KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

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15/01/20

Rapid bound-state formation of Dark Matter in the Early Universe

Tobias Binder

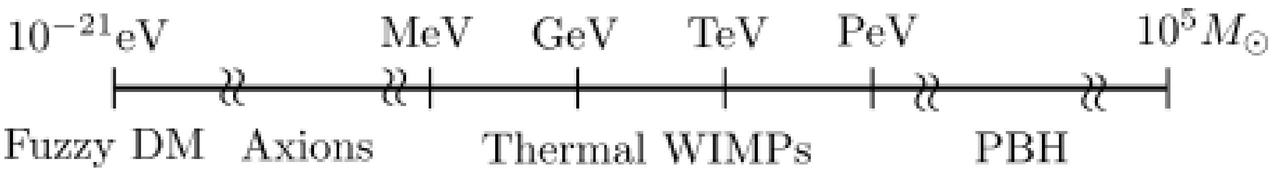
based on arxiv:**1910.11288**, arxiv:**2001.**[in prep.],

in collaboration with

Kyohei Mukaida and **Kalliopi Petraki**

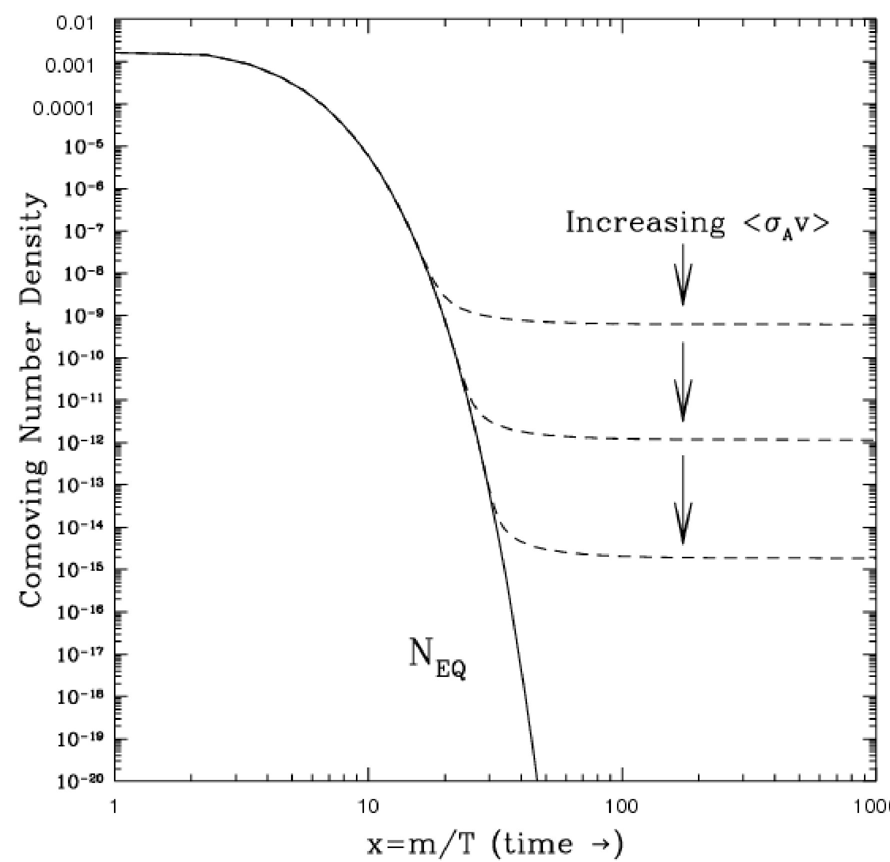
[**Burkhard Blobel**, and **Julia Harz**].

Thermally produced dark matter



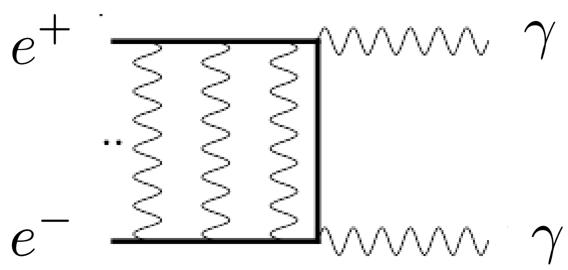
- One **leading hypothesis** for DM: Thermal WIMPs.
- **Testable** and final relic abundance **independent** of initial conditions.
- Strong constraints on coupling strength rule out many MeV-TeV mass realizations in thermal scenarios.
- TeV-scale and above still remains attractive and much less constrained.

Thermal production mechanism



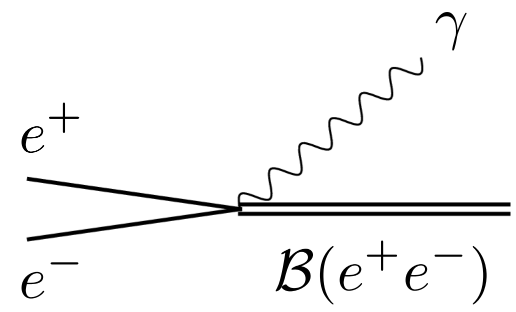
Quantum mechanical effects: Positronium

Sommerfeld-enhanced annihilation, formation and decay of bound states:



$$(\sigma v) = (\sigma v)_0 \times |\psi(r=0)|^2$$
$$\propto (\sigma v)_0 \, (\alpha/v)\,, \ \text{for } v \lesssim \alpha.$$

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$



Bound-state formation via on-shell photon emission.

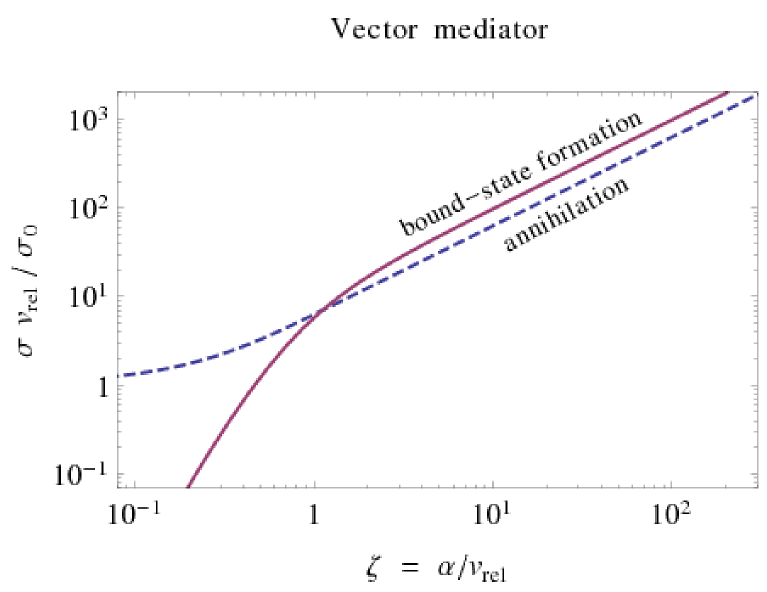
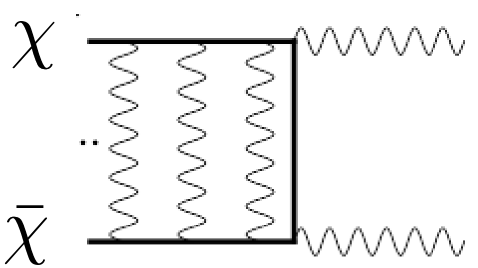
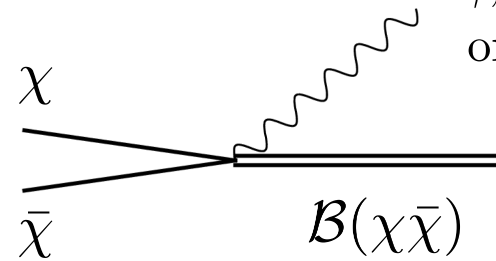


Figure taken from Petraki et al. 2015

Quantum mechanical effects: Dark Matter models

Sommerfeld-enhanced annihilation, formation and decay of bound states:



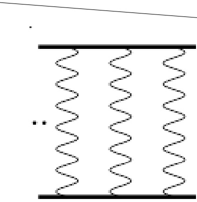


$\gamma, g, W^{\pm}, Z,$ or H **induced:**

- Minimal DM (includes Wino)
  J. Hisano *et al.* ‘03, ‘05, ‘06, Cirelli *et al.* ‘07, Mitridate *et al.* ‘17
- Co-annihilation with color-charged particles
  J. Ellis *et al.* ‘16, Kim&Laine ‘17, Harz&Petraki ‘18, S. Biodini *et al.* ‘19,’19,’19‘
- Higgs mediated bound states
  Harz&Petraki ‘18, S. Biodini ‘18

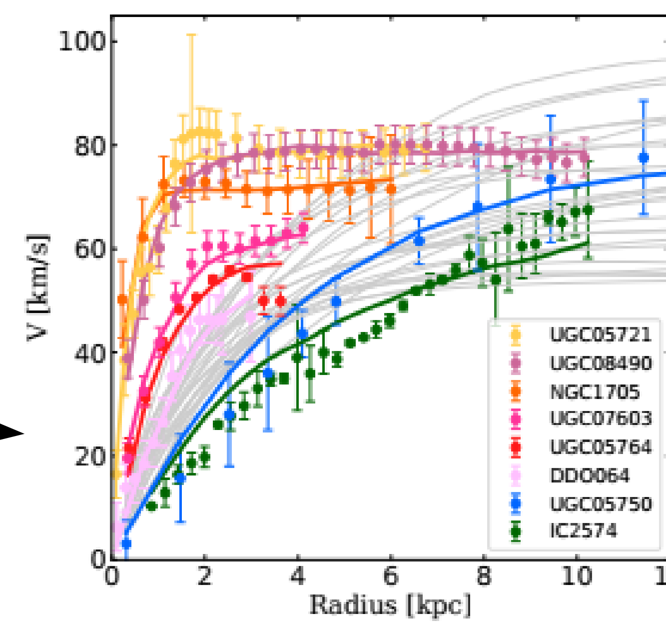
Or bottom-up motivated scenarios with exotic mediators:

- **S**elf-**I**nteracting **DM** with light mediators
  J. L. Feng *et al.* ‘10, von Harling&Petraki ‘14, …



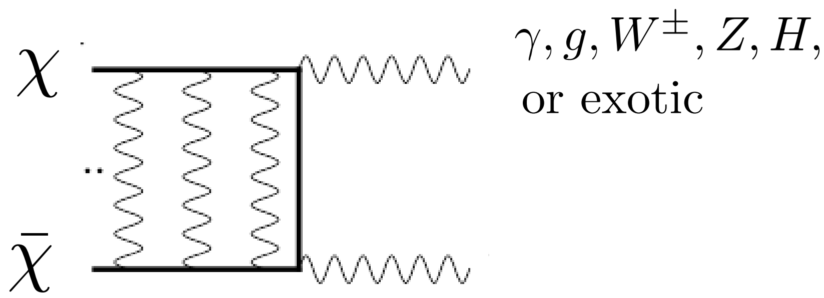
SIDM solves Diversity problem

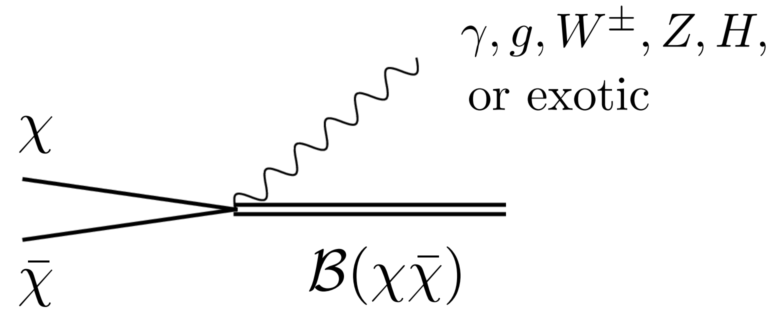
Kamada *et al.* ‘16,…, Kaplinghat *et al.* ‘19



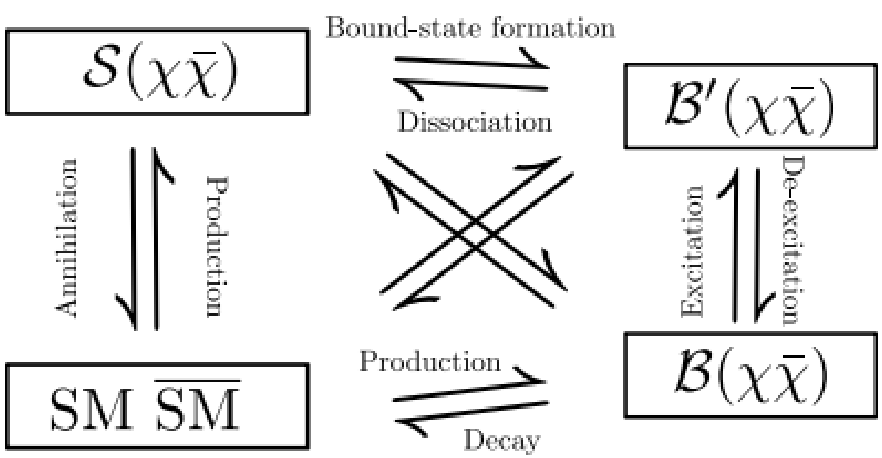
Quantum mechanical effects: Relic abundance

Sommerfeld-enhanced annihilation, formation and decay of bound states:



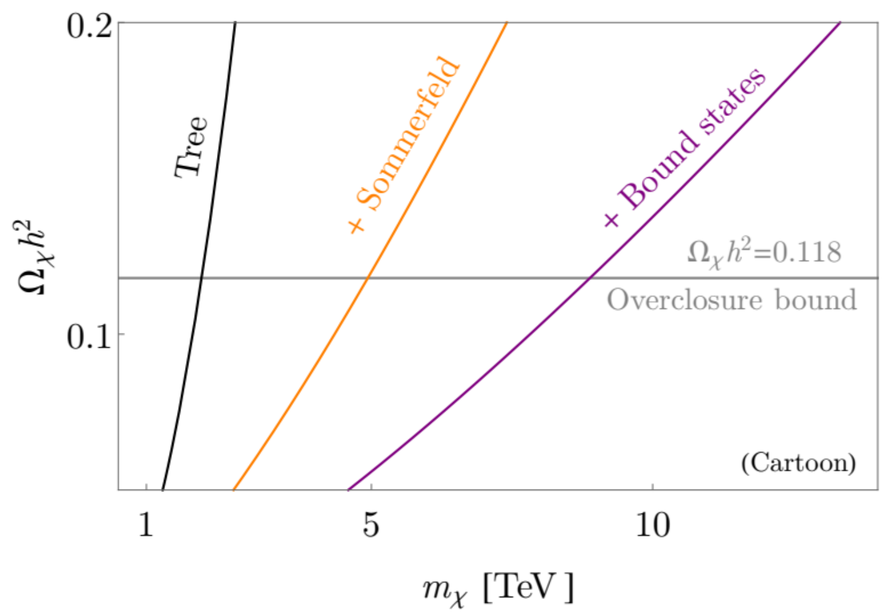


Complex chemical network:



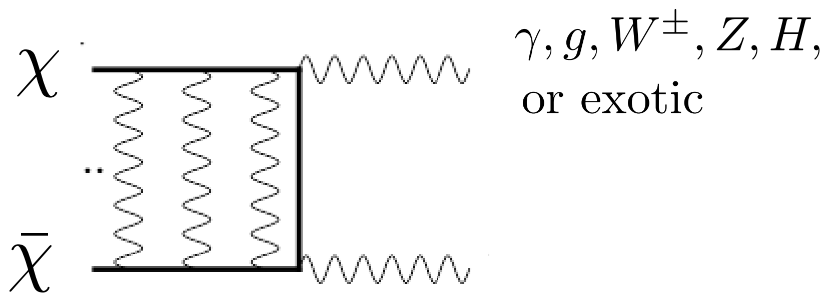
QM effects not included in any public relic density solver. (In progress…)

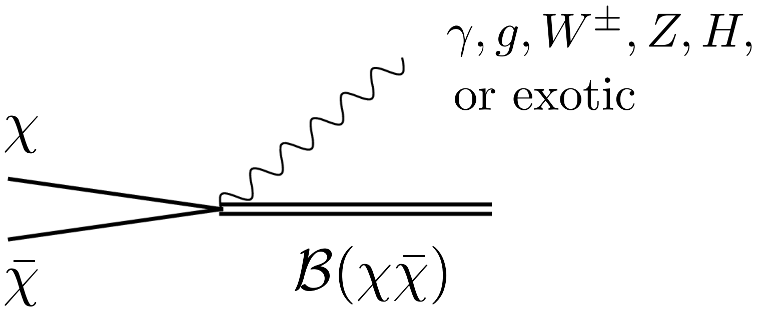
QM effects allow for **larger DM masses**:

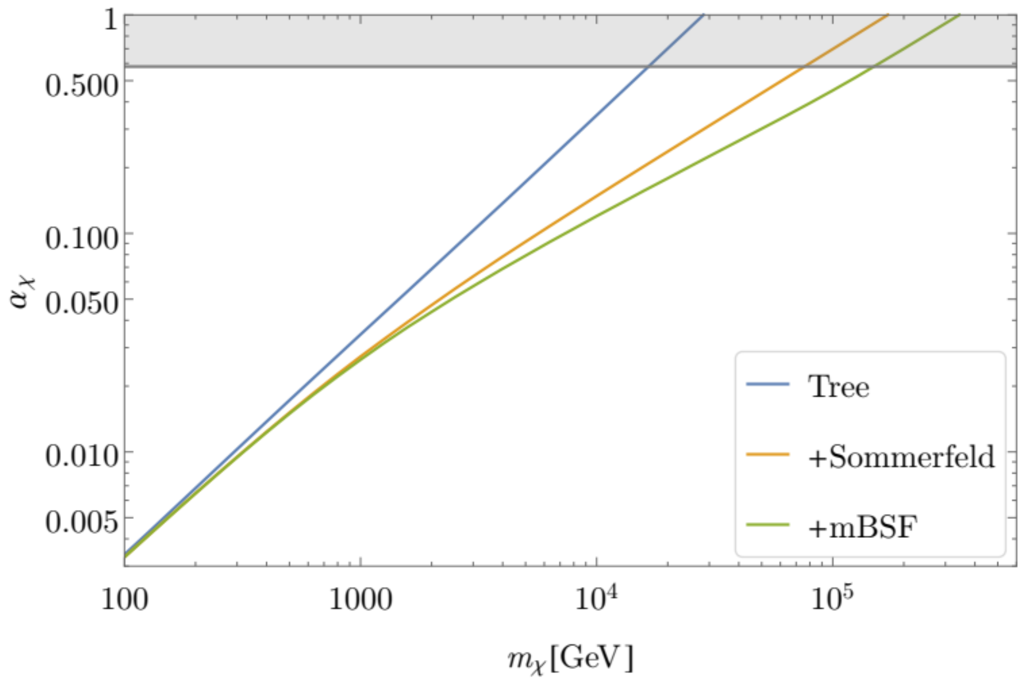


Quantum mechanical effects: Relic abundance

Sommerfeld-enhanced annihilation, formation and decay of bound states:

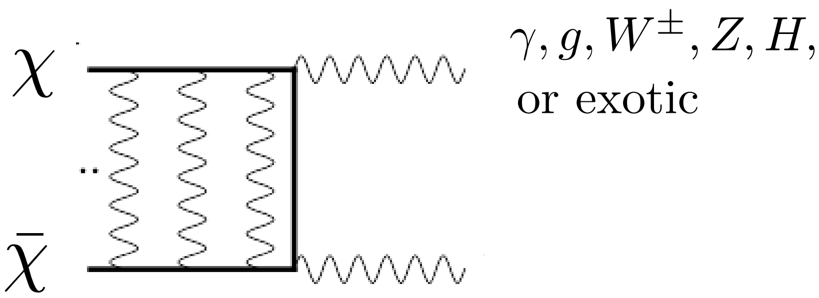


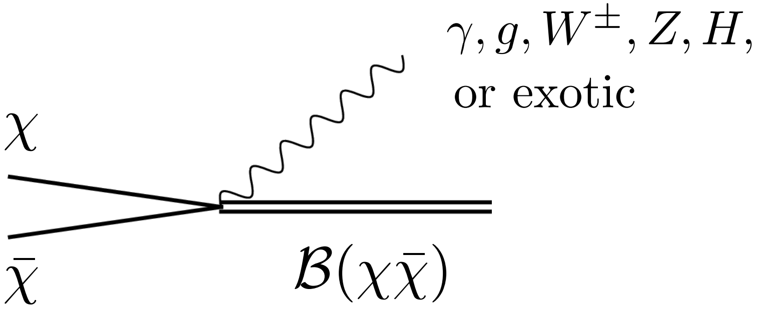


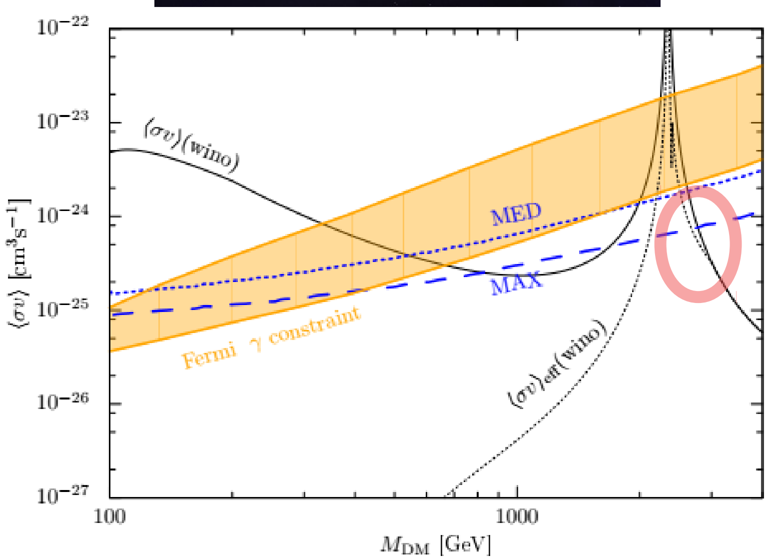


Quantum mechanical effects: Indirect Detection

Sommerfeld-enhanced annihilation, formation and decay of bound states:







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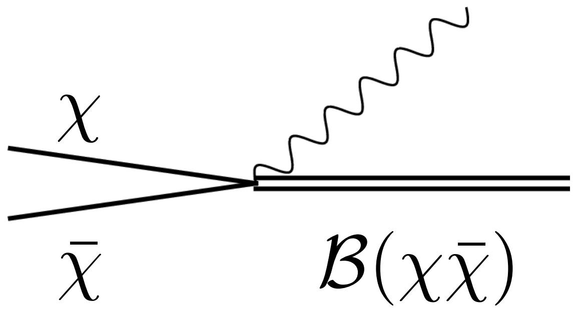
Indirect detection **sensitive to DM mass** due to Sommerfeld resonances. For constraining WIMPs reliably, we need to theoretical predict the relic abundance precisely!
(10% change in the Wino mass would result in 100 % change in the flux!)

BSF in a plasma: On-shell or off-shell mediator?

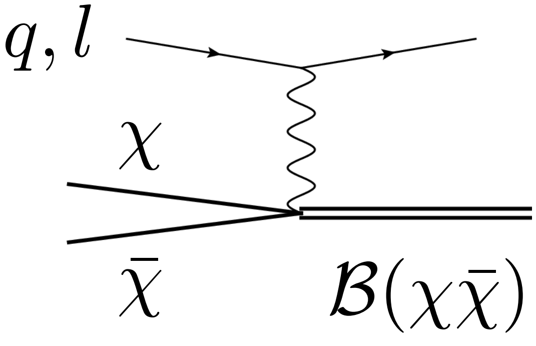
On-shell emission



(previous literature)

vs.

Virtual mediator contributions



(this talk)

3.) Which process dominates in the Early Universe?

2.) Cancellation of collinear divergences in massless mediator case?

1.) How can we systematically compute higher order BSF processes?

Generalized bound-state formation cross section

$$\mathcal{L} \supset g\bar{\chi}\gamma^\mu\chi V_\mu + g\bar{\psi}\gamma^\mu\psi V_\mu$$

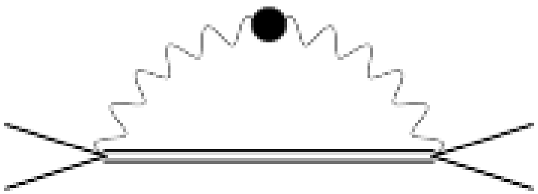
Starting from pNREFT and utilizing non-equilibrium QFT techniques, we derive

$$\dot{n} + 3Hn = -\sum_{\mathcal{B}} \langle \sigma_{\mathcal{B}}^{\mathrm{BSF}} v_{\mathrm{rel}} \rangle \left[n^2 - n_{\mathcal{B}} n_{\mathrm{eq}}^2 / n_{\mathcal{B}}^{\mathrm{eq}} \right] - \langle \sigma^{\mathrm{an}} v_{\mathrm{rel}} \rangle \left[n^2 - n_{\mathrm{eq}}^2 \right]$$

$$\dot{n}_{\mathcal{B}} + 3Hn_{\mathcal{B}} = \ldots$$

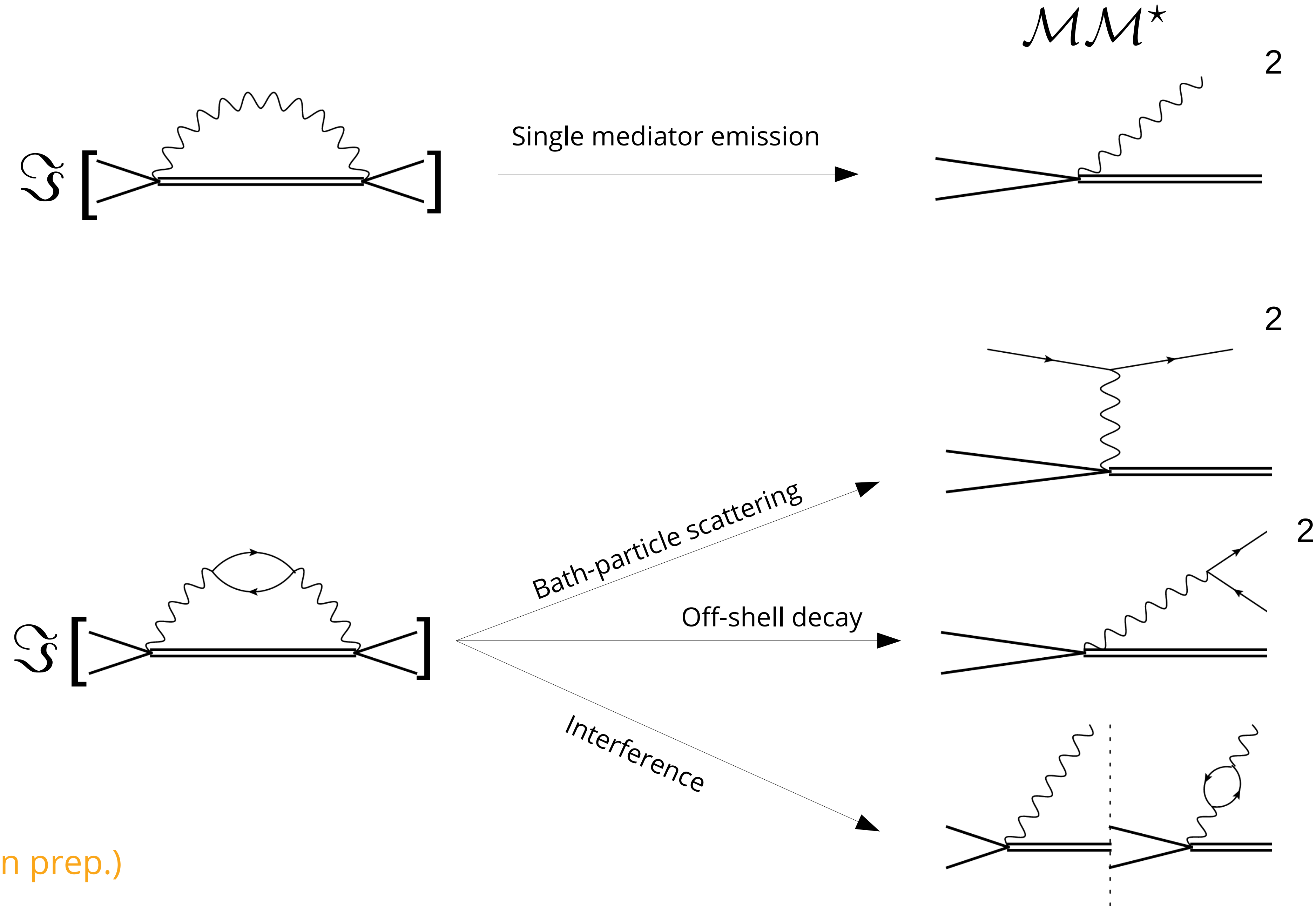
where

$$\sigma_{nlm}^{\mathrm{BSF}} v_{\mathrm{rel}} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D_{\mu\nu}^{-+}(\Delta E, \mathbf{p}) \sum_{\mathrm{spins}} \mathcal{T}_{\mathbf{k},nlm}^{\mu}(\Delta E, \mathbf{p}) \mathcal{T}_{\mathbf{k},nlm}^{\nu\star}(\Delta E, \mathbf{p}).$$



(in prep.)

Non-equilibrium QFT approach



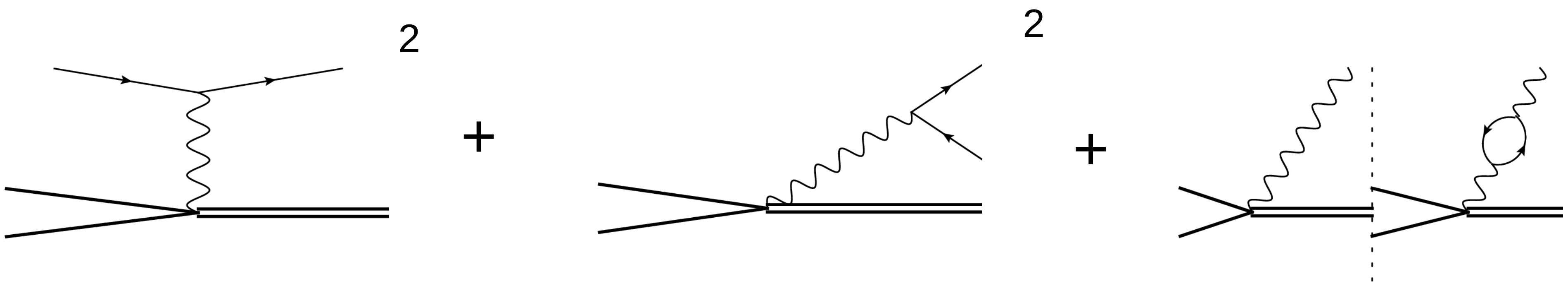
(in prep.)

NLO contributions

UV finite,
collinear divergent.

UV finite,
collinear divergent.

Vacuum part UV divergent,
collinear divergent.



= Finite in collinear direction, and UV finite after vacuum renormalization.

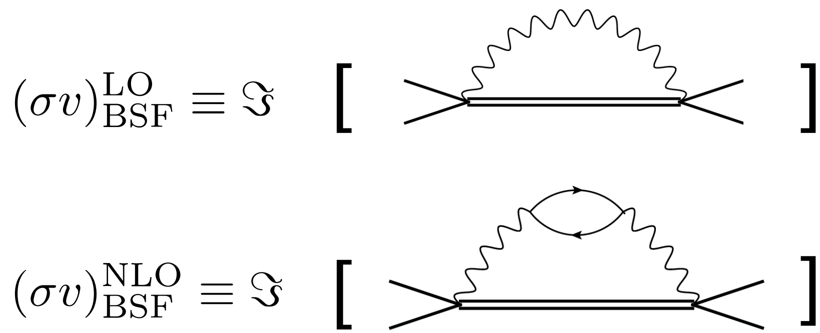
- Provide mathematical proof for cancellation of collinear divergences.
- Holds even for arbitrary phase-space distribution of bath particles, i.e. bath particles do not have to be in thermal equilibrium in order to guarantee finiteness in the forward scattering direction.
- (Bloch-Nordsieck theorem does not help here)

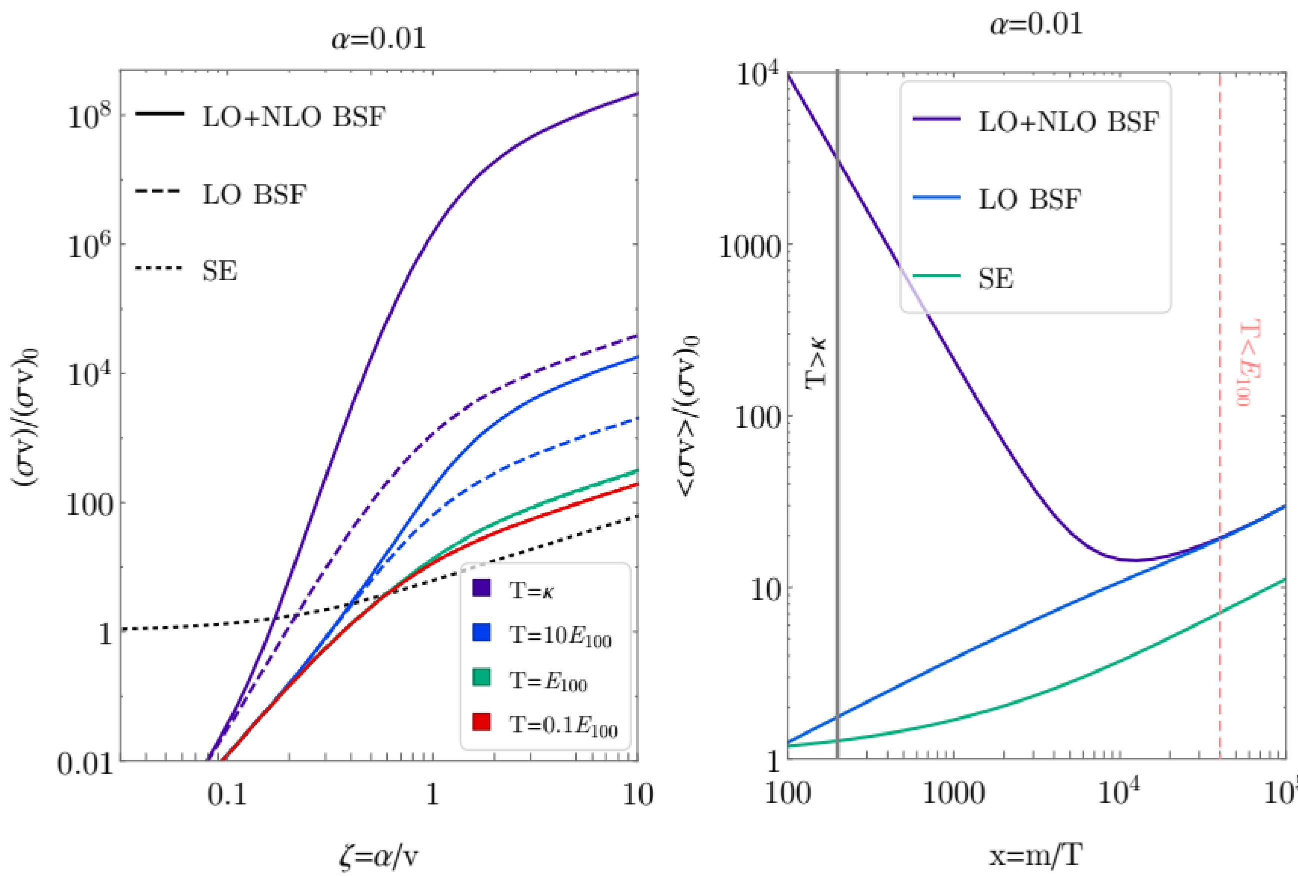
(in prep.)

Bound-state formation at NLO: massless case

- Interference terms **cancel collinear divergences,** resulting in a finite cross section.
- At high temperature **BSF via bath-particle scattering dominates over single mediator emission.**
- Variation of renormalization scale between DM mass and binding energy does not affect plot visually, hence Log-contributions are under control.

(in prep.)





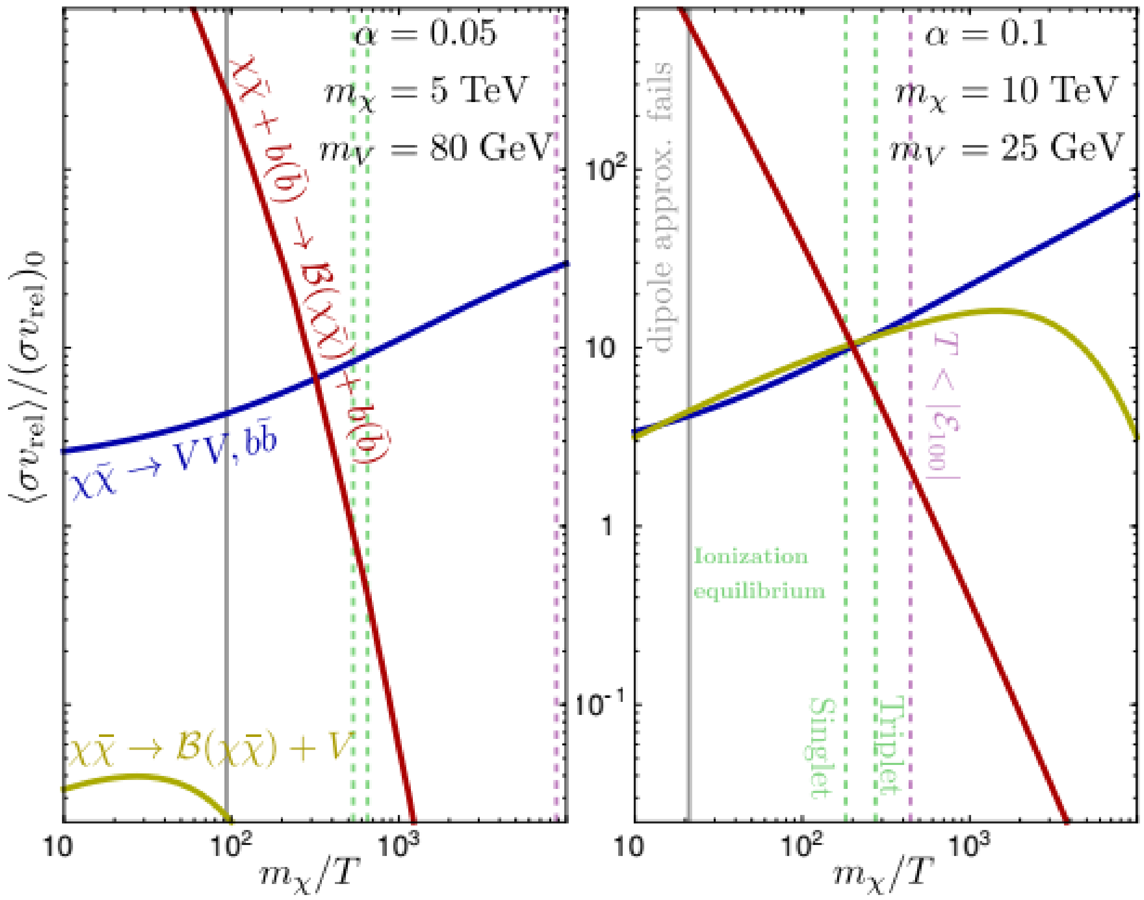
BSF via bath-particle scattering: massive case

Parametrically resembles:

no kimenatical block

Wino

Co-annihilation with
colored charged particles



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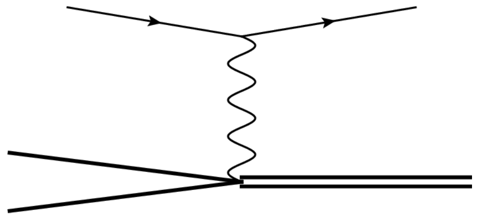
BSF via bath-particle scattering: massive case

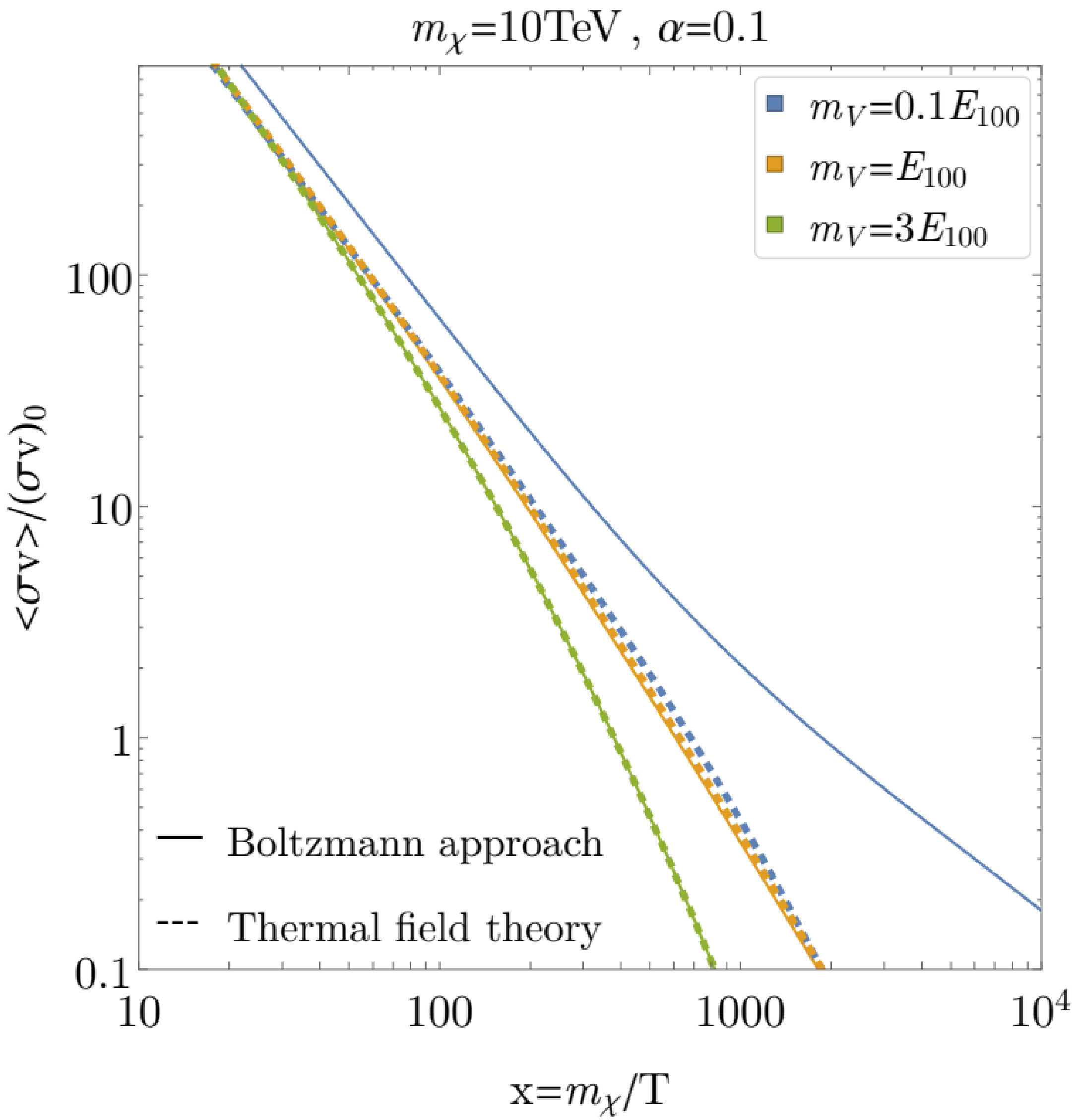


+ Interference

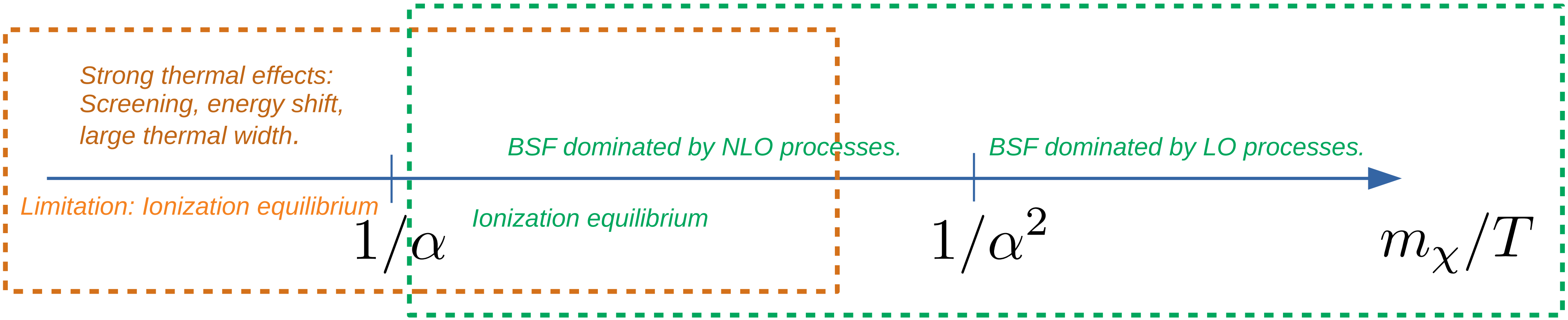
- Interference terms negligible for mediator masses much larger than binding energy: Boltzmann computation ok.

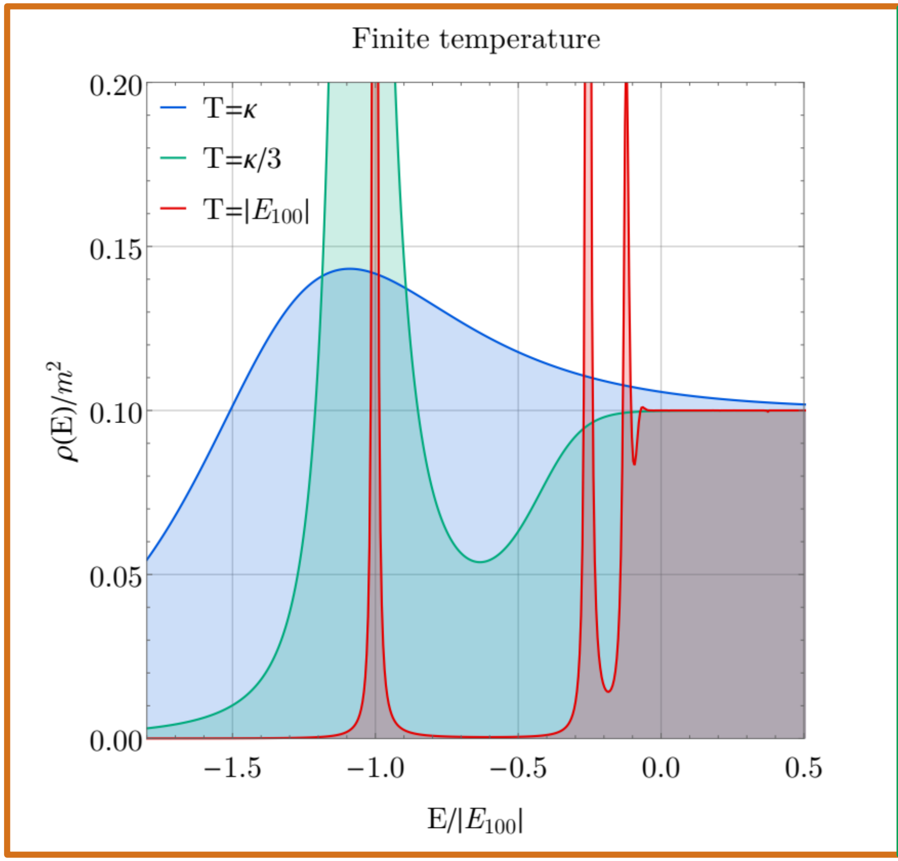
- **Thermal field theory approach required** for mediator masses smaller than or comparable to the binding energy.

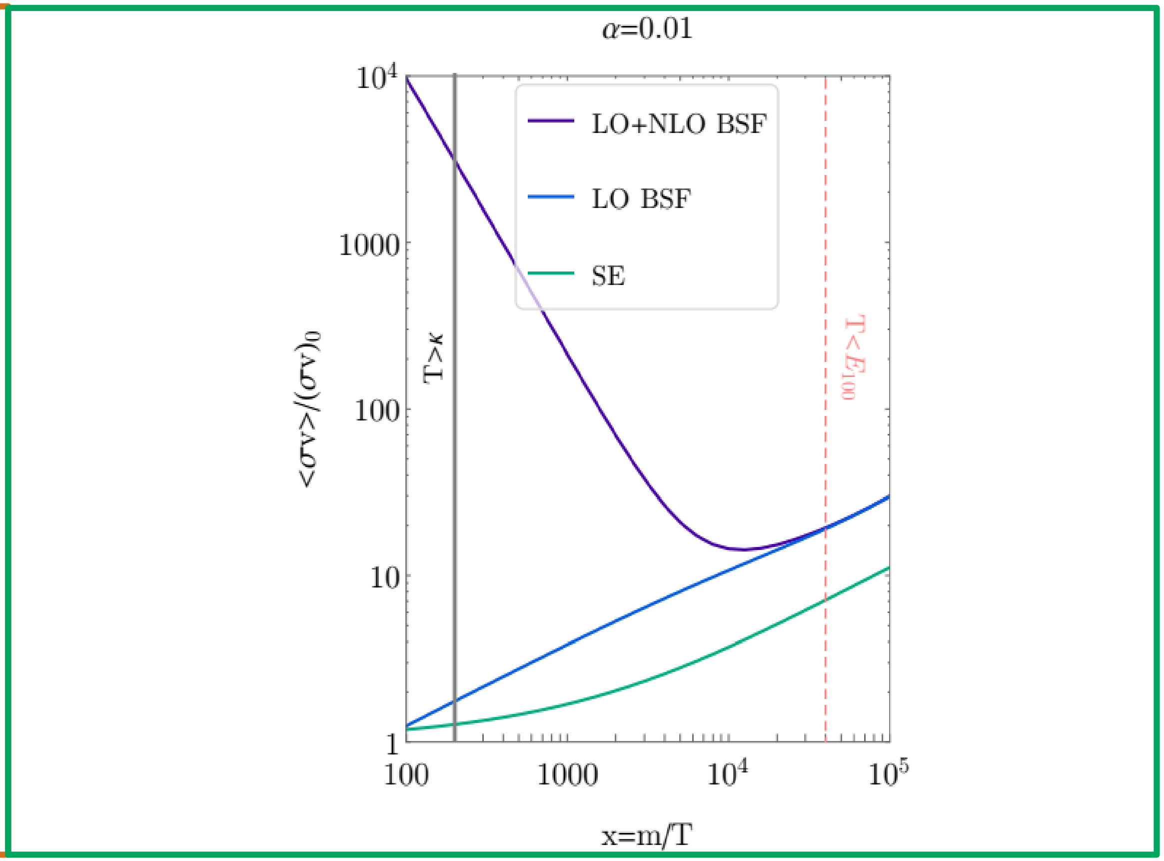
(in prep.)



Complete picture







→ **Ready to (re-)analyze heavy thermal relics!**

Summary and conclusion

Formal achievements:

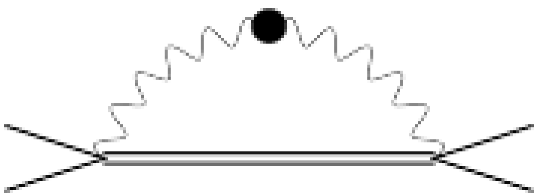
- Mathematical proof for **cancellation of collinear divergences**.
- More **complete description** of the DM freeze-out: from melting of bound states down to far below the decoupling from ionization equilibrium.

Phenomenological results and their implications:

- **Previous literature** considered BSF via **on-shell mediator emission** only.
- T>E_B: **dominant BSF channel** is **via bath-particle scattering**.
- Statement expected to be true also for non-abelian gauge or Yukawa theories.
- **Consequently, DM mass could be heavier than previously expected.**

(Eventually informs indirect searches and construction of future colliders)

Generalized bound-state formation cross section



$$\sigma^{\rm BSF}_{nlm} v_{\rm rel} = \int \frac{{\rm d}^3 p}{(2\pi)^3} D^{-+}_{\mu\nu}(\Delta E, \mathbf{p}) \sum_{\rm spins} \mathcal{T}^{\mu}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}) \mathcal{T}^{\nu\star}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}).$$

$$\mathcal{L} \supset g\bar{\chi}\gamma^\mu \chi V_\mu + g\bar{\psi}\gamma^\mu \psi V_\mu$$

Interacting two-point correlation fct.:

$$D^{-+}_{\mu\nu}(x,y) \equiv \langle V_\mu(x) V_\nu(y) \rangle$$
$$\langle ... \rangle = {\rm Tr}[e^{-H_{\rm env}/T} ...]$$

Kubo-Martin-Schwinger relation:

$$D^{-+}_{\mu\nu}(\Delta E, \mathbf{p}) = \left[1 + f_V^{\rm eq}(\Delta E)\right] D^{\rho}_{\mu\nu}(\Delta E, \mathbf{p})$$
$$D^{\rho}_{\mu\nu} = 2\Im \left[i D^R_{\mu\nu} \right]$$
$$D^R_{\mu\nu} = D^{R,0}_{\mu\nu} + D^{R,0}_{\mu\alpha} \Pi^{\alpha\beta}_R D^{R,0}_{\beta\nu} + ...$$

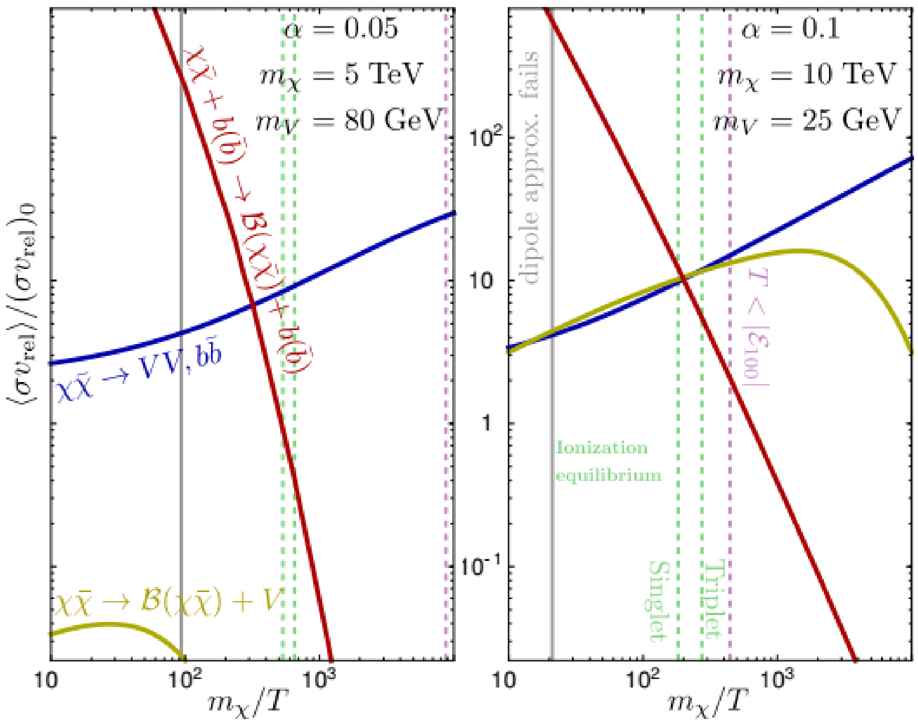
S-B transition matrix elements:

$$\mathcal{T}^{\mu}_{\mathbf{k},nlm}(P) \equiv (g_\chi g_{\bar{\chi}} 4 m_\chi^2 2M)^{-1/2} \mathcal{M}^{\mu}_{\mathbf{k},nlm}\Big|^{\rm NR}_{\rm dip}$$

$$\delta^4 \mathcal{M}^{\mu}_{\mathbf{k},nlm} = \int {\rm d}^4 x \; e^{iPx} \langle \mathcal{B}_{nlm} | \, g\bar{\chi}(x)\gamma^\mu \chi(x) \, | \mathcal{S}_{\mathbf{k}} \rangle$$

Well developed, see, e.g., Kallias works.

Implications of strongly enhanced BSF



Approx. number density eq. **[von Harling&Petraki '14]**:

$$\dot{n}_s + 3Hn_s = -\left[\langle\sigma v\rangle_{\rm an} + \frac{\Gamma_1\langle\sigma v\rangle_{\rm BSF}}{\Gamma_1 + \Gamma_{1\to s}}\right]\left(n_s^2 - n_s^{\rm eq}n_s^{\rm eq}\right)$$

$$\Gamma_{1\to s} = \langle\sigma v\rangle_{\rm BSF}\frac{n_s^{\rm eq}n_s^{\rm eq}}{n_1^{\rm eq}}$$

$$\Gamma_1 \ll \Gamma_{1\to s}$$

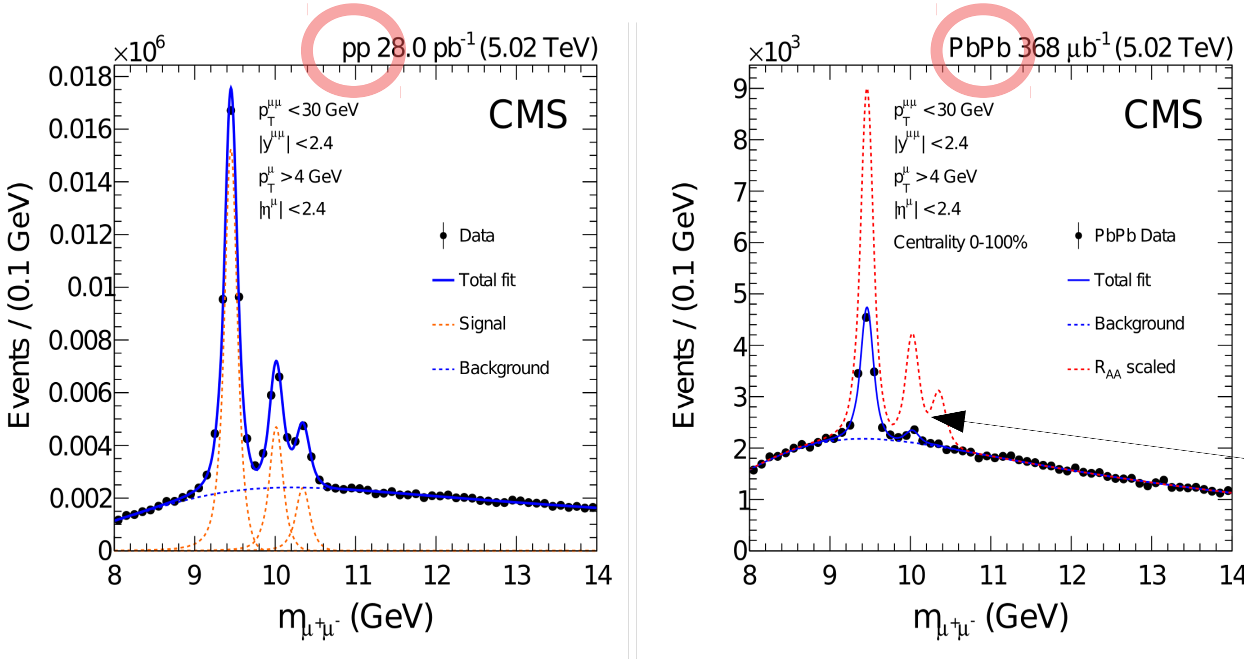
(Saha-) Ionization equilibrium [TB, Covi, Mukaida '18]:

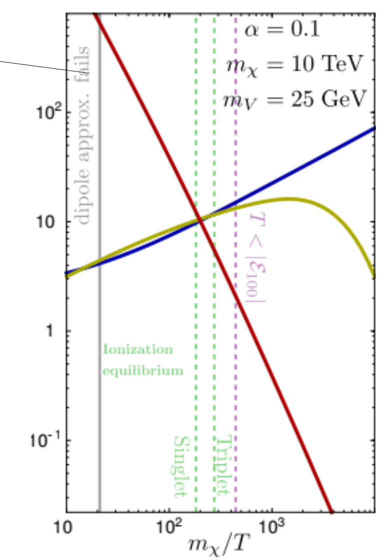
$$\dot{n}_s + 3Hn_s = -\left[\langle\sigma v\rangle_{\rm an} + \Gamma_1\frac{n_1^{\rm eq}}{n_s^{\rm eq}n_s^{\rm eq}}\right]\left(n_s^2 - n_s^{\rm eq}n_s^{\rm eq}\right)$$

→ Strongly enhanced BSF via bath-particle scattering leads to **ionization equilibrium,** where i) collision term is **independent of BSF cross section and DISS rate**, and ii) **effective depletion cross section takes maximum value** for fixed T.

Limitation

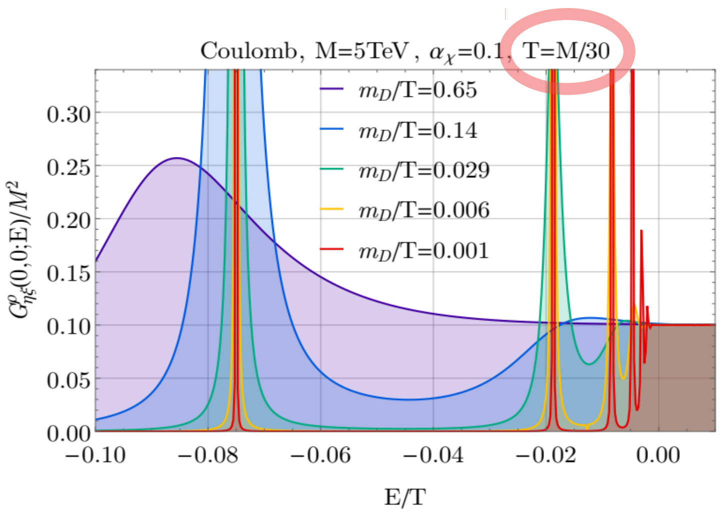
CMS collaboration, Phys.Lett. **B790** (2019) 270-293





Melting of bound states inside plasma environment observed.

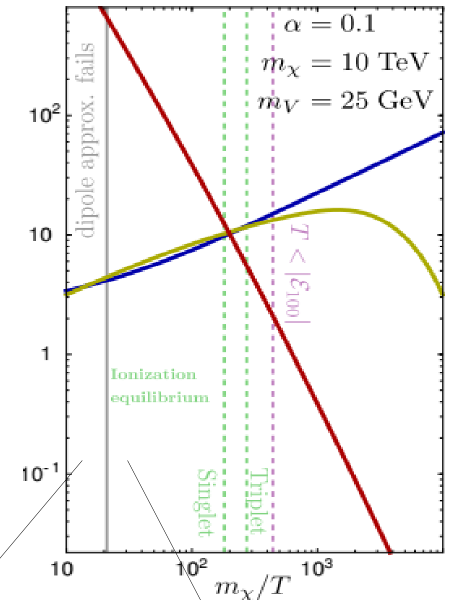
Complete picture



[TB, Covi, Mukaida ’18]

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\rm eq} = e^{-2\beta M}\int \frac{{\rm d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty}\frac{{\rm d}E}{(2\pi)} e^{-\beta E} G^\rho_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$



arxiv:**1911.**(in prep.)

arxiv:**1910.11288**

$$\sigma^{\rm BSF}_{nlm} v_{\rm rel} = \int \frac{{\rm d}^3 p}{(2\pi)^3} D^{-+}_{\mu\nu}(\Delta E, \mathbf{p}) \sum_{\rm spins} \mathcal{T}^{\mu}_{\mathbf{k},nlm}(\Delta E, \mathbf{p})\mathcal{T}^{\nu\star}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}).$$

Backup

1 BS+R.h.s of BS equation vanishes

$$\dot{n}_s + 3Hn_s = -\left[\langle\sigma v\rangle_{\rm an} + \frac{\Gamma_1 \langle\sigma v\rangle_{\rm BSF}}{\Gamma_1 + \Gamma_{1\to s}}\right]\left(n_s^2 - n_s^{\rm eq} n_s^{\rm eq}\right)$$

Coupled BEs:

$$\dot{n}_s + 3Hn_s = -\langle\sigma v\rangle_{\rm an}\left[n_s^2 - (n_s^{\rm eq})^2\right] - \sum_i \langle\sigma^i v\rangle_{\rm BSF}\left[n_s^2 - n_i (n_s^{\rm eq})^2/n_i^{\rm eq}\right],$$

$$\dot{n}_i + 3Hn_i = -\Gamma_i\left[n_i - n_i^{\rm eq}\right] + \langle\sigma^i v\rangle_{\rm BSF}\left[n_s^2 - n_i (n_s^{\rm eq})^2/n_i^{\rm eq}\right] - \sum_j \Gamma_{i\to j}\left[n_i - n_j n_i^{\rm eq}/n_j^{\rm eq}\right]$$

$\Gamma_1 \ll \Gamma_{1\to s}$

Radiative processes much faster than total number violating processes -> ionization equilibrium

[TB, Covi, Mukaida '18]

$$\dot{n} + 3Hn = -\left[\langle\sigma v\rangle_{\rm an} + \sum_i \Gamma_i \frac{n_i^{\rm eq}}{n_s^{\rm eq} n_s^{\rm eq}}\right]\left(\alpha^2 n^2 - n_s^{\rm eq} n_s^{\rm eq}\right)$$