Gravitational Lensing of GWs: Effect of Microlens Population in Lensing Galaxies

- To be arXived this week

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Motivation

- The probability of a GW signal encountering a massive isolated point lens is small relative to a population of stars (point lenses) in a galaxy.
- This is especially true when the signal has been strongly lensed by a galaxy, in which case the presence of multiple microlens on the path of GW is not only possible but inevitable.
- First dedicated study which incorporated the effect of a realistic stellar population was Diego et al., 2019, 2020. However, it mainly dealt with microlensing in intracluster regime, and high magnification values. [See talk by Diego]
- Our aim is to study the effects of microlensing when a signal gets strongly lensed by typical massive galaxies. Should one worry about ML effects in typical scenarios?

Introduction

[See talks by Takahashi, Diego and Dai]

(i) Strong Gravitational Lensing (Macrolensing) $\longrightarrow$ Geometric (ray) Optics $\longrightarrow$ Formation of multiple images (in time domain for GWs and in spatial domain for EMs).

- For a macrolensing magnification factor μ, both GW amplitude and GW-SNR increase by $\sqrt{\mu}$.

(ii) Gravitational Microlensing $\longrightarrow$ Wave Optics $\longrightarrow$ Interference (frequency dependent)

- Multiple images produced by a microlens are usually not resolved.
- Effect is significant when $\lambda_{GW} \gtrsim c\Delta\tau_{delay}$ ($\approx R_{Sch}$ for point lens).
- These microlenses can be stars and remnants in a galaxy or the intracluster medium or even primordial black holes (PBHs).
- *Frequency-dependent amplification and phase distortion.*

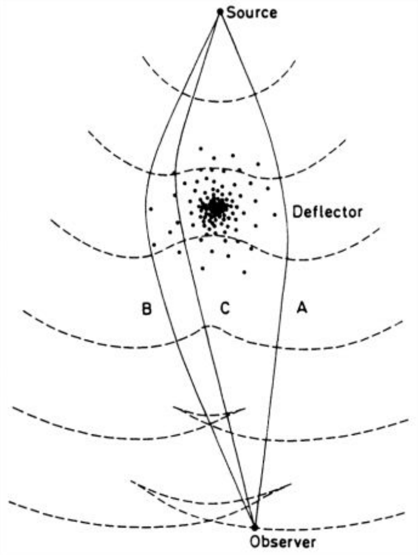

Figure 1: *Illustration of how multiple images form from the perspective of wave optics (Credit: [Schneider and Ehlers, 1992])*

Methodology

Generating a realistic stellar population using a stellar initial mass function (IMF).

↓

Evolving the population for an assumed time frame (between two redshifts).

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Estimating the stellar densities and macromodel magnifications that might be present in realistic scenarios near strongly lensed images.

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Developing code and generating several realizations of $F(f) = \frac{\Phi_{lensed}}{\Phi_{unlensed}}$

↓

Doing Parameter Estimation & Mismatch calculations.

↓

Statistical inference from these calculations.

Usually,

surface microlens density $\Sigma_{\bullet} \in \mathcal{O}(10) - \mathcal{O}\left(10^3\right)\ M_{\odot} pc^{-2}$ *and* $\mu \in \mathcal{O}(1) - \mathcal{O}\left(10^2\right)$.

- Amplification factor:

$$F(f) = \frac{\tilde{\Phi}^{L}_{obs.}(f)}{\tilde{\Phi}_{obs.}(f)}, \tag{1}$$

where $\tilde{\Phi}^{L}_{obs.}(f)$ and $\tilde{\Phi}_{obs.}(f)$ are the observed lensed and the unlensed GW amplitudes, respectively.

- Diffraction integral (obtained using the integral theorem of *Helmholtz and Kirchhoff*):

$$\boxed{F(f) = \frac{f}{i}\int d^2\mathbf{x}\ \exp[i2\pi f t_d(\mathbf{x},\mathbf{y})]};$$

where,

$$t_d(\mathbf{x},\mathbf{y}) = (1+z_\mathrm{d})\frac{\xi_0^2 D_s}{cD_\mathrm{d}D_\mathrm{ds}}\left[\frac{1}{2}(\mathbf{x}-\mathbf{y})^2 - \psi(\mathbf{x}) - \varphi(\mathbf{y})\right].$$

$$\psi(\mathbf{x}) = \psi_{\mathrm{SL}}(\mathbf{x}) + \psi_{\mathrm{ML}}(\mathbf{x})$$
$$\psi_{\mathrm{ML}}(\mathbf{x}) = \sum_i \frac{m_i}{M_0}\ln|\mathbf{x}-\mathbf{x}_i|,$$
$$\psi_{\mathrm{SL}}(\mathbf{x}) = \frac{\kappa}{2}\left(x_1^2+x_2^2\right) + \frac{\gamma_1}{2}\left(x_1^2-x_2^2\right) + \gamma_2 x_1 x_2,$$

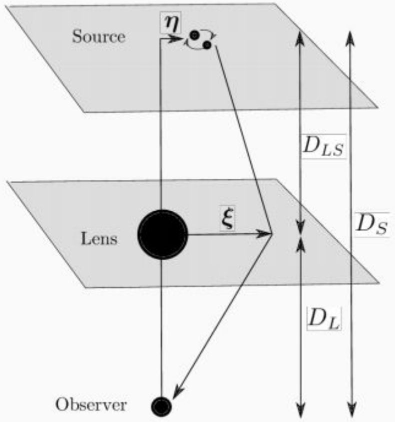


Figure 2: *Illustration of the gravitational lens configuration in the thin lens approximation.* The lensing configuration is described by the source displacement from the line-of-sight $\boldsymbol{\eta}$, the angular diameter distance from the observer to the source D_S, to the lens D_L and from lens to the source D_{LS} and by the relative position of the image in the image plane $\boldsymbol{\xi}$.(Credit: [Pagano et al., 2020])

Ingredients of our Model

- Macro lens mass model :
    - Singular Isothermal Ellipsoid, z_lens=0.5 and z_src=2.0

- Microlens model:
    - Stellar density profile: Sersic Profile (related info from Vernardos et al. 2019)
    - Stellar IMF: Chabrier
    - Evolution of single stellar population: 5-6 Gyr (Elbridge et al. 2017)
    - Binary population: Using Duchene & Krauss 2013
    - Final microlens population: stars and remnants (e.g. BH, NS, WD, MD)
    - Final microlens mass range: ~ 0.08 to 27 Msun

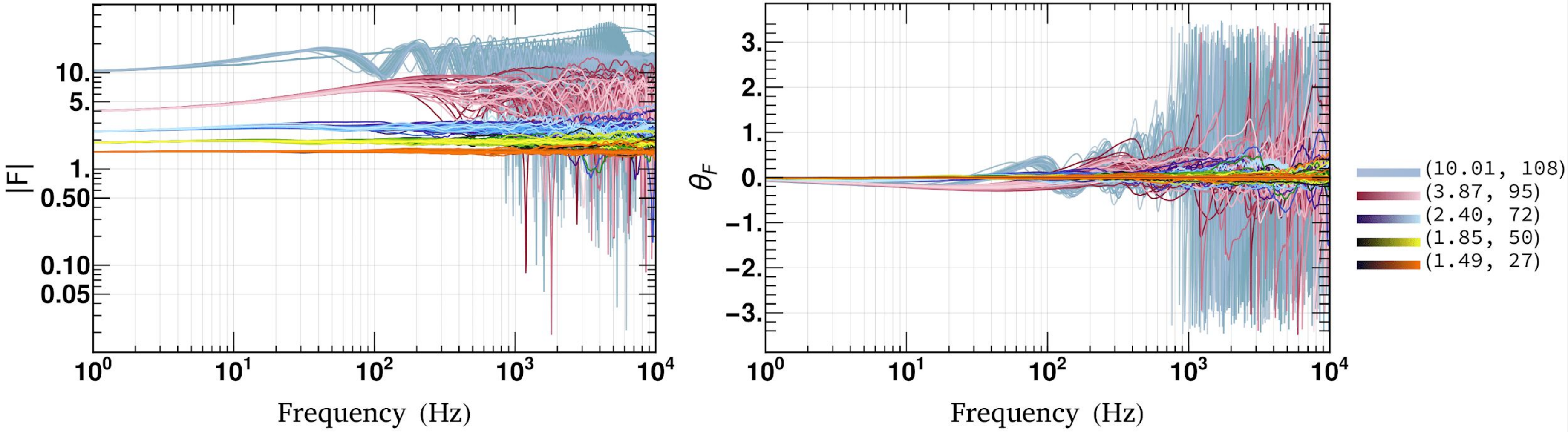
Lens Parameter Used

κ^{tot}	γ^{tot}	κ^{star}	$\sqrt{\mu}$	$\Sigma_{\bullet}$ ($M_{\odot}$ pc^{-2})
Minima				
0.276	0.276	0.013	1.49	27
0.354	0.354	0.024	1.85	50
0.412	0.412	0.035	2.40	72
0.467	0.467	0.046	3.87	95
0.495	0.495	0.052	10.01	108
Saddle points				
0.504	0.504	0.11	11.05	113
0.546	0.546	0.12	3.21	135
0.722	0.722	0.16	1.50	239

Details of the Simulations

- Generate simulations for
  a. type I (minima) and type II (saddle) images
  b. varying stellar densities
  c. varying SL magnification
  d. different IMFs
- Each simulation:
  a. Box size ~ 100 pc^2
  b. Patch size ~ 2.7 pc^2
  c. Total realisations: 36
  d. Source is at the center of each patch

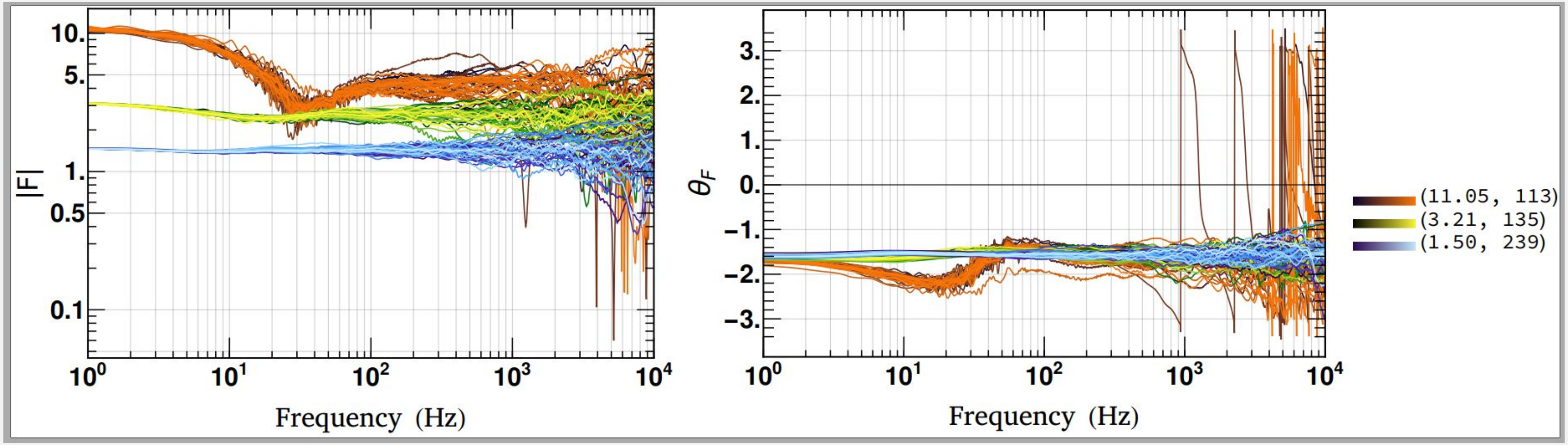
ML effects for minima images



where, $F(f) = |F|e^{i\theta_F}$.

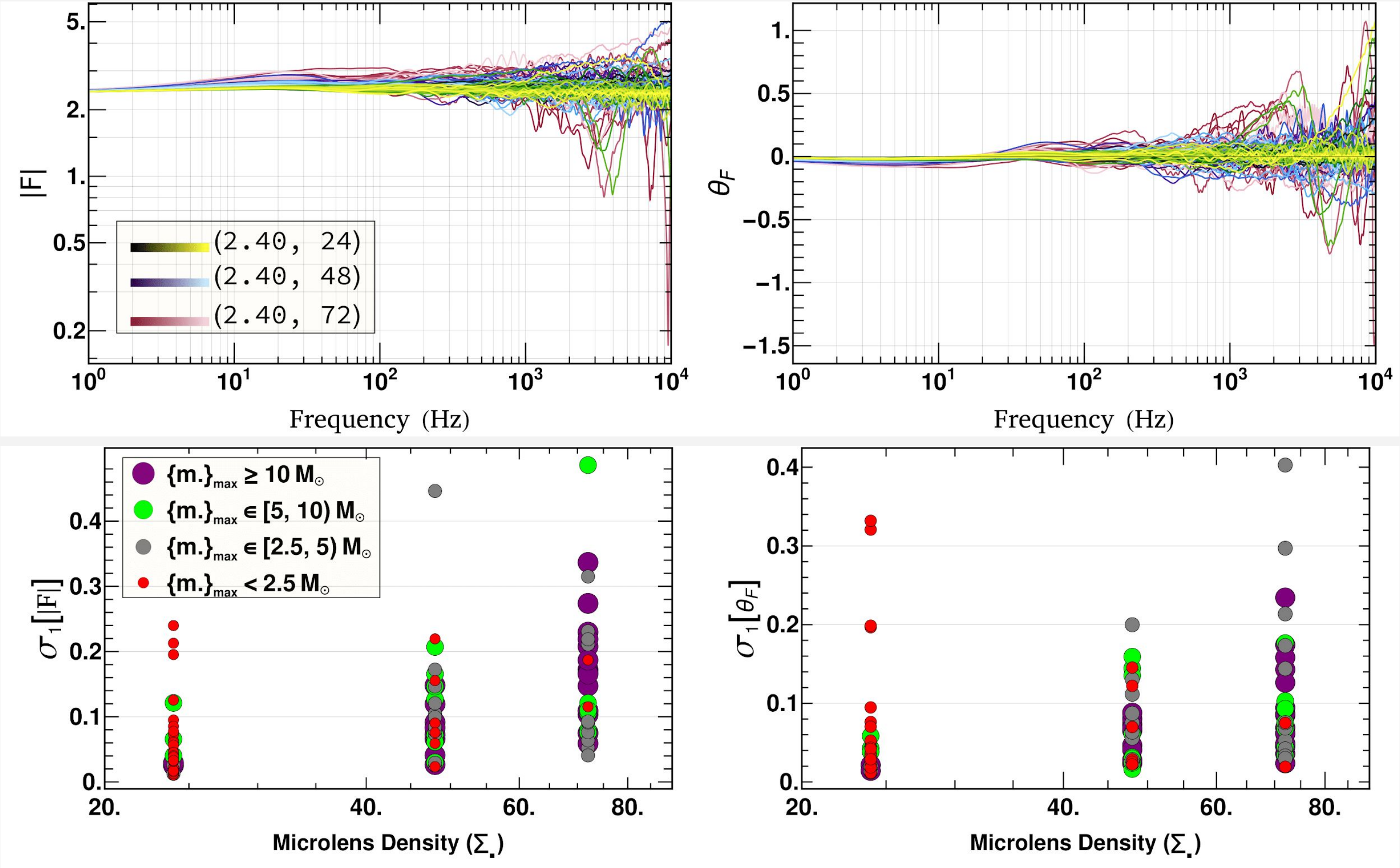
- Deviations become larger with increasing SL magnification
- On average, ML causes amplification in the curves for type-I (minima) macroimages
- For a given SL mag., with increasing stellar density, the no. of microimages also increases and hence the distortions.

ML effects for saddle images

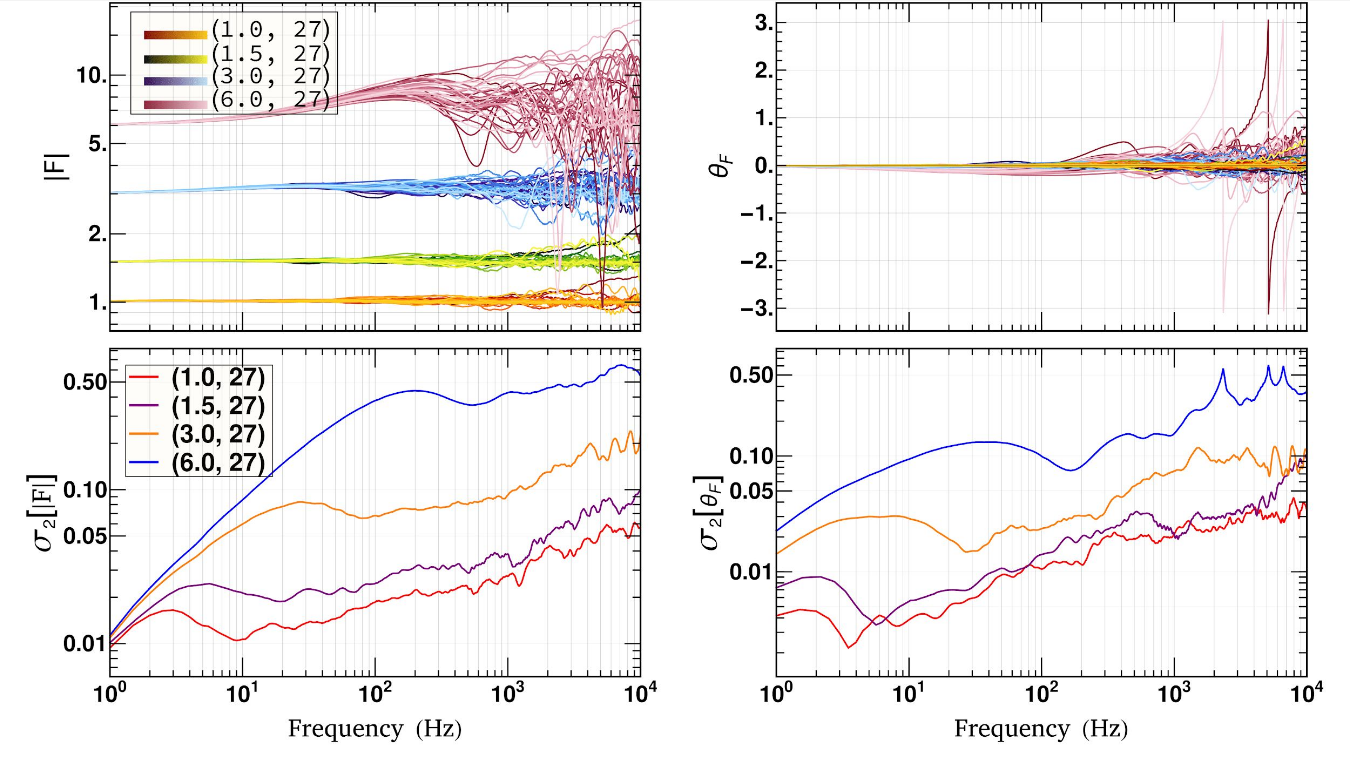


- Deviations are stronger with increasing SL magnification and at lower frequencies
- ML effects visible from relatively lower frequencies compared to minima-like macroimages.
- On average, we see deamplification due to ML
- We also recover Morse phase value of Pi/2 [See talk by Liang Dai]

Effect of varying Microlens densities

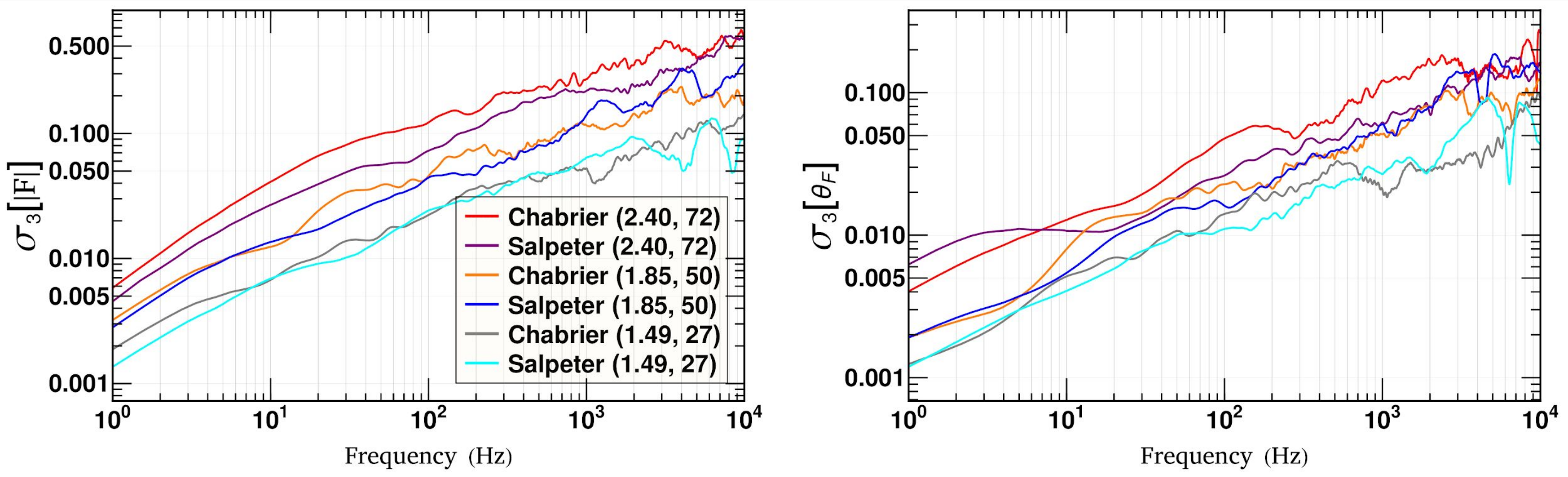


Effect of Macro-amplification (√μ) values



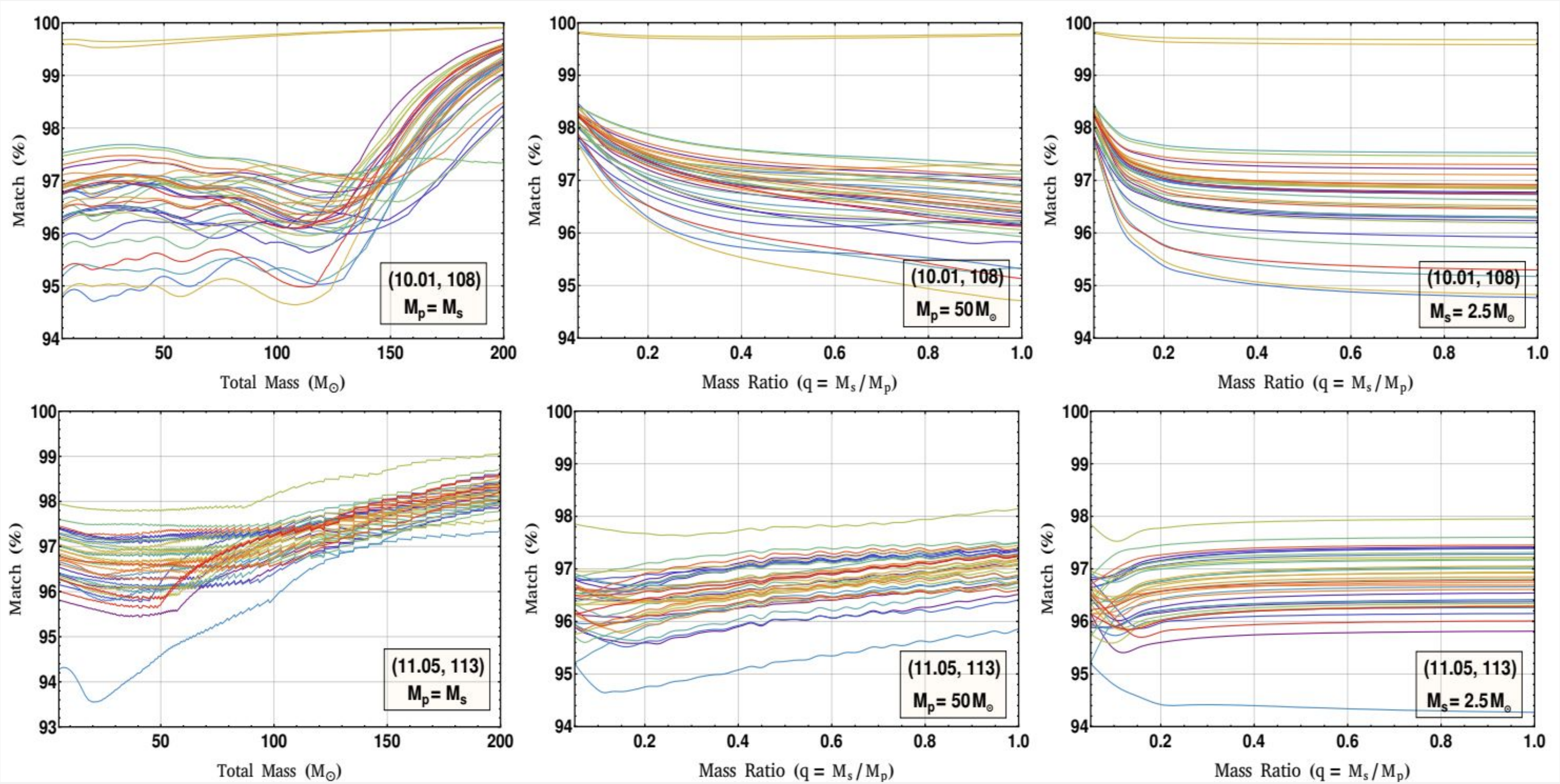
This is the most important factor for microlensing to be significant. ML effects rise steeply with increasing macro-magnification values. This is because density of microcaustics scale with increasing magnification values. [See talk by Diego]

Comparison of different IMFs



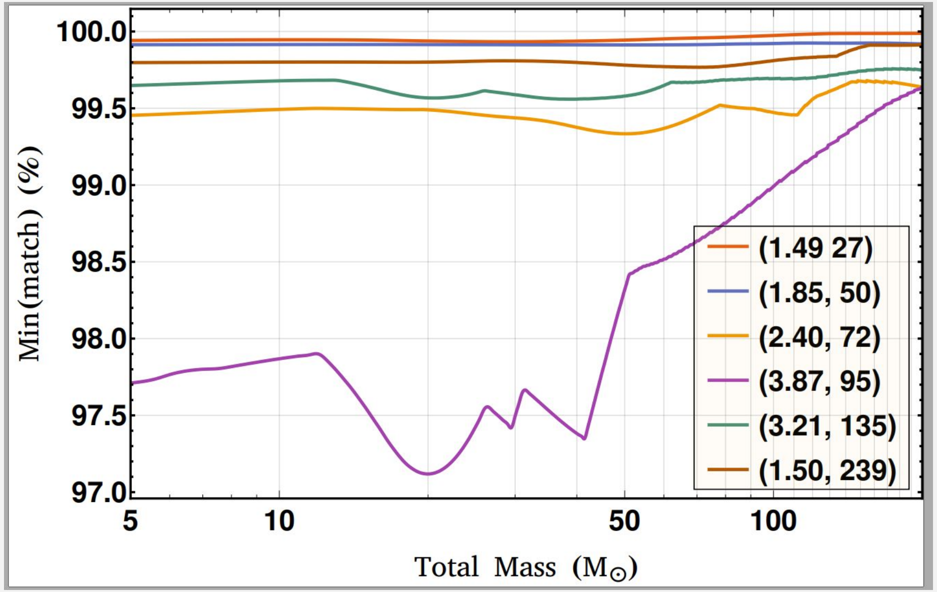
- High no. of low mass stars from Salpeter IMF tends to create steeper rise in distortions at higher frequencies than in the case of Chabrier IMF
- However, difference is not significant. Hence, our conclusions are not dependent on assumed IMF of our lens model.

Results of Mismatch analysis between lensed and unlensed GW signals



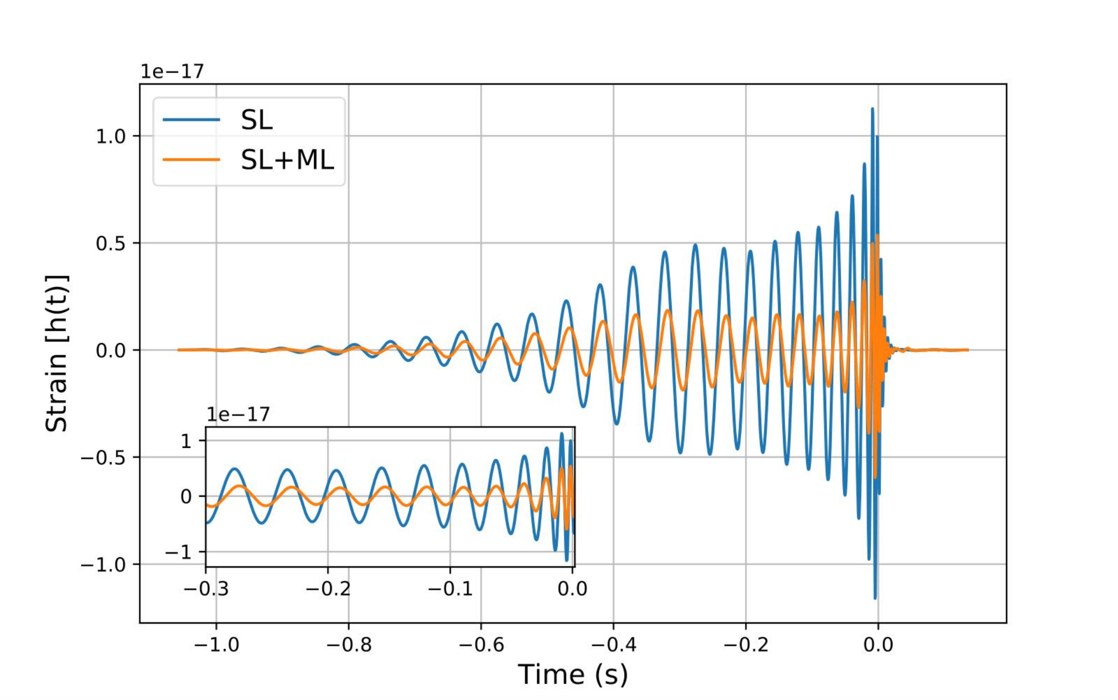
We compute match (normalized weighted inner product) between lensed and unlensed GW Waveforms. A match value $\lesssim$ 97% will be missed by current detectors (a maximum match of 97% is detection threshold for current detections). Although, we do not calculate maximum match here, we can observe that in some extreme magnification values, signals will definitely be missed.

Minimum Match at typical lensing values



Here we show minimum match obtained among our 36 realisations corresponding to each of the six typical cases listed as a function of total mass of the binary.

Effect on GW Waveform



Here we show an example of the distorted strongly lensed GW signal due to microlensing. The waveform corresponds to a 40+40=80 M_sun binary. And the ML effect is due to one of the realisations corresponding to saddle points: (11.05, 113). The mismatch obtained in this case is around 4.4 %.

Summary and Future Work

- We have calculated the ML effects using realistic stellar populations found in typical lensing galaxies.
- SL (macro) magnification plays the most dominant role in how strongly ML can distort the waveforms
- Minima tend to show (significant) amplification and saddles tend to show (significant) deamplification (particularly, at high SL magnification).
- Overall, ML effects for saddle-like macroimages are more significant.
- Mismatch (>3%) is rare but possible. May affect detection of highly magnified GW signals. For other typical cases, GW signal will not be missed but their inferred parameters might be degenrate with ML, e.g., spin misalignment, eccentricity, mass ratio, etc.
- No significant differences found between Chabrier and Salpeter IMF at higher frequencies.

Thank You