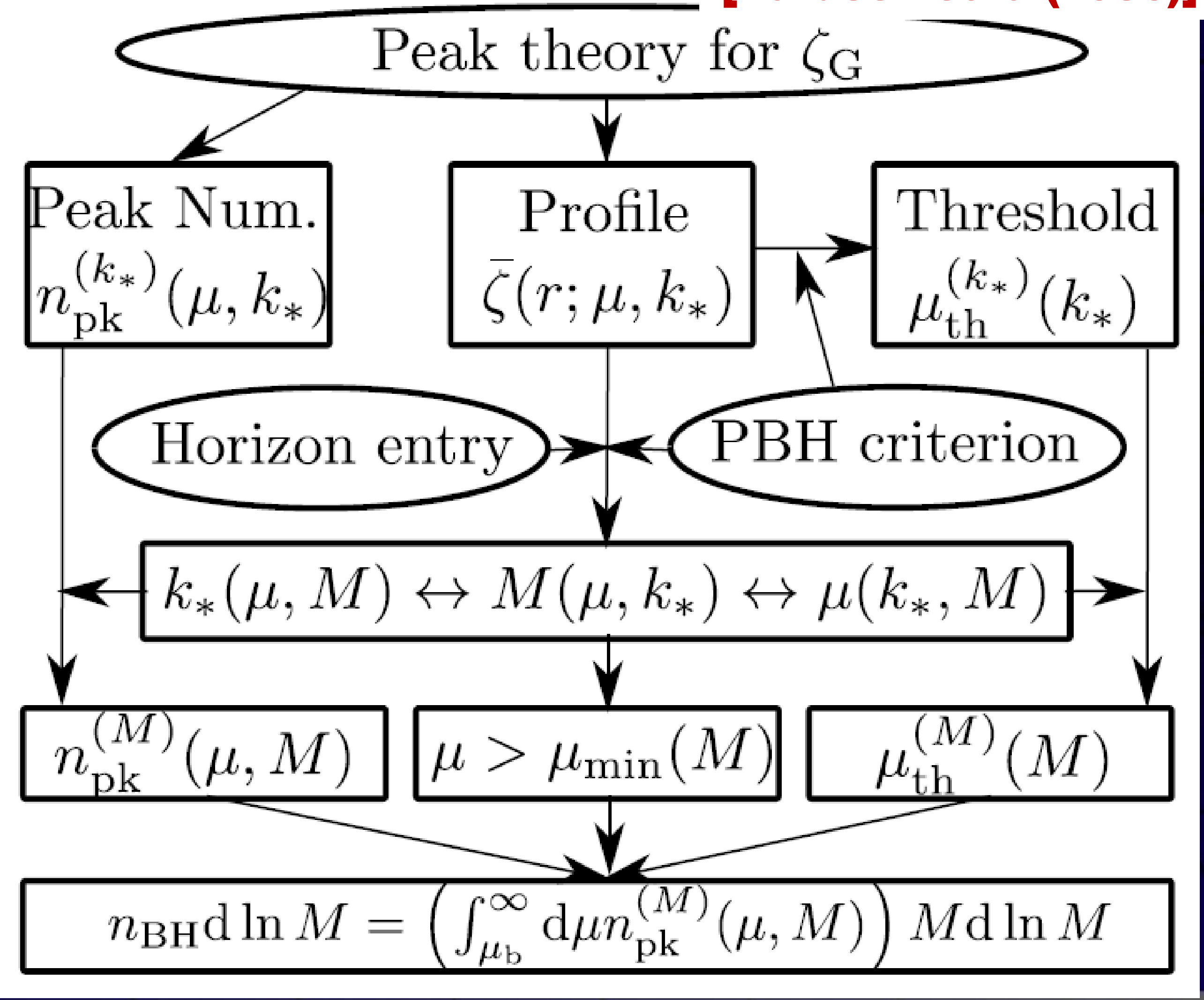
Variables for profile of ζ

◎**Variables:** $\mu=-\zeta|_{r=0}$, $k_*^2=\Delta\zeta|_{r=0}/\mu$



Flow chart

[Bardeen et. al(1986)]



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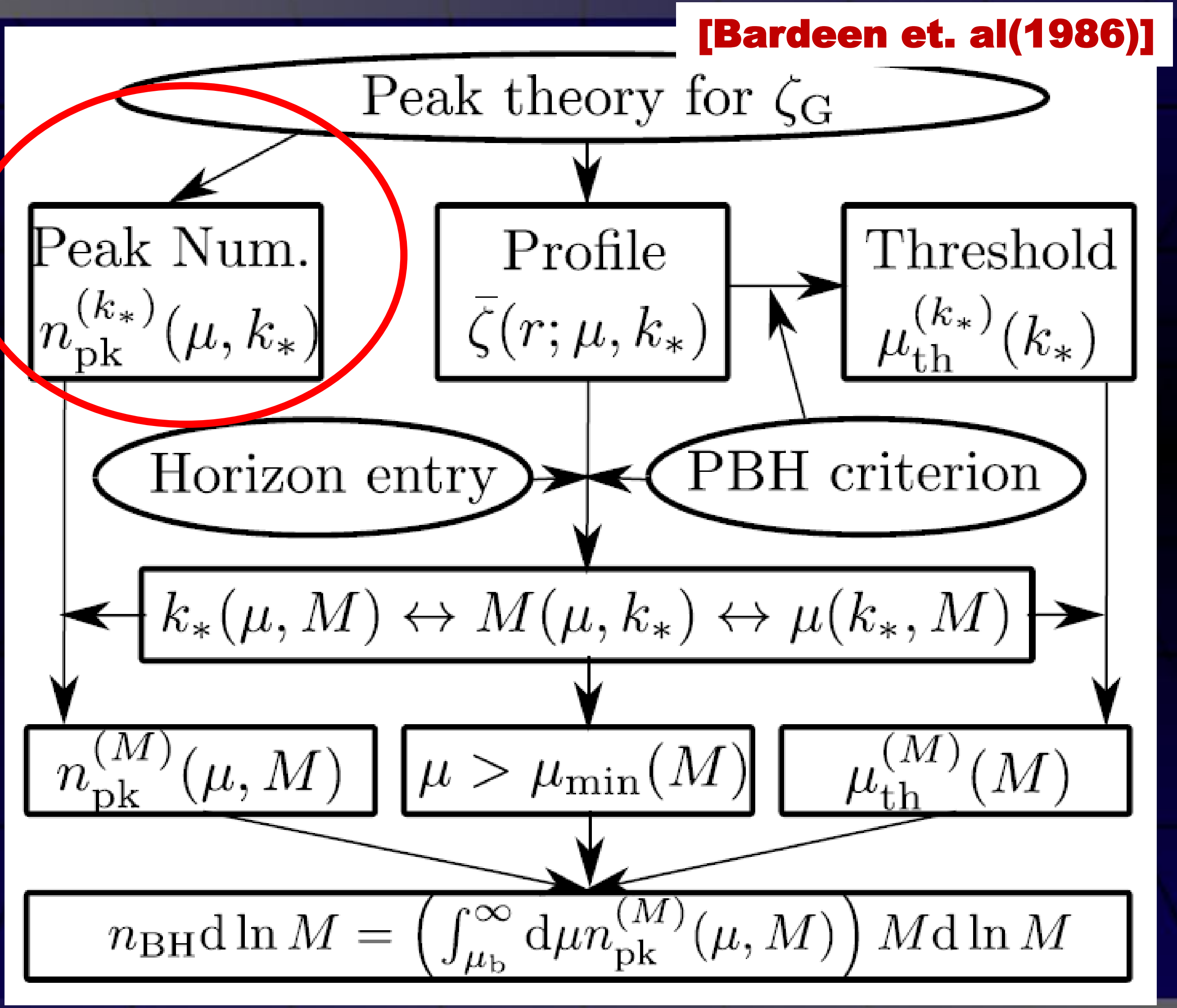
[Bardeen et. al(1986)]
Peak theory for ζ_G
Peak Num.
$n_{pk}^{(k_*)}(\mu, k_*)$
Profile
$\bar{\zeta}(r; \mu, k_*)$
Threshold
$\mu_{th}^{(k_*)}(k_*)$
Horizon entry
PBH criterion
$k_*(\mu, M) \leftrightarrow M(\mu, k_*) \leftrightarrow \mu(k_*, M)$
$n_{pk}^{(M)}(\mu, M)$
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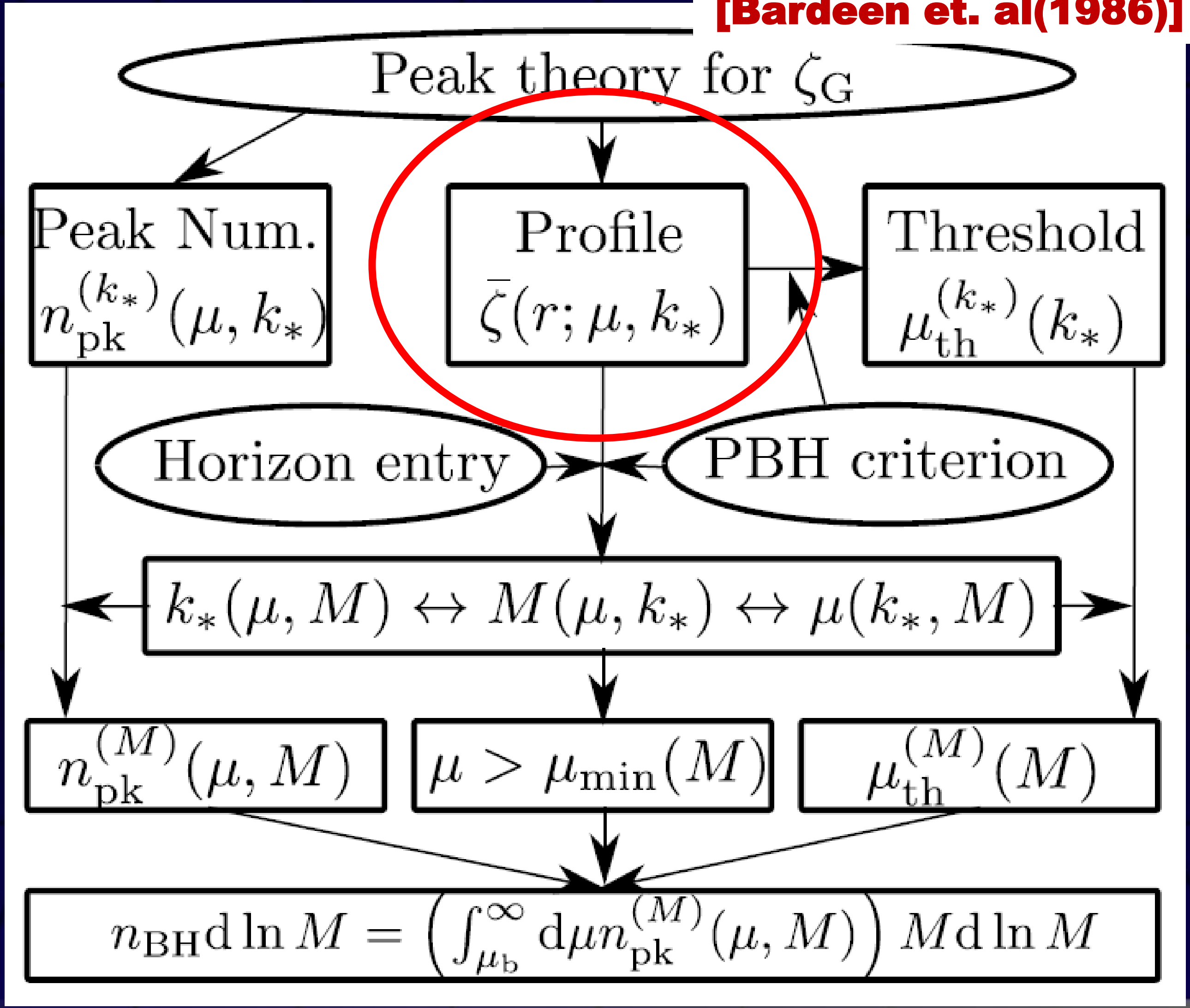


Typical peak profile

◎Typical spherical peak profile for a given set of (μ, k_*)

◎Mean value of $\zeta(r)$ with the conditional probability $P(\zeta(r)|\mu, k_*)$

[Bardeen et. al(1986)]

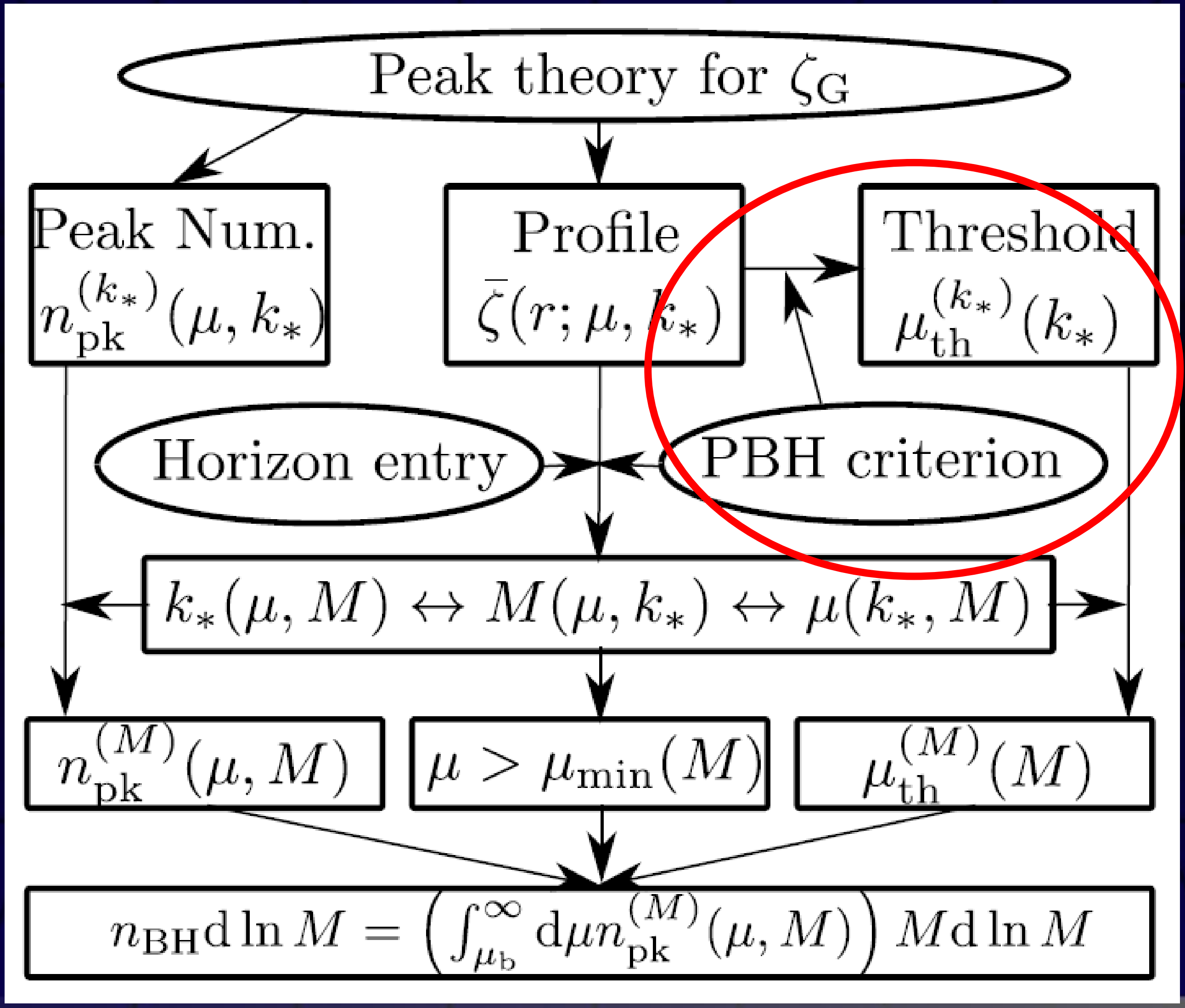
$$\bar{\zeta}(r;\mu,k_*) = \mu\left(-\frac{1}{1-\gamma^2}\left(\psi + \frac{\sigma_1^2}{\sigma_2^2}\Delta\psi\right) + \frac{k_*^2}{\gamma(1-\gamma^2)}\frac{\sigma_0}{\sigma_2}\left(\gamma^2\psi + \frac{\sigma_1^2}{\sigma_2^2}\Delta\psi\right)\right)$$

where $\psi(r) = \frac{1}{\sigma_0^2} < \zeta(r)\zeta_0 >= \frac{1}{\sigma_0^2}\int \frac{\mathrm{d}k}{k}\frac{\sin(kr)}{kr}\mathcal{P}(k)$

◎Variance

$$\frac{<\Delta\zeta(r)^2|\mu,k_*>}{\sigma_0^2} = 1 - \frac{\psi^2}{1-\gamma^2} - \frac{1}{\gamma^2(1-\gamma^2)}\left(2\gamma^2\psi + \frac{\sigma_1^2}{\sigma_2^2}\Delta\psi\right)\frac{\sigma_1^2}{\sigma_2^2}\Delta\psi$$
$$- \frac{5}{\gamma^2}\left(\frac{\psi'}{r} - \frac{\Delta\psi}{3}\right)^2 - \frac{1}{\gamma^2}\psi'^2 \sim \mathcal{O}(1) \Rightarrow \Delta\zeta(r)^2 \sim \sigma_0^2 \ll 1$$

Flow chart



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Compaction function

[Shibata, Sasaki(1999)]

◎Definition

$\mathcal{C} := \frac{\delta M}{R}$ δM:mass excess with the same R, R:areal radius

$$\delta M = M_{\rm MS}(r) - M_{\rm F}(re^{-\zeta})$$

Misner-Sharp mass: $M_{\rm MS}(r)$ MS mass in flat FLRW: $M_{\rm F}(r)$

◎$\mathcal{C}$ and averaged density perturbation $\bar{\delta}$

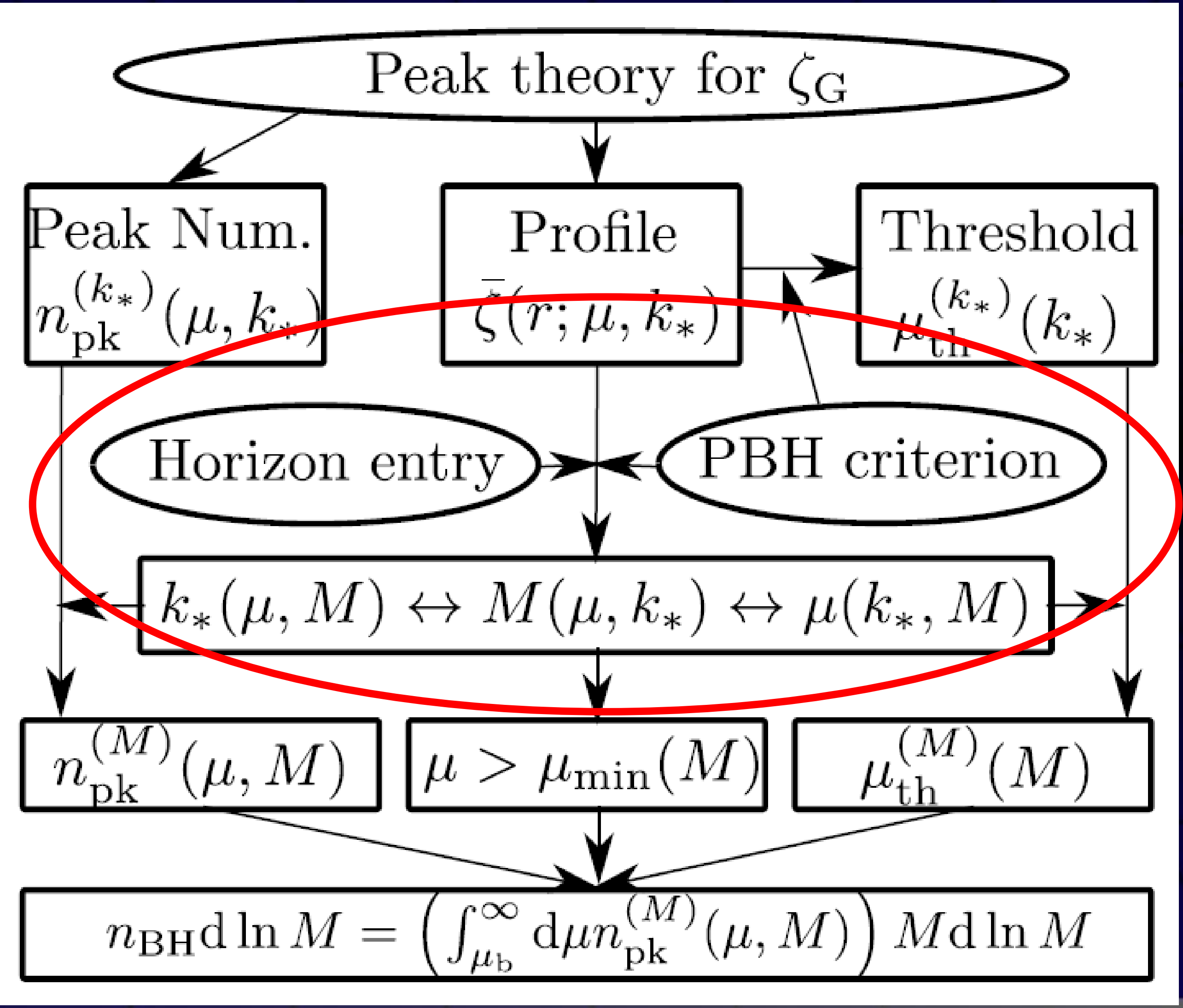
$$\mathcal{C} = \frac{1}{2}\bar{\delta}(HR)^2$$

◎Criterion [Shibata, Sasaki(1999)]

$\mathcal{C}(r_{\rm m}) > \frac{1}{2}\delta_{\rm th} = 0.267$ where $\mathcal{C} = \mathcal{C}_{max}$ at $r = r_m$

*See Escriva, Germani, Sheth(1907.13311) for more accurate criterion

Flow chart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Horizon entry and threshold

◎Estimation of the PBH mass for the typical profile

$$M(\mu,k_*)=\frac{1}{2}\alpha H^{-1}=\frac{1}{2}\alpha R\Big|_{r=\bar{r}_{\rm m}}=\frac{1}{2}\alpha a\bar{r}_{\rm m}e^{-\bar{\zeta}}=M_{eq}k_{eq}^2\bar{r}_{\rm m}^2(k_*)e^{-2\bar{\zeta}(\mu,k_*)}$$

↑ horizon entry

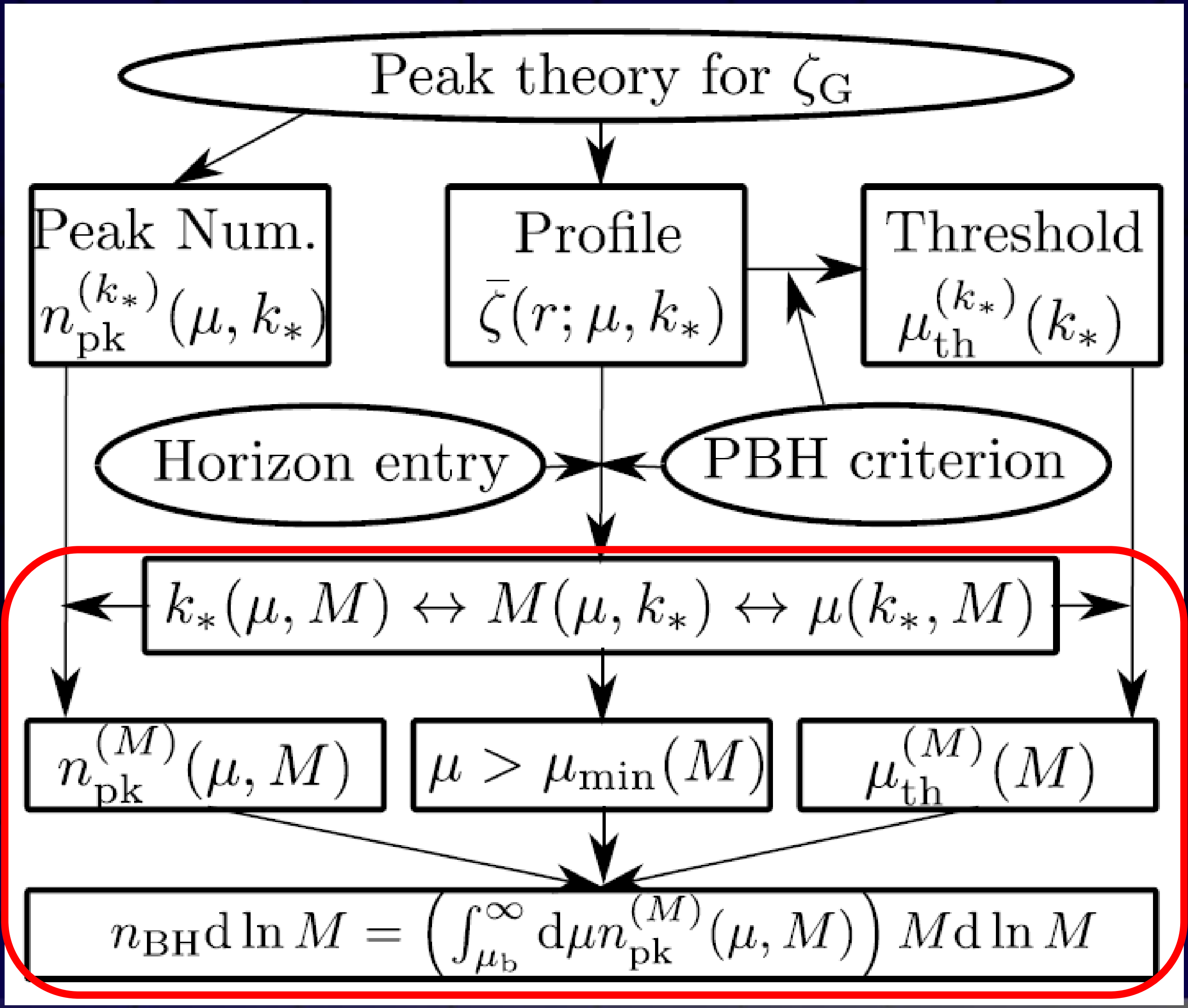
where we have assumed $\alpha\sim\mathcal{O}(1)$ factor

*note $\alpha=K(k_*)\left(\mu-\mu_{\rm th}(k_*)\right)^\gamma$ with $\gamma\simeq 0.36$
if we take into account the critical behavior

◎We obtain the relation of M, k_*, μ

$$M=M(\mu,k_*)\longleftrightarrow k_*=k_*(\mu,M)\longleftrightarrow \mu=\mu(M,k_*)$$

Flow chart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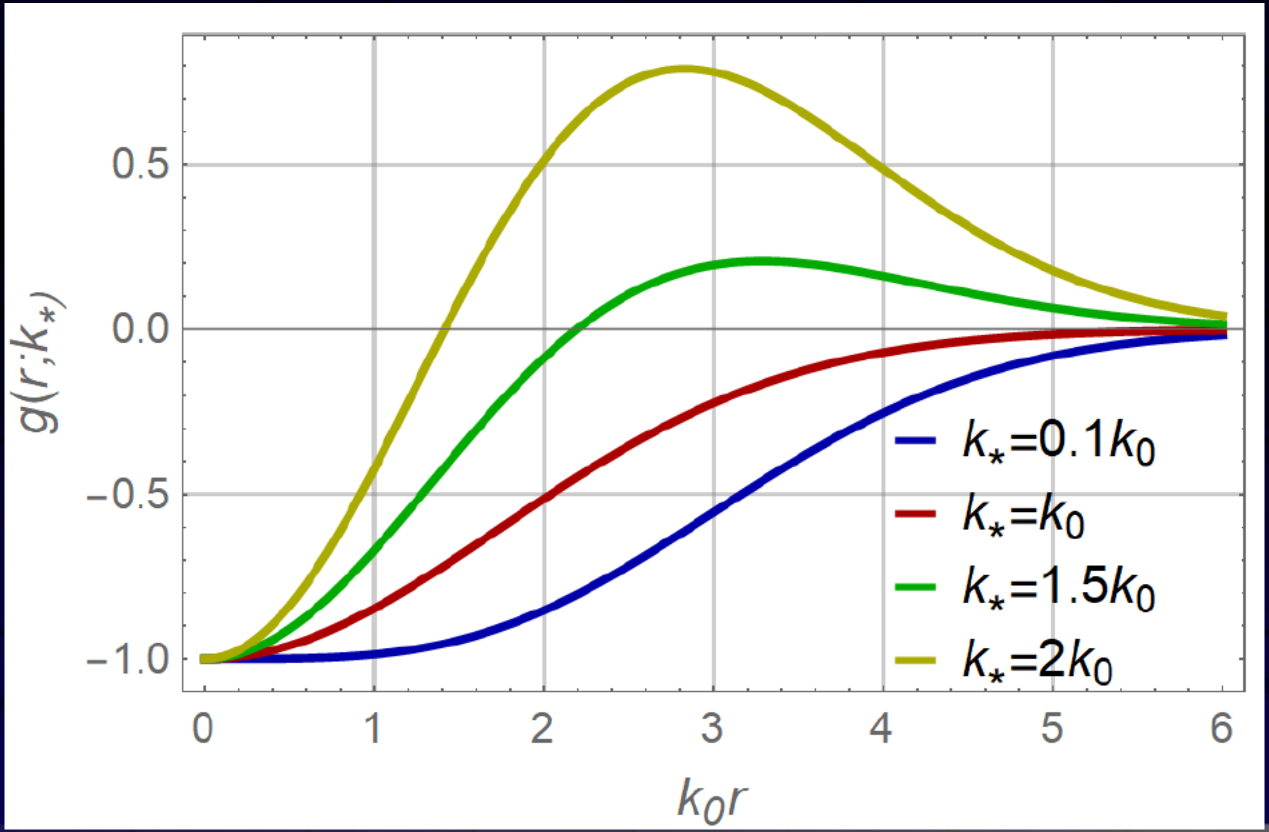
An Extended Spectrum

An extended $\mathcal{P}(k)$

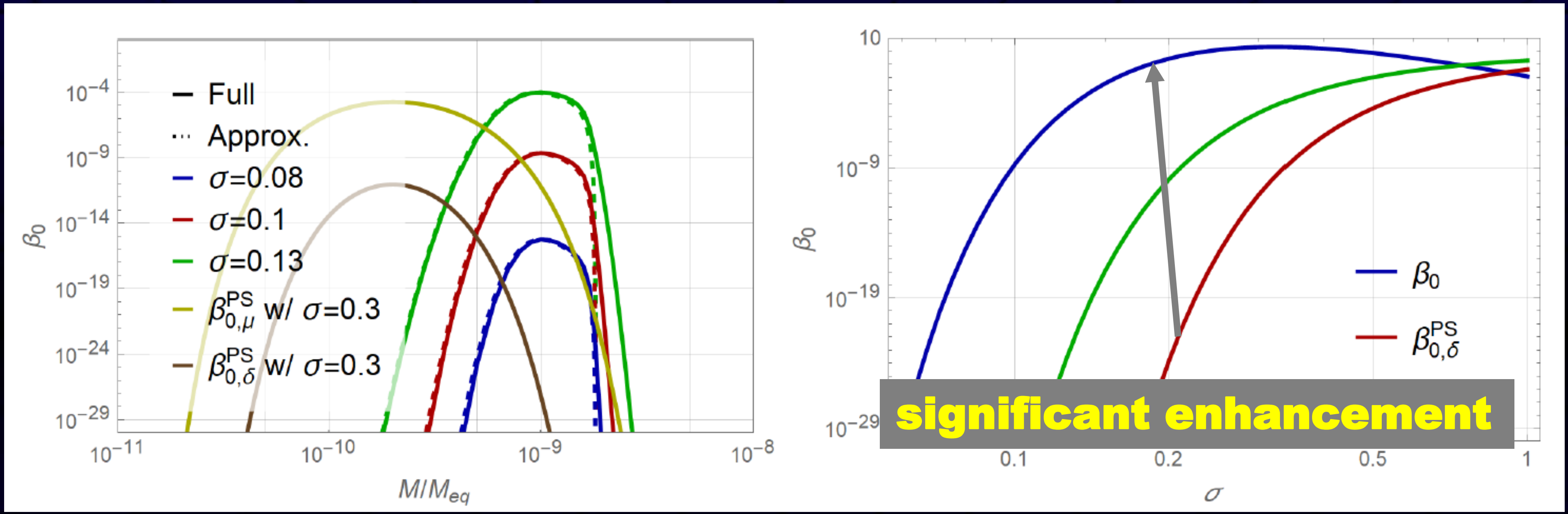
◎ $\mathcal{P}(k) = 3\sqrt{\frac{6}{\pi}}\sigma^2\left(\frac{k}{k_0}\right)^3 \exp\left(-\frac{3}{2}\frac{k^2}{k_0^2}\right)$

◎**Profile** $\bar{\zeta}(r;k_*)$



$\beta_0(M)$

◎ $k_0 = 10^5 k_{\rm eq}$



◎Spectrum shift by one order of mag.

$$\frac{M}{M_{\rm eq}} \simeq \frac{k_{\rm eq}^2}{k_0^2}\bar{r}_{\rm m}^2 k_0^2 e^{-2\mu\hat{\zeta}_{\rm m}} \sim \frac{k_{\rm eq}^2}{k_0^2} \times 10$$

◎Significantly larger abundance

- **Optimized criterion**
- **No suppression from a window function**

Note on window function

◎Don’t we need a Window function any more?

◎Two roles in the PS formalism

1. Smooth out the smaller scale inhomogeneity

2. Introduce the scale dependence of the mass spectrum

◎For our specific power spectrum

･No smaller scale inhomogeneity(single scale)

･The scale dependence is automatically induced by the random variable k_*, which characterizes the profile

◎We need a window function for a broad spectrum

Note on non-linearity

◎Non-linearity is taken into account in our procedure [arXiv:1805.03946]

◎Significance is reported in Kawasaki, Nakatsuka(1903.02994)
De Luca et al.(1904.00970)
Young, Musco, Byrnes(1904.00984)

・The nonlinearity decreases the PBH abundance

$$\delta = -\frac{8}{9}\frac{1}{a^2H^2}\, e^{5/2\zeta}\Delta e^{-\zeta/2}$$

$$\zeta_1^i = \partial_i\zeta|_{x=0}, \zeta_2 = \Delta\zeta|_{x=0}$$

Taylor expansion around a peak of δ →

$$\delta \simeq \frac{4}{9}\frac{e^{2\zeta_0}}{a^2H^2}\left[\zeta_2 - \frac{1}{2}\Sigma\left(\zeta_1^i\right)^2\right] + \mathcal{O}(x^2)$$

negative contribution from the non-linear term

◎But, taking into account the typical profile of a peak is much more significant than the non-linear effect when we compare it with the PS formalism

What we have done so far

A new procedure to estimate PBH abundance

◎Better motivated than PS

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◎Optimized criterion is implemented

◎No window function dependence for a narrow spectrum

CY,Harada,Garriga,Kohri[1805.03946]

with F_{NL}

CY,Gong,Yokoyama[1906.06790]

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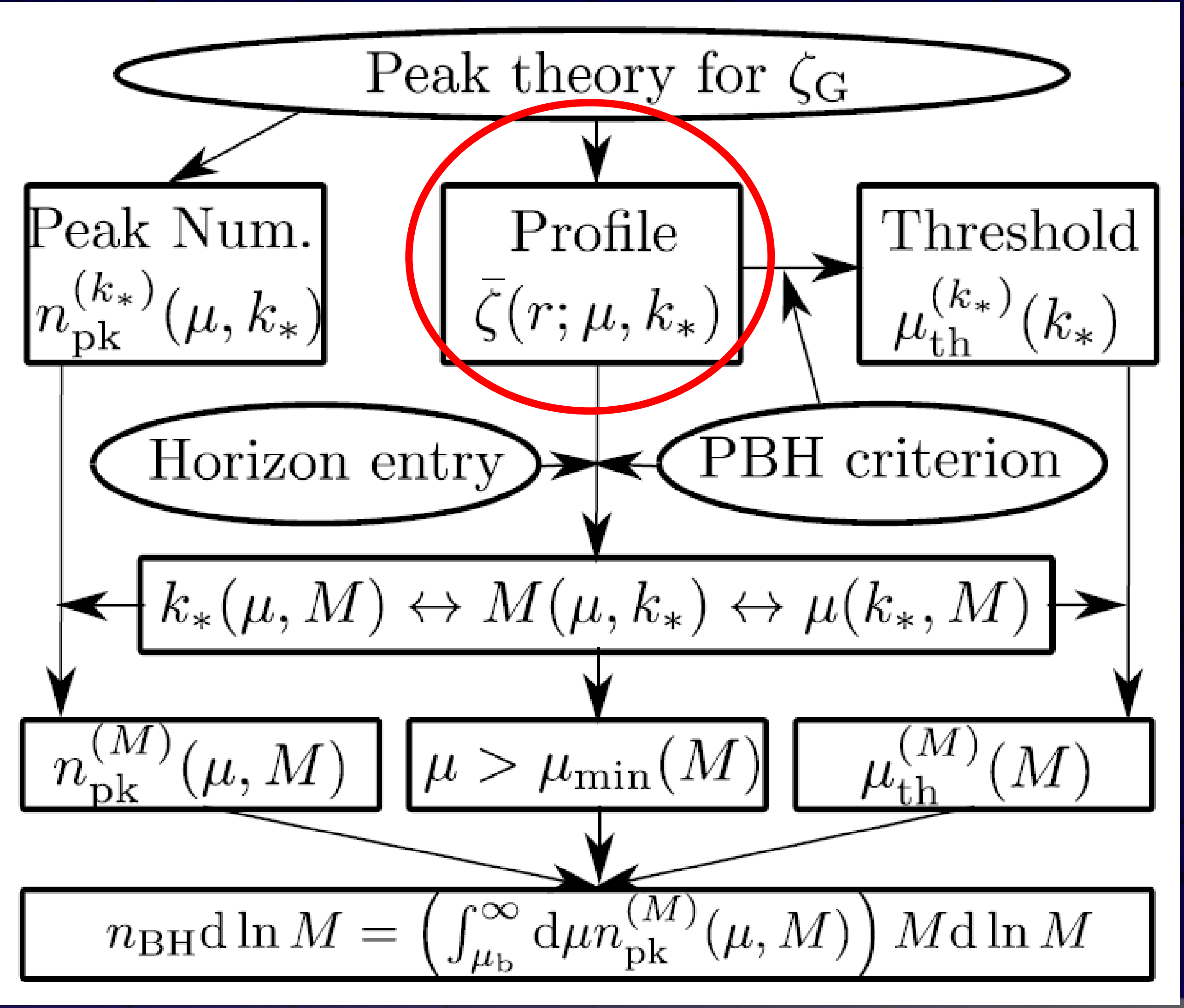
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Local type non-Gaussian

◎ζ from ζ_G

$$\zeta = \zeta_G - F_{NL}(\zeta_G^2 - \langle \zeta_G^2 \rangle)$$

◎Power spectrum

$$< \tilde{\zeta}_G^* \left(\vec{k}\right) \tilde{\zeta}_G \left(\vec{k}'\right) > = \frac{2\pi^2}{k^3} \mathcal{P}(k)(2\pi)^3 \delta\left(\vec{k} - \vec{k}'\right)$$

◎Conditional Prob.

$$P(\zeta_G(r); \mu, k_*) = \frac{1}{\sqrt{2\pi}\sigma_\zeta} \exp\left[-\frac{1}{2\sigma_\zeta^2}\left(\zeta_G(r) - \bar{\zeta}_G(r)\right)^2\right]$$

◎Typical prof. for $\zeta(r)$ with μ, k_*

$$-\bar{\zeta}(r; \mu, k_*) = -\int \zeta P(\zeta_G(r); \mu, k_*) d\zeta_G$$

$$= -\int \left[\zeta_G - F_{NL}\left(\zeta_G^2 - < \zeta_G^2 >\right)\right] P(\zeta_G; \mu, k_*) d\zeta_G$$

$$= -\bar{\zeta}_G + F_{NL}\left(\bar{\zeta}_G^2 + \sigma_\zeta^2 - \sigma_0^2\right)$$

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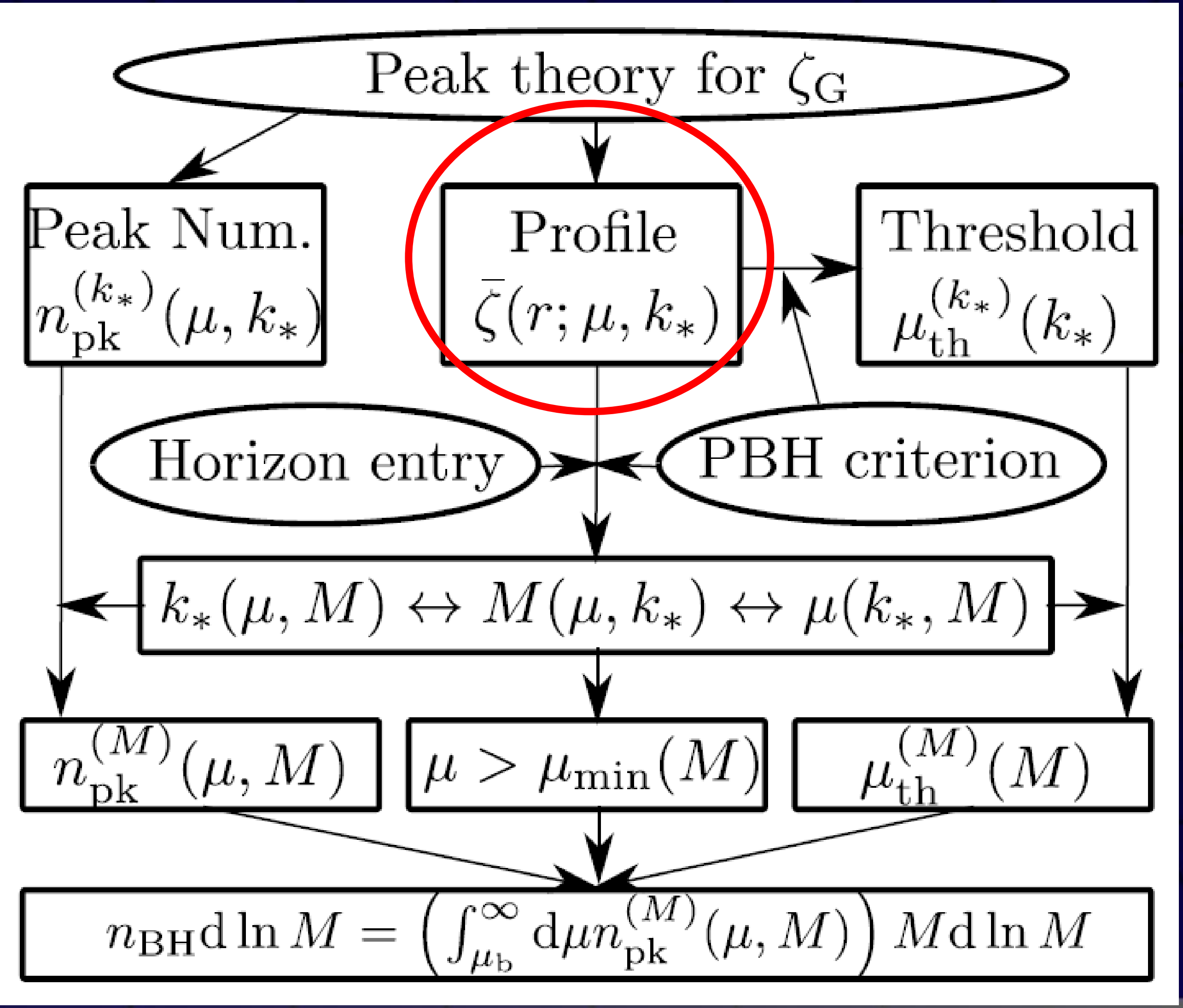

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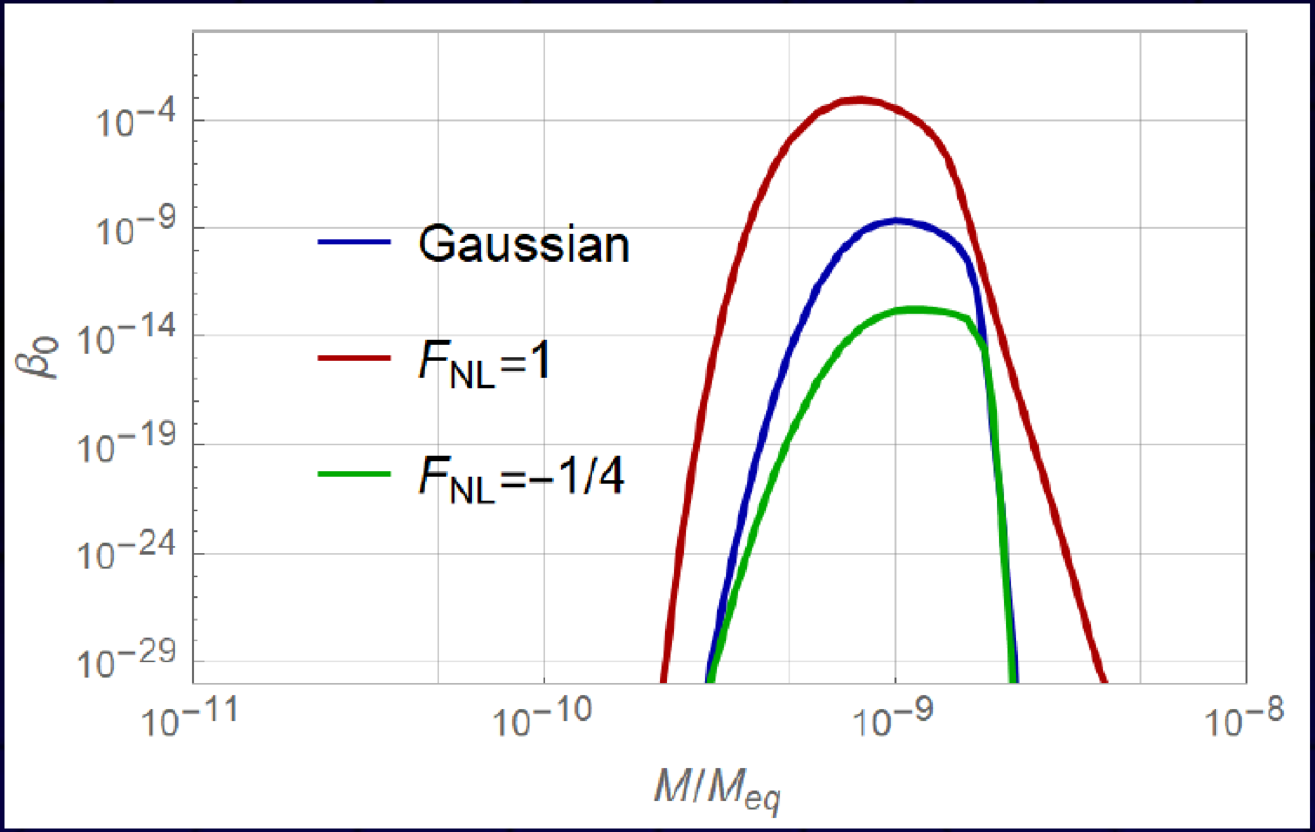
Result with non-G

◎Positive F_{NL} gives larger PBH abundance



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