Recent Studies on the propagation velocity of lensed gravitational waves

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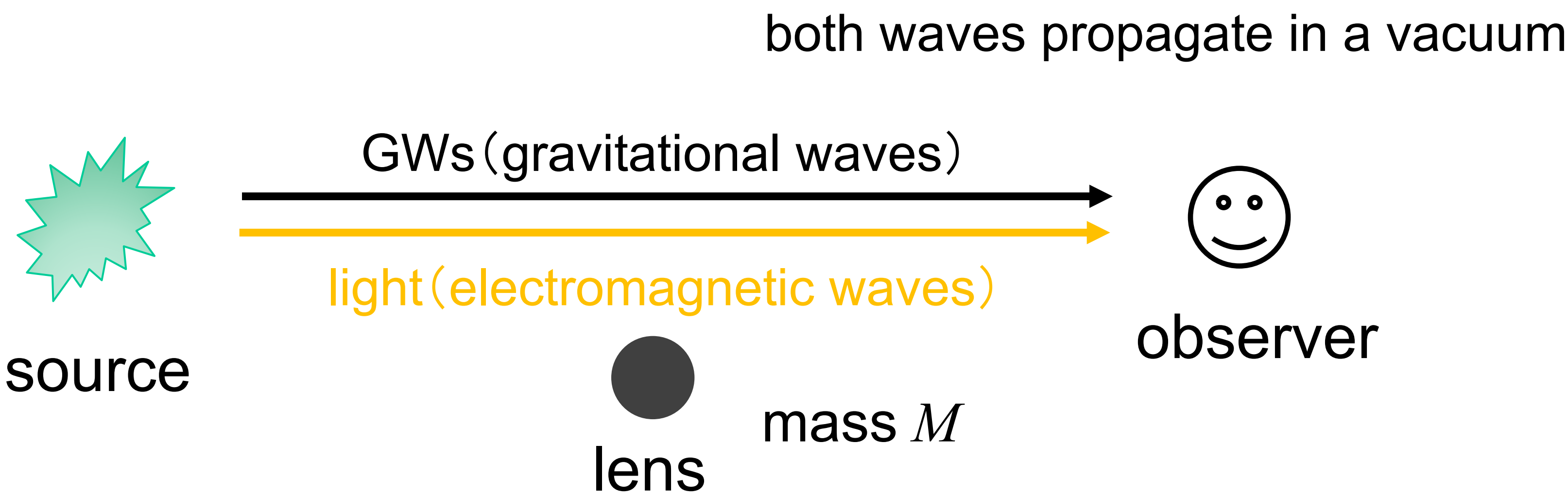
高橋龍一　　　　（弘前大学）

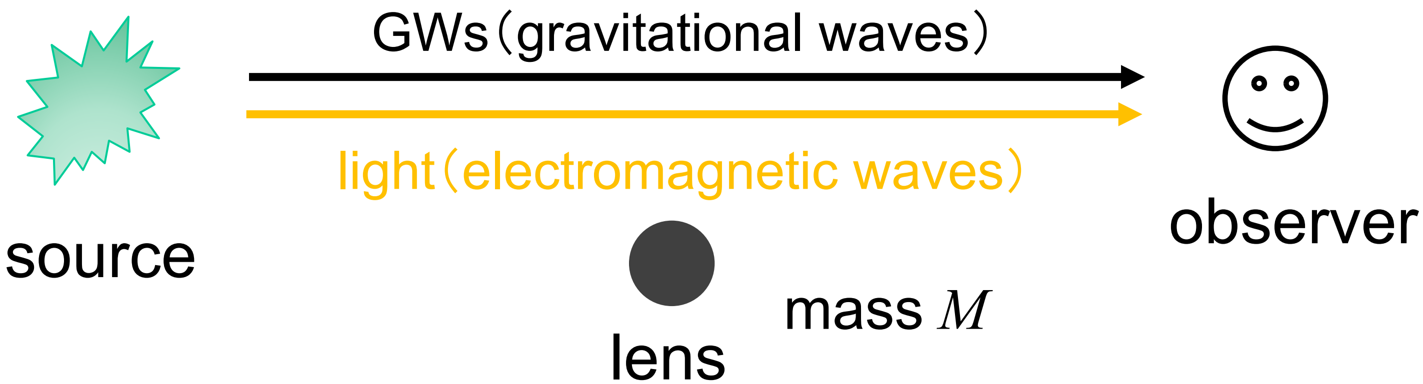
RT（2017）, Morita & Soda（2019）, Suyama（2020）, Ezquiaga+（2020）, He（2020）

Abstract



Question: If a source emitted GWs and light simultaneously and both waves propagated near a lens, does an observer receive them at the same time?

both waves propagate in a vacuum



light ← delayed due to the Shapiro time delay

valid <u>in geometrical optics</u>

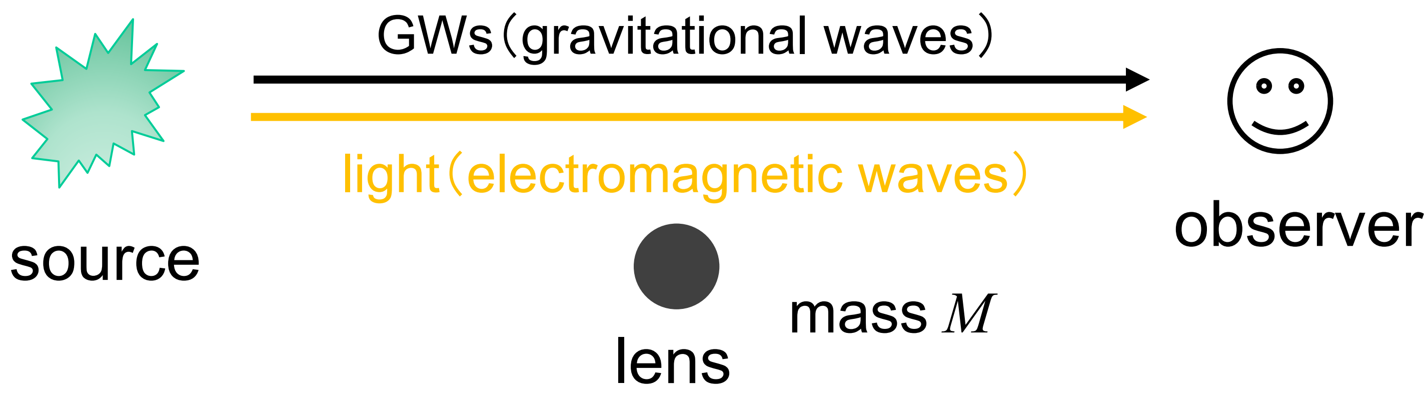
GWs ← delayed or not（?）

studied <u>in wave optics</u>

if the wavelength > the Schwarzschild radius of lens

$$M \lesssim 10^5 M_\odot \, (f/\mathrm{Hz})^{-1}$$

f: GW frequency



phase velocity : GWs **faster** than light （RT 2017）

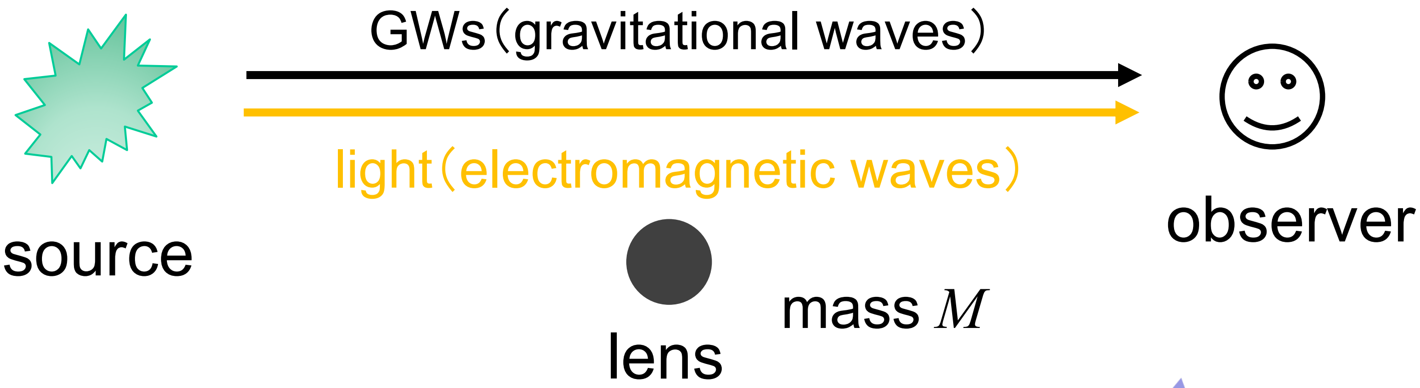
group velocity : GWs **faster** than light （Morita & Soda 2019）

wave-front velocity : GWs **equal** to or **slower** than light （Suyama 2020）

⇨ satisfy the causality of GR

apparent superluminality of phase & group velocities **caused by interference of GWs** （Ezquiaga+ 2020）

the arrival-time lag is a **real observable** in the far future



phase velocity : GWs **faster** than light (RT 2017)

group velocity : GWs **faster** than light (Morita & Soda 2019)

wave-front velocity : GWs **equal** to or **slower** than light (Suyama 2020)

all the results are correct

→ satisfy the causality of GR

apparent superluminality of phase & group velocities **caused by interference of GWs** (Ezquiaga+ 2020)

the arrival-time lag is a **real observable** in the far future

Introduction

（e.g. Andersson+ 2013; Rosswog 2015）

Promising sources emitting both GWs and light

NS: neutron star
BH: black hole

- NS-NS or NS-BH merger
  → short gamma ray burst
  target of ground- & space-based detectors
- Supernova
  target of ground-based detectors
- Massive BH binary
  may emit light if the binary is embedded in accretion disk
  target of space-based detectors & pulsar timing arrays

Lensed GW signal not yet confirmed at present （Hannuksela+ 2019）

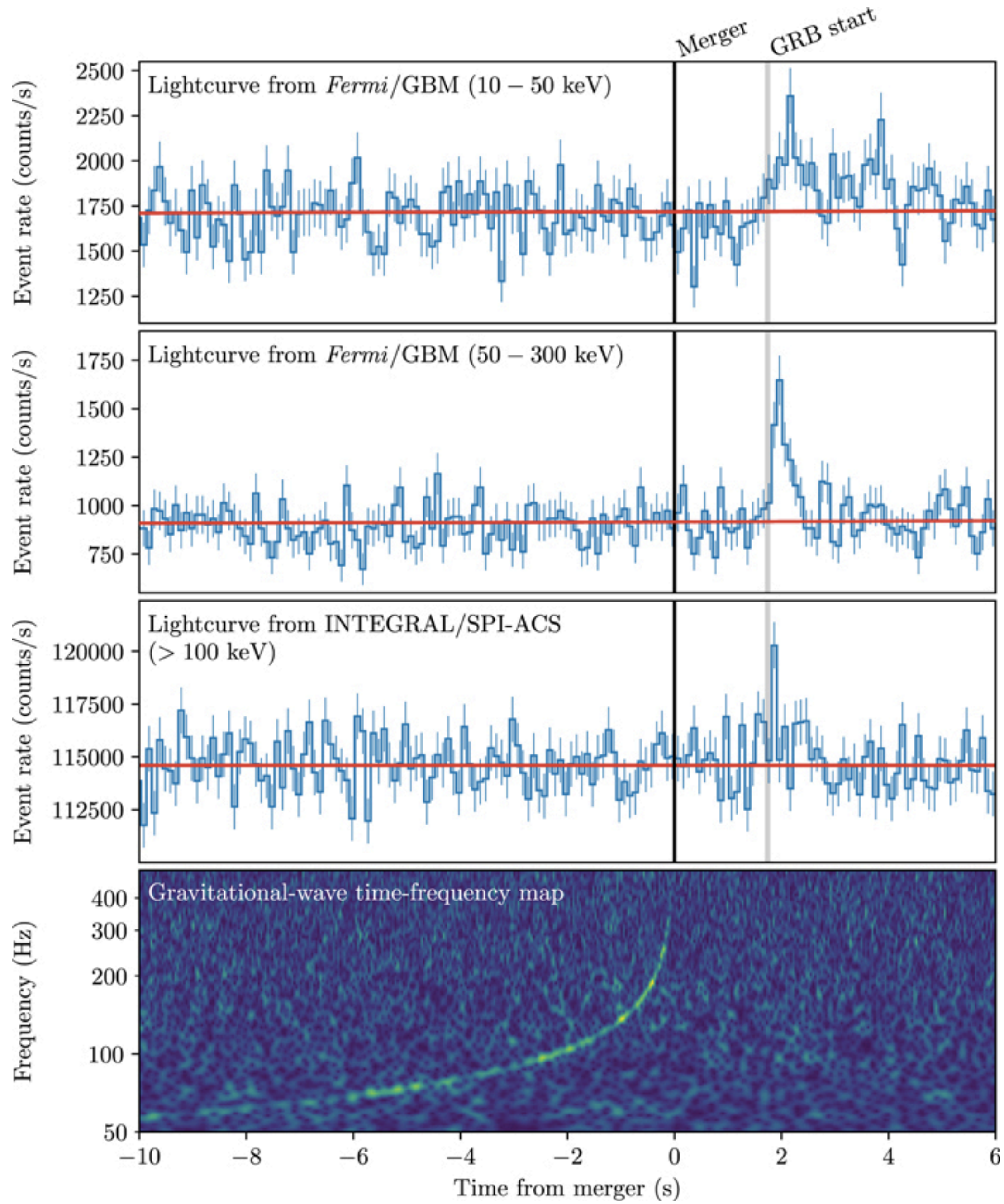
GW170817

detected in both GWs & light

neutron star binary merger detected by LIGO/VIRGO
(Abbott+ 2017)

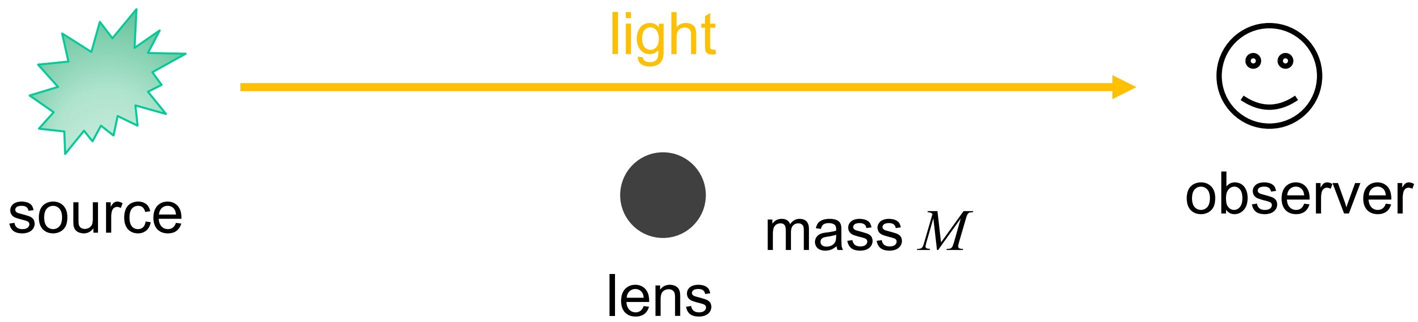
X & gamma-ray signals detected ~1.7s later than the binary merger
(LIGO collaboration+ 2017)

GRB associated the merger



2. Theory

Gravitational lensing of light

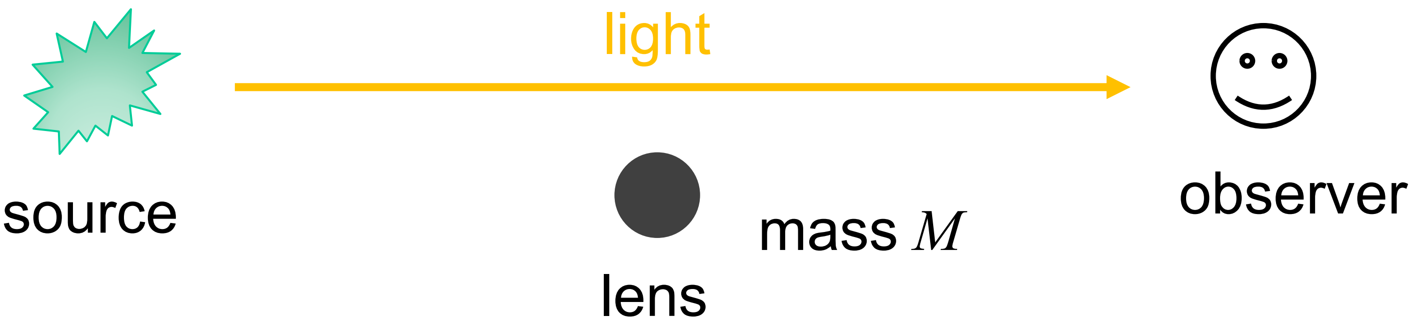


wavelength of light << typical lens size

→ geometrical optics valid

2. Theory

Gravitational lensing of light



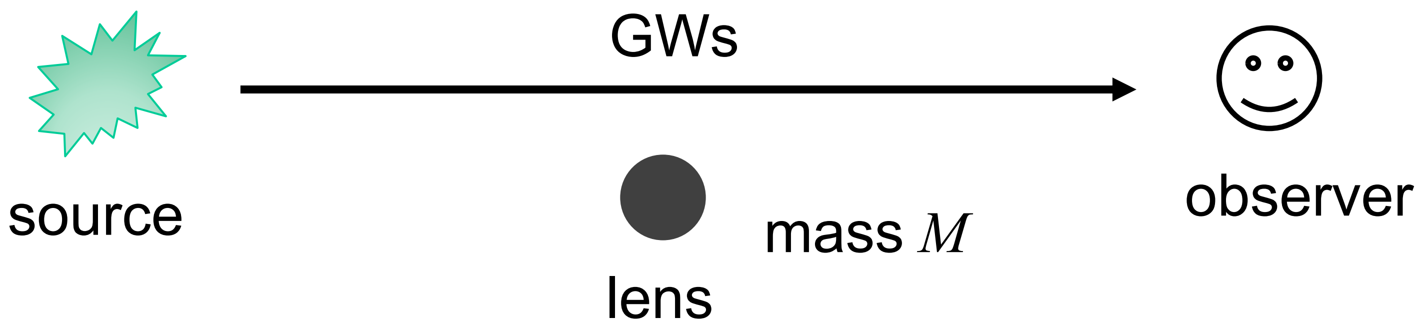
Light experiences the Shapiro time delay

$$t_{\mathrm{M}} \simeq \frac{4GM}{c^3} = \frac{2 \times \text{Schwarzschild radius of the lens}}{c}$$

$$\approx 1\,\mathrm{sec}\left(\frac{M}{10^5 M_\odot}\right)$$

↑ independent of frequency

Gravitational lensing of GWs



GW wavelength > Schwarzschild radius of the lens

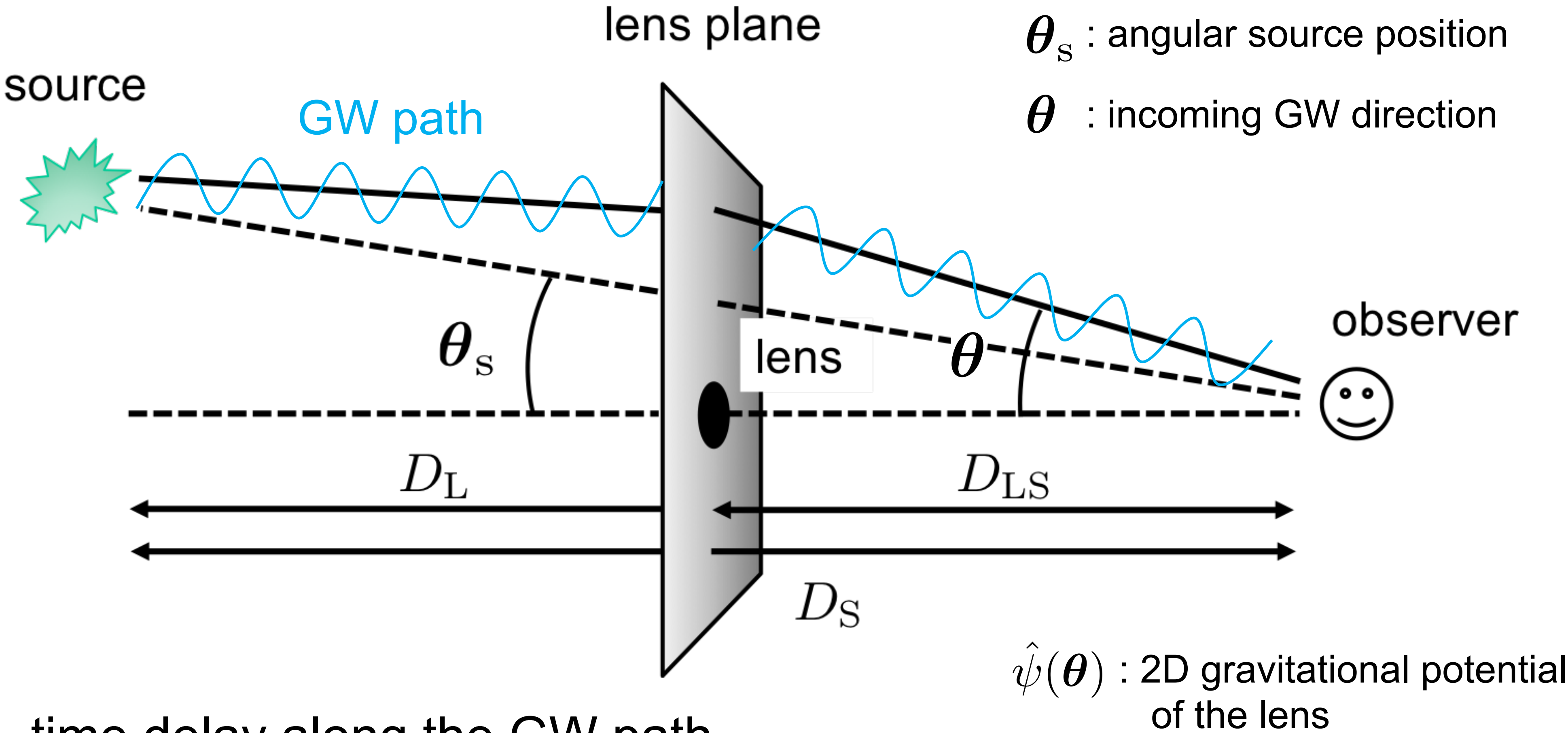
→ wave optics should be used

$$M < 10^5 M_\odot \left(\frac{f}{\mathrm{Hz}}\right)^{-1}$$

$f \approx 100$ Hz for LIGO/Virgo

$\approx 10^8$ Hz for pulsar timing arrays

refer to talks by Jose Diego, Liang Dai & Anuj Mishra



time delay along the GW path

$$t_{\rm d}(\boldsymbol{\theta}, \boldsymbol{\theta}_{\rm s}) = \frac{1}{c}\left[\underbrace{\frac{D_{\rm L} D_{\rm S}}{2 D_{\rm LS}} |\boldsymbol{\theta} - \boldsymbol{\theta}_{\rm s}|^2}_{\text{geometrical time delay}} - \underbrace{\frac{\hat{\psi}(\boldsymbol{\theta})}{c}}_{\text{Shapiro time delay}}\right]$$

point mass lens assumed

Lensed waveform given by the Kirchhoff diffraction integral

(Schneider, Ehlers, Falco 1992)

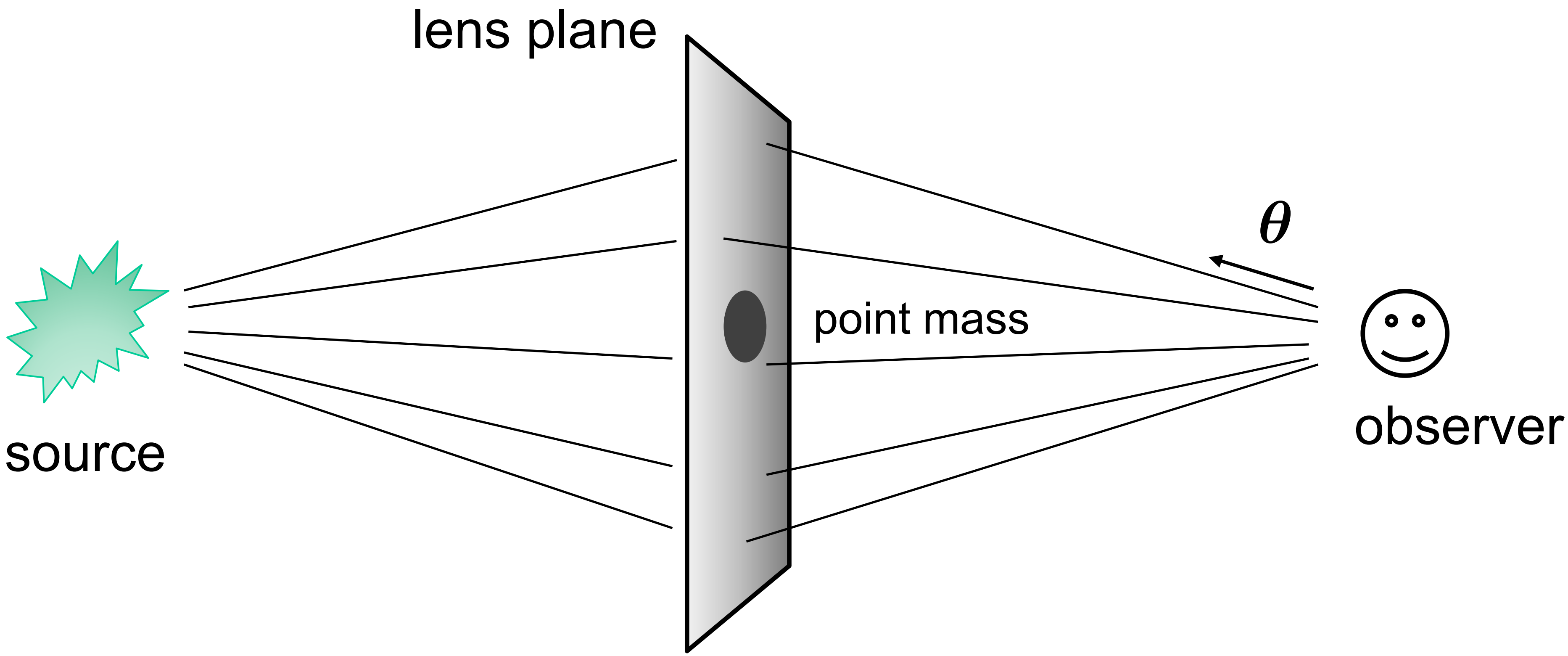
lensed waveform unlensed waveform

$$\tilde{h}^{\rm L}(f) = F(f;\boldsymbol{\theta}_{\rm s})\tilde{h}(f)$$

Amplification factor

$$F(f;\boldsymbol{\theta}_{\rm s}) = \frac{D_{\rm L} D_{\rm S}}{c D_{\rm LS}} \frac{f}{i} \int d^2\theta \, \exp\left[2\pi i f t_{\rm d}(\boldsymbol{\theta},\boldsymbol{\theta}_{\rm s})\right]$$

In wave optics

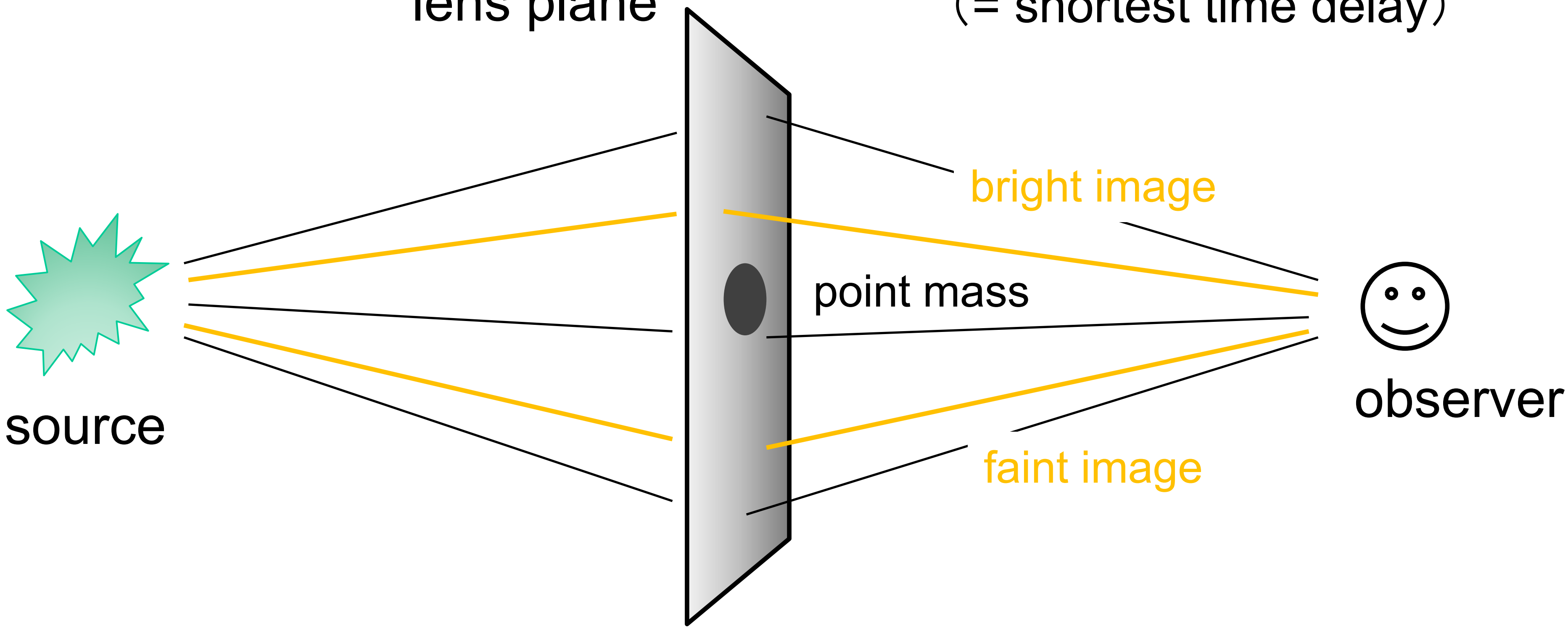


lensed GWs = superposition of many GW paths

In geometrical optics

bright image forms at minimum $t_{\rm d}$

(= shortest time delay)



lensed light = sum of two paths (bright & faint images)

➡ arrival-time difference $\Delta t_{\rm d}$ between GWs and the bright image will be shown

Results

THE ASTROPHYSICAL JOURNAL, 835:103 (9pp), 2017 January 20
doi:10.3847/1538-4357/835/1/103

ARRIVAL TIME DIFFERENCES BETWEEN GRAVITATIONAL WAVES AND ELECTROMAGNETIC SIGNALS DUE TO GRAVITATIONAL LENSING

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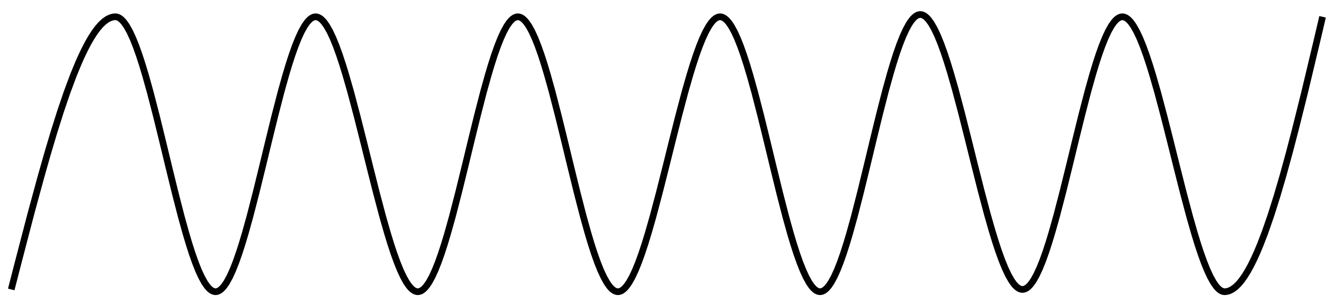
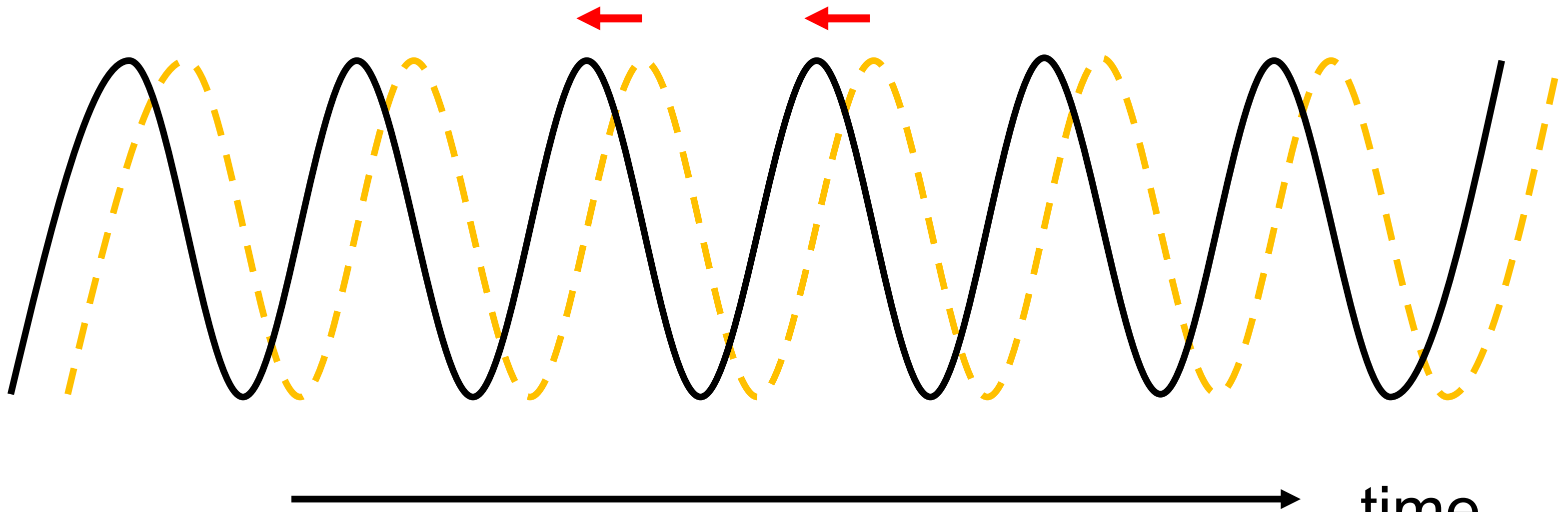
propagation of a monochromatic wave
in the presence of a point mass lens



phase shift for monochromatic waves (RT 2017)

$$\Delta t_{\mathrm{d}} = t_{\mathrm{d,GWs}} - t_{\mathrm{d,light}} > 0$$

GW phases appear in advance



solid black : wave optics

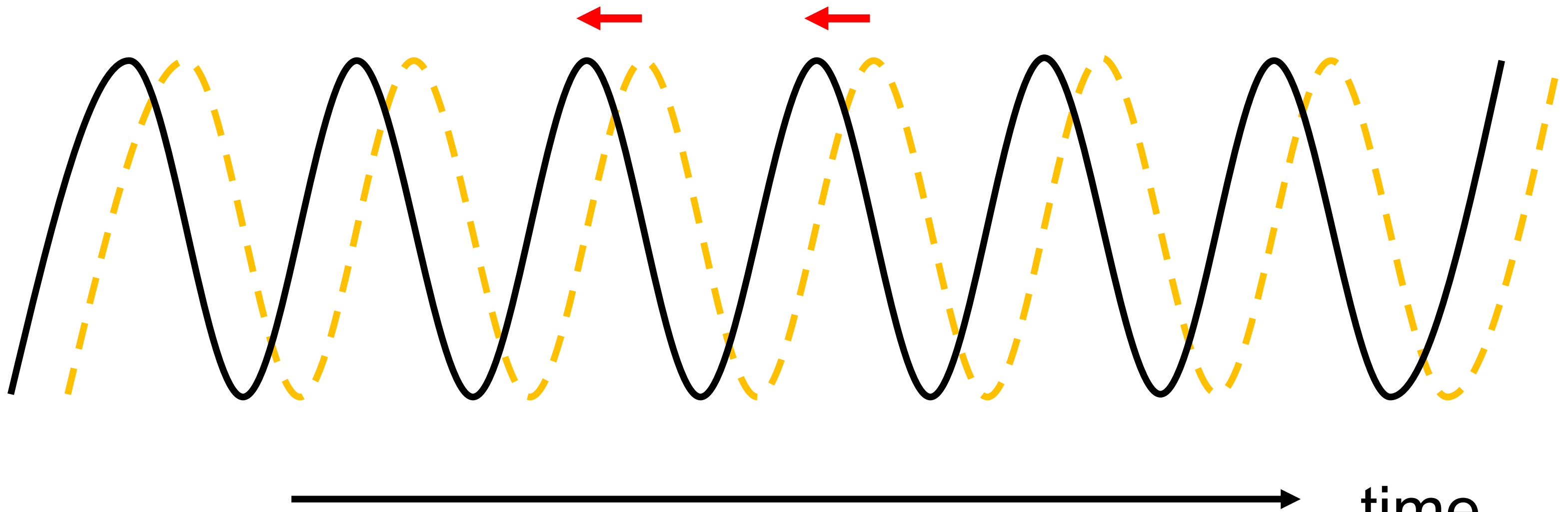
dashed orange : geometrical optics (for the bright image)

GW propagation is “superluminal”

phase shift for monochromatic waves (RT 2017)

$$\Delta t_{\mathrm{d}} = t_{\mathrm{d,GWs}} - t_{\mathrm{d,light}} > 0$$

GW phases appear in advance



solid black : wave optics

dashed orange : geometrical optics (for the bright image)

GW propagation is "~~superluminal~~"

apparently superluminal (Ezquiaga+ 2020) the arrival time difference defined in phase can reach

$$\Delta t_{\mathrm{d}} \simeq 0.1\,\mathrm{sec}\left(\frac{f}{\mathrm{Hz}}\right)^{-1}$$

when the wavelength is comparable to the Schwarzschild radius
& the impact parameter is smaller than the Einstein radius

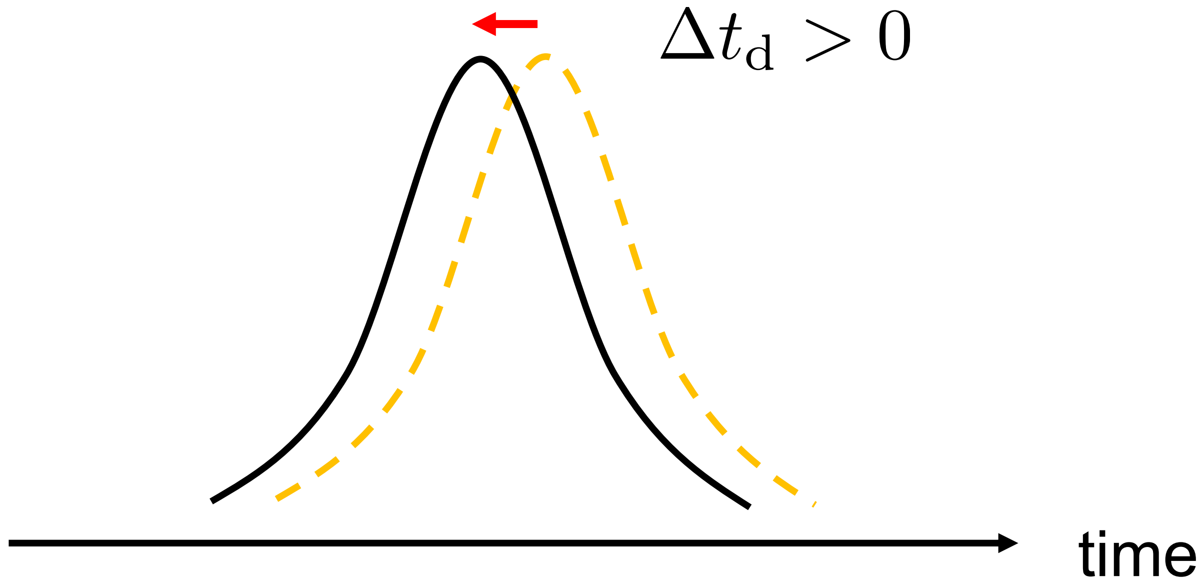
$$\Delta t_{\mathrm{d}} \sim 1\,\mathrm{msec}\left(\frac{f}{100\mathrm{Hz}}\right)^{-1}$$ for ground-based detectors

$$\sim 2\,\mathrm{min}\left(\frac{f}{\mathrm{mHz}}\right)^{-1}$$ for space-based detectors

$$\sim 4\,\mathrm{months}\left(\frac{f}{10^{-8}\mathrm{Hz}}\right)^{-1}$$ for pulsar timing arrays

the time difference is more prominent for lower GW frequency

propagation of a Gaussian wave packet （Morita & Soda 2019）



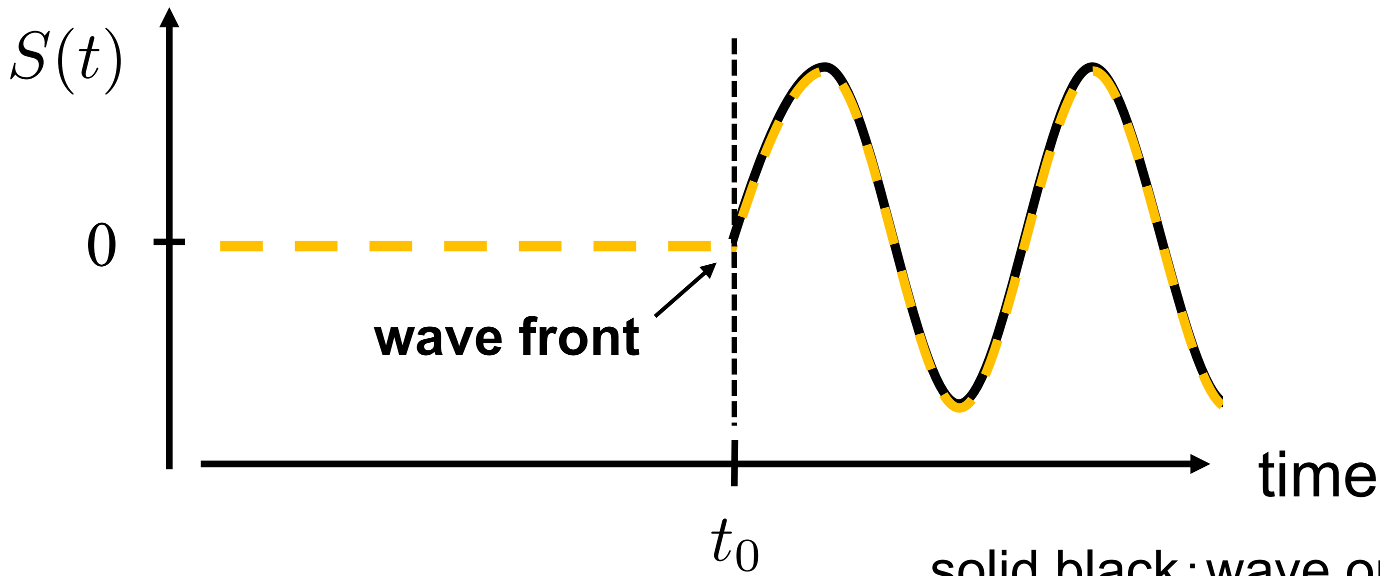
solid black：wave optics
dashed orange：geometrical optics
（for the bright image）

GW packet arrives **earlier** than the packet of light

but the time lag $\Delta t_{\rm d}$ is smaller than that in phase velocity propagation velocity of wave front （Suyama 2020）

$S(t) = 0$ $\quad t < t_0$

$S(t) \neq 0$ （any waveform OK） $\quad t > t_0$



solid black：wave optics

dashed orange：geometrical optics （for the bright image）

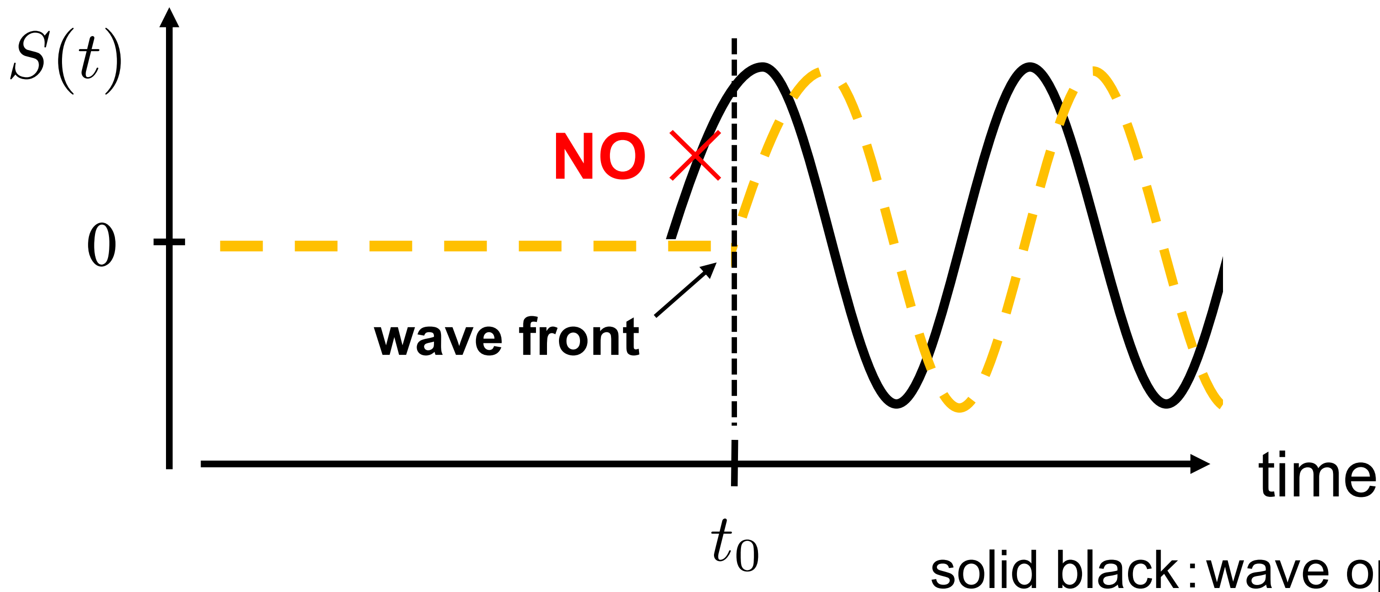
GWs **never** arrive at $t < t_0$ （← independent of frequency）

consistent with the causality of GR

propagation velocity of wave front (Suyama 2020)

$S(t) = 0$ $t < t_0$

$S(t) \neq 0$ (any waveform OK) $t > t_0$

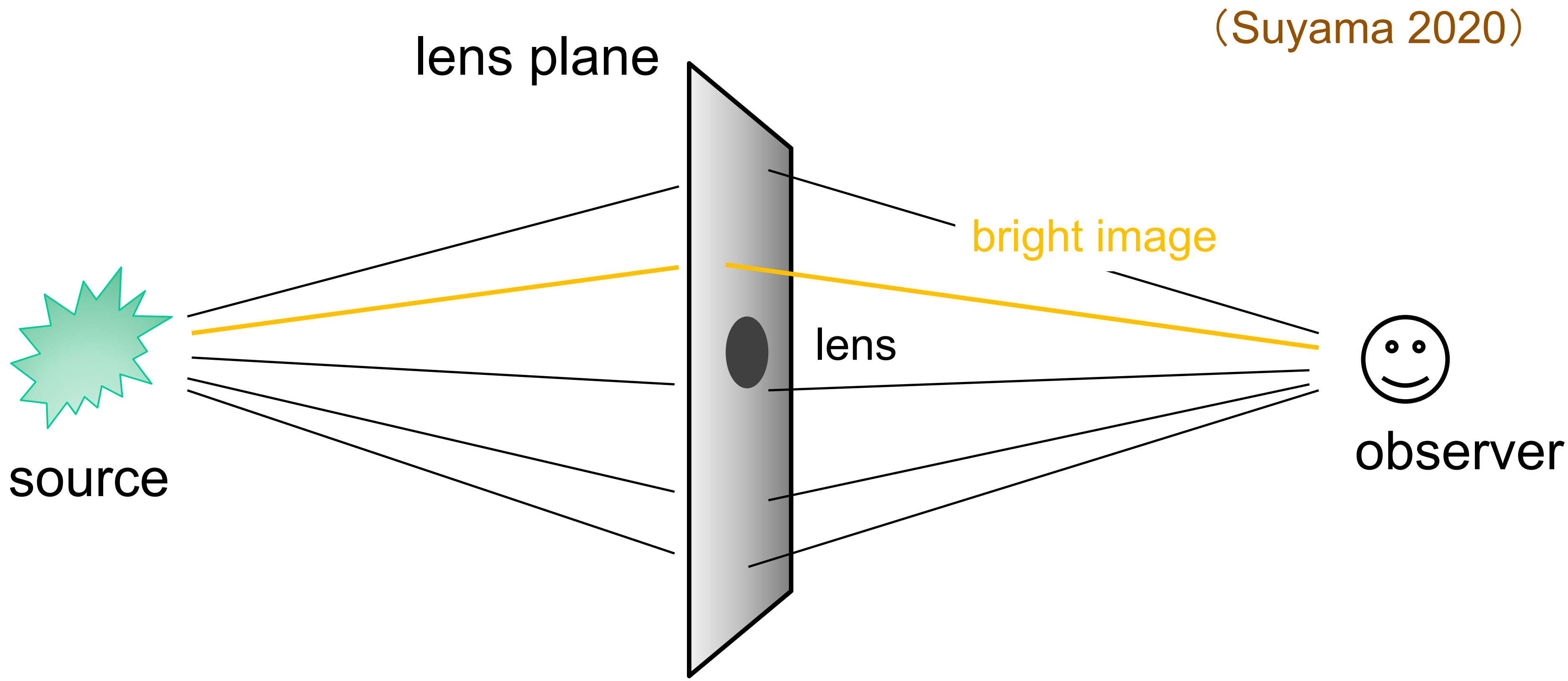


solid black : wave optics

dashed orange : geometrical optics (for the bright image)

GWs **never** arrive at $t < t_0$ (← independent of frequency)

consistent with the causality of GR



lensed GWs = superposition of many GW paths

any path **cannot** arrive **earlier** than the bright image of light
(the minimum time delay)

arXiv:2005.10702

Apparent Superluminality of Lensed Gravitational Waves

Jose María Ezquiaga,[1,*] Wayne Hu,[1,2,†] and Macarena Lagos[1,‡]

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[2] *Department of Astronomy & Astrophysics, The University of Chicago, Chicago, IL 60637, USA*

(Dated: May 22, 2020)

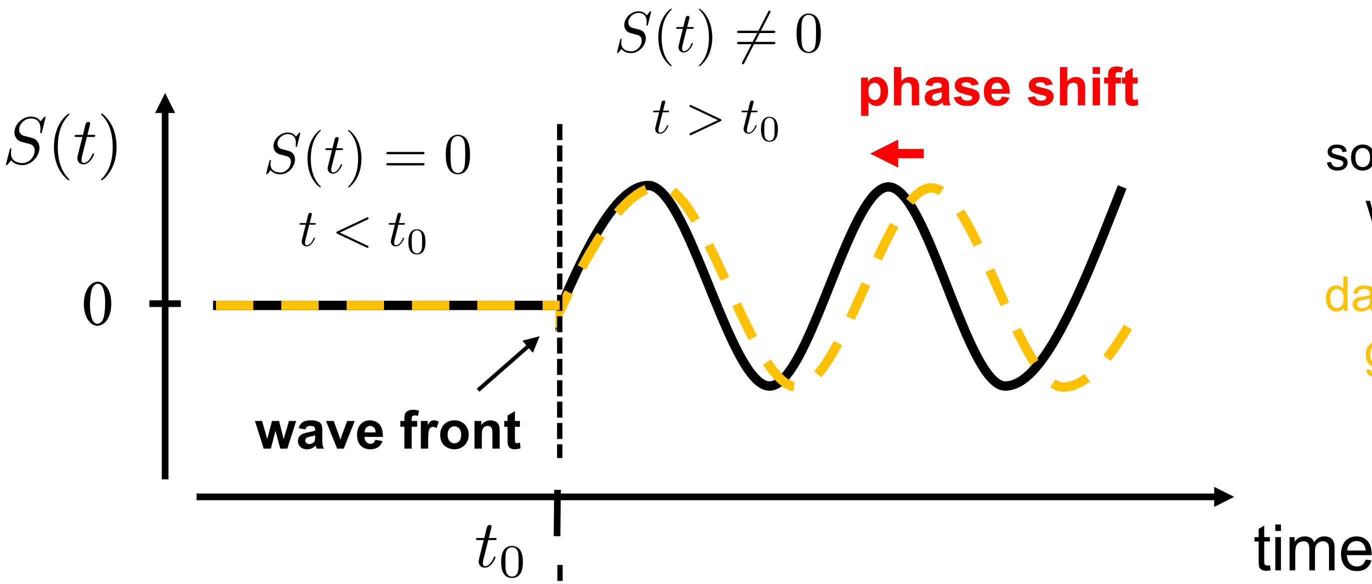
This paper confirmed all the previous results by

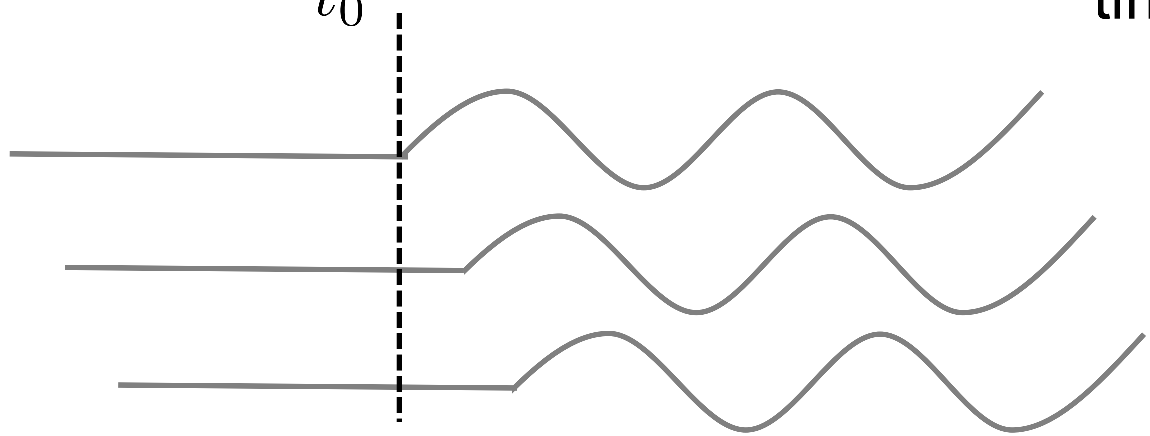
RT 2017, Morita & Soda 2019 and Suyama 2020

Interference between lensed GW paths causes **apparent superluminal propagation** of GWs observed in phase & group velocities

Why GW phases appear in advance? （Ezquiaga+ 2020）

→ interference among multiple images cause phase modulation

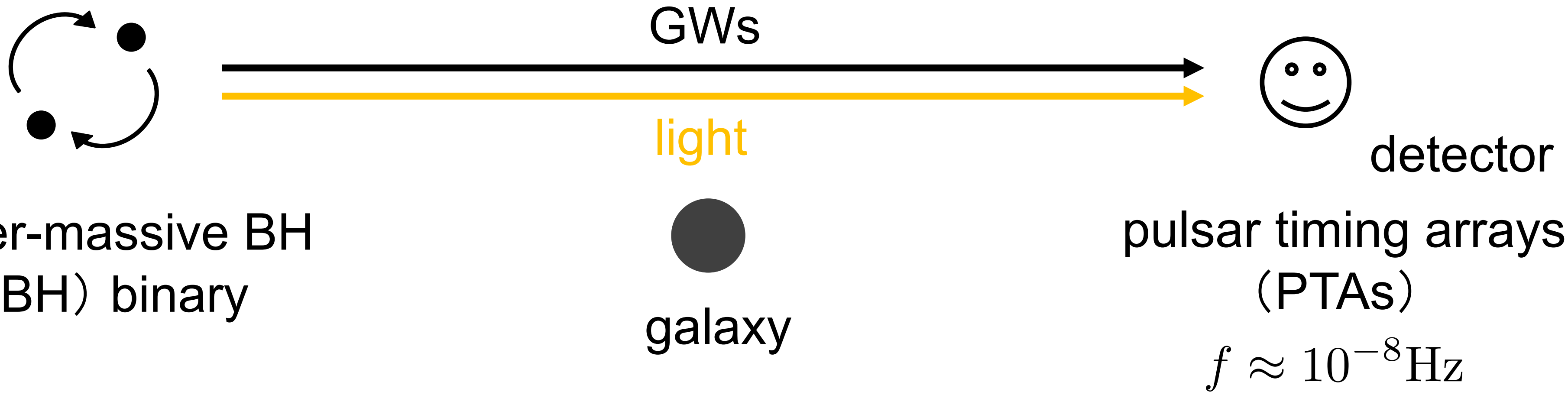




GWs = superposition of many signals

Measurement of the orbital phase difference in SMBH binary

(RT 2017)



source

Super Massive Black Hole Binaries at z=0.2-2

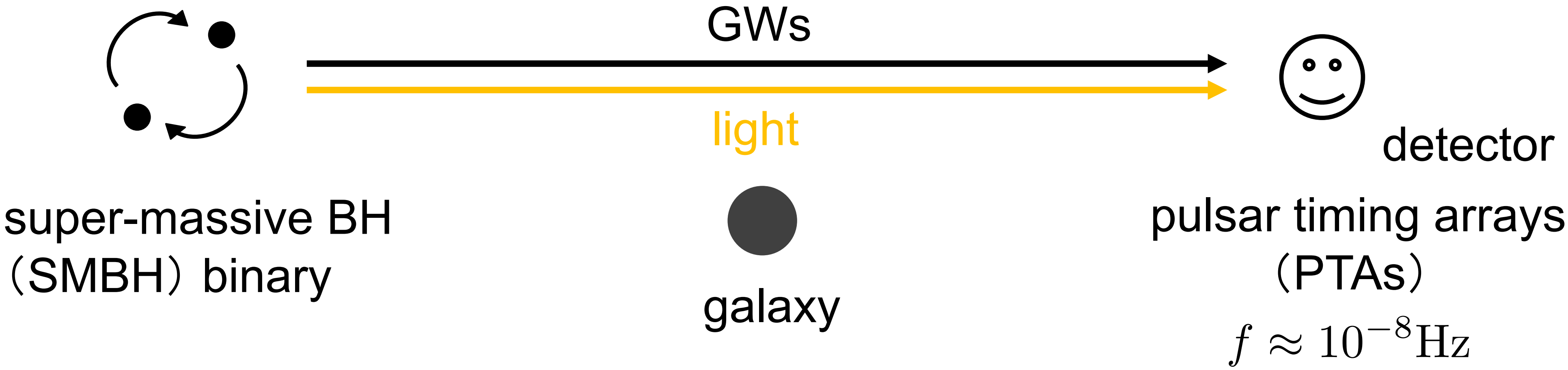
Future pulsar timing arrays may detect 500-1000 sources in both GW and X-ray signals (Sesana+ 2012)

The orbital motion may be observed in both GW and x-ray detectors

→ **measure the orbital phase difference**

Measurement of the orbital phase difference in SMBH binary

(RT 2017)



lens

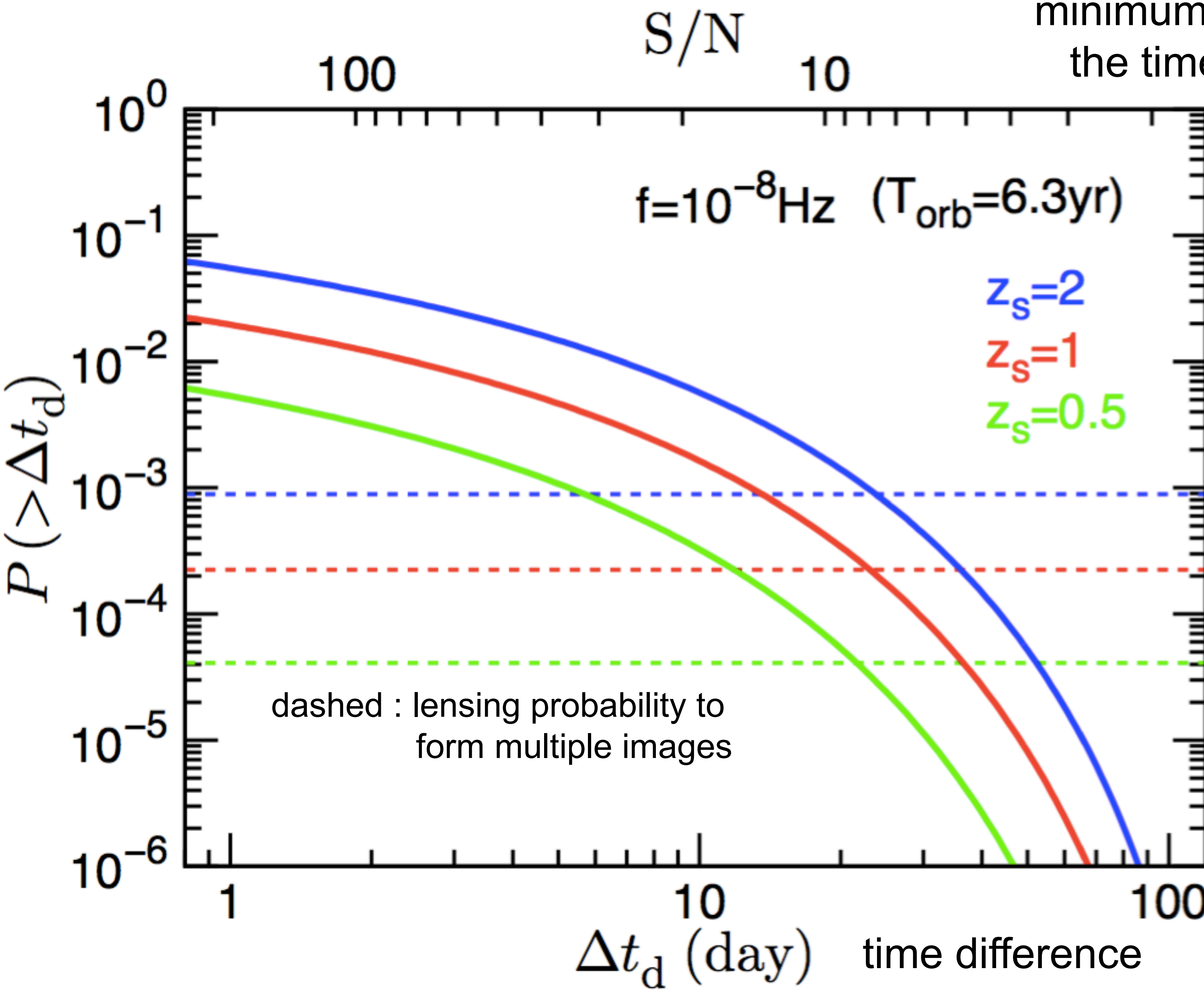
galaxy

typical mass $$M = 8 \times 10^{11} M_\odot \left(\frac{f}{10^{-8}\mathrm{Hz}} \right)^{-1}$$

typical arrival-time lag $$\Delta t_\mathrm{d} \sim 4\,\mathrm{months} \left(\frac{f}{10^{-8}\mathrm{Hz}} \right)^{-1}$$

lensing probability $$\sim 0.1\,\% - 1\,\%$$

Lensing probability



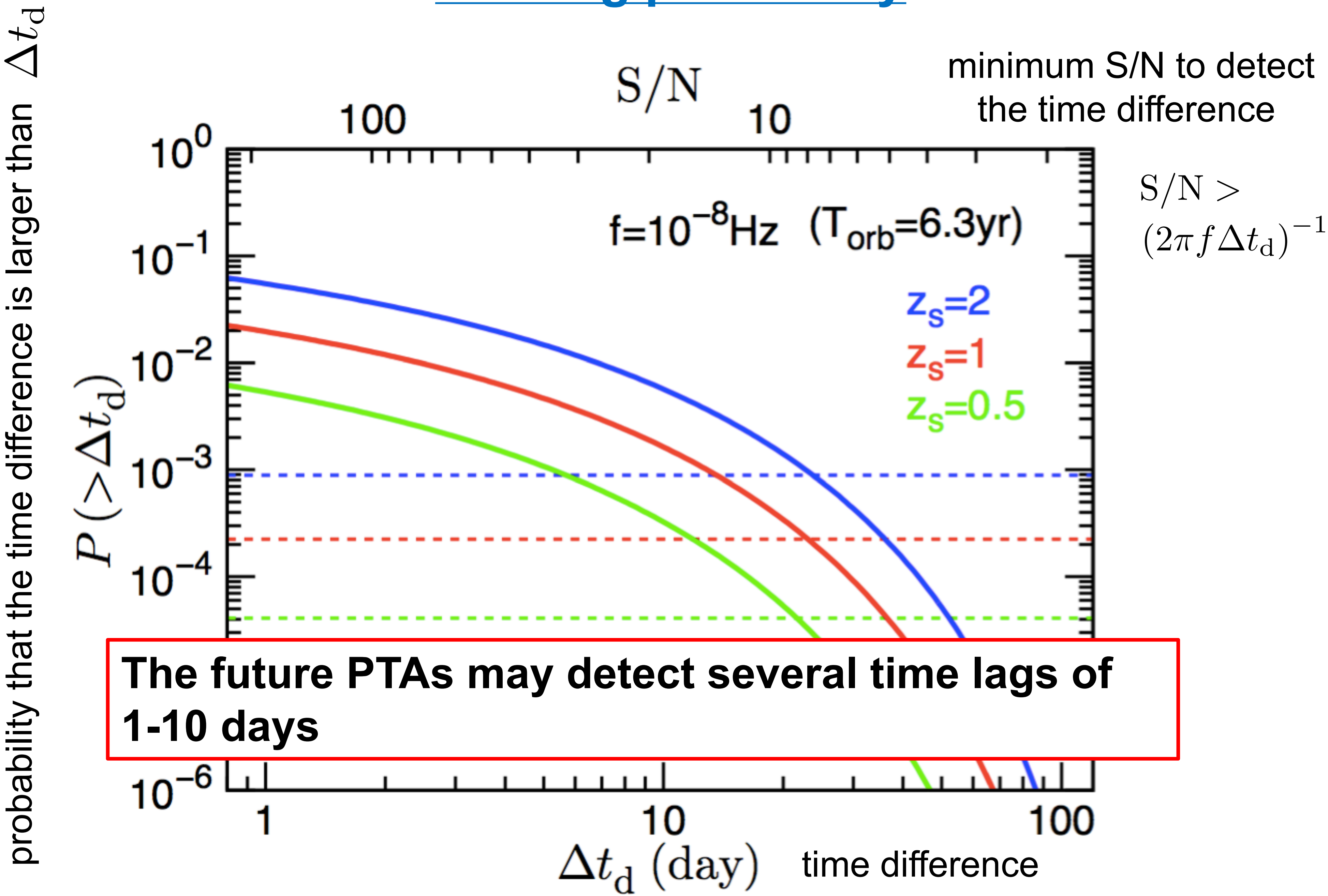
probability that the time difference is larger than Δt_d

minimum S/N to detect the time difference

$\mathrm{S/N} > (2\pi f \Delta t_\mathrm{d})^{-1}$

time difference

Lensing probability



The future PTAs may detect several time lags of 1-10 days

Conclusions

- Comparison of the propagation velocities of lensed GWs (in wave optics) and lensed light (in geometrical optics)

  GWs **faster** than light in **phase & group velocities**
  GWs **equal** to or **slower** than light in **wave-front velocity**

  RT (2017), Morita & Soda (2019), Suyama (2020), Ezquiaga+ (2020)

- The arrival time difference in phase can reach

$$\Delta t_{\rm d} \sim 0.1{\rm s}\,(f/{\rm Hz})^{-1}$$

f : GW frequency

- Future pulsar timing arrays may detect arrival time differences by measuring the orbital phase differences between GW/light signals in SMBH binaries

KOBE-COSMO-19-20

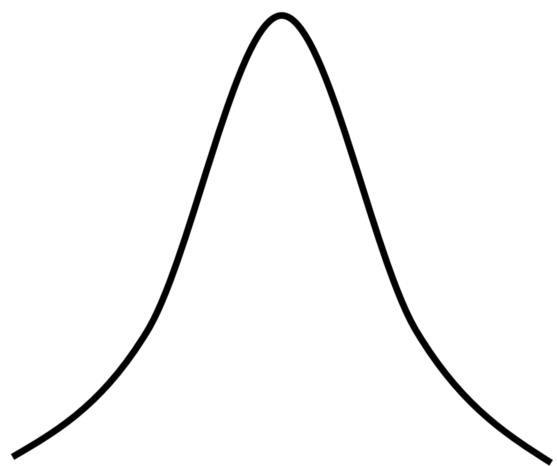
arXiv:1911.07435

Arrival Time Differences of Lensed Massive Gravitational Waves

Takuya Morita* and Jiro Soda†

Department of Physics, Kobe University, Kobe 657-8501, Japan

propagation of a Gaussian wave packet



3. Measurement of the phase difference

In gravitational lensing of light, the following two lenses are common

	distant galaxy	**star in Milky Way**
lensing probability	$\sim 0.1\,\% - 1\,\%$	$\sim 10^{-6}$
lens mass	$\approx 10^{12} M_\odot$	$\sim M_\odot$
arrival-time difference in phase	$\sim \mathrm{months} \left(\frac{M}{10^{12} M_\odot} \right)$	$\sim 10^{-5}\mathrm{s} \left(\frac{M}{M_\odot} \right)$
GW frequency (must in wave-optics regime)	$\lesssim 10^{-7}\mathrm{Hz} \left(\frac{M}{10^{12} M_\odot} \right)^{-1}$	$\lesssim 10^{5}\mathrm{Hz} \left(\frac{M}{M_\odot} \right)^{-1}$

Lensed waveform in high frequency limit

(Schneider, Ehlers, Falco 1992)

Amplification factor (given by the Kirchhoff diffraction integral)

$$F(f;\boldsymbol{\theta}_{\rm s}) = \frac{D_{\rm L} D_{\rm S}}{c D_{\rm LS}} \frac{f}{i} \int d^2\theta \, \exp\left[2\pi i f t_{\rm d}(\boldsymbol{\theta}, \boldsymbol{\theta}_{\rm s})\right]$$

In high frequency limit $f t_{\rm d}(\boldsymbol{\theta}, \boldsymbol{\theta}_{\rm s}) \gg 1$

stationary points of the phase only contribute the integral

(i.e., the Fermat principle)

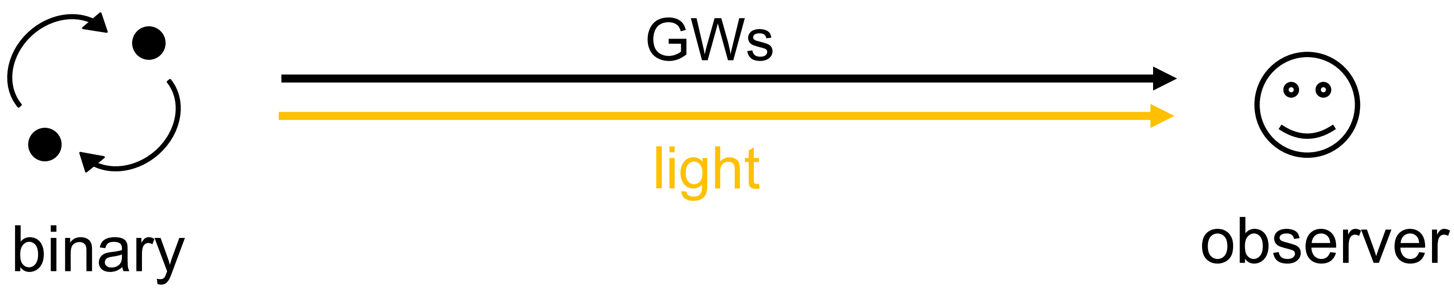
$\nabla_\theta \, t_{\rm d}(\boldsymbol{\theta}, \boldsymbol{\theta}_{\rm s}) = 0$ ⇨ two solutions $\boldsymbol{\theta}_\pm$

corresponding to bright & faint images

Constraint on the propagation speed of GWs

→ a test of general relativity

1）orbital phase difference in binary



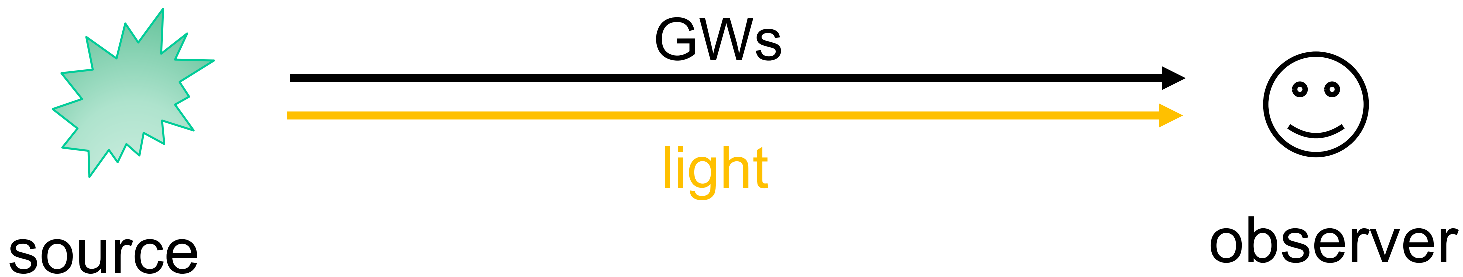
Orbital phase differences between the GW/light signals in a white dwarf binary can be used to constrain the propagation speed of GWs.

（Larson & Hiscock 2000; Cutler+ 2003; Cooray & Seto 2004）

Constraint on the propagation speed of GWs

→ a test of general relativity

2) short gamma-ray burst / supernova



Arrival-time difference between the GW/light signals from SGRB or SN can be used to measure the velocity difference.

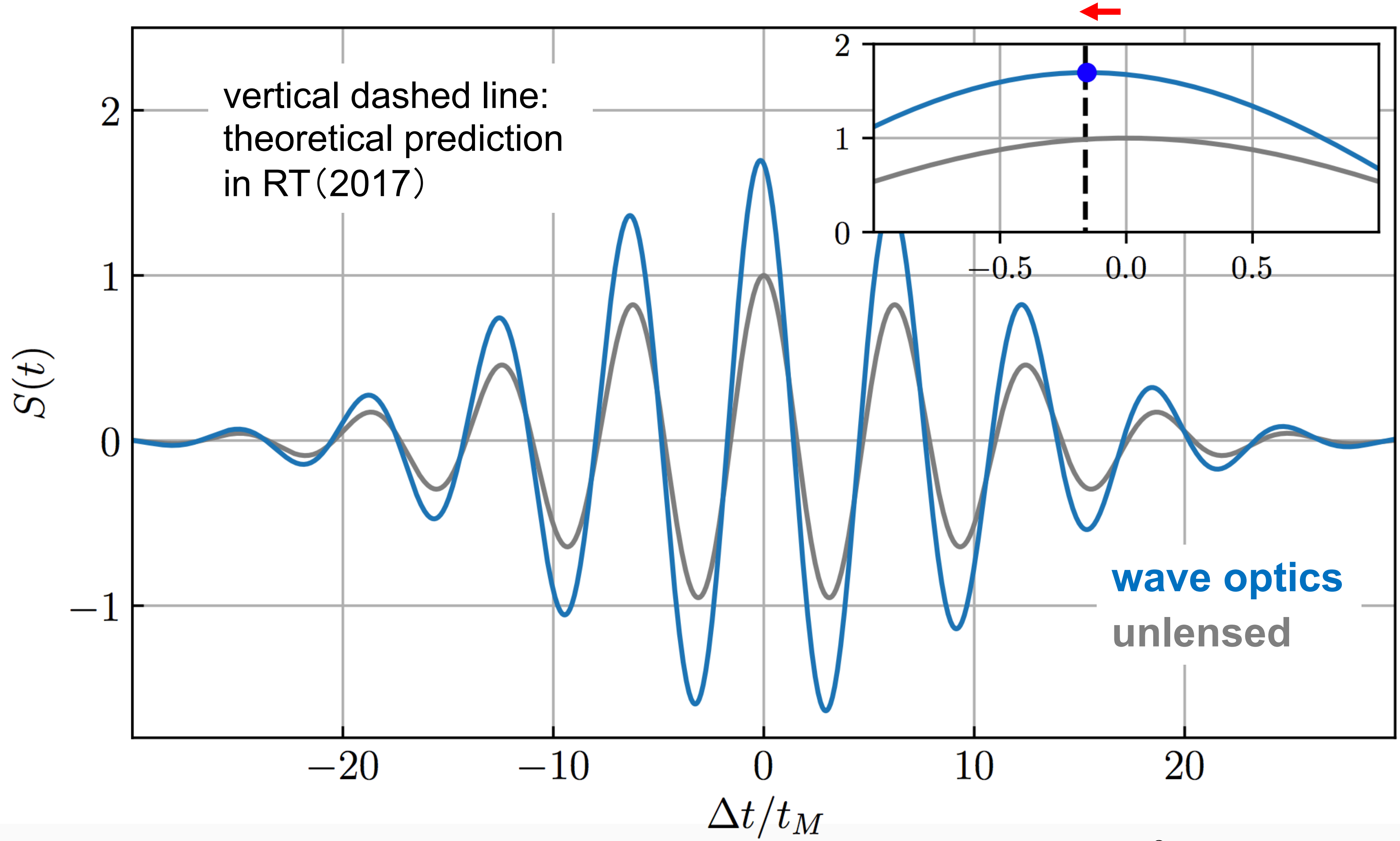
The intrinsic time lag of emissions is required.

(Nishizawa & Nakamura 2014; Nishizawa 2016)

phase velocity

(Ezquiaga+ 2020)

GW peak appears in advance



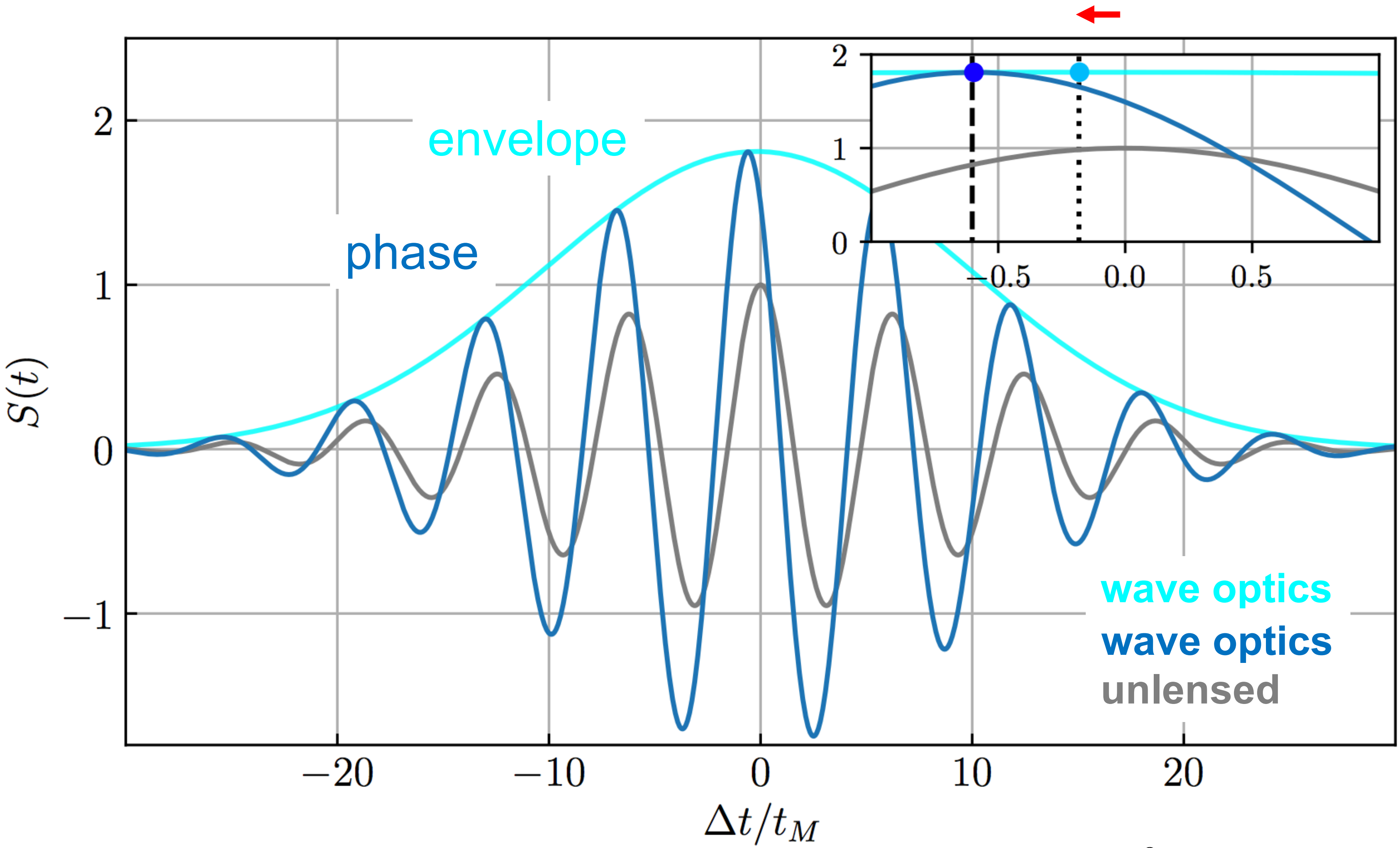
Δt : time relative to the arrival of bright light image

$t_M = 4GM/c^3$

= 2*Schwarzschild radius/c

group velocity

envelope of GWs also appears in advance



Δt : time relative to the arrival of bright light image

$t_M = 4GM/c^3$
= 2*Schwarzschild radius/c

Lensed waveform in time domain

lensed waveform unlensed waveform

$$h^{\mathrm{L}}(t) = \frac{1}{2\pi}\frac{D_{\mathrm{L}}D_{\mathrm{S}}}{D_{\mathrm{LS}}}\frac{d}{dt}\int d^2\theta\, h\left(t - t_{\mathrm{d}}(\boldsymbol{\theta}, \boldsymbol{\theta}_{\mathrm{s}})\right)$$

arrival time difference defined by the group velocity

$$\Delta t_{\rm d}(f) \equiv t_{\rm d,light} - t_{\rm d,GWs}(f)$$

bright light image (independent of f)

GWs

$$t_{\rm d,GWs}(f)|_{\rm group} = \left(1 + f\frac{d}{df}\right) t_{\rm d,GWs}(f)|_{\rm phase}$$

GW time delay in group velocity

GW time delay in phase velocity

arrival time difference defined in phase

$$\Delta t_{\mathrm{d}}(f) \equiv t_{\mathrm{d,light}} - t_{\mathrm{d,GWs}}(f)$$

bright light image (independent of f)

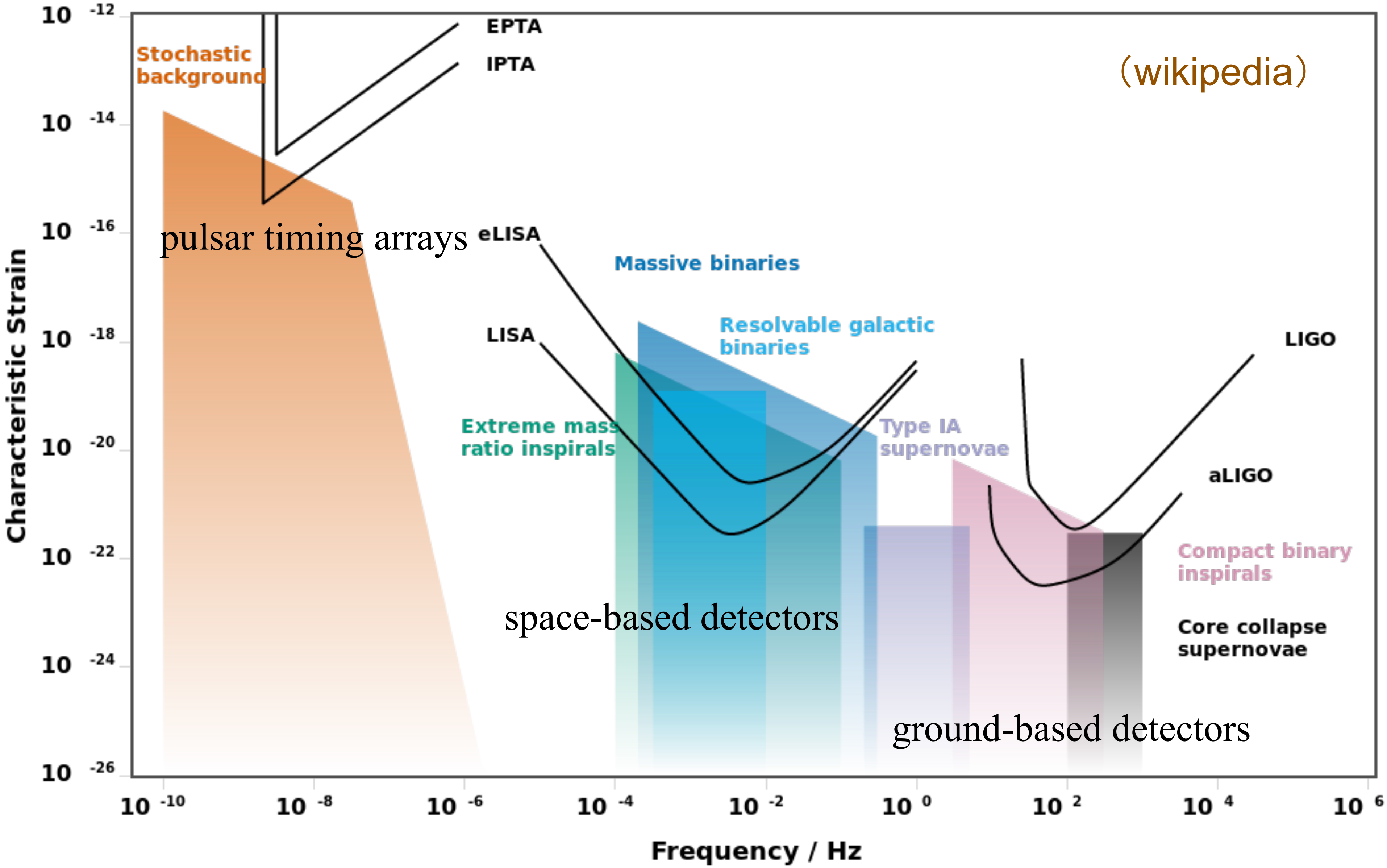
GWs

$$t_{\mathrm{d,GWs}}(f) \equiv -\frac{i}{2\pi f} \ln \left[\frac{F(f;\boldsymbol{\theta}_{\mathrm{s}})}{|F(f;\boldsymbol{\theta}_{\mathrm{s}})|} \right]$$

$\Delta t_{\mathrm{d}} > 0$: GWs faster than light

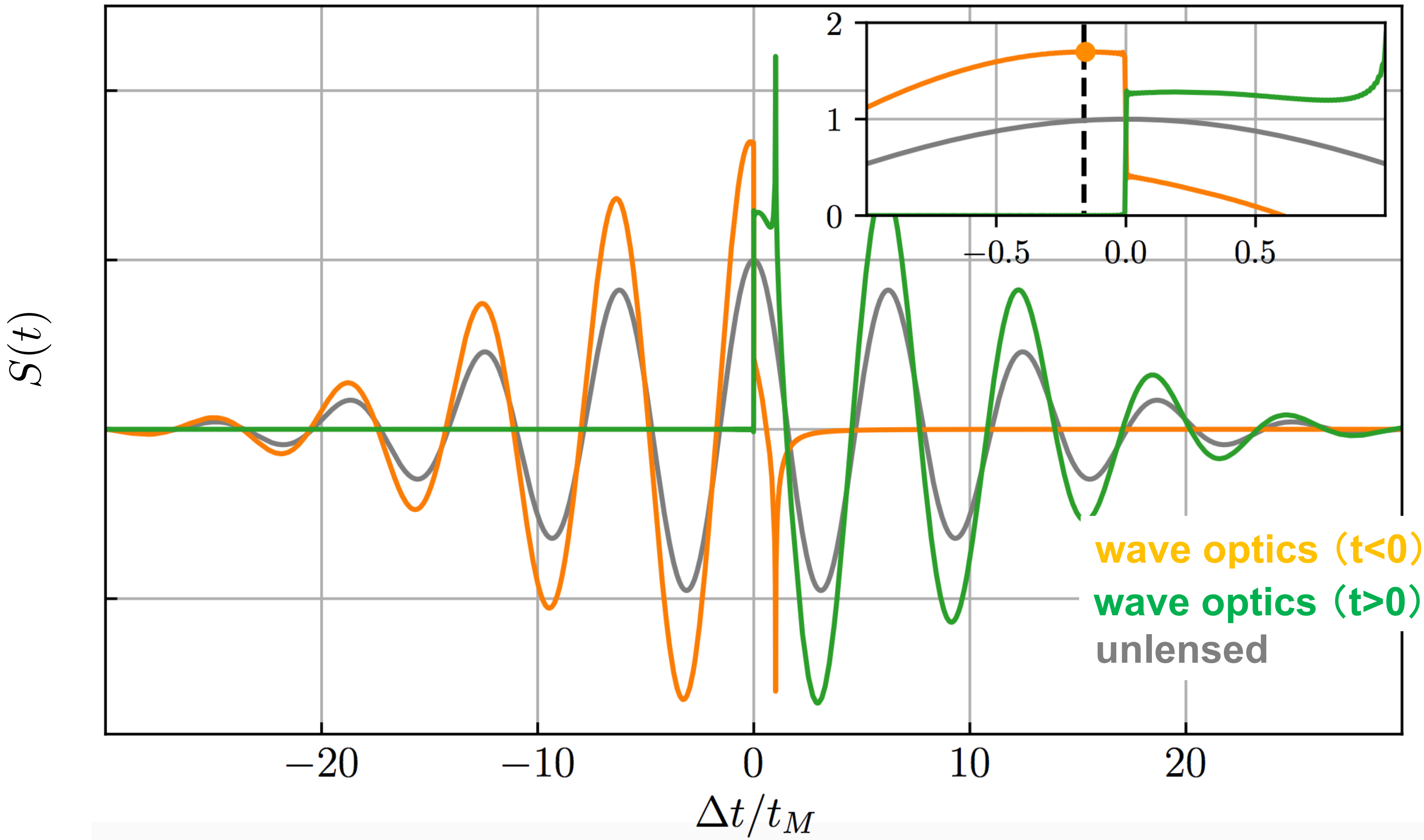
(< 0) (slower)

several GW detectors now in operation or planned, over a wide frequency range from nHz to kHz



GWs splitted into t<0 (orange) and t>0 (green) in wave optics

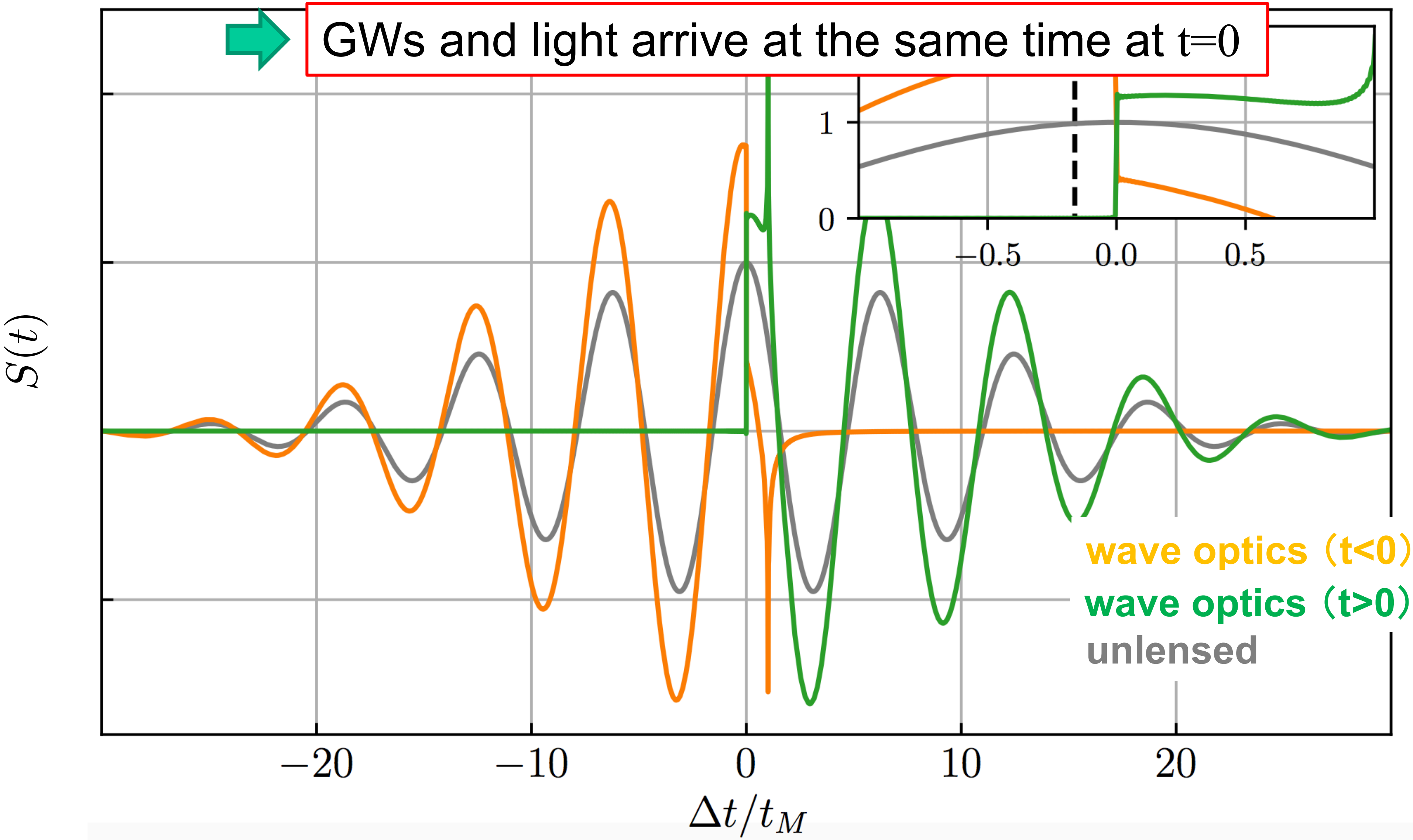
If GWs arrive earlier than light, the green signal arrives at t<0



GWs splitted into t<0 (orange) and t>0 (green) in wave optics

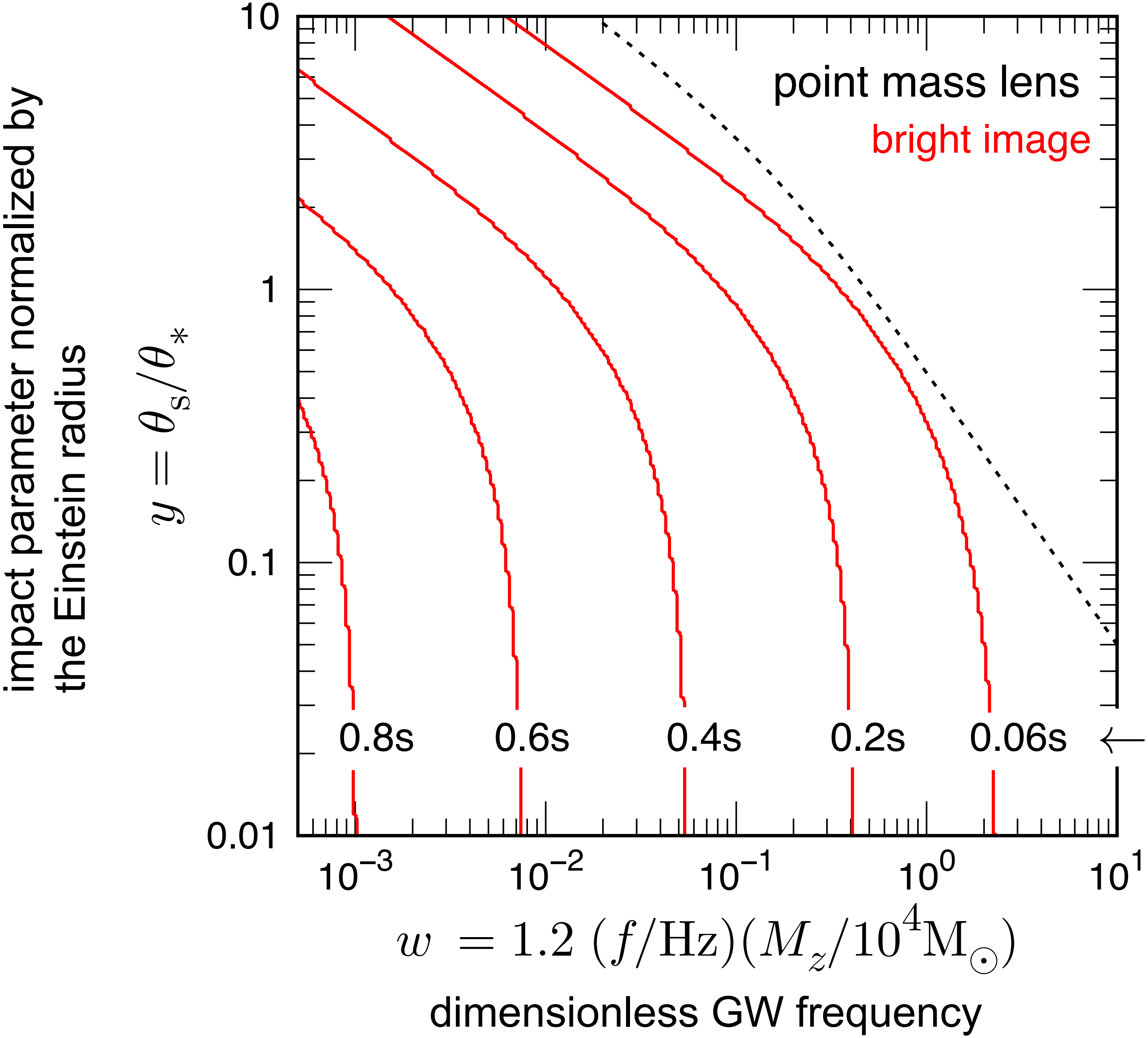
If GWs arrive earlier ~~than light, the green~~ signal arrives at t<0

GWs and light arrive at the same time at t=0



contour plot of the arrival time difference

(RT 2017)



$\leftarrow \Delta t_d$ for $M = 10^4 M_\odot$

$$\Delta t_d \propto \left(\frac{M}{10^4 M_\odot}\right)$$

Degeneracy between binary's and lens parameters
in the lensed waveform

$$\tilde{h}(f) = A f^{-7/6} e^{i\Phi(f)}$$

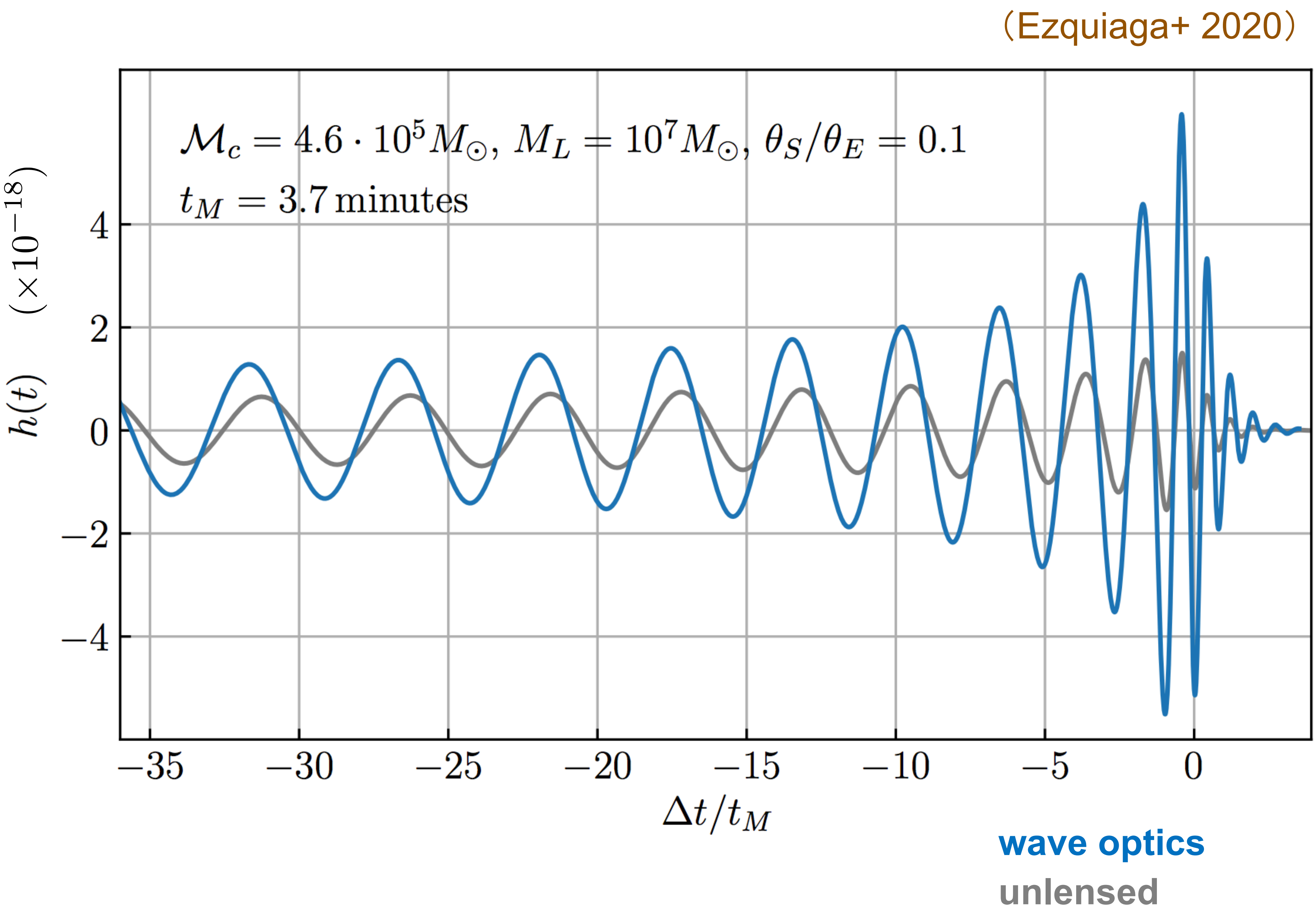
（Liang Dai+ 2018）

lens model parameters（such as mass, impact parameter, density profile）may degenerate with spin-orbit precession or eccentricity of the binary

lensing modulates the amplitude and phase

spin-orbit precession and eccentricity similarly modulate both

while masses, aligned spin, and tidal deformability modulate the phase only


(Ezquiaga+ 2020)
$\mathcal{M}_c = 4.6 \cdot 10^5 M_\odot,\ M_L = 10^7 M_\odot,\ \theta_S/\theta_E = 0.1$
$t_M = 3.7\,\mathrm{minutes}$
$h(t) \quad (\times 10^{-18})$
4
2
0
−2
−4
−35
−30
−25
−20
−15
−10
−5
0
$\Delta t / t_M$
wave optics
unlensed

2005.10485

Unveiling the wave nature of gravitational-waves with simulations

Jian-hua He*

School of Astronomy and Space Science, Nanjing University, Nanjing 210093, P. R. China and
Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, Nanjing 210093, China

The author solved the partial derivative equation
for the lensed wave propagation in time domain
(instead of the Kirchhoff diffraction integral)

⇨ GWs faster than light

Lensing effects on the polarization of GWs

In geometrical optics,

the polarization is parallelly transported along the null geodesic

In quasi-geometrical optics, （Harte 2019; Cusin & Lagos 2020）

the propagation equation was solved in a series of 1/λ

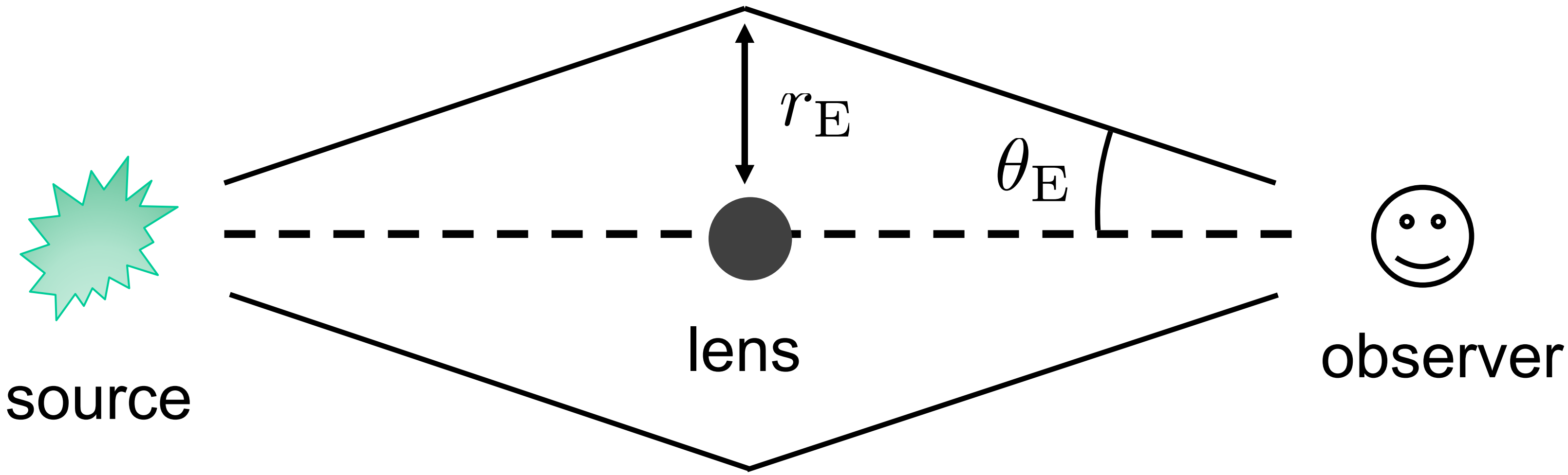
$$h^{\mathrm{L}}_{\mu\nu}(f) = h^{\mathrm{L,geo}}_{\mu\nu}(f) + h^{\mathrm{L,(1)}}_{\mu\nu}(f) + O(1/\lambda^2)$$

geometrical optics

leading correction $O(1/\lambda)$

In wave optics, ???

source, lens, and observer placed on straight line

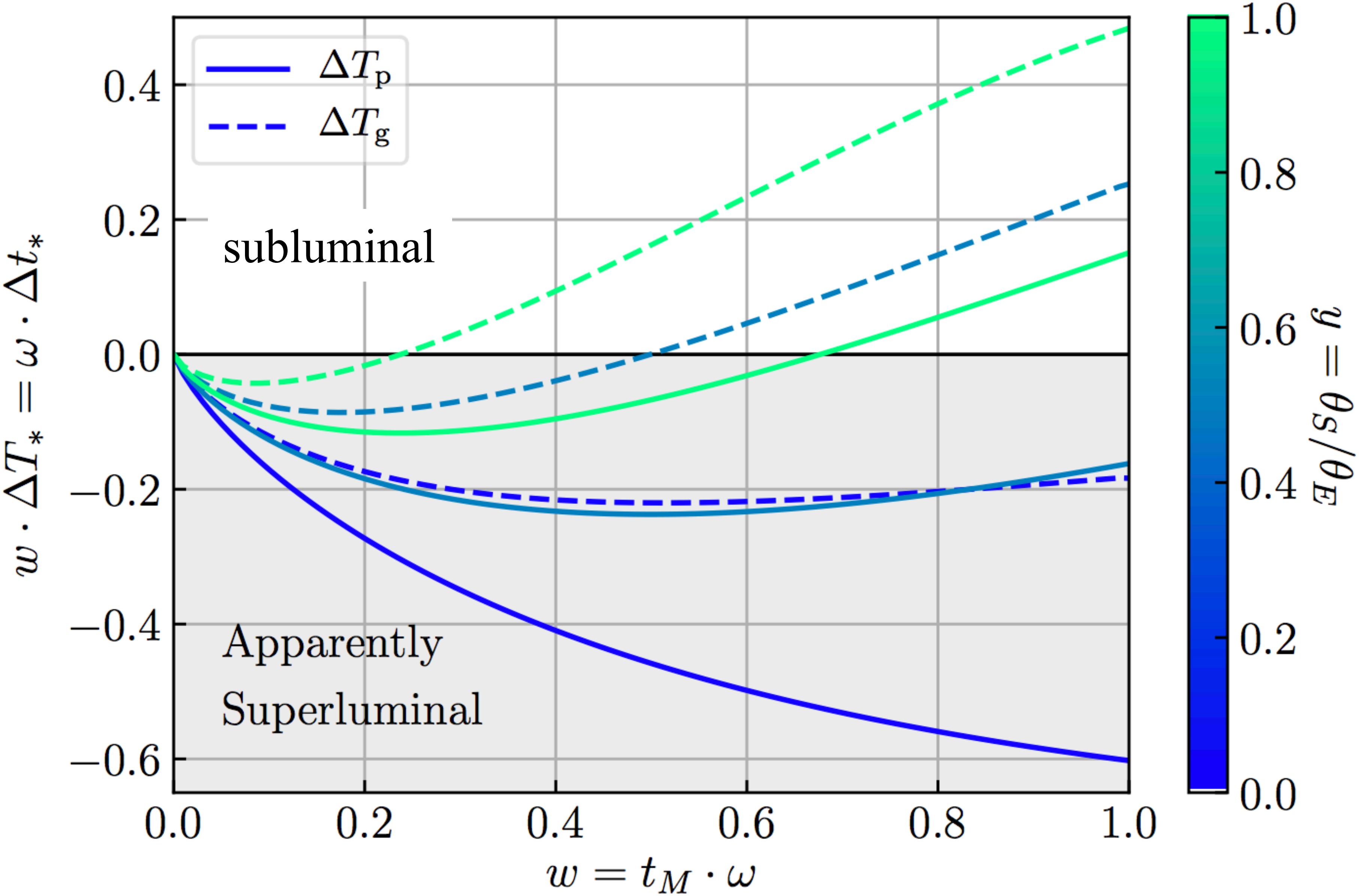


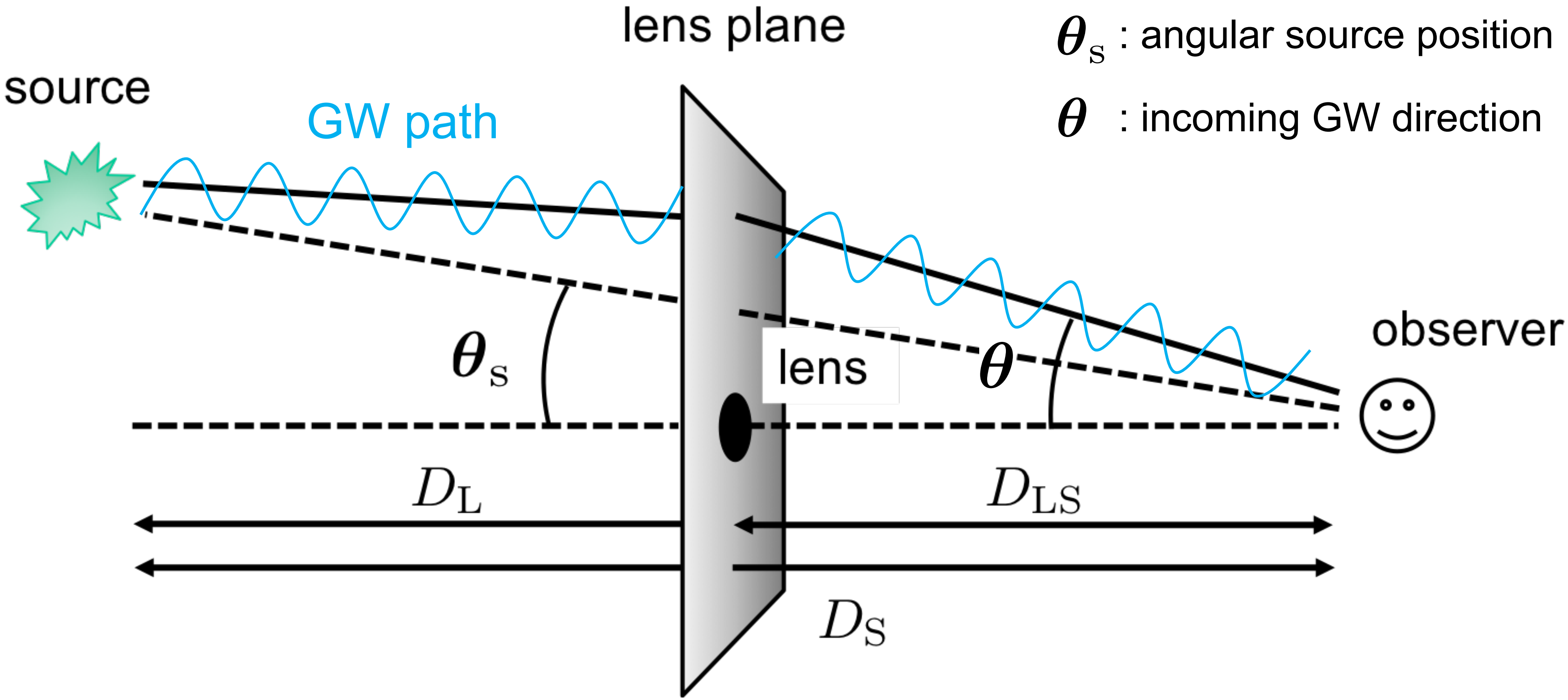
r_{E} : Einstein radius

θ_{E} : angular Einstein radius solid : Δtd in phase velocity

dashed: Δtd in group velocity

(Ezquiaga+ 2020)





D_L, D_S, D_{LS} : distances between the source, lens and observer

gravitational potential of the lens confined on the lens plane

(i.e., the thin lens approximation)

Source properties

(Abbott+ PRL 2017)

	Low-spin priors ($\|\chi\| \leq 0.05$)	High-spin priors ($\|\chi\| \leq 0.89$)
Primary mass m_1	1.36–$1.60\,M_\odot$	1.36–$2.26\,M_\odot$
Secondary mass m_2	1.17–$1.36\,M_\odot$	0.86–$1.36\,M_\odot$
Chirp mass $\mathscr{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55^\circ$	$\leq 56^\circ$
Using NGC 4993 location	$\leq 28^\circ$	$\leq 28^\circ$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400