Phase Shifts of Lensed Gravitational Waves

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Time-Domain Cosmology with Strong Lensing
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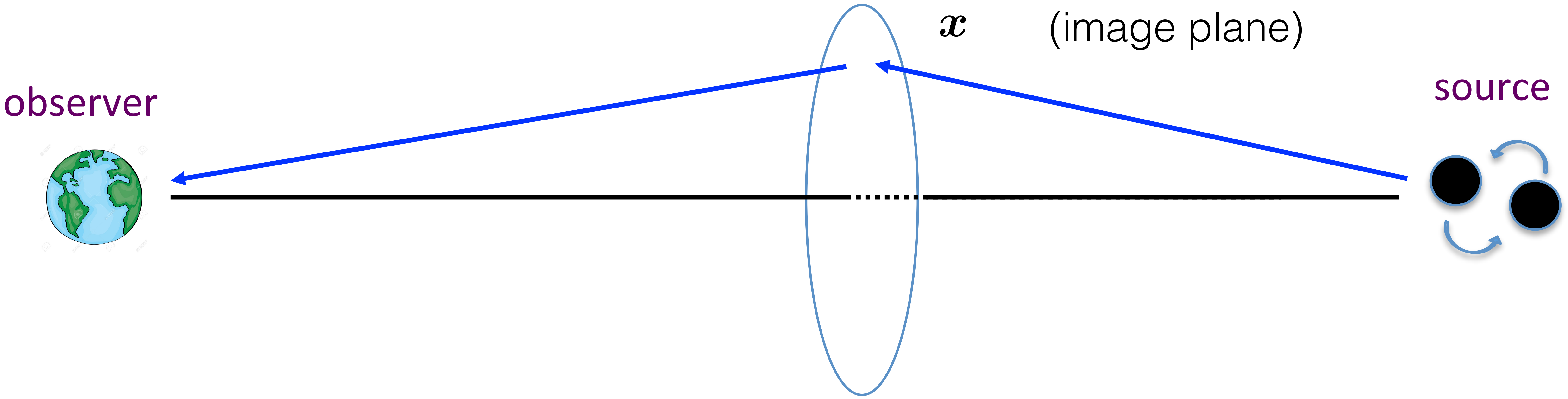
Lensing of Gravitational Waves



Wave amplification by lensing $$h_{\rm len}(\omega) = F(\omega)\, h(\omega)$$

Diffraction integral $$F(\omega) = \frac{\omega}{2\pi\, i} \int \mathrm{d}^2\boldsymbol{x}\, e^{i\,\omega\,\tau(\boldsymbol{x})}$$

e.g. Schneider, Ehlers & Falco (1992)

Fermat potential (travel time)

e.g. Nakamura (1998); Takahashi & Nakamura (2003)

Diffraction regime: delay time ~ $\frac{G\,M_L}{c^3} \lesssim \frac{2\pi}{\omega}$ (see Takahashi's and Mishra's talk)

Geometric (or ray-optics) regime: delay time ~ $\frac{G\,M_L}{c^3} \gg \frac{2\pi}{\omega}$

Geometrical (ray-optics) regime

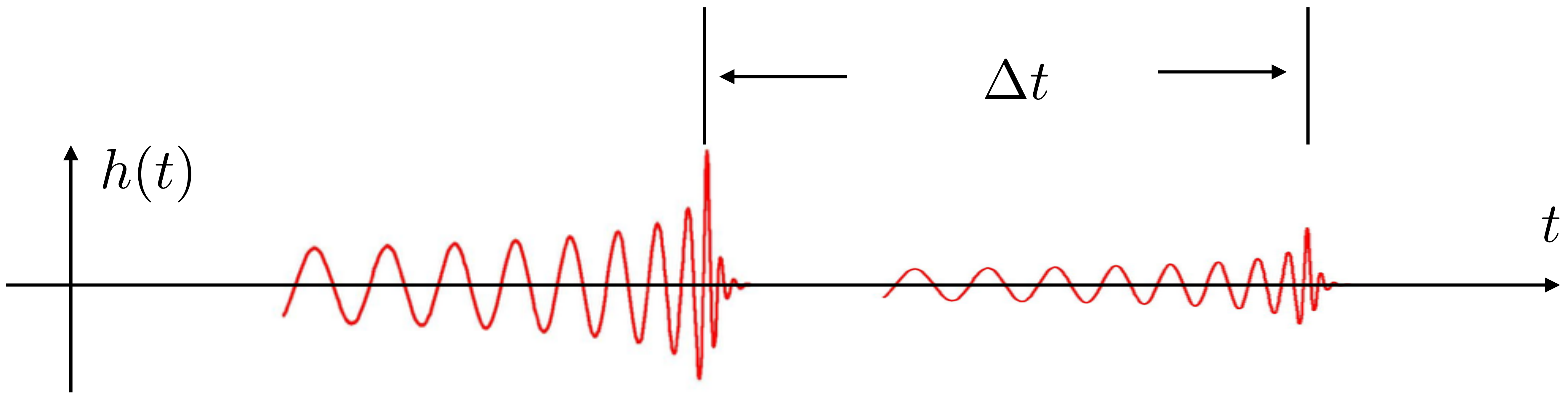
Cosmological GW sources most likely lensed by **galaxy/cluster scale** lenses: geometric theory of lensing

flux magnification

$$F(\omega) \approx \sum_I |\mu(\boldsymbol{x}_I)|^{1/2} \, e^{i\,\omega\,\tau(\boldsymbol{x}_I)} \, e^{i\,\mathrm{sgn}(\omega)\,n_I\,\pi/2}$$

time delay   Morse phase

frequency independent amplification and time delay



Biased mass and redshift inferred

$$M\,(1+z) = M'\,(1+z')$$

$$\frac{\sqrt{\mu}}{d_L\,(z)} = \frac{1}{d_L\,(z')}$$

e.g. Dai, Venumadhav & Sigurdson (2017); Oguri (2018); Broadhurst, Diego & Smoot 1802.05273

Difficult to tell if any single GW event is lensed !

Morse phase shift

Morse phase

$$F(\omega) \approx \sum_I |\mu(\boldsymbol{x}_I)|^{1/2} e^{i\,\omega\,\tau(\boldsymbol{x}_I)} \boxed{e^{i\,\mathrm{sgn}(\omega)\,n_I\,\pi/2}}$$

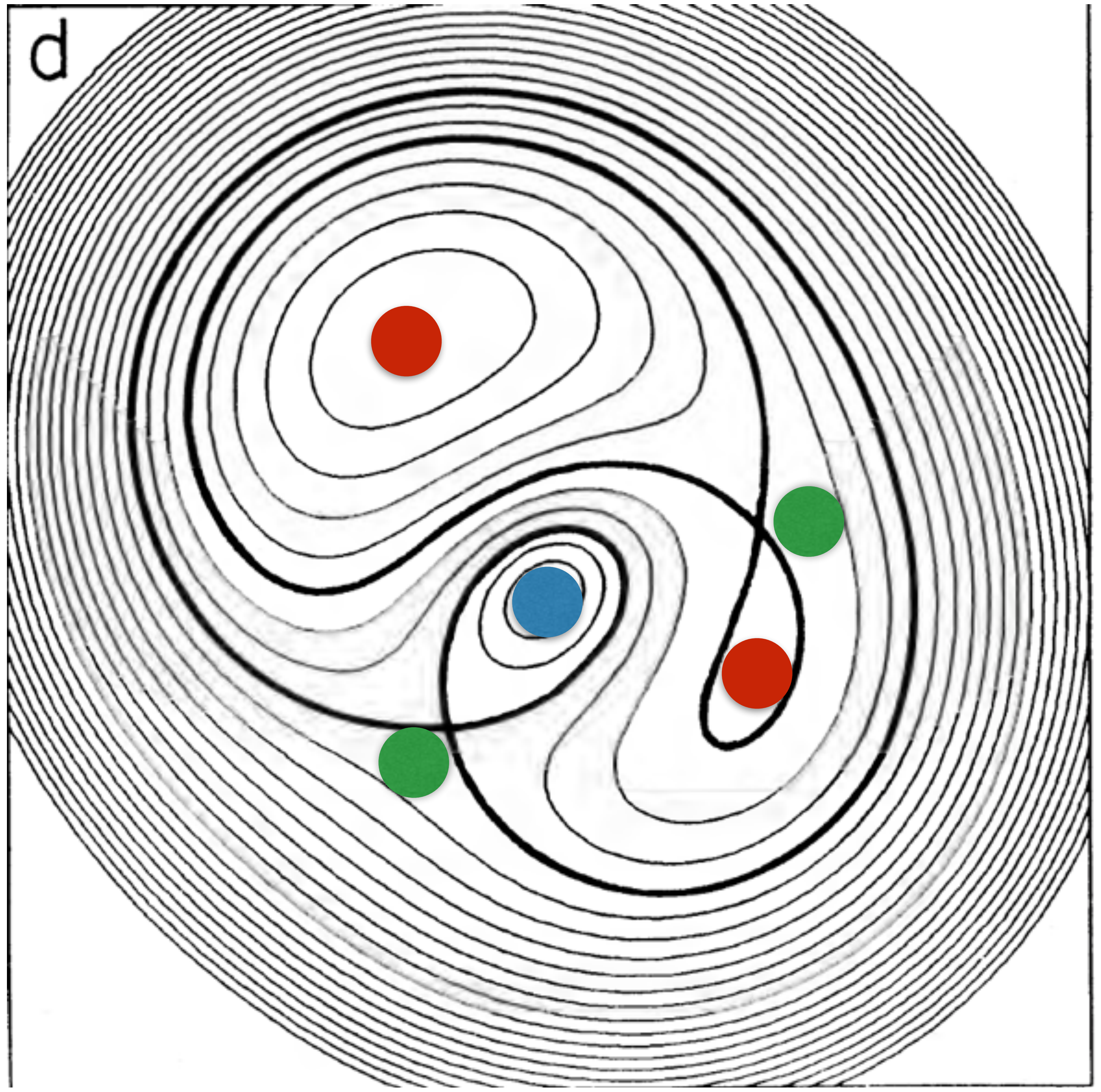
$$n_I = \#\text{ of negative eigenvals} = \begin{cases} 0 & \textbf{minima} \\ 1 & \textbf{saddle points} \\ 2 & \textbf{maxima} \end{cases}$$

Binary black holes on **circular orbit** with dominant **(2,2)** radiation mode: degenerate with **orbital phase**

Dai & Venumadhav 1702.04724; Ezquiaga et al 2008.12814

$$\Delta\varphi = \frac{\pi}{4} \times \begin{cases} 0 & \textbf{minima} \\ 1 & \textbf{saddle points} \\ 2 & \textbf{maxima} \end{cases}$$

minima
saddle points
maxima



Blandford & Narayan (1986)

What to learn from lensed BBHs?

- Probe BBH sources beyond the usual detection horizon (in particular in the era of 2G detectors)

  Buscicchio et al (2020); Mukherjee et al (2021)

- Opportunity to associate a BBH source with a host galaxy, or even localize it within the host galaxy

  Hannuksela, Collett, Caliskan & Li (2020);
  Yu, Zhang & Wang (2020)

- Probe profile of galaxy or cluster lenses complementary to lensed EM sources: central images; faint images;

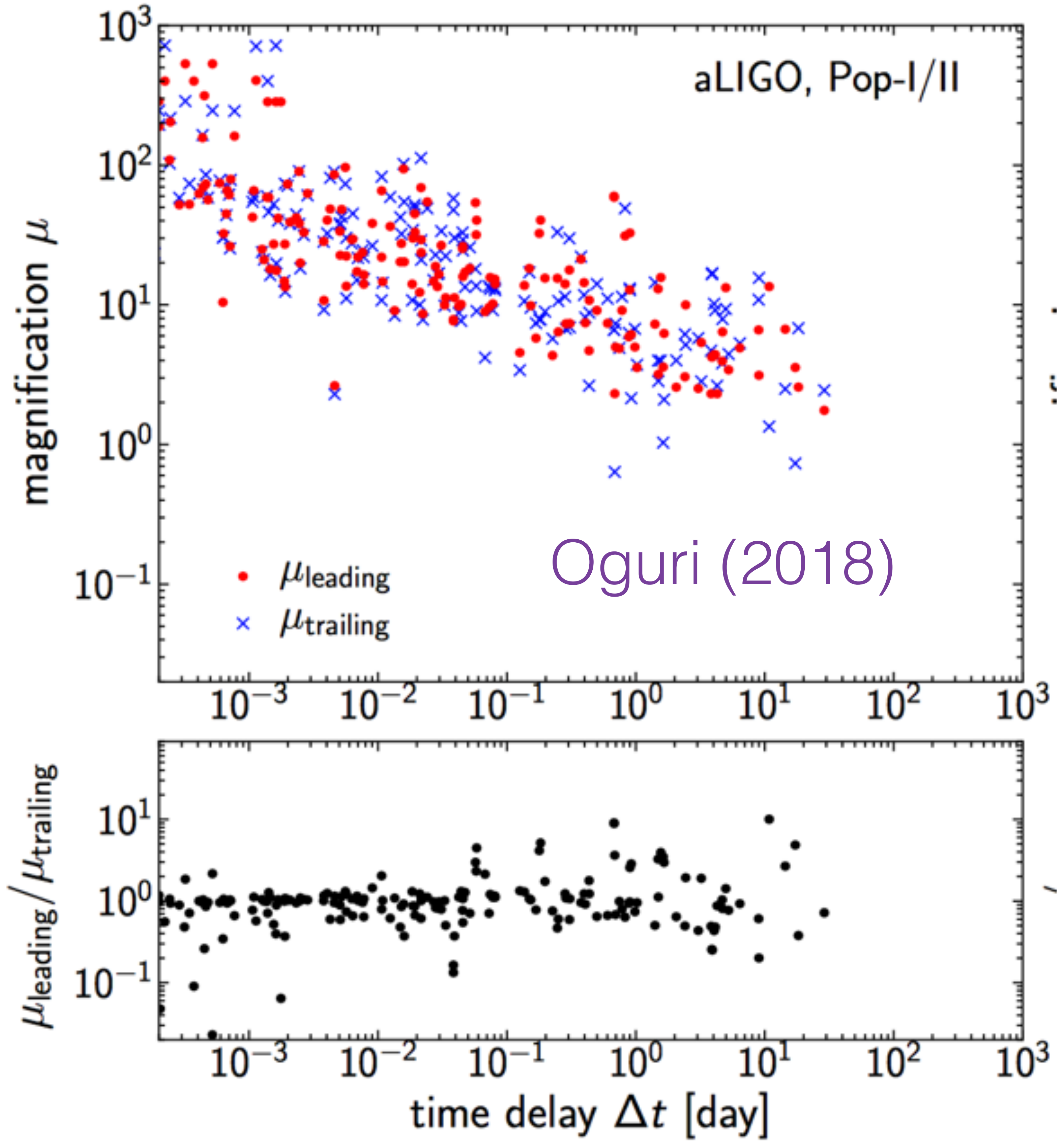
- Probe lens substructure through wave diffraction effects; e.g. stellar mass objects; small-scale clustering of the Dark Matter.

  Dai, Li, Zackay, Mao & Lu (2018)
  Diego, Hannuksela, Kelly, Pagano, Broadhurst, Kim, Li & Smoot (2019)
  Oguri & Takahashi (2020)

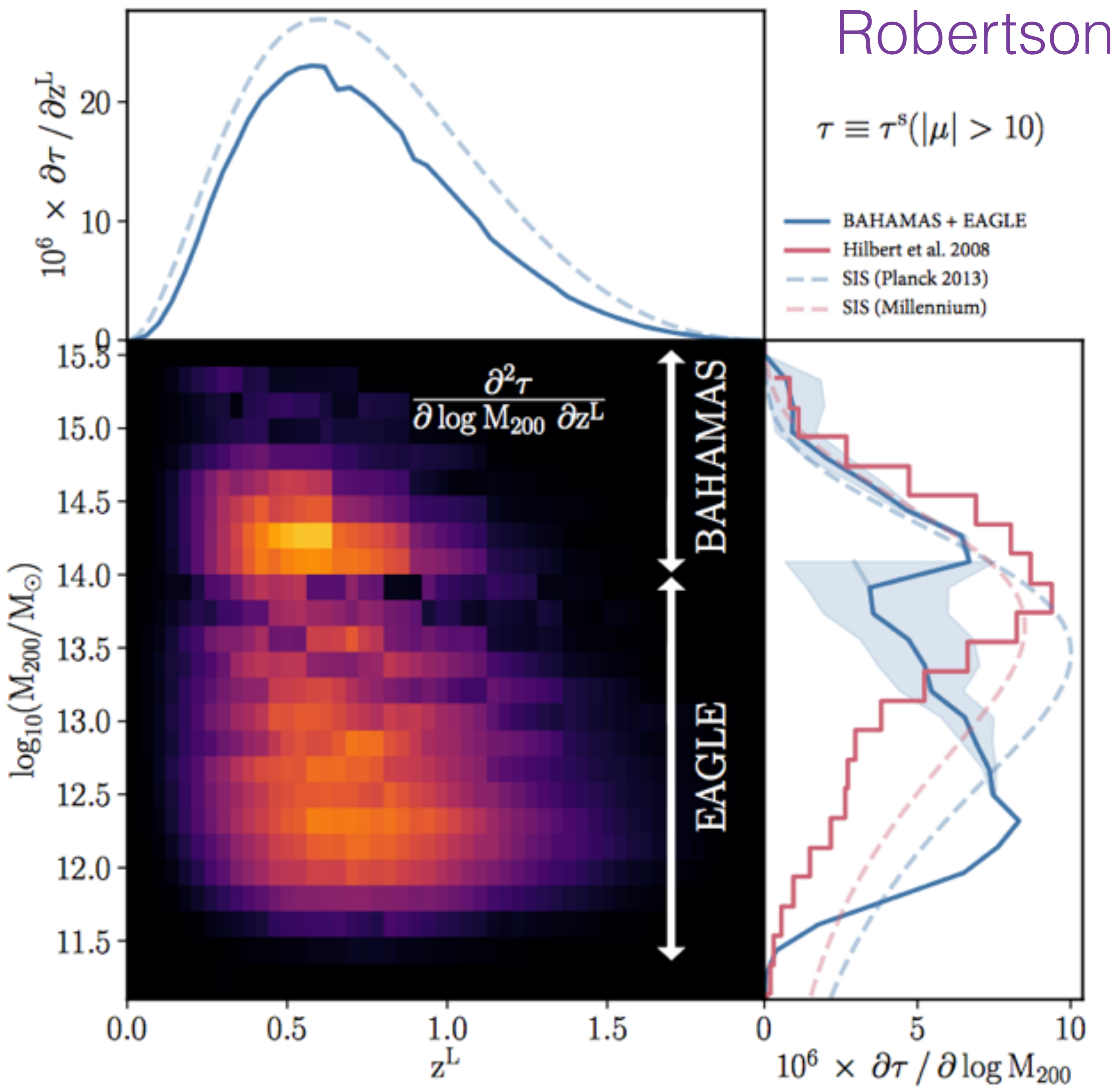
Rate of lensing by galaxies or clusters

e.g. Dai, Venumadhav & Sigurdson (2017); Ng, Wong, Broadhurst & Li (2018); Li, Mao, Zhao & Lu (2018); Oguri (2018); Hannuksela++ (2019); Contigiani (2020);

Models of lenses and BBH population;
~ 0.01 yr^{-1} at O1/O2; **~ 0.1 —1 yr^{-1}** at fully upgraded LIGO/Virgo; **~ O(100-1000) yr^{-1}** at 3rd gen. detectors



Robertson++ (2020)

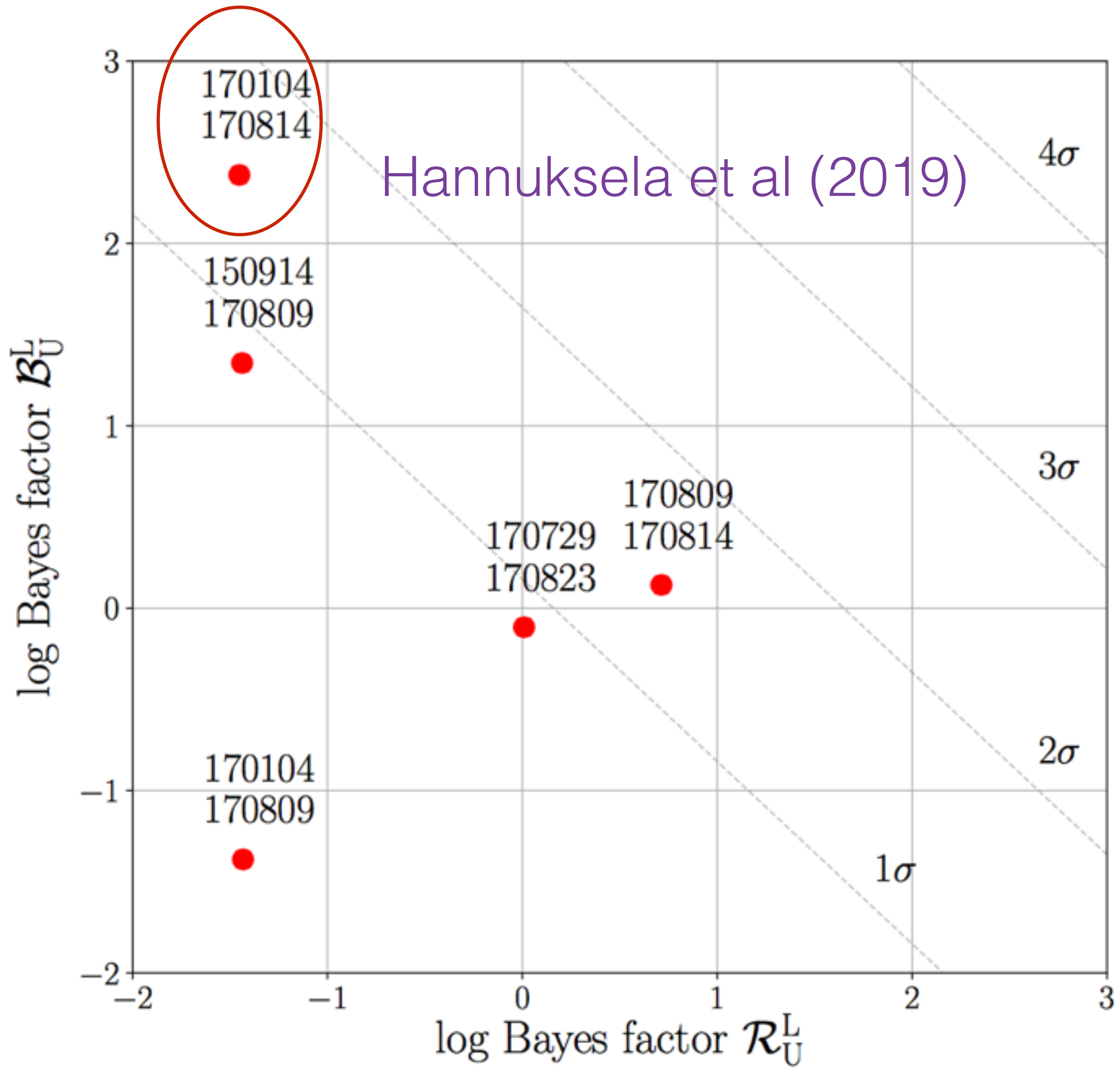


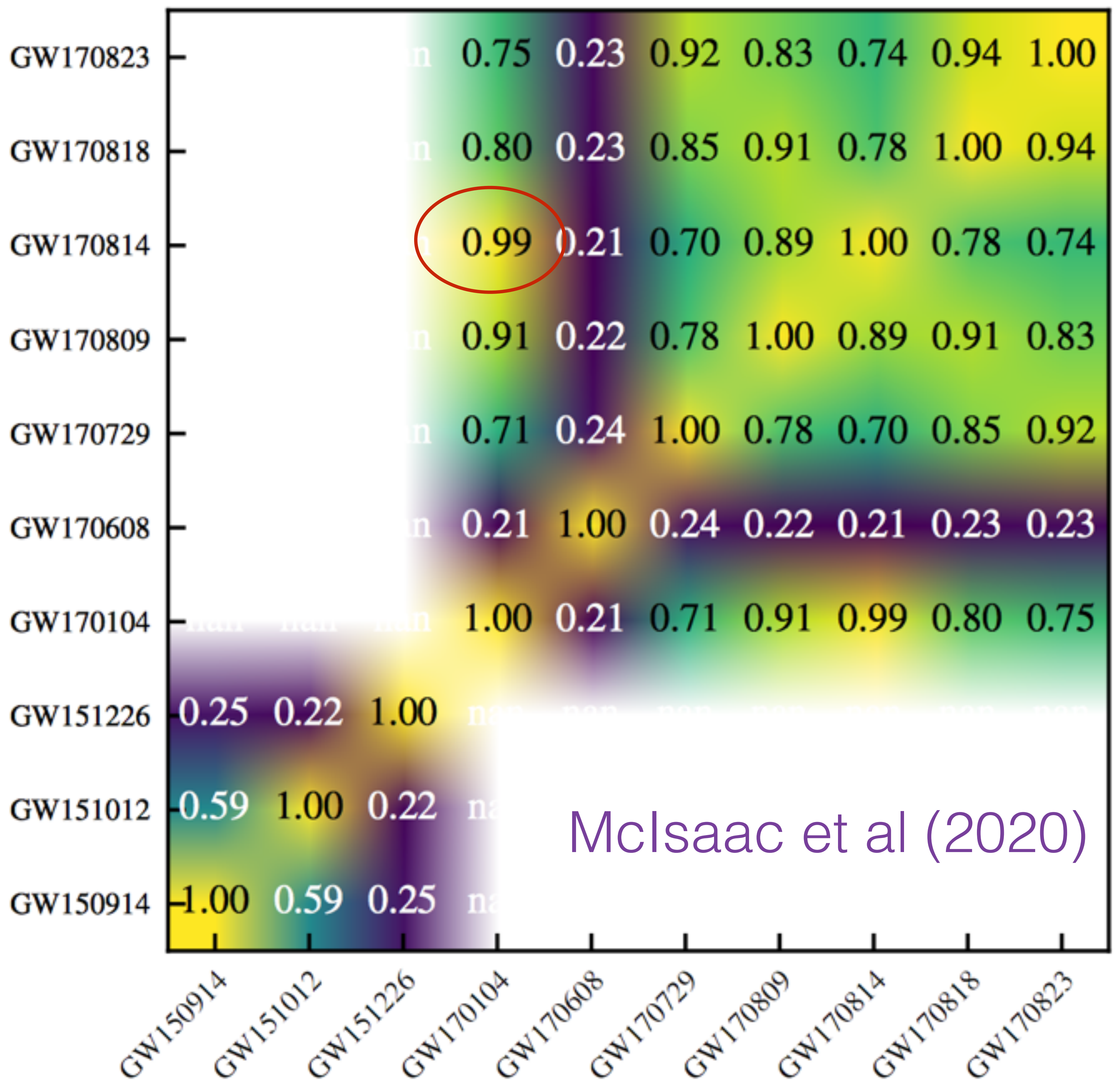
including cluster-scale lenses

for source at z = 2

Search for multiple images in O1/O2

Parameter coincidence quantified by Bayes evidence ratios (excluding time, amplitude, and orbital phase) Haris et al (2018)

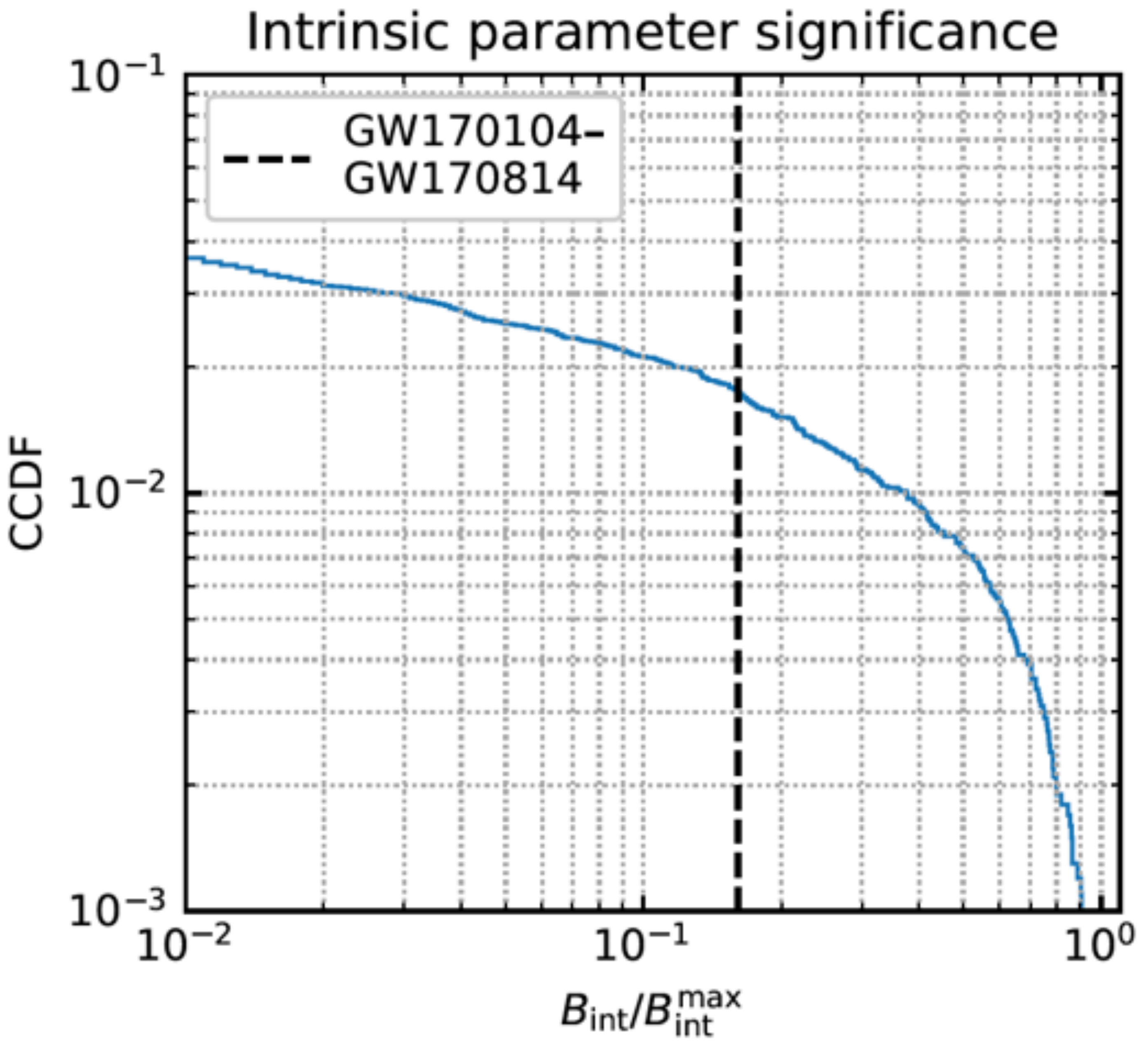




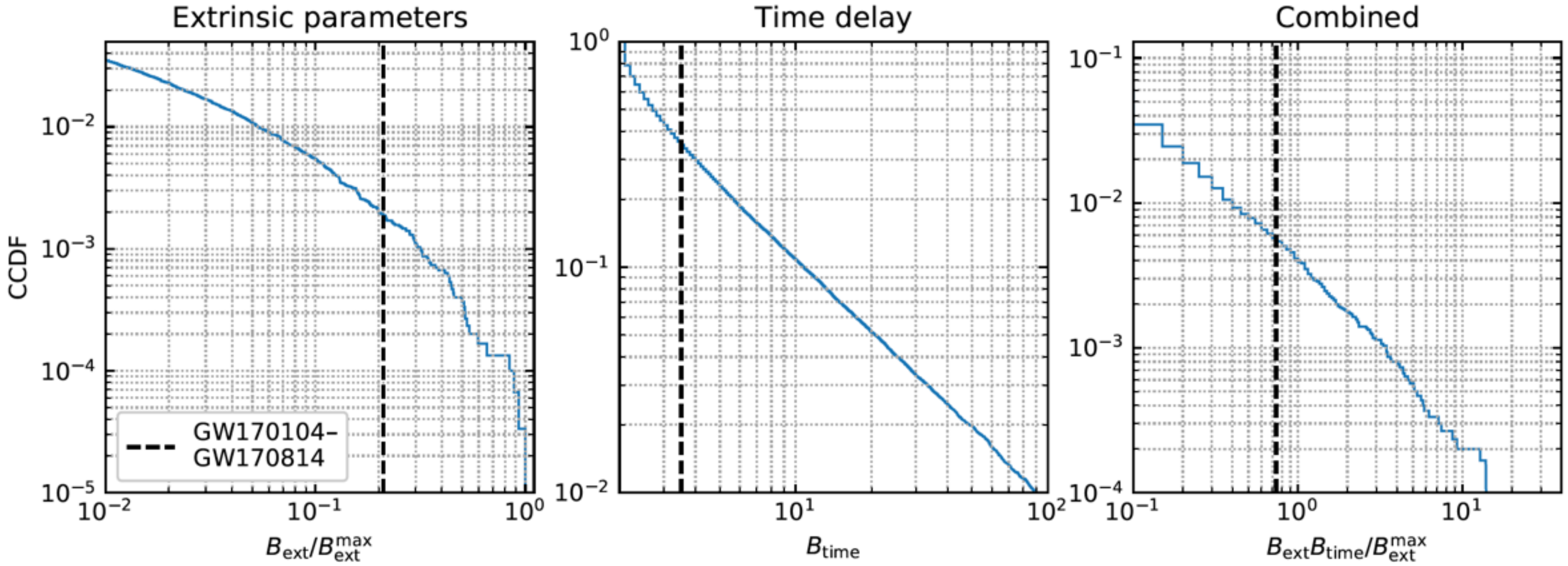
However, Morse phase shifts not accounted for

Independent investigation into GW170104 / GW170814

Dai, Zackay, Venumadhav, Roulet, Zaldarriaga 2007.12709



Coincidence in mass and spin (intrinsic) parameters
p-value ~ 0.02



Coincidence in extrinsic parameters (sky location, source orbital orientation, etc.)
p-value ~ 0.002

Look-elsewhere effect:
pick the “best” event pair
No greater than a factor of **~ 15**

Single-template search for sub-threshold signals

If lensing if true, there are possibly additional lensed images.

Use the **best-fit waveform** informed from GW170814 and GW170104 for a single-template search. Use extrinsic parameters informed from GW170814 and GW170104 (especially RA, DEC); this is implemented as a coherent score:

$$S \propto \sum_{s\in\Pi_{\rm pair}} \sum_{\phi_M, t_0} \int {\rm d}D_L \, \mathcal{L}\,(s, \phi_M, t_0, D_L) \; P(D_L).$$

Overall **~ 10^4** reduction in look-elsewhere effect and **~ 20%** more sensitive in strain amplitude.

Somewhat surprisingly, we found one sub-threshold signal, GWC170620, with a false alarm probability **1.3%** determined empirically from time slides

Relative Morse phases

Morse phase manifests itself as a change in the orbital phase $\Delta\varphi = \frac{1}{2}\Delta\phi_M$

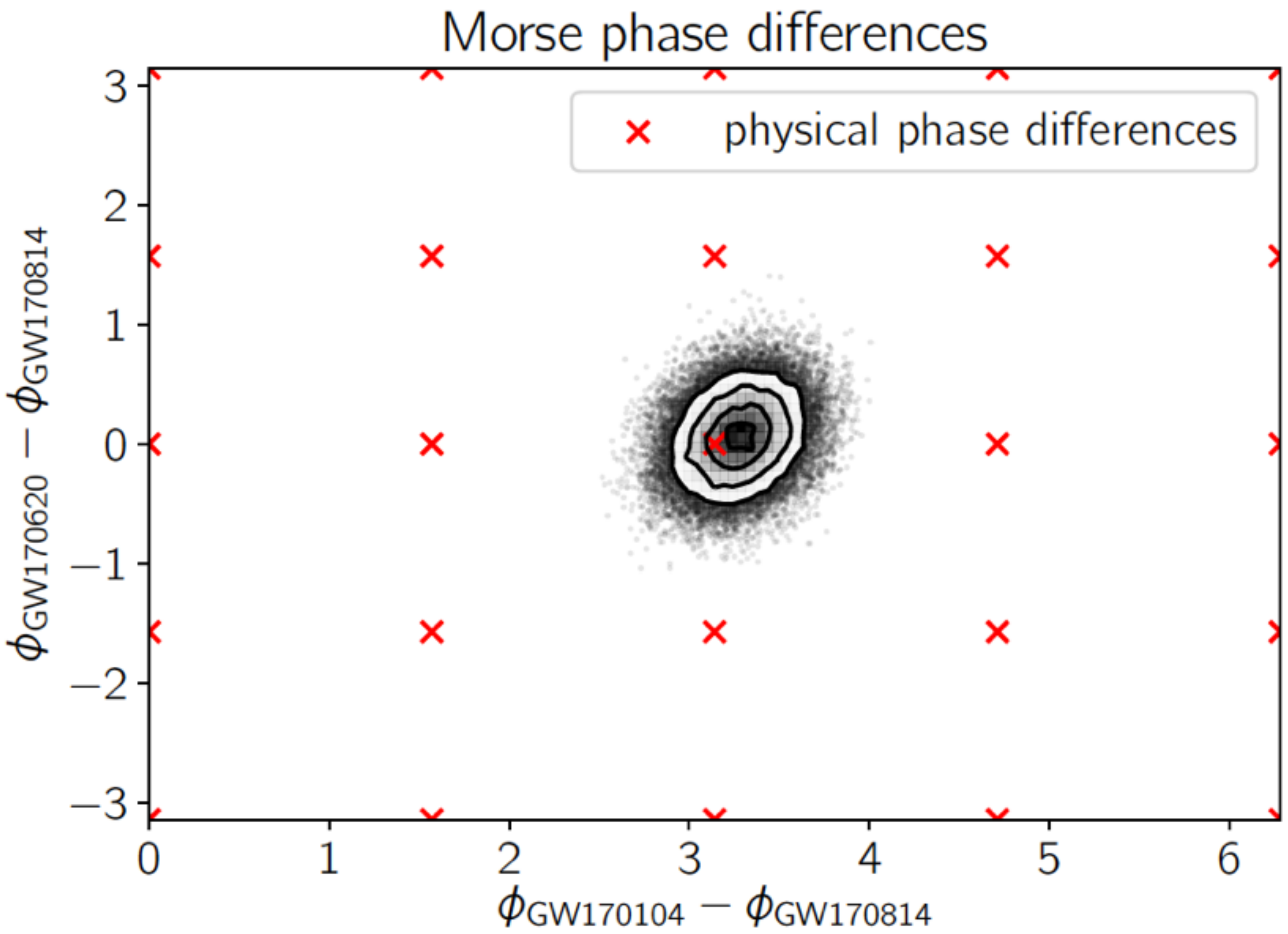
$$h = F_+(\alpha, \delta, \psi)\, h_+ + F_\times(\alpha, \delta, \psi)\, h_\times$$

$$h_+ \propto \frac{1}{2}\,(1 + \cos^2 \iota)\, e^{2\,i\,\varphi}$$

$$h_\times \propto -i\,\cos\iota\, e^{2\,i\,\varphi}$$

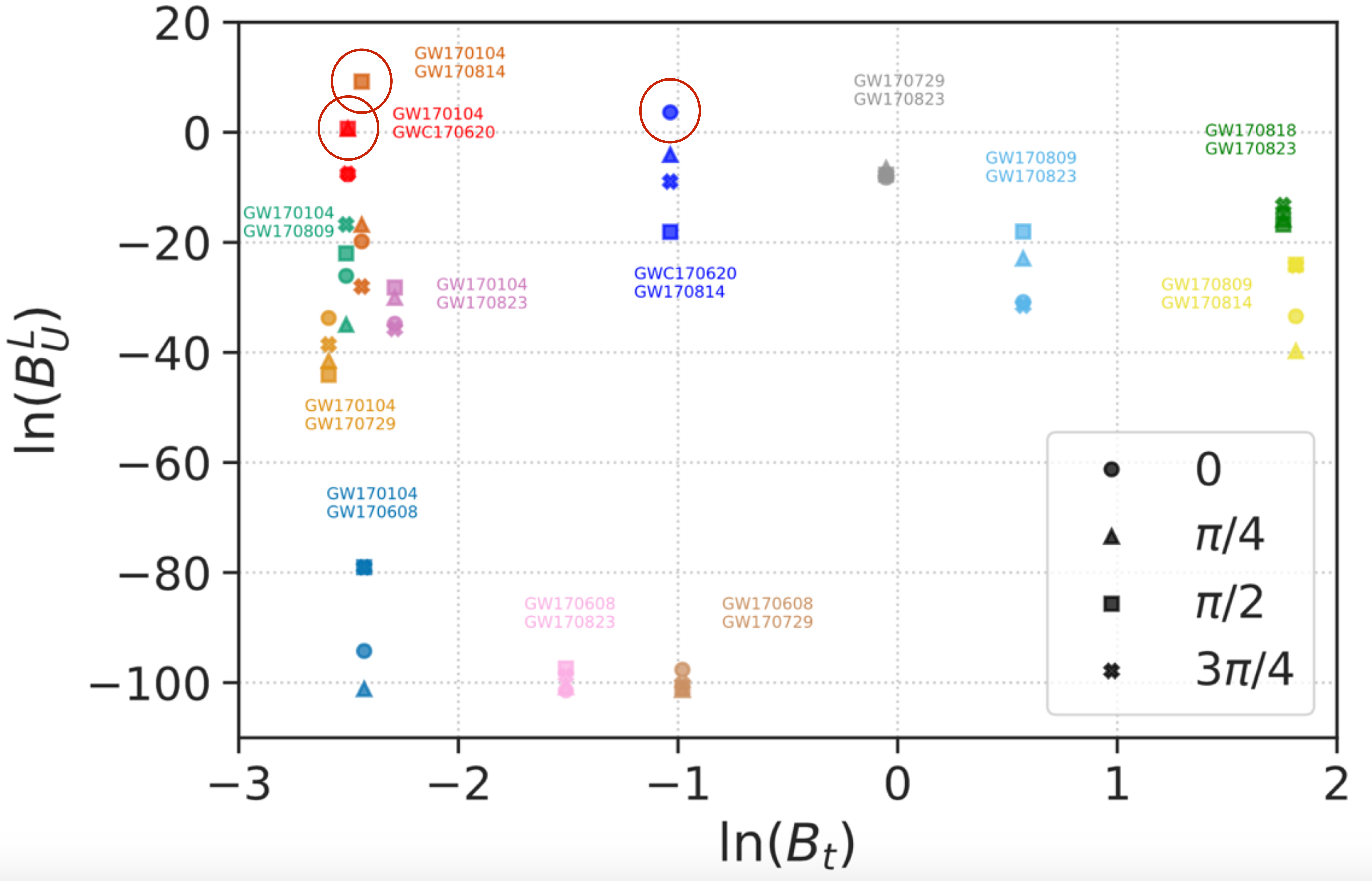
Orbital phase for any *individual* event poorly constrained due to degeneracy with other extrinsic parameters.

Under lensing hypothesis, multiple events share the same extrinsic parameters, hence relative Morse phases can manifest as **apparent orbital phases differing by discrete amounts**.



Checks on GW170814, GW170104 and GWC170620

Liu, Hernandez & Creighton 2009.06539



Joint parameter estimations with Morse phase shifts accounted for

Confirmed significant coincidence in intrinsic + extrinsic parameters + Morse phases

Properties of candidate lensed images

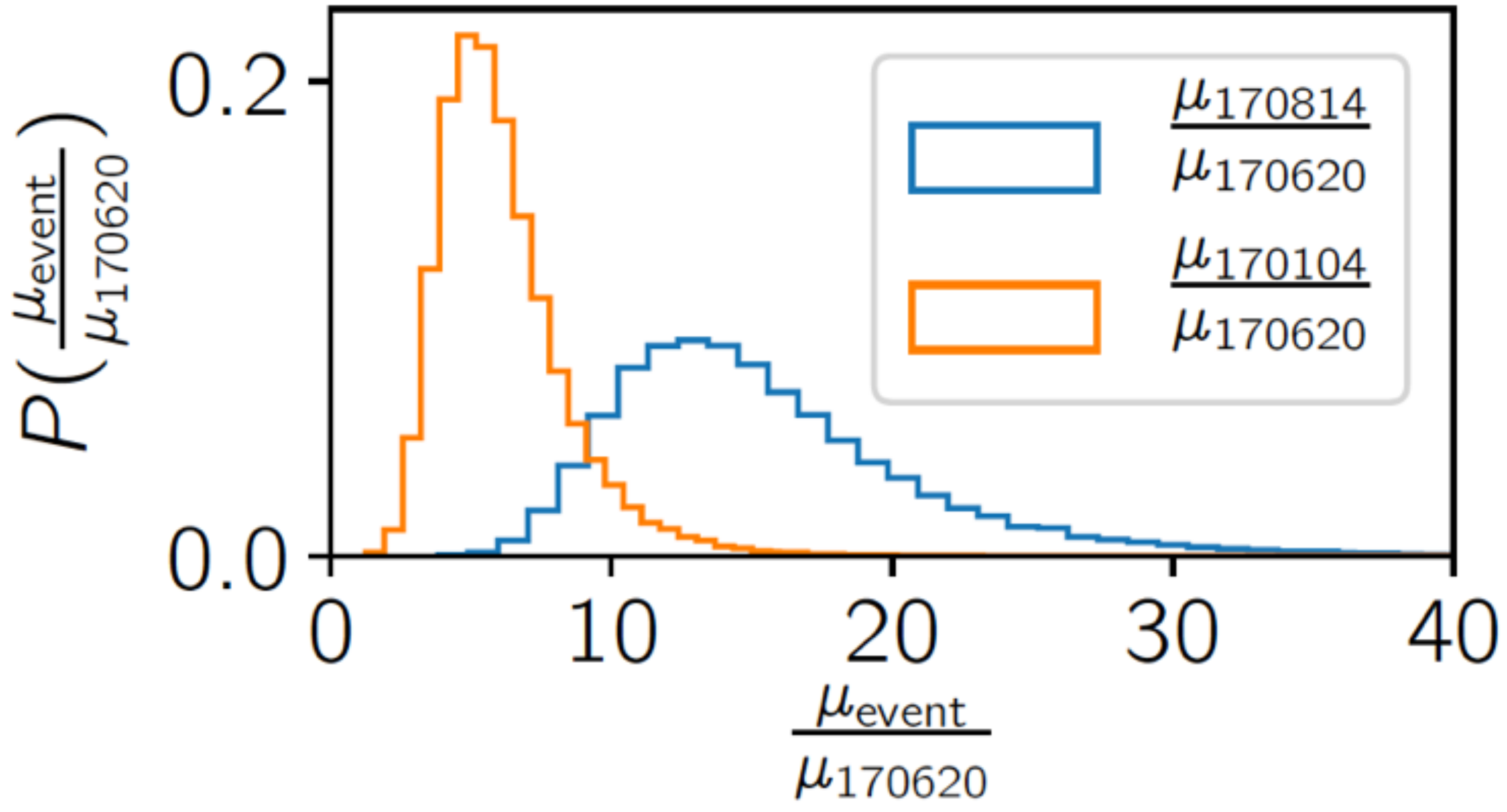
GW170814, GW170104 and GWC170620 have network SNRs ~18, ~14, 7.8

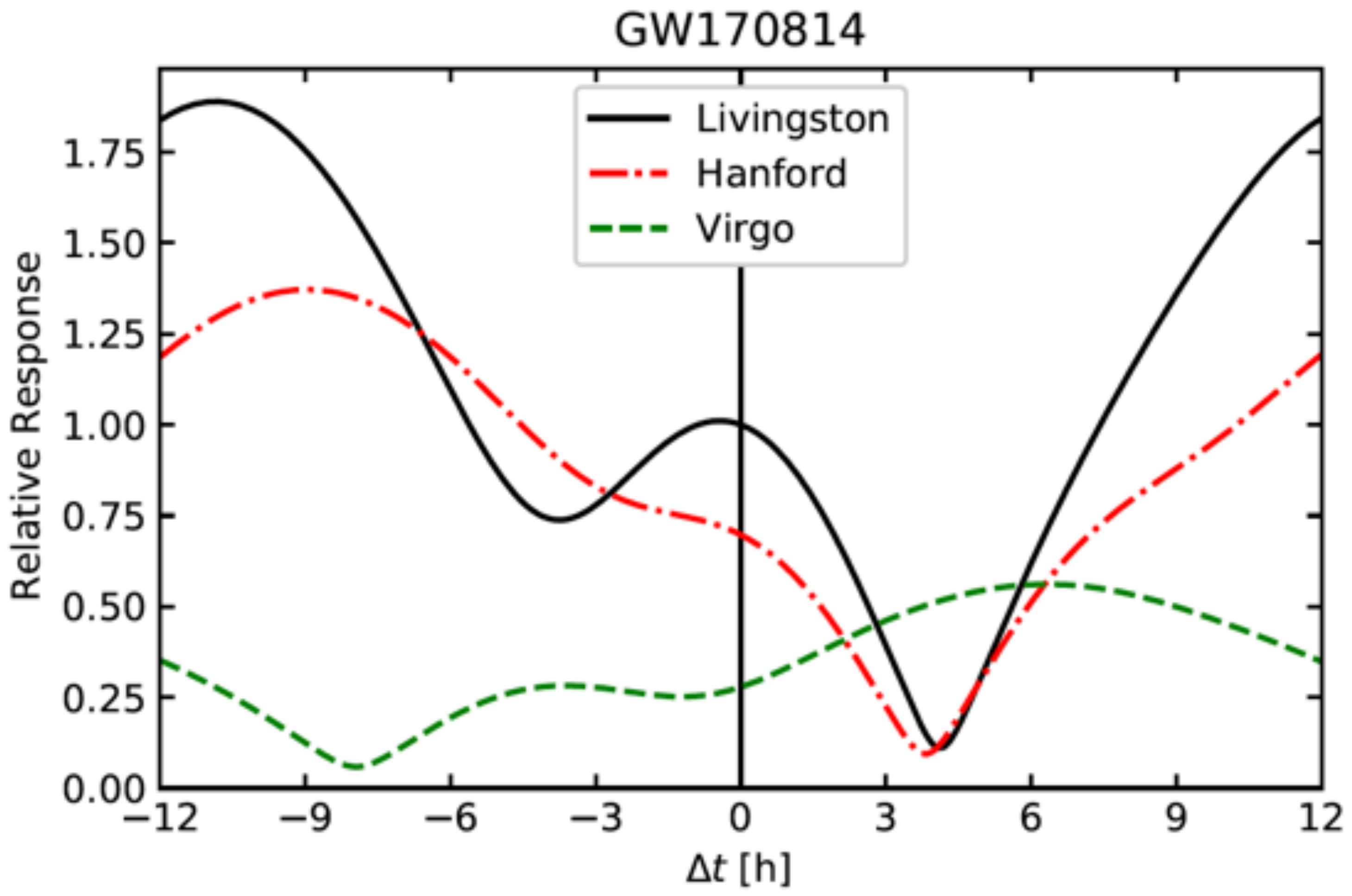
strain amplification ratios ~ 1 : 0.4 : 0.25

flux magnification ratios ~ 1 : 1/6 : 1/15

(Absolute magnifications undetermined.)

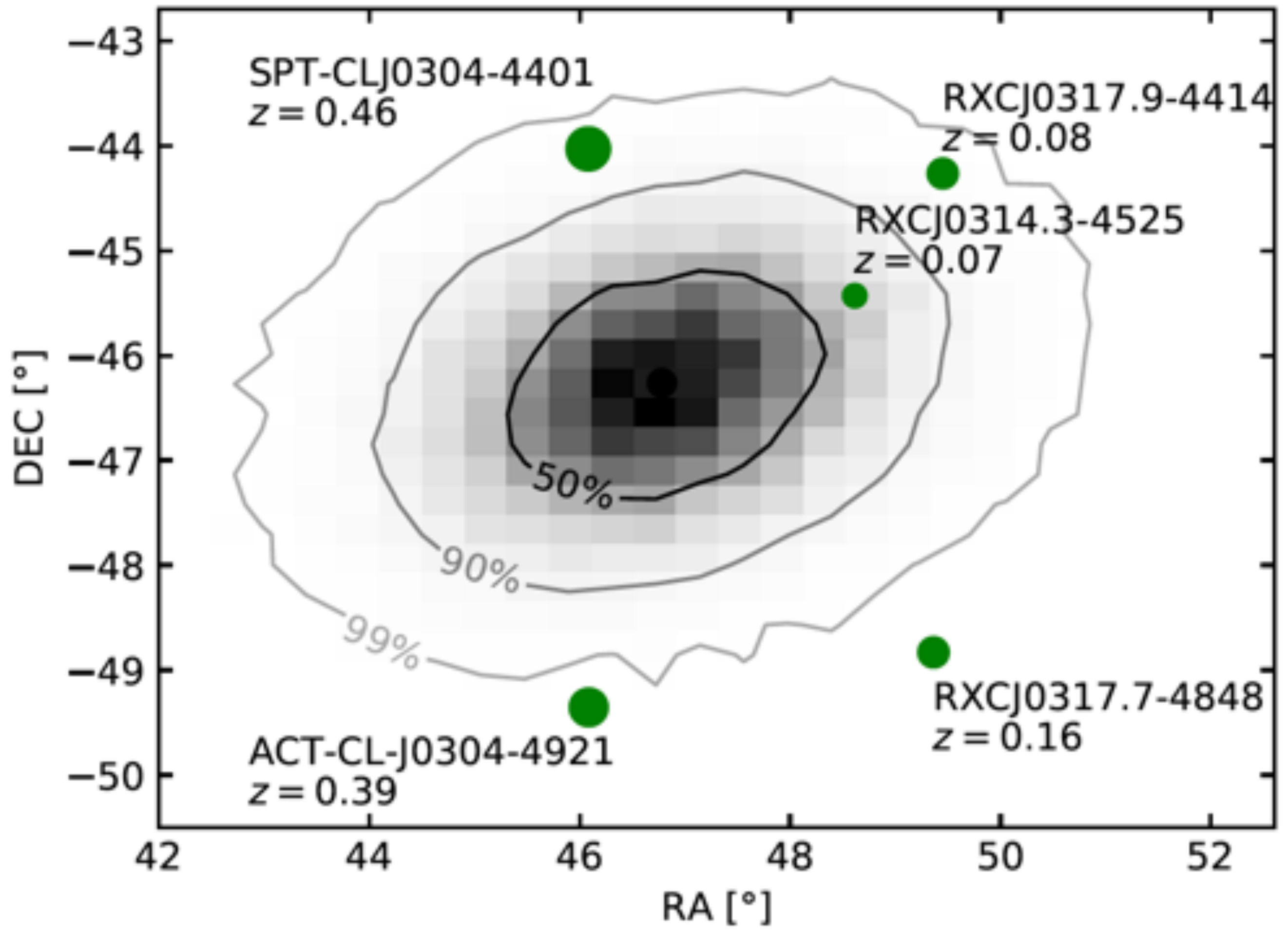
Long time delays on the order of months; need a galaxy cluster scale lens





No comparably loud counter-images within hours of GW170814, expect for a blind "window"!

(see Broadhurst, Diego & Smoot 1901.03190)



localized to ~ 16 deg^2

Difficulty with image configuration

Three types of images:
minimum (L), saddle point (S), maximum (H)

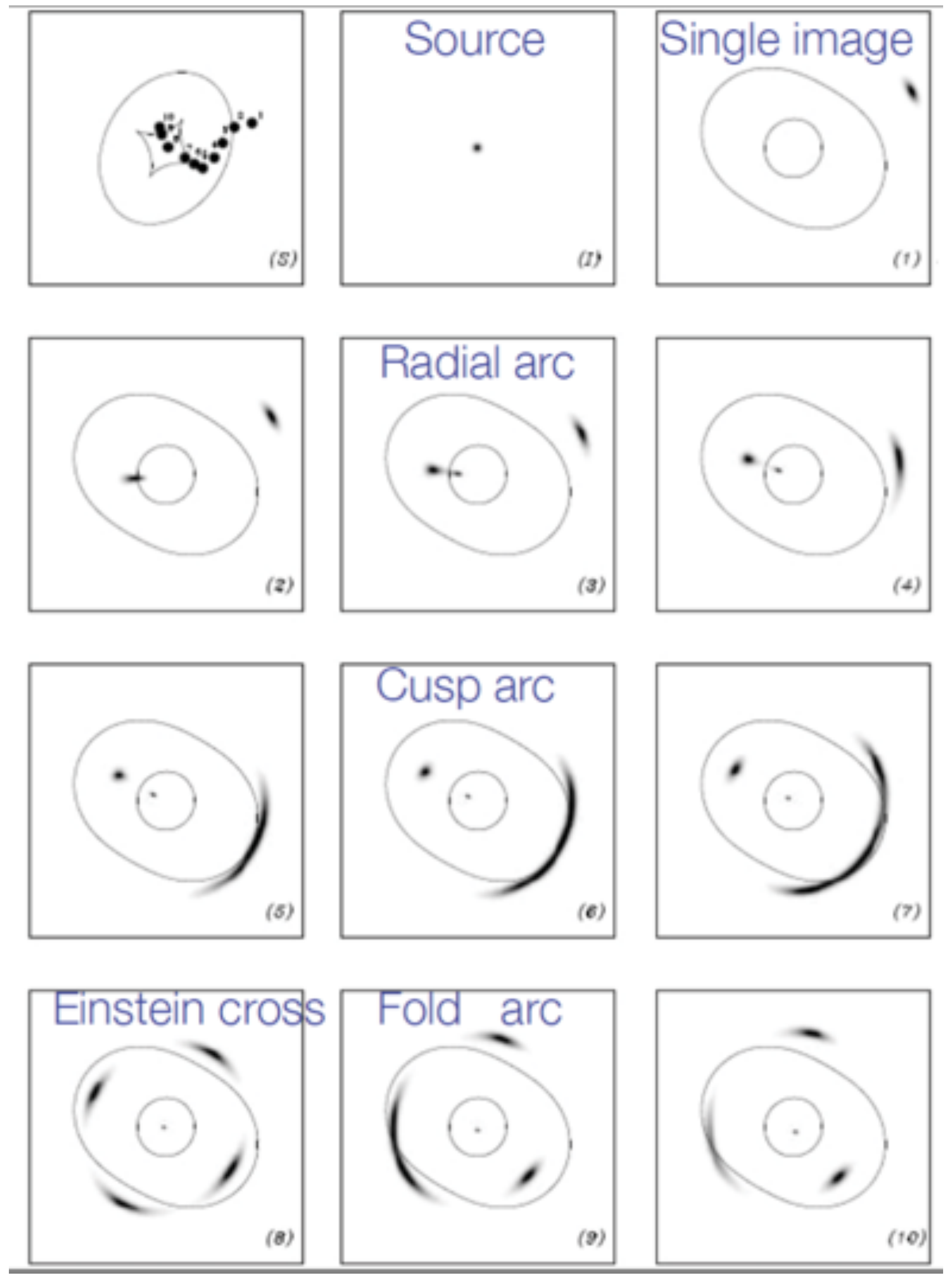
GW170104, GWC170620, GW170814 must be either **L, H, H** or **H, L, L**

S images must have been missed from n(L) + n(H) - n(S) = 1. This however is not too strange given the fraction of Hanford-Livingston coincident times

L images have (absolute) magnification factors > 1, so **at least one H image is significantly magnified** (?!) (GW170814 for **L, H, H**, or GW170104 for **H, L, L**). This is peculiar!

$$z_S^{\rm HLL} > 0.26$$
$$z_S^{\rm LHH} > 0.13$$

Likely **zs < 0.7** because no counter image within 1 hr of GW170814.



Kneib (1993)

Einstein quad: central de-magnified H image
Radial arc: counter S image with short delay

Lesson learned from O1/O2

- **Relative Morse phases are measurable!** The combination of time delays, magnification ratios, and Morse phases tightly constrains any viable lens.

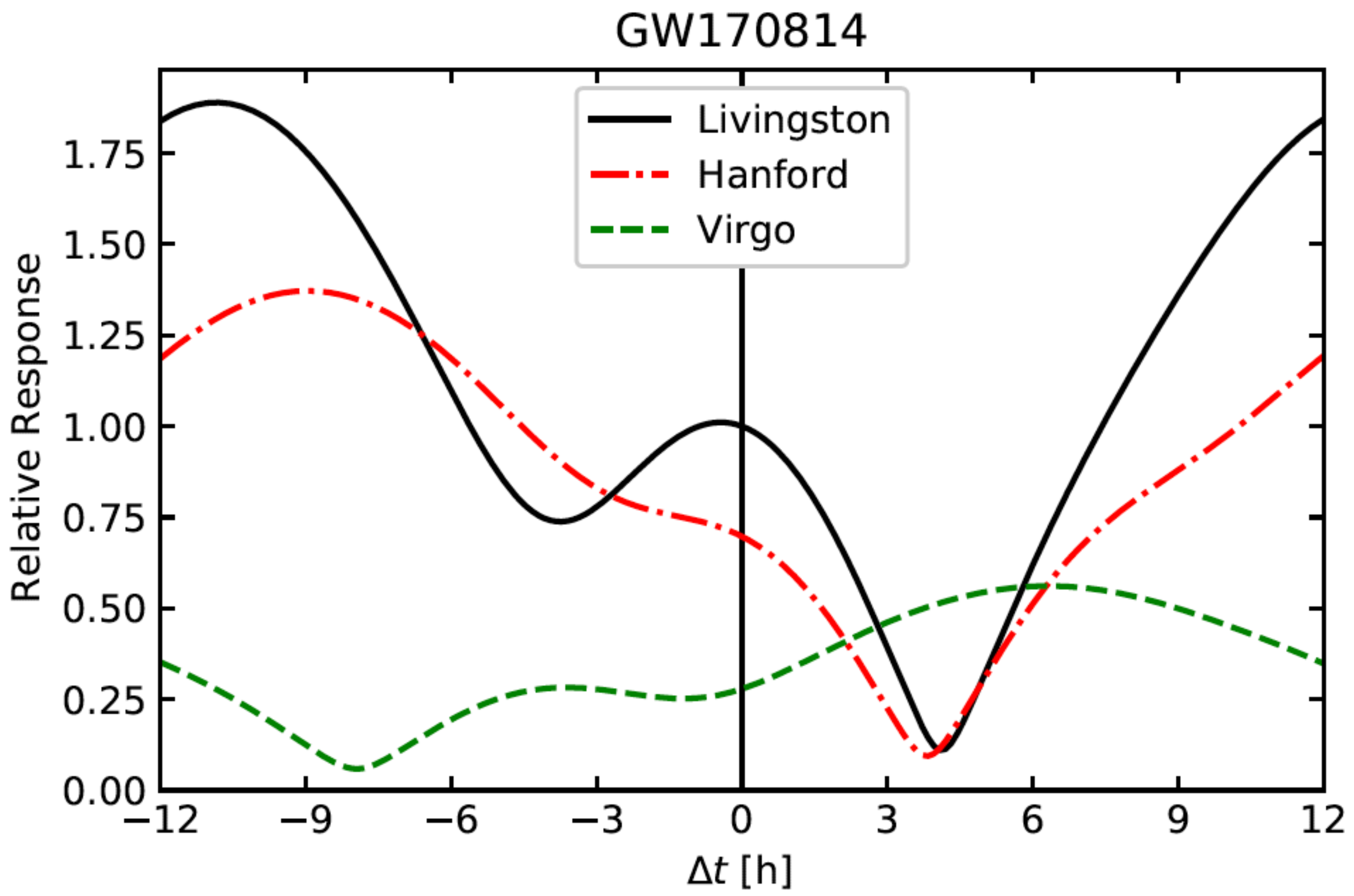
- Due to detector antenna patterns, even very loud lensed images can hide in noise. Conversely, very faint lensed images can get lucky to be detectable.

- **If the triplet is a statistical fluke, we are currently limited by false alarms.** Getting more events at current measurement quality won't help. Need better measurement!
  - (1) better sky localization from at least 3 detectors
  - (2) more short time-delay (hours, days) candidates
  - (3) smaller error bars on intrinsic parameters and extrinsic parameters from better sensitivity at low frequencies
  - (4) detection of distant lower-mass BBH events (much better chirp mass measurement)
  - (5) S image of high SNR signals (higher harmonics & spin-orbit precession)

Wang, Lo, Li & Chen 2101.08264

The End

Detector responses play tricks



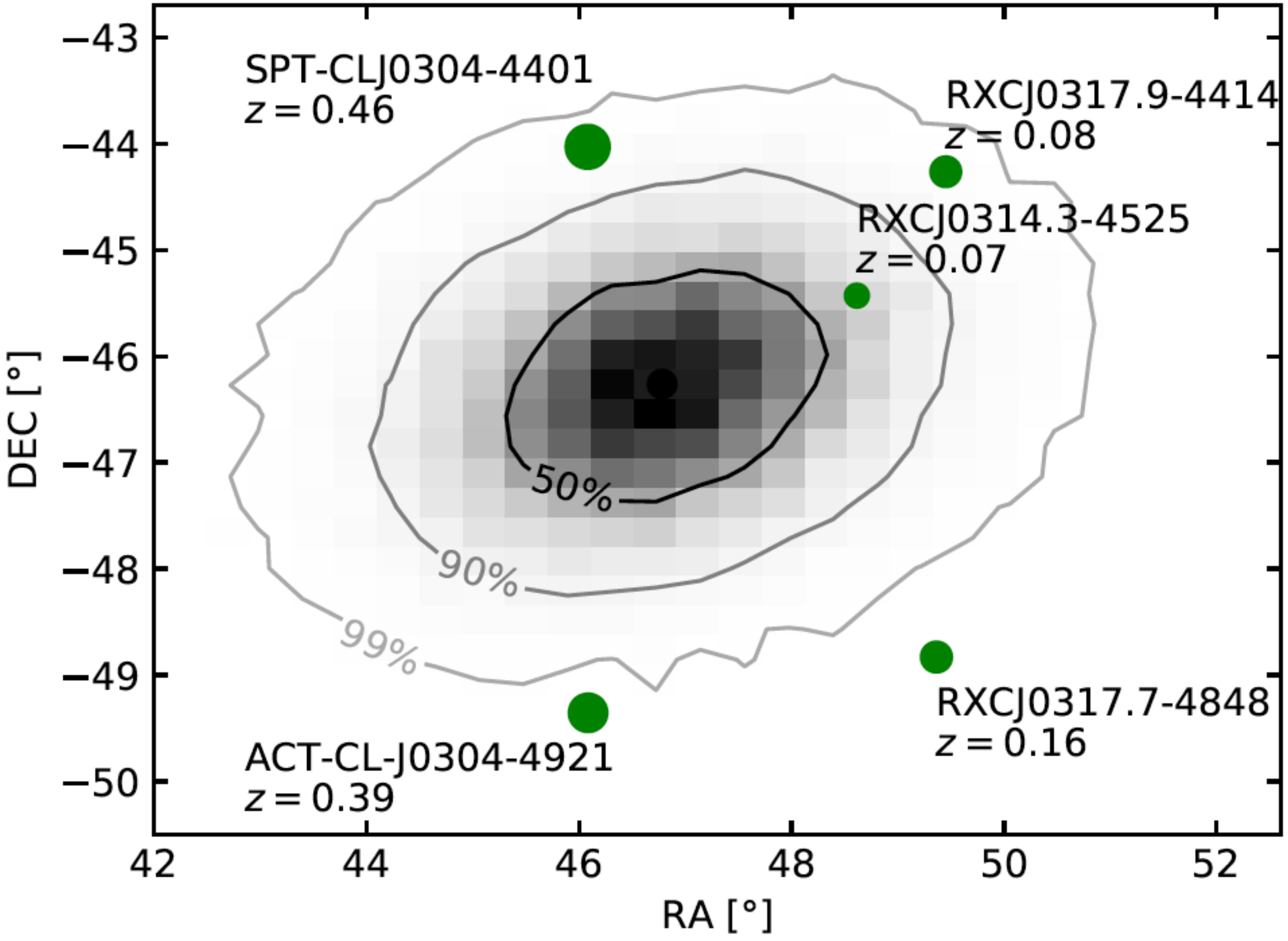
Strain amplitude response

About 4—5 hrs after GW170814, LIGO detectors became almost blind toward that direction on the sky, and Virgo was more sensitive !

Broadhurst, Diego & Smoot 1901.03190

Lesson learned:
an image equally loud as GW170814 could have hidden in the Gaussian noise.

Localization From Joint PE



localized to ~ 16 deg^2

Time delays ~ months require a DM halo of galaxy group or galaxy cluster scale

Galaxy group or cluster lenses can have more complicated structure.

Perhaps the peculiar image configuration not completely out of the question.