# TDCOSMO IV: Hierarchical time-delay cosmography - joint inference of the Hubble constant and galaxy density profiles

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**PhD student**

*(Affiliations can be found after the references)*

Accepted XXX. Received YYY; in original form ZZZ

## ABSTRACT

The H0LiCOW collaboration inferred via strong gravitational lensing time delays a Hubble constant value of $H_0 = 73.3^{+1.7}_{-1.8}$ km s$^{-1}$Mpc$^{-1}$, describing deflector mass density profiles by either a power-law or stars (constant mass-to-light ratio) plus standard dark matter halos. The mass-sheet transform (MST) that leaves the lensing observables unchanged is considered the dominant source of residual uncertainty in $H_0$. We quantify any potential effect of the MST with a flexible family of mass models that directly encodes it and are hence maximally degenerate with $H_0$. Our calculation is based on a new hierarchical Bayesian approach in which the MST is only constrained by stellar kinematics. The approach is validated on mock lenses generated from hydrodynamic simulations. We first apply the inference to the TDCOSMO sample of 7 lenses (6 from H0LiCOW) and measure $H_0 = 74.5^{+5.6}_{-6.1}$ km s$^{-1}$Mpc$^{-1}$.
Secondly, in order to further constrain the deflector mass density profiles, we add imaging and spectroscopy for a set of 33 strong gravitational lenses from the SLACS sample. For 9 of the 33 SLAC lenses, we use resolved kinematics to constrain the stellar anisotropy. From the joint hierarchical analysis of the TDCOSMO+SLACS sample, we measure $H_0 = 67.4^{+4.1}_{-3.2}$ km s$^{-1}$Mpc$^{-1}$. This measurement assumes that the TDCOSMO and SLACS galaxies are drawn from the same parent population. The **blind** H0LiCOW, TDCOSMO-only and TDCOSMO+SLACS analyses are in mutual statistical agreement. The TDCOSMO+SLACS analysis prefers marginally shallower mass profiles than H0LiCOW or TDCOSMO-only. Without relying on the form of the mass density profile used by H0LiCOW, we achieve a ~5% measurement of $H_0$. While our new hierarchical analysis does not statistically invalidate the mass profile assumptions by H0LiCOW – and thus their $H_0$ measurement relying on those – it demonstrates the importance of understanding the mass density profile of elliptical galaxies. The uncertainties on $H_0$ derived in this paper can be reduced by physical or observational priors on the form of the mass profile, or by additional data. The full analysis is available here.

**Validated! Blind! Public!**

# Simon Birrer, Stanford University

**https://github.com/TDCOSMO/hierarchy_analysis_2020_public**

Longer talk on youtube: https://www.youtube.com/watch?v=QrdqbZv_tBs **arXiv:2007.02941**

# TDCOSMO IV: Hierarchical time-delay cosmography - joint inference of the Hubble constant and galaxy density profiles

**assumptions**

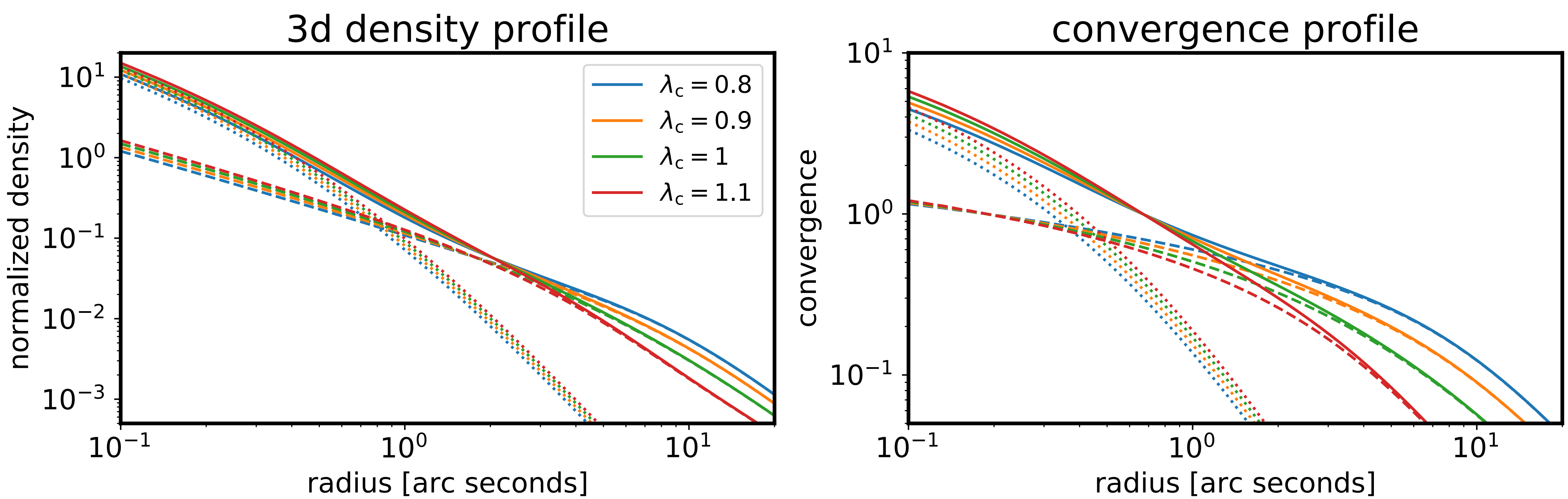
**+**

**data**

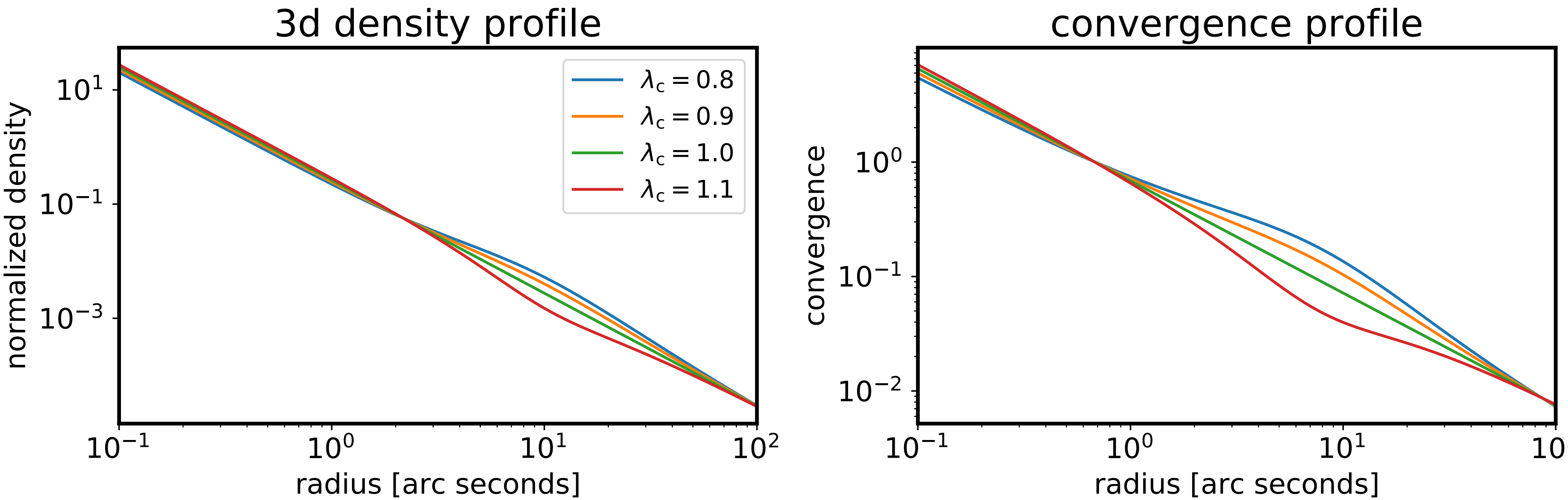
**assumptions**

**+**

**data**



## Section 2: Cosmography from individual lenses and the mass-sheet degeneracy




$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

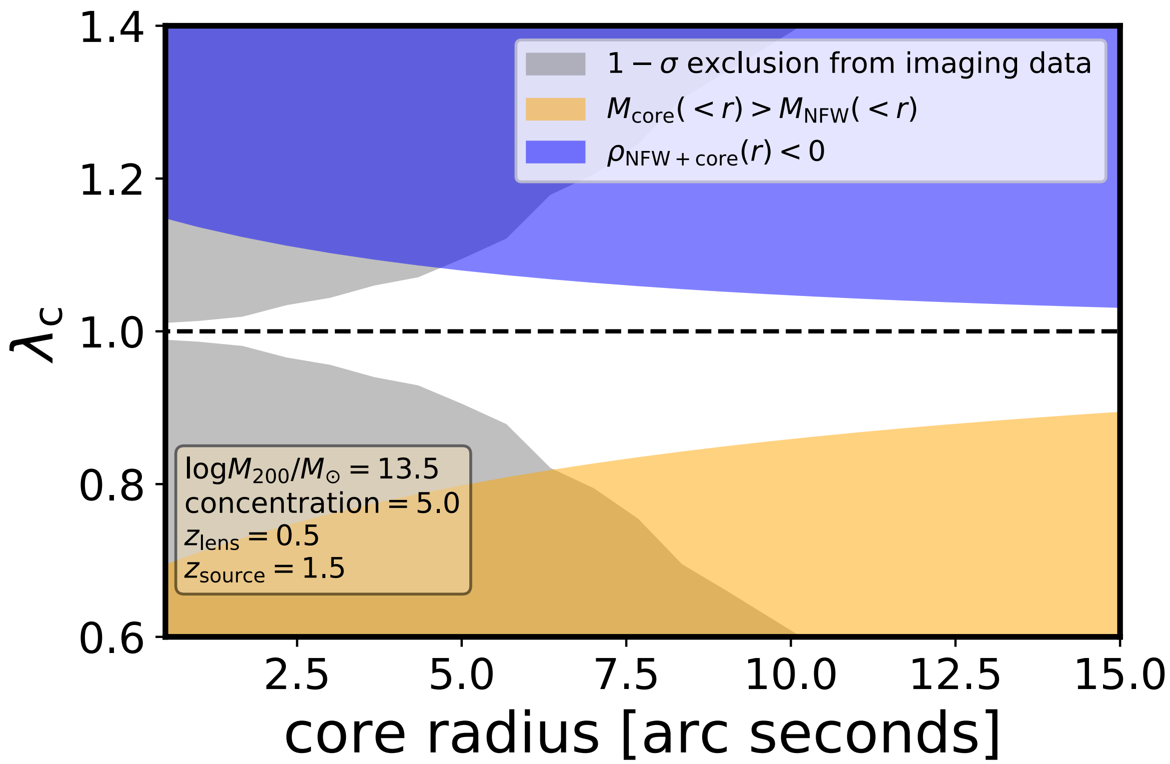
$$\lambda\boldsymbol{\beta} = \boldsymbol{\theta} - \lambda\boldsymbol{\alpha}(\boldsymbol{\theta}) - (1-\lambda)\boldsymbol{\theta}$$

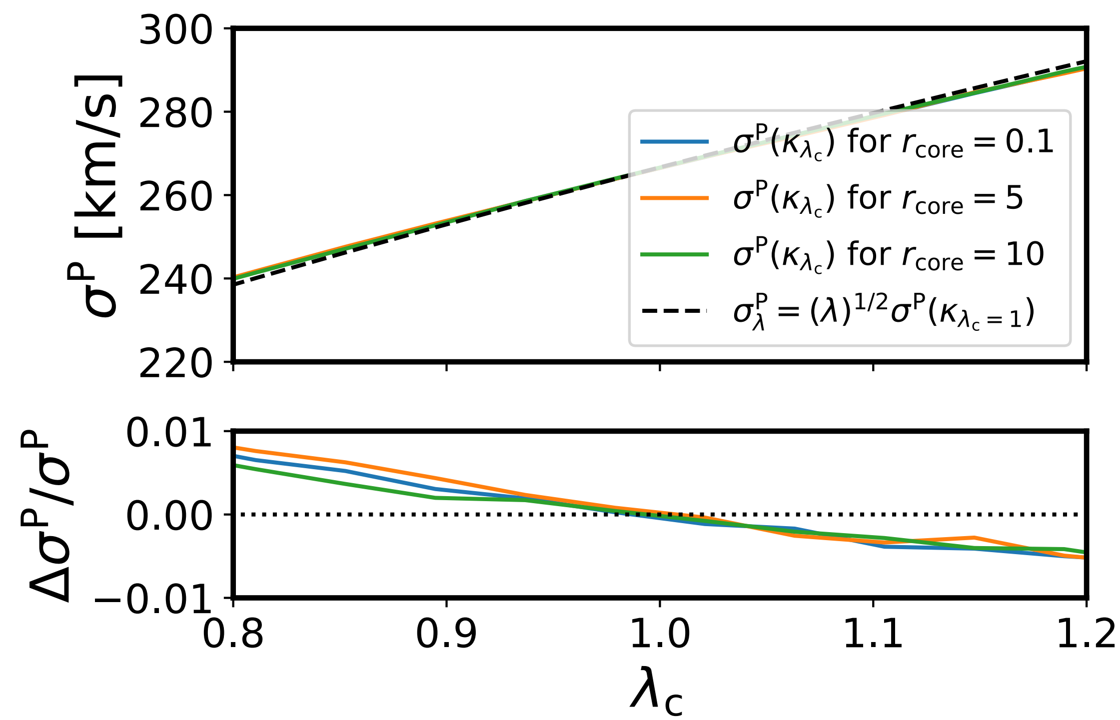
$$\kappa_\lambda(\theta) = \lambda \times \kappa(\theta) + (1-\lambda)$$

$$H_{0\,\lambda} = \lambda H_0$$

Gorenstein+1988, Kochanek 2002, Saha&Williams 2006,
Kochanek 2006, Read+2007, Schneider&Sluse 2013/2014,
Coles+2014, Xu+2016, Birrer+2016, Unruh+2017,
Sonnenfeld 2018, Wertz+2018, Kochanek 2020a, b,Blum+2020

## Section 2: Cosmography from individual lenses and the mass-sheet degeneracy




**Imaging data leaves a significant part of density profiles unconstraint!**

**Kinematics allows to constrain MST!**

### LOS convergences:

$$D^{\rm lens}_{\Delta t} \equiv (1-\kappa_{\rm ext}) D^{\rm bkg}_{\Delta t}$$

$$1-\kappa_{\rm ext} = \frac{(1-\kappa_{\rm d})(1-\kappa_{\rm s})}{(1-\kappa_{\rm ds})}$$

$$D^{\rm lens}_{\rm d} = (1-\kappa_{\rm d}) D^{\rm bkg}_{\rm d}$$

$$D^{\rm lens}_{\rm s} = (1-\kappa_{\rm s}) D^{\rm bkg}_{\rm s}$$

$$D^{\rm lens}_{\rm ds} = (1-\kappa_{\rm ds}) D^{\rm bkg}_{\rm ds}$$

### Observables:

$$\Delta t_{{\rm AB}\,\lambda} = \lambda \Delta t_{\rm AB}$$

$$\sigma^{\rm P}_{v\,\lambda} = \sqrt{\lambda}\sigma^{\rm P}_v$$

### Angular diameter distances:

$$D_{\Delta t\,\lambda} = \lambda^{-1} D_{\Delta t}$$

$$(D_{\rm s}/D_{\rm ds})_\lambda = \lambda^{-1} D_{\rm s}/D_{\rm ds}$$

$$D_{{\rm d}\,\lambda} = D_{\rm d}$$

## Section 3: Hierarchical Bayesian cosmography

**As individual lenses do not allow for precise measurements of MST components and H0, population level parameters and priors are necessary!**

$$p(\boldsymbol{\pi}|\{\mathcal{D}_i\}_N) \propto \mathcal{L}(\{\mathcal{D}_i\}_N|\boldsymbol{\pi})p(\boldsymbol{\pi}) = \int \mathcal{L}(\{\mathcal{D}_i\}_N|\boldsymbol{\pi},\boldsymbol{\xi})p(\boldsymbol{\pi},\boldsymbol{\xi})d\boldsymbol{\xi} = \int \prod_i^N \mathcal{L}(\mathcal{D}_i|\boldsymbol{\pi},\boldsymbol{\xi})p(\boldsymbol{\pi},\boldsymbol{\xi})d\boldsymbol{\xi}$$

$$p(\boldsymbol{\pi}|\{\mathcal{D}_i\}_N) \propto \int \prod_i \mathcal{L}(\mathcal{D}_i|D_{\mathrm{d,s,ds}},\boldsymbol{\xi}_{\mathrm{pop}})p(\boldsymbol{\pi},\boldsymbol{\xi}_{\mathrm{pop}})d\boldsymbol{\xi}_{\mathrm{pop}}$$

individual lens likelihood for a **given hyper-parameter value**

prior defined on the **population level**

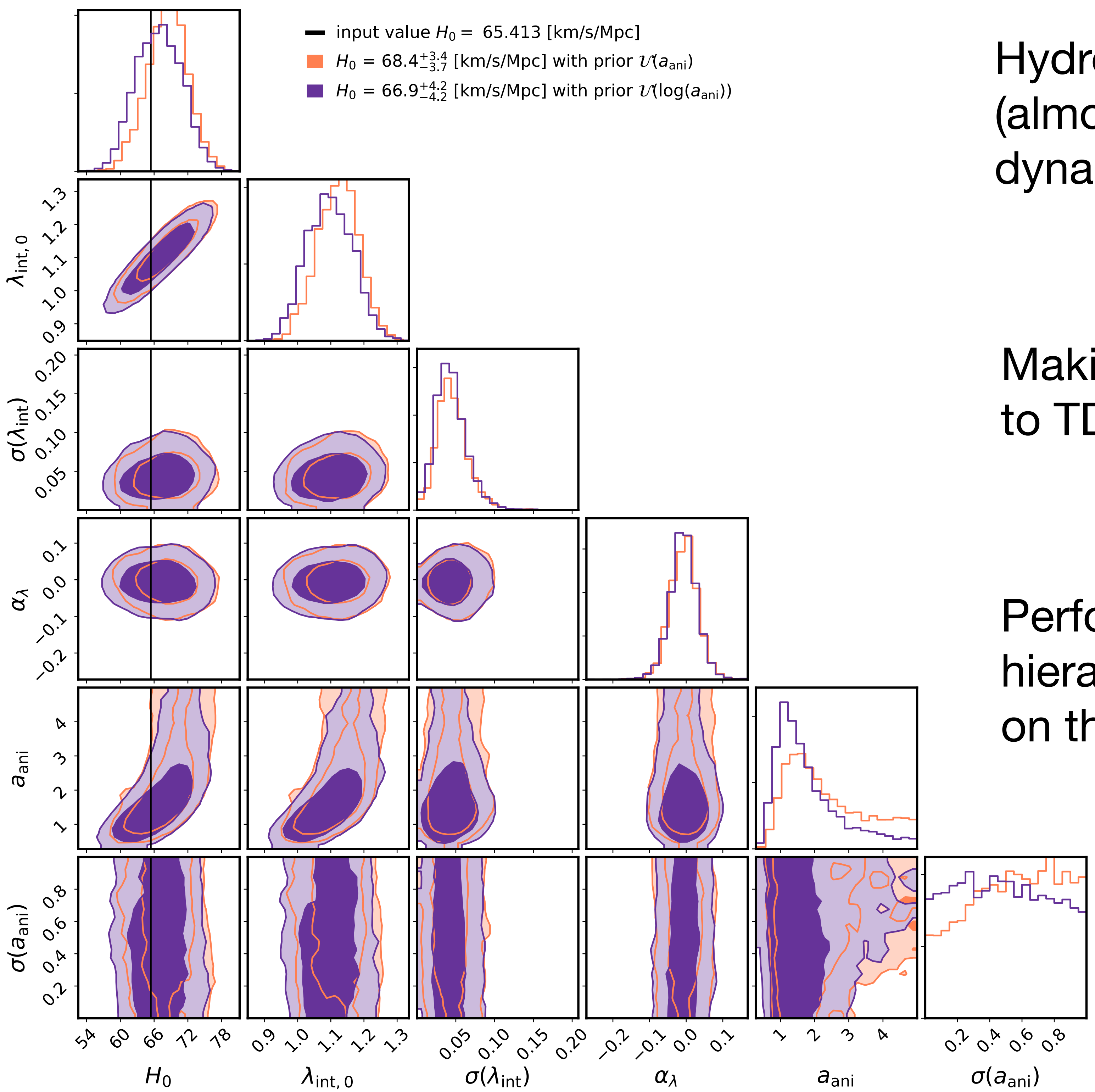
### Population level hyper-parameters:

| name | prior | description |
|---|---|---|
| Cosmology (Flat $\Lambda$CDM) | | |
| $H_0$ [km s$^{-1}$Mpc$^{-1}$] | $\mathcal{U}([0, 150])$ | Hubble constant |
| $\Omega_{\mathrm{m}}$ | $= 0.27$ | current normalized matter density |
| Mass profile | | |
| $\lambda_{\mathrm{int},0}$ | $\mathcal{U}([0.5, 1.5])$ | internal MST population mean for $r_{\mathrm{eff}}/\theta_{\mathrm{E}} = 1$ |
| $\alpha_{\lambda}$ | $\mathcal{U}([-1, 1])$ | slope of $\lambda_{\mathrm{int}}$ with $r_{\mathrm{eff}}/\theta_{\mathrm{E}}$ of the deflector (Eqn. 50) |
| $\sigma(\lambda_{\mathrm{int}})$ | $\mathcal{U}([0, 0.2])$ | 1-$\sigma$ Gaussian scatter in $\lambda_{\mathrm{int}}$ at fixed $r_{\mathrm{eff}}/\theta_{\mathrm{E}}$ |
| Stellar kinematics | | |
| $\langle a_{\mathrm{ani}}\rangle$ | $\mathcal{U}([0.1, 5])$ or $\mathcal{U}(\log([0.1, 5]))$ | scaled anisotropy radius (Eqn. 51, 52) |
| $\sigma(a_{\mathrm{ani}})$ | $\mathcal{U}([0, 1])$ | $\sigma(a_{\mathrm{ani}})\langle a_{\mathrm{ani}}\rangle$ is the 1-$\sigma$ Gaussian scatter in $a_{\mathrm{ani}}$ |
| Line of sight | | |
| $\langle \kappa_{\mathrm{ext}}\rangle$ | $= 0$ | population mean in external convergence of lenses |
| $\sigma(\kappa_{\mathrm{ext}})$ | $= 0.025$ | 1-$\sigma$ Gaussian scatter in $\kappa_{\mathrm{ext}}$ |

details on the likelihood can be found in Appendix C

Likelihood and sampling with hierArc: https://github.com/sibirrer/hierArc

# Section 4: Validation on the time-delay lens modeling challenge



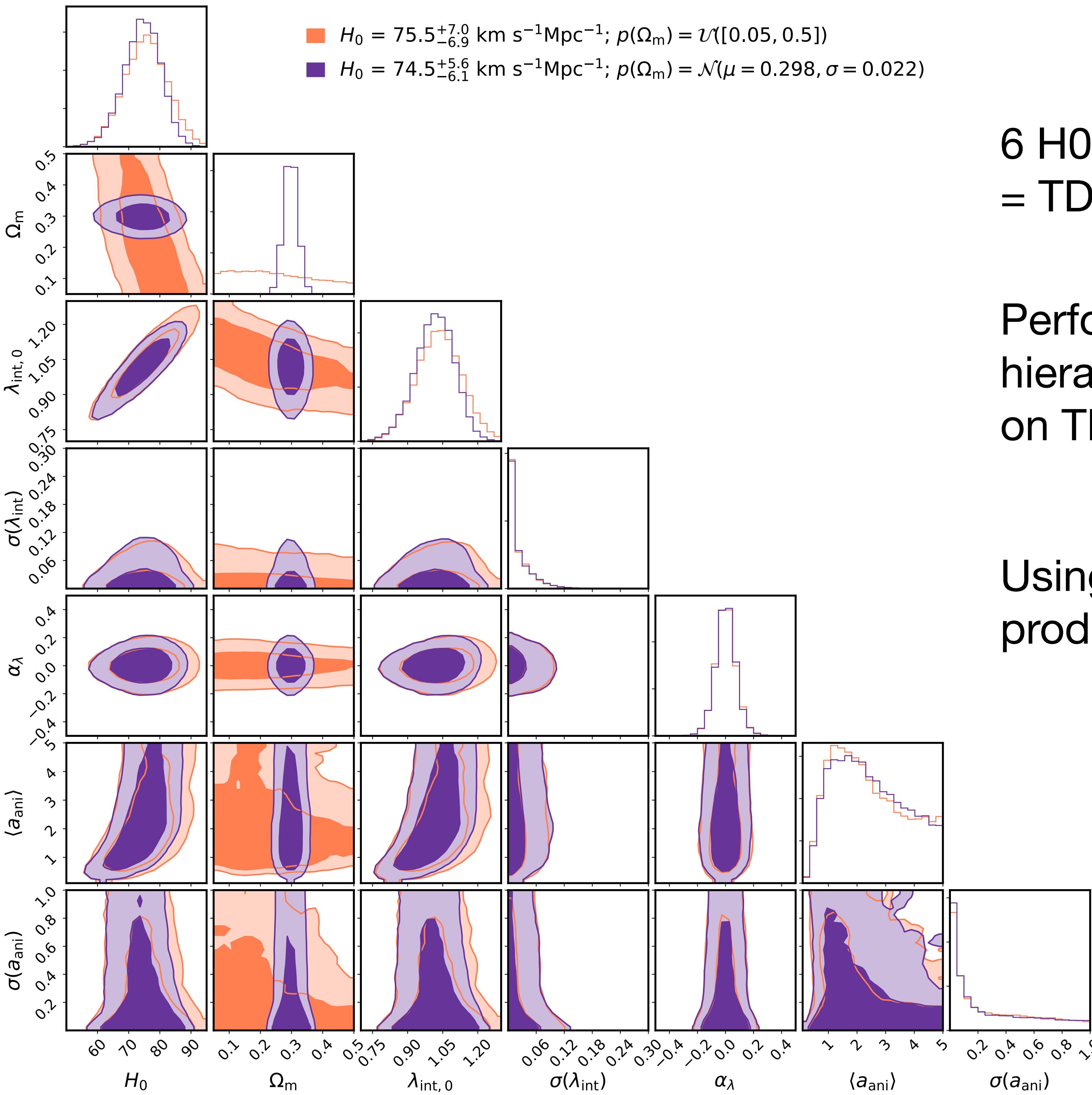

Hydrodynamic simulations with (almost) self-consistent dynamics and lensing quantities

Making us of the **blind** submission to TDLMC rung3 by EPFL team

Performing the **identical** hierarchical analysis as performed on the real data (Section 5-7)

TDLMC: Ding et al 2020

# Section 5: TDCOSMO mass profile and H0 inference

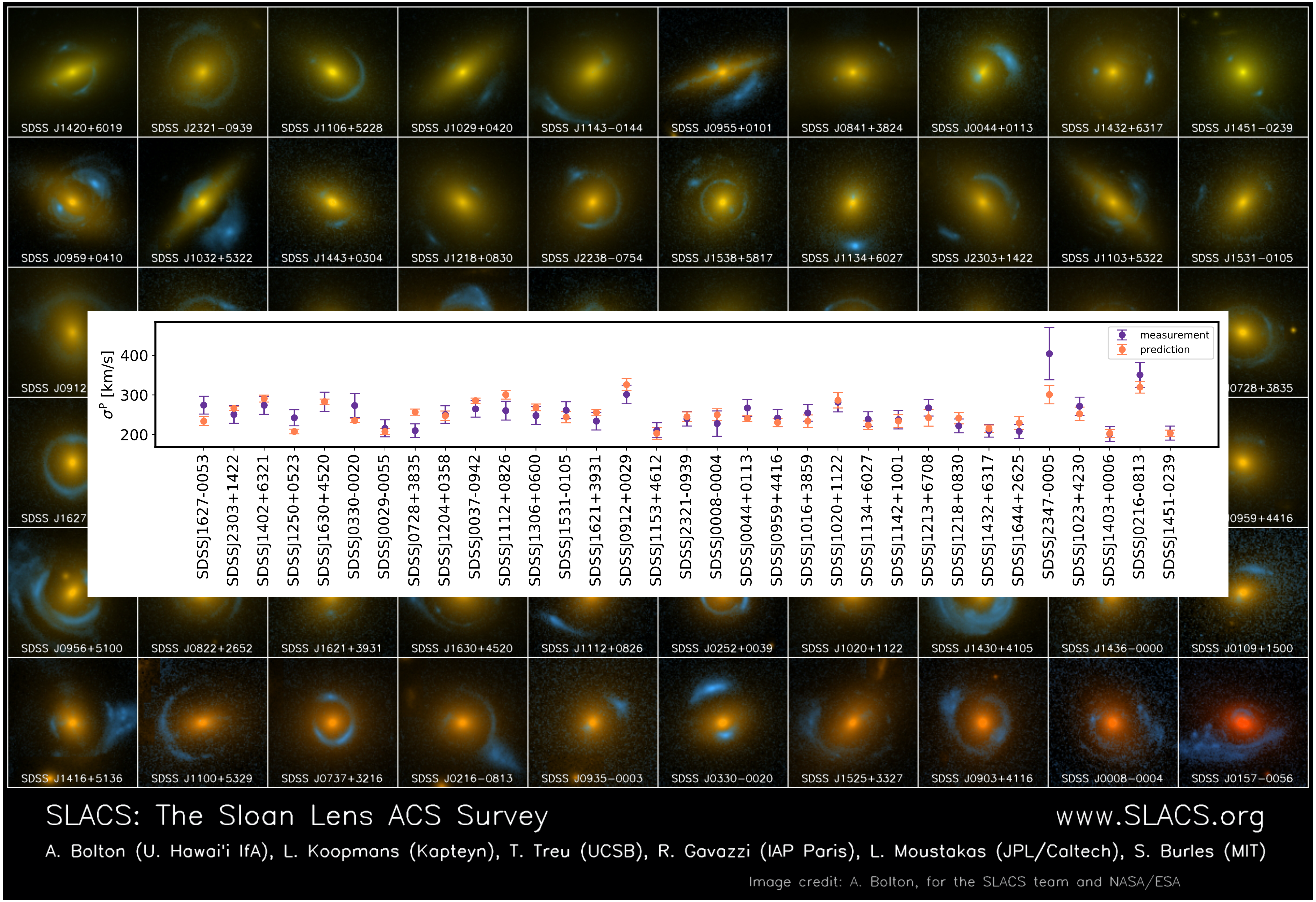



6 H0LiCOW + 1 STRIDES lens = TDCOSMO 7

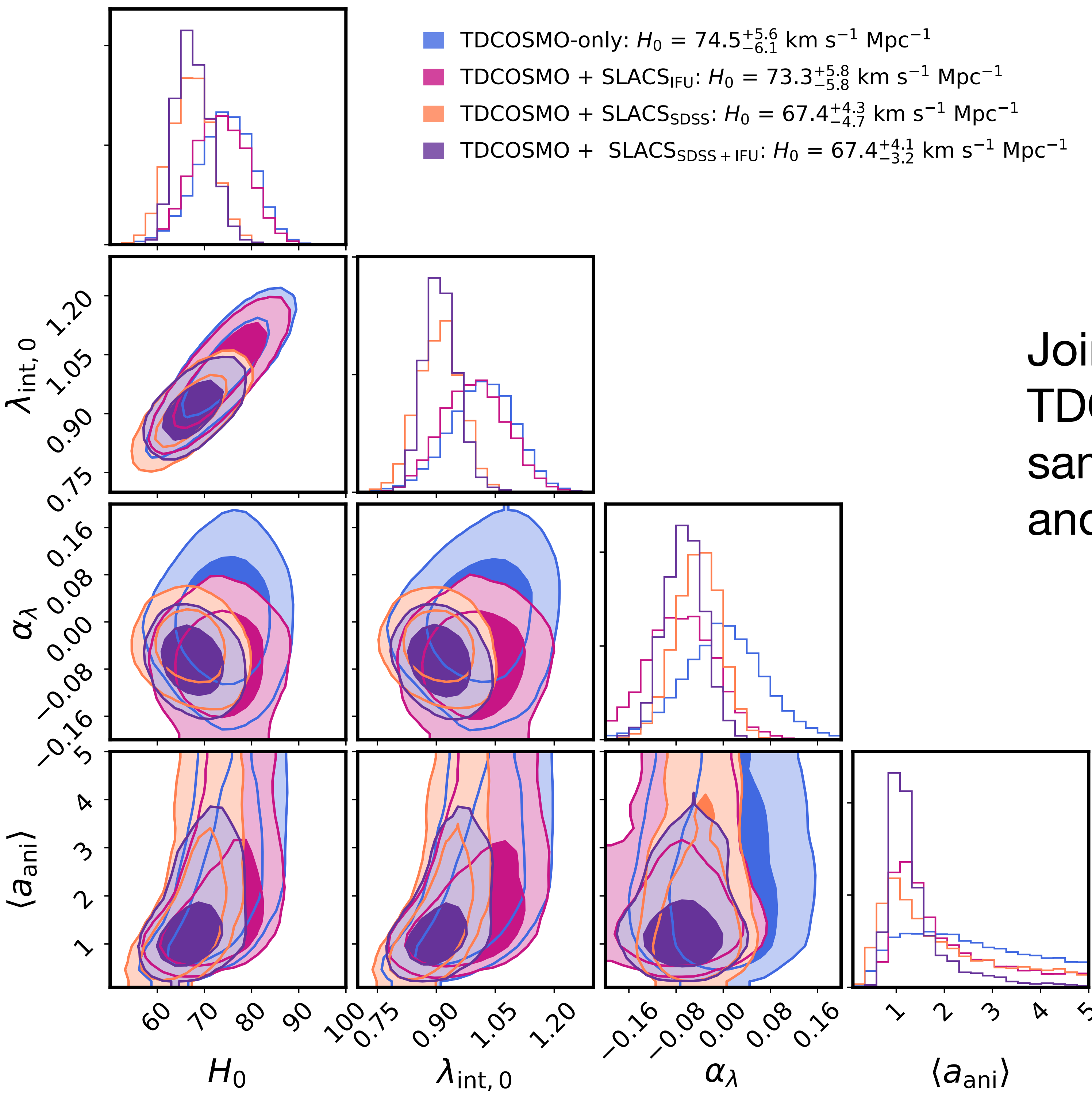
Performing the **identical** hierarchical analysis as performed on TDLMC

Using constraints and data products of previous analyses

# Section 6: SLACS analysis of galaxy density profiles



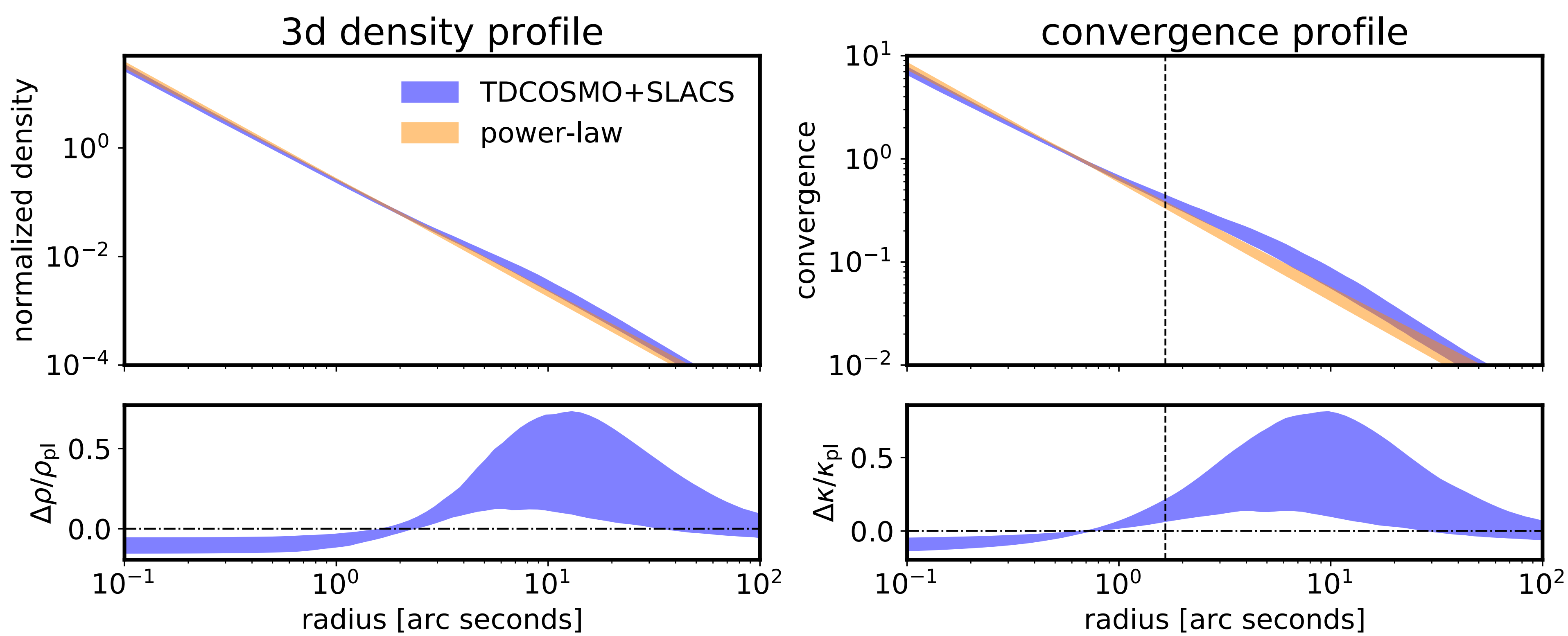
# Section 7: Hierarchical analysis of TDCOSMO+SLACS

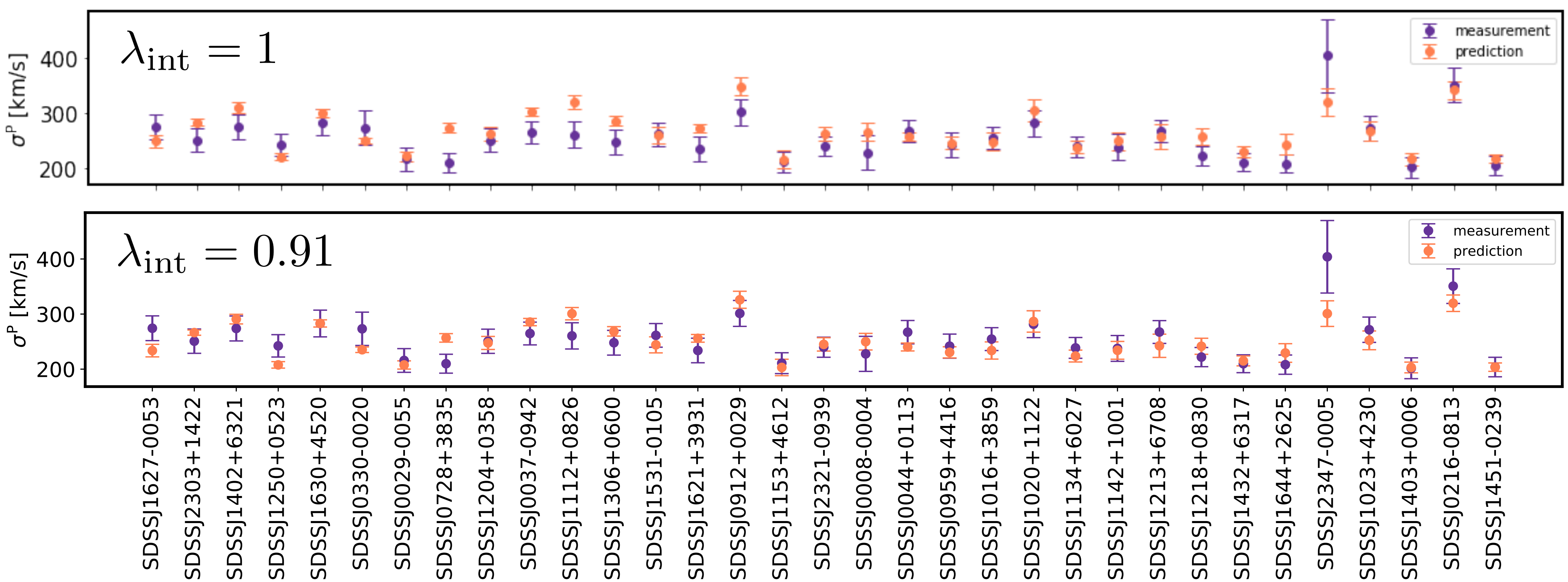


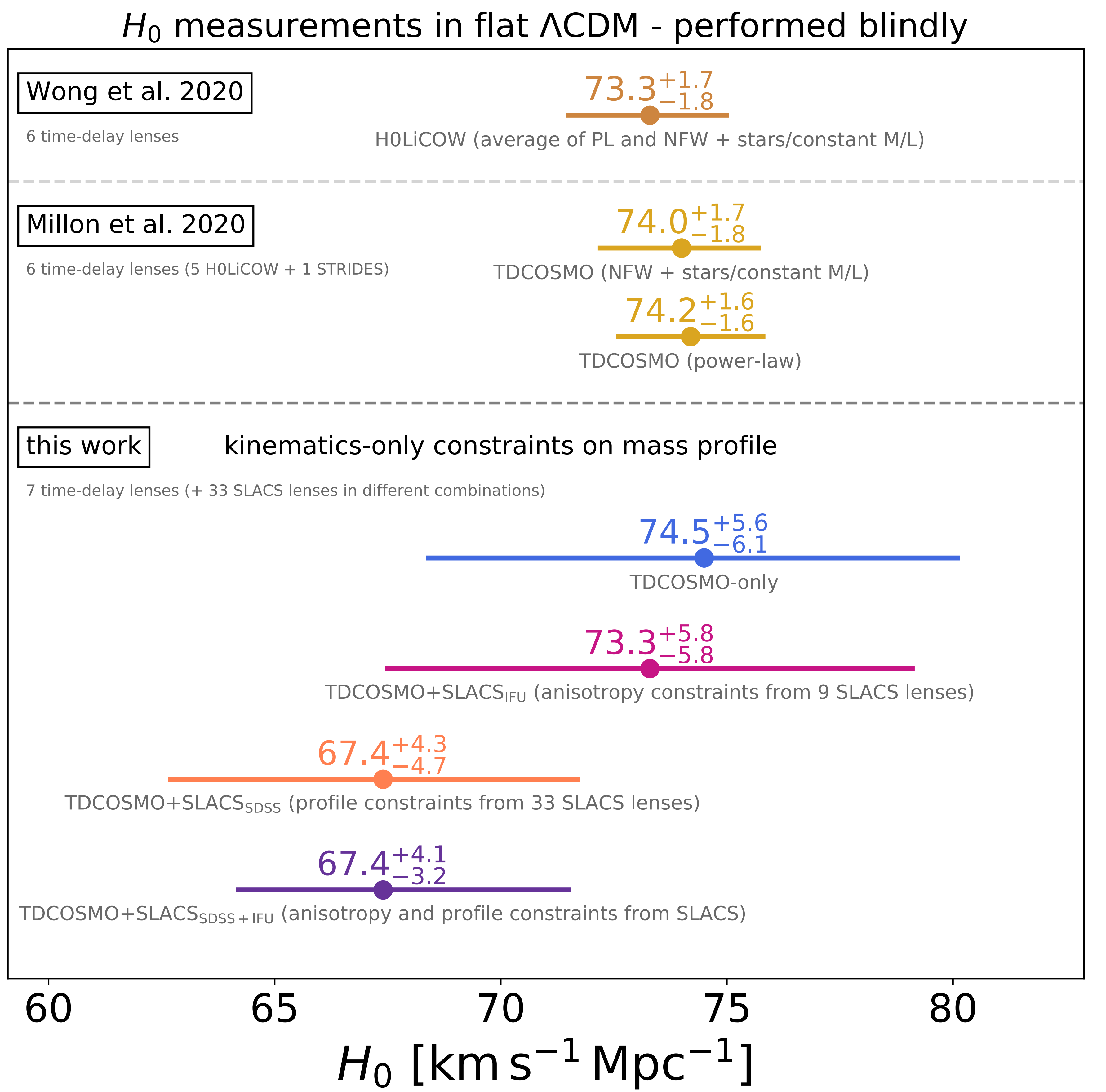

7 time-delay lenses from TDCOSMO
33 galaxy-galaxy lenses from SLACS
9 VIMOS IFU data sets

Joint population-level inference of TDCOSMO+SLACS (assuming same population of galaxy density and stellar anisotropy populations)

# Constraining galaxy density profiles with lensing and kinematics



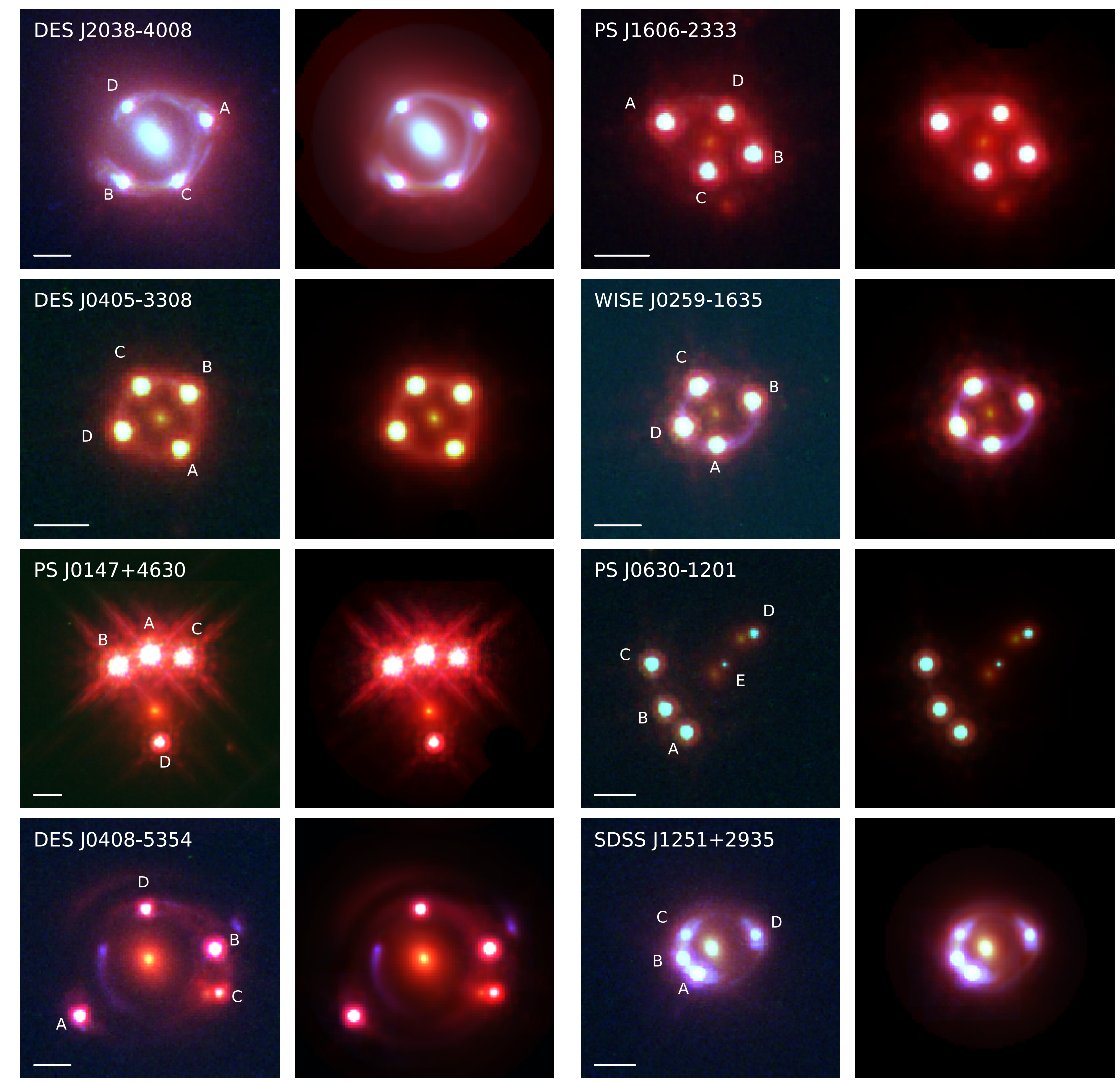





**SB** et al. 2020, TDOSMSO IV

# Way forward 1:
# data on time delay lenses

- spatially resolved stellar kinematics
  (i.e. VLT MUSE, Keck KCWI)
- improving kinematics measurement and modeling
  (mitigating errors on the population level)
- increase sample size of time-delay lenses
  (discovery, monitoring, high-resolution imaging, spectroscopy)




Shajib, **SB**+2018, STRIDES collaboration

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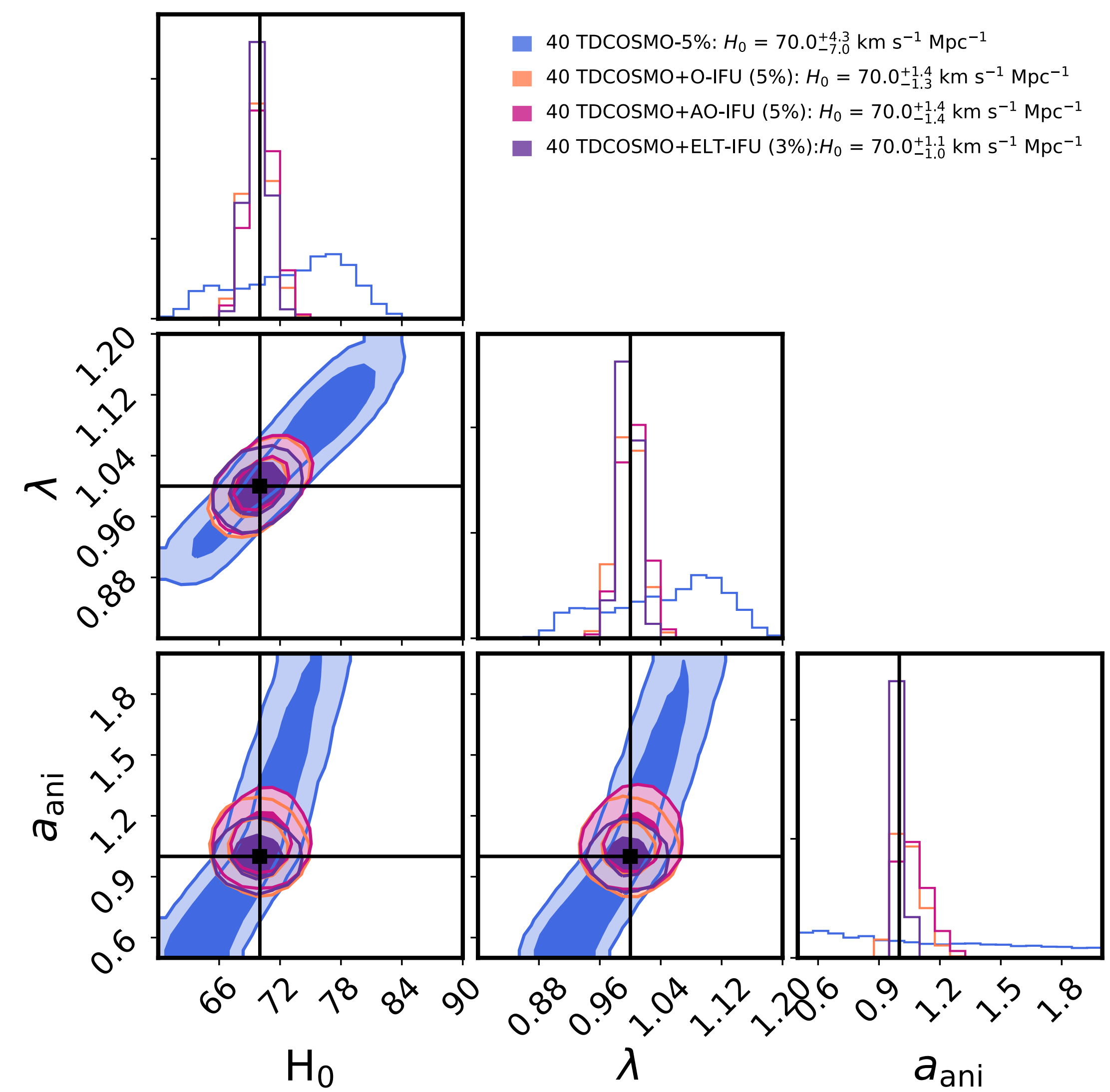
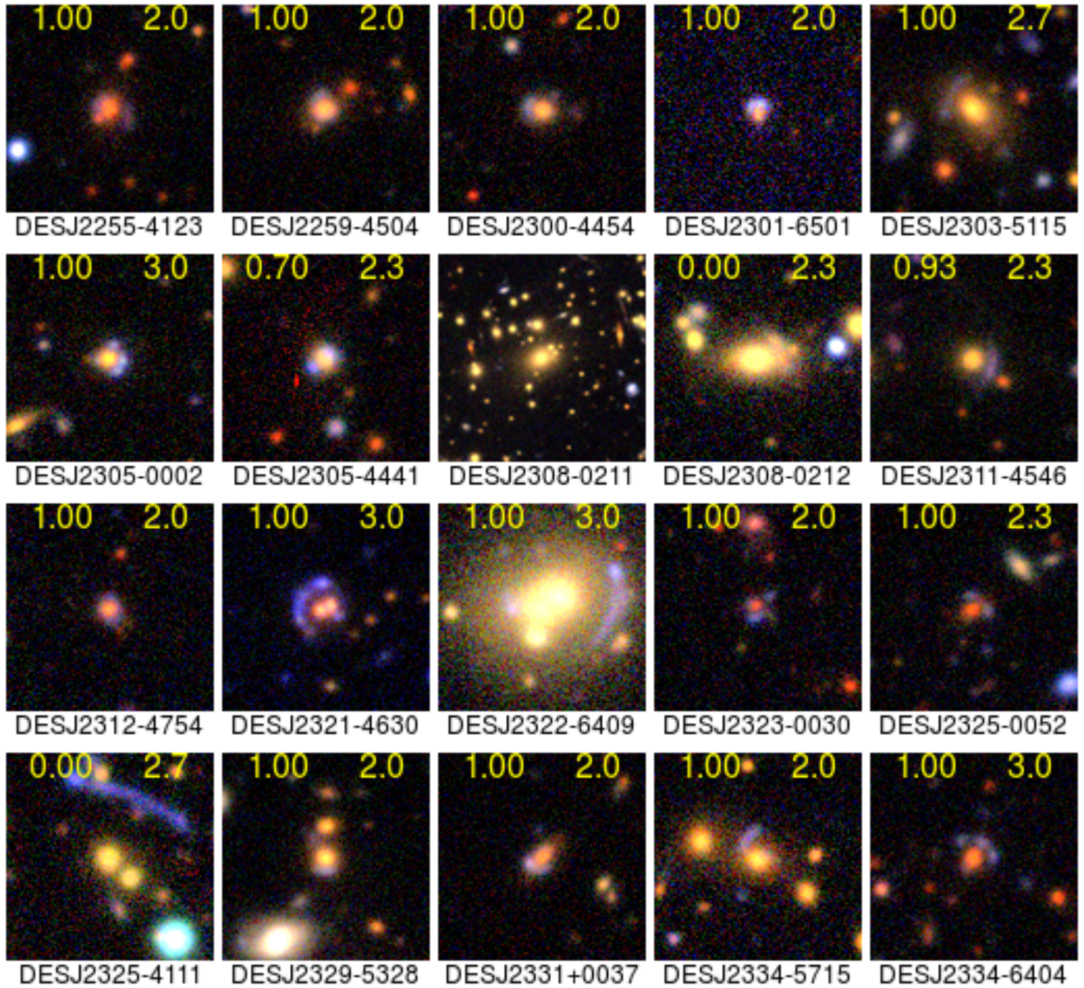



**SB** & Treu 2020, TDCOSMO V

# Way forward 2: adding external data sets

- external lensing sample matching precisely TDCOSMO
(same redshift, deflector morphology etc)
- increase sample size of galaxy-galaxy lenses (Rubin, Euclid, Roman observatories will discover 10’000+ lenses)
- add kinematic information from local elliptical galaxies
(SAURON, ATLAS3D, MASSIVE, …)




Jacobs+2019, DES collaboration

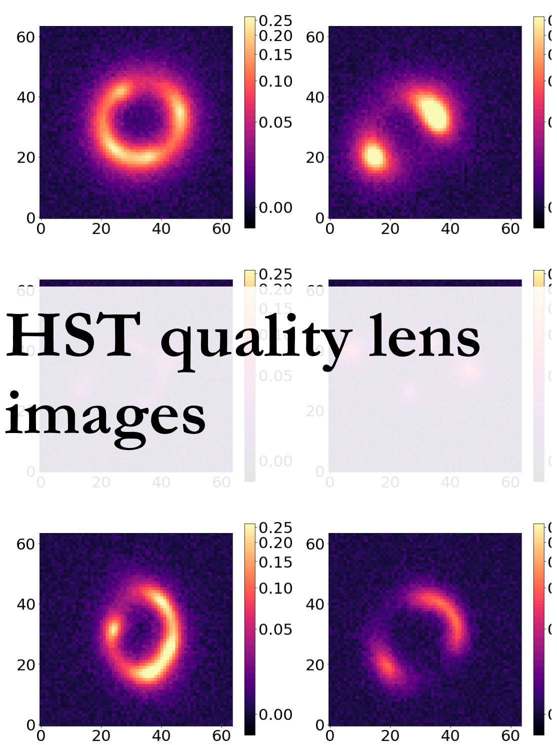
see also Birrer & Treu 2020, arXiv:2008.06157

# Way forward 3: challenge yourself!

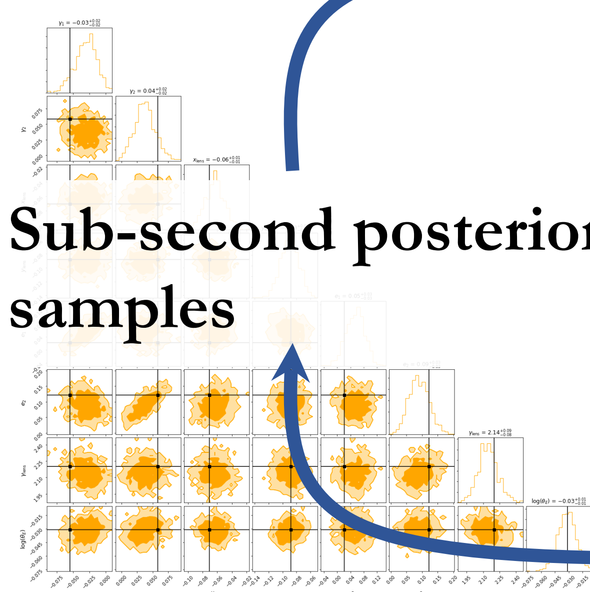
- Improve simulation products for better validation
  (full line-of-sight ray-tracing)
- Blind analysis challenges
  (blind data challenges for the community - as realistic as possible)
- Keep analysis blind!
  (continue assessing systematics regardless of the outcome of the experiment - challenge our intuition and assumptions)
- Open source
  (provide the full end-to-end analysis open source)

# Hierarchical Inference of Strong Lenses with Bayesian Neural Networks

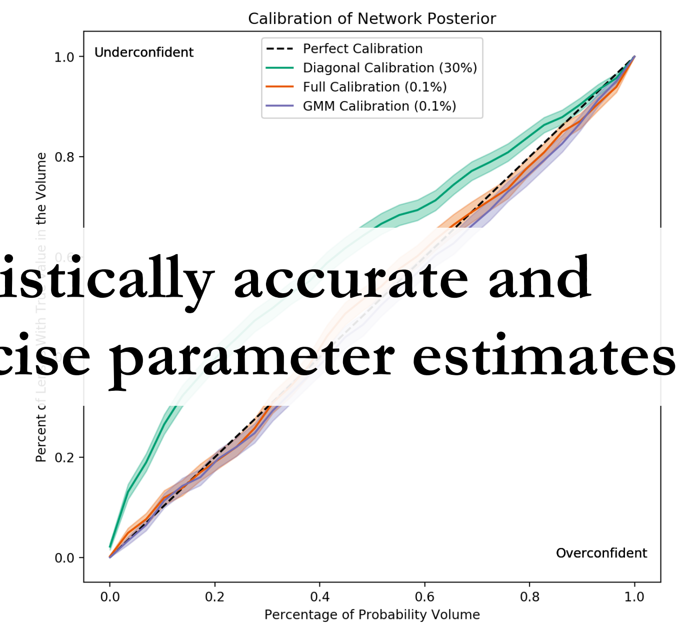
Assessing speed and accuracy of methods to scope with future data sets!



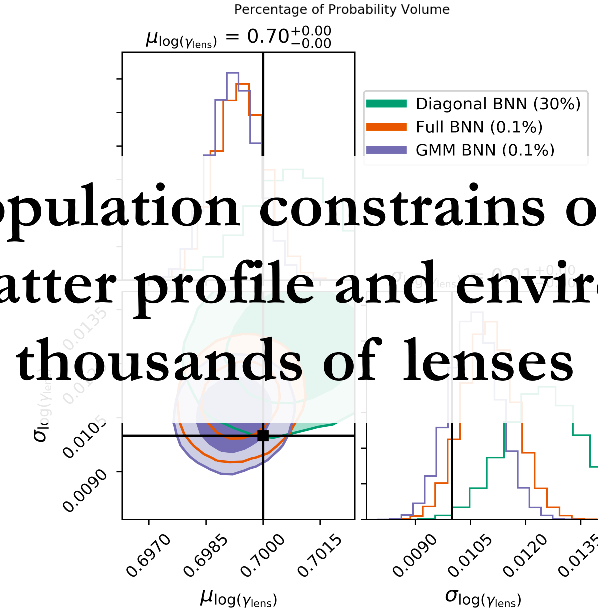
**HST quality lens images**



**Sub-second posterior samples**




**Statistically accurate and precise parameter estimates**




**Population constrains on matter profile and environment of thousands of lenses**

Carena-Wagner, Park, **SB** et al. 2020
Park, Carena-Wagner, **SB** et al. 2020
LSST-DESC collaboration

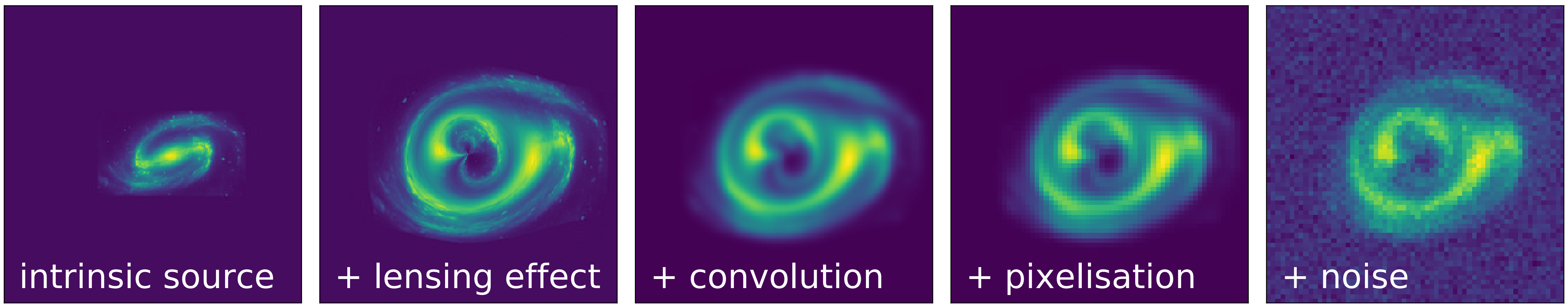
https://github.com/jiwoncpark/baobab
https://github.com/swagnercarena/ovejero
https://github.com/jiwoncpark/h0rton

# Software

sibirrer / lenstronomy

Unwatch 13 | Star 65 | Fork 29

Code | Issues 8 | Pull requests 2 | Actions | Projects | Wiki | Security | Insights | Settings




pypi package 1.3.0 | build passing | docs passing | coverage 97% | license MIT | arXiv 1803.09746

**Full software, scripts and data released for Birrer+19, 20**

**Astropy affiliated!** astropy — A Community Python Library for Astronomy

The development is coordinated on GitHub and contributions are welcome. The documentation of `lenstronomy` is available at readthedocs.org and the package is distributed over PyPI.

## Installation

**SB** et al. 2015, **SB** & Amara 2018

```
$ pip install lenstronomy --user
```

**https://github.com/sibirrer/lenstronomy**

# Affiliated packages

Here is an (incomplete) list of packages and wrappers that are using lenstronomy in various ways for specific scientific applications:

- baobab: Training data generator for hierarchically modeling of strong lenses with Bayesian neural networks.
- dolphin: Automated pipeline for lens modeling based on lenstronomy.
- hierArc: Hierarchical Bayesian time-delay cosmography to infer the Hubble constant and galaxy density profiles in conjunction with lenstronomy.
- lenstruction: Versatile tool for cluster source reconstruction and local perturbative lens modeling.
- SLITronomy: Updated and improved version of the Sparse Lens Inversion Technique (SLIT), developed within the framework of lenstronomy.
- LSSTDESC SLSprinkler: The DESC SL (Strong Lensing) Sprinkler adds strongly lensed AGN and SNe to simulated catalogs and generates postage stamps for these systems.
- lensingGW: A Python package designed to handle both strong and microlensing of compact binaries and the related gravitational-wave signals.
- ovejero: Conducts hierarchical inference of strongly-lensed systems with Bayesian neural networks.
- h0rton: H0 inferences with Bayesian neural network lens modeling.
- deeplenstronomy: Tool for simulating large datasets for applying deep learning to strong gravitational lensing.

These packages come with their own documentation and examples - so check them out!

**https://github.com/sibirrer/lenstronomy/blob/master/AFFILIATEDPACKAGES.rst**

**List of contributors - thank you very much!**

**https://github.com/sibirrer/lenstronomy/blob/master/AUTHORS.rst**