Anomalies in an EFT: The On-Shell Way

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Work in Progress with C. Csaki, A. Gomes

Recap: what is an Anomaly?

- A classical symmetry, violated at the quantum level
- Corresponds to non-invariance of the path integral measure
- Example: the abelian anomaly for an axial current

$$\mathcal{L} = -\frac{1}{4} F^{\alpha}_{\mu\nu} F^{\alpha\mu\nu} - \overline{\psi} \not{D} \psi$$

Axial rotation: $U(x) = e^{i\epsilon(x) q \gamma_5}$

$$\psi(x) \to \psi'(x) = U(x)\psi(x)$$

$\xrightarrow{\text{Noether procedure}}$

$$J_5^{\mu} = iq\,\overline{\psi}\gamma^{\mu}\gamma_5\psi$$

Classically conserved

Quantum mechanically:

$$\partial_\mu \left\langle J_5^{\mu}(x) \right\rangle = \frac{q^3}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}(x)$$

Anomalous Ward identity

Gauge Anomalies - the Big No-No

- Gauging a symmetry $\longrightarrow$ coupling to a current

$$S = \int \mathcal{D}A \, e^{i\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\mu \langle J^\mu \rangle\right)}$$

- If $\partial_\mu \langle J^\mu \rangle \neq 0$ gauge invariance is broken

- (If A is massless) we cannot establish equivalence of Lorentz and Unitary gauge

$\longrightarrow$ Cannot quantize Lorentz invariant theory

Solution in SM:

Anomaly cancellation!
Charges of all SM fermions conspire to cancel all triangle diagrams

Anomaly Cancelation : a Toy Example

D'Hoker and Farhi 84'

- $SU(2)_L$ x $U(1)_R$ gauge theory with N=2n generations of Weyl fermions

$$\psi_L^i(\mathbf{2})_{y_i} \qquad \psi_R^i = \left(\psi_{R1}^i(\mathbf{1})_{y_i+\frac{1}{2}},\, \psi_{R2}^i(\mathbf{1})_{y_i-\frac{1}{2}}\right)$$

- The Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{1\mu\nu}^2 - \frac{1}{2}\,\mathrm{Tr}\,F_{2\mu\nu}^2 + \sum_{i=1}^{N}\left[i\overline{\psi}_L^i \not{D}_L^i \psi_L^i + i\overline{\psi}_R^i \not{D}_R^i \psi_R^i\right]$$

$$D_{L;\mu}^i = \partial_\mu + ig_2 A_\mu + ig_1 y_i B_\mu \qquad D_{R;\mu}^i = \partial_\mu + ig_1\left(y_i + \tau_3\right) B_\mu$$

- Fermions couple chirally $\longrightarrow$ potential $U(1)_R^3$ and $SU(2)_L xU(1)_R$ anomalies

Anomaly Cancelation : a Toy Example

D'Hoker and Farhi 84'

- Fermions couple chirally $\longrightarrow$ potential $U(1)_R{}^3$ and $SU(2)_L{}^2 x U(1)_R$ anomalies

U(1)^3 anomaly:

$$\partial_\mu \left\langle J^\mu_{\mathrm{U}(1)} \right\rangle_{\mathrm{U}(1)^2} = -\frac{g_1^2}{48\pi^2} F_{1\mu\nu}\tilde{F}_1^{\mu\nu} \sum \left[2y_i^3 - (y_i + \frac{1}{2})^3 - (y_i - \frac{1}{2})^3 \right]$$

Mixed anomaly:

$$\partial_\mu \left\langle J^\mu_{\mathrm{U}(1)} \right\rangle_{\mathrm{SU}(2)^2} = -\frac{g_2^2}{24\pi^2} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu \operatorname{Tr} \left[A_\nu \partial_\kappa A_\lambda + \frac{1}{2} i g_2 A_\nu A_\kappa A_\lambda \right] \sum y_i$$

$$\partial_\mu \left\langle J^{\mu\, a}_{\mathrm{SU}(2)} \right\rangle_{\mathrm{SU}(2)\times \mathrm{U}(1)} = -\frac{g_1\, g_2}{24\pi^2} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu \operatorname{Tr} \left[\tau^a \left(2B_\nu \partial_\kappa A_\lambda + \frac{1}{2} i g_2 B_\nu A_\kappa A_\lambda \right) \right] \sum y_i$$

- Anomalies cancel if $\sum y_i = 0$

Toy Example: Higgsing the Theory

D'Hoker and Farhi 84'

- What happens if we Higgs the theory?
- Introduce Higgs $\phi(\mathbf{2})_{\frac{1}{2}}$ with a VEV that breaks $SU(2)_L \times U(1)_R \to U(1)_V$
- Choose Higgs quartic, gauge couplings and Yukawas such that

$$m_Z,\, m_W,\, m^f_{i<N},\, \ll \mathrm{E} \ll m^f_N,\, m_h$$

- Integrate the radial mode and the Nth generation of fermions

$$\downarrow$$

EFT for A, Z, W with N-1 "massless" fermions and $\sum y_i \neq 0$

Is this EFT consistent? How can we quantize in a Lorentz invariant way?

Anomaly Cancelation in the EFT

D'Hoker and Farhi 84'
Goldstone and Wilczek 81'

- Is the EFT consistent? Missing Nth generation of fermions
- D'Hoker and Farhi:
  - Integrating out radial mode leaves an EFT for the Goldstone matrix U
  - Integrating out the Nth generation generates gauged WZW (or GW) terms
  - The gauged WZW cancel the anomaly from the N-1 light fermions

- Effective action:

$$\mathcal{S}_{\mathrm{EFT}} = \underbrace{\mathcal{S}_{\mathrm{nl\sigma m}}(A_\mu, B_\mu, \psi^i, U)} + \Gamma_{\mathrm{WZW}}(U) + \int d^4x\, g_1\, y_N\, B_\mu J^\mu_{\mathrm{GW}}$$

Fixed by the nonlinear realization of SU(2)xU(1)

$U \in \dfrac{\mathrm{SU}(2)_L \times U(1)_R}{\mathrm{U}(1)_V}$ is the Goldstone matrix

Anomaly Cancelation in the EFT

D'Hoker and Farhi 84'
Goldstone and Wilczek 81'

- Improved Goldstone-Wilczek term $\int d^4x\, g_1\, y_N\, B_\mu \hat{J}^\mu_{\rm GW}$ where

$$J^\mu_{\rm GW} = \frac{1}{24\pi^2}\epsilon^{\mu\alpha\beta\gamma}{\rm Tr}\Bigg[\underbrace{U^\dagger D_\alpha U U^\dagger D_\beta U U^\dagger D_\gamma U - i\frac{3g_2}{2}F_{2\alpha\beta}D_\gamma U U^\dagger - i\frac{3g_1}{2}F_{1\alpha\beta}\tau_3 U^\dagger D_\gamma U}_{\text{Cancels the } U(1)_R^3 \text{ and } SU(2)_L^2 \times U(1)_R \text{ anomalies}}$$

$$\underbrace{-g_2^2\left(A_\alpha F_{2\beta\gamma} - \frac{1}{2} i g_2 A_\alpha A_\beta A_\gamma\right)}\Bigg]$$

Local counterterm that does not cancel the $SU(2)_L{}^2 xU(1)_R$ anomaly but only shifts it only to the $U(1)_R$. Analogous to the regulator ambiguity of the chiral anomaly - we can choose to preserve $SU(2)_L$ or $U(1)_R$, but not both.

Anomaly Cancelation in the EFT

D'Hoker and Farhi 84'

Main point: The U(1)^3 and U(1)xSU(2)^2 anomalies of the GW current are exactly equal to those of the integrated out Nth fermion generation, and the anomalies are canceled.

$$\partial_\mu \left\langle J^{\mu\, a}_{\mathrm{SU(2)}} \right\rangle_{\mathrm{SU(2)\times U(1)}} = -\frac{g_1\, g_2}{24\pi^2}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu \, \mathrm{Tr}\left[\tau^a \left(2B_\nu\partial_\kappa A_\lambda + \frac{1}{2} i g_2 B_\nu A_\kappa A_\lambda\right)\right] \sum_i^{N-1} y_i + \delta S_{GW} = 0$$

From the local counterterm that makes the mixed anomaly violate U(1)$_R$ but not SU(2)$_L$

$$\partial_\mu \left\langle J^{\mu}_{\mathrm{U(1)}} \right\rangle_{\mathrm{U(1)^2}} = \frac{1}{32\pi^2} g_1^2 F_{1\mu\nu}\tilde{F}_1^{\mu\nu} \sum_i^{N-1} y_i + \partial_\mu\, J^\mu_{\mathrm{GW}}|_{A=0} = 0$$

$$\partial_\mu \left\langle J^{\mu}_{\mathrm{U(1)}} \right\rangle_{\mathrm{SU(2)^2}} = -\frac{g_2^2}{24\pi^2}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu \, \mathrm{Tr}\left[\left(2B_\nu\partial_\kappa A_\lambda + \frac{1}{2} i g_2 B_\nu A_\kappa A_\lambda\right)\right] \sum_i^{N-1} y_i + \partial_\mu J^\mu_{\mathrm{GW}}|_{B=0} = 0$$

From the Goldstone dependent terms that cancel the anomalies

Quick Summary

- Gauge anomalies are bad (... or were bad in 1984)

- Gauge anomalies in an EFT
  - Start in the UV with an anomaly free theory
  - Higgs the theory and integrate out one heavy generation of fermions
  - The EFT in anomaly free
  - The anomaly from the light fermions is canceled by the GW current

Is this a *gauge independent* statement?

Another Look at EFT Anomaly Cancelation

- Back to the improved GW current:

$$J^\mu_{\rm GW} = \frac{1}{24\pi^2}\epsilon^{\mu\alpha\beta\gamma}{\rm Tr}\left[U^\dagger D_\alpha U U^\dagger D_\beta U U^\dagger D_\gamma U - i\frac{3g_2}{2}F_{2\alpha\beta}D_\gamma U U^\dagger - i\frac{3g_1}{2}F_{1\alpha\beta}\tau_3 U^\dagger D_\gamma U \right.$$
$$\left. -g_2^2\left(A_\alpha F_{2\beta\gamma} - \frac{1}{2}ig_2 A_\alpha A_\beta A_\gamma\right)\right]$$

- Go to unitary gauge: U is eaten, left only with the local counterterm

$$g_1 B_\mu J^\mu_{\rm GW} = -\frac{g_1 g_2^2}{24\pi^2}\epsilon^{\mu\alpha\beta\gamma}B_\mu{\rm Tr}\left[\left(A_\alpha F_{2\beta\gamma} - \frac{1}{2}ig_2 A_\alpha A_\beta A_\gamma\right)\right]$$

But this counterterm only *shifts* the mixed anomaly to the U(1)...

What cancels the anomalies?

The Plot Thickens

- Well known paper by Preskill 91':

Gauge Anomalies in an Effective Field Theory

  - There is no problem with quantizing an anomalous gauge theory, as long as the gauge bosons are *massive* (the theory is in the Higgs phase or "spontaneously broken").

  - D'Hoker and Farhi's anomaly cancelation is a *gauge artifact*
  - The theory in the Higgs phase has a cutoff Λ related to the GB mass

Preskill's Argument

- Massive U(1) coupled to a single Weyl fermion

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mu^2}{2}A_\mu A^\mu + i\bar{\psi}_L \not{D}\psi$$

Under $A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \omega$ the theory has an anomaly $\frac{e^2 q^3}{48\pi^2}\omega F_{\mu\nu}\tilde{F}^{\mu\nu}$

We can cancel the anomaly by introducing a gauge artifact field b:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left(\partial_\mu b - \mu A_\mu\right)^2 + i\bar{\psi}_L \not{D}\psi - \frac{e^3 q^3}{48\pi^2 \mu}\, b\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

The anomaly cancels if $b \to b + \frac{\mu}{e}\omega$ under a gauge transformation

Preskill's Argument

- In the b theory, we can always go to unitary gauge and set b=0, and the Lagrangian reduces to the original, anomalous Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mu^2}{2}A_\mu A^\mu + i\bar{\psi}_L \not{D}\psi$$

- The two theories, with and without the b field, are identical in unitary gauge, and so all of their physical observables are the same

- Consequently, there is *no physical difference* between an anomalous massive theory, and a massive theory with "WZW" anomaly cancellation

A Pause for Confusion

- By Preskill's argument, an *anomalous* EFT is equivalent to an *anomaly canceled* EFT. But what about the Ward identities? Anomalous or not?

- Also...
    - Didn't we say that anomalous gauge theories cannot be quantized?
    - Apparently *massive* anomalous gauge theories *can* be quantized

Our research question:

What's the fundamental difference between the massless and massive theory that only allows to quantize the latter?

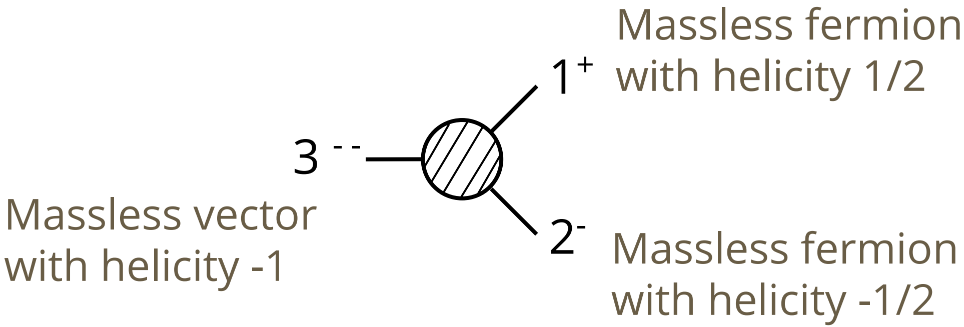
We want a manifestly gauge invariant answer!

On-Shell Methods: a Manifestly Gauge Invariant Formalism

- To understand the consistency of massive anomalous gauge theories, we first focus on the inconsistency of massless ones
- This inconsistency should arise in a *gauge invariant* way, i.e. in scattering amplitudes
- The *on-shell formalism* allows us to compute tree and loop level scattering amplitudes without introducing any action / gauge freedom
- We first review the formalism, and then arrive at the on-shell notion of a gauge anomaly as *tension between locality and unitarity @ 1-loop*

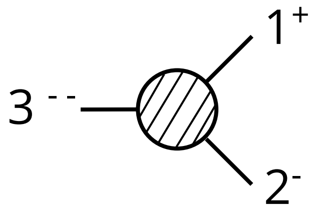
On-Shell Methods: a Manifestly Gauge Invariant Formalism

- In the *on-shell formalism*, a theory is specified not by an action, but by the representations of the scattering particles under the Lorentz group (actually little group)
- The basic building blocks are tree-level three-point amplitudes



Extremely Quick Intro to Spinor-Helicity

- 3-pt amplitudes are uniquely determined by their little group transformation. i.e. saying that a helicity -1 vector scatters with helicity ±1/2 fermion is enough to fix the amplitude (up to color factors)



$$= \frac{\langle 23\rangle^2}{\langle 12\rangle}$$

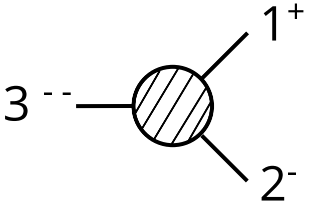
The spinor-helicity variables are defined by $p_i^\mu\,\bar{\sigma}_\mu^{\dot{\alpha}\alpha} = |i\rangle^{\dot{\alpha}}\,[i|^\alpha$ or explicitly

$$|i\rangle^{\dot{\alpha}} = \sqrt{2E_i}\begin{pmatrix} \cos\theta_i/2 \\ e^{i\phi_i}\sin\theta_i/2 \end{pmatrix}$$ with little group weight -1/2

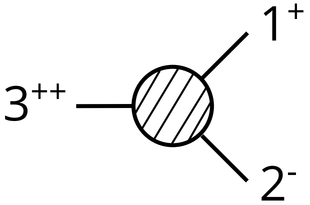
$$[i|^{\alpha} = \sqrt{2E_i}\begin{pmatrix} \cos\theta_i/2 \\ e^{-i\phi_i}\sin\theta_i/2 \end{pmatrix}$$ with little group weight +1/2

Rules For Forming Helicity Amplitudes

- The little group transformation of the amplitude is known and has to be saturated by stacking up spinor-helicity variables.
- Dotted and undotted indices have to be contracted separately, with $\epsilon^{\dot{\alpha}\dot{\beta}},\ \epsilon^{\alpha\beta}$
- A spinor in the denominator has the opposite little group weight (helicity)



$$= \frac{\langle 23\rangle^2}{\langle 12\rangle}$$



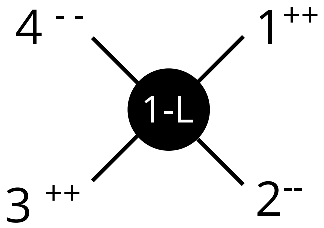
$$= \frac{[13]^2}{[12]}$$

- Locality requires that all amplitudes factorize on their poles. Since 3-pt amplitudes can't factorize, they cannot have poles. Dimensional analysis further reduces them to these unique expressions.

Massless 4-Vector 1-loop Amplitude

Chen, Huang and McGady 14’

- A theory with a single Weyl Fermion coupled to massless vectors is inconsistent
- The inconsistency arises in the 1-loop 4-vector amplitudes
    - Constructed in a manifestly *unitary* way, these amplitudes cannot be also *local*
    - This is seen using *generalized unitarity*



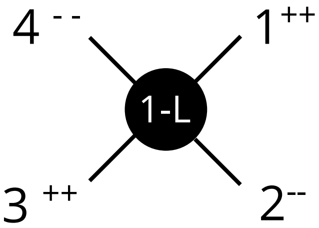
$$= \underbrace{\frac{\langle 24\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}}_{\text{Little group factor } A^{PT} \text{ (Parke-Taylor)}} \underbrace{F\left(s_{12}, s_{13}, s_{14}\right)}_{\text{Function of Mandelstams determined by unitarity cuts up to rational terms}}$$

How do we determine F?

Massless 4-Vector 1-loop Amplitude

Bern, Dunbar, Kosower 95'
Forde 07'

Expand amplitude in basis of all possible loop integrals (like Passarino-Veltman)



$$= A^{\mathrm{PT}} \Big[c_4 I_4\left(s_{12}, s_{14}\right) + \left(c_3^{(12)} + c_3^{(34)}\right) I_3\left(s_{12}\right)$$

$$+ \left(c_3^{(14)} + c_3^{(23)}\right) I_3\left(s_{14}\right) + c_2^{(12)} I_2\left(s_{12}\right) + c_2^{(14)} I_2\left(s_{14}\right) + R(s_{12}, s_{13}, s_{14}) \Big]$$

The $I_i\left(s\right)$ are known master integrals with unique branch cuts. We can extract the coefficients c_i by performing unitarity cuts on both sides. R is a rational term, unfixed by unitarity alone.

Massless 4-Vector 1-loop Amplitude

Bern, Dunbar, Kosower 95'
Forde 07'

Let's extract c_4 by performing three unitarity cuts on both sides of the expansion

On the RH side we have

$$c_4\, I_4\,(s_{12}, s_{14}) \;=\; -\,ic_4 \int \frac{d^4\ell}{(2\pi)^4}\, \frac{1}{\ell^2(\ell-p_1)^2(\ell-p_2)^2(\ell+p_4)^2}$$

Cutting the integral means replacing:

$$\frac{i}{(\ell-x)^2} \to 2\pi\,\delta\left[(\ell-x)^2\right]$$

$$c_4\, I_4\,(s_{12}, s_{14}) \to -c_4 \int d^4\ell\,\delta\left[\ell^2\right]\,\delta\left[(\ell-p_1)^2\right]\,\delta\left[(\ell-p_2)^2\right]\delta\left[(\ell+p_4)^2\right]$$

Massless 4-Vector 1-loop Amplitude

Bern, Dunbar, Kosower 95'
Forde 07'

Next, we eliminate the delta functions by choosing an appropriate parametrization for ℓ

$$\ell^\mu = t\frac{\langle 1|\bar{\sigma}^\mu|4]}{2} \quad \text{such that} \quad \ell^2 = (\ell - p_1)^2 = (\ell + p_4)^2 = 0,$$

and solve for t so that $(\ell - p_2)^2 = 0$:

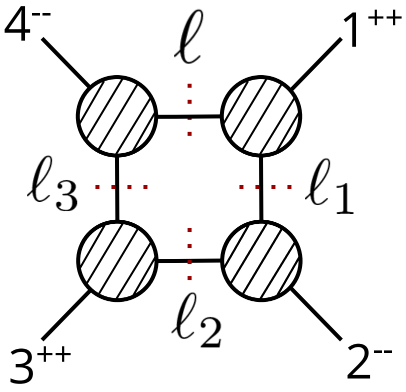
$$\boxed{t = -\frac{\langle 4|2|1]}{s_{13}}} \longrightarrow c_4\, I_4 \;\to\; -c_4 \int d^4\ell \prod_i \delta\left[\ell_i\right] = -c_4$$

Massless 4-Vector 1-loop Amplitude

Bern, Dunbar, Kosower 95'
Forde 07'

On the RH side, we also cut $\ell^2 = (\ell - p_1)^2 = (\ell + p_4)^2 = 0$

By *cutting rules*, the amplitude has the form



$$= \frac{1}{2} A_1 \, A_2 \, A_3 \, A_4$$

where the A_i are tree level 3-pt amplitudes involving the external vector and the internal Weyl Fermion

Massless 4-Vector 1-loop Amplitude

Bern, Dunbar, Kosower 95'
Forde 07'

In our case

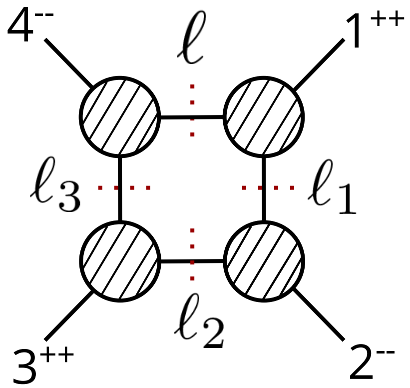
$$A_1\,A_2\,A_3\,A_4 = \frac{[\ell\,1]^2}{[\ell\ell_1]} \times \frac{\langle\ell_1\,2\rangle^2}{\langle\ell_1\ell_2\rangle} \times \frac{[\ell_2\,3]^2}{[\ell_2\ell_3]} \times \frac{\langle\ell_3\,4\rangle^2}{\langle\ell_3\ell\rangle}$$

$$|\ell\rangle = t\,|1\rangle \qquad |\ell] = |4]$$

$$|\ell_1\rangle = t\,|1\rangle \qquad |\ell_1] = -t^{-1}\,|1] + |4]$$

$$|\ell_2\rangle = t\,|1\rangle + |4\rangle \qquad |\ell_2] = \frac{s_{12}}{s_{14}}\left(t^{-1}\,|1] - |4]\right)$$

$$|\ell_3\rangle = t\,|1\rangle + |4\rangle \qquad |\ell_3] = |4]$$



For $t = -\dfrac{\langle 4|2|1]}{s_{13}}$ all satisfy $\ell_i^2 = 0$

Massless 4-Vector 1-loop Amplitude

Bern, Dunbar, Kosower 95’
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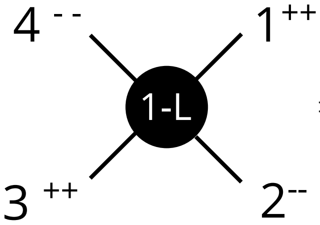
Similarly, we obtain all other coefficients in



$$= A^{\mathrm{PT}}\Big[c_4 I_4\left(s_{12}, s_{14}\right) + \left(c_3^{(12)} + c_3^{(34)}\right) I_3\left(s_{12}\right) + \left(c_3^{(14)} + c_3^{(23)}\right) I_3\left(s_{14}\right) + c_2^{(12)} I_2\left(s_{12}\right) + c_2^{(14)} I_2\left(s_{14}\right) + R(s_{12}, s_{13}, s_{14})\Big]$$

$$\begin{aligned}
c_4^L &= \frac{s_{12}^2 s_{14}^4}{2s_{13}^4} \quad , \quad c_4^R = \frac{s_{12}^4 s_{14}^2}{2s_{13}^4}\\
c_3^{L,(14)} = c_3^{L,(23)} &= \frac{s_{12} s_{14}^4}{2s_{13}^4} \quad , \quad c_3^{R,(14)} = c_3^{R,(23)} = \frac{s_{12}^3 s_{14}^2}{2s_{13}^4}\\
c_3^{L,(34)} = c_3^{L,(12)} &= \frac{s_{14}^3 s_{12}^2}{2s_{13}^4} \quad , \quad c_3^{R,(34)} = c_3^{R,(12)} = \frac{s_{14} s_{12}^4}{2s_{13}^4}\\
c_2^{(12)} &= \frac{s_{14}\left(2s_{14}^2 - 5s_{12}s_{14} - s_{12}^2\right)}{6s_{13}^3} \quad , \quad c_2^{R,(12)} = \frac{s_{14}\left(2s_{14}^2 + 7s_{12}s_{14} + 11s_{12}^2\right)}{6s_{13}^3}\\
c_2^{(14)} &= \frac{s_{12}\left(2s_{12}^2 + 7s_{12}s_{14} + 11s_{14}^2\right)}{6s_{13}^3} \quad , \quad c_2^{R,(14)} = \frac{s_{12}\left(2s_{12}^2 - 5s_{12}s_{14} - s_{14}^2\right)}{6s_{13}^3} \,. \quad \text{(C.41)}
\end{aligned}$$

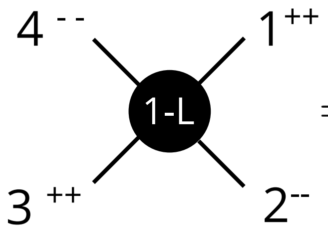
Coefficients for LH/RH fermions running in the loop. Since the vectorlike contribution cannot be anomalous, we set

$$c_i = c_i^L - c_i^R$$

Massless 4-Vector 1-loop Amplitude

Bern, Dunbar, Kosower 95’
Forde 07’

Similarly, we obtain all other coefficients in



$$= A^{\mathrm{PT}}\left[c_4 I_4\left(s_{12}, s_{14}\right)+\left(c_3^{(12)}+c_3^{(34)}\right) I_3\left(s_{12}\right)\right.$$
$$\left.+\left(c_3^{(14)}+c_3^{(23)}\right) I_3\left(s_{14}\right)+c_2^{(12)} I_2\left(s_{12}\right)+c_2^{(14)} I_2\left(s_{14}\right)+\underbrace{R(s_{12}, s_{13}, s_{14})}\right]$$

$$\begin{aligned}
c_4^L &= \frac{s_{12}^2 s_{14}^4}{2 s_{13}^4} \quad , \quad c_4^R = \frac{s_{12}^4 s_{14}^2}{2 s_{13}^4} \\
c_3^{L,(14)} = c_3^{L,(23)} &= \frac{s_{12} s_{14}^4}{2 s_{13}^4} \quad , \quad c_3^{R,(14)} = c_3^{R,(23)} = \frac{s_{12}^3 s_{14}^2}{2 s_{13}^4} \\
c_3^{L,(34)} = c_3^{L,(12)} &= \frac{s_{14}^3 s_{12}^2}{2 s_{13}^4} \quad , \quad c_3^{R,(34)} = c_3^{R,(12)} = \frac{s_{14} s_{12}^4}{2 s_{13}^4} \\
c_2^{(12)} &= \frac{s_{14}\left(2 s_{14}^2-5 s_{12} s_{14}-s_{12}^2\right)}{6 s_{13}^3} \quad , \quad c_2^{R,(12)} = \frac{s_{14}\left(2 s_{14}^2+7 s_{12} s_{14}+11 s_{12}^2\right)}{6 s_{13}^3} \\
c_2^{(14)} &= \frac{s_{12}\left(2 s_{12}^2+7 s_{12} s_{14}+11 s_{14}^2\right)}{6 s_{13}^3} \quad , \quad c_2^{R,(14)} = \frac{s_{12}\left(2 s_{12}^2-5 s_{12} s_{14}-s_{14}^2\right)}{6 s_{13}^3} \, . \quad \text{(C.41)}
\end{aligned}$$

What about the rational term?

Can’t determine from cuts!

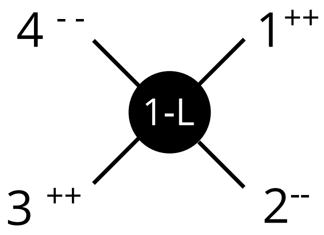
Coefficients for LH/RH fermions running in the loop. Since the vectorlike contribution cannot be anomalous, we set

$$c_i = c_i^L - c_i^R$$

Touching Base

Chen, Huang and McGady 14’

- We look for an inconsistency in the on shell construction of the 1-loop, massless 4-vector amplitude (on shell manifestation of a “gauge anomaly”)
- Using generalized unitarity, we can nail down the functional form of



up to an unknown rational term.

- Further constraint: *locality* implies that the full 1-loop amplitude, including the rational term, factorizes correctly on all of its poles
- For our inconsistent theory, *no rational term* can lead to correct factorization

Constraints on the Rational Term

Chen, Huang and McGady 14'

- First of all, our amplitude is *color ordered* so external legs cannot cross and we cannot have a pole in the u (or s_{13}) channel. Taking the limit $s_{13} \to 0$, we have

$$\lim_{s_{13}\to 0} A^{1-\text{loop}} = \lim_{s_{13}\to 0} A^{\text{PT}}\left[\sum c_i I_i + R\right] = \lim_{s_{13}\to 0} A^{\text{PT}}\left[-\frac{s_{12}}{s_{13}} + R\right]$$

- Demanding a sign flip under the cyclic shift $A(1^{++},2^{--},3^{++},4^{--}) \to A(4^{--},1^{++},2^{--},3^{++})$ we arrive at the only viable rational term:

$$R = \frac{s_{12} - s_{14}}{2 s_{13}}$$

- But $A^{\text{PT}} = \frac{[13]^2 \langle 24\rangle^2}{s_{12}s_{14}}$, and so the rational term modifies the residues of the full amplitude in the s_{12} and s_{14} channels.

Constraints on the Rational Term

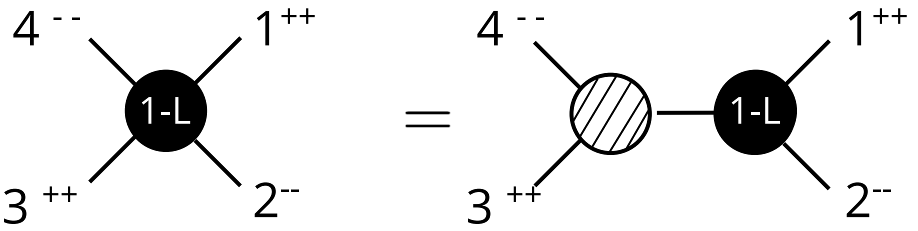
Chen, Huang and McGady 14’

- The extra s_{12} residue from the rational term is:

$$\underset{s_{12}\to 0}{\mathrm{Res}}\, A^{\mathrm{PT}} R \;=\; \frac{[13]^2 \langle 24\rangle^2}{2 s_{14}}$$

- By locality, this residue should be a product of two 3-pt amplitudes:

$$\underset{s_{12}\to 0}{\mathrm{Res}}$$



- However, there are no possible 3-pt amplitudes that can yield this residue

The On Shell Inconsistency of a Massless Anomalous Gauge Theory

- The inconsistency of a massless gauge anomalous theory arises in the 1-loop, 4-vector amplitude
- Constructed in a manifestly unitary way, there is no choice of rational term that could lead to consistent factorization on all channels
- From an on-shell perspective, the tension is between *unitarity* and *locality*
- This is different from our field theory intuition, where the gauge anomaly signals a disconnect between *unitarity* and *Lorentz invariance* (different gauges)

Chen, Huang and McGady 14'

The On Shell Consistency of a Massive Anomalous Gauge Theory

- The demonstrate the consistency of the massive theory, we developed a formalism for generalized unitarity with *massive* external vectors. This is a fusion of the generalized unitarity formalism of Forde 07' and the massive amplitude formalism of Arkani-Hamed, Huang and Huang 17'.

- Massive particles correspond to **bolded** spinors $|\mathbf{i}\rangle^{\dot\alpha}{}_I$, $[\mathbf{i}|^{J\alpha}$, transforming as □ of their SU(2) little group, and defined so that

$$p_i^\mu \bar\sigma_\mu^{\dot\alpha\alpha} = |\mathbf{i}\rangle^{\dot\alpha}{}_I \, [\mathbf{i}|^{I\alpha}$$

This work, 19'

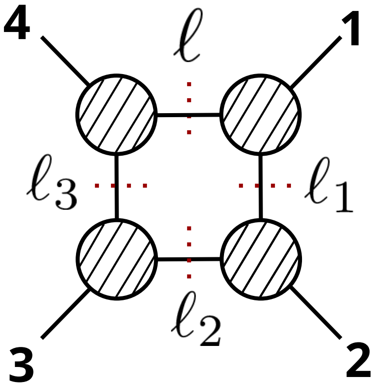
A 1-Loop Massive Amplitude

In this work we perform the first generalized unitarity calculation in the massive amplitude formalism. Expanding in the master integral basis, we have as usual

$$A^{1-\text{loop}}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}) = \sum_i c_i \, I_i \left(s_{12}, s_{14}\right) + R$$

Once again we calculate c_4 by doing a quadruple cut

Example: Calculation of c_4



$$= \frac{1}{m^4} \langle \ell \mathbf{1} \rangle [\mathbf{1} \ell_1] \times \langle \ell_1 \mathbf{2} \rangle [\mathbf{2} \ell_2] \times$$

$$\times \langle \ell_2 \mathbf{3} \rangle [\mathbf{3} \ell_3] \times \langle \ell_3 \mathbf{4} \rangle [\mathbf{4} \ell]$$

$$= \frac{1}{m^4} \langle \mathbf{4} | \ell | \mathbf{1}] \langle \mathbf{1} | \ell_1 | \mathbf{2}] \langle \mathbf{2} | \ell_2 | \mathbf{3}] \langle \mathbf{3} | \ell_3 | \mathbf{4}]$$

And we just need to solve for $\ell^2 = \ell_1^2 = \ell_2^2 = \ell_3^2 = 0$

Calculation of c_4

To solve the cut conditions, we need to express ℓ in a basis of massless vectors constructed from external *massive* momenta. Following Forde 07' we define

$$\gamma = -p_1 \cdot p_4 - \sqrt{\Delta}, \quad \Delta = (p_1 \cdot p_4)^2 - m^4$$

$$p_1^{\flat,\mu} = \frac{\gamma}{\gamma^2 - m^4}\left(\gamma p_1^\mu + m^2 p_4^\mu\right) \qquad p_4^{\flat,\mu} = -\frac{\gamma}{\gamma^2 - m^4}\left(m^2 p_1^\mu + \gamma p_4^\mu\right)$$

with $\left(p_1^\flat\right)^2 = \left(p_4^\flat\right)^2 = 0\,.$

Calculation of c_4

Now we can compute c_4

$$c_4 = \frac{1}{2}\sum_{t_\pm}\frac{1}{m^4}\langle\mathbf{4}|\ell|\mathbf{1}]\,\langle\mathbf{1}|\ell_1|\mathbf{2}]\,\langle\mathbf{2}|\ell_2|\mathbf{3}]\,\langle\mathbf{3}|\ell_3|\mathbf{4}]$$

By substituting ℓ that satisfies $\ell_i^2 = 0$:

$$\langle\mathbf{a}|\,\ell\,|\mathbf{b}] = x\left\langle\mathbf{a}|p_1^\flat + p_4^\flat|\mathbf{b}\right] + \hat{t}\left\langle\mathbf{a}|1^\flat 2 4^\flat|\mathbf{b}\right] + \frac{x^2}{A_t\hat{t}}\left\langle\mathbf{a}|4^\flat 2 1^\flat|\mathbf{b}\right]$$

$$\begin{aligned}\langle\mathbf{1}|\ell_1|\mathbf{2}] &= \langle\mathbf{1}|\ell|\mathbf{2}] - \langle\mathbf{1}|1|\mathbf{2}]\\ \langle\mathbf{2}|\ell_2|\mathbf{3}] &= \langle\mathbf{2}|\ell|\mathbf{3}] - \langle\mathbf{2}|1|\mathbf{3}] - \langle\mathbf{2}|2|\mathbf{3}]\\ \langle\mathbf{3}|\ell_3|\mathbf{4}] &= \langle\mathbf{3}|\ell|\mathbf{4}] + \langle\mathbf{3}|4|\mathbf{4}]\end{aligned}$$

$$x \equiv \frac{m^2}{m^2+\gamma}$$

$$\hat{t}_\pm = \frac{1}{\gamma s_{13}}\left(p_1\cdot p_4 \pm \sqrt{\Delta}\sqrt{1-\frac{4m^4}{s_{12}s_{14}}}\right)$$

Status: Generalized Unitarity for 4-Massive Vectors

- We've calculated all of the coefficients

$$c_4,\ c_3^{(12)},\ c_3^{(34)},\ c_3^{(14)},\ c_3^{(23)},\ c_2^{(12)},\ c_2^{(14)}$$

  using our generalized unitarity formalism for massive external particles

- All of the coefficients match their correct massless limits
- We are still working on extracting the s_{13} pole and finding the rational term which cancels it (lots of algebra!)

Expectations from Calculation

- We know from the field theory side that the massive anomalous theory *is* consistent, so we expect to find a rational term that can factorize on all the poles

- It would be interesting to understand if there's a spurious s_{13} pole that can be resolved by a rational term, without modifying all of the other residues, or if there's no s_{13} pole to begin with.

- Next, we plan to study how the spurious poles emerge in the $m \to 0$, or alternatively the high energy limit. This will provide us with a natural cutoff for the massive EFT, analogous to the one explored by Preskill

- Most of all, we want to gain *physical intuition* why massive theory is consistent, while the massless one isn't. Is there a way to know in advance that the massive theory resolved the tension between *unitarity* and *locality*, without all the gory detail?

Summary

- We presented a counterintuitive field theory argument (due to Preskill) that massive anomalous gauge theories are equivalent to massive “anomaly canceled” theories

- We asked the gauge invariant question why massless theories are inconsistent while massive ones are

- We are close to a technical resolution of the question: the spurious poles arising in the 1-loop 4-vector amplitude should disappear in the massive theory. The tension between *Unitarity* and *locality* is resolved.

- We are still wondering about the gauge invariant physics that makes one theory consistent, while the other one isn’t. Perhaps a close look at the poles will provide a more fundamental resolution of this issue.

Thank You!